Quantum many-body adiabaticity and the topological Thouless pump

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111 Quantum adiabatic evolution and the Quantum Adiabatic Theorem (QAT)

Quantum Adiabatic Theorem (QAT) – colloquially: A system evolving under a time-dependent Hamiltonian can be kept arbitrarily close to the Hamiltonian's instantaneous ground state provided that the parameters of the Hamiltonian vary *slowly enough*.

Key question: What is the exact meaning of *"slowly enough"*?

The question is tricky even for few-level systems [3, 7].

We address this question in a particularly involved case of many-body systems. This is highly relevant for

- adiabatic quantum computers and quantum annealers,
- **topological quantum pumps** [5, 1],
- quasi-Bloch oscillations of a mobile impurity in a 1D fluid [4].

Notations and definitions.

- Driven quantum system can be described by a Hamiltonian $\hat H_\lambda$, where $\lambda = \lambda(t)$ is a time-dependent parameter.
- Simplest case: linear driving, $\lambda(t) = \Gamma t$, Γ being the driving rate. In general, $\Gamma = \partial\lambda/\partial t$.
- Use λ instead of t as the evolution parameter. Schrodinger equation:

$$i\,\Gamma\,\frac{\partial}{\partial\lambda}\Psi_\lambda = \hat H_\lambda\,\Psi_\lambda. \qquad (1)$$

- Define the *instantaneous* ground state of the system, Φ_λ:

$$\hat H_\lambda\,\Phi_\lambda = E_\lambda\,\Phi_\lambda. \qquad (2)$$

- The system is initially prepared in the instantaneous ground state:

$$\Psi_{\lambda=0} = \Phi_{\lambda=0}$$

.

- One calls the evolution adiabatic as long as Ψ_λ remains close to Φ_λ, or in other words if the fidelity

$$\mathcal F(\lambda) = |\langle\Phi_\lambda|\Psi_\lambda\rangle|^2$$

is close to one.

QAT – rigorously: For however small $\varepsilon > 0$ and arbitrary target λ there exists Γ small enough that

$$1 - \mathcal F(\lambda) < \varepsilon.$$

Under closer investigation, two complementary questions can be posed:

Q1. ***For a given driving rate Γ, how long the adiabaticity can be maintained with a given accuracy ε?***

Q2. ***How small should the driving rate Γ be for a given target λ and a given accuracy ε?***

For a large many-body system, we answer **Q1** and give a necessary condition for **Q2** [2].

222 Orthogonality catastrophe

Colloquially. Given a many-body Hamiltonian $\hat H_\lambda$, two ground states corresponding to slightly different λ's can become nearly orthogonal with growing size of the system, N [Anderson, 1967].

Rigorously. *Orthogonality overlap* in the leading order in λ:

$$\mathcal C(\lambda) \equiv |\langle\Phi_\lambda|\Phi_0\rangle|^2 = e^{-C_N\lambda^2}. \qquad (3)$$

The orthogonality catastrophe takes place whenever $C_N \to \infty$ in the thermodynamic limit (TL), $N \to \infty$. The behavior of C_N is determined by the type of driving and the gap.

	Local driving	Bulk driving
gapless	$C_N \sim \log N$	$C_N \sim N$
gapped	$\lim_{N\to\infty} C_N$ is finite	$C_N \sim N$

333 Key idea and main technical result

Key idea: In a many-body system subject to orthogonality catastrophe

$$\boxed{\mathcal F(\lambda) \simeq \mathcal C(\lambda)}$$

up to times sufficiently long for the adiabaticity to completely break down.

Main technical result:

$$\boxed{|\mathcal F(\lambda) - \mathcal C(\lambda)| \le \mathcal R_\lambda \equiv \Gamma^{-1}\int_0^\lambda \sqrt{\langle \hat H_{\lambda'}^2\rangle_0 - \langle \hat H_{\lambda'}\rangle_0^2}\,d\lambda',} \qquad (4)$$

where $\langle\cdots\rangle_0 \equiv \langle\Psi_0|\cdots|\Psi_0\rangle$.

In an important special case of $\hat H_\lambda = \hat H_0 + \lambda\hat V$, one obtains

$$R_\lambda = \lambda^2\,\delta V_N/(2\Gamma) \;\text{ with }\; \delta V_N \equiv \sqrt{\langle\hat V^2\rangle_0 - \langle\hat V\rangle_0^2}.$$

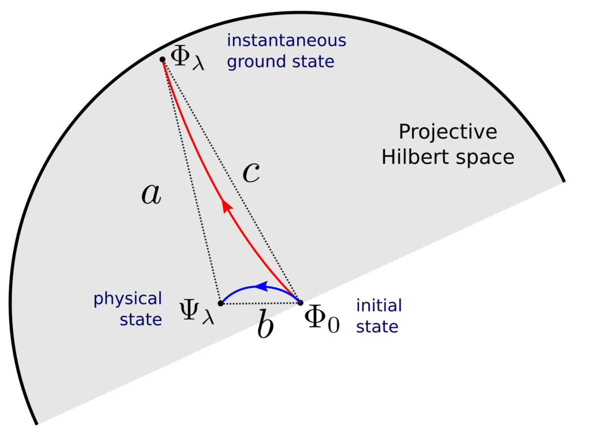


Figure 1: Triangle inequality resulting in the estimate Eq. (4). States are shown as points in the projective Hilbert space. The red trajectory shows the evolution of the instantaneous ground state, Eq. (2), while the blue trajectory corresponds to the physical evolution given in Eq. (1). The length of the side b is bounded by the quantum speed limit, while the length of the side c approaches the maximally possible distance of 1 in the large N limit.

4444 Adiabaticity breakdown time – addressing Q1

Define the adiabaticity breakdown time t_* and parameter $\lambda_* \equiv \lambda(t_*)$:

$$\mathcal F(\lambda_*) = \frac{1}{e}.$$

According to (4) and (3),

$$\boxed{\lambda_* = 1/\sqrt{C_N}}$$

up to small corrections as long as $\mathcal R(C_N^{-1/2}) \ll 1$. The latter is guaranteed for sufficiently large system since

$$\frac{\delta V_N}{C_N} \to 0 \quad\text{for}\quad N \to \infty.$$

5555 Necessary condition for adiabaticity (Q2)

If the orthogonality catastrophe is present, the adiabaticity can be maintained for finite systems only as long as $\mathcal R(\lambda_*)$ is large enough to make inequality (4) trivial. This entails a ***necessary adiabatic condition***:

$$\Gamma_N < \frac{\delta V_N}{2C_N}\,\frac{1}{1 - e^{-1} - \varepsilon}.$$

6666 Quantized transport in Thouless pump

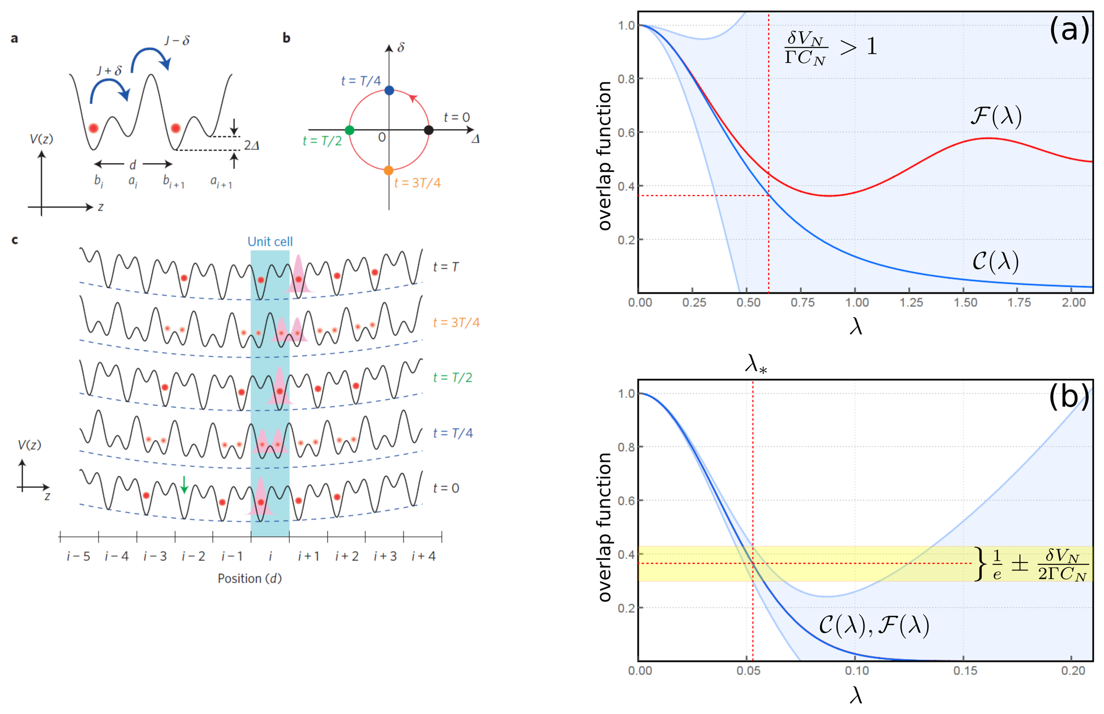


Figure 2: Left: Thouless pump, figure from Ref. [5]. The pump transfers exactly one particle per cycle from left to right despite the fact that the coordinates of local minima of the potential remain constant. Right - evolution of the ground state fidelity in the parameter space of the Hamiltonian (5) for (a) $N = 10$, and (b) $N = 1000$ particles. The shaded region is the one, which has to contain the $\mathcal F(\lambda)$ curve due to the inequality (4). For $N = 10$ the inequality (4) does not impose a meaningful upper bound on the fidelity, and therefore has nothing to say about the relationship between the fidelity and the orthogonality catastrophe. In contrast, in panel (b) ($N = 1000$) the bounds imposed by the inequality (4) are tight enough, moreover, the two curves, $\mathcal C(\lambda)$ and $\mathcal F(\lambda)$ are so close that look indistinguishable.

Rice-Mele model: N fermions in a time-dependent tight-binding lattice:

$$H_{\rm RM} = \sum_{j=1}^{N}\Big[-(J+\delta)a_j^\dagger b_j - (J-\delta)a_j^\dagger b_{j+1} + \text{h.c.}\Big] + \sum_{j=1}^{N}\Delta(a_j^\dagger a_j - b_j^\dagger b_j). \qquad (5)$$

$$\Delta(\lambda) = \Delta_{\max}\cos\lambda, \quad \delta(\lambda) = \delta_{\max}\sin\lambda$$

At the point $\lambda = \pi/4$

$$C_N = \frac{N\Delta^2}{16J\delta}, \quad \delta V_N^{\rm RM} = \sqrt N\Delta.$$

Adiabaticity breakdown time:

$$\boxed{t_* = \frac{1}{\Gamma\sqrt N}\frac{4\sqrt{J\delta}}{\Delta}}$$

Necessary adiabatic condition:

$$\boxed{\Gamma_N < \frac{1}{\sqrt N}\frac{16J\delta}{\Delta}}$$

777 Two modes of operation of Thouless pump

Quantum many-body adiabaticity in a strict sense (3) is implied in the original derivation by Thouless [6]. But is it is really necessary for quantization of transport? Yes and No. This depends on the mode of operation of the Thouless pump.

- Mode 1: A single cycle is performed, the transferred charge is measured immediately after the cycle is over. Many-body quantum adiabaticity is *not required*.
- Mode 2: Pump is operated continuously (one cycle after another) in a stationary state, charge transferred per cycle is measured. Many-body quantum adiabaticity is *indispensable*.
- Mode 2′: A single cycle is performed, but the transferred charge is measured after large time after the cycle is over. Many-body quantum adiabaticity is *indispensable*.

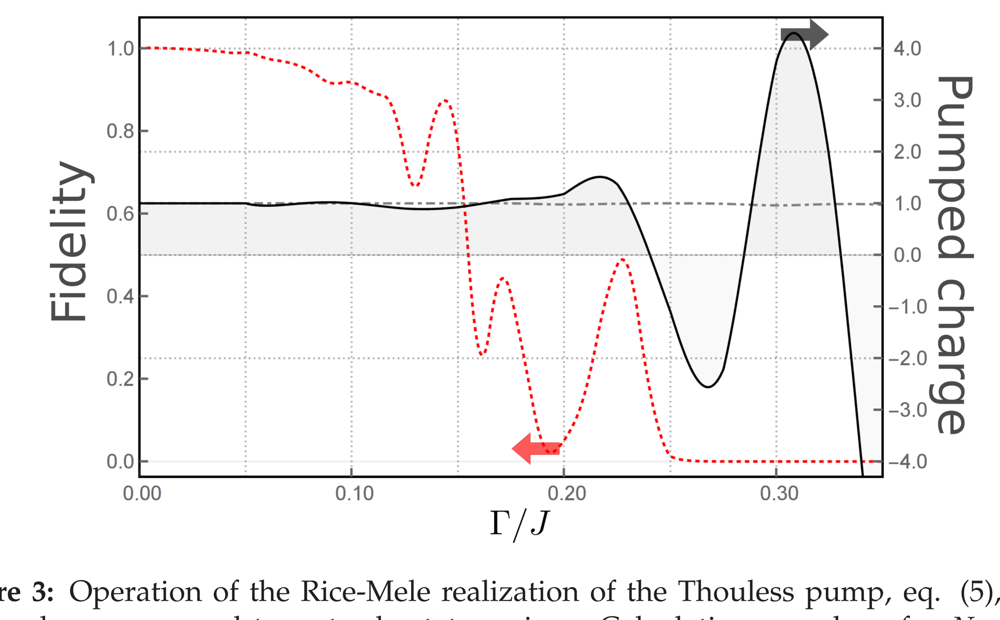
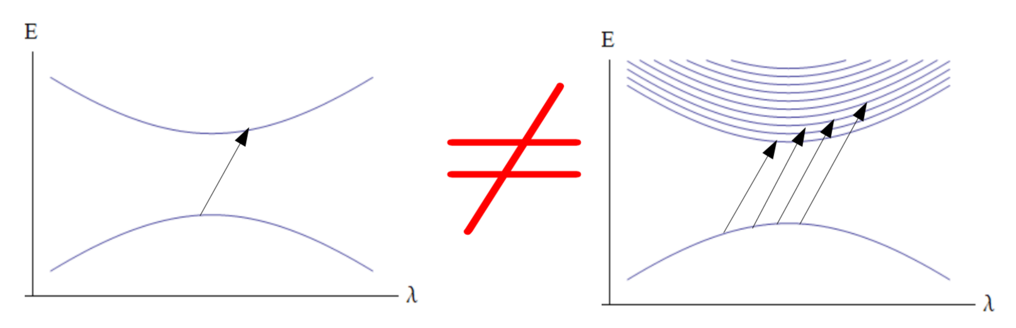


Figure 3: Operation of the Rice-Mele realization of the Thouless pump, eq. (5), in the first cycle as compared to a steady-state regime. Calculations are done for $N = 1000$ and different values of the driving rate, Γ. The cycle is given by $\Delta = (J/2)\sin\lambda$, $\delta = (J/2)\cos\lambda$ with $\lambda = \Gamma t$. Initially the system is in equilibrium. Dash-dotted gray – charge transferred in the first cycle. Dashed red – the adiabatic fidelity $\mathcal F$ at the end of the first cycle. Solid black – charge transferred per cycle in a continuous regime after the stationary state is reached. One can see that the charge transferred in the first cycle is quantized even when the many-body adiabaticity has gone completely ($\mathcal F \simeq 0$), while the quantization of the charge in the continuous regime disappears as soon as the many-body adiabaticity is broken.

Qualitative reason: When the adiabaticity is broken, $\delta N \sim \sqrt N$ elementary excitations are produced during one cycle.

- Mode 1: Only $(vT/a)\delta N/N \sim 1/\sqrt N$ of these excitations leave the pump through its end points during the first cycle. Here v is the average velocity of excitations. Their effect is *negligible* in the thermodynamic limit.
- Mode 2: In a stationary state number of excitations which leave the pump per cycle is equal to the number of excitations created per cycle due to driving, $\delta N \sim \sqrt N$. Their effect *diverges* in the thermodynamic limit.

8888 Simple but important corollary



In a two-level system adiabaticity is governed by the gap (Landau-Zener). Adiabatic condition reads

$$\Gamma \ll \Delta E_{\min}.$$

A Landau-Zener-type guess is typically **completely wrong** for bulk driven many-body systems! For a gapped system the adiabatic condition reads

$$\Gamma \ll f_N\,\Delta E_{\min} \quad\text{with}\quad \lim_{N\to\infty} f_N = 0.$$

999 Summary and concluding remarks

- Adiabaticity breakdown in many-body systems is closely related to the orthogonality catastrophe.
- The adiabaticity breakdown time t_* vanishes in the thermodynamic limit.
- Scaling of t_* with the number of particles, N, is determined by the nature of driving (global or local) and by presense/absense of gapless excitations.
- Contrary to the common belief, whenever driving is of bulk type, even a finite gap is not able to protect adiabaticity in the thermodynamic limit!
- Necessary adiabatic condition for finite systems derived – the most stringent to date, to the best of our knowledge!
- Quantum many-body adiabaticity is mandatory for the quantized particle transport in a topological Thouless pump operated continuously. However, adiabaticity breakdown does not spoil quantization in the first few cycles, if initially the pump is in the ground state. In other words, the quantization of the transport under nonadiabatic conditions is necessarily a transient phenomenon.

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