

Embedding the Petersen Graph on the Cross Cap

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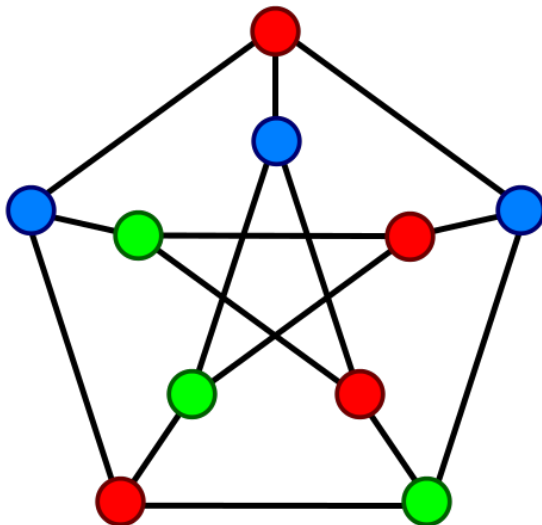
Petersen Graph

- ▶ Special Case of the Kneser Graph ($KG_{5,2}$)
- ▶ Chromatic number of $KG_{n,k}$ is $n - 2k + 2$
- ▶ Will have intersecting edges in any realization in \mathbb{R}^2
- ▶ Smallest bridgeless cubic graph with no three-edge-coloring

Donald Knuth about the Petersen Graph

[The Petersen Graph is] a remarkable configuration that serves as a counterexample to many optimistic predictions about what might be true for graphs in general.

The Petersen Graph and a Three-Coloring



The Cross Cap

- ▶ Homeomorphic to $\mathbb{R}P^2$, the real projective plane
- ▶ Obtained by identifying border points of the a two-cover of D^2
- ▶ Cannot be realized without self-intersection in \mathbb{R}^3

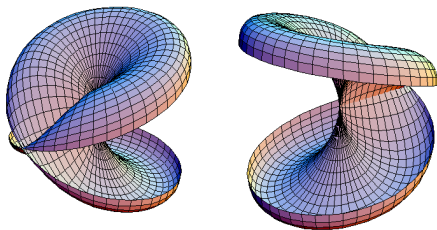


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Plan

- ▶ Research graph-theoretical properties of the Petersen Graph
- ▶ Research topological properties of the Cross Cap
- ▶ Create a realization of the Cross Cap in Maya
- ▶ Embed the Petersen Graph with a suitable coloring in the Cross Cap realization
- ▶ Create an insightful animation of the embedding