MATH 2070 HOMEWORK 4

1. Find the interval of existence for the following IVPs

(a)
$$t^2y'' + ty' + 6y = 0, y(-1) = 2, y'(-1) = 3$$

(b)
$$y'' - (\cot(t - \pi/6))y' + (\ln(2t + 3))y = e^{5t}, y(1) = 0, y'(1) = 2$$

2. Check that y_1, y_2 are solutions of the ODE. Check also that y_1, y_2 are linearly independent.

(a)
$$y_1 = e^{2t} \sin t$$
, $y_2 = e^{2t} \cos t$, $y'' - 4y' + 5y = 0$.

(b)
$$y_1 = e^{3t}, y_2 = te^{3t}, y'' - 6y' + 9y = 0$$

(c)
$$y_1 = t^3, y_2 = t^4, t^2y'' - 6ty' + 12y = 0$$

3. Find the general solution to the following ODEs

(a)
$$y'' - 7y' + 12y = 0$$

(b)
$$y'' - 5y' - 6y = 0$$

(c)
$$y'' + 10y' + 23y = 0$$

4. Find the solution to the following IVP and determine the long term behavior. (Hint: look at where the limit goes)

(a)
$$y'' - 25y = 0, y(0) = 3, y'(0) = -9$$

(b)
$$y'' - y' = 0, y(0) = 3, y'(0) = 2$$

(c)
$$6y'' + 5y' - 4y = 0, y(0) = 0, y'(0) = 0$$