MATH 2070 HOMEWORK 9

- (1) Use the first translation theorem to find the following Laplace transforms
 - (a) $\mathcal{L}\{(t+t^{10})e^{-6t}\}.$
 - (b) $\mathcal{L}\{(t-1)^2e^{-2t}\}.$
 - (c) $\mathcal{L}\{(1-e^{-2t})^2\cos 5t\}$.
- (2) Use the second translation theorem to find the following Laplace transforms.
 - (a) $\mathcal{L}\{e^{2-t}\mathcal{U}(t-2)\}.$
 - (b) $\mathcal{L}\{(3t+1)\mathcal{U}(t-1)\}.$
 - (c) $\mathcal{L}\{\sin t\mathcal{U}(t-\pi/2)\}.$
- (3) Use the second translation theorem to find the following inverse Laplace transforms.
- (a) $\mathcal{L}^{-1}\left\{\frac{(1+e^{-2s})^2}{s+2}\right\}$. (b) $\mathcal{L}^{-1}\left\{\frac{se^{-\pi s/2}}{s^2+4}\right\}$. (c) $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\}$. (4) Use Laplace transform to solve the following IVPs
- - (a) $\frac{dy}{dt} + 2y = f(t), y(0) = 0, f(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$ (b) $y'' + 2y = f(t), y(0) = 0, y'(0) = 0, f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$