MATH 2070 HOMEWORK 1

1. Draw the direction field for the following ODEs.

$$y' = 4 - y$$

$$y' = y + 2$$

$$y' = (y + 1)(y - 2)$$

2. Classify the following differential equations by order and linearity

$$y^{(100)} + y' = 6$$

$$y''' + t^2 y'' + t^4 y' - (\cos t)y = \sqrt{t}$$

$$y' = y + \sin y$$

3. Check if the given functions $y(t)$ are solutions to the corresponding ODEs

$$y(t) = \tan t, y' = 1 + y^2$$

$$y(t) = e^t + e^{-2t}, y'' + y' - 2y = 0$$

$$y(t) = 1/t, t^2 y'' - 2y = 0$$

$$y(t) = \sin 2t, y'' + y = \sin 2t$$

4. Find the general solutions to the following ODEs

$$y' + 3y = t + e^{-2t}$$

$$(1 + t^2)y' + 4ty = (1 + t^2)^{-2}$$

and determine the long term behaviors of the solution.

5. (Bonus) For the ODE

$$ty' + (t - 1)y = -e^{-t}$$

Investigate the following sets of initial values

$$y(0) = 0 \text{ and } y(0) = 1$$

What happens in each case?

6. (no need to write) Review the techniques of integration and make sure you are comfortable to u-substitution, integrating by parts and integration of $\cos^2 x, \sin^2 x$.