MATH 2070 HOMEWORK 8

(1) Find the Laplace transform or inverse Laplace transforms

(a) $f(t) = (1 + e^{-2t})^2 + 5t^2$. Find $\mathscr{L}\{f(t)\}$.

(b) $f(t) = \cos 5t + \sin 2t$. Find $\mathscr{L}\{f(t)\}$.

(c) $f(t) = e^t \sinh t + e^{-t} \cosh t$. Find $\mathscr{L}\{f(t)\}$.

(d) $F(s) = \dfrac{s^2 + 1}{s(s-1)(s+1)(s-2)}$. Find $\mathscr{L}^{-1}\{F(s)\}$.

(e) $F(s) = \dfrac{1}{s^4 - 9}$. Find $\mathscr{L}^{-1}\{F(s)\}$.

(f) $F(s) = \dfrac{6s + 3}{s^4 + 5s^2 + 4}$. Find $\mathscr{L}^{-1}\{F(s)\}$. (Hint: This one has to be done by usual partial fractions).

(2) Use Laplace transform to solve the following IVPs

(a) $\begin{cases} \dfrac{dy}{dt} - y = 1 \\ y(0) = 0 \end{cases}$

(b) $\begin{cases} y' + 6y = e^{4t}, \\ y(0) = 2. \end{cases}$

(c) $\begin{cases} y'' - 4y' = 6e^{3t} - 3e^{-t}, \\ y(0) = 1, y'(0) = -1. \end{cases}$

(d) $\begin{cases} 2y''' + 3y'' - 3y' - 2y = e^{-t} \\ y(0) = 0, y'(0) = 0, y''(0) = 1. \end{cases}$