EECS 489, Spring 2012

Problem Set 7: Interaction Control

Due 4/24/12

Consider our familiar 2 Link robotic manipulator. To analyze issues in force control, it is necessary to include realistic transmission dynamics. Assume that we can read the encoders and know the positions of the motor shafts, but the actual link positions will lag or lead the motor depending on link inertias, contact forces and transmission compliance.

The link positions will be denoted as $\underline{q_l}$ and the motor positions as $\underline{q_m}$. The transmissions have a linear stiffness K_{pt} and damping K_{vt}. As such, the motors and links have the following dynamics:

$$motor\ dynamics: \frac{d}{dt}([B_m]\dot{q}_m) = \underline{\tau}_m + K_{pt}\left(\underline{q_l} - \underline{q_m}\right) + K_{vt}\left(\underline{\dot{q}_l} - \underline{\dot{q}_m}\right)$$

$$link\ dynamics: \frac{d}{dt}([B_l]\dot{q}_l) = K_{pt}\left(\underline{q_m} - \underline{q_l}\right) + K_{vt}\left(\underline{\dot{q}_m} - \underline{\dot{q}_l}\right) + J^T \underline{F}_{on\_hand}$$

In the above, the motor angles are expressed in terms of the output angles of an idealized transmission. The difference between a motor angle and its corresponding link angle is due to transmission compliance.

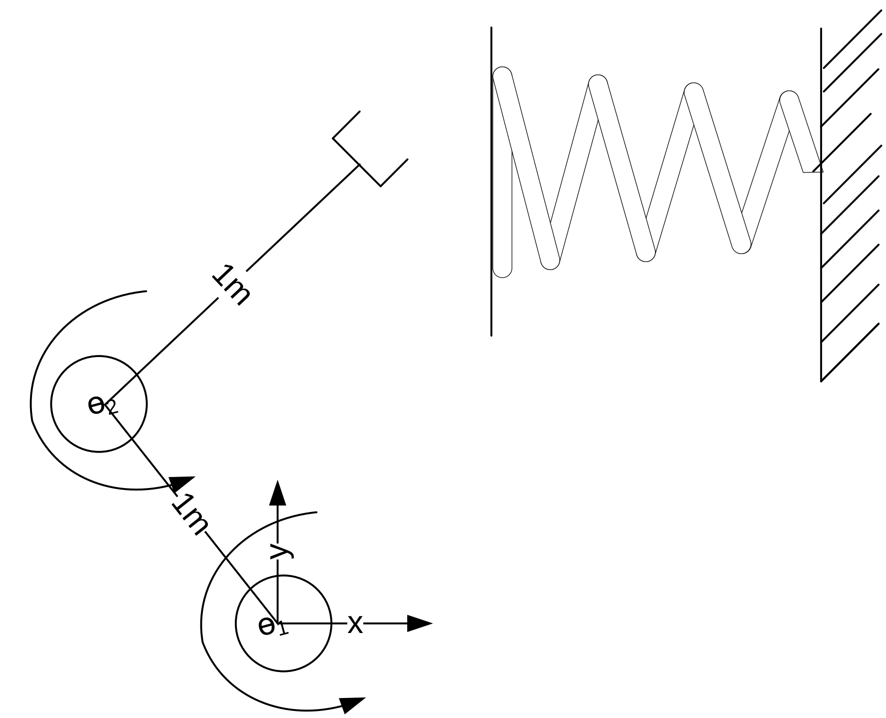
Typically, rotation sensors (e.g. encoders) are mounted directly to the motors. Thus the motor angles are known directly, but the link angles are not directly measured.

We will model contact dynamics with the environment in terms of a lumped-parameter nonlinear stiffness. Per the above figure, our environment model is conceptually equivalent to interacting with a mattress, where the mattress is oriented vertically (parallel to the y axis) and the mattress surface is at a distance of 1.2m along the x direction from the robot's base frame. The mattress equivalent models the environment with stiffness and damping coefficients K_e and K_{ve}. Our environment therefore exerts reaction forces on the robot's hand depending on the x and y motion of the hand as follows:

$$if\ x > 1.2m, \qquad \underline{F}_{on\_hand} = \begin{bmatrix} -K_e(x - 1.2) - K_{ve}\dot{x} \\ -K_{ve}\dot{y} \end{bmatrix}$$

$$if\ x \leq 1.2m, \qquad \underline{F}_{on\_hand} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note that our robot's hand can slide along the surface of the mattress, but this sliding results in viscous drag parallel to the surface. In this problem, we are ignoring gravity (e.g., gravity might act in the z direction, out of the plane of the robot's motion).



We would like our robot to exert a 1Newton force in the X direction on the environment. We can use the following control law:

$$\underline{\tau} = J^T[\underline{F}_{des} + K_{pf}\left(\underline{F}_{des\_on\_environ} - (-\underline{F}_{sensed\_on\_hand})\right) - K_{vf}\left(J\underline{\dot{q}}_m\right)]$$

Implement this control law by editing the provided function:

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function tau=force_controller(Fdes,Fe,Jp,qmdot)
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Ideally you would like to be able to control using $-K_{vf}(\underline{\dot{F}})$; however, this cannot be directly measured, so you must use the motor tachs $\underline{\dot{q}}$ to approximate this. Also, one needs to be careful with the sign of sensed force. The desired force is expressed in terms of the force to be exerted on the environment. Sensed force should thus have the same meaning, which is equal and opposite to the force exerted on the hand by the environment. In the above formula, $\underline{F}_{des\_on\_environ}$ is the desired force to be exerted on the environment, and $\underline{F}_{sensed\_on\_hand}$ is the reaction force of the environment, sensed as a force acting on the robot's hand.

A simulator has been provided for your use. Note that the robot and environment states must be initialized. To use the simulator, you should provide motor torques, τ_m , to the simulator at 1kHz and the simulator will perform an integration at 10kHz and report motor positions and velocities ($\underline{q}_m, \underline{\dot{q}}_m$) as well as the force sensed at the end effecter $\underline{F}_{sensed}$. The simulator will keep track of $\underline{q}_l$ and $\underline{\dot{q}}_l$ internally (as well as return these values). The simulator function has an optional second parameter that will reset the robot state to

$X = \begin{bmatrix} 1.99 \\ 1.2 \end{bmatrix}$ $\dot{X} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ when “reset” is set to true.

The following parameters describe the robot, the environment and the desired force. These values are set in the provided code.

B_m	$\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$
B_l	$\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$
K_{ve}	1.0 N/m/s
K_{vt}	$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$
$F_{desired}$	1N
K_e	10N/m
K_pt	$\begin{bmatrix} 4000 & 0 \\ 0 & 4000 \end{bmatrix}$

1. Recommend a set of gains K_{pf} and K_{vf} that are stable and approach steady state at a reasonable rate.

2. With the control gains set to the following, what is the maximum environment stiffness K_e for which the control remains stable?

K_{pf}	$\begin{bmatrix} \mathbf{100} & \mathbf{0} \\ \mathbf{0} & \mathbf{10} \end{bmatrix}$
K_{vf}	$\begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix}$
K_{pt}	$\begin{bmatrix} 4000 & 0 \\ 0 & 4000 \end{bmatrix}$

3. Find the minimum transmission stiffness K_{pt} for which control remains stable.
   Use gains K_{pf} and K_{vf} from problem (1) and environment stiffness of Ke=10N/m.

4. Comment on the results.