CIS 121 Fei Liu

Homework 08

Recitation 201

Collaborators:

Q1.

(a) The random variable R_i is actually a binomial random variable. There are n trials, and we are looking for k successes, and $n-k$ failures, where a success represents the jth element being inserted into $A[i]$, where $0 \leq j \leq n$, and a failure represents the jth element being inserted into any other slot. Each trial's probability of success is $\frac{1}{n}$, and each trial's probability of failure is $\frac{n-1}{n}$. Using the binomial distribution, $Pr(R_i = k) = \binom{n}{k}(\frac{1}{n})^k(\frac{n-1}{n})^{n-k}$.

(b) I can simply start with the expression I found in part a for $Pr(R_i = k)$ and manipulate it until I get $(\frac{4}{k})^k$:

1. $\binom{n}{k}(\frac{1}{n})^k(\frac{n-1}{n})^{n-k} \leq (\frac{en}{k})^k(\frac{1}{n})^k(\frac{n-1}{n})^{n-k}$ because of the inequality $\binom{n}{k} \leq (\frac{en}{k})^k$ given in the problem.
2. $(\frac{en}{k})^k(\frac{1}{n})^k(\frac{n-1}{n})^{n-k} = (\frac{e}{k})^k(\frac{n-1}{n})^{n-k}$.
3. $(\frac{e}{k})^k(\frac{n-1}{n})^{n-k} < (\frac{e}{k})^k$ because $(\frac{n-1}{n})^{n-k} < 1$.
4. $(\frac{e}{k})^k \leq (\frac{4}{k})^k$ because $e < 4$.

I have proved via a series of inequalities that $Pr(R_i = k)$ is less than or equal to $(\frac{4}{k})^k$.

Q2. Here's how I got my answer: First, let me treat the n distinct elements as n indistinguishable elements. The number of ways to distribute the n indistinguishable elements amongst the m slots in the hash table can be found using stars and bars. The n elements are the stars and the m slots are the bars. The total number of ways to do this is $\binom{n+m-1}{n}$.

There has to be more distinct hash tables than this because within every bin, I can permute the positions of every element in it, where as the stars bars technique counts every each of these permutations as one arrangement due to the indistinguishable stars. For every distribution of indistinguishable elements, the first spot in this distribution can be any of n distinct elements, the second spot can be any of $n-1$ remaining distinct elements, the third spot can be any of $n-2$ remaining distinct elements, ..., the nth spot is the last remaining element. Because the element placed in any given spot does not affect the element placed in any other given spot in the distribution, by the multiplication rule, the total number of ways to permute n elements in n spots is simply $n!$.

Because the distribution of the elements as well as their permutation within the hash table are independent, by the multiplication rule, the total number of distinct hash tables that can result from n distinct elements and m slots is $\binom{n+m-1}{n} * n!$.

Q3.

(a) Here's the hash table that I got:

$$12|83|null|3|39|52|18|29|null|null|46|23$$

(b) Here's the hash table that I got:

$$12|83|null|3|52|29|18|null|39|null|46|23$$

(c) Here's the hash table that I got:

$$12|39|null|3|52|29|18|83|null|null|46|23$$