Problem 4.5

mixture of Poisson distribution

$$p(X=k|\theta) = \sum_{j=1}^{k} \pi_j \frac{1}{k!} e^{-\lambda_j} \lambda_j^k$$

$$p(X, Z|\pi, \lambda) = \prod_{i=1}^{N} \prod_{j=1}^{K} (\pi_j \frac{1}{x_i!} e^{-\lambda_j} \lambda_j^{x_i})^{z_{ij}}$$

$$\ln p(X, Z|\pi, \lambda) = \sum_{i=1}^{N} \sum_{j=1}^{K} [z_{ij} (\ln \pi_j - \ln(x_i!) - \lambda_j + x_i \ln \lambda_j)]$$

① E-Step

$$E[z_{ij}] = \frac{\pi_j \frac{1}{x_i!} e^{-\lambda_j} \lambda_j^{x_i}}{\sum_{j=1}^{K} \pi_j \frac{1}{x_i!} e^{-\lambda_j} \lambda_j^{x_i}} = \gamma(z_{ij})$$

② M-Step

$$l = E_Z[\ln P(X, Z|\pi, \lambda)] = \sum_{i=1}^{N} \sum_{j=1}^{K} [\gamma(z_{ij}) (\ln \pi_j - \ln(x_i!) - \lambda_j + x_i \ln \lambda_j)]$$

for λ_j : $$\frac{\partial l}{\partial \lambda_j} = \sum_{i=1}^{N} [\gamma(z_{ij}) (-1 + \frac{x_i}{\lambda_j})] = 0$$

$$-\sum^{N} \gamma(z_{ij}) + \frac{\sum_{i=1}^{N} \gamma(z_{ij}) x_i}{\lambda_j} = 0$$

$$\lambda_j = \frac{\sum_{i=1}^{N} \gamma(z_{ij}) x_i}{\sum_{i=1}^{N} \gamma(z_{ij})}$$

for π_j : $\underset{\lambda_j}{\operatorname{argmax}}\ l$

s.t. $\sum_{j=1}^{K} \pi_j = 1$

$$L = l + \alpha\left(\sum_{j=1}^{K}\pi_j - 1\right) = \sum_{i=1}^{N}\sum_{j=1}^{K}\left[r(z_{ij})\left(\ln\pi_j - \ln(x_i!) - \lambda_j + x_i \ln\lambda_j\right)\right] + \alpha\left(\sum_{j=1}^{K}\pi_j - 1\right)$$

$$\frac{\partial L}{\partial \alpha} = \sum_{j=1}^{K}\pi_j - 1 = 0 \Rightarrow \sum_{j=1}^{K}\pi_j = 1$$

$$\frac{\partial L}{\partial \pi_j} = \sum_{i=1}^{N} r(z_{ij})\frac{1}{\pi_j} + \alpha = 0 \Rightarrow \pi_j = \frac{\sum_{i=1}^{N} r(z_{ij})}{-\alpha}$$

$$\sum_{j=1}^{K}\pi_j = \sum_{j=1}^{K}\frac{\sum_{i=1}^{N} r(z_{ij})}{-\alpha} = 1$$

$$-\alpha = \sum_{j=1}^{K}\sum_{i=1}^{N} r(z_{ij})$$

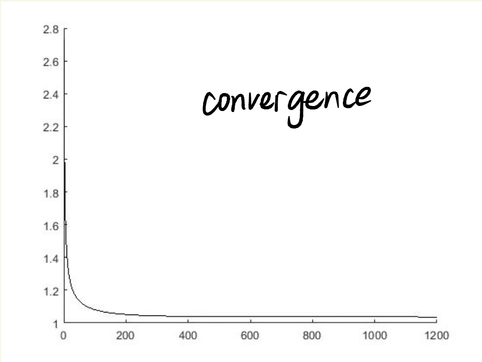
$$\therefore \pi_j = \frac{\sum_{i=1}^{N} r(z_{ij})}{\sum_{j=1}^{K}\sum_{i=1}^{N} r(z_{ij})}$$

① $K=1$, it's the same with nomal Poissons distribution.

② $K \geq 2$, write a matlab program
after 500 iteration, show that

London

K	λ	π
1	0.929	1
2	[0.919, 1.072]	[0.936, 0.064]
3	[0.926, 1.088, 1.088]	[0.98, 0.005, 0.015]
4	[0.916, 1.067, 1.067, 1.067]	[0.917, 0.072, 0.007, 0.004]
5	[0.894, 1.037, 1.037, 1.037, 1.037]	[0.758, 0.140, 0.061, 0.039, 0.002]



Conclusions: we can see that there is one biggest π_k, so we can suggest there is a specific targeting of area.

Antwerp

K	λ	π
1	0.896	1
2	[2.195, 0.230]	[0.339, 0.661]
3	[2.195, 2.195, 0.230]	[0.309, 0.030, 0.661]
4	[2.374, 0.525, 0.525, 0]	[0.278, 0.236, 0.214, 0.272]
5	[2.195, 2.195, 2.195, 2.195, 0.23]	[0.161, 0.086, 0.064, 0.028, 0.661]