P6.4

(a) $p(r=H, s=T) = p(r=H|s=T)\,p(s=T) = \theta_2(1-\alpha)$

$p(r=T, s=H) = p(r=T|s=H)\,p(s=H) = \theta_1\alpha$

so we can get a table :

r \ s	H	T
H	HH $(1-\theta_1)\alpha$	HT $\theta_2(1-\alpha)$
T	TH $\theta_1\alpha$	TT $(1-\theta_2)(1-\alpha)$

so I guess heads if $p(HH) \geq p(HT)$

$$(1-\theta_1)\alpha \geq \theta_2(1-\alpha)$$

$$\alpha \geq \frac{\theta_2}{1-\theta_1+\theta_2}$$

guess tails if $\alpha < \frac{\theta_2}{1-\theta_1+\theta_2}$

(b) when $\theta_1=\theta_2=\theta$

the judgement will compare α with θ directly, this means that you should believe your friend's report if $\alpha \geq \theta$

(c) guess heads if $p(s=H|r_1,r_2\cdots r_n) \geq p(s=T|r_1,r_2\cdots r_n)$

$$p(r_1,r_2\cdots r_n|s=H)\,p(s=H) \geq p(r_1,r_2\cdots,r_n|s=T)\,p(s=T)$$

suppose $r_1\cdots r_n$ have k times heads, $n-k$ times tails

so $p(r_1|s=H)\,p(r_2|s=H)\cdots p(r_n|s=H)\cdots \geq p(r_1|s=T)\cdots p(r_n|s=T)\,p(s=T)$

$$\theta_1^{(n-k)}(1-\theta_1)^k\alpha \geq \theta_2^k(1-\theta_2)^{n-k}(1-\alpha)$$

$$\alpha \geq \frac{1}{1+\left(\frac{1-\theta_1}{\theta_2}\right)^k\left(\frac{\theta_1}{1-\theta_2}\right)^{n-k}}$$

and tails otherwise

(d) if $\theta_1=\theta_2$ and all reports are heads $(k=n)$, so that $\alpha \geq \frac{1}{1+\left(\frac{1-\theta}{\theta}\right)^n}$