P 9.1

(a) the Lagranglan for this problem is $L = \|x - x_a\|^2 - \lambda(w^T x + b)$

$\frac{\partial L}{\partial \lambda} = -(w^T x + b) = 0 \Rightarrow f(x) = 0$

$\frac{\partial L}{\partial x} = 2(x - x_a) - \lambda w = 0 \Rightarrow 2x - 2x_a = \lambda w$

$\downarrow$

$2(w^T x - w^T x_a) = \lambda \|w\|^2$

$2(f(x) - f(x_a)) = \lambda \|w\|^2 \Rightarrow \lambda = \frac{-2f(x_a)}{\|w\|^2}$

and then, $x - x_a = \frac{-f(x_a)}{\|w\|^2} w$

the length-squared of this vector is $\|x - x_a\|^2 = \frac{f(x_a)^2}{\|w\|^4} \|w\|^2$

so $\|x - x_a\| = \frac{|f(x_a)|}{\|w\|}$

(b) let $x_a = 0$. we have $\|x\| = \frac{|w^T 0 + b|}{\|w\|} = \frac{|b|}{\|w\|}$

(c) see question (a) we have got $\hat{x} = x_a - \frac{f(x_a)}{\|w\|^2} w$

P9.5

(a) the soft-margin constraint is $y_i(w^T x_i + b) \geqslant 1 - \xi_i \Rightarrow \xi_i \geqslant 1 - y_i(w^T x_i + b)$

because $\xi_i \geqslant 0$

so $\xi_i \geqslant \max(0, 1 - y_i(w^T x_i + b)$

(b) $E(w, b) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(w^T x_i + b)) \propto \sum_{i=1}^{n} \max(0, 1 - y_i(w^T x_i + b)) + \lambda \|w\|^2$

so $L_{svm}(z_i) = \max(0, 1 - y_i(w^T x_i + n)) = \max(0, 1 - z_i)$