Problem 2.1

(a) maximum-likehood function is

$$l(\lambda) = \sum_{i=1}^{N} \ln p(x=k_i|\lambda) = \sum_{i=1}^{N} \ln\left(\frac{1}{k_i!} e^{-\lambda} \lambda^{k}\right) = \sum_{i=1}^{N} \left[-\ln(k_i!) - \lambda + k_i \ln\lambda\right]$$

$$= -\ln\left(\prod_{i=1}^{N} k_i!\right) - N\lambda + \left(\sum_{i=1}^{N} k_i\right)\ln\lambda$$

Solve: ① $l'(\lambda) = -N + \frac{1}{\lambda}\sum_{i=1}^{N} k_i = 0$

$$\Rightarrow \hat{\lambda} = \frac{1}{N}\sum_{i=1}^{N} k_i$$

② $l''(\lambda) = -\frac{1}{\lambda^2}\sum_{i=1}^{N} k_i < 0$ valid ✓

③ $\because 0 \le k_i \le N$

$\therefore 0 \le \sum_{i=1}^{N} k_i \le N^2 \Rightarrow 0 \le \lambda \le N$ valid ✓

(b) $\text{Bias}[\hat{\lambda}] = E[\hat{\lambda}] - \lambda = E\left[\frac{1}{N}\sum_{i=1}^{N} k_i\right] - \lambda$

$$= \frac{1}{N}\sum_{i=1}^{N} E[k_i] - \lambda = \frac{N\lambda}{N} - \lambda = 0$$

$\therefore$ the ML estimate is unbiased

$$\text{var}(\hat{\lambda}) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{N} k_i\right) = \frac{1}{N}\text{var}\left(\sum_{i=1}^{N} k_i\right) = \frac{1}{N}\text{var}(k_i) = \frac{\lambda}{N}$$

(3)

$$\hat{\lambda} = \frac{1}{N}\sum_{i=1}^{N} k_i = \frac{1}{576}(229\times 0 + 211\times 1 + 93\times 2 + 35\times 3 + 7\times 4 + 1\times 5)$$

$$= 0.93$$

(4) $p(X=k \mid 0.93) = \frac{1}{k!} e^{-0.93} (0.93)^k$

1°	number of cells with	0 hits	$N\times p(X=0 \mid 0.93) = 227$
2°	⋮	1	$N\times p(X=1 \mid 0.93) = 211$
3°	⋮	2	$N\times p(X=2 \mid 0.93) = 98$
4°	⋮	3	$N\times p(X=3 \mid 0.93) = 30$
5°	⋮	4	$N\times p(X=4 \mid 0.93) = 7$
6°	⋮	5	$N\times p(X=5 \mid 0.93) = 1$

I can make the conclusion that the flying bombs obey poisson distribution.