

P8.5

(a) because the number of misclassified training points is the number of points with $z_i < 0$

$$\text{so } R_{\text{emp}} = \sum_{i=1}^n L_0(z_i) \quad \text{where } L_0(z_i) = \begin{cases} 0, & z_i \geq 0 \\ 1, & z_i < 0 \end{cases}$$

(b) because the perceptron minimizes the error over the misclassified points

$$\text{so } E(w) = \sum_{i \in \text{misclassified}} -z_i = \sum_{i=1}^n \begin{cases} 0, & z_i \geq 0 \\ -z_i, & z_i < 0 \end{cases} = \sum_{i=1}^n \max(0, -z_i)$$

$$\therefore L_P(z_i) = \max(0, -z_i)$$

$$\begin{aligned} \text{(c)} \quad E(w) &= \sum_{i=1}^n (w^T x_i - y_i)^2 = \sum_{i=1}^n \frac{1}{y_i^2} (y_i w^T x_i - y_i)^2 = \sum_{i=1}^n (y_i w^T x_i - 1)^2 \quad \because y_i^2 = 1 \\ &= \sum_{i=1}^n (z_i - 1)^2 \end{aligned}$$

$$\therefore L_{LS}(z_i) = (z_i - 1)^2$$

$$\text{(d) we have } E(w) = - \sum_{i=1}^n \begin{cases} \log \sigma(w^T x_i) & , y_i = 1 \\ \log \sigma(-w^T x_i) & , y_i = -1 \end{cases}$$

$$= - \sum_{i=1}^n \log \sigma(y_i w^T x_i)$$

$$= \sum_{i=1}^n - \log \frac{1}{1 + \exp(-y_i w^T x_i)}$$

$$= \sum_{i=1}^n \log(1 + e^{-z_i})$$

$$\therefore L_{LR}(z_i) = \log(1 + e^{-z_i}) \propto \frac{1}{\log 2} \log(1 + e^{-z_i})$$