香港城市大學
City University of Hong Kong
電腦科學系
Department of
Computer Science

Metaheuristics for Vehicle Routing Problems

Fei Liu (柳斐)

Department of Computer Science
City University of Hong Kong

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Outline

1. Vehicle Routing Problem (VRP)
2. Metaheuristics for VRP
3. Conclusion

WIKIPEDIA

The **vehicle routing problem (VRP)** is a **combinatorial optimization** and **integer programming** problem which asks "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?".

Paolo Toth

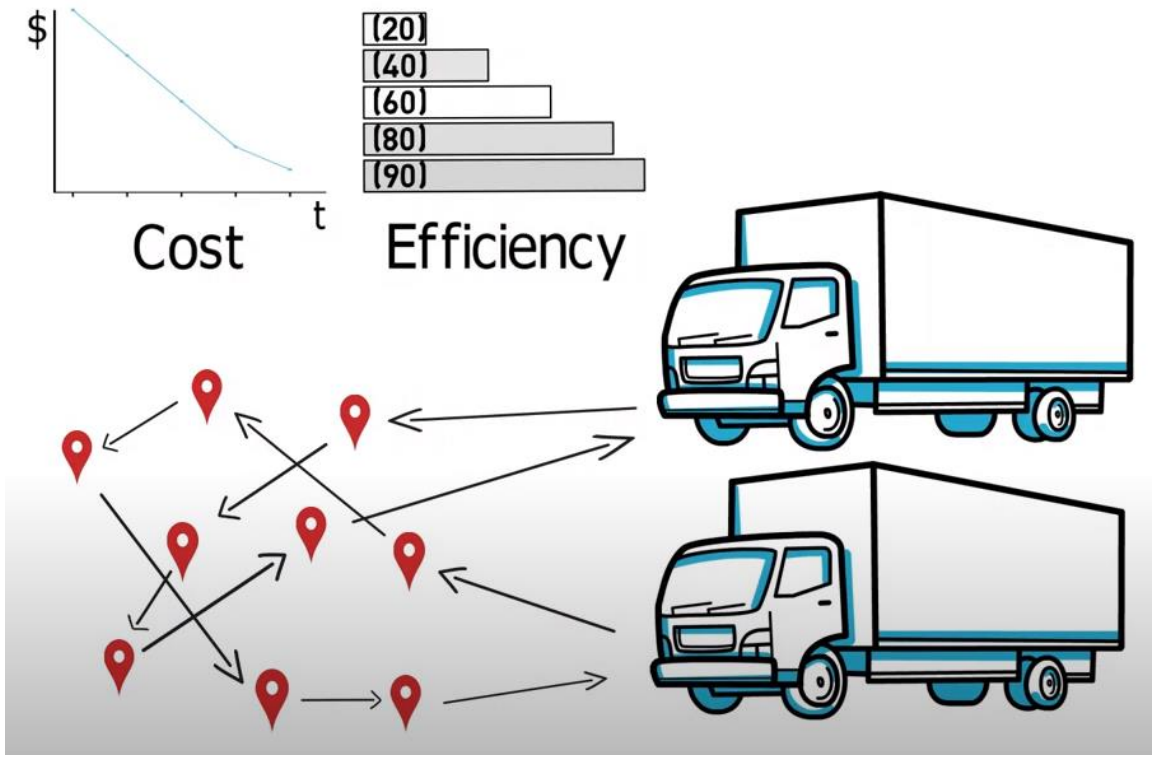
A generic verbal definition of the *family of vehicle routing problems* can be the following:

Given: A set of *transportation requests* and a *fleet of vehicles*.

The problem is then to find a plan for the following:

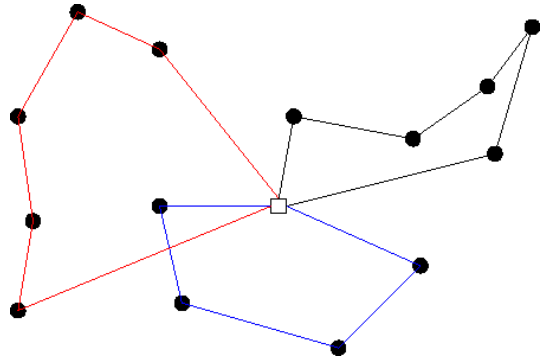
Task: Determine a set of *vehicle routes* to *perform* all (or some) transportation requests with the given vehicle fleet *at minimum cost*; in particular, decide which *vehicle handles which requests in which sequence* so that all *vehicle routes* can be *feasibly* executed.

- Toth, Paolo, and Daniele Vigo, eds. **Vehicle routing: problems, methods, and applications**. Society for Industrial and Applied Mathematics, 2014.



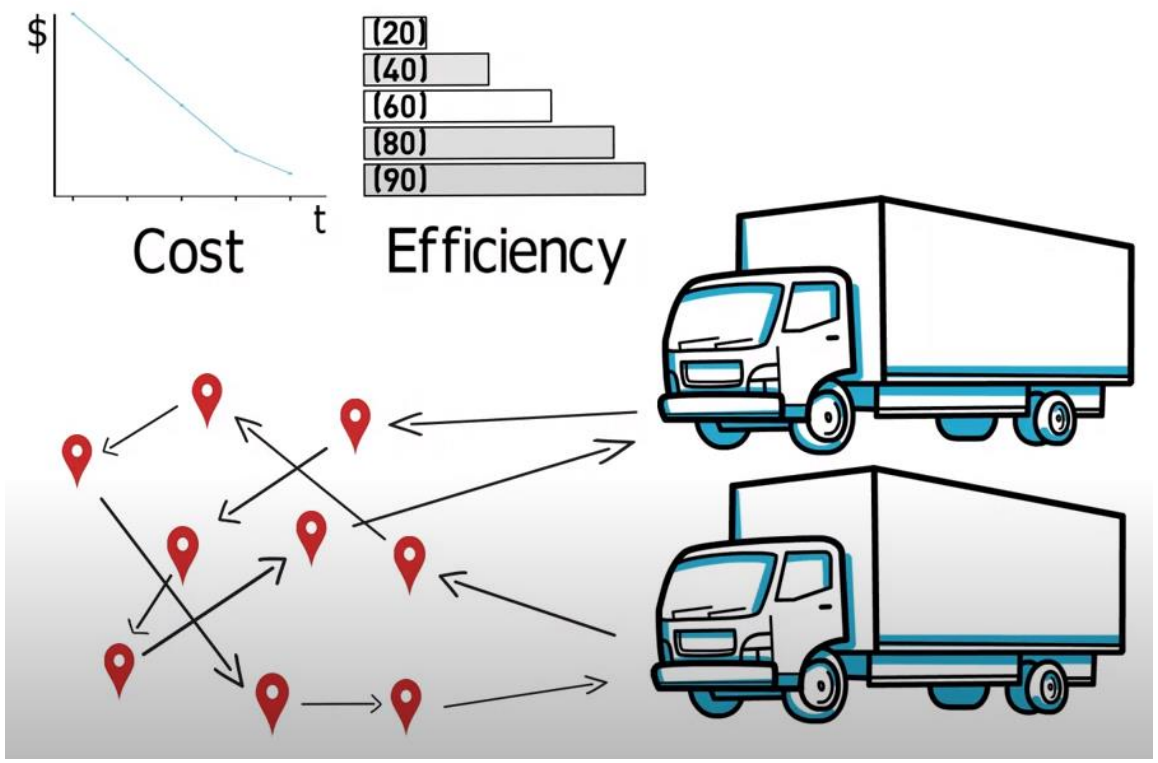
- https://www.youtube.com/watch?v=OKMssWdC0I0

Vehicle Routing Problem (VRP)



Four components

- **Network**
- **Sites to be visited (customers to serve, tasks to process, etc.)**
- **Fleet of vehicles**
- **Depot(s)**



Objectives: Cost & Efficiency

Constraints:
- Limited capacities
- Many depots
- Customer demand
- Time windows
- Precedence and synchronization
- …

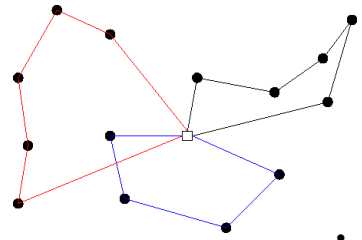
Typical VRPs:
- Capacitated Vehicle Routing Problem (CVRP)
- Vehicle Routing Problem with Backhauls (VRPB)
- Vehicle Routing Problem with Split Deliveries (VRPSD)
- Vehicle Routing Problem with Multiple Depot (VRPMD)
- Vehicle Routing Problem with Time Windows (VRPTW)
- …

- https://www.youtube.com/watch?v=OKMssWdC0I0

Dantzig and Ramser (**1959**) the first to introduce **"Truck Dispatching Problem"**

Clarke and Wright (**1964**) generalized this problem to a linear optimization problem **"Vehicle Routing Problem"**

Lenstra and Rinnooy Kan (**1981**) proved VRP is **an NP-hard problem**, exact algorithms are only efficient for small problems



time

1950 1960 1970 1980 2000 2021

1. Constructive Heuristics：

Savings heuristic

sweep algorithm

2. Improvement Heuristics：

K-opt

λ-interchange

3. Exact algorithms：

Branch and Bound

Cutting Plane

Network-flows

Dynamic Programming

4. Metaheuristics：

Tabu search

Simulated Annealing

Local search methods

GRASP

Genetic Algorithm

Ant Colony Algorithm

Partical Swarm Optimization

Reinforcement Learning

Perhaps the most famous heuristic of this category is the Clarke and Wright (**1964**) savings heuristic

The development of exact algorithms for the VRP took off in **1981** with the publication of two papers by Christofides

The development of modern heuristics for the VRP really started in the **1990s** with the advent of metaheuristics.

- Laporte, Gilbert, Paolo Toth, and Daniele Vigo. "**Vehicle routing: historical perspective and recent contributions.**" (2013): 1-4.

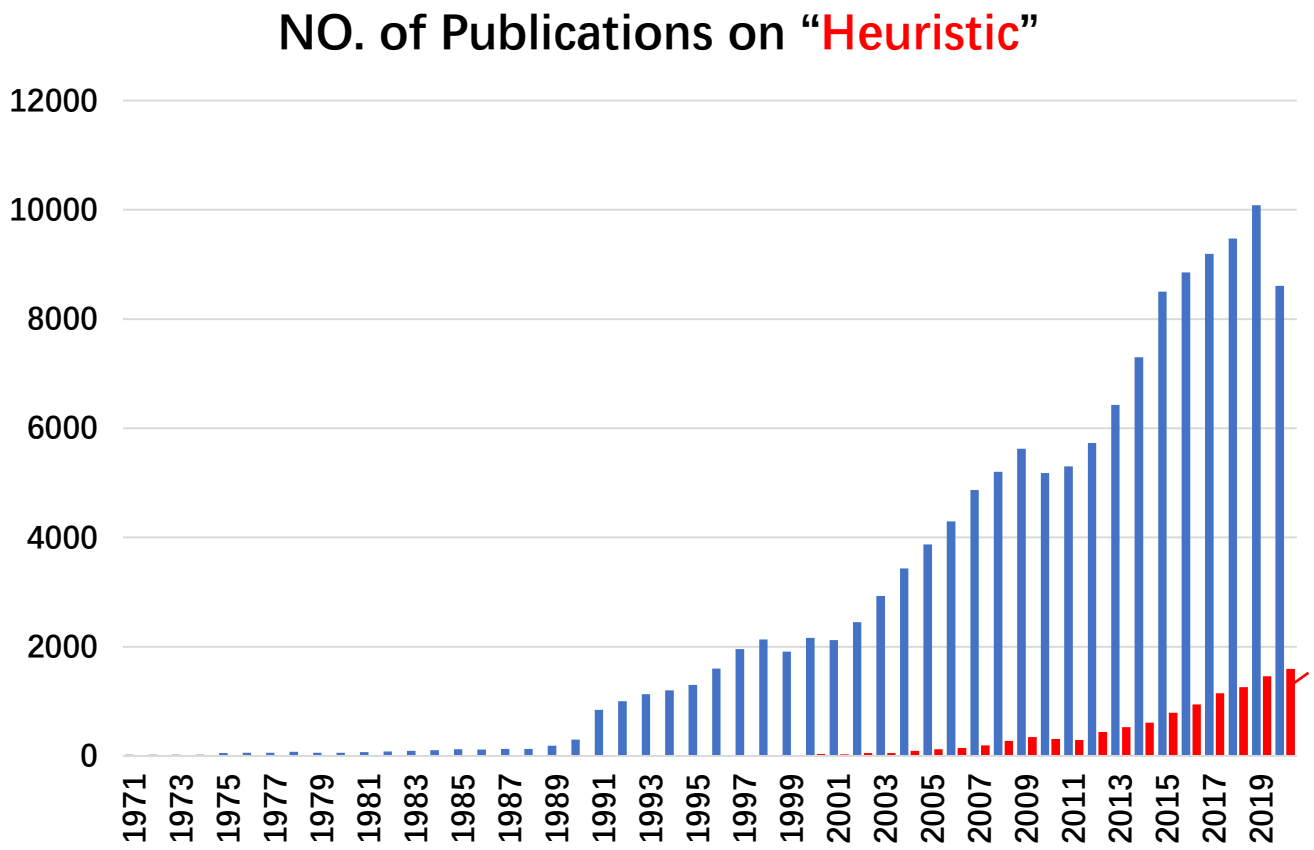
Overview of **applied methods**

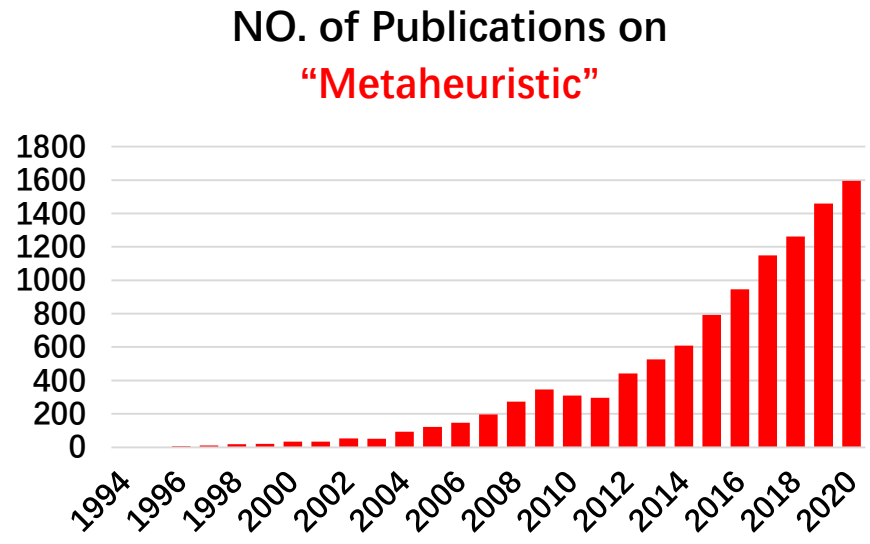
Applied Method	Number of articles (total = 144)	Relative presence
Metaheuristic	104	72.22%
Classical Heuristic	26	18.06%
Exact Method	20	13.89%
Simulation	7	4.86%
Real-time solution methods	6	4.17%

Overview of **VRP variants**

Variant	Number of articles (total = 144)	Relative presence
CVRP (Capacitated)	128	88.89%
VRPTW (Time Window)	57	39.58%
HVRP (Heterogeneous)	27	18.75%
MDVRP (Multi Depot)	18	12.50%
VRPPB (Backhauls)	17	11.81%
SDVRP (Split Deliveries)	16	11.11%
DVRP (Dynamic)	15	10.42%
PVRP (Periodic)	14	9.72%
VRPSD (Stochastic Demands)	13	9.03%
VRRSPD (Simultaneous Pickup and Delivery)	12	8.33%
OVRP (Open)	9	6.25%
TDVRP (Time Dependent)	7	4.86%
MCVRP (Multi-Compartment)	5	3.47%
CCVRP (Cumulative)	3	2.08%

- Braekers, Kris, Katrien Ramaekers, and Inneke Van Nieuwenhuyse. "**The vehicle routing problem: State of the art classification and review**." Computers & Industrial Engineering 99 (2016): 300-313.

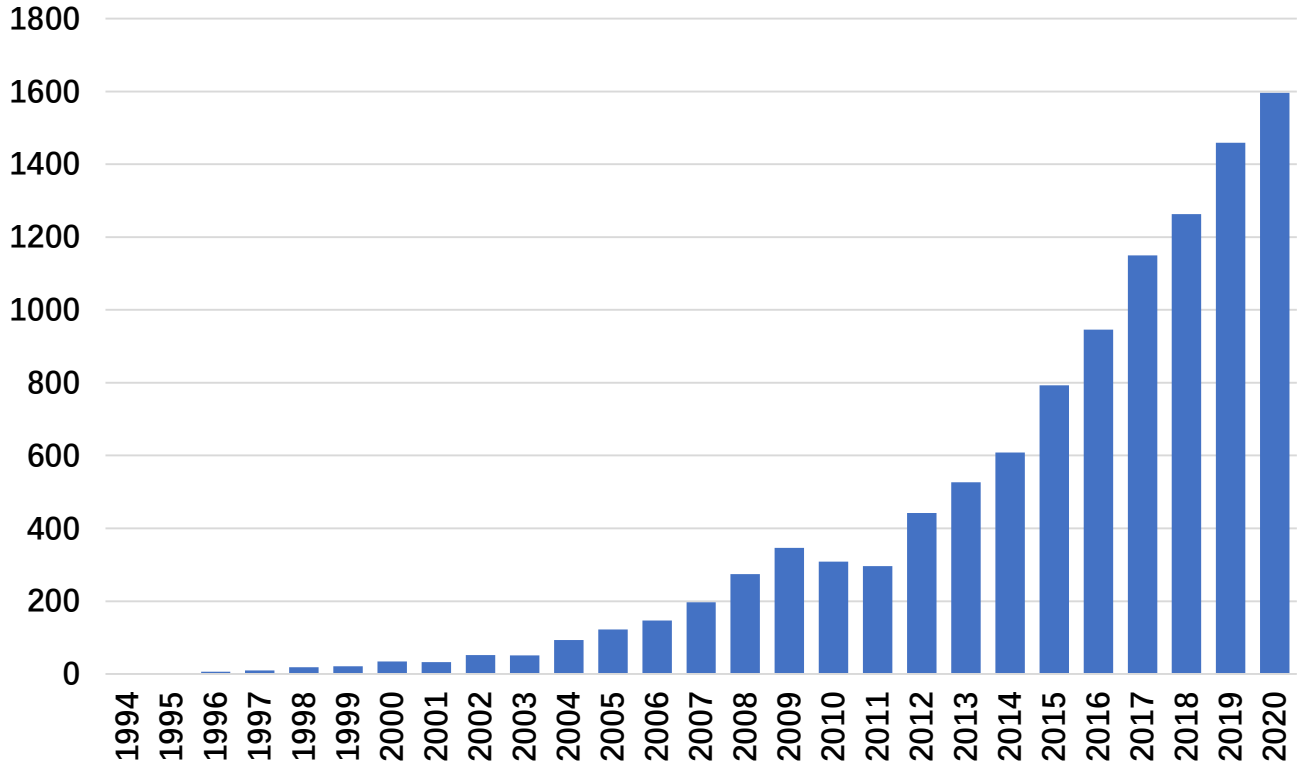




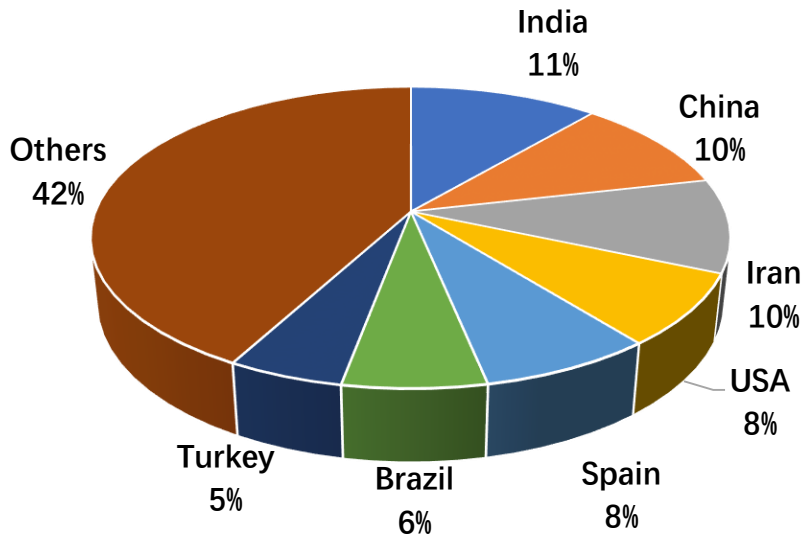
Search Topic "Heuristic" on Web of Science Core Collection

- http://apps.webofknowledge.com/WOS_GeneralSearch_input.do?product=WOS&search_mode=GeneralSearch&SID=C2BORLnb5dsieTRqkbq&preferencesSaved=

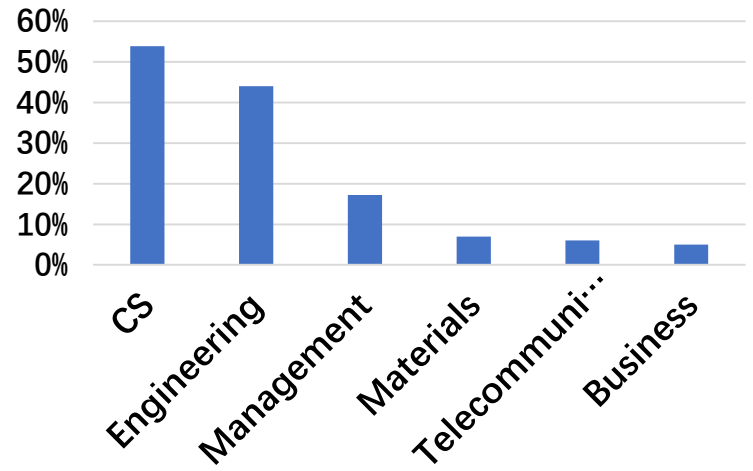
NO. of Publications on "Metaheuristic"



Countries



Fields



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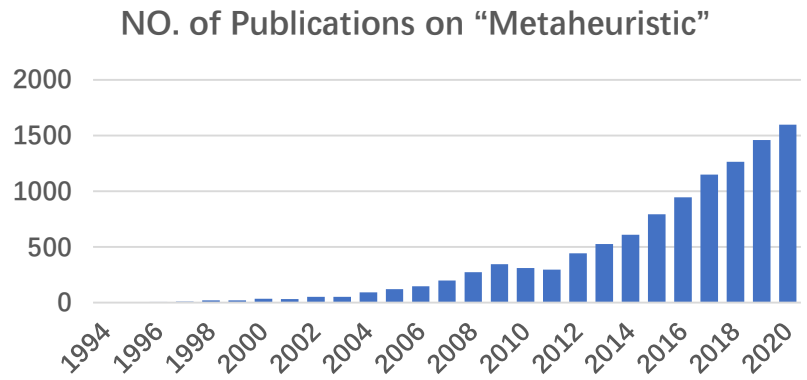
- http://apps.webofknowledge.com/WOS_GeneralSearch_input.do?product=WOS&search_mode=GeneralSearch&SID=C2BORLnb5dsieTRqkbq&preferencesSaved=



- **The pre-theoretical period (until 1940)** not formally studied.
- **The early period (1940-1980)** the formal studies appear.
- **The method-centric period (1980-2000)** truly takes on
- **The framework-centric period (2000-now)** insight grows
- **The scientific period (the future)**

time

1950 | 1960 | 1970 | 1980 | 2000 | 2021

- **Pattern Search**
- **…**

- **Evolutionary Programming**
- **Simplex method**
- **Evolution strategies**
- **Random search**
- **…**

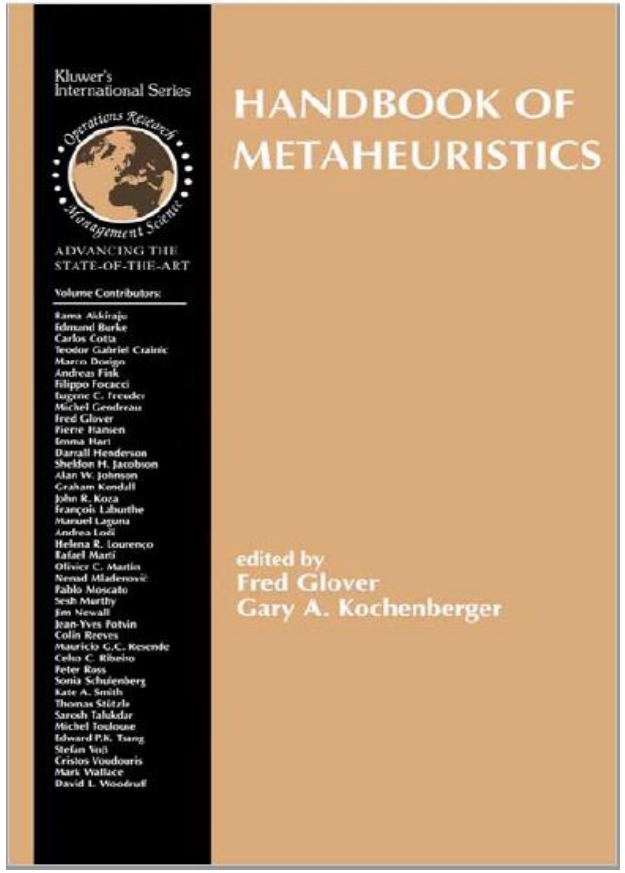
- **Tuning control parameters**
- **Scatter search**
- **Genetic algorithms**
- **Simulated annealing**
- **Genetic programming**
- **Tabu search**
- **Artificial immune systems**
- **…**

- **Differential evolution**
- **CMA-ES & EAD**
- **No free lunch (1997)**
- **Partical swarm optimization**
- **Ant colony optimization**
- **…**

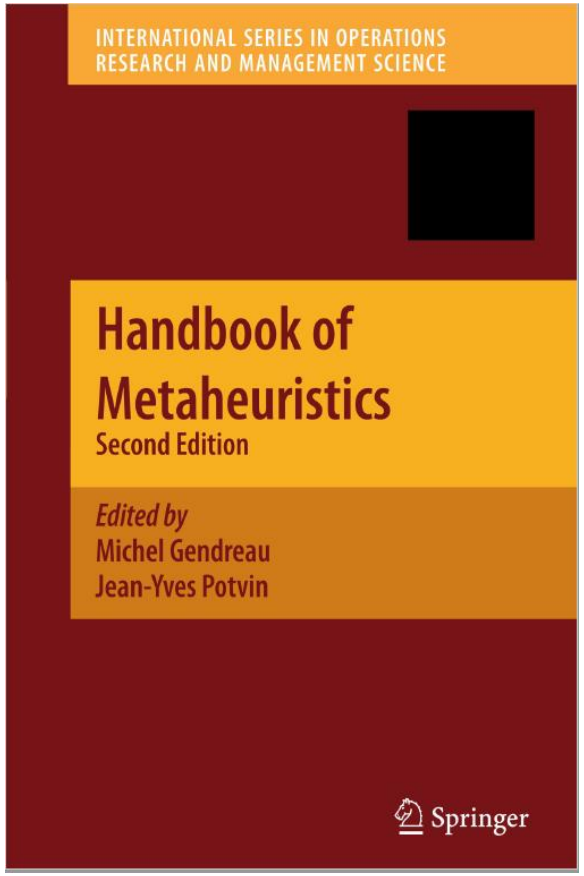
- **…**
- **…**

▪ Sorensen, Kenneth, Marc Sevaux, and Fred Glover. "**A history of metaheuristics**." arXiv preprint arXiv:1704.00853 (2017).

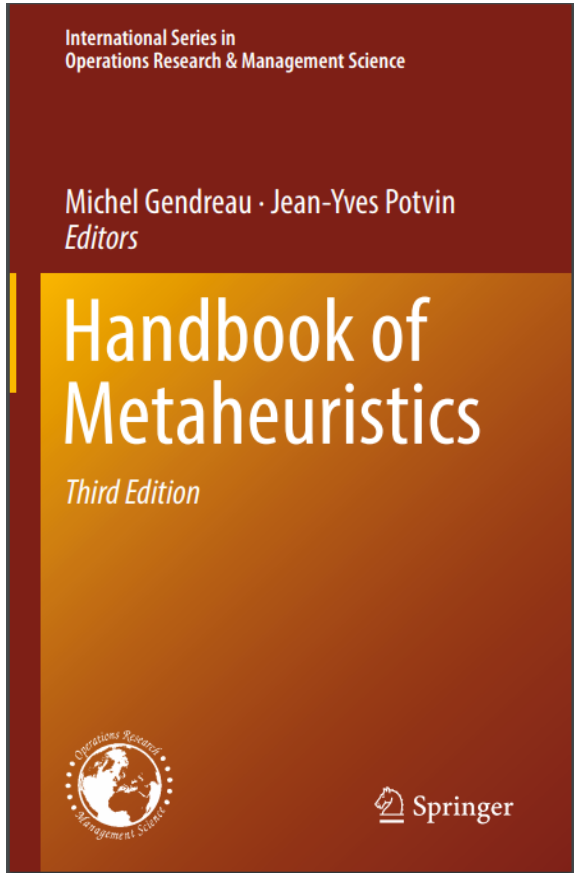
First Edition 2003



Second Edition 2010



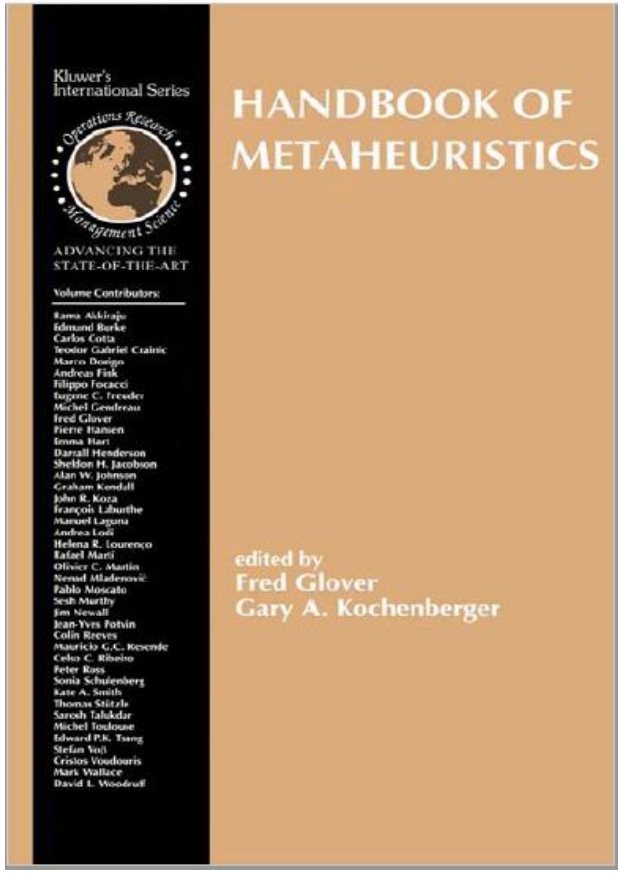
Third Edition 2019



Prof.
Fred W. Glover

Prof.
MICHEL GENDREAU

First Edition 2003



CONTENTS

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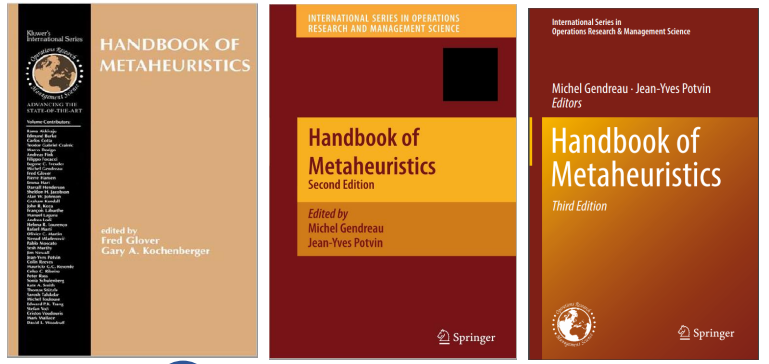
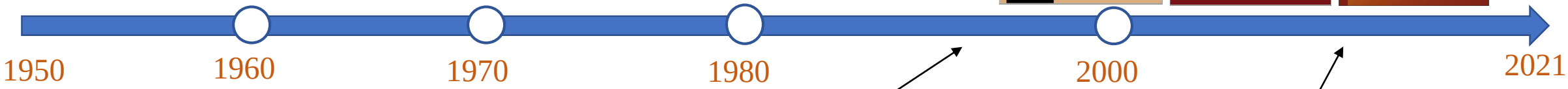
First Edition Second Edition Third Edition



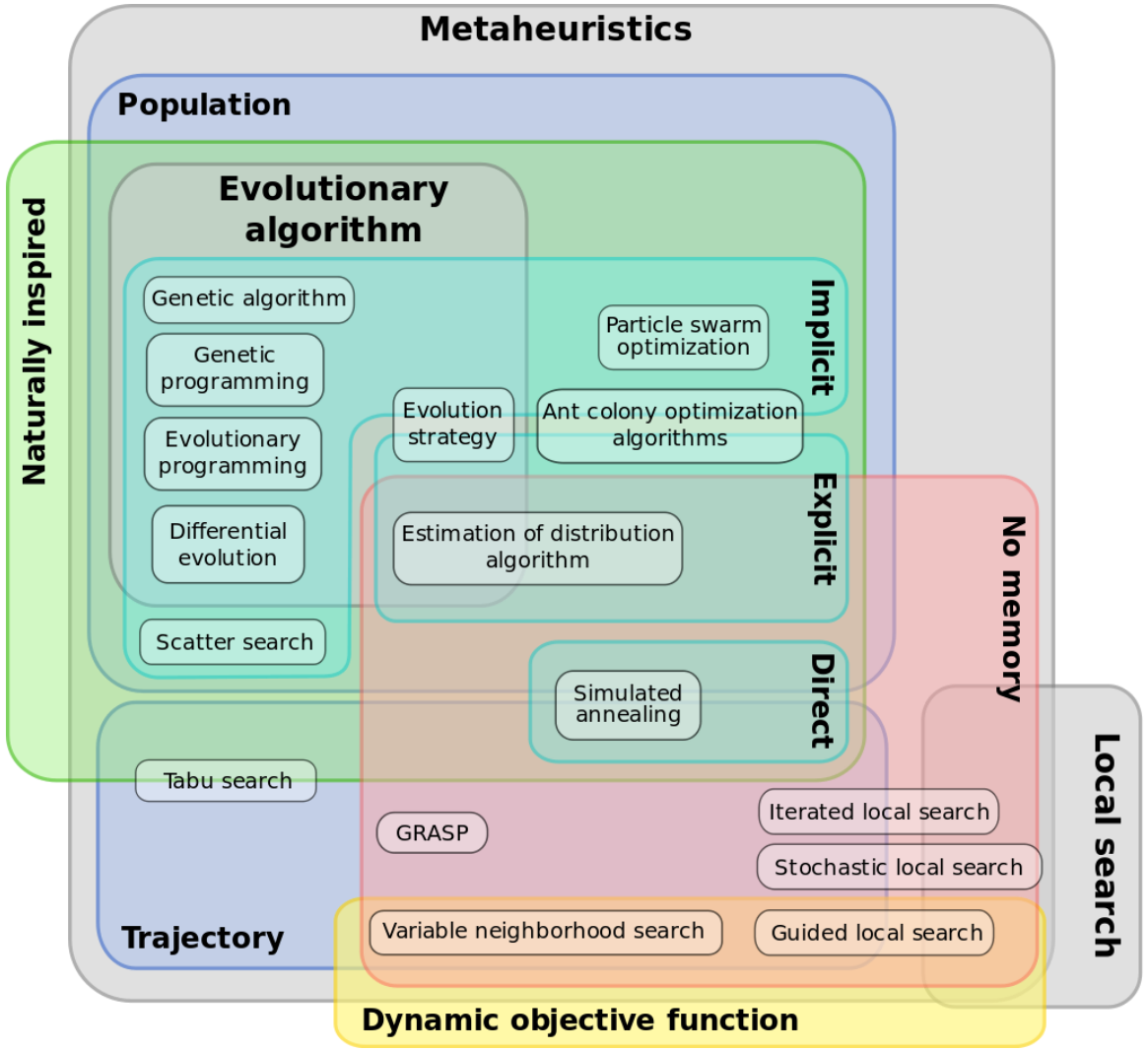


First Edition:

- Tabu search
- Genetic algorithm
- Grasp
- Local Search methods
- Ant Colony Optimization
- …

Second & Third Editions:

- A Classification of **Hyper-heuristic** Approaches: Revisit
- Reactive Search Optimization: **Learning While Optimization**
- **Automated Design** of Metaheuristic Algorithms



- https://en.wikipedia.org/wiki/Metaheuristic#cite_note-nojhan07-8

Classification # 1

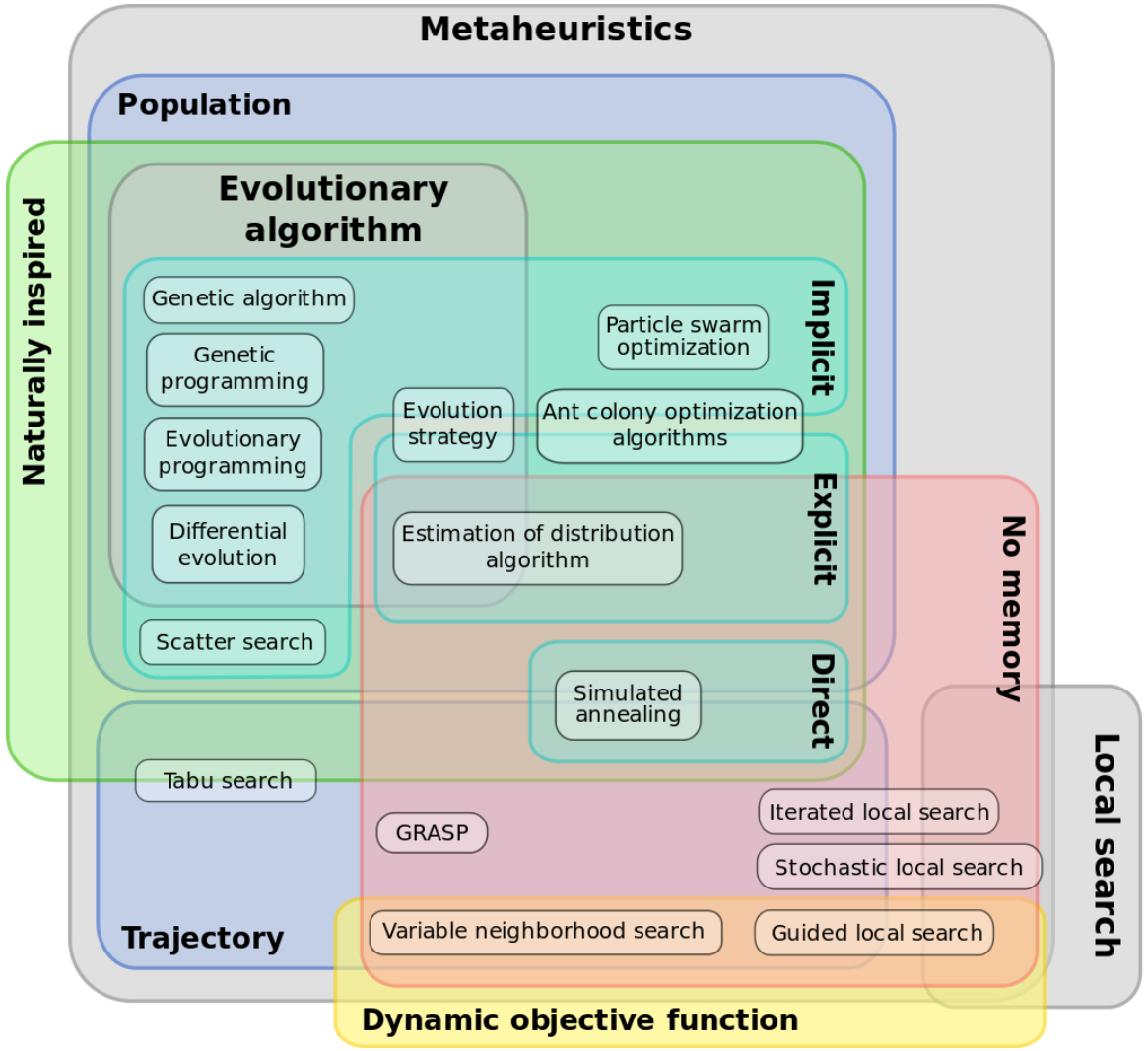
Whether it evolves **one or multiple solutions**

Population-based

Genetic algorithm
Ant colony optimization
Differential evolution
…

Trajectory

Tabu search
Simulated annealing
Iterated local search
…



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Classification # 2

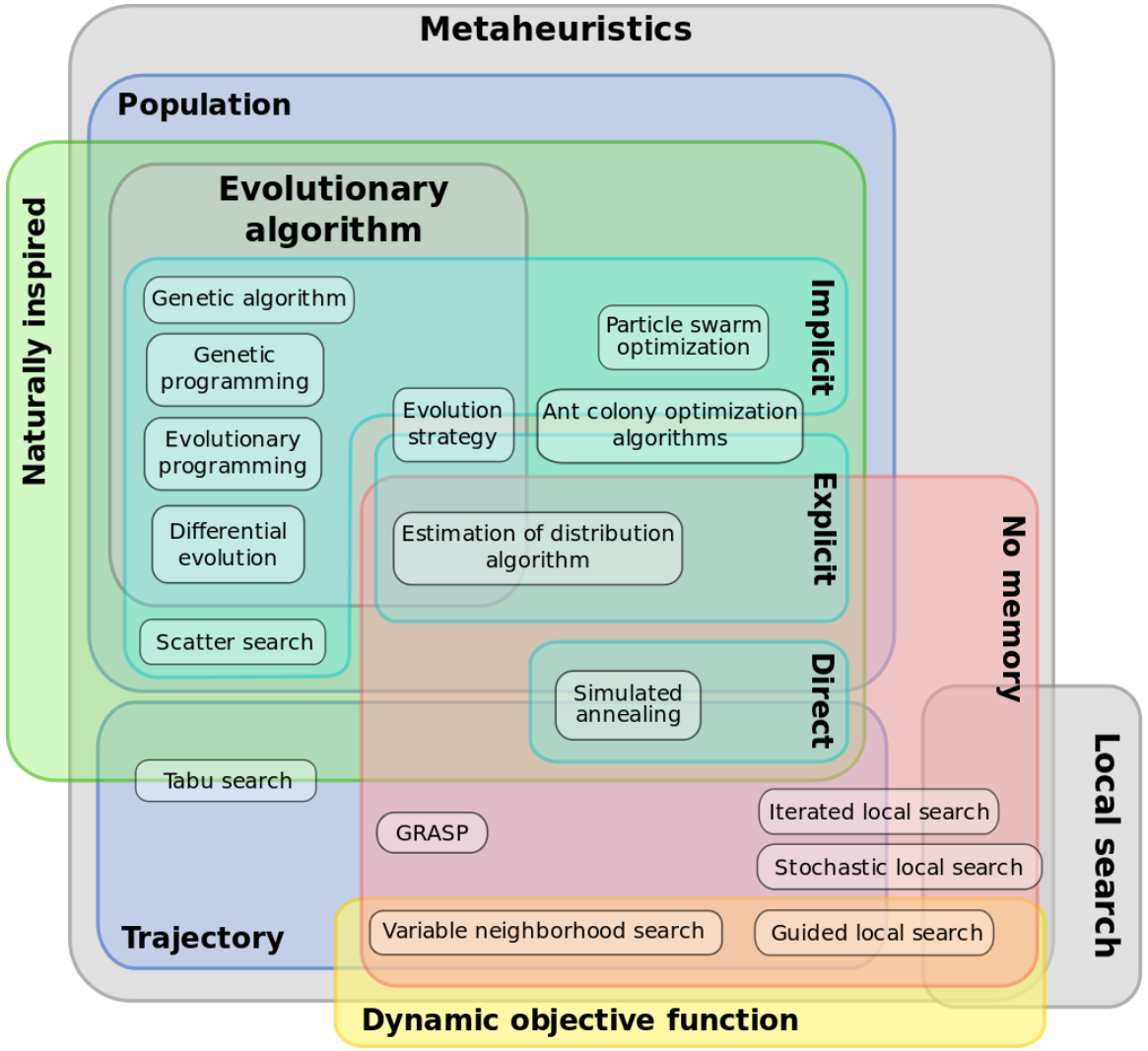
Whether it **remembers the search history**

Using memory

Tabu search
Ant colony optimization
…

Memory-less

Simulated annealing
Grasp
Most local search
…



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Classification # 3

Whether it is **based on inspiration**

Nature

Genetic algorithm
Ant colony optimization
…

Non-nature

Grasp
Most local search
…

Other classifications:

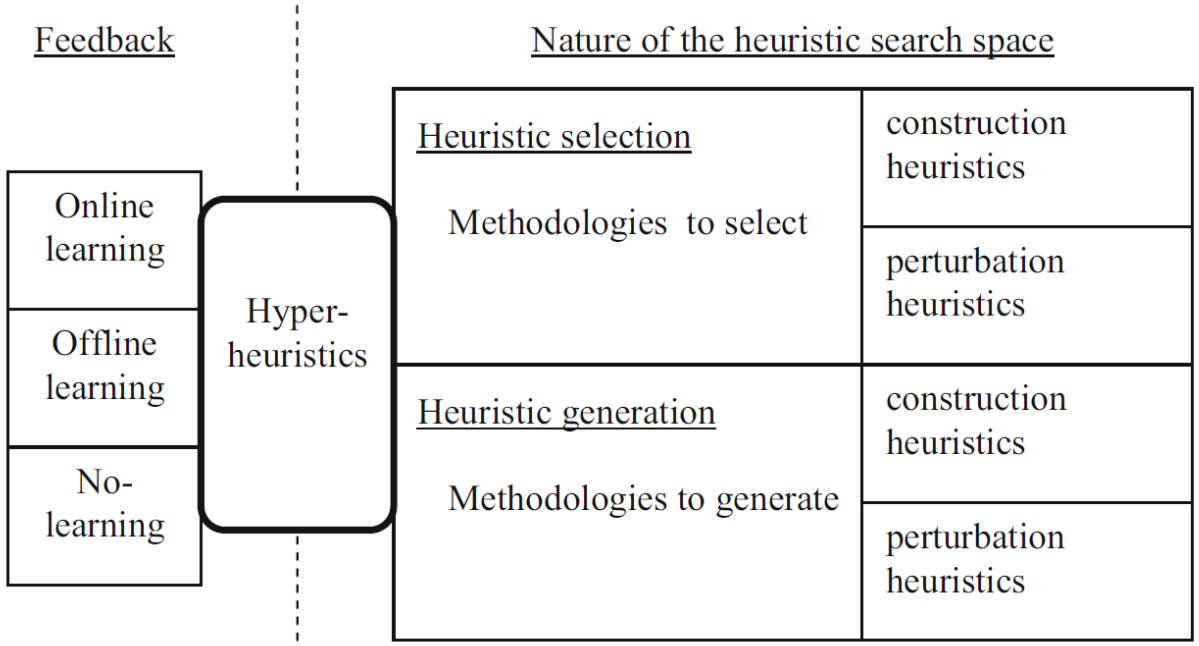
- **Global / Local**
- **One neighbourhood / Multiple neighborhoods**
- **Static objective / Dynamic objective**
- **…**

UNIVERSITY OF LEICESTER

Prof.
Edmund Kieran Burke

- 2003 Hyper-heuristics: **An emerging direction** in modern search technology
- 2010 **A classification** of hyper-heuristic approaches
- 2013 Hyper-heuristics: **A survey of the state of the art**
- 2019 **A classification** of hyper-heuristic approaches: **revisited**
- 2020 **Recent advances** in selection hyper-heuristics

A hyper-heuristic is an automated methodology for selecting or generating heuristics to solve computational search problems.



- https://le.ac.uk/about/governance-and-management/vc-office/academic-leadership-team/deputy-vc

UNIVERSITY OF TRENTO

Prof.
Roberto Battiti

- 1996 Reactive search: Toward self-tuning heuristics
- 2008 Reactive search and intelligent optimization
- 2010 Reactive search optimization: learning while optimizing
- 2019 Reactive search optimization: learning while optimizing

Reactive Search Optimization has to do with *learning for optimizing*

Reactive Neighborhood	Reactive Annealing
Reactive Objective	Reactive Population

Examples:

- Variable neighborhood descent
- Non-monotonic cooling schedules

- https://webapps.unitn.it/du/en/Persona/PER0004438/Didattica
- https://intelligent-optimization.org/

ULB UNIVERSITÉ LIBRE DE BRUXELLES

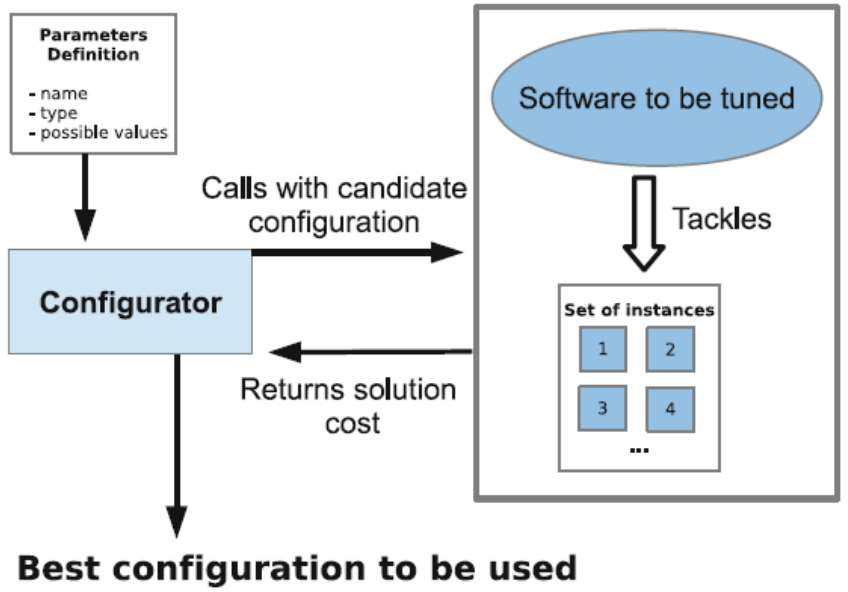
Prof.
Thomas Stützle

- 2009 ParamILS: an automatic algorithm configuration framework
- 2015 Automated algorithm configuration: advances and prospects
- 2018 Automated design of metaheuristic
- 2020 Automated algorithm configuration and design

- https://www.ulb.be/fr/thomas-stutzle
- https://iridia.ulb.ac.be/~stuetzle/

Example:

- Automatic algorithm configuration

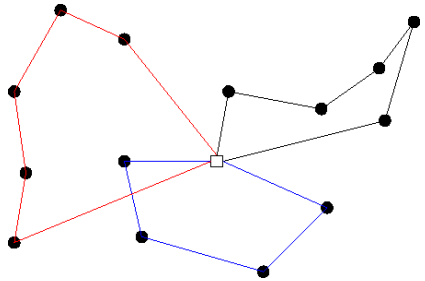


Conclusion

1. Time line A: Algorithms for VRP
2. Time line B: Metaheuristics
3. Time line C: Handbook of Metaheuristics

Conclusion

Time line A: Algorithms for VRP



(1959)
"Truck Dispatching Problem"

(1964)
"Vehicle Routing Problem"

(1981)
It is an NP-hard problem

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Savings heuristic

sweep algorithm

2. Improvement Heuristics:

K-opt

λ-interchange

3. Exact algorithms:

Branch and Bound

Cutting Plane

Network-flows

Dynamic Programming

4. Metaheuristics:

Tabu search

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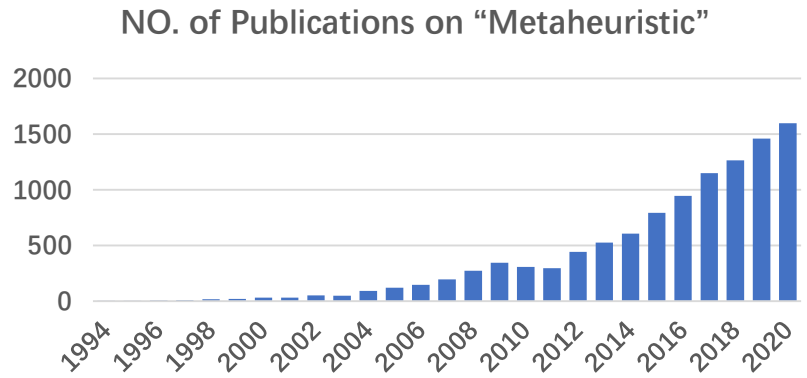
Reinforcement Learning

Conclusion

Time line B: Metaheuristics



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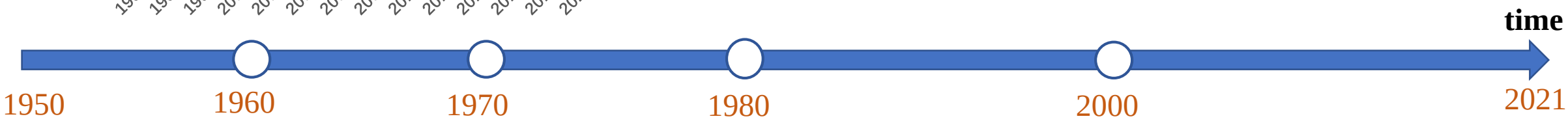

- **Pattern Search**
- **…**

- **Evolutionary Programming**
- **Simplex method**
- **Evolution strategies**
- **Random search**
- **…**

- **Differential evolution**
- **CMA-ES & EAD**
- **No free lunch**
- **Partical swarm optimization**
- **Ant colony optimization**
- **…**

- **Metaheuristic**

Conclusion

Time line C: Handbook of Metaheuristics

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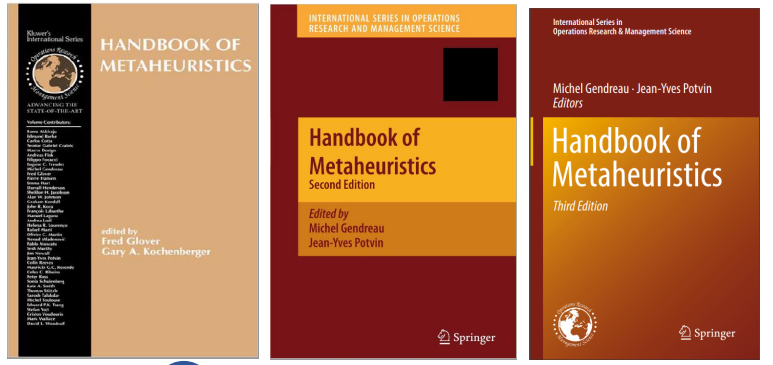
First Edition Second Edition Third Edition



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- Laporte, Gilbert, Paolo Toth, and Daniele Vigo. "**Vehicle routing: historical perspective and recent contributions.**" (2013): 1-4.
- Labadie, Nacima, et al. **Metaheuristics for vehicle routing problems**. ISTE Limited, 2016.
- Sorensen, Kenneth, Marc Sevaux, and Fred Glover. "**A history of metaheuristics**." arXiv preprint arXiv:1704.00853 (2017).
- Glover, Fred W., and Gary A. Kochenberger, eds. **Handbook of metaheuristics**. Springer Science & Business Media, 2006.
- Gendreau, Michel, and Jean-Yves Potvin, eds. **Handbook of metaheuristics**. New York: Springer, 2010.
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香港城市大學
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Thanks!

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Date: January 19, 2021