

# A Metaheuristic Approach for 3L-SDVRP

---

Fei Liu, Qingling Zhu

Department of Computer Science  
City University of Hong Kong

March 11, 2021

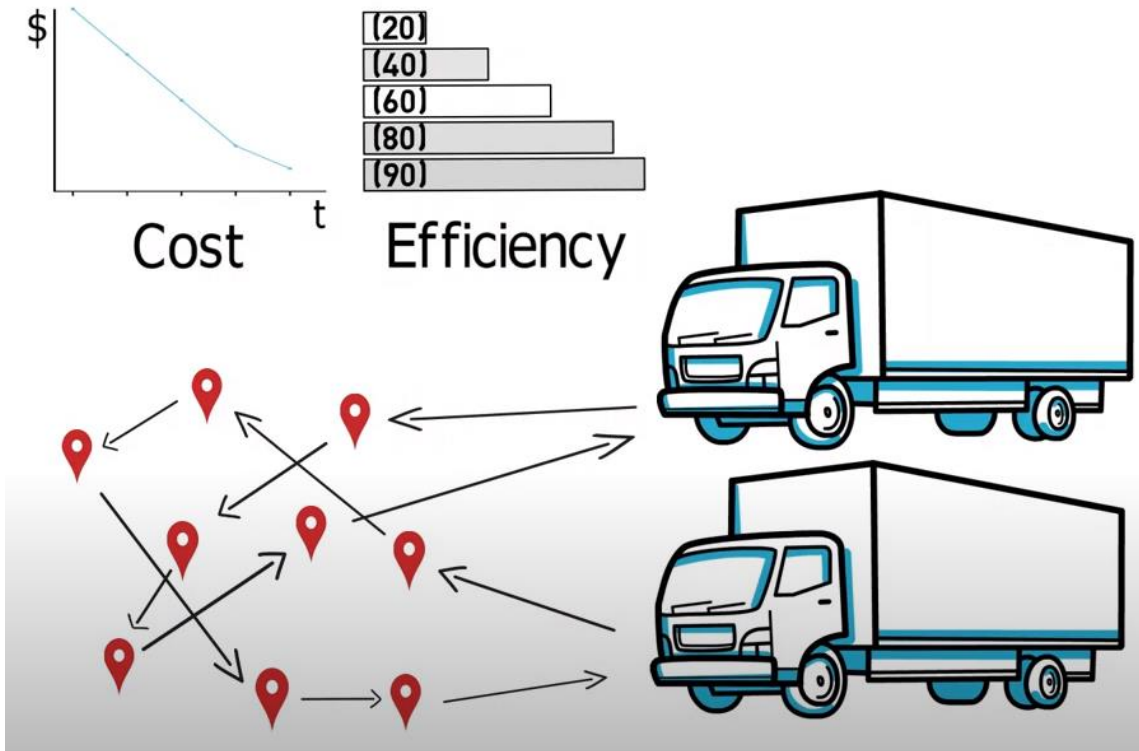
# Outline

1. EMO2021 Competition – Huawei 3L-SDVRP
2. A Metaheuristic Approach for 3L-SDVRP
3. Conclusion

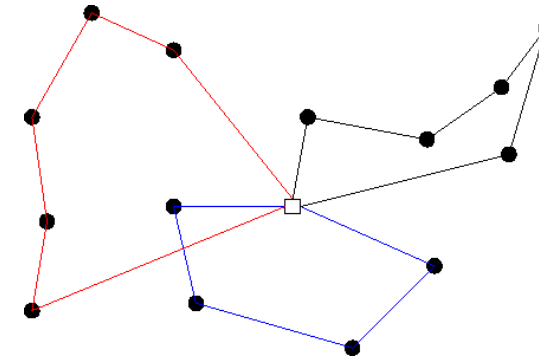
# Outline

- 1. EMO2021 Competition – Huawei 3L-SDVRP**
2. A Metaheuristic Approach for 3L-SDVRP
3. Conclusion

# Vehicle Routing Problem (VRP)



## Vehicle Routing Problem (VRP)

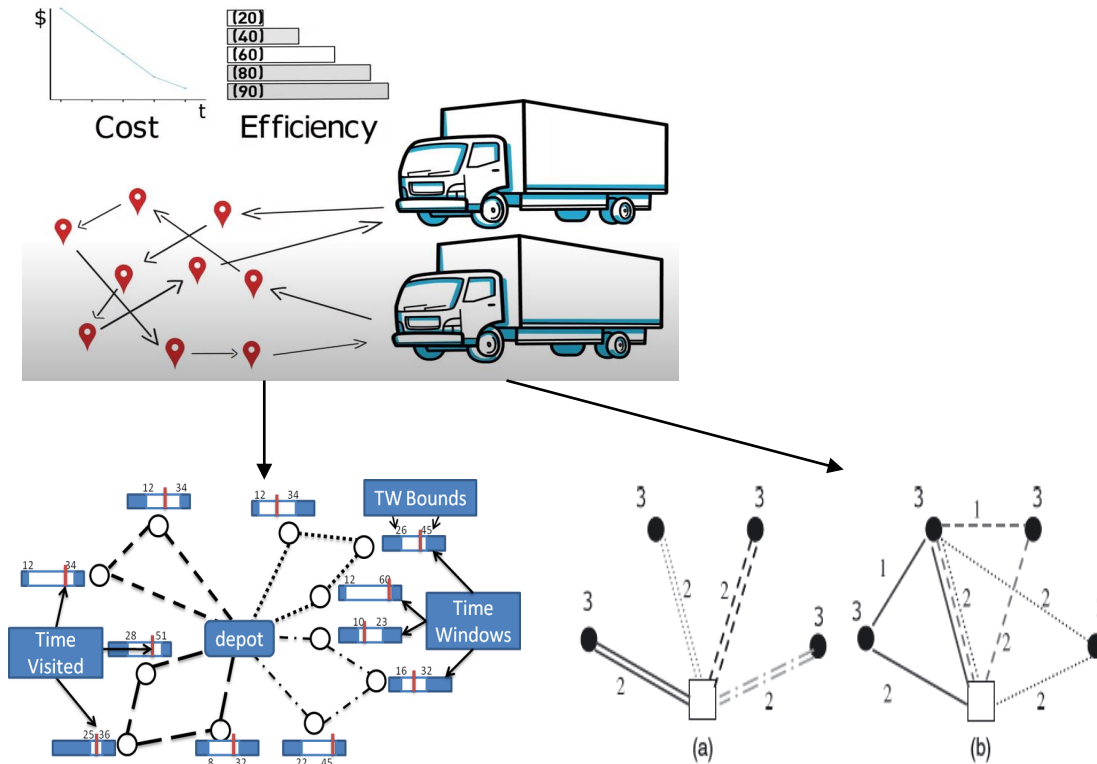


### Four components

- Network
- Sites to be visited (customers to serve, tasks to process, etc.)
- Fleet of vehicles
- Depot(s)

• <https://www.youtube.com/watch?v=OKMssWdC0I0>

# Variants of VRP



## Objectives:

Cost & Efficiency

## Constraints:

Limited capacities

Many depots

Customer demand

Time windows

Precedence and synchronization

...

## Typical VRPs:

Capacitated Vehicle Routing Problem (CVRP)

Vehicle Routing Problem with Backhauls (VRPB)

Vehicle Routing Problem with Split Deliveries (VRPSD)

Vehicle Routing Problem with Multiple Depot (VRPMD)

Vehicle Routing Problem with Time Windows (VRPTW)

...

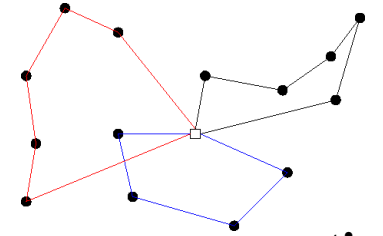
- <https://www.youtube.com/watch?v=OKMssWdC0I0>

# Methods for VRP

Dantzig and Ramser (1959)  
the first to introduce “**Truck Dispatching Problem**”

Clarke and Wright (1964)  
generalized this problem to a  
linear optimization problem  
“**Vehicle Routing Problem**”

Lenstra and Rinnooy Kan (1981)  
proved VRP is an **NP-hard problem**, exact algorithms are only  
efficient for small problems



## 1. Constructive Heuristics:

Savings heuristic  
Sweep algorithm

## 2. Improvement Heuristics:

K-opt  
 $\lambda$ -interchange

## 3. Exact algorithms:

Branch and Bound  
Cutting Plane  
Network-flows  
Dynamic Programming

## 4. Metaheuristics:

Tabu search  
Simulated Annealing  
Local search methods  
Partical Swarm Optimization  
Ant Colony Algorithm

Genetic Algorithm  
GRASP

## 5. Machine Learning:

Reinforcement Learning  
Pointer network

Perhaps the most famous heuristic of  
this category is the Clarke and Wright  
(1964) savings heuristic

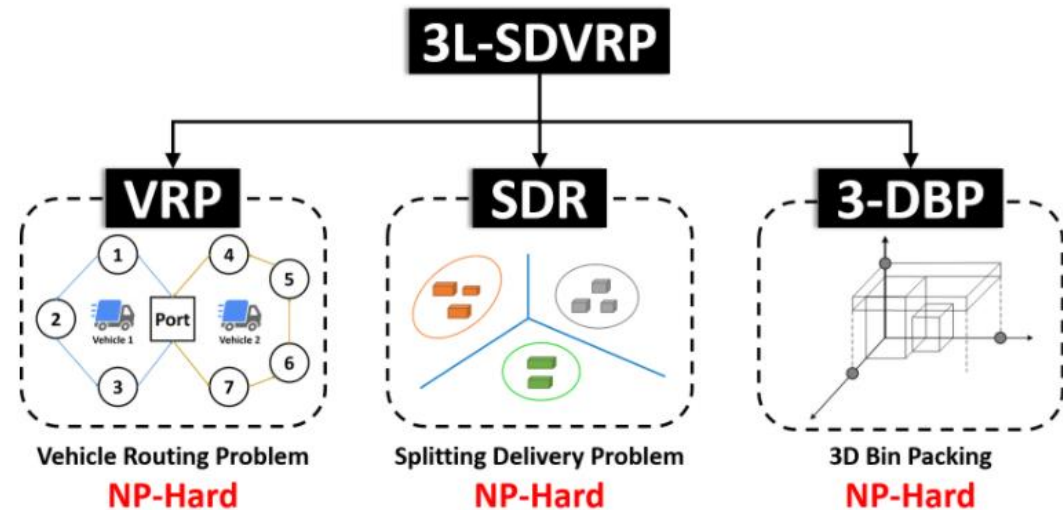
The development of exact algorithms for  
the VRP took off in 1981 with the  
publication of two papers by Christofides

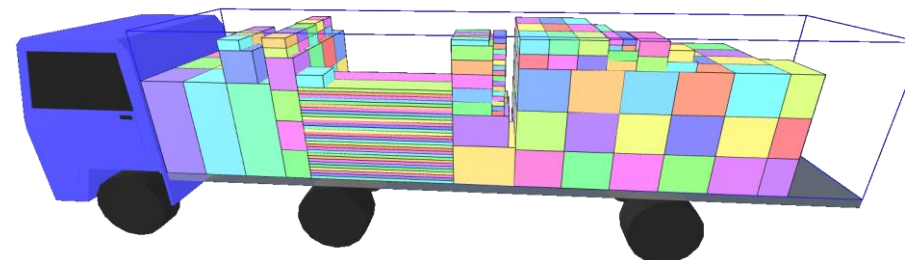
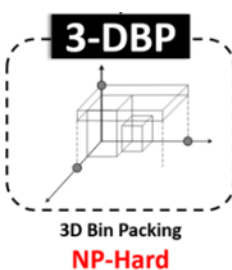
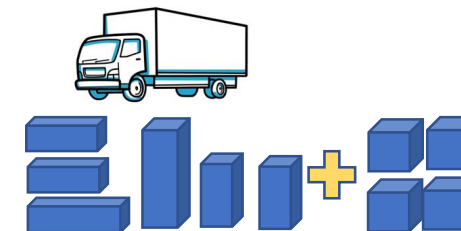
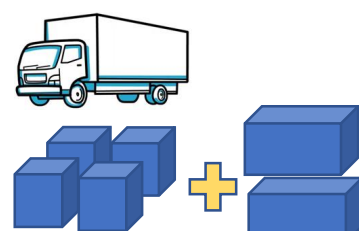
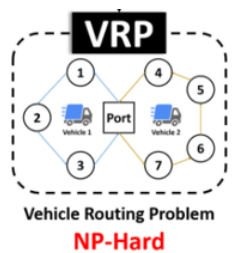
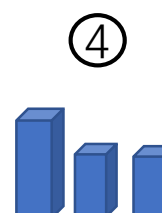
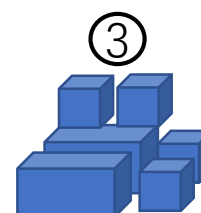
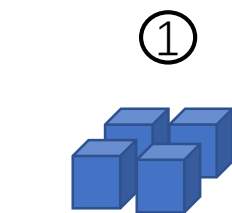
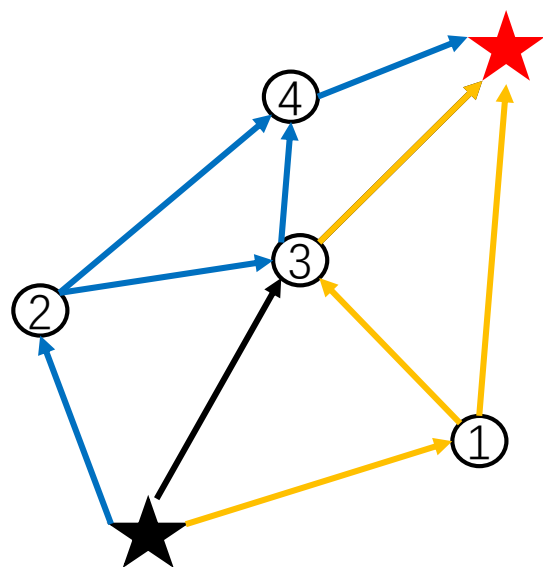
The development of modern heuristics for  
the VRP really started in the 1990s with the  
advent of metaheuristics.

## EMO2021 Competition – Huawei 3L-SDVRP

It involves multiple-pickup-points, single-delivery-point, many types of cargoes, multiple kinds of trucks with containers, and various routes to the port or airport.

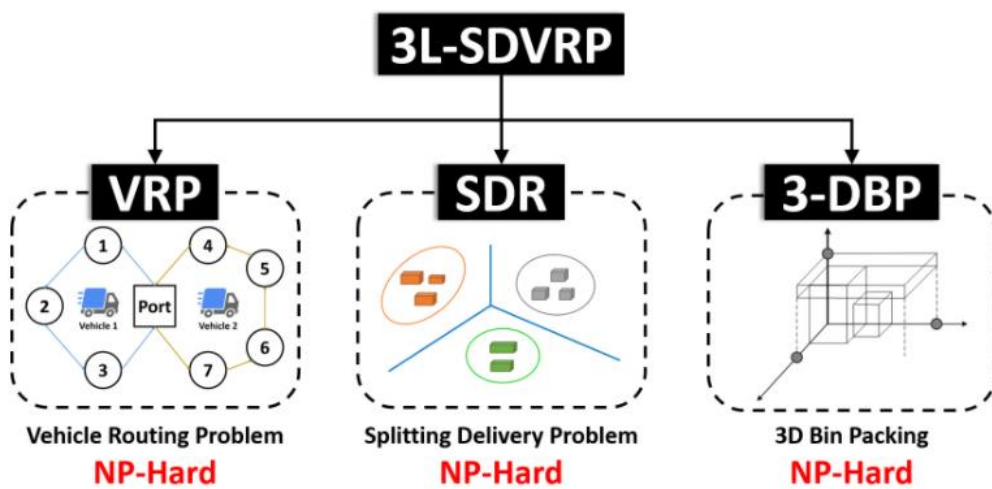
The two objectives of the MOCO problem are to maximize the loading rate as well as to minimize the traveling distance under specific constraints and requirements.







# EMO2021 Competition – Huawei 3L-SDVRP



## Objective functions

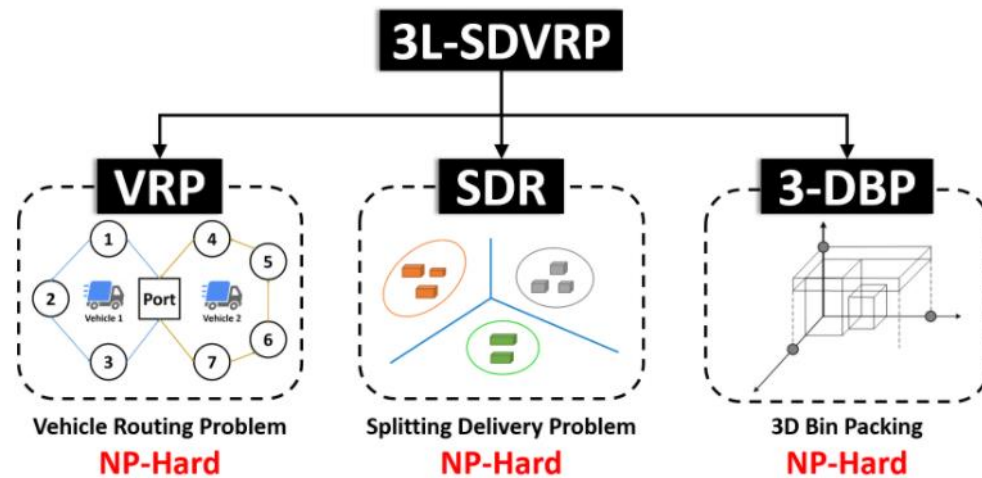
- $f_1 = 1 - \sum_{i=1}^n \text{loading\_rate}_i / n$ ,  
where  $\text{loading\_rate}_i = \max(v\_rate_i, w\_rate_i)$ ,  
 $v\_rate_i$  = the total volume of all the items in truck  $i$  / the size of the truck  $i$ ,  
 $w\_rate_i$  = the total weight of all the items in truck  $i$  / the weight capacity of the truck  $i$ .
- $f_2 = \sum_{i=1}^n \text{truck\_distance}_i$   
where  $\text{truck\_distance}_i$  is the distance traveled by the truck  $i$ .

## Constraints

This problem includes two types of constraints: vehicle routing and 3-D loading,

- valid route constraints: each route must start from the starting point and end at the delivery point
- bonded warehouse constraints: the bonded warehouse is a special pickup point that can only be visited by empty trucks. Hence, if a pickup point is marked as a bonded warehouse, it must be the first of any route that contains it
- single visit constraints: each truck can only visit each pickup point once
- none-overlapping constraints: items cannot overlap in any dimension
- none-splitting constraints: one item can only be loaded in one truck
- supporting constraints: the item must have sufficient supporting surface, i.e., the supporting area should be greater than 80% of the bottom area of the item
- weight capacity constraints: the total weight of the cargoes loaded on one truck cannot exceed the weight capacity of the truck
- size constraints: all the items must be loaded inside the container
- one-time loading constraints: once a truck leaves a pickup point, the items loaded at the pickup point cannot be moved anymore

# Approaches for 3L-SDVRP



The three approaches can hardly solve 3D-SDVRP directly

Exact algorithms

Heuristics

Machine Learning

# Approaches for 3L-SDVRP

Applied Data Science Track Paper KDD 2018, August 19-23, 2018, London, United Kingdom

## A Data-Driven Three-Layer Algorithm for Split Delivery Vehicle Routing Problem with 3D Container Loading Constraint

Xijun Li  
Shanghai Jiao Tong University  
Noah's Ark Lab of Huawei  
lixijun@sjtu.edu.cn

Mingxuan Yuan  
Noah's Ark Lab  
Huawei Technologies  
yuan.mingxuan@huawei.com

Di Chen  
Noah's Ark Lab  
Huawei Technologies  
di.chen@huawei.com

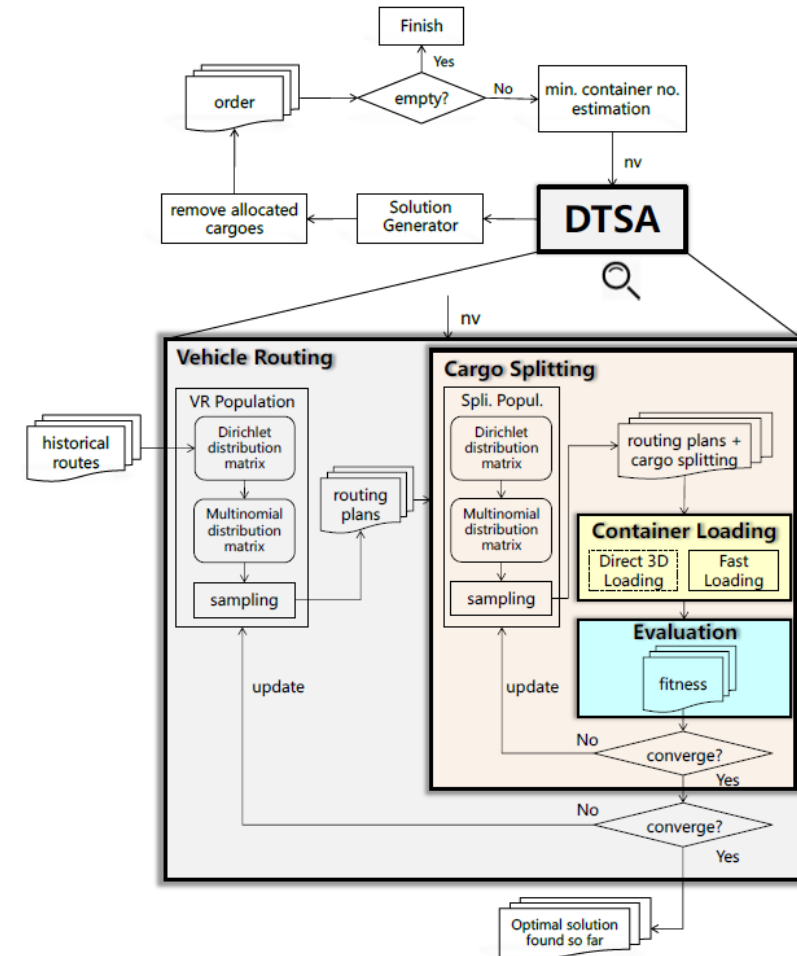
Jianguo Yao  
Shanghai Jiao Tong University  
jianguo.yao@sjtu.edu.cn

Jia Zeng  
Noah's Ark Lab  
Huawei Technologies  
Zeng.Jia@huawei.com

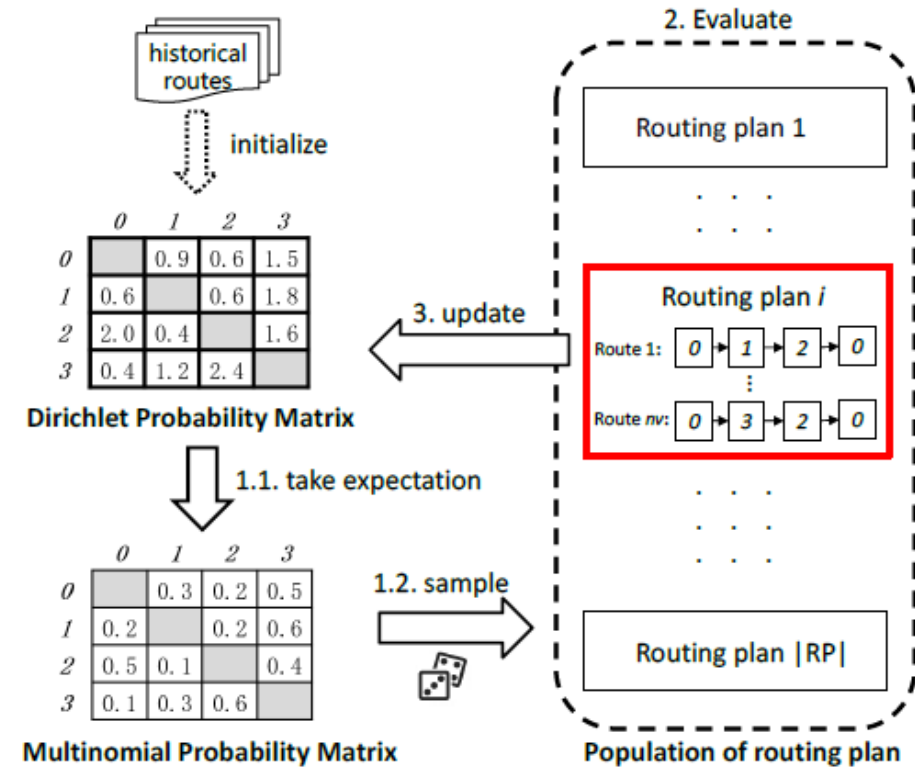
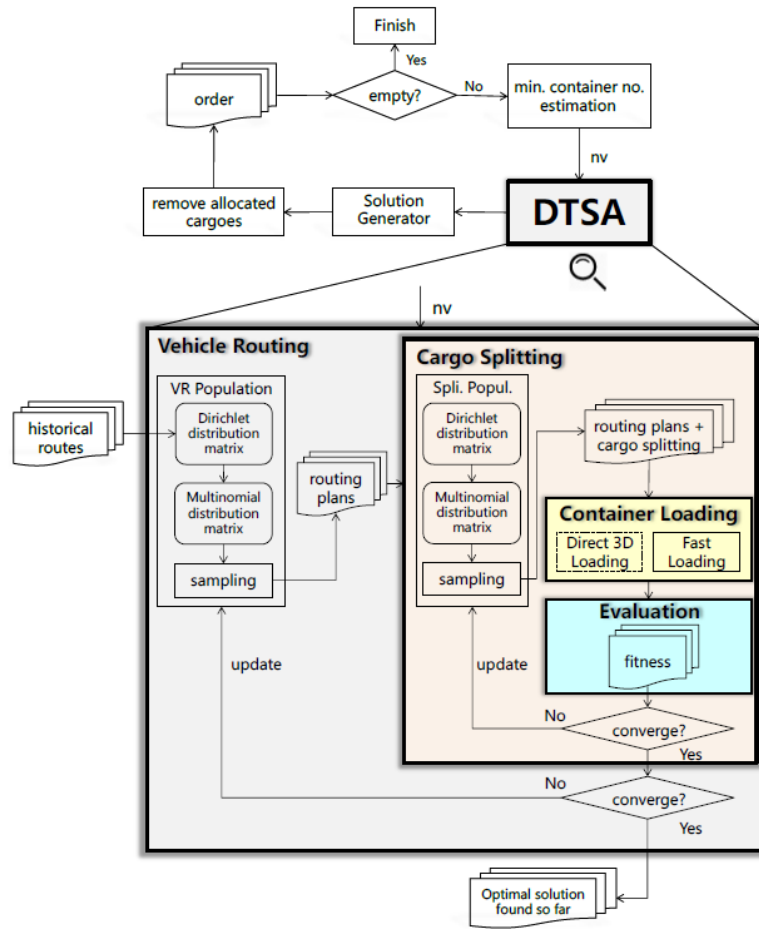
1. First propose 3L-SDVRP  
(Split Delivery Vehicle Routing Problem with 3D Loading Constraints )

2. Three-Layer method

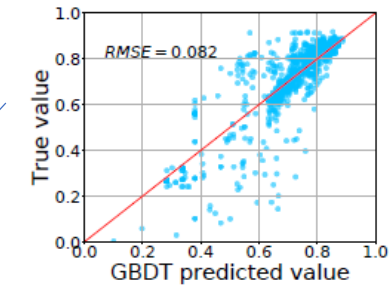
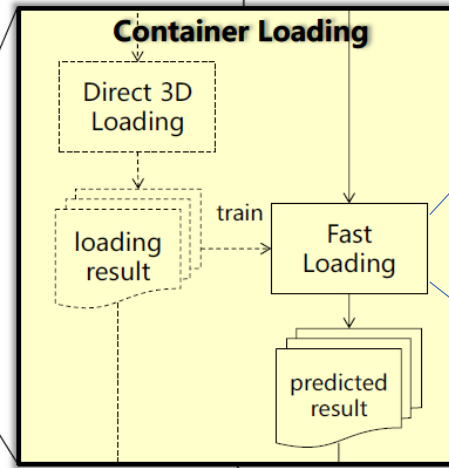
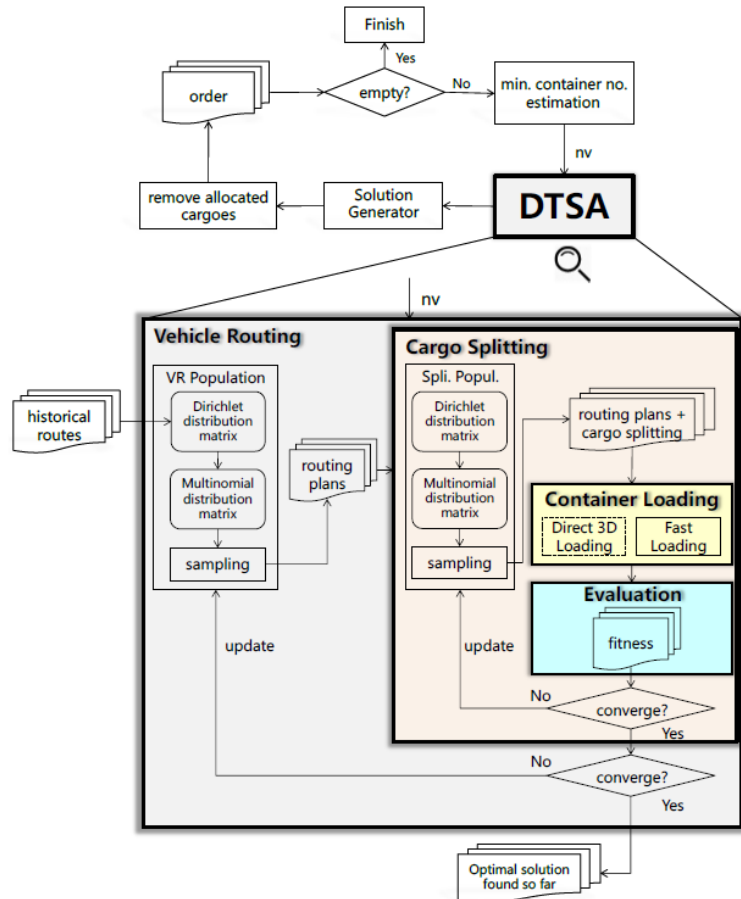
- Out-layer: vehicle routing
- Mid-layer: cargo splitting
- Inner-layer: container loading



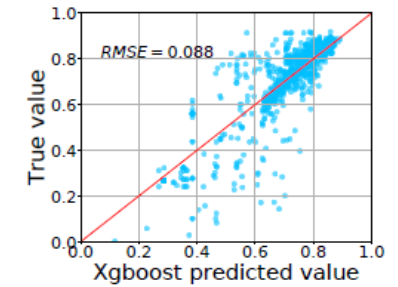
# Approaches for 3L-SDVRP



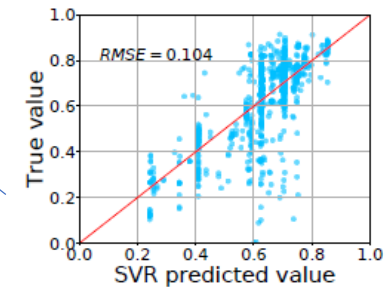
# Approaches for 3L-SDVRP



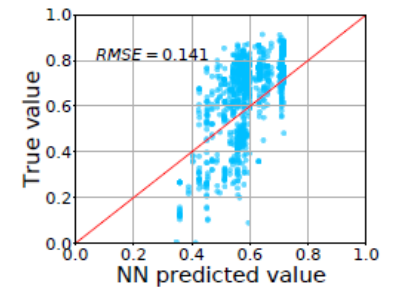
(a) GBDT



(b) Xgboost

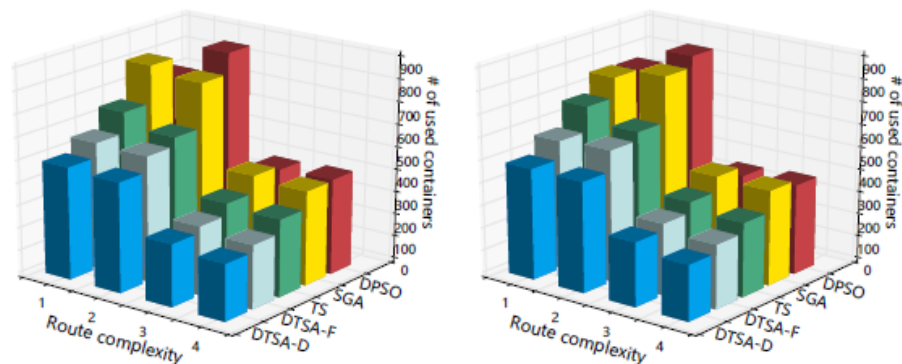


(c) SVR

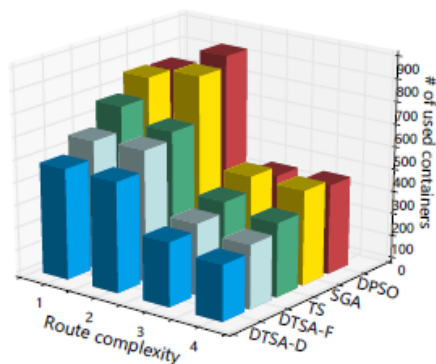


(d) NN

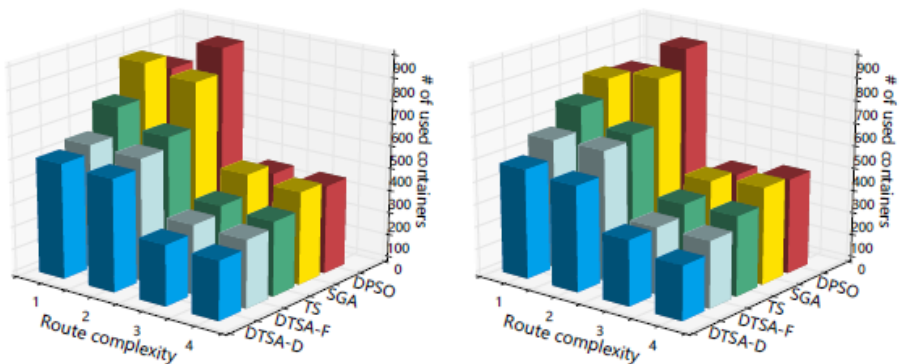
# Approaches for 3L-SDVRP



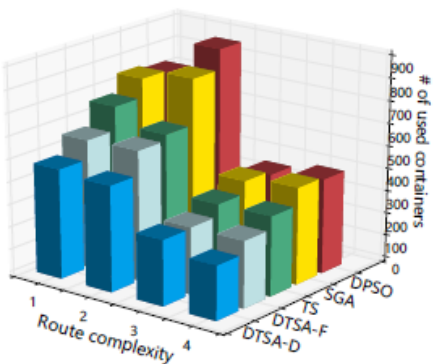
(a) NSGA



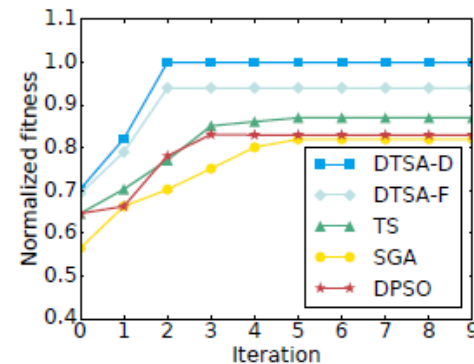
(b) SPEA



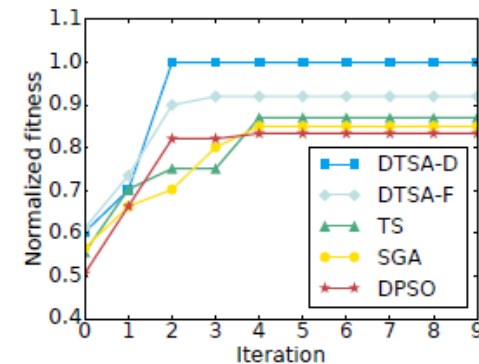
(c)  $I_{\epsilon+}$



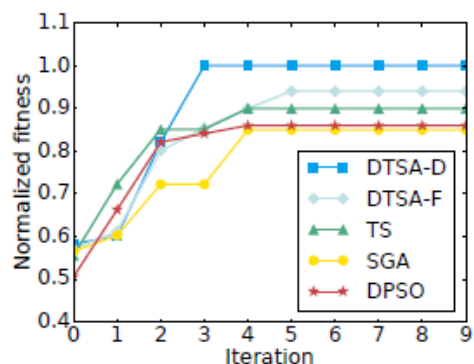
(d) MOEA/D



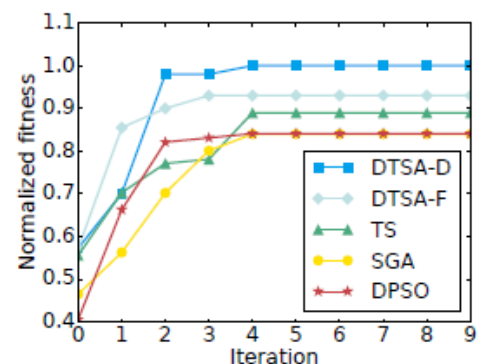
(a) NSGA



(b) SPEA



(c)  $I_{\epsilon+}$



(d) MOEA/D

# Approaches for 3L-SDVRP

European Journal of Operational Research 282 (2020) 545–558



Contents lists available at ScienceDirect  
European Journal of Operational Research  
journal homepage: [www.elsevier.com/locate/ejor](http://www.elsevier.com/locate/ejor)



Production, Manufacturing, Transportation and Logistics

The Split Delivery Vehicle Routing Problem with three-dimensional loading constraints



Andreas Bortfeldt<sup>a</sup>, Junmin Yi<sup>b,\*</sup>

<sup>a</sup> Otto-von-Guericke-University Magdeburg, Germany  
<sup>b</sup> Xiamen University of Technology, China

## 1. Introduce two variance of 3L-SDVRP

- 3L-SDVRP with *forced* splitting  
A delivery is only split if the demand of a customer cannot be transported by a single vehicle
- 3L-SDVRP with *optional* splitting

## 2. Packing first hybrid algorithm

- a local search algorithm for routing
- a genetic algorithm and several construction heuristics for packing.

---

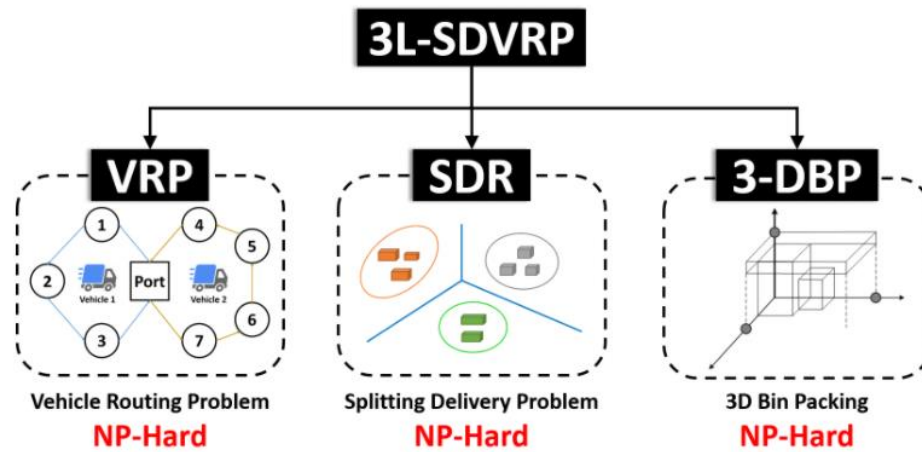
**Algorithm 1** Overview of the hybrid algorithm.

---

```
1: SDVRLH2 (in: problem data, parameters, out: best solution  $s_{\text{best}}$ )  
   // Packing step  
2: for each customer  $i$  do  
3:   generate patterns for customer  $i$  (1C-FLPs where necessary, one 1C-SP)  
   by container loading GA  
4: endfor  
5: for selected customer pairs  $(i, j)$  do  
6:   generate 2C-SP pattern for customer pair  $(i, j)$  by construction  
   heuristics  
7: endfor  
   // Routing step  
8: for each customer  $i$   
9:   generate as many direct trips  $0 \rightarrow i \rightarrow 0$  as 1C-FLPs for customer  $i$  do  
   exist  
10: endfor  
11: if only forced splits allowed then  
12:   solve remaining CVRP by local search  
13: else // optional splits allowed  
14:   solve remaining SDVRP by (modified) local search  
15: endif  
   // Final step  
16: prepare solution  $s_{\text{best}}$  consisting of best achieved routing plan and  
   related 3D packing patterns  
17: end.
```

---





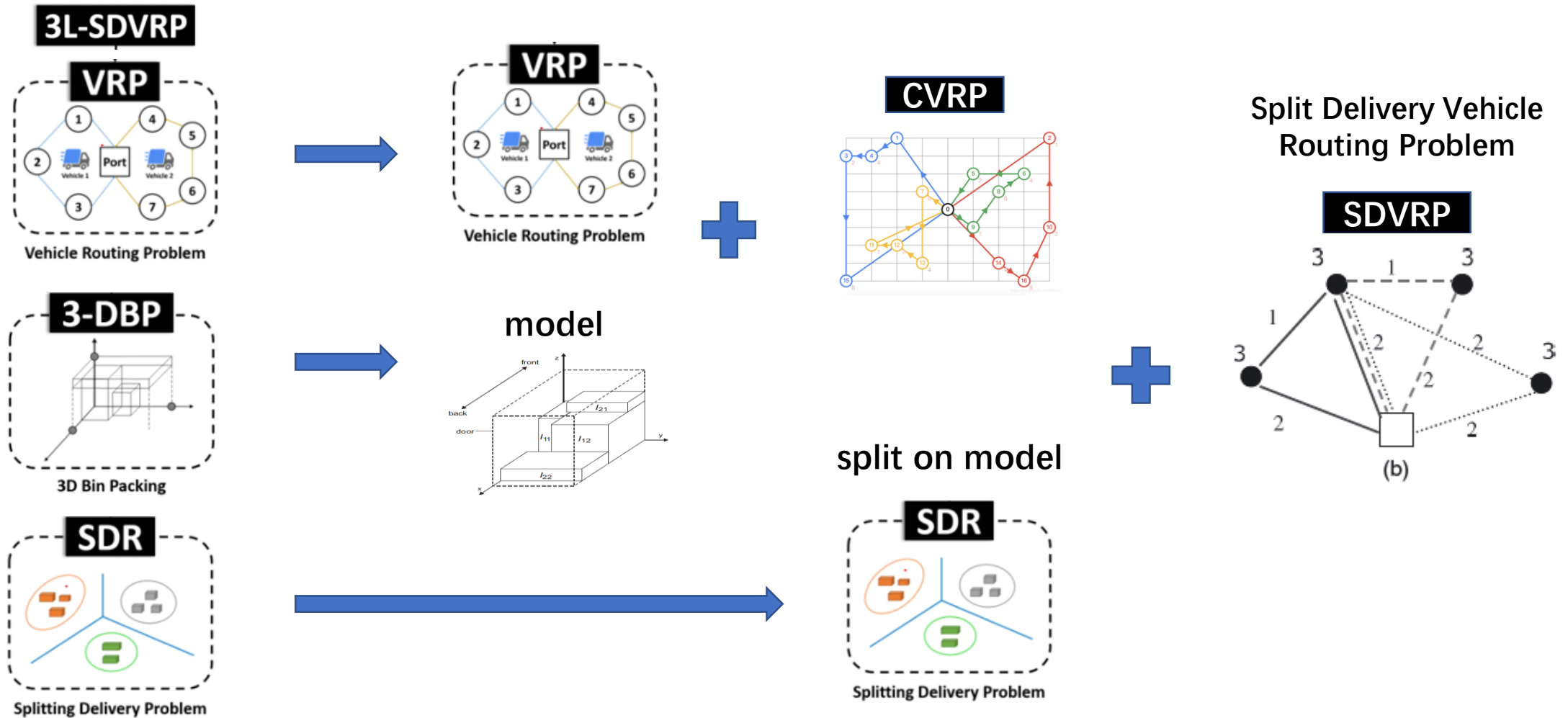
1. Proposed in 2018
2. Complex and time-consuming
3. There are three approaches:
  - Routing first
  - Packing first
  - Mixed routing & packing



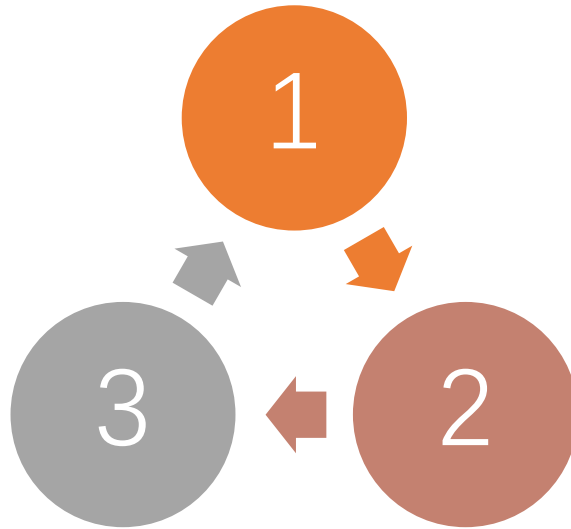
# Outline

1. EMO2021 Competition – Huawei 3L-SDVRP
- 2. A Metaheuristic Approach for 3L-SDVRP**
3. Conclusion

# A Multiobjective Metaheuristic Approach for 3L-SDVRP

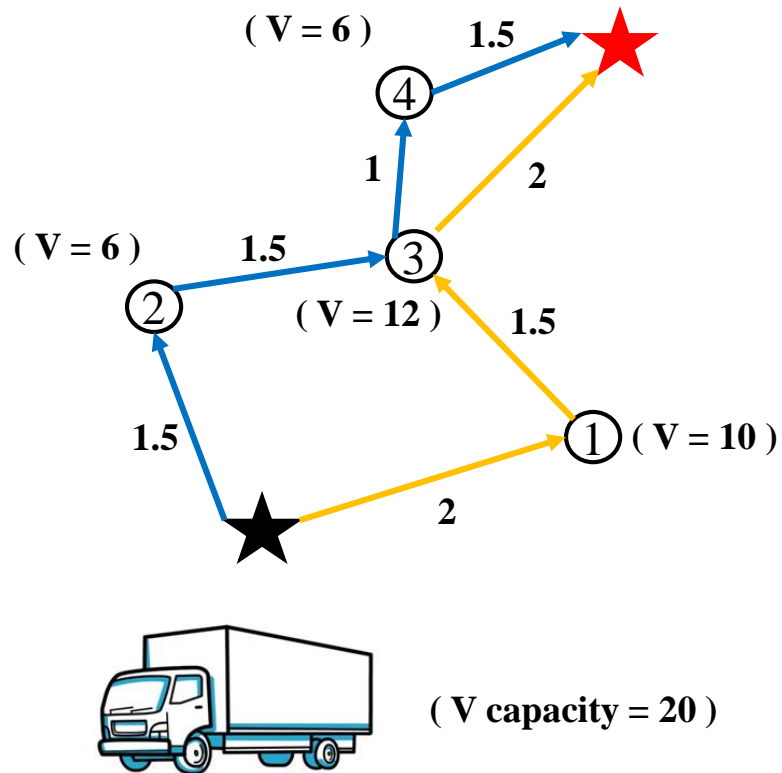


# A Multiobjective Metaheuristic Approach for 3L-SDVRP



1. Objectives
2. Design space
3. Search method

# A Multiobjective Metaheuristic Approach for 3L-SDVRP



## Objectives:

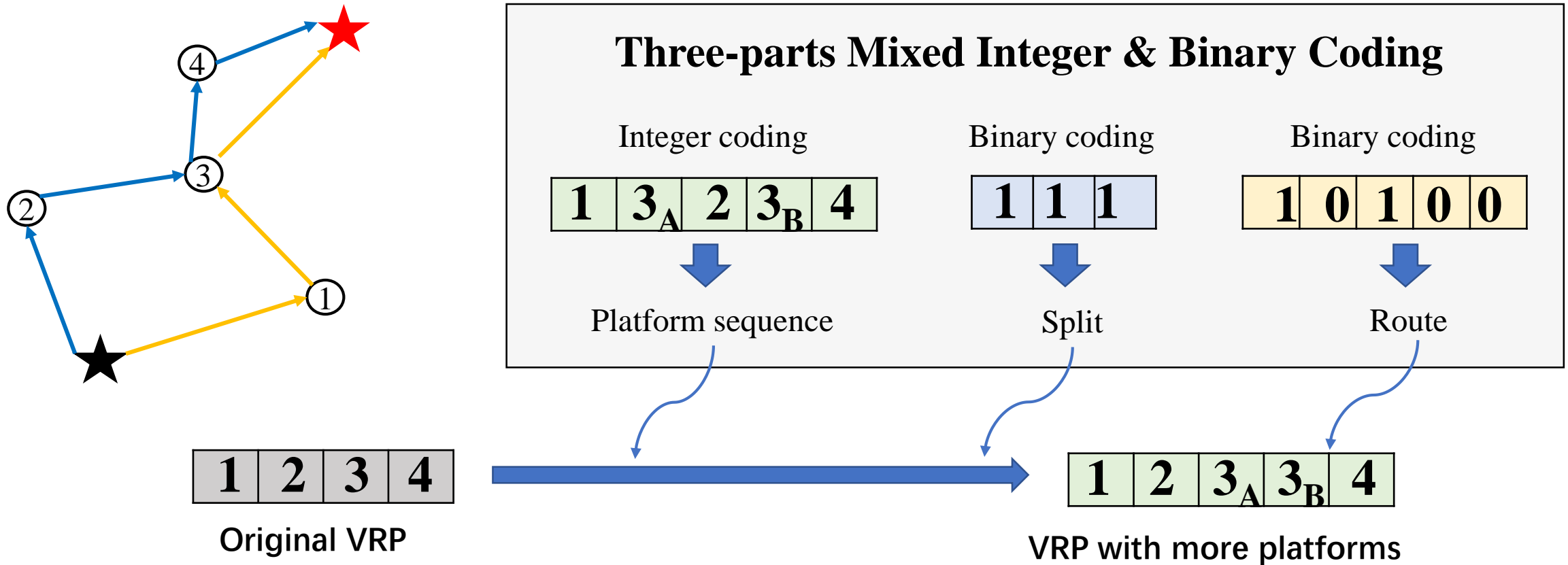
- $f_1 = 1 - \sum_{i=1}^n \text{loading\_rate}_i / n$ ,  
where  $\text{loading\_rate}_i = \max(v\_rate_i, w\_rate_i)$ ,  
 $v\_rate_i$  = the total volume of all the items in truck  $i$  / the size of the truck  $i$ ,  
 $w\_rate_i$  = the total weight of all the items in truck  $i$  / the weight capacity of the truck  $i$ .
- $f_2 = \sum_{i=1}^n \text{truck\_distance}_i$   
where  $\text{truck\_distance}_i$  is the distance traveled by the truck  $i$ .

## Example:

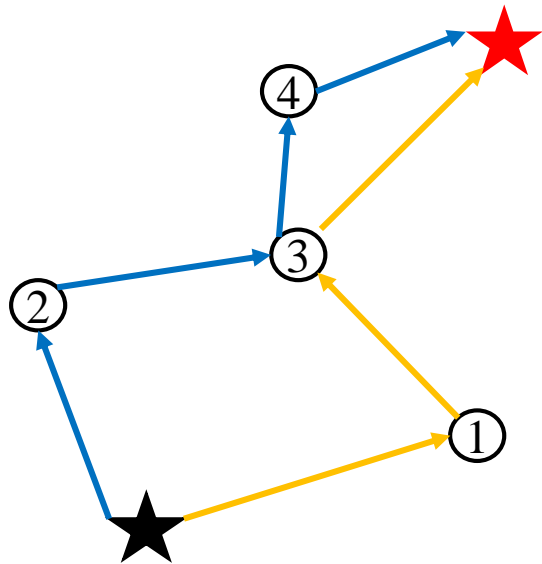
Objective 1 :  $1 - ( (10+6)/20 + (6+6+6)/20 ) / 2 = 0.15$

Objective 2 :  $( 2+1.5+2 ) + ( 1.5+1.5+1+1.5 ) = 11$

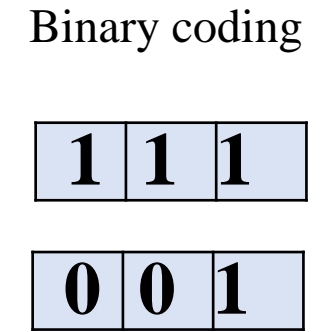
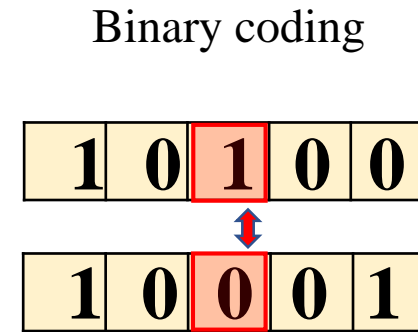
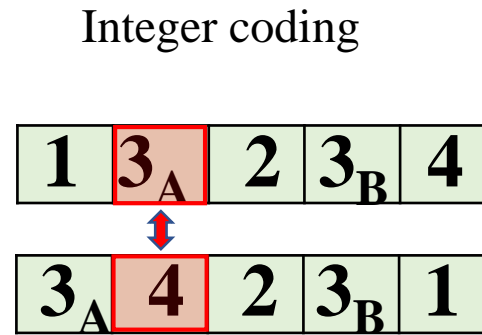
# A Multiobjective Metaheuristic Approach for 3L-SDVRP



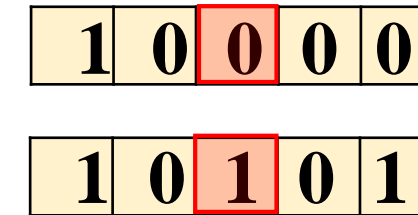
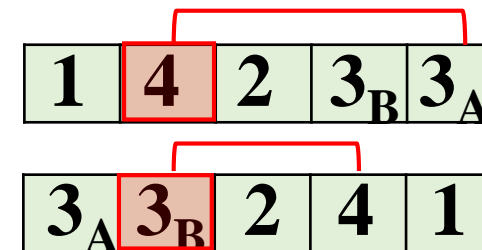
# A Multiobjective Metaheuristic Approach for 3L-SDVRP



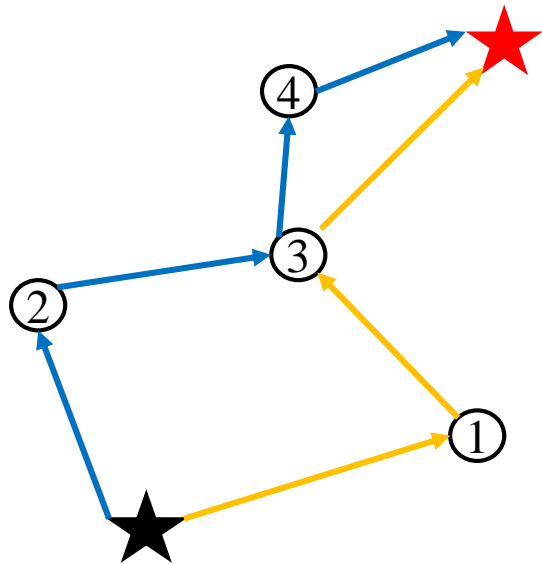
Parents



Offspring



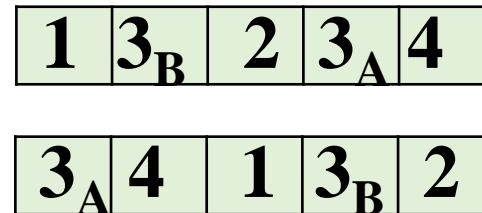
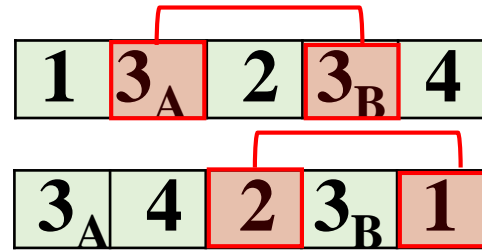
# A Multiobjective Metaheuristic Approach for 3L-SDVRP



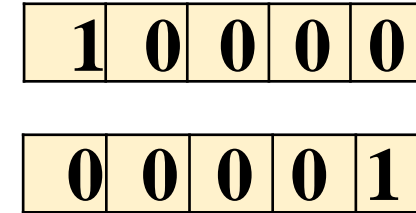
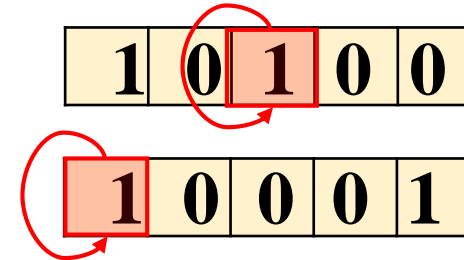
Parents

Offspring

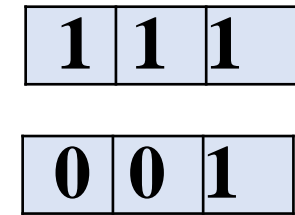
Integer coding



Binary coding

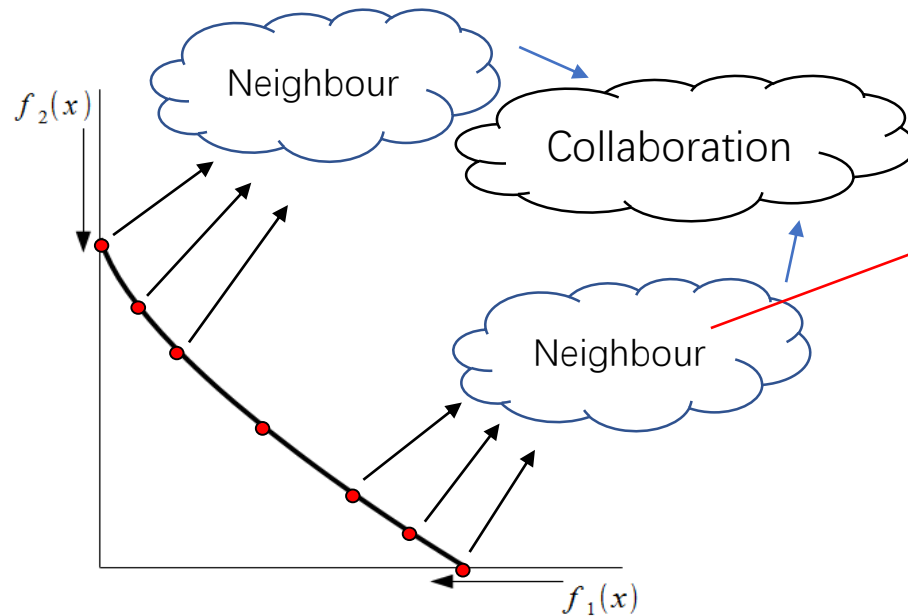


Binary coding



# A Multiobjective Metaheuristic Approach for 3L-SDVRP

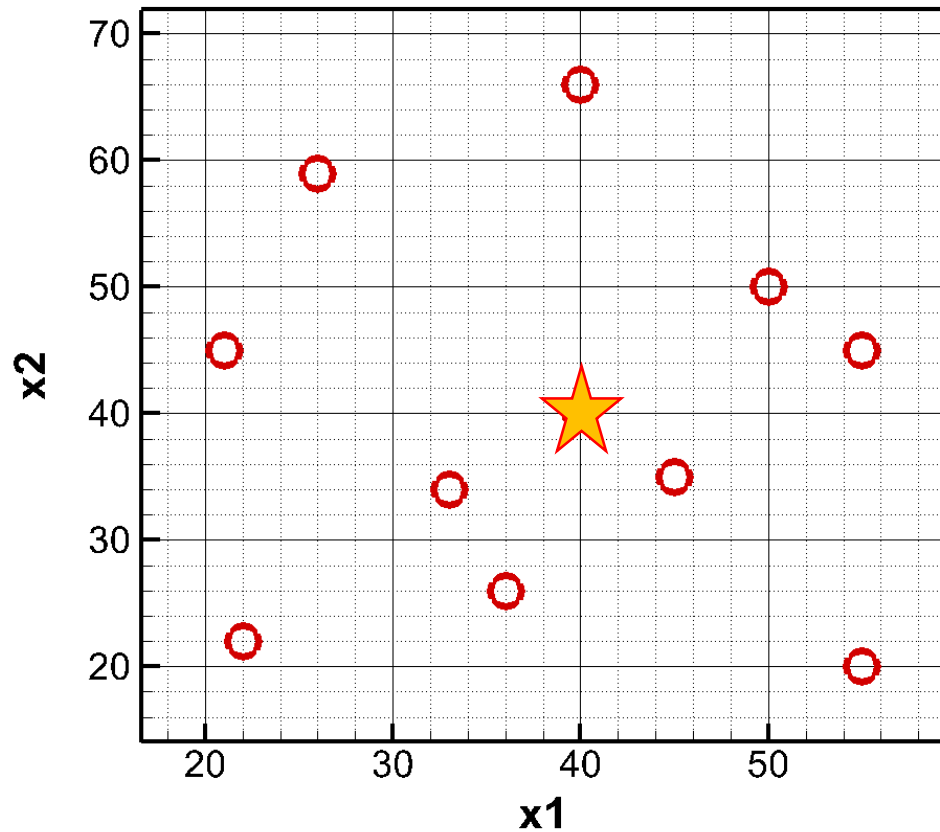
## MOEA/D



- | Integer coding                      | Binary coding | Binary coding |
|-------------------------------------|---------------|---------------|
| 1 3 <sub>A</sub> 2 3 <sub>B</sub> 4 | 1 0 1 0 0     | 1 1 1         |
- Mating select from neighbour with  $ps$
  - Swap mixed integer & binary
  - Mutation mixed integer & binary
  - Replacement replace neighbour with  $pr$



## A simple SDVRP test instance



( capacity = 50 )



( capacity = 60 )

{22, 22},	18
{36, 26},	26
{21, 45},	11
{45, 35},	30
{55, 20},	21
{33, 34},	19
{50, 50},	15
{55, 45},	16
{26, 59},	29
{40, 66},	26
{55, 65},	37

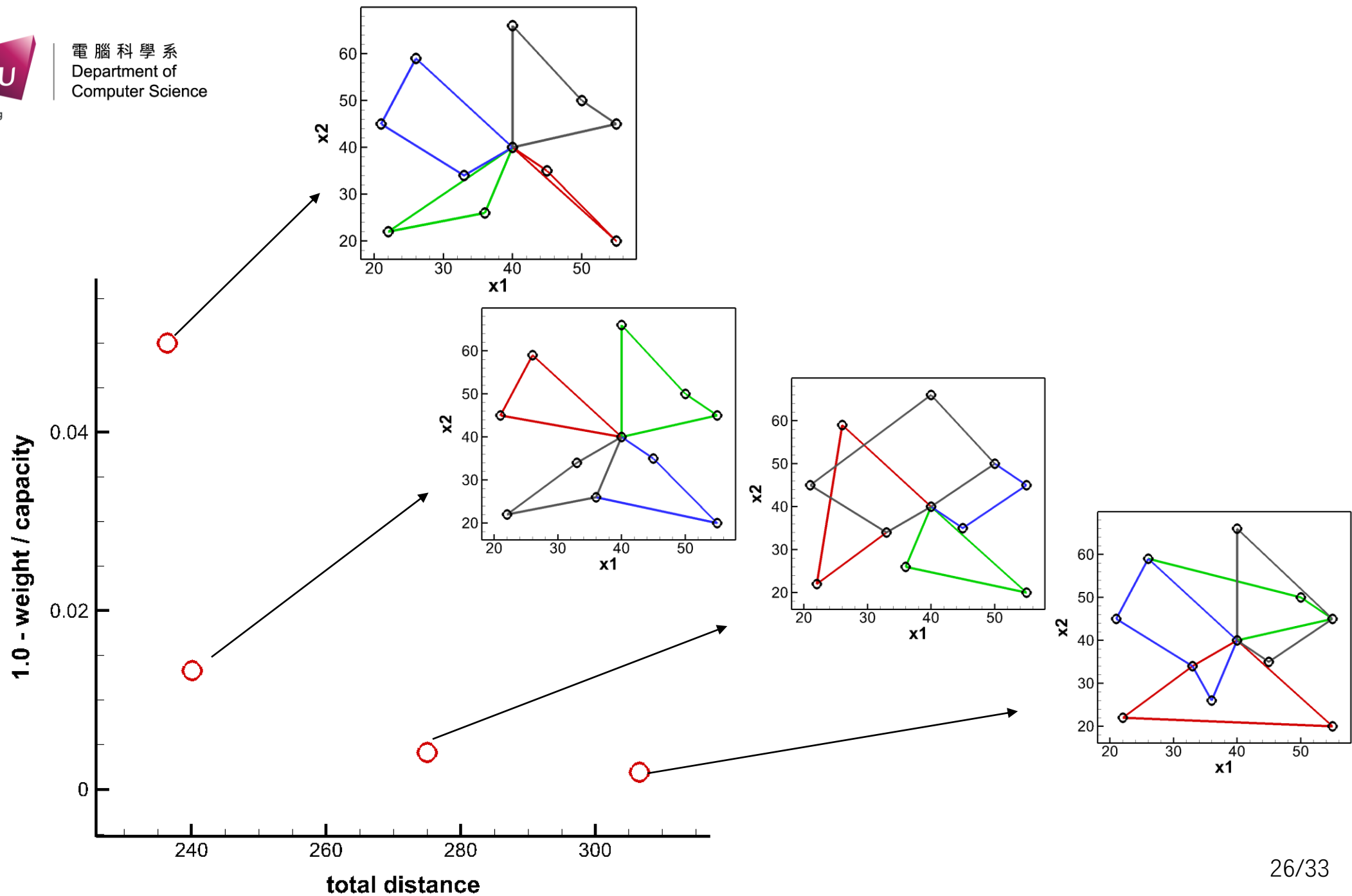
**Two objectives:**

$$f_1 = 1 - \sum_{i=1}^n \text{loading\_rate}_i / n$$

$$f_2 = \sum_{i=1}^n \text{truck\_distance}_i$$

**Constraints:**

- Start & end nodes
- Max capacity
- 2 Splits



# A Multiobjective Metaheuristic Approach for 3L-SDVRP

## Advantages:

1. Adaptive& Flexible

Adaptive to different size of VRP problems, different split number and vehicle type ...

2. Extensible

Combine with 3D bin packing method or used in other framework.

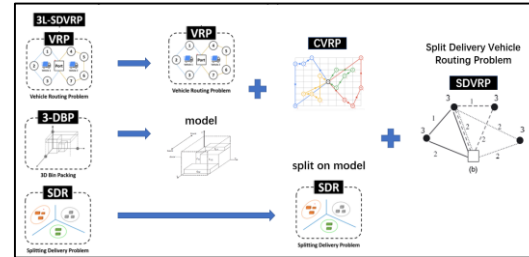
3. Efficient

In general, more efficient than exactly method.

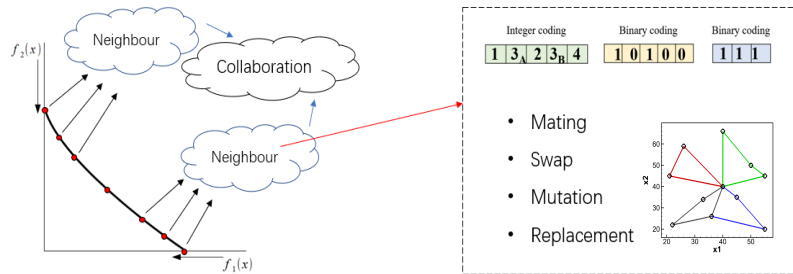
4. Constraints

Constraints can be integrated easily.

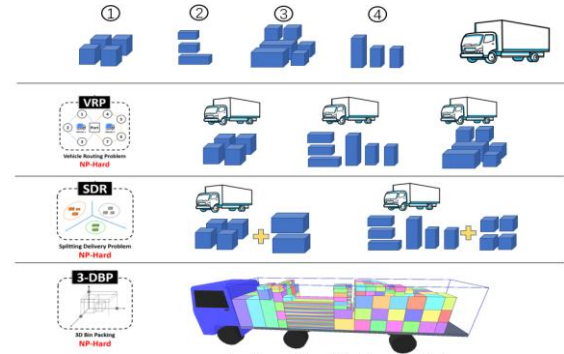
# A Multiobjective Metaheuristic Approach for 3L-SDVRP



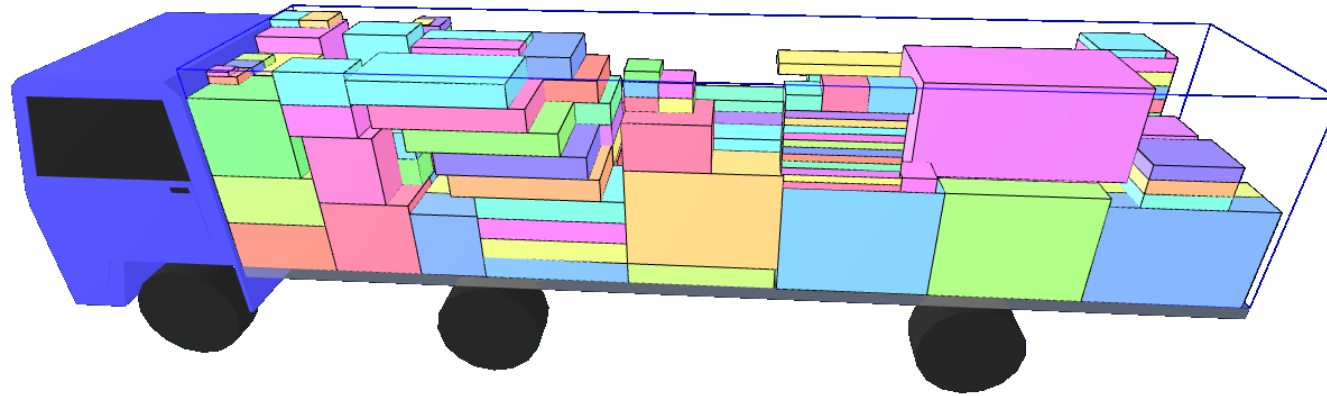
## Proposed Multiobjective Metaheuristic



- Model for packing
- Constraints

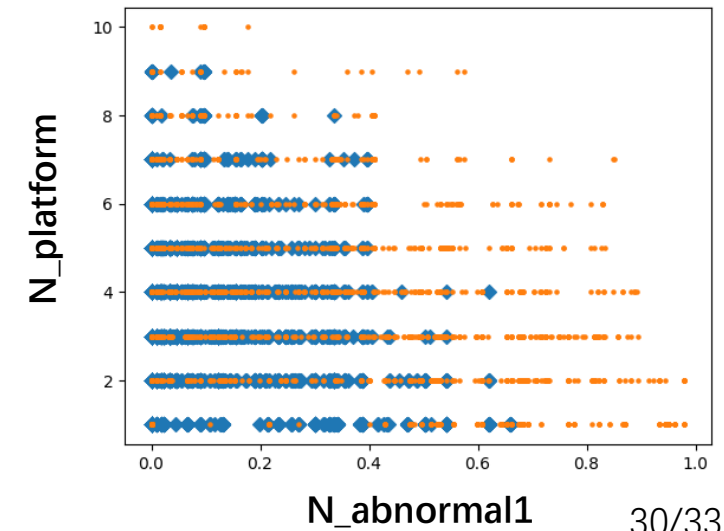
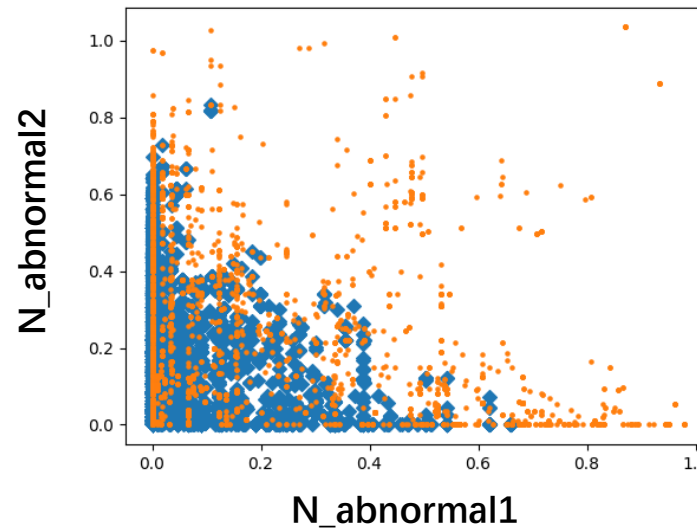
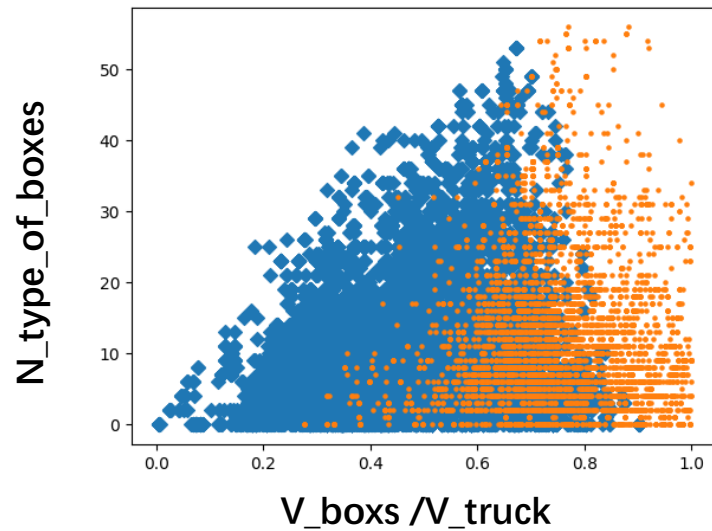
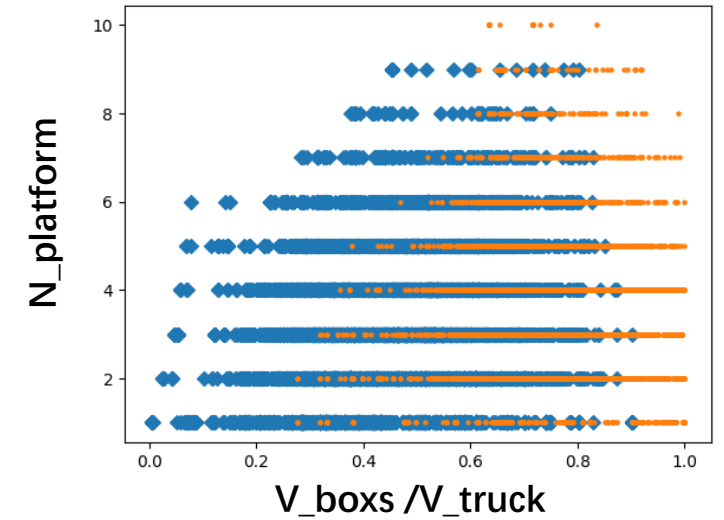
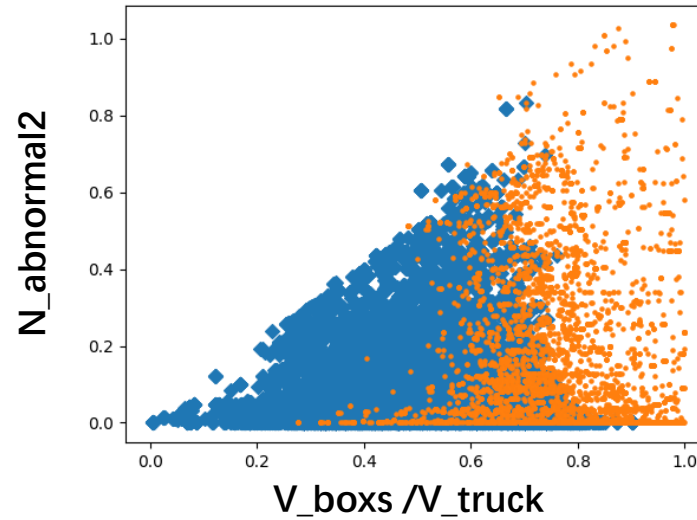
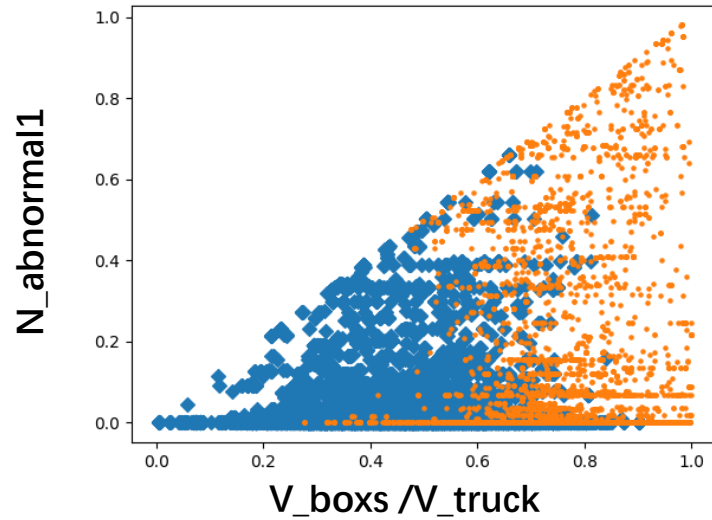


## A Multiobjective Metaheuristic Approach for 3L-SDVRP (model for 3D bin packing)

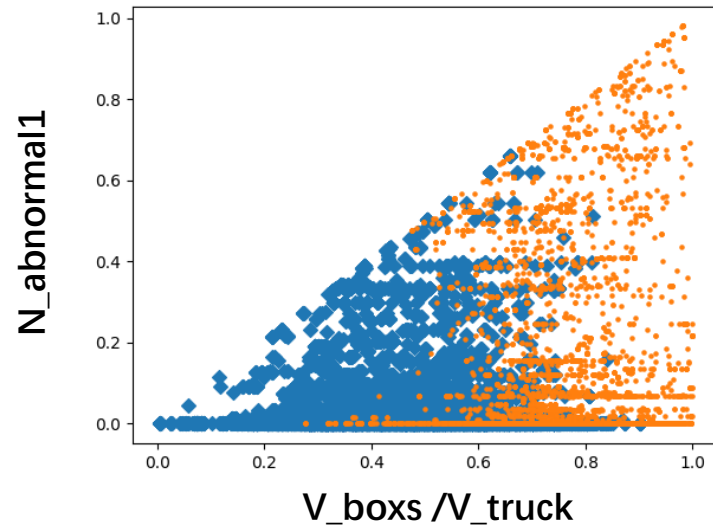
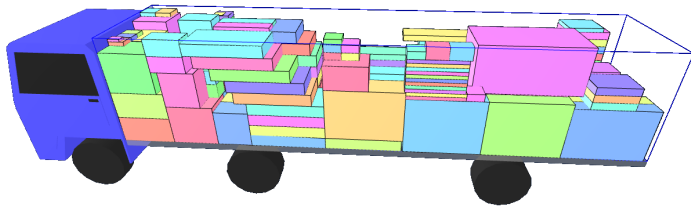


- |   |   |
|---|---|
| • $V_{\text{boxes}} / V_{\text{truck}}$ | Total volume of boxes / volume of truck   |
| • $W_{\text{boxes}} / W_{\text{truck}}$ | Total weight of boxes / capacity of truck |
| • $N_{\text{platform}}$                 | Number of platform                        |
| • $N_{\text{type\_of\_box}}$            | Number of types of boxes                  |
| • $N_{\text{abnormal}}$                 | Number of abnormal boxes                  |
| • ...                                   | ...                                       |

- 50000 data samples with label on if it is a successful packing
- 6 features
- $V_{\text{box}} / V_{\text{truck}}$  is the most important feature



# A Multiobjective Metaheuristic Approach for 3L-SDVRP (model for 3D bin packing)



80%-90%

- SVM, NN, linear or ...
  - Accuracy is not the most important thing ?
  - Regression or Classification
  - How to define features
- ↓
- Volume ratio is much more significant than other features
  - Multi-level volume ratio is used as the model

# A Multiobjective Metaheuristic Approach for 3L-SDVRP (constraints)

## Constraints

This problem includes two types of constraints: vehicle routing and 3-D loading,

- valid route constraints: each route must start from the starting point and end at the delivery point
- bonded warehouse constraints: the bonded warehouse is a special pickup point that can only be visited by empty trucks. Hence, if a pickup point is marked as a bonded warehouse, it must be the first of any route that contains it
- single visit constraints: each truck can only visit each pickup point once
- none-overlapping constraints: items cannot overlap in any dimension
- none-splitting constraints: one item can only be loaded in one truck
- supporting constraints: the item must have sufficient supporting surface, i.e., the supporting area should be greater than 80% of the bottom area of the item
- weight capacity constraints: the total weight of the cargoes loaded on one truck cannot exceed the weight capacity of the truck
- size constraints: all the items must be loaded inside the container
- one-time loading constraints: once a truck leaves a pickup point, the items loaded at the pickup point cannot be moved anymore

Routing constraints

- **Valid route constraints**
- **Single visit constraints**
- **Weight capacity constraints**
- **Bonded warehouse constraints**

implicit

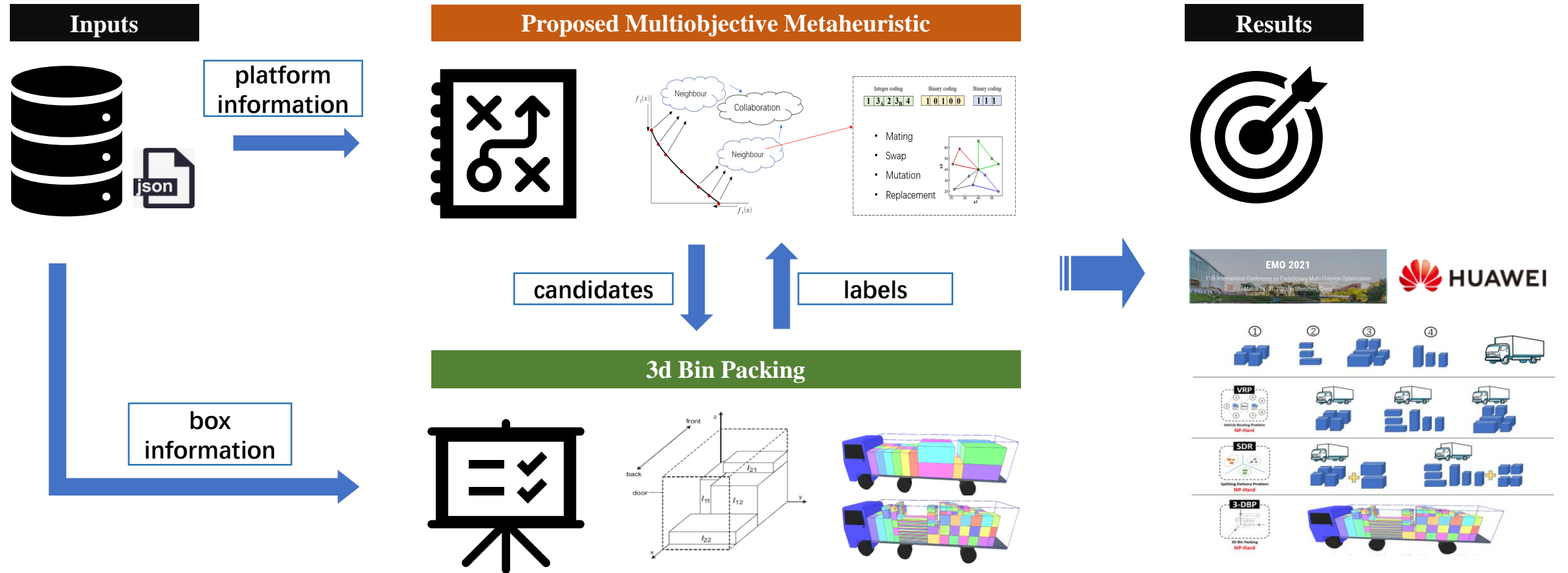
explicit

Packing constraints

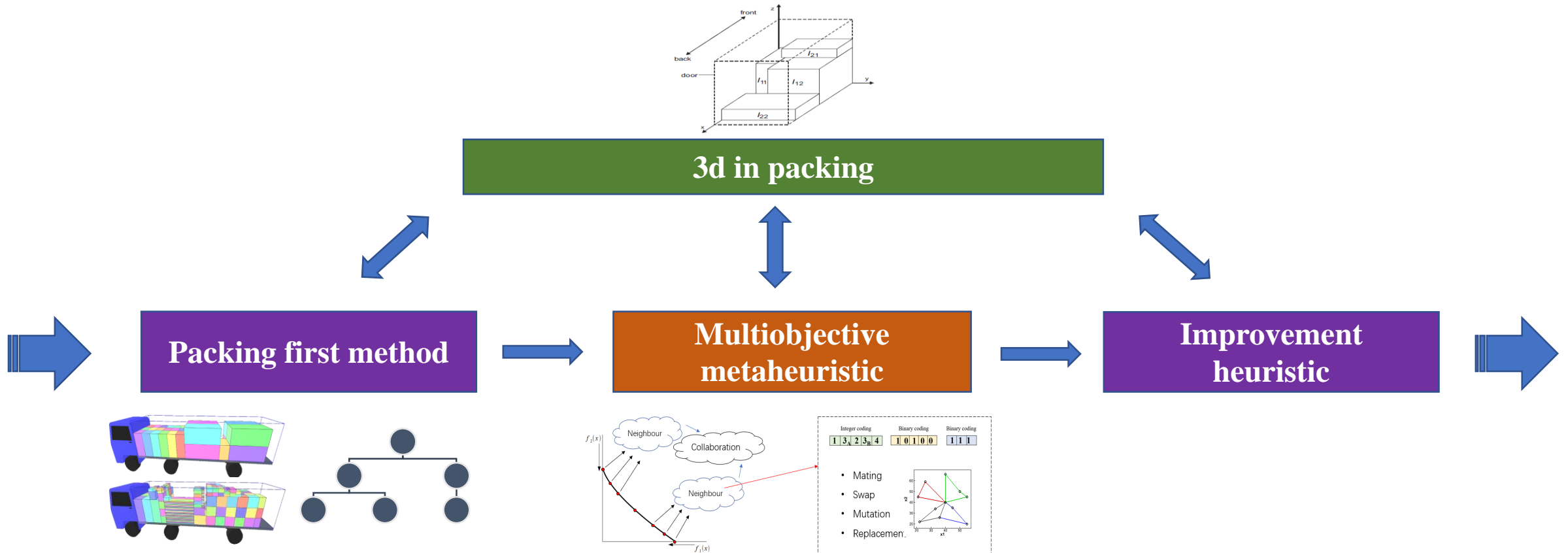
- **none-overlapping constraints**
- **None-splitting constraints**
- **Size constraints**
- **One-time loading constraints**
- **Supporting constraints**



# Solution for Huawei 3L-SDVRP



# Final mixed version for Huawei 3L-SDVRP



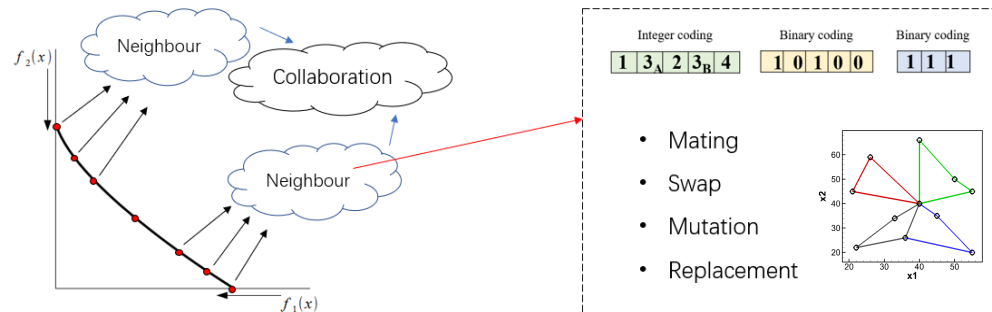
- <https://www.noahlab.com.hk/logistics-ranking/#/ranking>

# Outline

1. EMO2021 Competition – Huawei 3L-SDVRP
2. A Metaheuristic Approach for 3L-SDVRP
- 3. Conclusion**

# Conclusion

- A metaheuristic approach is proposed for 3L-SDVRP
- The method is validated on a simple instance and the EMO competition
- Results show the flexibility of metaheuristic



## Future works

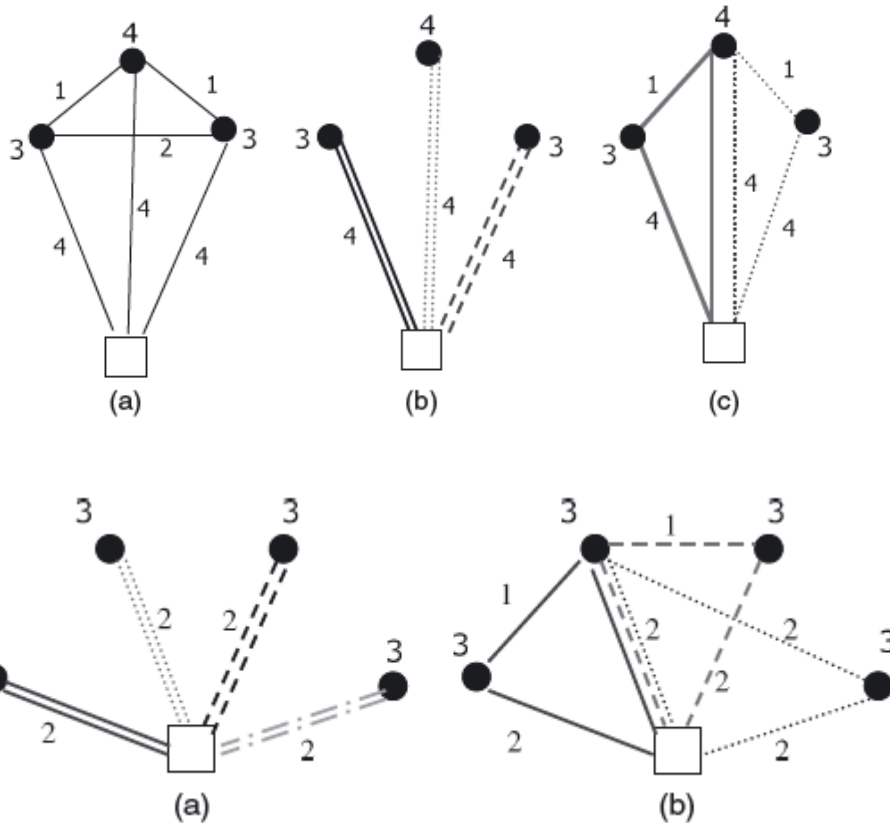
- General encoding of design space & encoding of exploration and exploitation operators in metaheuristic
- Use offline & online knowledge in VRP and SDR
- Experiments on benchmark SDVRP instances, middle & large scale SDVRP, and compare with other methods
- ...

# A Multiobjective Metaheuristic Approach for 3L-SDVRP

Thanks!

- **Fei Liu, Qingling Zhu**
- **March 1, 2021**
- **Email: [fliu36-c@my.cityu.edu.hk](mailto:fliu36-c@my.cityu.edu.hk)**

## Some conclusions on SDVRP



- The delivery cost reductions obtained when allowing split deliveries appear to be due primarily to the ability to reduce the number of delivery routes.
- The largest benefits are obtained when the mean demand is greater than half but less than three quarters of the vehicle capacity.
- The benefits from allowing split deliveries mainly depends on the relation between mean demand and vehicle capacity and on demand variance; there does not appear to be a dependence on customer locations.

- Archetti, Claudia, Martin WP Savelsbergh, and M. Grazia Speranza. "To split or not to split: That is the question." *Transportation Research Part E: Logistics and Transportation Review* 44.1 (2008): 114-123.
- Archetti, Claudia, and Maria Grazia Speranza. "Vehicle routing problems with split deliveries." *International transactions in operational research* 19.1-2 (2012): 3-22.

## Some conclusions on SDVRP

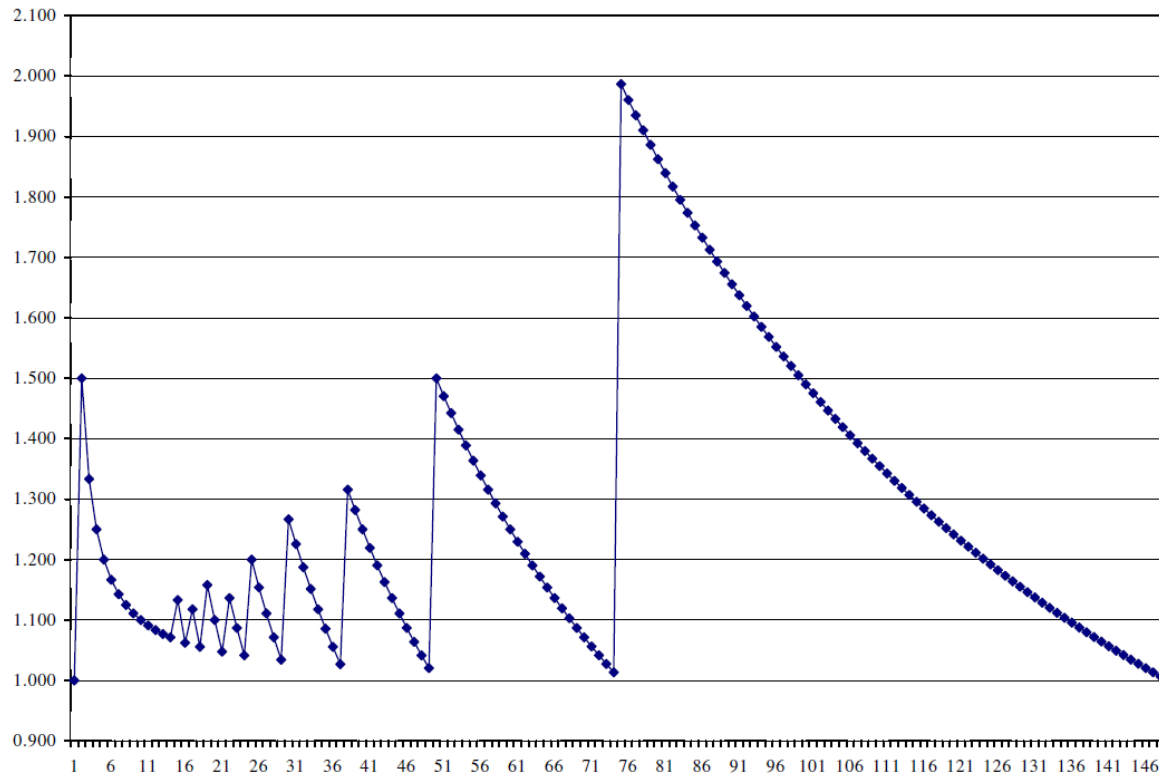
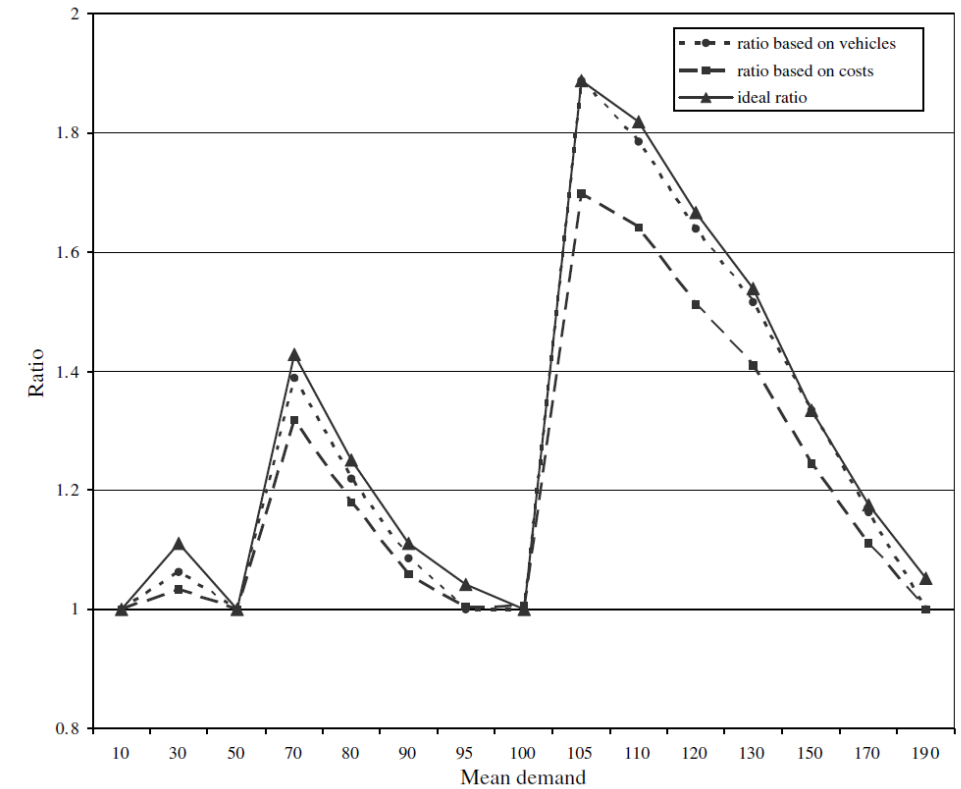


Fig. 1. Ratio  $\frac{r(\text{VRP})}{r(\text{SDVRP})}$  as a function of  $d$  for an instance with 149 customers and vehicles with capacity 149.





# Some examples on coding for VRP

## coding

- Natural number coding

For example:

Chromosome:

(1 4 6 5 3 2 7)

Corresponding string:

(1 4 6 5 1 3 2 7 1)

## crossover

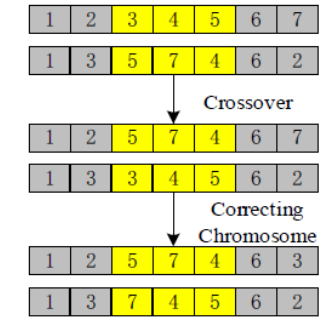


Figure 1. Chromosome Crossover

## mutation

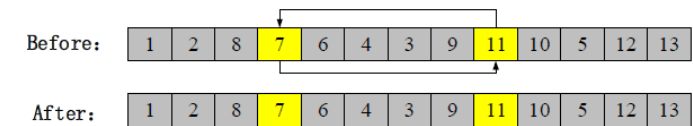


Figure 2. Chromosome Mutation

## selection

- copying the best individuals, weeding out the worst individuals.

- Gao, Zhao, Guohua Sun, and Zhenglei Yuan. "Genetic Algorithm to the Split Delivery Vehicle Routing Problem." *2019 6th International Conference on Systems and Informatics (ICSAI)*. IEEE, 2019.