Theoretical Models of Power Spectrum

The papers for this code:
V12: https://arxiv.org/pdf/1207.0839
V13: https://arxiv.org/pdf/1308.6294
O14: https://arxiv.org/pdf/1312.4214v2
H19: https://arxiv.org/pdf/1906.02875
Q is D , in this software 'D' is a function, so we instead use Q.
PL is the linear power spectrum.
$\mathcal{H}= a$ H where a is the scale factor, H is the Hubble parameter.
Pmnc denotes the loop terms defined in C.Howlett 2019 (H19).
Pmn   denotes the loop terms defined in Vlah12 (V12).
We want to check if there is any bugs in Pmnc of H19.

Sec.1: Comparing the expressions of the power spectrum of H19 to those of V12 and O14

1.1: The density power spectrum

This is the density power spectrum of H19, see the Eq.A2 of H19:

In[1]:=
```
Pδc = p00c + μ^2 ( 2 p01c + p02c + p11c) +
   μ^4 (p03c + p04c + p12c + p13c + 1/4 × p22c); (* Eq.A2 of H19 *)
ExpandAll[Pδc]
```
展开全部

Out[2]= $$\text{p00c} + 2\,\text{p01c}\,\mu^2 + \text{p02c}\,\mu^2 + \text{p11c}\,\mu^2 + \text{p03c}\,\mu^4 + \text{p04c}\,\mu^4 + \text{p12c}\,\mu^4 + \text{p13c}\,\mu^4 + \frac{\text{p22c}\,\mu^4}{4}$$

On the other hand, the expression of density power of V12 is given by :

In[3]:=
```
Pδ = p00 + (k μ/𝓗)^2 p11 + 1/4 × (k μ/𝓗)^4 p22 +
  2 ((-ⅈ k μ)/𝓗 p01 + (-1/2 × (k μ/𝓗)^2) p02 + ⅈ/6 × (k μ/𝓗)^3 p03 + (-ⅈ/2 × (k μ/𝓗)^3) p12 +
    (-1/6 × (k μ/𝓗)^4 p13) + 1/24 × (k μ/𝓗)^4 p04); (* Eq.2.23 of V12 *)
ExpandAll[Pδ]
```
展开全部

Out[4]=
$$p00 - \frac{2\, i\, k\, p01\, \mu}{\mathcal{H}} - \frac{k^2\, p02\, \mu^2}{\mathcal{H}^2} + \frac{k^2\, p11\, \mu^2}{\mathcal{H}^2} + \frac{i\, k^3\, p03\, \mu^3}{3\, \mathcal{H}^3} - \frac{i\, k^3\, p12\, \mu^3}{\mathcal{H}^3} + \frac{k^4\, p04\, \mu^4}{12\, \mathcal{H}^4} - \frac{k^4\, p13\, \mu^4}{3\, \mathcal{H}^4} + \frac{k^4\, p22\, \mu^4}{4\, \mathcal{H}^4}$$

Comparing the above Pδ to Pδc, we can find the relations between Pmn of H19 and Pmn of V12, which are given by :

In[5]:=
```
Solve[p00c == p00, p00c]
```
解方程

$$\text{Solve}\left[2\, p01c\, \mu^2 == -\frac{2\, i\, k\, p01\, \mu}{\mathcal{H}},\ p01c\right]$$
解方程

$$\text{Solve}\left[p02c\, \mu^2 == -\frac{k^2\, p02\, \mu^2}{\mathcal{H}^2},\ p02c\right]$$
解方程

$$\text{Solve}\left[p11c\, \mu^2 == \frac{k^2\, p11\, \mu^2}{\mathcal{H}^2},\ p11c\right]$$
解方程

$$\text{Solve}\left[p03c\, \mu^4 == \frac{i\, k^3\, p03\, \mu^3}{3\, \mathcal{H}^3},\ p03c\right]$$
解方程

$$\text{Solve}\left[p04c\, \mu^4 == \frac{k^4\, p04\, \mu^4}{12\, \mathcal{H}^4},\ p04c\right]$$
解方程

$$\text{Solve}\left[p12c\, \mu^4 == -\frac{i\, k^3\, p12\, \mu^3}{\mathcal{H}^3},\ p12c\right]$$
解方程

$$\text{Solve}\left[p13c\, \mu^4 == -\frac{k^4\, p13\, \mu^4}{3\, \mathcal{H}^4},\ p13c\right]$$
解方程

$$\text{Solve}\left[\frac{p22c\, \mu^4}{4} == \frac{k^4\, p22\, \mu^4}{4\, \mathcal{H}^4},\ p22c\right]$$
解方程

```
Clear["Global`*"]
```
清除

Out[5]= {{p00c → p00}}

Out[6]=
$$\left\{\left\{p01c \to -\frac{i\, k\, p01}{\mathcal{H}\, \mu}\right\}\right\}$$

Out[7]= $\left\{\left\{p02c \to -\frac{k^2\, p02}{\mathcal{H}^2}\right\}\right\}$

Out[8]= $\left\{\left\{p11c \to \frac{k^2\, p11}{\mathcal{H}^2}\right\}\right\}$

Out[9]= $\left\{\left\{p03c \to \frac{i\, k^3\, p03}{3\, \mathcal{H}^3\, \mu}\right\}\right\}$

Out[10]=

$\left\{\left\{p04c \to \frac{k^4\, p04}{12\, \mathcal{H}^4}\right\}\right\}$

Out[11]=

$\left\{\left\{p12c \to -\frac{i\, k^3\, p12}{\mathcal{H}^3\, \mu}\right\}\right\}$

Out[12]=

$\left\{\left\{p13c \to -\frac{k^4\, p13}{3\, \mathcal{H}^4}\right\}\right\}$

Out[13]=

$\left\{\left\{p22c \to \frac{k^4\, p22}{\mathcal{H}^4}\right\}\right\}$

1.2: The momentum power spectrum

This is the momentum power spectrum of H19, see Eq.A3 of H19:

In[15]:= $Pmc = \frac{\mathcal{H}^2}{k^2} \times \left(p11c + \mu^2 \times (2\, p12c + 3\, p13c + p22c)\right);$ (* Eq.A3 of H19 *)

```
ExpandAll[Pmc]
```
展开全部

Out[16]=

$$\frac{p11c\, \mathcal{H}^2}{k^2} + \frac{2\, p12c\, \mathcal{H}^2\, \mu^2}{k^2} + \frac{3\, p13c\, \mathcal{H}^2\, \mu^2}{k^2} + \frac{p22c\, \mathcal{H}^2\, \mu^2}{k^2}$$

On the other hand, the expression of momentum power of O14 is given by :

In[17]:= $\text{Solve}\left[\left(\frac{k\,\mu}{\mathcal{H}}\right)^2 Pm == \left(\frac{k\,\mu}{\mathcal{H}}\right)^2 p11 - 2\, i \left(\frac{k\,\mu}{\mathcal{H}}\right)^3 p12 - \left(\frac{k\,\mu}{\mathcal{H}}\right)^4 p13 + \left(\frac{k\,\mu}{\mathcal{H}}\right)^4 p22,\ Pm\right]$
解方程

(* Eq.4.2 of O14 *)

Out[17]=

$$\left\{\left\{Pm \to \frac{p11\, \mathcal{H}^2 - 2\, i\, k\, p12\, \mathcal{H}\, \mu - k^2\, p13\, \mu^2 + k^2\, p22\, \mu^2}{\mathcal{H}^2}\right\}\right\}$$

In[18]:=
$$Pm = \frac{p11\,\mathcal{H}^2 - 2\,i\,k\,p12\,\mathcal{H}\,\mu - k^2\,p13\,\mu^2 + k^2\,p22\,\mu^2}{\mathcal{H}^2};$$

```
(*it should be taken from the above eq*)
ExpandAll[Pm]
```
展开全部

Out[19]=
$$p11 - \frac{2\,i\,k\,p12\,\mu}{\mathcal{H}} - \frac{k^2\,p13\,\mu^2}{\mathcal{H}^2} + \frac{k^2\,p22\,\mu^2}{\mathcal{H}^2}$$

Comparing the above Pm to Pmc, we can find the relation between Pmn of H19 and Pmn of V12 :

In[20]:=
$$\text{Solve}\left[\frac{p11c\,\mathcal{H}^2}{k^2} == p11,\ p11c\right]$$
解方程

$$\text{Solve}\left[\frac{2\,p12c\,\mathcal{H}^2\,\mu^2}{k^2} == -\frac{2\,i\,k\,p12\,\mu}{\mathcal{H}},\ p12c\right]$$
解方程

$$\text{Solve}\left[\frac{3\,p13c\,\mathcal{H}^2\,\mu^2}{k^2} == -\frac{k^2\,p13\,\mu^2}{\mathcal{H}^2},\ p13c\right]$$
解方程

$$\text{Solve}\left[\frac{p22c\,\mathcal{H}^2\,\mu^2}{k^2} == \frac{k^2\,p22\,\mu^2}{\mathcal{H}^2},\ p22c\right]$$
解方程

```
Clear["Global`*"]
```
清除

Out[20]=
$$\left\{\left\{p11c \to \frac{k^2\,p11}{\mathcal{H}^2}\right\}\right\}$$

Out[21]=
$$\left\{\left\{p12c \to -\frac{i\,k^3\,p12}{\mathcal{H}^3\,\mu}\right\}\right\}$$

Out[22]=
$$\left\{\left\{p13c \to -\frac{k^4\,p13}{3\,\mathcal{H}^4}\right\}\right\}$$

Out[23]=
$$\left\{\left\{p22c \to \frac{k^4\,p22}{\mathcal{H}^4}\right\}\right\}$$

The above equations are the same as the Eqs of Sec 1.1 .

1.3: The cross power spectrum

This is the cross power spectrum of H19:

In[25]:=
```
Pδmc = ⅈ 𝓗/k × μ ×
  (p01c + p02c + p11c + μ^2 × (3/2 × p03c + 2 p04c + 2 p12c + 3 p13c + 1/2 × p22c));
ExpandAll[Pδmc]
```
展开全部

Out[26]=
$$\frac{i\,\text{p01c}\,\mathcal{H}\,\mu}{k}+\frac{i\,\text{p02c}\,\mathcal{H}\,\mu}{k}+\frac{i\,\text{p11c}\,\mathcal{H}\,\mu}{k}+\frac{3\,i\,\text{p03c}\,\mathcal{H}\,\mu^3}{2\,k}+\frac{2\,i\,\text{p04c}\,\mathcal{H}\,\mu^3}{k}+\frac{2\,i\,\text{p12c}\,\mathcal{H}\,\mu^3}{k}+\frac{3\,i\,\text{p13c}\,\mathcal{H}\,\mu^3}{k}+\frac{i\,\text{p22c}\,\mathcal{H}\,\mu^3}{2\,k}$$

In practice, Pδmc should be the imaginary part of the above equation, i.e.
$\text{P}\delta\text{mc} = \frac{\mathcal{H}\mu}{k}\left(\text{p01c} + \text{p02c} + \text{p11c} + \mu^2\left(\frac{3}{2}\,\text{p03c} + 2\,\text{p04c} + 2\,\text{p12c} + 3\,\text{p13c} + \frac{1}{2}\,\text{p22c}\right)\right)$.
On the other hand, the expression of cross power of O14 is given by :

In[27]:=
```
Solve[(-ⅈ k μ/𝓗) Pδm ==
  -ⅈ k μ/𝓗 p01 + (k μ/𝓗)^2 p11 - (k μ/𝓗)^2 p02 + ⅈ/2 (k μ/𝓗)^3 p03 - 2 ⅈ (k μ/𝓗)^3 p12 +
   1/6 (k μ/𝓗)^4 p04 - (k μ/𝓗)^4 p13 + 1/2 (k μ/𝓗)^4 p22, Pδm] (* Eq.4.1 of O14 *)
```
解方程

Out[27]=
$$\left\{\left\{\text{P}\delta\text{m}\to\frac{1}{6\,\mathcal{H}^3}\left(3\,\left(2\,\text{p01}\,\mathcal{H}^3-k^2\,\text{p03}\,\mathcal{H}\,\mu^2+4\,k^2\,\text{p12}\,\mathcal{H}\,\mu^2\right)+i\,\left(-6\,k\,\text{p02}\,\mathcal{H}^2\,\mu+6\,k\,\text{p11}\,\mathcal{H}^2\,\mu+k^3\,\text{p04}\,\mu^3-6\,k^3\,\text{p13}\,\mu^3+3\,k^3\,\text{p22}\,\mu^3\right)\right)\right\}\right\}$$

In[28]:=
```
Pδm = 1/(6 𝓗^3) (3 (2 p01 𝓗^3 - k^2 p03 𝓗 μ^2 + 4 k^2 p12 𝓗 μ^2) +
     ⅈ (-6 k p02 𝓗^2 μ + 6 k p11 𝓗^2 μ + k^3 p04 μ^3 - 6 k^3 p13 μ^3 + 3 k^3 p22 μ^3));
 (*this should be taken from the above eq.*)
ExpandAll[Pδm]
```
展开全部

Out[29]=
$$\text{p01}-\frac{i\,k\,\text{p02}\,\mu}{\mathcal{H}}+\frac{i\,k\,\text{p11}\,\mu}{\mathcal{H}}-\frac{k^2\,\text{p03}\,\mu^2}{2\,\mathcal{H}^2}+\frac{2\,k^2\,\text{p12}\,\mu^2}{\mathcal{H}^2}+\frac{i\,k^3\,\text{p04}\,\mu^3}{6\,\mathcal{H}^3}-\frac{i\,k^3\,\text{p13}\,\mu^3}{\mathcal{H}^3}+\frac{i\,k^3\,\text{p22}\,\mu^3}{2\,\mathcal{H}^3}$$

Comparing the above Pδm to Pδmc we can find the the relation between Pmn of H19 and Pmn of V12:

In[30]:=
```
Solve[ⅈ p01c 𝓗 μ / k == p01, p01c]
Solve[ⅈ p02c 𝓗 μ / k == -ⅈ k p02 μ / 𝓗, p02c]
Solve[ⅈ p11c 𝓗 μ / k == ⅈ k p11 μ / 𝓗, p11c]
Solve[3 ⅈ p03c 𝓗 μ^3 / (2 k) == -k^2 p03 μ^2 / (2 𝓗^2), p03c]
Solve[2 ⅈ p04c 𝓗 μ^3 / k == ⅈ k^3 p04 μ^3 / (6 𝓗^3), p04c]
Solve[2 ⅈ p12c 𝓗 μ^3 / k == 2 k^2 p12 μ^2 / 𝓗^2, p12c]
Solve[3 ⅈ p13c 𝓗 μ^3 / k == -ⅈ k^3 p13 μ^3 / 𝓗^3, p13c]
Solve[ⅈ p22c 𝓗 μ^3 / (2 k) == ⅈ k^3 p22 μ^3 / (2 𝓗^3), p22c]
Clear["Global`*"]
```
(Solve: 解方程; Clear: 清除)

Out[30]=

$$\left\{\left\{p01c \to -\frac{i\, k\, p01}{\mathcal{H}\, \mu}\right\}\right\}$$

Out[31]=

$$\left\{\left\{p02c \to -\frac{k^2\, p02}{\mathcal{H}^2}\right\}\right\}$$

Out[32]=

$$\left\{\left\{p11c \to \frac{k^2\, p11}{\mathcal{H}^2}\right\}\right\}$$

Out[33]=

$$\left\{\left\{p03c \to \frac{i\, k^3\, p03}{3\, \mathcal{H}^3\, \mu}\right\}\right\}$$

Out[34]=

$$\left\{\left\{p04c \to \frac{k^4\, p04}{12\, \mathcal{H}^4}\right\}\right\}$$

Out[35]=

$$\left\{\left\{p12c \to -\frac{i\, k^3\, p12}{\mathcal{H}^3\, \mu}\right\}\right\}$$

Out[36]=

$$\left\{\left\{p13c \to -\frac{k^4\, p13}{3\, \mathcal{H}^4}\right\}\right\}$$

Out[37]=

$$\left\{\left\{p22c \to \frac{k^4\, p22}{\mathcal{H}^4}\right\}\right\}$$

The above equations are the same as the Eqs of Sec1.1.

1.4: Over all the relation between Pmnc and Pmn are:

We finally have the following relations between Pmnc of H19 and Pmn of Vlah12 :

In[39]:=

$$p00c = p00;\ p01c = -\frac{i\, k\, p01}{\mathcal{H}\, \mu};\ p02c = -\frac{k^2\, p02}{\mathcal{H}^2};\ p03c = \frac{i\, k^3\, p03}{3\, \mathcal{H}^3\, \mu};\ p04c = \frac{k^4\, p04}{12\, \mathcal{H}^4};$$

$$p11c = \frac{k^2\, p11}{\mathcal{H}^2};\ p12c = -\frac{i\, k^3\, p12}{\mathcal{H}^3\, \mu};\ p13c = -\frac{k^4\, p13}{3\, \mathcal{H}^4};\ p22c = \frac{k^4\, p22}{\mathcal{H}^4};$$

```
Clear["Global`*"]
```
清除

Sec.2: Validating the loop terms Pmn

2.1: P00

In[42]:=

```
P11dd = PL;(* linear power spectrum in V12*)
P22dd = 2 I00;(* Eq3.2 of V12*)
```

$P13dd = 3\, k^2\, PL\, J00;$ (* Eq3.2 of V12*)

$P00 = Q^2\ P11dd + Q^4\ (P22dd + 2\, P13dd);$ (* Eq3.1 of V12*)

$$Phh00 = b1^2\ P00 + Q^4 \times \left(2\, b1\ (b2\, K00 + bs\, Ks00) + \frac{1}{2}\left(b2^2\, K01 + bs^2\, Ks01\right) + b2\, bs\, Ks02 + 2\, b3nl\, \sigma3^2\, PL\right);$$ (* Eq2.28 of V13*)

Important Notice : In the above equation, do not fogort the prefactor ' Q^4 ' for the term of $2\, b1\, (b2K00 + bsKs00) + \frac{1}{2}\left(b2^2\, K01 + bs^2\, Ks01\right) + b2bsKs02 + 2\, b3nl\sigma3^2\, PL$

In[47]:=

```
Phh00 = FullSimplify[ExpandAll[Phh00]];
```
完全简化 展开全部

```
P00c = FullSimplify[Phh00 ]
```
完全简化

Out[48]=

$$2\, b1\ (b2\, K00 + bs\, Ks00)\ Q^4 + b1^2\, Q^2\ \left(PL + 2\ \left(I00 + 3\, J00\, k^2\, PL\right)\, Q^2\right) + \frac{1}{2}\, Q^4\ \left(b2^2\, K01 + bs^2\, Ks01 + 2\, b2\, bs\, Ks02 + 4\, b3nl\, PL\, \sigma3^2\right)$$

The corrected P00 of H19 should be:

In[49]:=

```
P00cor =
  b1^2 Q^2 (PL + 2 Q^2 (I00 + 3 k^2 PL J00)) + 2 b1 Q^4 (b2 K00 + bs Ks00 + (b3nl PL σ3^2)/b1) +
   Q^4 (1/2 b2^2 K01 + 1/2 bs^2 Ks01 + b2 bs Ks02);
ExpandAll[P00cor] - ExpandAll[P00c]
展开全部              展开全部
Clear["Global`*"]
清除
```

Out[50]=

```
0
```

2.2: P01

In[52]:=

```
P11δθ = -f ℋ PL;(*Eq.3.7 of V12 . *)
P22δθ = -2 f ℋ I01;(*Eq.3.7 of V12*)
P13δθ = -3 f ℋ k^2 PL J01;(*Eq.3.7 of V12*)
Pδθ = Q^2 P11δθ + Q^4 (P22δθ + 2 P13δθ );(*Eq.3.6 of V12*)
AA211 = -2 f ℋ μ/k I10;(*Eq.3.9 of V12*)
AA121 = -2 f ℋ μ k PL (3 J10 + 1/2 (σv^2 + σ3^2)); (*Eq.3.9 of V12*)
AA112 = f ℋ μ k PL (σv^2 + σ3^2); (*Eq.3.9 of V12*)
A01 = Q^4 (AA211 + AA112 + AA121);(*Eq.3.8 of V12*)
α = - μ ⅈ / k ;
P01 = - ⅈ μ/k Pδθ - ⅈ A01;(*Eq.3.4 of V12*)
Phh01 = b1^2 P01 + b1 (1 - b1 ) α Pδθ +
   f ℋ Q^4 × ( α (b2 K10 + bs Ks10) + α b1 (b2 K11 + bs Ks11) + α b3nl σ3^2 PL);
 (*Eq.2.29 of V13 , Again, in this equation,
do not fogort the pre-factor 'fℋQ^4' for the term of 'α(b2K10+bsKs10)+
 αb1(b2K11+bsKs11)+αb3nlσ3^2PL'. *)
P01c = FullSimplify[ExpandAll[ - (ⅈ k Phh01)/(ℋ μ) ]]
       完全简化        展开全部
```

Out[63]=

```
f Q^2 (2 b1^2 (I10 + 3 J10 k^2 PL) Q^2 +
   b1 (PL + (2 I01 - b2 K11 - bs Ks11 + 6 J01 k^2 PL) Q^2) - Q^2 (b2 K10 + bs Ks10 + b3nl PL σ3^2))
```

The P01 of H19 is correct :

In[64]:=
```
P01cor =
  f b1 Q^2 (PL + 2 Q^2 (I01 + b1 I10 + 3 k^2 PL (J01 + b1 J10)) - b2 Q^2 K11 - bs Q^2 Ks11) -
   f Q^4 (b2 K10 + bs Ks10 + b3nl σ3^2 PL);
ExpandAll[P01cor ] - ExpandAll[P01c]
```
展开全部 展开全部
```
Clear["Global`*"]
```
清除

Out[65]=
```
0
```

2.3: P02

In[67]:=
```
A211 = (f ℋ / k)^2 (I02 + μ^2 I20); (*Eq.3.34 of V12*)
A121 = (f ℋ)^2 PL (J02 + μ^2 J20); (*Eq.3.34 of V12*)
A112 = A121; (*Eq.3.34 of V12*)
A02 = Q^4 (A211 + A121 + A112); (*Eq.3.33 of V12*)
B1111 = - (f ℋ)^2 PL σv^2 ; (*Eq.3.34 of V12*)
B02 = Q^4 B1111 ; (*Eq.3.33 of V12,do not fogort Q^4*)
P02 = - (A02 + B02) (*Eq.3.31 of V12. I moved Q^4 to A02 and B02. *);
```
虚数单位
```
P02‾ = P02 - f^2 PL Q^4 ℋ^2 σv^2;(*see the texts below Eq. 3.38 of V12,
omitting the velocity dispersion part from P02*)
Phh02 = b1 P02‾ + phh00 σv^2 - (k^-2 ℋ^2 f^2 Q^4) × ( b2 K20 + bs Ks20);
(*Eq.2.31 of V13. phh00=p00c.  In this equation,
```
输入行
```
do not fogort the pre-factor 'k^-2ℋ^2f^2Q^4' for '(b2K20+bsKs20)'.  *)
phh00 = p00c;
Phh02 = FullSimplify[ExpandAll[Phh02]];
```
完全简化 展开全部
```
P02c = - k^2 Phh02 / ℋ^2 ;
P02c = FullSimplify[ExpandAll[P02c]]
```
完全简化 展开全部

Out[79]=

$$f^2 Q^4 \left(b2\, K20 + bs\, Ks20 + b1 \left(I02 + I20\, \mu^2 + 2\, k^2\, PL \left(J02 + J20\, \mu^2\right)\right)\right) - \frac{k^2\, p00c\, \sigma v^2}{\mathcal{H}^2}$$

The corrected P02 of H19 should be

In[80]:=
```
P02cor = f^2 b1 Q^4 (I02 + μ^2 I20 + 2 k^2 PL (J02 + μ^2 J20)) -
   1/(ℋ^2 f^2) f^2 k^2 σv^2 p00c - (-1) × f^2 Q^4 (b2 K20 + bs Ks20 ) ;(* phh00=p00c *)
ExpandAll[P02cor] - ExpandAll[P02c]
```
展开全部 展开全部

Out[81]=
```
0
```

Important Notice : If we define: $\sigma_v{}^{\rm vlah} \equiv \mathcal{H} f\, \sigma_v{}^{\rm Howlett}$ for the equations in Vlah’s paper, then we have:

In[82]:=
```
σv = ℋ f σvc; (*σv is Vlah, σvc is Howlett.*)
P02cnew = FullSimplify[ExpandAll[P02c]];
          完全简化       展开全部
P02cor = f^2 b1 Q^4 (I02 + μ^2 I20 + 2 k^2 PL (J02 + μ^2 J20))
   - f^2 k^2 σvc^2 p00c - (-1) × f^2 Q^4 (b2 K20 + bs Ks20) ; (*H19*)
ExpandAll[P02cor] - ExpandAll[P02cnew]
展开全部              展开全部
Clear["Global`*"]
清除
```

Out[85]=

0

So, the above is the corrected P02 of H19 is correct .

2.4: P03

In[87]:=

$$\text{phh01} = \frac{\text{p01c}}{-\frac{i\,k}{\mathcal{H}\,\mu}};$$

$$\text{Phh03} = 3\ \text{phh01}\ \sigma v^2\ ;\ (*\text{Eq.2.40 of V13}\quad *)$$

(* ON the other hand, we also have: *)

$$\text{Phh03} = \frac{\text{p03c}}{\frac{i\,k^3}{3\,\mathcal{H}^3\,\mu}};$$

(*So, we can solve the equation: *)

$$\text{Solve}\left[\frac{\text{p03c}}{\frac{i\,k^3}{3\,\mathcal{H}^3\,\mu}} == 3\ \frac{\text{p01c}}{-\frac{i\,k}{\mathcal{H}\,\mu}}\ \sigma v^2,\ \text{p03c}\right]$$

解方程

Out[90]=

$$\left\{\left\{\text{p03c} \to -\frac{k^2\ \text{p01c}\ \sigma v^2}{\mathcal{H}^2}\right\}\right\}$$

Again, if we define: $\sigma_v^{\text{vlah}} \equiv \mathcal{H} f\, \sigma_v^{\text{Howlett}}$ for the equations in Vlah’s paper, then we have:

In[91]:=

$$\sigma v\ =\ \mathcal{H}\ f\ \sigma vc;$$

$$\text{P03c} = \text{ExpandAll}\left[-\frac{k^2\ \text{p01c}\ \sigma v^2}{\mathcal{H}^2}\right]$$

展开全部

Out[92]=

$$-f^2\ k^2\ \text{p01c}\ \sigma vc^2$$

The P03 of H19 is correct :

In[93]:=
```
P03cor = - f^2 k^2 σvc^2 p01c ;
ExpandAll[P03cor] - ExpandAll[P03c]
Clear["Global`*"]
```
展开全部 展开全部 清除

Out[94]=

0

2.5: P04

In[96]:=

$$A211 = \left(\frac{f\mathcal{H}}{k}\right)^2 (I02 + \mu^2 I20);\ (*Eq.3.34\ of\ V12*)$$

```
A121 = (f 𝓗)^2 PL (J02 + μ^2 J20); (*Eq.3.34 of V12*)
A112 = A121; (*Eq.3.34 of V12*)
A02 = Q^4 (A211 + A121 + A112); (*Eq.3.33 of V12*)
σv = 𝓗 f σvc; (* Again, we re-define σ_v use 𝓗fσ_v*)
σ4 = 𝓗 f σ4c ;
B1111 = - (f 𝓗)^2 PL σv^2 ; (*Eq.3.34 of V12*)
B02 = Q^4 B1111 ; (*Eq.3.33 of V12*)
P02 = - (A02 + B02) (*Eq.3.31 of V12. I moved Q^4 to A02 and B02. *);
```
虚数单位

$$\overline{P02} = P02 - f^2 PL\, Q^4 \mathcal{H}^2 \sigma v^2;\ (*\text{see the texts below Eq. 3.38 of V12, omitting the velocity dispersion part from P02}*)$$

$$Phh04 = 6\, b1\, \overline{P02}\, \sigma v^2 + b1\, b1\, phh00\, (3\, \sigma v^4 + \sigma 4^4);\ (*\ Eq.2.47\ of\ V13*)$$

```
phh00 = p00c;
```

$$P04c = FullSimplify\left[ExpandAll\left[\frac{k^4\, Phh04}{12\, \mathcal{H}^4}\right]\right]$$

完全简化 展开全部

Out[108]=

$$\frac{1}{12} b1\, f^4 k^2 \left(-6\, Q^4 \left(I02 + I20\, \mu^2 + 2\, k^2\, PL\, (J02 + J20\, \mu^2)\right) \sigma vc^2 + b1\, k^2\, p00c\, (\sigma 4c^4 + 3\, \sigma vc^4)\right)$$

The corrected P04 of H19 should be :

In[109]:=

$$P04cor = \left(-\frac{1}{2}\right) f^4\, b1\, k^2\, \sigma vc^2\, Q^4 \left(I02 + \mu^2 I20 + 2\, k^2\, PL\, (J02 + \mu^2 J20)\right) + \frac{1}{4} f^4\, b1^2\, k^4\, p00c \left(\sigma vc^4 + \sigma 4c^4 \times \frac{1}{3}\right);\ (*\ phh00=p00c\ *)$$

```
ExpandAll[P04cor] - ExpandAll[P04c] (* phh00=p00c *)
Clear["Global`*"]
```
展开全部 展开全部 清除

Out[110]=

0

In the $\sigma 4c^4 \times \frac{1}{3}$, $\frac{1}{3}$ can be absorbed into σ4c, so P04 of H19 is correct.

2.6: P11

In[112]:=

```
P11tt = ℋ^2 f^2 PL;(*Eq.3.21 of V12*)
P22tt = 2 ℋ^2 f^2 I11;(*Eq.3.21 of V12*)
P13tt = 3 ℋ^2 f^2 k^2 PL J11;(*Eq.3.21 of V12*)
Ptt = Q^2 P11tt + Q^4 ( P22tt + 2 P13tt) ; (*Eq.3.20 of V12*)
Bs211 = 2 ℋ^2 f^2 μ k PL (3 J10 + 1 / 2 (σv^2 + σ3^2)); (*Eq.3.23 of V12*)
Bs121 = - ℋ^2 f^2 μ k PL (σv^2 + σ3^2); (*Eq.3.23 of V12*)
Bs112 = 2 ℋ^2 f^2 μ/k I22; (*Eq.3.23 of V12*)
B11 = Q^4 ( Bs211 + Bs112 + Bs121); (*Eq.3.22 of V12*)
C1111 = ℋ^2 f^2 k^-2 (I31 + I13 μ^2); (*Eq.3.24 of V12*)
C11 = Q^4 C1111; (* do not fogort Q^4 *)
P11 = μ^2/k^2 Ptt + 2 μ/k B11 + C11;(*Eq.3.18 of V12*)
Phh11 = P11 + ( (b1 - 1) + (b1 - 1)) μ/k B11 + (b1 b1 - 1 ) C11 ;
(*Eq.2.35 of V13*)
P11c = FullSimplify[ExpandAll[k^2 Phh11/ℋ^2]]
```

完全简化 展开全部

Out[124]=

$$f^2 Q^2 \left(PL\,\mu^2 + Q^2 \left(b1^2\,I31 + \left(2\,I11 + b1\,(b1\,I13 + 4\,I22) + 6\,(2\,b1\,J10 + J11)\,k^2\,PL\right)\mu^2\right)\right)$$

The P11 of H19 is correct :

In[125]:=

```
P11cor = f^2 Q^2 (μ^2 ( PL + Q^2 ( 2 I11 + 4 b1 I22 + b1^2 I13 + 6 k^2 PL (J11 + 2 b1 J10))) +
    b1^2 Q^2 I31);
ExpandAll[P11cor] - Expand[P11c]
Clear["Global`*"]
```

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清除

Out[126]=

0

2.7: P12

In[128]:=

```
(*we need to fistly calculate P01 of V12*)
P11δθ = -f ℋ PL;(*Eq.3.7 of V12, the Hubble parametter aH is ignored. *)
P22δθ  = -2 f ℋ I01;(*Eq.3.7 of V12*)
P13δθ = -3 f ℋ k^2 PL J01;(*Eq.3.7 of V12*)
Pδθ = Q^2 P11δθ + Q^4 (P22δθ + 2 P13δθ);(*Eq.3.6 of V12*)
AA211 = -2 f ℋ μ/k I10;(*Eq.3.9 of V12*)
AA121 = -2 f ℋ μ k PL (3 J10 + 1/2 (σvc^2 + σ3^2));
(*Eq.3.9 of V12, These should be σvc,
because the growth rate is taken outside sigmav in Vlah2012. *)
AA112 = f ℋ μ k PL (σvc^2 + σ3^2); (*Eq.3.9 of V12. These should be σvc,
because the growth rate is taken outside sigmav in Vlah2012.*)
A01 = Q^4 (AA211 + AA112 + AA121);(*Eq.3.8 of V12*)
α = - μ ⅈ / k;
P01 = FullSimplify[ExpandAll[- ⅈ μ/k Pδθ - ⅈ A01]];(*Eq.3.4 of V12*)
      └完全简化       └展开全部

(*Then, we calculate P02*)
k11 = k μ;
As211 = -(f ℋ)^3/k^2 (I12 + μ^2 I21); (*Eq.3.44 of V12*)
As121 = -(f ℋ)^3 PL (J02 + μ^2 J20); (*Eq.3.44 of V12*)
As112 = As121; (*Eq.3.44 of V12*)
A12 = Q^4 (As211 + As121 + As112);(*Eq.3.43 of V12*)
B12 = Q^4 (f ℋ)^3 μ k^-3 (I03 + μ^2 I30) ;  (*Eq.3.44   of V12, do not forgot Q^4 *)
C12 = Q^4 (f ℋ)^3 PL σvc^2 ;  (*Eq.3.44 of V12. Should be σvc as well*)
P12 = - ⅈ/k^2 (k11 A12 + k^2 B12 + k11 C12);
(*Eq.3.41 of V12*)
phh01 = p01c/(-(ⅈ k)/(ℋ μ));
Phh12 = P12 - ⅈ (b1 - 1) B12  - (phh01 - Q^2 P01) σv^2;
(*Eq2.38 of V13. For consistency,
                 └For循环
this should have a Q^2. Because in C12 we are affectively fitting sigmav at z=
                           └因为
 0,then rescaling to z=z.But here we have rescaled the P01 contains P_L(z=0),
while the sigmav is still being evaluated at z=z.*)
σv = f ℋ σvc;
P12c = FullSimplify[ExpandAll[-(ⅈ k^3 Phh12)/(ℋ^3 μ)]]
       └完全简化        └展开全部
```

Out[149]=

$$f^2 \left(f Q^4 \left(I12 + I21\,\mu^2 - b1 \left(I03 + I30\,\mu^2\right) + 2 k^2 PL \left(J02 + J20\,\mu^2\right)\right) + k^2 \left(-p01c + 2 f \left(I01 + I10 + 3 \left(J01 + J10\right) k^2 PL\right) Q^6\right) \sigma vc^2\right)$$

The corrected P12 of H19 should be :

In[150]:=

$$\text{P12cor} = f^3 Q^4 \left(\text{I12} + \text{I21}\,\mu^2 - \text{b1}\left(\text{I03} + \text{I30}\,\mu^2\right) + 2 k^2 \text{PL}\left(\text{J02} + \text{J20}\,\mu^2\right)\right) - f^2 k^2 \sigma\text{vc}^2\,\text{p01c} + 2 f^3 k^2 \left(\text{I01} + \text{I10} + 3\,(\text{J01} + \text{J10})\,k^2\,\text{PL}\right) Q^4 \,(Q\,\sigma\text{vc})^2;$$

```
ExpandAll[P12cor] - Expand[P12c]
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Clear["Global`*"]
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Out[151]=

0

2.8: P13

In[153]:=

$$\text{phh11} = \frac{\text{p11c}}{\frac{k^2}{\mathcal{H}^2}};$$

$$\text{Phh13} = 3\ \text{phh11}\,\sigma\text{v}^2;\quad \text{(* Eq.2.43 of V13 *)}$$

(* ON the other hand, we also have: *)

$$\text{Phh13} = \frac{\text{p13c}}{-\frac{k^4}{3\mathcal{H}^4}};$$

(*So, we can solve the equation: *)

$$\text{Solve}\left[\frac{\text{p13c}}{-\frac{k^4}{3\mathcal{H}^4}} == 3\ \frac{\text{p11c}}{\frac{k^2}{\mathcal{H}^2}}\ \sigma\text{v}^2,\ \text{p13c}\right]$$

解方程

Out[156]=

$$\left\{\left\{\text{p13c} \to -\frac{k^2\,\text{p11c}\,\sigma\text{v}^2}{\mathcal{H}^2}\right\}\right\}$$

In[157]:=

$$\sigma\text{v} = f\,\mathcal{H}\,\sigma\text{vc};$$

$$\text{P13c} = \text{ExpandAll}\left[-\frac{k^2\,\text{p11c}\,\sigma\text{v}^2}{\mathcal{H}^2}\right]$$

展开全部

Out[158]=

$$-f^2 k^2\,\text{p11c}\,\sigma\text{vc}^2$$

The P13 of H19 is correct :

In[159]:=

$$\text{P13cor} = -f^2 k^2\,\sigma\text{vc}^2\,\text{p11c};$$

```
ExpandAll[P13cor] - ExpandAll[P13c]
展开全部              展开全部
Clear["Global`*"]
清除
```

Out[160]=

0

2.9: P22

In[162]:=

```
A211 = (f 𝓗/k)^2 (I02 + μ^2 I20); (*Eq.3.34 of V12*)
A121 = (f 𝓗)^2 PL (J02 + μ^2 J20); (*Eq.3.34 of V12*)
A112 = A121; (*Eq.3.34 of V12*)
A02 = Q^4 (A211 + A121 + A112); (*Eq.3.33 of V12*)
σv = f 𝓗 σvc;
B1111 = -(f 𝓗)^2 PL σv^2; (*Eq.3.34  of V12*)
B02 = Q^4 B1111; (*Eq.3.33 of V12,do not fogort Q^4*)
P02 = -(A02 + B02) (*Eq.3.31 of V12. I moved Q^4 to A02 and B02. *);
                                  虚数单位
P02‾ = Simplify[ExpandAll[P02 - f^2 PL Q^4 𝓗^2 σv^2]];
        化简
(*Finally, calculate P22*)
P22‾ = (1/16 f^4 Q^4 μ^4 (I23 + 2 μ^2 I32 + μ^4 I33)) / (1/4 (k μ/𝓗)^4); (*Eq.3.48 of V12*)
Phh00Pb22 = 2 phh00 σv^4; (*texts bellow Eq.3.50 of V12.   *)
phh00 = p00c;
phh02 = p02c / (-k^2/𝓗^2);
Phh22 = P22‾ + b1 σv^2 P02‾ + phh02 σv^2 + Phh00Pb22; (*Eq.2.45 of V13      *)
(* The following p02c is taken from Sec2.3. Please check
 it if the calculation of P02 of Sec2.3 has been updated. *)
p02c = f^2 b1 Q^4 (I02 + μ^2 I20 + 2 k^2 PL (J02 + μ^2 J20))
    - f^2 k^2 σvc^2 p00c - (-1) × f^2 Q^4 (b2 K20 + bs Ks20);
P22c = FullSimplify[ExpandAll[k^4 Phh22 / 𝓗^4]]
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```

Out[177]=

$$\frac{1}{4} f^4 \left(Q^4 \left(I23 + 2\, I32\, \mu^2 + I33\, \mu^4\right) - 4 k^2 Q^4 \left(b2\, K20 + bs\, Ks20 + 2\, b1 \left(I02 + I20\, \mu^2 + 2 k^2\, PL \left(J02 + J20\, \mu^2\right)\right)\right) \sigma vc^2 + 12 k^4\, p00c\, \sigma vc^4\right)$$

The P22 of H19 is correct:

In[178]:=

```
P22cor = 1/4 f^4 Q^4 (I23 + 2 μ^2 I32 + μ^4 I33) + f^4 k^4 σvc^4 p00c -
    f^2 k^2 σvc^2 ( 2 p02c - f^2 Q^4 (b2 K20 + bs Ks20) ) ; (* phh00=p00c *)
ExpandAll[P22cor] - ExpandAll[P22c]
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Clear["Global`*"]
清除
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Out[179]=

0

Sec.3: Summary

The corrected Pmn are:

$$P00cor = b1^2\, Q^2 \left(PL + 2\, Q^2 \left(I00 + 3\, k^2\, PL\, J00\right)\right) + 2\, b1\, Q^4 \left(b2\, K00 + bs\, Ks00 + \frac{b3nl\, PL\, \sigma 3^2}{b1}\right) + Q^4 \left(\frac{1}{2}\, b2^2\, K01 + \frac{1}{2}\, bs^2\, Ks01 + b2\, bs\, Ks02\right)$$

$$P02cor = f^2\, b1\, Q^4 \left(I02 + \mu^2\, I20 + 2\, k^2\, PL \left(J02 + \mu^2\, J20\right)\right) - f^2\, k^2\, \sigma vc^2\, p00c - (-1) \times f^2\, Q^4\, (b2\, K20 + bs\, Ks20)$$

$$P12cor = f^3\, Q^4 \left(I12 + I21\, \mu^2 - b1\, \left(I03 + I30\, \mu^2\right) + 2\, k^2\, PL\, \left(J02 + J20\, \mu^2\right)\right) - f^2\, k^2\, \sigma vc^2\, p01c + 2\, f^3\, k^2 \left(I01 + I10 + 3\, (J01 + J10)\, k^2\, PL\right) Q^4\, (Q\, \sigma vc)^2 ;$$

The end of the code.