



# BLP Lattice Structure

Example

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10 October 2022



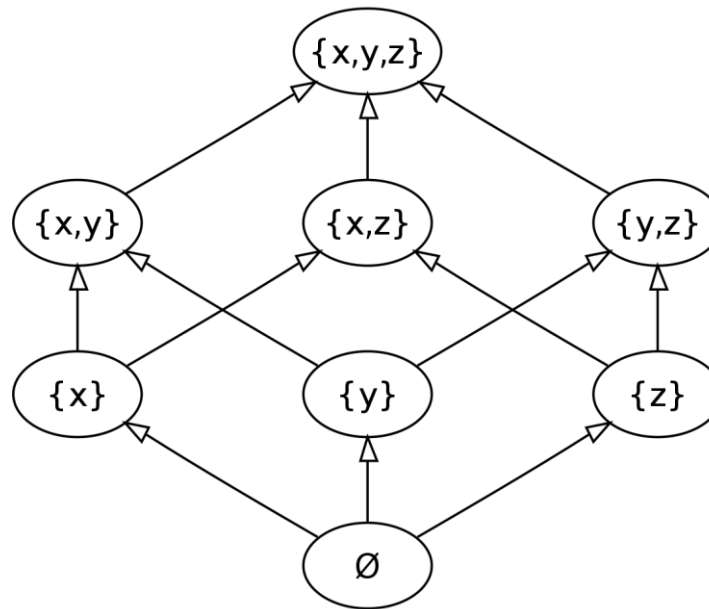
# Given the following access control matrix:

|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |

Generate a BLP lattice-structured system where the objects and subjects are appropriately levelled to give access consistent with the access control matrix shown.

# What is a lattice?

- A **lattice** is a **partially ordered** set (POSET) in which every pair of elements has both a **least upper bound** and a **greatest lower bound**.



# What is a partially order relation?

$$R = \{(a, b) \in A \times A \mid a \mid b\} \quad A \in \mathbb{Z}$$

Reflexive:

if for all  $a \in A$ ,  $(a, a) \in R$

$a \mid a$  and  $a \in A$

Antisymmetric:

if  $(a, b) \in R$  and  $(b, a) \in R$  then  $a = b$

$a \mid b$  then  $b \mid a$  only if  $a = b$

Transitive:

if  $(a, b) \in R$  and  $(b, c) \in R$  then  $(a, c) \in R$

$a \mid b$  and  $b \mid c$  then  $a \mid c$

# BLP properties (Rules)

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- **Ss-property**
  - Subject  $S(n)$  can WRITE object  $O(n)$  iff level of clearance of subject  $L(S)$  is less than or equal the level of clearance of the object  $L(O)$ , that is,  $L(S) \leq L(O)$ , and the subject has permission to WRITE the object.
- **\*-property**
  - Subject  $S(n)$  can READ object  $O(n)$  iff level of clearance of subject  $L(S)$  is greater than or equal (dominant) the level of object  $L(O)$ , that is,  $L(S) \geq L(O)$ , and the subject has permission to READ the object.
- **Discretionary**
  - Subject  $S(n)$  can discretionarily transfer his/her authorization to Subject at a different clearance level (subject to organization policy).

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|    | O1 | O2 | O3 | O4 | O5 |
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| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |

O1, O2

*↖ greatest lower bound*

Subjects from every level can read objects O1 and O2 -> O1 and O2 are dominated by all subject.  
Hence O1 and O2 must be at the lowest point.

# Given the following access control matrix:

|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |

S1, O3

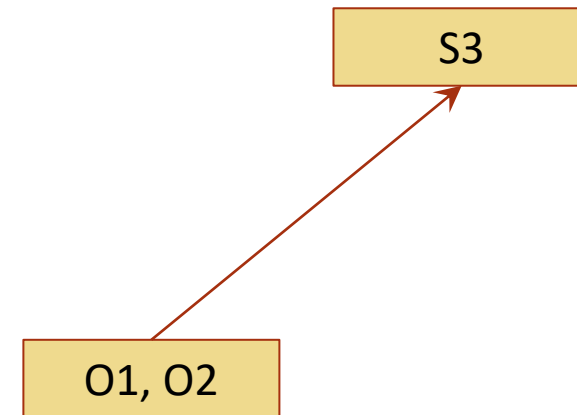
O1, O2

Subject S1 can read and write object O3, hence S1 and O3 must be at the same level (can put them together.)

# Given the following access control matrix:

|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |

S1, O3

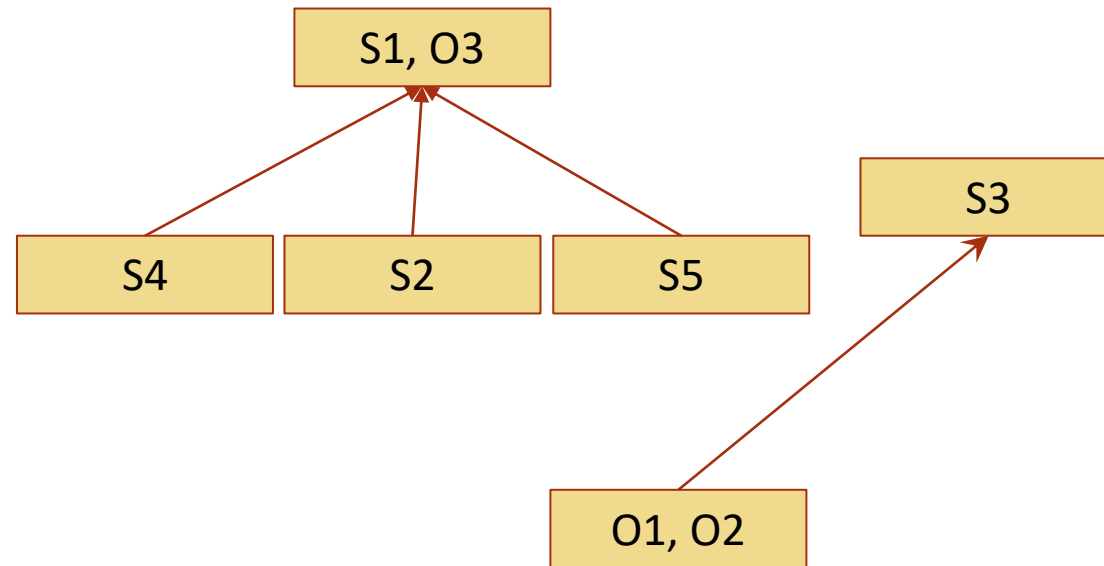


Subject S3 can read only objects O1 and O2, hence subject S3 can only dominate object O1 and O2.



# Given the following access control matrix:

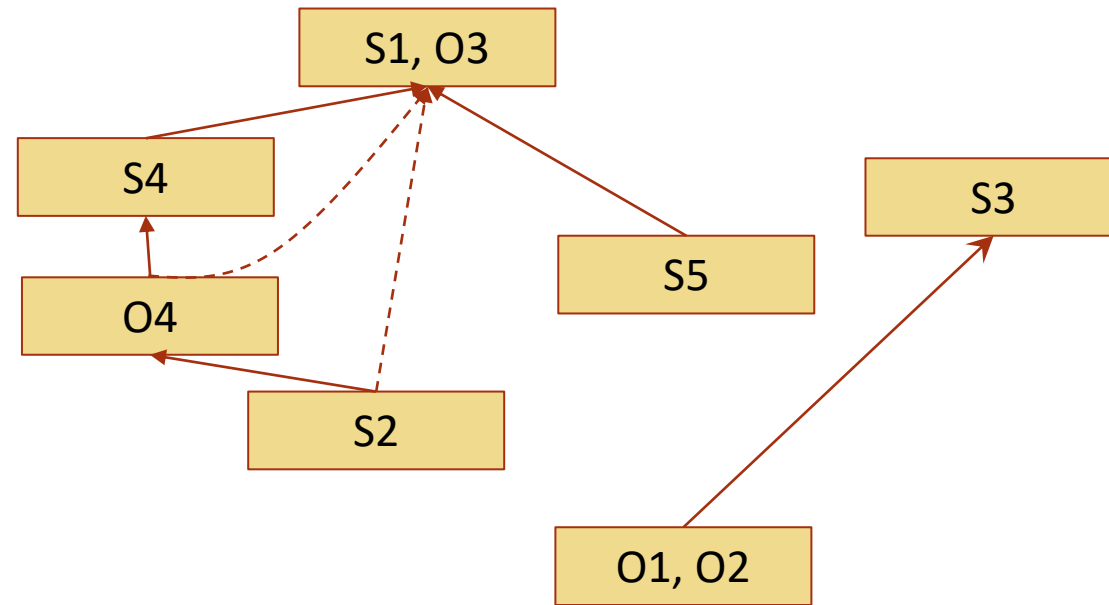
|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |



S1 and O3 must dominate subjects S2, S4 and S5 in order to allow writing.

# Given the following access control matrix:

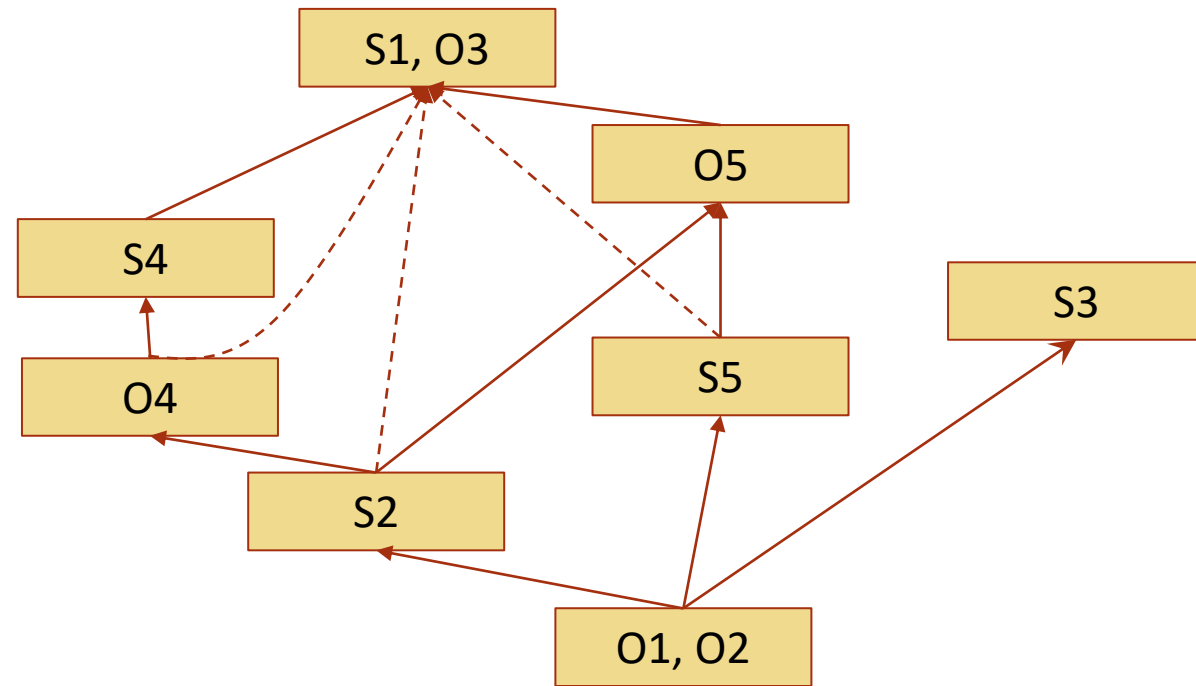
|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |



Subjects S2 and S4 are not at the same level due to the different behaviour with respect to O4.

# Given the following access control matrix:

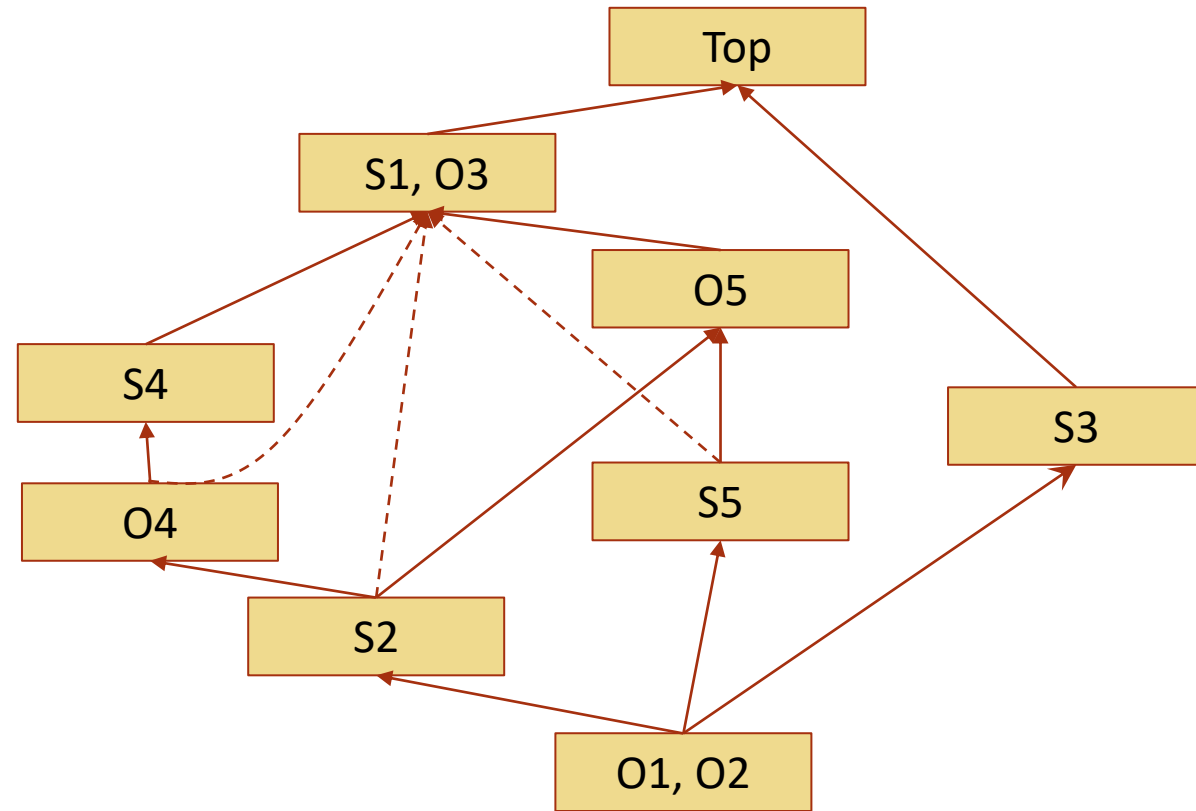
|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |



Complete the other dominance.

# Given the following access control matrix:

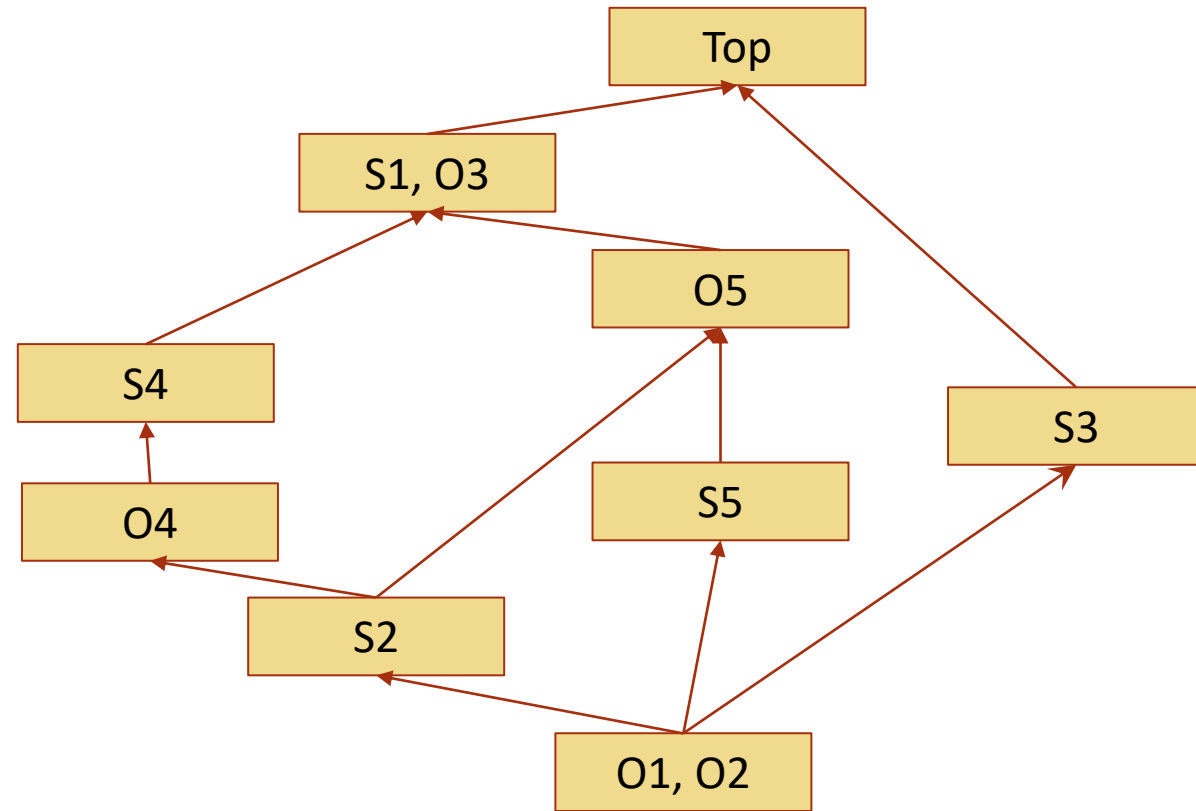
|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |



Introduce a top level to complete the lattice.

# Given the following access control matrix:

|    | O1 | O2 | O3 | O4 | O5 |
|----|----|----|----|----|----|
| S1 | R  | R  | RW | R  | R  |
| S2 | R  | R  | W  | W  | W  |
| S3 | R  | R  |    |    |    |
| S4 | R  | R  | W  | R  |    |
| S5 | R  | R  | W  |    | W  |



Final complete lattice.