

Explain what is a Zero Knowledge Proof.

Zero knowledge proof is a **protocol**, in cryptography, that allows one party to convince another party that he/she knows a secret (e.g., his/her private key) without revealing to the other party the secret. One example is as follow:

$$y = g^x \bmod p$$

$$H(g^a y^b) = a$$

1) random z

$$2) a = H(g^z)$$

$$3) a + xb = z \bmod p$$

$$xb = z - a \bmod p$$

$$b = (z - a)x^{-1} \bmod p$$

Output: (a, b) such that $H(g^a y^b) = a$

Proof:

$$H(g^a y^b) = H(g^a (g^x)^b) = H(g^{a+bx})$$

The scenario explanation:

- (i) Alice knows x , her private key.
- (ii) Alice computes $y = g^x \bmod p$ and passes y to Bob.
- (iii) To prove to Bob that Alice indeed knows the value of x without revealing the value of x to Bob, Alice does the following:
 - i. Chooses a random value z
 - ii. Computes two values a and b :
 - $a = H(g^z)$, and
 - $a + xb = z \bmod p$
 $xb = z - a \bmod p$
 $b = (z - a)x^{-1} \bmod p$

and output (a, b) such that $H(g^a y^b) = a$

- (iv) Alice sends $H(g^a y^b) = a$ to Bob. Note: The value of a is computed involving the value of x .
- (v) Bob, in order to convince that Alice knows x , Bob, will randomly chooses two values for a and b and asks Alice to compute $H(g^a y^b)$.

- (vi) Bob will verify that $H(g^a y^b) = a$, and after a few verification (with different values of a and b), Bob will be convince that Alice indeed knows the value of x , because both the values of a and y in $H(g^a y^b)$ are computed involving the value of x .