A fully coupled elasto-plastic damage model applied to anisotropic materials

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Abstract : In this paper, an elastoplastic energy-based anisotropic damage model for ductile fracture is described. A calibration method is presented. The potential applicability of this model is illustrated by a numerical example of deep drawing process realised on two steels.

Keywords : Damage, Anisotropy, Metal sheet, Calibration

1. Introduction

This model, initially developed by Zhu and Cescotto [1], is devoted to predict the damage growth and fracture appearance in ductile materials. It has been developed in the case of sheets, especially for deep drawing processes [2].
Important characteristics of this macroscopic model are its easy parameters identification, the anisotropic evolution of the damage and plastic surfaces computed from energy equivalence assumption..

2. Model description

This damage model takes place in the frame of the continuum theory of damage.
The damage in the material is represented by a variable $\underline{D}$ corresponding to an average material degradation affecting stiffness, strength, anisotropy. It reflects various types of damage at the micro-scale level such as nucleation, growth and coalescence of voids and micro-cracks.
In the present model, $\underline{D}$ is a vector of three components, the damage in each othotropic direction of the sheet. The well known concept of effective stress is used :

$$\overline{\underline{\sigma}} = \underline{\underline{M}}(\underline{D})\underline{\sigma} \qquad (1)$$

$$\underline{\underline{M}}(\underline{D}) = \mathrm{diag}\left[\frac{1}{1-D_1} \quad \frac{1}{1-D_2} \quad \frac{1}{1-D_3} \quad \frac{1}{\sqrt{(1-D_2)(1-D_3)}} \quad \frac{1}{\sqrt{(1-D_1)(1-D_3)}} \quad \frac{1}{\sqrt{(1-D_1)(1-D_2)}} \right] \qquad (2)$$

The principle of energy equivalence is taken into account. It states that the complementary elastic energy stored in the damaged material has the same form as the one for a fictitious undamaged material except that the true stress tensor is replaced by the effective stress tensor.

$$W_e(\underline{\sigma}, \underline{D}) = W_e(\overline{\underline{\sigma}}, \underline{D}) \qquad (3)$$

The general structure of the constitutive equations is furnished by the thermodynamic theory of irreversible processes. As the thermodynamic force $\underline{\sigma}$ (Cauchy or true stress tensor) is associated to the elastic strain $\underline{\varepsilon}_e$, a thermodynamic force $\underline{Y}$ can be associated to the damage tensor $\underline{D}$ thanks to the Helmholtz free energy $\rho\psi(\varepsilon,T,D)$:

$$\underline{Y} = \rho\frac{\partial\psi}{\partial\underline{D}} = \frac{\partial W_e(\underline{\varepsilon}_e, T, \underline{D})}{\partial\underline{D}} = -\frac{\partial W_e(\underline{\sigma}, T, \underline{D})}{\partial\underline{D}} \qquad (4)$$

$\underline{Y}$ is called the damage energy release rate. The forces associated to the cumulated plastic strain α and cumulated damage β are respectively R and B, respectively the plastic hardening and the damage strengthening thresholds. With the hypotheses of uncoupling between mechanical plastic and damage dissipations, the second law of thermodynamics yields for an isothermal process :

$$\underline{\sigma} : \dot{\underline{\varepsilon}}_p - R\dot{\alpha} \geq 0 \qquad -\underline{Y}\dot{\underline{D}} - B\dot{\beta} \geq 0 \qquad (5)$$

This induces the existence of a plastic dissipative potential and a damage dissipative potential, chosen in this associated theory frame, as the plastic yield criterion ($F_p = 0$) and a damage evolution criterion ($F_d = 0$).

2.1 Anisotropic elasticity and damage

When the material is damaged, the constitutive elastic law is given hereafter :

$$\underline{\sigma} = \overline{\underline{\underline{C}}_e} \, \varepsilon_e \tag{6}$$

$\overline{\underline{\underline{C}}_e}$ is the elastic stiffness matrix of the damaged material. Using the principle of energy equivalence, we have :

$$\overline{\underline{\underline{C}}_e} = \underline{\underline{M}}^{-1}(\underline{D}) \underline{\underline{C}}_e \, \underline{\underline{M}}^{-1}(\underline{D}) \tag{7}$$

2.2 Anisotropic plastic yield surface

The plastic yield surface is chosen as the Hill's one :

$$F_p(\underline{\sigma},\underline{D},R) = F_p(\overline{\underline{\sigma}},R) = \overline{\sigma_{eq}} - R_0 - R(\alpha) = 0 \tag{8}$$

with R_0 the initial elastic stress threshold and $\overline{\sigma_{eq}}$ the effective anisotropic equivalent stress :

$$\overline{\sigma_{eq}} = \left\{ \frac{1}{2}(\overline{\sigma} - \overline{\underline{\gamma}})^t \underline{\underline{H}}(\overline{\sigma} - \overline{\underline{\gamma}}) \right\}^{\frac{1}{2}} = \left\{ \frac{1}{2}(\underline{\sigma} - \underline{\gamma})^t \overline{\underline{\underline{H}}}(\underline{\sigma} - \underline{\gamma}) \right\}^{\frac{1}{2}} \tag{9}$$

where $\underline{\gamma}$ is the back stress tensor and $\underline{\underline{H}}$ the plastic characteristic Hill tensor for the fictitious undamaged material :The components of this matrix (F, G, H, L, M and N) are parameters characterising the current state of plastic anisotropy. For a strain-hardening material, the uniaxial stress in one direction varies with increasing of plastic strain, and therefore the anisotropic parameters should also vary, since they are function of the current yield stress. To determine them for the current state, we consider that the plastic work should be the same in each direction (three orthotropic directions and three shear planes).

2.2 Damage evolution law and damage surface

By analogy to the plasticity, a damage criterion, chosen as a quadratic homogeneous function of the damage energy release rate $\underline{Y}$, is proposed [3] :

$$F_d = Y_{eq} - B_0 - B(\beta) = 0 \tag{10}$$

with the equivalent damage energy release rate Y_{eq} defined thanks to the damage characteristic tensor $\underline{\underline{J}}$:

$$Y_{eq} = \left\{ \frac{1}{2} \underline{Y}^t \underline{\underline{J}} \, \underline{Y} \right\}^{\frac{1}{2}} \tag{11}$$

A suitable tensor $\underline{\underline{J}}$, simple enough to be applied and able to describe the damage growth, has been proposed [1]:

$$\underline{\underline{J}} = 2 \begin{bmatrix} J_1 & \sqrt{J_1 J_2} & \sqrt{J_1 J_3} \\ \sqrt{J_1 J_2} & J_2 & \sqrt{J_2 J_3} \\ \sqrt{J_1 J_3} & \sqrt{J_2 J_3} & J_3 \end{bmatrix} \tag{12}$$

In the case of damage hardening materials, the equivalent damage energy release rate increases with increasing of the total damage growth. As for the $\underline{\underline{H}}$ matrix components, the anisotropic parameters should also vary. Again,

we suppose that for a current state of damage, the damage work done in each direction should be the same. In the case of a linear damage hardening characterised by its slope Dt, we have :

$$J_i = \left(\frac{Y_{eq}}{Y_i}\right)^2 = \frac{Y_{eq}^2}{\left(\dfrac{Dt_i}{Dt_{eq}}\right)\left(Y_{eq}^2 - Y_{0eq}^2\right) + Y_{0i}^2} \qquad (13)$$

with i = 1 to 3 (the three principal direction of an orthotropic material)
In this model, the reference direction is the rolling direction and $J_1 = 1$

3. Calibration of the model

The model needs to be provided in the initial elastic parameters, six <u>effective</u> stress-strain curves and three damage curves.

The identification of all the parameters of this model can be realised only with tensile tests. These tests are characterised by α, angle between the rolling direction of the sheet and the axial direction of the sample.

3.1 Elastic parameters

The elastic parameters are deduced from tensile tests in the directions $\alpha = 0°$, 45° and 90°. The following hypotheses have to be done :

$$E_3 = \frac{(E_1 + E_2)}{2} \quad \text{and} \quad G_{13} = G_{23} = G_{12} \qquad (14)$$

3.2 Plastic parameters

Uniaxial tensile tests are realised in the domain of large displacements, for $\alpha = 0°$, 15°, 30°, 45°, 60°, 75° and 90°. A statistical method is used [4]. It consists, for a given plastic work level, in minimising a functional of the corresponding stresses and Lankford coefficients in the seven directions, leading to the Hill's parameters F, G, H and N (N = M = L for sheets). The stress-strain curves in the thickness direction and in the shear plane 1-2 can so be deduced. The shear curves in the planes 1-3 and 2-3 are supposed to be equal to the one in the plane 1-2.

3.3 Damage parameters

Three damage curves are necessary. As the hypothesis of linear behaviour is done, they are characterised by the an initial value of Y (damage starting with entrance in plasticity), Y_0, and the slope Dt. From the theory, we have the following relationship for an uniaxial test in direction i :

$$Y_i = \frac{\sigma_i^2}{E_i(1 - D_i)^3} \qquad (15)$$

To find the effective curves, the damage values associated to a given stress value are computed. It corresponds to the resolution of the system hereafter:

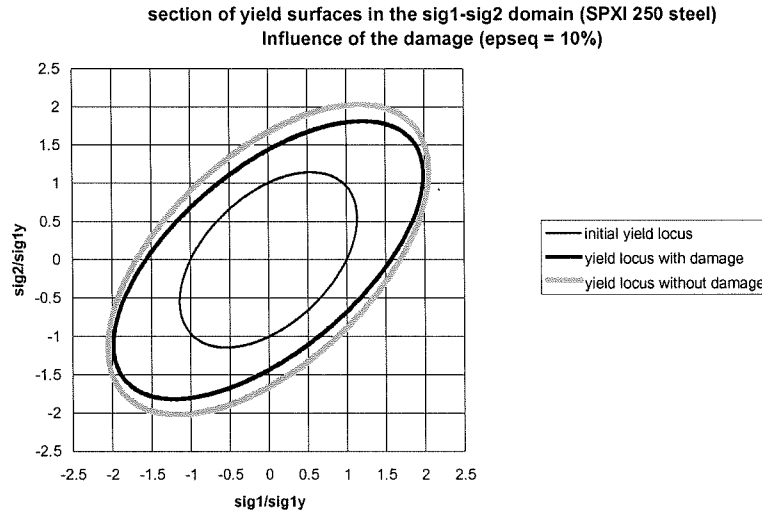$$Y_i = \frac{\sigma_i^2}{E_i(1 - D_i)^3} = Dt_i D_i + Y_{i0} \qquad (16)$$

This yields to a function $\sigma_i(D_i)$ presenting a maximum stress value for a precise value of D_i. Physically, $\sigma_{i\,max}$ must have the same value as the maximum of stress on the real stress-strain curve. Therefore, Dt_i can be easily deduced.
It leads to the conclusion that **no particular damage test is necessary** in the case of a linear damage curve.

4 Validation

The model has been implemented in a non-linear finite element code
The predictions of the model can be illustrated by the following figures, describing the effect on an uniaxial tensile loading.

section of yield surfaces in the sig1-sig2 domain (SPXI 250 steel)
Influence of the damage (epseq = 10%)

This first figure compares the plastic surface obtained by using the true stresses (with damage) and the one which is defined in the fictitious case of no damaging phenomenon (using effective stresses). As it can be observed, the elastic zone is reduced by the damage.
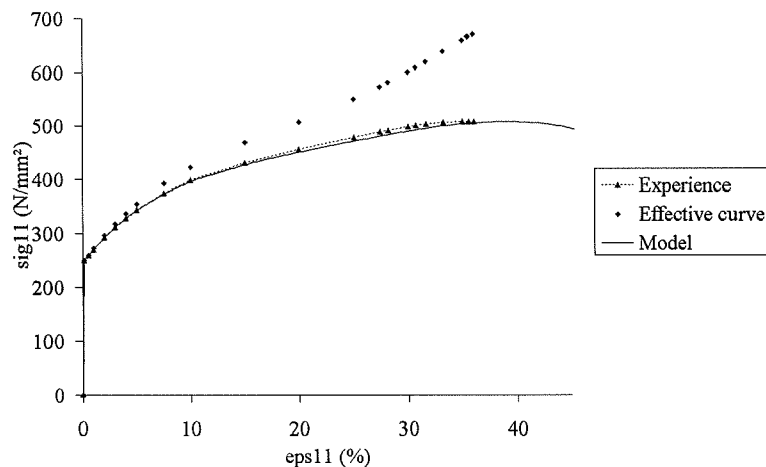
The second figure illustrates the stress evolution during a tensile test in the rolling direction (SPXI 250 steel). It can be observed that the simulated curve is rather near the experimental one. It decreases after reaching the maximum of stress, corresponding to a high damage growth (striction appearance).

5. References

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