

## A NEW DUCTILE DAMAGE EVOLUTION MODEL

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A few ductile damage evolution models based on continuum damage mechanics have been developed in the literature. Some of them are expressed in terms of dissipation potential  $\phi^*$  and the continuum damage variable 'D' as follows:

*Lemaitre's damage model [1]*

$$\phi^* = \left( \frac{S_o}{S_o + 1} \right) \left( \frac{-y}{S_o} \right)^{S_o + 1} \dot{p} \quad (1)$$

$$D = \left( \frac{-y}{S_o} \right)^{S_o + 1} \dot{p} = \frac{D_c}{(\epsilon_R - \epsilon_o)} \left[ \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_m}{\sigma_{eq}} \right)^2 \right] p^{2/m} \dot{p} \quad (2)$$

For proportional loading

$$D = \frac{D_c}{(\epsilon_R - \epsilon_o)} < \left[ \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_m}{\sigma_{eq}} \right)^2 \right] p - \epsilon_o > \quad (3)$$

This model of ductile plastic damage developed on a thermodynamic and effective stress concept is linear with equivalent strain and shows a strong effect of triaxiality.

*W.H. Tai's damage model [2,3]*

$$\phi^* = \frac{1}{2} S_o \left( -\frac{y}{S_o} \right)^2 D \dot{p} \quad (4)$$

$$\dot{D} = m \frac{1}{(\epsilon_R^m - \epsilon_o^m)} \ln \left( \frac{D_c}{D_o} \right) f \left( \frac{\sigma_m}{\sigma_{eq}} \right) p^{2/n} dp \quad (5)$$

where

$$f\left(\frac{\sigma_m}{\sigma_{eq}}\right) = \left[ \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_m}{\sigma_{eq}}\right)^2 \right]$$

and

$$m = \frac{2+n}{n}$$

$$D = D_o \exp \left[ \frac{\ln(D_c/D_o)}{\epsilon_R^m - \epsilon_o^m} < p^m f\left(\frac{\sigma_m}{\sigma_{eq}}\right) - \epsilon_o^m > \right] \quad (6)$$

Tai's model was found suitable for low-carbon steel No. 20 and A3. It was also suggested that the model is suitable for small range of triaxiality ratio.

*Wang Tie-Jun's damage model [4]*

$$\phi^* = \frac{S_o}{2} \left( -\frac{y}{S_o} \right)^2 \frac{\dot{p}}{p^{2n} \left( 1 - \frac{p}{p_c} \right)^{1-\alpha}} \quad (7)$$

$$\dot{D} = \frac{(D_c - D_o)}{\epsilon_R} \left( 1 - \frac{\epsilon_o}{\epsilon_R} \right)^{-\alpha} f\left(\frac{\sigma_m}{\sigma_{eq}}\right) \frac{1}{\left( 1 - \frac{p}{p_c} \right)^\alpha} \quad (8)$$

$$D = D_c - \frac{(D_c - D_o)}{\left( 1 - \frac{\epsilon_o}{\epsilon_R} \right)^\alpha} \left[ 1 - f\left(\frac{\sigma_m}{\sigma_{eq}}\right) \frac{p}{\epsilon_R} \right]^\alpha \quad (9)$$

A coefficient  $\alpha$  has been introduced in the model, where  $\alpha = 1$ , gives a linear model. A nonlinear model can be generated by suitably choosing  $\alpha$  other than 1.

*The damage model proposed by the authors*

A new dissipation potential has been suggested by the authors as the basis for developing an exponential damage evolution model. The final expression for the damage evolution has been developed and given in terms of five material parameters [ $D_o$ ,  $D_c$ ,  $\epsilon_o$  and  $\epsilon_r$ , and the hardening parameter  $n$ ]. The model has been verified with the experimental curves reported by G. LeRoy et al. [5] in Fig. 1.

$$\phi^* = \frac{S_o}{2} \left( -\frac{y}{S_o} \right)^2 \left[ \frac{1}{(1-D)^n} - (1-D) \right] \dot{p} \quad (10)$$

$$D = A \cdot \left( \frac{2+n}{n(n+1)} \right) f \left( \frac{\sigma_m}{\sigma_{eq}} \right) \left[ \frac{1}{(1-D)^n} - (1-D) \right] p^{2n} \dot{p} \quad (11)$$

$$D = 1 - \left\{ 1 - [1 - (1 - D_o)^{n+1}] \exp \left( A \cdot \left\langle p^m f \left( \frac{\sigma_m}{\sigma_{eq}} \right) - \epsilon_o^m \right\rangle \right) \right\}^{1/n+1}$$

where

$$A = \frac{\ln \left[ \frac{1 - (1 - D_c)^{n+1}}{1 - (1 - D_o)^{n+1}} \right]}{(\epsilon_r^m - \epsilon_o^m)} \quad (12)$$

G. LeRoy et al. conducted tensile tests on four spheroidized carbon steel specimens to study the effect on nucleation and growth of voids. For each material, void nucleation strain, rupture strain, initial void fraction and void fraction and rupture strain and the hardening parameters have been listed. A graphical representation of the areal fraction of voids as a function of strain based on experimental results (Fig. 1) shows exponential variation. The author's damage model compares well with experimental results for 1045 steel and 1015 steel. There is little difference for 1090 steel.

The following comments can be made.

Lemaitre's model cannot predict exponential damage evolution as shown in Fig. 1.

W.H. Tai's model can predict exponential damage evolution, but cannot predict a linear one.

Wang Tie-Jun's model can predict both linear and nonlinear damage evolution, but needs an additional material dependent parameter  $\alpha$  which is to be selected by a trial and error approach.

The model presented by the authors can predict exponential damage evolution well as demonstrated in Fig. 1. Also, it needs only the same number of parameters as the W.H. Tai model. However, the model has been found to be sensitive to stress triaxiality. The range of applicability is being investigated. Also, in its present form it cannot predict a linear damage evolution like Wang Tie-Jun's model. This aspect is also being examined.

## REFERENCES

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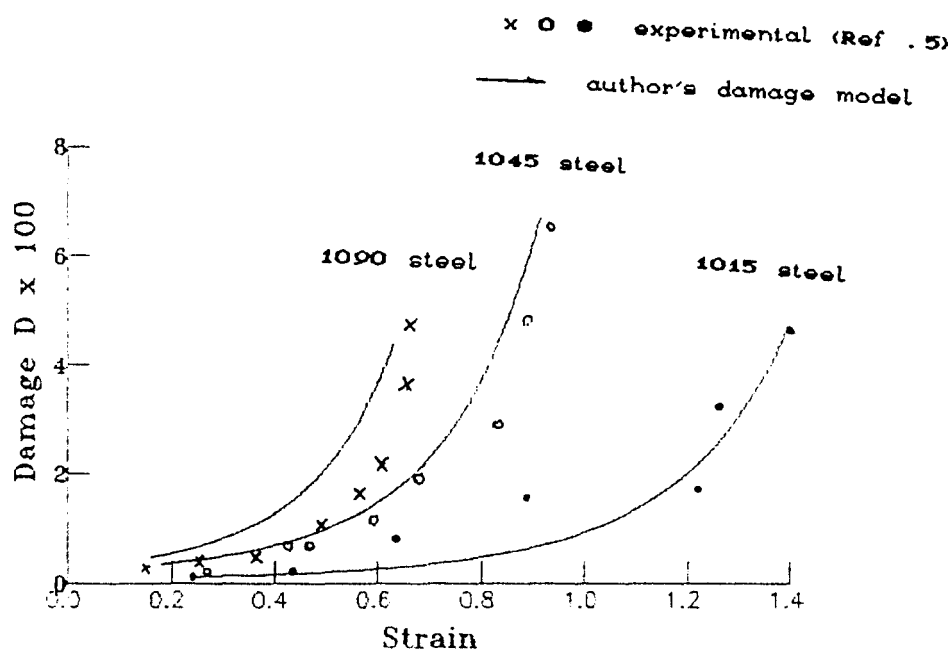


Figure 1. Comparison of author's damage model with experimental results for spheroidized steel.