# Peridynamics-Based Fracture Animation for Elastoplastic Solids

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Figure 1: here is ball crack wall, brittle and ductile, isotropic and anisotropic

#### Abstract

In this paper, we exploit the use of peridynamics theory for graphical animation of material deformation and fracture. We present a new meshless framework for elastoplastic constitutive modeling that contrasts with previous approaches in graphics. Our peridynamics-based elastoplasticity model represents deformation behaviors of materials with high realism. We validate the model by varying the material properties and performing comparisons with FEM simulations. Besides, the integral-based nature of peridynamics makes it trivial to model material discontinuities, which outweighs differential-based methods in both accuracy and ease of implementation. We propose a simple strategy to model fracture in the setting of peridynamics discretization. We demonstrate that the fracture criterion combined with our elastoplasticity model could realistically produce ductile fracture as well as brittle fracture. Our work is the first application of peridynamics in graphics that could create a wide range of material phenomena including elasticity, plasticity, and fracture. The complete framework provides an attractive alternative to existing methods for producing modern visual effects.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation

# 1. Introduction

The simulation of deformable materials has been an important research topic in computer graphics for decades, since the early work by Terzopoulos and colleagues [TPBF87]. One of the strongest driving forces behind the active research is the persistently growing needs for higher realism from the visual effects industry. Materials in real-world exhibit complex behaviors, such as coupled elastoplastic deformations, fracture, etc. The complicated material behaviors are difficult to be virtually replicated by any single method despite the numerous ones that have been developed thus far. Existing approaches generally excel at some phenomena but would stumble (if not fail) at others. For instance, mesh-based methods [MG04, ITF04, TSIF05, SB12] are a good choice to simulate elastic deformations whereas not preferred for phenomena that involve topological changes. Particle-based methods [MCG03, PKA\*05,SSC\*13] are considered suitable for modeling topological changes, however the inherent loss of connectivity information would cause undesirable numerical fracture [LZLW11, ZZL\*16] while simulating large deformations.

We build on recent developments of peridynamics theory in the computational physics community [Sil00, SEW\*07, Mit11, ELP13, MO14] and propose a novel framework for graphical animation of varied deformation behaviors and fracture. Our aim is to enrich available options of simulation techniques for easier and better animation production. Peridynamics was first adopted to animation applications by Levine et al. [LBC\*14]. They described a simple spring-mass system to handle brittle fracture of solids. In contrast, we handle elastoplasticiy, brittle fracture, and ductile fracture in a single framework. To this end, we propose several novel contributions in this work. We first present an elastoplastic constitutive model in the peridynamics-based framework with simple extension to anisotropy, and the model is validated against results produced by FEM. Furthermore, we show that both brittle and ductile fracture phenomena can be naturally represented with nearly no effort by integrating a simple fracture criterion into this material model. This is due to the integral-based formulation of peridynamics, in which forces at a material point are computed by gathering contributions from all material points in its interaction range through integration. On the other hand, methods based on classical continuum mechanics formulate force computations with partial differential equations that fail to be applicable directly on singularities such as a crack. This feature makes our peridynamics-based framework more attractive over existing approaches for producing animations that involve fracture. Lastly, our method is simple to implement and trivially parallelizable, providing a useful alternative to previous methods for animation production.

### 2. Related Work

A large body of literature has been devoted to physical simulation of natural phenomena as a result of active research. A complete literature review is beyond the scope of this paper. In the following we comment only on the representative works most related to ours.

Elastoplasticity Animation The modeling of deformable plasticity in graphics dates back to the pioneering work by Terzopoulos and Fleischer [TF88]. O'Brien and colleagues [OBH02] incorporated a similar additive plasticity model into a finite element simulation to animate ductile fracture. The strain measure was decomposed into two components, where one is due to elastic deformation and the other due to plastic deformation. Müller et al. [MKN\*04] applied this model in their point-based animation framework and simulated plastic behaviors of objects. Irving et al. [ITF04] presented a multiplicative formulation of plasticity and pointed out that their model was better handling finite plastic deformation than the additive one. In contrast to the additive model, they decomposed the deformation gradient into two components through multiplication. The multiplicative model was extensively used by later works to animate phenomena that involve plasticity. Bargteil et al. simulated large viscoplastic flow [BWHT07], Gerszewski and his colleagues animated elastoplastic solids [GBB09], and Stomakhin et al. modeled plasticity of snow [SSC\*13], just to name a few. Unfortunately, neither of the above plasticity models applies in the peridynamics framework because there is no concept of strain nor deformation gradient in the integral-based formulation. As a result, we present a new constitutive model for peridynamics in this work to animate elastoplastic solids.

Fracture Animation Numerous methods have been proposed on fracture animation [MBP14, WWD15] because the stunning phenomenon of fracture and failure is an indispensable visual element in animated movies and video games. Early approaches use simple schemes to model fracture, such as the finite difference method [TF88], the mass-spring system [NTB\*91], and the masspoint constraint system [SWB01]. O'Brien and colleagues [OH99] adopted techniques from continuum mechanics and presented a FEM-based method to simulate brittle fracture of solids. They later extended their method to ductile fracture by incorporating a plasticity model [OBH02]. Müller et al. [MMDJ01] employed a quasi-static finite element analysis to animate brittle fracture of stiff materials undergoing collisions. Parker et al. [PO09] presented some useful techniques for real-time simulation of fracture in game environment. One major issue in FEM-based methods is the generation of fracture patterns on meshes, which would alter

the underlying mesh topology. Early methods typically made use of simple separation along mesh element boundaries [NTB\*91, MMA99, SWB01, MMDJ01] or even element deletion [FDA02]. Mesh subdivision prior to splitting could somewhat increase the available geometric details [MK00, BG00], whereas this tended to introduce elements with poor aspect ratios. Allowing failure along more arbitrary paths could generate more geometrically rich fracture patterns [NF99, OH99, OBH02], albeit with the expense of complicated re-meshing. Molino et al. [MBF04] proposed a virtual node algorithm to avoid the complexity of re-meshing, where elements were duplicated into partially filled counterparts with virtual nodes. The virtual node algorithm was frequently used by subsequent works on fracture animation [BHTF07] and mesh cutting [SDF07, WJST14] due to its simplicity compared to re-meshing methods. Other representative mesh-based methods resorted to modal analysis [GMD13] and pure geometric mesh decompositions [MCK13,SO14] for real-time brittle fracture. Most recently, several works explored the boundary element method for rigid body fracture [ZBG15, HW15] where only surface meshes were employed for both representation and computation.

In contrast to mesh-based approaches, meshless methods are generally considered as a better solution for animating topological changes. Based on the moving least square (MLS) meshless framework by Müller et al. [MKN\*04], Pauly and colleagues [PKA\*05] developed a novel meshless method for fracture animation of elastoplastic solids. Steinemann et al. [SOG09] employed surface mesh representation in meshless framework and presented a novel surface tracking technique to efficiently split the meshless deforming objects. Inspired by the rigid body assumption for simulating brittle fracture, Liu et al. [LHLW11] employed quasi-static analysis in a meshless local Petrov-Galerkin framework. Stomakhin et al. modeled the fracture of snow using a meshless material point method [SSC\*13]. Hegemann et al. [HJST13] combined a level set based mesh embedding technique with the material point method to animate dynamic ductile fracture.

Peridynamics The peridynamics theory was first proposed by Silling [Sil00] as a nonlocal reformulation of classical solid mechanics. It contrasts with classical (local) theory in that the state of a material point is influenced by not necessarily the material points located in its immediate vicinity, but also those over long distances. The governing equations of the peridynamics theory are spatial integral equations instead of partial differential equations. The theory was further developed by subsequent works [SEW\*07, ELP13], and its applications to the engineering field such as multiscale material modeling [ABL\*08, SC14] and fracture modeling [AXS06, SWAB10, SA14] were studied. A comprehensive review of the research literature in the computational physics community is beyond our scope, we refer the readers to the book by Madenci and Oterkus [MO14]. Levine et al. [LBC\*14] first introduced peridynamics to graphics for fracture animation. Their method was limited to brittle fracture of isotropic elastic materials with a single Poisson ratio of 0.25. Our work, on the other hand, is a complete framework that models elastoplasticity and anisotropy under various parameter settings, representing brittle and ductile fracture with high realism.

#### 3. Background

This section describes background of peridynamics theory, and conceptual comparsion with traditional particle methods.0.5 page.

#### 4. Overview

This section briefly describes the algorithm pipeline.

# 5. Elastoplastic Model

This section introduces the constitutive model. 1.5 page.

# 6. Fracture

This section describes the fracture criterion, and the mesh split strategy. 0.5 page.

#### 7. Results

This section demonstrates some results. 1.5 page.

#### 8. Discussions

This section concludes the paper, and discusses the limitations. 0.5 page.

## References

- [ABL\*08] ASKARI E., BOBARU F., LEHOUCQ R., PARKS M., SILLING S., WECKNER O.: Peridynamics for multiscale materials modeling. In Journal of Physics: Conference Series (2008), vol. 125, IOP Publishing, p. 012078. 2
- [AXS06] ASKARI E., XU J., SILLING S.: Peridynamic analysis of damage and failure in composites. In 44th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada. Reston, VA: AIAA (2006). 2
- [BG00] BIELSER D., GROSS M. H.: Interactive simulation of surgical cuts. In *Proceedings of the 8th Pacific Conference on Computer Graphics and Applications* (Washington, DC, USA, 2000), PG '00, IEEE Computer Society, pp. 116–. 2
- [BHTF07] BAO Z., HONG J.-M., TERAN J., FEDKIW R.: Fracturing rigid materials. *IEEE Transactions on Visualization and Computer Graphics* 13, 2 (Mar. 2007), 370–378. 2
- [BWHT07] BARGTEIL A. W., WOJTAN C., HODGINS J. K., TURK G.: A finite element method for animating large viscoplastic flow. ACM Trans. Graph. 26, 3 (July 2007).
- [ELP13] EMMRICH E., LEHOUCQ R. B., PUHST D.: Peridynamics: a nonlocal continuum theory. In Meshfree Methods for Partial Differential Equations VI. Springer, 2013, pp. 45–65. 1, 2
- [FDA02] FOREST C., DELINGETTE H., AYACHE N.: Removing tetrahedra from a manifold mesh. In Computer Animation, 2002. Proceedings of (2002), IEEE, pp. 225–229. 2
- [GBB09] GERSZEWSKI D., BHATTACHARYA H., BARGTEIL A. W.: A point-based method for animating elastoplastic solids. In *Proceedings* of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation (New York, NY, USA, 2009), SCA '09, ACM, pp. 133–138.
- [GMD13] GLONDU L., MARCHAL M., DUMONT G.: Real-time simulation of brittle fracture using modal analysis. Visualization and Computer Graphics, IEEE Transactions on 19, 2 (2013), 201–209. 2

- [HJST13] HEGEMANN J., JIANG C., SCHROEDER C., TERAN J. M.: A level set method for ductile fracture. In *Proceedings of the 12th ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (New York, NY, USA, 2013), SCA '13, ACM, pp. 193–201. 2
- [HW15] HAHN D., WOJTAN C.: High-resolution brittle fracture simulation with boundary elements. ACM Trans. Graph. 34, 4 (July 2015), 151:1–151:12. 2
- [ITF04] IRVING G., TERAN J., FEDKIW R.: Invertible finite elements for robust simulation of large deformation. In *Proceedings of the 2004* ACM SIGGRAPH/Eurographics Symposium on Computer Animation (Aire-la-Ville, Switzerland, Switzerland, 2004), SCA '04, Eurographics Association, pp. 131–140. 1, 2
- [LBC\*14] LEVINE J. A., BARGTEIL A. W., CORSI C., TESSENDORF J., GEIST R.: A peridynamic perspective on spring-mass fracture. In *Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (Aire-la-Ville, Switzerland, Switzerland, 2014), SCA '14, Eurographics Association, pp. 47–55. 1, 2
- [LHLW11] LIU N., HE X., LI S., WANG G.: Meshless simulation of brittle fracture. Comput. Animat. Virtual Worlds 22, 2-3 (Apr. 2011), 115–124. 2
- [LZLW11] LIU N., ZHU F., LI S., WANG G.: Anisotropic kernels for meshless elastic solids. In *Proceedings of the 2011 12th International Conference on Computer-Aided Design and Computer Graphics* (Washington, DC, USA, 2011), CADGRAPHICS '11, IEEE Computer Society, pp. 349–356. 1
- [MBF04] MOLINO N., BAO Z., FEDKIW R.: A virtual node algorithm for changing mesh topology during simulation. *ACM Trans. Graph. 23*, 3 (Aug. 2004), 385–392. 2
- [MBP14] MUGUERCIA L., BOSCH C., PATOW G.: Fracture modeling in computer graphics. *Computers & Graphics* 45 (2014), 86–100. 2
- [MCG03] MÜLLER M., CHARYPAR D., GROSS M.: Particle-based fluid simulation for interactive applications. In *Proceedings of the 2003* ACM SIGGRAPH/Eurographics Symposium on Computer Animation (Aire-la-Ville, Switzerland, Switzerland, 2003), SCA '03, Eurographics Association, pp. 154–159. 1
- [MCK13] MÜLLER M., CHENTANEZ N., KIM T.-Y.: Real time dynamic fracture with volumetric approximate convex decompositions. ACM Trans. Graph. 32, 4 (July 2013), 115:1–115:10. 2
- [MG04] MÜLLER M., GROSS M.: Interactive virtual materials. In Proceedings of Graphics Interface 2004 (School of Computer Science, University of Waterloo, Waterloo, Ontario, Canada, 2004), GI '04, Canadian Human-Computer Communications Society, pp. 239–246. 1
- [Mit11] MITCHELL J. A.: A nonlocal, ordinary, state-based plasticity model for peridynamics. SAND report 7597 (2011). 1
- [MK00] MOR A. B., KANADE T.: Modifying soft tissue models: Progressive cutting with minimal new element creation. In *Proceedings* of the Third International Conference on Medical Image Computing and Computer-Assisted Intervention (London, UK, UK, 2000), MICCAI '00, Springer-Verlag, pp. 598–607.
- [MKN\*04] MÜLLER M., KEISER R., NEALEN A., PAULY M., GROSS M., ALEXA M.: Point based animation of elastic, plastic and melting objects. In *Proceedings of the 2004 ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (Airela-Ville, Switzerland, Switzerland, 2004), SCA '04, Eurographics Association, pp. 141–151. 2
- [MMA99] MAZARAK O., MARTINS C., AMANATIDES J.: Animating exploding objects. In *Proceedings of the 1999 Conference on Graphics Interface* '99 (San Francisco, CA, USA, 1999), Morgan Kaufmann Publishers Inc., pp. 211–218. 2
- [MMDJ01] MÜLLER M., MCMILLAN L., DORSEY J., JAGNOW R.: Real-time simulation of deformation and fracture of stiff materials. In Proceedings of the Eurographic Workshop on Computer Animation and Simulation (New York, NY, USA, 2001), Springer-Verlag New York, Inc., pp. 113–124. 2

- [MO14] MADENCI E., OTERKUS E.: Peridynamic theory and its applications. Springer, 2014. 1, 2
- [NF99] NEFF M., FIUME E.: A visual model for blast waves and fracture. In *Proceedings of the 1999 Conference on Graphics Interface* '99 (San Francisco, CA, USA, 1999), Morgan Kaufmann Publishers Inc., pp. 193–202. 2
- [NTB\*91] NORTON A., TURK G., BACON B., GERTH J., SWEENEY P.: Animation of fracture by physical modeling. Vis. Comput. 7, 4 (July 1991), 210–219. 2
- [OBH02] O'BRIEN J. F., BARGTEIL A. W., HODGINS J. K.: Graphical modeling and animation of ductile fracture. ACM Trans. Graph. 21, 3 (July 2002), 291–294. 2
- [OH99] O'BRIEN J. F., HODGINS J. K.: Graphical modeling and animation of brittle fracture. In *Proceedings of the 26th Annual Conference on Computer Graphics and Interactive Techniques* (New York, NY, USA, 1999), SIGGRAPH '99, ACM Press/Addison-Wesley Publishing Co., pp. 137–146. 2
- [PKA\*05] PAULY M., KEISER R., ADAMS B., DUTRÉ P., GROSS M., GUIBAS L. J.: Meshless animation of fracturing solids. *ACM Trans. Graph.* 24, 3 (July 2005), 957–964. 1, 2
- [PO09] PARKER E. G., O'BRIEN J. F.: Real-time deformation and fracture in a game environment. In *Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (New York, NY, USA, 2009), SCA '09, ACM, pp. 165–175.
- [SA14] SILLING S. A., ASKARI A.: Peridynamic model for fatigue cracking. Tech. rep., Sandia National Laboratories (SNL-NM), Albuquerque, NM (United States), 2014. 2
- [SB12] SIFAKIS E., BARBIC J.: Fem simulation of 3d deformable solids: A practitioner's guide to theory, discretization and model reduction. In *ACM SIGGRAPH 2012 Courses* (New York, NY, USA, 2012), SIGGRAPH '12, ACM, pp. 20:1–20:50. 1
- [SC14] SILLING S. A., Cox J. V.: Hierarchical multiscale method development for peridynamics. SAND Report 18565 (2014). 2
- [SDF07] SIFAKIS E., DER K. G., FEDKIW R.: Arbitrary cutting of deformable tetrahedralized objects. In *Proceedings of the 2007 ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (Aire-la-Ville, Switzerland, Switzerland, 2007), SCA '07, Eurographics Association, pp. 73–80. 2
- [SEW\*07] SILLING S. A., EPTON M., WECKNER O., Xu J., ASKARI E.: Peridynamic states and constitutive modeling. *Journal of Elasticity* 88, 2 (2007), 151–184. 1, 2
- [Sil00] SILLING S.: Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids* 48, 1 (2000), 175 – 209. 1, 2
- [SO14] SCHVARTZMAN S. C., OTADUY M. A.: Fracture animation based on high-dimensional voronoi diagrams. In Proceedings of the 18th Meeting of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games (New York, NY, USA, 2014), I3D '14, ACM, pp. 15–22. 2
- [SOG09] STEINEMANN D., OTADUY M. A., GROSS M.: Splitting meshless deforming objects with explicit surface tracking. Graph. Models 71, 6 (Nov. 2009), 209–220. 2
- [SSC\*13] STOMAKHIN A., SCHROEDER C., CHAI L., TERAN J., SELLE A.: A material point method for snow simulation. ACM Trans. Graph. 32, 4 (July 2013), 102:1–102:10. 1, 2
- [SWAB10] SILLING S., WECKNER O., ASKARI E., BOBARU F.: Crack nucleation in a peridynamic solid. *International Journal of Fracture 162*, 1-2 (2010), 219–227. 2
- [SWB01] SMITH J., WITKIN A., BARAFF D.: Fast and controllable simulation of the shattering of brittle objects. *Comput. Graph. Forum* 20, 2 (2001), 81–90,. 2
- [TF88] TERZOPOULOS D., FLEISCHER K.: Modeling inelastic deformation: Viscolelasticity, plasticity, fracture. SIGGRAPH Comput. Graph. 22, 4 (June 1988), 269–278. 2

- [TPBF87] TERZOPOULOS D., PLATT J., BARR A., FLEISCHER K.: Elastically deformable models. *SIGGRAPH Comput. Graph.* 21, 4 (Aug. 1987), 205–214. 1
- [TSIF05] TERAN J., SIFAKIS E., IRVING G., FEDKIW R.: Robust quasistatic finite elements and flesh simulation. In *Proceedings of* the 2005 ACM SIGGRAPH/Eurographics Symposium on Computer Animation (New York, NY, USA, 2005), SCA '05, ACM, pp. 181–190.
- [WJST14] WANG Y., JIANG C., SCHROEDER C., TERAN J.: An adaptive virtual node algorithm with robust mesh cutting. In *Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation* (Aire-la-Ville, Switzerland, Switzerland, 2014), SCA '14, Eurographics Association, pp. 77–85. 2
- [WWD15] WU J., WESTERMANN R., DICK C.: A survey of physically based simulation of cuts in deformable bodies. *Comput. Graph. Forum 34*, 6 (Sept. 2015), 161–187. 2
- [ZBG15] ZHU Y., BRIDSON R., GREIF C.: Simulating rigid body fracture with surface meshes. ACM Trans. Graph. 34, 4 (July 2015), 150:1–150:11. 2
- [ZZL\*16] ZHU F., ZHAO J., LI S., TANG Y., WANG G.: Dynamically enriched mpm for invertible elasticity. *Comput. Graph. Forum* (2016). In Press. 1