Peridynamics-Based Fracture Animation for Elastoplastic Solids: Supplementary Technical Document

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1 Introduction

This document presents the derivation of the constitutive model proposed in the paper. Section 2 introduces some preliminaries for the derivation and explains why the classical continuum elasticity theory could be regarded as a special case of peridynamics. Section 3 lists the common procedures to derive the peridynamics formulation of a general hyperelastic constitutive model in continuum mechanics. Finally, section 4 presents the derivation of the model introduced in the paper, which is a *nonlocal* extension of the linear elastic model in the *local* theory.

2 Preliminaries

2.1 Peridynamics equations of motion and the discretization

As is introduced in the paper, in peridynamics the governing equation for any material point located at $\mathbf{x}$ is formulated as below:

$$\rho\ddot{\mathbf{u}}(\mathbf{x}) = \int_{H_{\mathbf{x}}} [\mathbf{T} < \mathbf{x}' - \mathbf{x} > - \mathbf{T} < \mathbf{x} - \mathbf{x}' >] dV_{\mathbf{x}} + \mathbf{b}(\mathbf{x}). \tag{1}$$

The state of material point at $\mathbf{x}$ is influenced by the possibly infinite number of material points $\mathbf{x}'$ that belong to its family $H_{\mathbf{x}}$. When the continuum is discretized into particles, the integral in Equation 1 is replaced with the summations of particles within the horizon $\delta_{\mathbf{x}}$:

$$\rho\ddot{\mathbf{u}}(\mathbf{x}) = \sum_{\mathbf{x}', \mathbf{x}' \in H_{\mathbf{x}}} [\mathbf{T} < \mathbf{x}' - \mathbf{x} > - \mathbf{T} < \mathbf{x} - \mathbf{x}' >] V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}), \tag{2}$$

where $V_{\mathbf{x}'}$ is the volume of the discrete particle and its value depends on the distribution of particles.

2.2 Peridynamics for local interactions

In the limiting case where the horizon $\delta_{\mathbf{x}}$ approaches 0, the material point $\mathbf{x}$ interacts only with its immediate neighbors. See Figure 1, the material point with label k interacts with the other six material points in the immediate vicinity denoted as $(k-l)$,$(k+l)$,$(k-m)$,$(k+m)$,$(k-n)$,and $(k+n)$. This conforms to the classical continuum mechanics, see the book [Bon08] for reference.

In the context of local interactions, Equation 2 for material point k is represented in the following form:

$$\rho_{(k)}\ddot{\mathbf{u}}_{(k)} = \sum_{j=k-l,k+l,k-m,k+m,k-n,k+n} (\mathbf{t}_{(k)(j)} - \mathbf{t}_{(j)(k)}) V_{(j)} + \mathbf{b}_{(k)}, \tag{3}$$

where $\mathbf{t}_{(k)(j)}$ denotes the internal force density that material point j exerted on point k, and $\mathbf{t}_{(j)(k)}$ is the other way around.

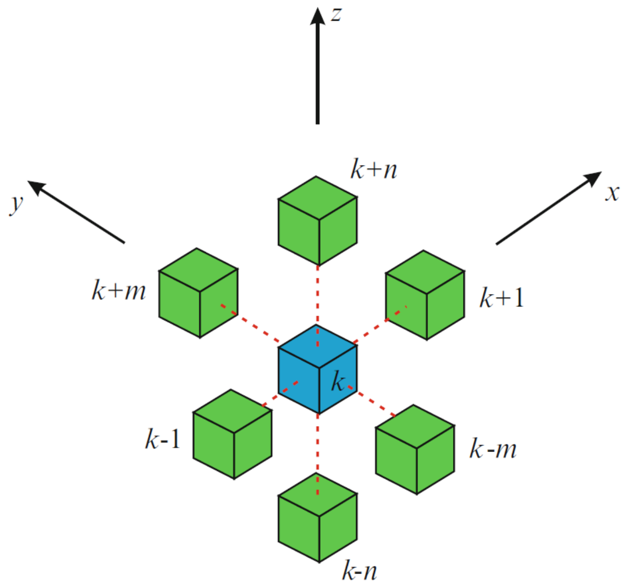

Figure 1: Material point k interacts with its immediate neighborhood. Image from [MO14].

2.3 Strain energy density and internal force density

3 Derivation for general hyperelastic materials

4 Derivation for linear elastic materials

References

[Bon08] Bonet J.: *Nonlinear continuum mechanics for finite element analysis*. Cambridge University Press, Cambridge, UK New York, 2008.

[MO14] Madenci E., Oterkus E.: *Peridynamic theory and its applications*. Springer, 2014.