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Peridynamics-Based Fracture Animation for Elastoplastic Solids

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1. Revision Summary

We have carefully revised our submission according to the suggestions of the reviewers. The major changes include (but not limited to):

1. Blablabla.

2. Answers to Reviewers' Comments

Here we respond to the comments made by each reviewer.

Reviewer #1:

Q: *More explicit discussion on damping is preferred. Particularly, it is very discouraging to know the proposed method needs 'air damping' to stabilize the simulation.*

We apologize for the imprecise description in prior revision note that causes the misunderstanding that we need damping forces to stabilize the simulation, which is not as a matter of fact. We used air damping to slow the motion of objects down so that the length of the video would not be too long, which is indeed a bad idea. However, we only used it in very few of our examples, namely Figure 1(c)(d) and Figure 3. We submitted a video of the example in Figure 3 to support our point, where the settings were identical as before but the damping forces were removed. The simulation is still stable, only that it takes more time for the armadillo to reach rest. We also applied Laplacian smoothing of velocity to a few examples, where the smoothed velocity of a particle is computed as $\mathbf{v}_{\text{new}} = \mathbf{v}_{\text{old}} + \lambda \mathbf{L}(\mathbf{v}_{\text{old}})$. The smoothing is almost negligible since we set λ to very small value of 0.001. We admit that the use of damping forces in our experiment is quite casual, but we argue it is not a necessity for our method. Please refer to the undamped example in the supplementary video.

Reviewer #2:

Q: *Limitations for meshing and explicit fracture comparison to other methods.*

Blablabla.

Reviewer #3:

Q: *How much does the interaction radius parameter δ affect the simulation quality? Do the comparisons hold up for any value of δ? Or does it need to be tuned? How sensitive is the range of values for which peridynamics gives good result, and are there rules of thumb for how δ should be selected?*

We provide a supplement video that compares four different δ values for a bending beam. It shows that the bending beam with δ values of 1.0λ and 1.5λ are visually undistinguished, which suggests that the simulation results are less sensitive to the parameter δ.

Note that peridynamics could converge to the classical continuum mechanics when δ approaching 0 according to [WA05]. So we use a δ value of 1.0λ for most FEM comparisons, which amounts to that we consider only the 1-ring neighbors similar to FEM. Finetuning the δ value is also viable to get a better result. For example, in Figure 13, we adjust it to 1.38λ to achieve our best result.

As for the selection of δ, we recommend a value range from 1.0λ to 2.0λ. A grater δ could result in inadequate family members for those particles close to boundary, thus causing the material behave more softer due to the insufficient force contribution from its family, which can be seen from a bending beam of a δ value 3.0λ in the right-bottom of supplement video. In this case, a correction for boundary particle could be taken into account.

Q: *There are missing steps for discretization. How do we go from equation 1 to discrete equations of motion, where $\mathbf{x}$ and $\mathbf{x}'$ are particles rather than arbitrary points in the material?*

Blablabla.

Q: *Can't the use of the weights defined in equation 4 be interpreted as just a discretization of divergence, except over the entire horizon rather than over an element neighborhood?*

To our minds, we do not think the weight function in Peridynamics amounts to a discretization of divergence in FEM. In fact, by comparing the governing equations in both theories, we could easily derive that the divergence of stress in FEM is equivalent to integration of force density over

horizon, which is

$$\nabla \cdot \sigma = \int_{H_{\mathbf{x}}} [\mathbf{T}\langle \mathbf{x}', \mathbf{x} \rangle - \mathbf{T}\langle \mathbf{x}, \mathbf{x}' \rangle] dV_{\mathbf{x}'} \quad (1)$$

From discretization, we could naturally have

$$\nabla \cdot \sigma = \sum_{H_{\mathbf{x}}} [\mathbf{T}\langle \mathbf{x}', \mathbf{x} \rangle - \mathbf{T}\langle \mathbf{x}, \mathbf{x}' \rangle] V_{\mathbf{x}'} \quad (2)$$

We note that it is the force summation over horizon that should be interpreted as a discretization of divergence. Although weight functions are encoded in force density, ω is used only to compute the force summation in a weighted way, which is inherently different from other methods such as SPH where weight functions are simutaneously used to calculate quantities' first-order or second-order derivatives (e.g. $\nabla \cdot \mathbf{A_i} = -\frac{1}{\rho_i} \sum_j (\mathbf{A_i} - \mathbf{A_j}) \cdot \nabla \omega_{ij}$)

That is, in our formulation, what involved here are just displacements of material points, instead of the spatial derivative of the displacements, which consequently lead to the biggest difference between Peridynamic and other mesh(FEM) or meshless method.

In addition, ω is only associated with positions in reference space, rather than positions in deformed space, so we could pre-gurantee its value rationality by appropriately populating our material points in advance.

Q: *Correction for typos.*

Blablabla.

Q: *Lack discussion for XFEM-based methods in the related work.*

Blablaba.

Q: *How is equation 17 used, exactly? what are "rest_bond_num" and "initial_bond_num" in the definition of ϕ?*

Equation 17 amounts to the cauchy stretch with plastic deformation reasonably excluded, then we further improve it to Equation 18 by incorporating the weight function and increasing critical value through ϕ as fracture proceeding. "rest_bond_num" is the rest active bonds during fracture and "initial_bond_num" is the initial total bonds before fracture. We have modify this part of statements in the paper to eliminate the ambiguities.

Q: *Unreasonable statements about the relationship between rotation artifacts, geometric linearity and stress-strain linearity.*

Blablabla.

Q: *How many elements were used for each example? How did the performance compare to the performance of peridynamics?*

Since we position our particles to the center of elements, the elements and the particles are of the same size in our simulations. We did't detailly compare the performance between FEM and Peridynamics, considering it could be affected by a lof of factors such as integration method and implementation, but we found in our experiments that the efficients of these two method are comparable.

Q: *Question on the supplemental video (random cube)?*

Blablabla.

References

[WA05] WECKNER O., ABEYARATNE R.: The effect of long-range forces on the dynamics of a bar. *Journal of the Mechanics and Physics of Solids 53*, 3 (2005), 705 – 728. 1