Mathematical Formulas for Geomview's Motion Model

Mark Phillips & Nathaniel Thurston

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m = coord sys of moving object; $[x]_m$ = coords of x in m
c = coord sys of center object; $[x]_c$ = coords of x in c
f = coord sys of frame object; $[x]_f$ = coords of x in f
U = universal coord sys; $[x]_U$ = coords of x in U

T_m changes m coords to U coords: $[x]_m \cdot T_m = [x]_U$
T_c changes c coords to U coords: $[x]_c \cdot T_c = [x]_U$
T_f changes f coords to U coords: $[x]_f \cdot T_f = [x]_U$

1. Let p = origin of c: $[p]_c = [0,0,0,1]$. Compute coords of p in f:

$$\begin{aligned} [p]_f &= [p]_c \cdot T_c \cdot T_f^{-1} && (1)\\ &= [p]_U \cdot T_f^{-1} && (2) \end{aligned}$$

2. Let f' be f translated to have same center as c:

$$\begin{aligned} P &= \text{translation taking } [0,0,0,1] \text{ to } [p]_f && (3)\\ T_f' &= P \cdot T_f && (4)\\ &= \text{change } f' \text{ coords to } U \text{ coords: } [x]_{f'} \cdot T_{f'} = [x]_U && (5) \end{aligned}$$

3. $T_R = T_m \cdot T_{f'}^{-1}$ = change f' coords to m coords

4. T_l = local transform

4'. invert T_l if moving object is an ancestor of frame object

5. Compute the transform in m:

$$\begin{aligned} T &= T_R \cdot T_l \cdot T_m^{-1} && (6)\\ &= T_m \cdot T_f^{-1} \cdot P^{-1} \cdot T_l \cdot P \cdot T_f \cdot T_m^{-1} && (7) \end{aligned}$$