Exchangeable random variables

detailed balance in time-reversible Markov chains

$$\pi(z)\kappa(\tilde{z} \mid z) = \pi(\tilde{z})\kappa(z \mid \tilde{z})$$

- q **is any proposal distribution**

- **only need to know π up to a normalizing constant**

Metropolis—Hastings kernel from x to y:

propose x^* from $q(\,\cdot \mid x)$, then set

$$y = \begin{cases} x^* & \text{with prob. } \alpha, \\ x & \text{with prob. } 1-\alpha, \end{cases} \qquad \alpha = \min\left(1, \frac{\pi(x^*)q(x \mid x^*)}{\pi(x)q(x^* \mid x)}\right)$$

$Z \sim \pi$, find conditional distribution kernel $\kappa(\,\cdot \mid z)$ such that if $\tilde{Z} \mid Z \sim \kappa(\,\cdot \mid Z)$, $(Z, \tilde{Z}) \overset{d}{=} (\tilde{Z}, Z)$

Equivalently, find conditional density $\kappa(\cdot \mid z)$ such that

(only worry about conditional exchangeability, because the other condition, knockoff symmetry, is easy to satisfy)

$$(X_j, \tilde{X}_j) = (\tilde{X}_j, X_j) \mid X_{-j}, \tilde{X}_{1:(j-1)}, 1 \leq j \leq p$$

d

Exchangeable random variables

(only worry about conditional exchangeability, because the other condition, knockoff symmetry, is easy to satisfy)

$$(X_j, \tilde{X}_j) \stackrel{d}{=} (\tilde{X}_j, X_j) \mid X_{-j}, \tilde{X}_{1:(j-1)}, 1 \leq j \leq p$$

$Z \sim \pi$, find conditional distribution kernel $\kappa(\,\cdot \mid z)$ such that if $\tilde{Z} \mid Z \sim \kappa(\,\cdot \mid Z)$, $(Z, \tilde{Z}) \stackrel{d}{=} (\tilde{Z}, Z)$

Equivalently, find conditional density $\kappa(\,\cdot \mid z)$ such that $\pi(z)\kappa(\tilde{z} \mid z) = \pi(\tilde{z})\kappa(z \mid \tilde{z})$

detailed balance in
time-reversible Markov chains

Metropolis—Hastings kernel from x to y:

propose x^* from $q(\,\cdot \mid x)$, then set

$$y = \begin{cases} x^* & \text{with prob. } \alpha, \\ x & \text{with prob. } 1 - \alpha, \end{cases} \qquad \alpha = \min\left(1, \frac{\pi(x^*)q(x \mid x^*)}{\pi(x)q(x^* \mid x)}\right)$$

- **q is any proposal distribution**
- **only need to know π up to a normalizing constant**

1. Introduction
    - Variable selection and model-X knockoffs
    - Knockoff sampling is difficult
2. Characterizing knockoff distributions
    - The characterization theorem
    - Connection to Markov chain Monte Carlo (MCMC)
3. **Metropolized knockoff sampling (Metro)**
    - How it works
    - Time complexity and graphical structure
4. Good proposals inspired by the MCMC literature
    - Covariance-guided proposal
    - Multiple-try Metropolis (MTM)
5. Simulation results
6. Discussion