1. With proof, find the ordered pair of values (*a*, *b*) which satisfy the equation below?

$$\sum_{k=1}^{2n}(ak+b) = 7n^2 + 3n$$

Simplify the left hand side in terms of a and b to get to this point:

$$\sum_{k=1}^{2n}(ak+b) = 7n^2 + 3n$$

$$\frac{a(2n)(2n+1)}{2} + b(2n) = 7n^2 + 3n$$

$$an(2n+1) + 2bn = 7n^2 + 3n$$

$$2an^2 + (2b+a)n = 7n^2 + 3n$$

In order for this equation to always be true, we have to equate coefficients, giving us the two following simultaneous equations:

$$2a = 7 \qquad\qquad 2b + a = 3$$

Solving for the first equation we find that a = $\frac{7}{2}$. Plugging this into the second equation, we have

$$2b + \frac{7}{2} = 3$$

$$2b = -\frac{1}{2}$$

$$b = -\frac{1}{4}$$

Thus, the desired ordered pair (a, b) is ($\frac{7}{2}$, $-\frac{1}{4}$).

2. What is the simplified closed form of the following summation in terms of n? Please show each step of work. (Note: the bounds on the inner summation are NOT a misprint!!!)

$$\sum_{a=0}^{n}\left(\sum_{b=a}^{a} 4b\right)$$

$$\sum_{a=0}^{n}\left(\sum_{b=a}^{a} 4b\right) = \sum_{a=0}^{n} 4a = \frac{4n(n+1)}{2} = 2n(n+1)$$

3. What is the closed form of the following summation? Your solution should be a function in terms of n. For full credit work must be shown.

$$\sum_{i=0}^{n}\sum_{j=0}^{i} 2^j$$

$$\sum_{i=0}^{n}\sum_{j=0}^{i} 2^j = \sum_{i=0}^{n}(2^{i+1}-1)$$

$$= \sum_{i=0}^{n} 2^{i+1} - \sum_{i=0}^{n} 1$$

$$= \sum_{i=0}^{n}(2)2^i - (n+1)$$

$$= 2\sum_{i=0}^{n} 2^i - (n+1)$$

$$= 2(2^{n+1}-1) - (n+1)$$

$$= 2^{n+2} - 2 - n - 1$$

$$= 2^{n+2} - n - 3$$

4. Using the fact that if $x \neq 1$, then $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$, for positive integers n, determine the following summation, in terms of n (assume n is a positive integer). Express your answer as a fraction, where the numerator has two terms.

$$\sum_{i=2n+1}^{3n} 4^i$$

First, notice that we can factor out 4^{2n+1} from each term of our summation. Next, we can re-index the summation by noticing that inside the new sum, the terms are $4^0 + 4^1 + \ldots + 4^{n-1}$. Formally, we set j = i - (2n+1).

$$\sum_{i=2n+1}^{3n} 4^i = 4^{2n+1} \sum_{i=2n+1}^{3n} 4^{i-(2n+1)}$$

$$= 4^{2n+1} \sum_{j=0}^{n-1} 4^j$$

$$= 4^{2n+1} (\frac{4^n - 1}{4 - 1})$$

$$= \frac{4^{3n+1} - 4^{2n+1}}{3}$$

Another way to solve the sum is to take the sum from i=1 to 3n, and subtract from it the same sum form i=1 to 2n. If we proceed in this way, we'll get $\frac{4^{3n+1}-1}{4-1} - \frac{4^{2n+1}-1}{4-1}$, after evaluating both sums. Notice that both terms equal to one-third (first -, second +) cancel out and that we arrive at the same answer as above