Solution of bilevel optimization problems using the KKT approach

Stephan Dempe, Susanne Franke

The optimistic version of the bilevel optimization problem reads as

$$\min_{x,y}\{F(x,y) : G(x) \leq 0,\ (x,y) \in \mathbf{gph}\Psi\}, \tag{0.1}$$

where $\mathbf{gph}\Psi$ is the solution set mapping of the so-called lower level problem

$$\Psi(x) = \ \operatorname*{Argmin}_y \ \{f(x,y) : g(x,y) \leq 0\}. \tag{0.2}$$

assumes lower lvl problem is cvx

All functions $F, f, g_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R},\ i = 1,\dots,p$ and $G : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$ are assumed to be smooth, the functions $y \mapsto f(x,y)$ and $y \mapsto g_i(x,y),\ i = 1,\dots,p$ are convex. Problem (0.1), (0.2) has many applications in very different fields, see e.g. [1]. The problem is a nonconvex, nonsmooth optimization problem. This can be seen if we consider e.g. the linear bilevel optimization problem where the feasible set of (0.1) equals the union of faces of a polyhedron describing the feasible set of the lower level problem, see [3].

One method often used is to replace the lower level problem by its Karush-Kuhn-Tucker conditions provided some regularity condition is satisfied. This results in

Slater's condition enforces strong duality

$$\begin{aligned} F(x,y) &\to \min_{x,y,u}, \\ G(x) &\leq 0, \\ \nabla_y L(x,y,u) &= 0, \\ g(x,y) &\leq 0, \\ u &\geq 0, \\ u^\top g(x,y) &= 0, \end{aligned} \tag{0.3}$$

where $L(x,y,u) = f(x,y) + u^\top g(x,y)$ denotes the Lagrange function of (0.2). In [2] it is shown that both problems (0.1) and (0.3) are equivalent if global optimal solutions are computed. But, problem (0.3) is a nonconvex optimization problem for which the Mangasarian-Fromovitz constraint qualification is violated at every feasible point [5]. One promising approach for solving such problems uses a relaxation of the complementarity slackness conditions of (0.3), see [6]. This approach can be shown to converge to certain stationary solution of (0.3). Unfortunately, stationary solutions of (0.3) are in general not related to stationary solutions of the bilevel problem (0.1), see [2]. Using one more approximation, Mersha [4] was able to show convergence to Bouligand stationary solutions of (0.1) under very restrictive assumptions.

In the talk we will show that local optimal solutions of the problem

$$
\begin{array}{rcl}
F(x,y) & \to & \min\limits_{x,y,u} \\
G(x) & \leq & 0 \\
\|\nabla_y L(x,y,u)\| & \leq & \varepsilon_1 \\
g(x,y) & \leq & 0 \\
u & \geq & 0 \\
-u^\top g(x,y) & \leq & \varepsilon_2
\end{array}
\tag{0.4}
$$

converge for $\varepsilon \downarrow 0$ to local optimal solutions of (0.1) under weak assumptions. Problem (0.4) can be solved using standard solution algorithms. Using variational analysis we will also show that stationary solutions of (0.4) converge to C-stationary solutions of (0.3) for $\varepsilon \downarrow 0$ provided the MPEC-MFCQ is satisfied for (0.3).

References

[1] S. Dempe, *Annotated bibliography on bilevel programming and mathematical programs with equilibrium constraints*, Optimization **52** (2003), 333–359.

[2] S. Dempe and J. Dutta, *Is bilevel programming a special case of a mathematical program with complementarity constraints?*, Mathematical Programming **131** (2012), 37–48.

[3] S. Dempe, V. Kalashnikov, G.A. Pérez-Valdés, and N. Kalashnykova, *Bilevel programming problems: Theory, algorithms and application to energy networks*, Springer-Verlag, 2015.

[4] A.G. Mersha, *Solution methods for bilevel programming problems*, Ph.D. thesis, TU Bergakademie Freiberg, 2008.

[5] H. Scheel and S. Scholtes, *Mathematical programs with equilibrium constraints: stationarity, optimality, and sensitivity*, Mathematics of Operations Research **25** (2000), 1–22.

[6] S. Scholtes, *Convergence properties of a regularization scheme for mathematical programs with complementarity constraints*, SIAM J. on Optimization **11** (2001), 918–936.