Вариант 13

Условие:

$$\begin{cases} u_{xx} + 2\sin x u_{xy} + \sin^2 x u_{yy} + \cos x u_y = 0 \\ u|_{x=0} = y^2 \\ u_x|_{x=0} = y^3 \end{cases}$$

$a_{11}a_{22} = 1 \cdot \sin^2 x - \sin^2 x = 0$ - параболическое уравнение

Характеристическое уравнение:

$$dy^2 - 2\sin x \, dydx + \sin^2 x \, dx^2 = 0$$

$$\left(\frac{dy}{dx}\right)^2 - 2\sin x \left(\frac{dy}{dx}\right) + \sin^2 x = 0$$

$$\frac{D}{4} = \sin^2 x - \sin^2 x = 0$$

$$\frac{dy}{dx} = \sin x$$

$$y = -\cos x + C_1$$

$$\begin{cases} \xi = y + \cos x \\ \eta = x \end{cases}$$

$$\xi_x = -\sin x$$

$$\xi_y = 1$$

$$\eta_x = 1$$

$$\eta_y = 0$$

$$\xi_{xy} = 0$$

$$\eta_{xy} = 0$$

$$\xi_{xx} = -\cos x$$

$$\xi_{yy} = 0$$

$$\eta_{xx} = 0$$

$$\eta_{yy} = 0$$

$$u_x = \xi_x u_\xi + \eta_x u_\eta = -\sin x u_\xi + u_\eta$$

$$u_y = u_\xi$$

$$u_{xy} = \xi_x\xi_y u_{\xi\xi} + (\xi_x\eta_y + \xi_y\eta_x)u_{\xi\eta} + \eta_x\eta_y u_{\eta\eta} + \xi_{xy}u_\xi + \eta_{xy}u_\eta$$

$$u_{xx} = \sin^2 x u_{\xi\xi} - 2\sin x u_{\xi\eta} + u_{\eta\eta}$$

$$u_{xx} = \sin^2 x u_{\xi\xi} - 2\sin x u_{\xi\eta} + u_{\eta\eta}$$

$$u_{yy} = u_{\xi\xi}$$

$$u_{xy} = -\sin x u_{\xi\xi} + u_{\xi\eta}$$

$$(\sin^2 x u_{\xi\xi} - 2\sin x u_{\xi\eta} + u_{\eta\eta}) + 2\sin x(-\sin x u_{\xi\xi} + u_{\xi\eta}) + \sin^2 x u_{\xi\xi} + \cos x u_\xi = 0$$

$$u_{\eta\eta} = -\cos x u_\xi$$

$$(-\sin x u_\xi + u_\eta)|_{x=0} = y^3 \Rightarrow u_\eta|_{x=0} = y^3$$