Задание 1

Условие:

$$f(z) = \frac{1}{z(z+2)}, z_0 = -1$$

Решение:

Т.к. -2 и 0 симметричны относительно -1, необходимо рассмотреть 2 области

$$z - z_0 = q$$

$$f(z) = \frac{1}{(z+1-1)(z+1+1)} = \frac{1}{(q-1)(q+1)} = \frac{1}{q^2-1} = -\frac{1}{1-q^2}$$

$$t = q^2$$

$$f(z) = -\frac{1}{1-t} = -\sum_{n=0}^{\infty} t^n = \sum_{n=0}^{\infty} -t^n = \sum_{n=0}^{\infty} -q^{2n} = \sum_{n=0}^{\infty} -(z+1)^{2n}$$

$$\text{Для } |z+1| > 1:$$

$$f(z) = \frac{1}{z(z+2)} = \frac{1}{(q-1)(q+1)} = \frac{1}{2}\frac{1}{q-1} - \frac{1}{2}\frac{1}{1+q} =$$

$$\frac{1}{q-a} = \frac{\frac{1}{q}}{1-\frac{a}{q}} = \frac{1}{q}\cdot\frac{1}{1-\frac{a}{q}} = \frac{1}{q}\sum_{n=0}^{\infty}\left(\frac{a}{q}\right)^n = \sum_{n=0}^{\infty}\frac{a^n}{q^{n+1}}$$

$$= \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{q^{n+1}} - \sum_{n=0}^{\infty}\frac{(-1)^n}{q^{n+1}}\right) = \frac{1}{2}\sum_{n=0}^{\infty}\frac{1-(-1)^n}{q^{n+1}} = \sum_{n=0}^{\infty}\frac{1}{2}\cdot\frac{1-(-1)^n}{(z+1)^{n+1}} = \sum_{n=0}^{\infty}\frac{1}{(z+1)^{2(n+1)}}$$

Задание 2

Условие:

$$f(z) = z^3 \cos\frac{1}{z^2}$$

Решение:

$$\text{Особые точки:}$$

$$z = 0 : \lim_{z\to 0} z^3 \cos\frac{1}{z^2} = \nexists$$

$$z^3\cos\frac{1}{z^2} = z^3\cdot\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{1}{z^2}\right)^{2n}}{(2n)!} = z^3\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n)!z^{4n}} = \sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n)!z^{4n-3}}$$

$$c_{-1} = -1\cdot\frac{1}{2!} = -\frac{1}{2}$$

$$\operatorname*{Res}_{z=0} f = -\frac{1}{2}$$

$$z = \infty: \text{ т.к. сумма всех вычетов равна 0, то} \operatorname*{Res}_{z=\infty} f = \frac{1}{2}$$

$$\text{Очевидно, что бесконечность - это полюс 3-его порядка}$$

Задание 3

Условие:

$$y = -x$$
$$w = \frac{z+1}{z-1}$$

Решение:

$$w(z-1) = z+1$$
$$(u+vi)(x+yi-1) = x+yi+1$$
$$ux - u - vy + i(vx - v + uy) = x + 1 + iy$$
$$\begin{cases} ux - u - vy = x+1 \\ vx - v + uy = y \\ y = -x \end{cases} \Rightarrow$$
$$\begin{cases} ux - u + vx = x+1 \\ vx - v - ux = -x \end{cases} \Rightarrow \begin{cases} x(u+v-1) = 1+u \\ x(v-u+1) = v \end{cases} \Rightarrow (1+u)(v-u+1) = v(u+v-1) \Rightarrow$$
$$v \cancel{-u} + 1 \cancel{+uv} - u^2 \cancel{+u} = \cancel{vu} + v^2 - v$$
$$1 = u^2 + v^2 - 2v$$
$$2 = u^2 + (v-1)^2 - \text{окружность}$$

Задание 4

Условие:

$$\frac{\operatorname{arctg}\frac{1}{2}}{2} \leq \mathrm{Re}z \leq \frac{\pi}{8}, \quad \omega = \operatorname{tg} z$$

Решение:

$$w = \operatorname{tg} z = \frac{e^{2iz}-1}{i(e^{2iz}+1)} - \text{ композиция линейной, экспоненты и дробно-линейной:}$$
$$w_1 = 2iz$$
$$w_2 = e^{w_1}$$
$$w = -i \cdot \frac{w_2 - 1}{w_2 + 1}$$
$$w_1 : \text{i повернёт полосу относительно } (0{,}0) \text{ против часовой на } \frac{\pi}{2}$$
$$2 \text{ "растянет" в 2 раза} \Rightarrow$$

$$w_1 :$$
$$\operatorname{arctg}\frac{1}{2} \leq \mathrm{Im}w_1 \leq \frac{\pi}{4}$$
$$w_2 :$$
$$w_2 = \exp(w_1) = \exp(\mathrm{Re}w_1 + i\mathrm{Im}w_1) = \exp(\mathrm{Re}w_1) \cdot \exp(i\mathrm{Im}w_1)$$
$$w_2 \text{ задаёт угол с следующим условием:}$$
$$\operatorname{arctg}\frac{1}{2} \leq \varphi \leq \frac{\pi}{4} \Rightarrow$$
$$\text{Границы:}$$
$$y = x$$
$$y = \frac{x}{2}$$
$$w :$$
$$w = -i \cdot \frac{w_2 - 1}{w_2 + 1}$$
$$(u+vi)(x+iy+1) = -i(x+iy-1)$$
$$ux + u - vy + i(uy + vx + v) = y + i(-x+1)$$
$$\begin{cases} ux + u - vy = y \\ uy + vx + v = -x + 1 \end{cases}$$
$$\text{Использую условия, задающие границы :}$$

$$u+vi=-i\cdot\frac{(1+i)x-1}{(1+i)x+1}$$
$$u+ux-vx+i(ux+vx+v)=x+i(1-x)$$
$$\begin{cases}u+ux-vx=x\\ ux+vx+v=1-x\end{cases}\Rightarrow\begin{cases}x(u-v-1)=-u\\ x(u+v+1)=1-v\end{cases}\Rightarrow$$
$$(1-v)(u-v-1)=-u(u+v+1)$$
$$u-v-1-vu+v^2+v=-u^2-uv-u$$
$$v^2+u^2+2u=1$$
$$v^2+(u+1)^2=2$$

Предыдущие попытки:

$$y=\alpha x$$
$$\begin{cases}xu+u-\alpha xv=\alpha x\\ \alpha xu+xv+v+x=1\end{cases}\Rightarrow\begin{cases}x(u-\alpha v-\alpha)=-u\\ x(\alpha u+v+1)=1-v\end{cases}\Rightarrow$$
$$(1-v)(u-\alpha v-\alpha)=-u(\alpha u+v+1)$$
$$u-\cancel{\alpha v}-\alpha\cancel{-vu}+\alpha v^2\cancel{+\alpha v}=-\alpha u^2\cancel{-uv}-u$$
$$\alpha v^2+\alpha u^2+2u=\alpha$$
$$v^2+\left(u+\frac{1}{\alpha}\right)^2=1+\frac{1}{\alpha^2}$$
$$\alpha=1:\ \ v^2+(u+1)^2=2$$
$$\alpha=\frac{1}{2}:\ \ v^2+(u+2)^2=5$$

Предыдущие попытки:

$$\omega=\operatorname{tg}(x+yi)$$
$$\begin{cases}y\in\mathbb{R}\\ \frac{\operatorname{arctg}\frac{1}{2}}{2}\le x\le\frac{\pi}{8}\\ \omega=\operatorname{tg}(x+iy)\\ \omega=u+vi\end{cases}\Rightarrow\begin{cases}y\in\mathbb{R}\\ \frac{\operatorname{arctg}\frac{1}{2}}{2}\le x\le\frac{\pi}{8}\\ u+vi=\frac{\sin(x+yi)}{\cos(x+iy)}=\frac{\sin x\operatorname{ch}y+\cos x\operatorname{sh}y\cdot i}{\cos x\operatorname{ch}y-\sin x\operatorname{sh}y\cdot i}=\frac{\operatorname{tg}x+\operatorname{th}y\cdot i}{1-\operatorname{tg}x\operatorname{th}y\cdot i}\end{cases}\Rightarrow$$
$$(1-\operatorname{tg}x\operatorname{th}y\cdot i)(u+vi)=\operatorname{tg}x+\operatorname{th}y\cdot i$$
$$u+\operatorname{tg}x\operatorname{th}y\,v+i(v-\operatorname{tg}x\operatorname{th}y\,u)=\operatorname{tg}x+\operatorname{th}y\,i$$
$$\begin{cases}u+\operatorname{tg}x\operatorname{th}yv=\operatorname{tg}x\\ v-\operatorname{tg}x\operatorname{th}yu=\operatorname{th}y\end{cases}\Rightarrow\begin{cases}u+v=\operatorname{tg}x+\operatorname{th}y\\ u-v+\operatorname{tg}x\operatorname{th}y(\operatorname{tg}x+\operatorname{th}y)=\operatorname{tg}x-\operatorname{th}y\end{cases}\Rightarrow$$
$$\begin{cases}u=\operatorname{tg}x-\operatorname{tg}x\operatorname{th}y(\operatorname{tg}x+\operatorname{th}y)\\ v=\operatorname{th}y+\operatorname{tg}x\operatorname{th}y(\operatorname{tg}x+\operatorname{th}y)\end{cases}$$
$$\begin{cases}y\in R\\ \frac{\operatorname{arctg}\frac{1}{2}}{2}\le x\le\frac{\pi}{8}\\ u=\operatorname{tg}x-\operatorname{tg}x\operatorname{th}y(\operatorname{tg}x+\operatorname{th}y)\\ v=\operatorname{th}y+\operatorname{tg}x\operatorname{th}y(\operatorname{tg}x+\operatorname{th}y)\end{cases}\Rightarrow\begin{cases}\theta\in[-1,1]\\ \operatorname{tg}\left(\frac{\operatorname{arctg}\frac{1}{2}}{2}\right)\le\xi\le\operatorname{tg}\frac{\pi}{8}\\ u=\xi-\xi\theta(\xi+\theta)\\ v=\theta+\xi\theta(\xi+\theta)\end{cases}$$
$$\eta=\operatorname{tg}(2\operatorname{arctg}(\xi))$$
$$\xi=\operatorname{tg}\left(\frac{\operatorname{arctg}\eta}{2}\right)$$
$$\begin{cases}-1\le\theta\le1\\ \frac{1}{2}\le\eta\le1\\ u=\operatorname{tg}\left(\frac{\operatorname{arctg}\eta}{2}\right)-\operatorname{tg}\frac{\operatorname{arctg}\eta}{2}\theta\left(\operatorname{tg}\frac{\operatorname{arctg}\eta}{2}+\theta\right)\\ v=\theta+\operatorname{tg}\frac{\operatorname{arctg}\eta}{2}\theta\left(\operatorname{tg}\frac{\operatorname{arctg}\eta}{2}+\theta\right)\end{cases}$$

$$\operatorname{tg} 2x = \frac{2\operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

$$(1 - t^2)p = 2t$$

$$t^2 + \frac{2}{p}t - 1 = 0$$

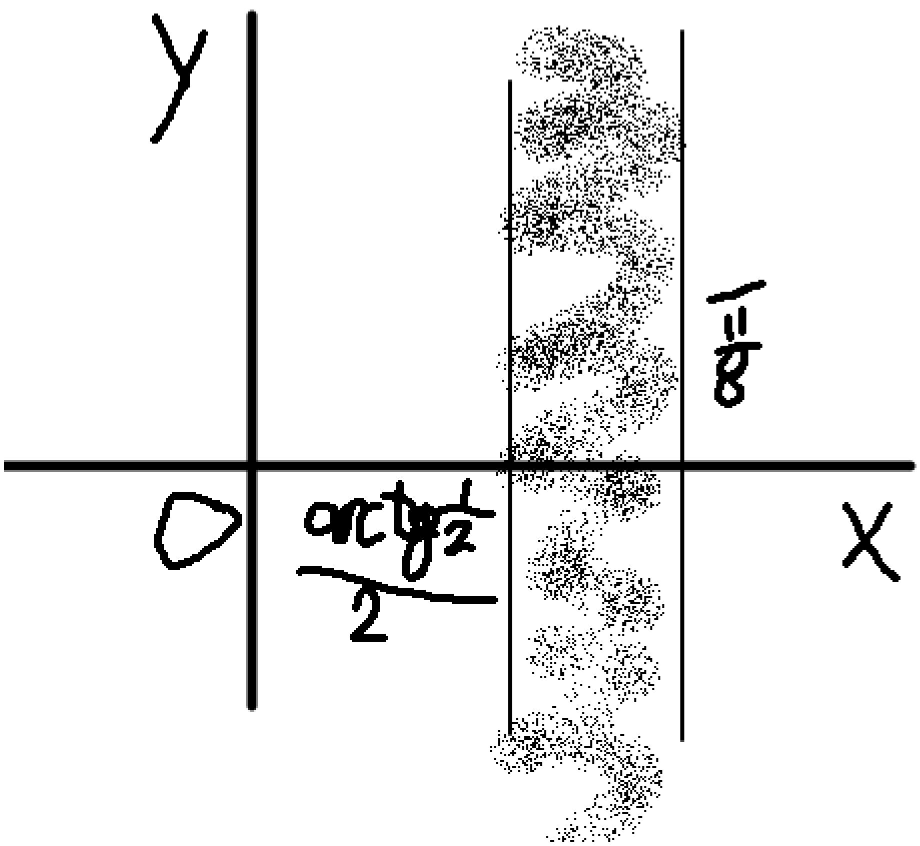
$$\frac{D}{4} = \frac{1}{p^2} + 1 = \frac{1 + p^2}{p^2}$$

$$t = -\frac{1 \pm \sqrt{1 + p^2}}{p} \Rightarrow \operatorname{tg}\frac{x}{2} = \frac{\sqrt{1 + \operatorname{tg}^2 x} - 1}{\operatorname{tg} x}$$

$$\frac{\sqrt{1 + \frac{1}{4}} - 1}{\frac{1}{2}} = \sqrt{5} - 2$$

$$\frac{\sqrt{1 + 1} - 1}{1} = \sqrt{2} - 1$$

$$\begin{cases} -1 \le \theta \le 1 \\ \sqrt{5} - 2 \le \xi \le \sqrt{2} - 1 \\ u = \xi - \xi\theta(\xi + \theta) \\ v = \theta + \xi\theta(\xi + \theta) \end{cases}$$



Из графических соображений понятно, что образ искомого множества лежит между кривыми, которые являются образами его границы.

$$\operatorname{tg}(\alpha+\beta i)=\frac{\sin(\alpha)\cos(\beta i)+\sin(\beta i)\cos(\alpha)}{\cos(\alpha)\cos(\beta i)-\sin(\alpha)\sin(\beta i)}=\frac{\sin\alpha\operatorname{ch}\beta+\cos\alpha\operatorname{sh}\beta i}{\cos\alpha\operatorname{ch}\beta-\sin\alpha\operatorname{sh}\beta i}=\frac{\operatorname{tg}\alpha+\operatorname{th}\beta i}{1-\operatorname{tg}\alpha\operatorname{th}\beta i}$$

$$\operatorname{tg}\left(\frac{\operatorname{arctg}\frac{1}{2}}{2}+yi\right)=\frac{\operatorname{tg}\left(\frac{\operatorname{arctg}\frac{1}{2}}{2}\right)+\operatorname{th}yi}{1-\operatorname{tg}\left(\frac{\operatorname{arctg}\frac{1}{2}}{2}\right)\operatorname{th}yi}=u+vi$$

$$T=\operatorname{tg}\left(\frac{\operatorname{arctg}\frac{1}{2}}{2}\right),\ \theta=\operatorname{th}y$$

$$\frac{T+\theta i}{1-T\theta i}=u+vi\Rightarrow T+\theta i=u+T\theta v+i(v-T\theta u)$$

$$\begin{cases}u+\theta Tv=T\\ v-\theta Tu=\theta\end{cases}\Rightarrow\begin{cases}u+v=\theta+T\\ u-v=-2\theta T(\theta+T)+T-\theta\end{cases}\Rightarrow\begin{cases}u=T-\theta T(\theta+T)\\ v=\theta+\theta T(\theta+T)\end{cases}$$

$$\operatorname{tg}2x=\frac{2\operatorname{tg}x}{1-\operatorname{tg}^2x}$$

$$(1-t^2)p=2t$$

$$t^2+\frac{2}{p}t-1=0$$

$$\frac{D}{4}=\frac{1}{p^2}+1=\frac{1+p^2}{p^2}$$

$$t=-\frac{1\pm\sqrt{1+p^2}}{p}\Rightarrow\operatorname{tg}\frac{x}{2}=\frac{\sqrt{1+\operatorname{tg}^2x}-1}{\operatorname{tg}x}$$

$$\frac{\sqrt{1+\frac{1}{4}}-1}{\frac{1}{2}}=\sqrt{5}-2$$

$$\frac{\sqrt{1+1}-1}{1}=\sqrt{2}-1$$

$$\begin{cases}u+\theta Tv=T\\ v-\theta Tu=\theta\end{cases}\Rightarrow\begin{cases}\theta=\frac{T-u}{Tv}\\ \theta=\frac{v}{1+Tu}\end{cases}\Rightarrow(T-u)(1+Tu)=Tv^2$$

$$T-u+T^2u-Tu^2=Tv^2$$

$$Tv^2+Tu^2+(1-T^2)u=T$$

$$v^2+u^2+2\cdot\frac{1-T^2}{2T}u+\frac{(1-T^2)^2}{4T^2}=T+\frac{(1-T^2)^2}{4T^2}$$

$$v^2+\left(u+\frac{1-T^2}{2T}\right)^2=T+\frac{(1-T^2)^2}{4T^2}$$

$$T=\sqrt{5}-2:T^2=9-4\sqrt{5}$$

$$v^2+\left(u+\frac{-8+4\sqrt{5}}{2\sqrt{5}-4}\right)^2=\sqrt{5}-2+2^2$$

$$v^2+(u+2)^2=\sqrt{5}+2$$

$$T=\sqrt{2}-1:T^2=3-2\sqrt{2}$$

$$v^2+\left(u+\frac{-2+2\sqrt{2}}{2\sqrt{2}-2}\right)^2=\sqrt{2}-1+1^2$$

$$v^2+(u+1)^2=\sqrt{2}$$

2 окружности с центрами в $(-2,0)$ и $(-1,0)$ и радиусами $\sqrt{\sqrt{5}+2}$ и $\sqrt{\sqrt{2}}$.

Задание 5

Условие:

$$\omega = \operatorname{Artg} z$$
$$z_0 = \frac{4}{3} i$$

Решение:

$$\omega = \operatorname{Artg} z_0 \Leftrightarrow \operatorname{tg} \omega = z_0$$
$$\begin{cases} \omega = u + vi \\ z_0 = \frac{4}{3} i \end{cases} \Rightarrow \operatorname{tg}(u + vi) = \frac{4}{3} i$$
$$\frac{\sin(u)\cos(vi) + \sin(vi)\cos(u)}{\cos(u)\cos(vi) - \sin(u)\sin(vi)} = \frac{\operatorname{tg} u + \operatorname{th} vi}{1 - \operatorname{tg} u \operatorname{th} vi} = \frac{4}{3} i$$
$$\operatorname{tg} u + \operatorname{th} vi = \frac{4}{3} \operatorname{tg} u \operatorname{th} v + \frac{4}{3} i$$
$$\begin{cases} \operatorname{tg} u = \frac{4}{3} \operatorname{tg} u \operatorname{th} v \\ \operatorname{th} v = \frac{4}{3} \end{cases} \Rightarrow \begin{cases} \operatorname{tg} u \left(\frac{16}{9} - 1\right) = 0 \\ \operatorname{th} v = \frac{4}{3} \end{cases} \Rightarrow \begin{cases} \operatorname{tg} u = 0 \\ \operatorname{th} v = \frac{4}{3} \end{cases} \Leftrightarrow \begin{cases} u = \pi k, k \in \mathbb{Z} \\ v = \operatorname{arth}\left(\frac{4}{3}\right) \end{cases}$$

Задание 6

Условие:

$$\sum_{n=0}^{\infty} \frac{(-1)^n (z - 2 + 2i)^n}{3^n \left(1 + \frac{1}{n}\right)^n}$$

Решение:

$$q = z - 2 + 2i$$
$$\cdot \sum_{n=0}^{\infty} \frac{|q|^n}{3^n \left(1 + \frac{1}{n}\right)^n}$$
$$a_n = \frac{|q|^n}{3^n \left(1 + \frac{1}{n}\right)^n}$$
$$\frac{a_{n+1}}{a_n} = \frac{|q|^{n+1}}{3^{n+1}\left(1 + \frac{1}{n+1}\right)^{n+1}} \cdot \frac{3^n \left(1 + \frac{1}{n}\right)^n}{|q|^n} = \frac{|q|}{3} \cdot \frac{1}{1 + \frac{1}{n+1}} \cdot \left(\frac{1 + \frac{1}{n}}{1 + \frac{1}{n+1}}\right)^n = \frac{|q|}{3} \cdot \frac{1}{1 + \frac{1}{n+1}} \cdot \left(\frac{(n+1)^2}{n(n+2)}\right)^n =$$
$$= \frac{|q|}{3} \cdot \frac{1}{1 + \frac{1}{n+1}} \cdot \left(1 + \frac{1}{n^2 + 2n}\right)^{(n^2 + 2n) \cdot \frac{1}{n+2}}$$
$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \frac{|q|}{3} \cdot \frac{1}{1 + \frac{1}{n+1}} \cdot e^{\frac{1}{n+2}} = \frac{|q|}{3} \Rightarrow |q| < 3 \text{ - область абсолютной сходимости}$$
$$|q| = 3:$$
$$\sum_{n=0}^{\infty} \frac{1}{\left(1 + \frac{1}{n}\right)^n}$$
$$b_n = \frac{1}{\left(1 + \frac{1}{n}\right)^n}$$
$$\lim_{n \to \infty} b_n = \frac{1}{e} \neq 0 \Rightarrow \text{не сходится даже условно}$$
$$3^2 = 9$$
$$2^2 + 2^2 = 8 < 9$$
$$3^2 + 0 = 9$$
$$3^2 + 1^2 > 9$$

Задание 7

Условие:

$$\oint_{|z|=8} \frac{\sin z}{z^4(z+4i)}dz$$

Решение:

$$f(z) = \frac{\sin z}{z^4(z+4i)}$$

Особые точки:

$z = 0 -$ полюс $n = 4$

$z = -4i$ - полюс $n = 1$

$z = \infty$ - лежит вне контура

$$\oint_L f(z)dz = -2\pi i \cdot \operatorname*{Res}_{z=\infty} f$$

$$\operatorname*{Res}_{z=\infty} f:$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\frac{1}{z+4i} = \sum_{n=0}^{\infty} \frac{(-4i)^n}{z^{n+1}}$$

$$f = \frac{1}{z^4}\left(\frac{1}{z} - \frac{4i}{z^2} - \frac{16}{z^3} + \frac{64i}{z^4} + \frac{256}{z^5} - \dots\right)\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \frac{z^9}{9!} - \frac{z^{11}}{11!} + \dots\right)$$

$$\operatorname*{Res}_{z=\infty} f = -\left(\frac{1}{5!} \cdot -4i - \frac{1}{7!} \cdot 64i + \dots + \frac{(-1)^n}{(2n+1)!} \cdot (-4i)^{2n+1}\right) = -\sum_{n=0}^{\infty} \frac{(-1)^n(-4i)^{2n+1}}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-i)^{2n+1}4^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{-i4^{2n+1}}{(2n+1)!} = -i\sum_{n=0}^{\infty} \frac{4^{2n+1}}{(2n+1)!} = -i \operatorname{sh} 4$$

$$\operatorname*{Res}_{z=\infty} f = i \operatorname{sh} 4$$

$$\oint_L f(z)dz = 2\pi \operatorname{sh} 4$$

Ответ неправильный

$$\operatorname*{Res}_{z=0} f = \frac{1}{6} \lim_{z\to 0} \left(\frac{\sin z}{z+4i} \right)'''$$

$$a' = \cos z$$

$$a'b = z\cos z + 4i\cos z$$

$$b' = 1$$

$$ab' = \sin z$$

$$a'b - ab' = z\cos z + 4i\cos z - \sin z$$

$$\left(\frac{z\cos z + 4i\cos z - \sin z}{z^2 + 8iz - 16} \right)''$$

$$a' = \cos z - z\sin z - 4i\sin z - \cos z = -z\sin z - 4i\sin z$$

$$a'b = -z^3\sin z - 12iz^2\sin z + 48z\sin z + 16\sin z$$

$$b' = 2z + 8i$$

$$ab' = 2z^2\cos z + 16iz\cos z - 32\cos z - 2z\sin z - 8i\sin z$$

$$a'b - ab' = -z^3\sin z - 12iz^2\sin z - 2z^2\cos z + 50z\sin z - 16iz\cos z + (16+8i)\sin z + 32\cos z$$

$$\left(\frac{-z^3\sin z - 12iz^2\sin z - 2z^2\cos z + 50z\sin z - 16iz\cos z + (16+8i)\sin z + 32\cos z}{z^4 + 16iz^3 - 96z^2 - 256iz + 256} \right)$$

$$a' = \cancel{-3z^2\sin z} - \cancel{z^3\cos z} - \cancel{24iz\sin z} - \cancel{12iz^2\cos z} - \cancel{4z\cos z} + \cancel{2z^2\cos z} + \cancel{50\sin z} + \cancel{50z\cos z}$$

$$- \cancel{16i\cos z} + \cancel{16iz\sin z} + \cancel{(16+8i)\cos z} - \cancel{32\sin z} =$$

$$(0)z^3\sin z + (-1)z^3\cos z + (-3)z^2\sin z + (-12i+2)z^2\cos z +$$

$$(-24i+16i)z\sin z + (-4+50)z\cos z + (50-32)\sin z + (-16i+16+8i)\cos z =$$

$$\cancel{-z^3\cos z} - \cancel{3z^2\sin z} + \cancel{(-12i+2)z^2\cos z} + (-24i+16i)z\sin z + 46z\cos z + 18\sin z + (16-8i)\cos z$$

$$a'b =$$

$$(-1)z^7\cos z +$$
$$(-16i - 12i + 2)z^6\cos z +$$
$$(+96 - 192i^2 + 32i + 46)z^5\cos z +$$
$$(+256i + 1152i - 192 + 736i_1 6 - 8i)z^4\cos z +$$
$$(-256 + 3072i^2 - 512i - 4416 + 256i - 128i^2)z^3\cos z +$$
$$(-3072i + 512 - 11776i - 1536 + 768i)z^2\cos z +$$
$$(11776 - 4096i + 2048i^2)z\cos z +$$
$$(4096 - 2048i)\cos z +$$
$$= -z^7\cos z +$$
$$(2 - 28i)z^6\cos z +$$
$$(334 + 32i)z^5\cos z +$$
$$(-176 + 2136i)z^4\cos z +$$
$$(-7616 - 256i)z^3\cos z +$$
$$(-1024 - 14080i)z^2\cos z +$$
$$(9728 - 4096)z\cos z +$$
$$(4096 - 2048)\cos z +$$

$$(0)z^7\sin z +$$
$$(-3)z^6\sin z +$$
$$(-48i - 24i + 16)z^5\sin z +$$
$$(288 - 384i^2 + 256i + 18)z^4\sin z +$$
$$(768i + 2304i - 1536 + 288i)z^3\sin z +$$
$$(-768 + 6144i^2 - 4096i - 1728)z^2\sin z +$$
$$(-6144i + 4096 - 4608i)z\sin z +$$
$$(4608)\sin z =$$
$$0 +$$
$$-3z^6\sin z +$$
$$(16 - 72i)z^5\sin z +$$
$$(690 + 256i)z^4\sin z +$$
$$(-1536 + 3360i)z^3\sin z +$$
$$(-8640 - 4096i)z^2\sin z +$$
$$(4096 - 10752)z\sin z +$$
$$4608\sin z$$

....

Задание 8

Условие:

$$\int_{C:\begin{cases}t\in[0,\pi]\\x=\pi-t\\y=t\end{cases}}\cos\overline{z}dz=$$

$$\cos\overline{z}=\cos(x-yi)=\cos x\cos yi+\sin x\sin yi=\cos x\operatorname{ch}y+\sin x\operatorname{sh}y\,i$$

$$=\int_C\cos x\operatorname{ch}ydx-\sin x\operatorname{sh}ydy+i\int_C\sin x\operatorname{sh}ydx+\cos x\operatorname{ch}ydy=$$

$$=\int_0^\pi\cos(\pi-t)\operatorname{ch}t\cdot-dt-\sin(\pi-t)\operatorname{sh}tdt+i\int_0^\pi\sin(\pi-t)\operatorname{sh}t\cdot-dt+\cos(\pi-t)\operatorname{ch}tdt=$$

$$=\int_0^\pi\cos t\operatorname{ch}t-\sin t\operatorname{sh}tdt+i\int_0^\pi-\sin t\operatorname{sh}t-\cos t\operatorname{ch}tdt=$$

$$\cos(t+it)=\cos t\operatorname{ch}t-\sin t\operatorname{sh}t\cdot i$$

$$=(1-i)\int_0^\pi(\cos t\operatorname{ch}t-\sin t\operatorname{sh}t\cdot i)dt=$$

$$=\frac{1-i}{1+i}\int_0^\pi\cos((1+i)t)d((1+i)t)=-i\sin((1+i)t)|_0^\pi=-i\sin(\pi+\pi i)=i\sin\pi i=-\operatorname{sh}\pi$$

Решение:

Задание 9

Условие:

Решение:

Задание 10

Условие:

Решение:

Задание 11

Условие:

$$\int_{\frac{\pi}{3}\to\ln_5}z\operatorname{sh}zdz$$

Решение:

$$(z\operatorname{ch}z)'=\operatorname{ch}z+z\operatorname{sh}z$$

$$(z\operatorname{ch}z-\operatorname{sh}z)'=\operatorname{ch}z+z\operatorname{sh}z-\operatorname{ch}z=z\operatorname{sh}z$$

$$\int_C z\operatorname{sh}zdz=z\operatorname{ch}z-\operatorname{sh}z|_{\frac{i\pi}{3}}^{\ln 5}=\ln 5\cdot\frac{e^{\ln 5}+e^{-\ln 5}}{2}-\frac{e^{\ln 5}-e^{-\ln 5}}{2}-\frac{i\pi}{3}\operatorname{ch}\left(\frac{i\pi}{3}\right)+\operatorname{sh}\left(\frac{i\pi}{3}\right)=$$

$$=\ln 5\cdot\frac{1}{2}\left(5+\frac{1}{5}\right)-\frac{1}{2}\left(5-\frac{1}{5}\right)-\frac{i\pi}{3}\cdot\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}=$$

$$=\frac{\ln 5}{2}\left(\frac{26}{5}\right)-\frac{12}{5}+i\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)$$