8/2/2025

Несколько типовых расчетов. Классификация уравнений - один из них.

$$a_{11}u_{xx} + 2a_{12}u_{yy} + a_{22}u_{zz} + b_1u_x + b_2u_y + cu = 0$$

$$a_{11}u_{xx} + 2a_{12}u_{yy} + a_{22}u_{zz} - \textit{содержит информацию о виде}$$

$$u_{xx} = \frac{\partial^2}{\partial x^2}u$$

$$a_{11}a_{22} - a_{12}^2 \begin{cases} < 0 \text{ - эллиптические} \\ = 0 \text{ - параболические} \\ > 0 \text{ - гиперболические} \end{cases}$$

Случай общего положения: Система обладает свойством. Параметры можно "шевелить" и свойство может меняться. Сколь угодно малое шевеление.
Пример:

$$\begin{cases} (3+\cos^2(y))u_{xx} - 2\sin y u_{xy} - u_{yy} + (2+\sin y - \cos y)u_x + u_y = 0 \\ \\ u|_{y=0} = x^2, u_y|_{y=0} = x \end{cases}$$

$$(3+\cos^2 y)(-1) - \sin^2 y = -4 < 0 - \textit{гиперболический}$$

Составим характеристическое уравнение:

$$(3+\cos^2 y)dy^2 + \sin(y)dydx - dx^2 = 0$$

$$\left(\frac{dx}{dy}\right)^2 - 2\sin(y)\frac{dx}{dy} - (3+\cos^2 y) = 0$$

$$\frac{D}{4} = \sin^2 y + (3+\cos^2 y) = 4$$

$$\frac{dx}{dy} = \sin y \pm 2$$

$$x \pm 2y + \cos y = C_{1,2}$$

$$\begin{cases} \xi = x - 2y + \cos y \\ \eta = x + 2y + \cos y \end{cases}$$

$$\xi\eta\zeta$$

$$u(x(\xi,\eta), y(\xi,\eta)) = u^\wedge(\xi,\eta)$$

$$u_x = u_\xi \xi_x + u_\xi \eta_x = u_\xi + u_\eta$$

$$u_y = u_\xi \xi_y + u_\eta \eta_y = u_\xi(-2-\sin y) + u_\eta(2-\sin y)$$

$$\frac{\partial}{\partial x}u_\xi = u_{\xi\xi}\xi_x + u_{\xi\eta}\eta_x$$

$$u_{xx} = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta}$$

$$u_{yy} = u_{\xi\xi}(-2-\sin y)^2 + 2u_{\xi\eta}(-2-\sin y)(2-\sin y) + u_{\eta\eta}(2-\sin y) = \dots$$

$$2 + \sin x - \cos y$$

$$1$$

$$(3+\cos^2 y)$$

$$-2\sin y$$

$$-1$$

$$(x,y)\to(\xi,\eta)$$

$$u_{\xi\xi}=3+\cos^2 y+4\sin y+2\sin^2 y-4-\sin^2 y-4\sin y=0$$

На дом

$$u_{\eta\eta}=?$$

<\На дом>

$$u_{\xi\eta}=2(3+\cos^2 y)-2\sin y\times(-2\sin y)+8-2\sin^2 y=16$$

$$u_{\eta\eta}=0$$

$$u_{\xi}=0$$

$$u_{\eta}=4$$

$$0u_{\xi\xi}+16u_{\xi\eta}+0u_{\eta\eta}+0u_{\xi}+4u_{\eta}=0$$

$$u_{\xi\eta}+\frac{u_\eta}{4}=0$$

$$u_\eta=z$$

$$z_\xi+\frac{z}{4}=0$$

$$z=C_1(\eta)e^{-\frac{1}{4}\xi}$$

$$u_\eta=C_1(\eta)e^{-\frac{1}{4}\xi}$$

$$u=e^{-\frac{1}{4}\xi}\varphi(\eta)+\psi(\xi)$$

$$u(\xi,\eta)=e^{-\frac{1}{4}(x-2y+\cos y)}\phi(x+2y+\cos y)+\psi(x-2y+\cos y)$$

$$\phi,\psi-\textit{произвольные функции}$$

$$y=0$$

$$u=e^{-\frac{1}{4}(x+1)}\phi(x+1)+\psi(x+1)=x^2$$

$$u_y=e^{-\frac{1}{4}(x-2y+\cos y)}\left(\frac{1}{2}+\frac{\sin y}{4}\right)\phi(x+2y+\cos y)+e^{-\frac{1}{4}(x-2y+\cos y)}\phi'(x+2y+\cos y)(2-\sin y)+$$

$$+\psi(x-2y+\cos y)(-2-\sin y)$$

$$y=0$$

$$\begin{cases}e^{-\frac{1}{4}(x+1)}\phi(x+1)+\psi(x+1)=x^2|\frac{d}{dx},\times 2\\ e^{-\frac{1}{4}(x+1)}\frac{1}{2}\phi(x+1)+e^{-\frac{1}{4}(x+1)}\phi'(x+1)2+\psi'(x+1)(-2)=x\end{cases}$$

$$4e^{-\frac{1}{4}(x+1)}\phi'(x+1)=5x$$

$$\phi(x+1)=\frac{5}{4}xe^{\frac{1}{4}(x+1)}$$

$$\phi(x+1)=5(x-4)e^{\frac{1}{4}(x+1)}+C$$

$$\psi(x+1) = x^2 - 5(x-4) - Ce^{-\frac{1}{4}(x+1)}$$

$$\phi(t_1) = 5e^{\frac{t_1}{4}}(t_1 - 4)$$

$$\xi(t_2) = t_2^2 - 7t_2 + 26 - Ce^{-\frac{t_2}{4}}$$

$$t_1 = x + 2y + \cos y$$

$$t_2 = x - 2y + \cos y$$

$$u(x,y) = \dots = 5e^y(x + 2y + \cos y - 5) + (x - 2y + \cos y)^2 - 7(x - 2y + \cos y) + 26$$

2 Семинар

$$y^6 u_{xx} - 2y^3 u_{xy} + u_{yy} - \frac{3}{y} z_y = 0$$

$$y^6 - (y^3)^2 = 0 - \textit{параболический тип}$$

Характеристическое уравнение

$$y^6 dy^2 + 2y^3 dxdy + dx^2 = 0$$

$$\frac{y^4}{4} + x = C$$

$$\begin{cases} \xi = x + \frac{y^4}{4} \\ \eta = y \ - \textit{нам так удобно} \end{cases}$$

На дом:

$$\xi_x = \dots$$

$$\xi_y = \dots$$

$$\eta_x = \dots$$

$$\eta_y = \dots$$

$$\dots$$

</На дом>
Лабораторная работа.
Графическая иллюстрация текущего параметра
Решение уравнения колебаний струны
Преобразование Фурье:

$$F[f](\xi) = \int_{-\infty}^{\infty} f(x) e^{-ix\xi} dx$$

Уравнение колебаний

$$\begin{cases} u_{tt} = a^2 u_{xx}, -\infty < x < \infty \\ u|_{t=0} = \phi(x), t > 0, \\ u_t|_{t=0} = \psi(x) \end{cases}$$

Характеристическое уравнение:

$$dx^2 - a^2 dt^2 = 0$$

$$x \pm at = C_{1,2}$$

$$\begin{cases} \xi = x - at \\ \eta = x + at \end{cases}$$

На дом:

$$u_{\xi\eta} = 0$$

<\На дом>

$$u = f(\xi) + g(\eta) = f(x - at) + g(x + at)$$

f, g - произвольные

Волна - процесс распространения состояния

$$\begin{cases} f(x) + g(x) = \phi(x) \\ -af'(x) + ag'(x) = \psi(x) \end{cases}$$

$$\begin{cases} f(x) + g(x) = \phi(x) \\ -f(x) + g(x) = \frac{1}{a}\int_{x_0}^{x} \psi(\zeta) d\zeta + C \end{cases}$$

$$\begin{cases} f(x) = \frac{\phi(x)}{2} - \frac{1}{2a}\int_{x_0}^{x} \psi(\zeta) d\zeta - \frac{C}{2} \\ g(x) = \frac{\phi(x)}{2} + \frac{1}{2a}\int_{x_0}^{x} \psi(\zeta) d\zeta + \frac{C}{2} \end{cases}$$

$$u(x,t) = \frac{\phi(x - at) + \phi(x + at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \psi(\zeta) d\zeta$$

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НУ:

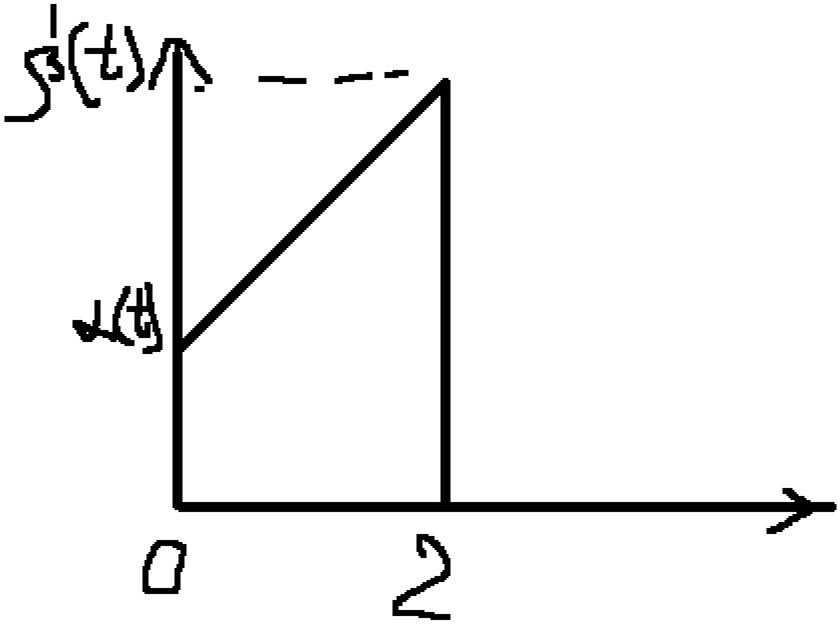
1. $\phi = 1 - |x|,\ \psi = 0$
2. $\phi = 0,\ \psi = 1$
   Desmos для илюстраций

$$u(x,t) = \frac{\phi(x - at) + \phi(x + at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \psi(\zeta) d\zeta$$

22/02/2025

Уравнение колебаний

$$\begin{cases} u_{tt} = a^2 u_{xx} + f(x,t), \qquad 0 < x < 2, t > 0 \\ u|_{x=0} = 1 \\ u|_{x=2} = t \\ u|_{t=0} = \varphi \\ u_t|_{t=0} = \psi \end{cases}$$
$$u = v + A(t)x + B(t)$$
$$u_t = v_t' + A'x + B'$$
$$u_{tt} = v_t'' + A''x + B''$$
$$u|_{t=0} = v|_{t=0} + A(0)x + B(0)$$



$$\mathcal{B}v = 0 \implies$$

$$\mathcal{B}(Ax + B) = \begin{pmatrix} 1 \\ t \end{pmatrix}$$

$$\begin{cases} A(t) \cdot 0 + B(t) = 1 \\ A(t) \cdot l + B(t) = t \end{cases}$$

$$A(t)x + B(t) = 1 + \frac{x}{2}(t-1)$$

$$u = v + \frac{t-1}{2}x + 1$$

$$u_t = v_t + \frac{1}{2}x$$

$$u_{tt} = v_{tt}(t)$$

$$u_x = v_x + \frac{t-1}{2}x$$

$$u_{xx} = v_{xx}$$

$$\begin{cases} v_{tt} = a^2 v_{xx} + f(x,t) = -A''x - B'' \\ v\big|_{x=0} = v\big|_{x=2} = 0 \\ v\big|_{t=0} = \varphi' - A(0)x - B(0) \\ v_t\big|_{t=0} = \psi - A'(0)x - B'(0) \end{cases}$$

$$f(x,t) = 3\sin\left(\frac{\pi x}{2}\right)$$

$$\varphi = -\frac{x}{2}$$

$$\psi = \frac{3x}{2}$$

$$\begin{cases} u = v + A(t)x + B(t) = v + \frac{t-1}{2}x + 1 \\ v_{tt} = u_{tt} + 3\sin\left(\frac{\pi x}{2}\right) \\ v\big|_{x=0} = v\big|_{x=2} = 0 \\ v\big|_{t=0} = -1 \\ v_t\big|_{t=0} = x \end{cases}$$

$$\begin{cases} X_n(x) = \sin\left(\frac{\pi n x}{2}\right) \\ \lambda_n = \left(\frac{\pi n}{2}\right)^2, n \in \mathbb{Z}_{>0} \end{cases}$$

$$v(x, y) = \sum_{n=1}^{N} T_n(t) X_n(t)$$

$$\begin{cases} T_n'' + a^2 \lambda_n T_n = f_n(t) \\ T_n(0) = \hat{\varphi}_n \\ T_n'(0) = \hat{\psi}_n \end{cases}$$

$$\begin{cases} T_1'' + a^2 \left(\frac{\pi}{2}\right)^2 T_1 = 3 \\ T_1'(0) = \hat{\varphi}_1 \\ T_1'(0) = \hat{\psi}_1 \end{cases}$$

$$\sum_{n=1}^{\infty} \hat{\varphi}_n X_n(x) \sim \hat{\varphi}(x)$$

$$\sum_{n=1}^{\infty} \hat{\psi}_n X_n(x) \sim \hat{\psi}(x)$$

Повторение разложения в ряд Фурье

$$1 \sim \sum_{n=1}^{N} b_n \sin \frac{\pi n x}{2}, l = 2$$

$$b_n = \frac{2}{2} \int_0^2 1 \cdot \sin \frac{\pi n x}{2} dx = \frac{2}{\pi n} \cos \frac{\pi n x}{2} |_0^2 = \frac{2(1 - (-1)^n)}{\pi n}$$

$$b_1 = \frac{4}{\pi}$$

$$b_2 = 0$$

$$b_3 = \frac{4}{3\pi}$$

$$x \sim \sum_{n=1}^{N} b_n \sin \frac{\pi n x}{2}, l = 2$$

$$b_n = \frac{2}{2} \int_0^2 x \sin \frac{\pi n x}{2} dx = -\frac{2}{\pi n} \int_0^2 x d \left(\cos \frac{\pi n x}{2} \right) = -\frac{2}{\pi n} \left(x \cos \frac{\pi n x}{2} |_0^2 - \int_0^2 \cos \frac{\pi n x}{2} dx \right) =$$

$$-\frac{2}{\pi n} \left(2 \cancelto{(-1)^n}{\cos \pi n} - \frac{2}{\pi n} \sin \frac{\pi n x}{2} |_0^2 \right) = \frac{4(-1)^{n+1}}{\pi n} + \left(\frac{2}{\pi n} \right)^2 \cdot 0 = \frac{4(-1)^{n+1}}{\pi n}$$

Мои попытки сделать лабу по урматфизу:

$$v_{t=0} = 3 \sin \pi x$$

Разложить x на $\cos$. Коэффициенты будут как n^2

$$\begin{cases} T_n'' + a^2\left(\frac{\pi n}{2}\right)^2 T_n = 3\sin\left(\frac{\pi x}{2}\right) \\ T_n(0) = C_1 = -\frac{2(1-(-1)^n)}{\pi n} \\ T_n'(0) = C_2 = -\frac{4(-1)^n}{\pi n} \end{cases}$$

$$y = T_n$$

$$y = A\sin\left(\frac{a\pi n}{2}x\right) + B\cos\left(\frac{a\pi n}{2}x\right)$$

$$y(0) = B = -\frac{2(1-(-1)^n)}{\pi n}$$

$$y' = \frac{a\pi n}{2}\left(A\cos\left(\frac{a\pi n}{2}x\right) - B\sin\left(\frac{a\pi n}{2}x\right)\right)$$

$$y'(0) = \frac{a\pi n}{2}A = -\frac{4(-1)^n}{\pi n} \implies$$

$$T_n = \frac{8(-1)^{n+1}}{a\pi^2 n^2}\sin\left(\frac{a\pi n}{2}x\right) + -\frac{2(1-(-1)^n)}{\pi n}\cos\left(\frac{a\pi n}{2}x\right)$$

$$\begin{cases} v_{tt} = a^2 u_{tt} \\ v|_{x=0} = v|_{x=2} = 0 \\ v|_{t=0} = 3\sin\pi x \\ v_t|_{t=0} = 0 \end{cases}$$

$$v = T(t)X(x)$$

$$T''X = a^2$$

$$\{$$

$$\begin{cases} u = v \\ v_{tt} = u_{tt} \\ v|_{x=0} = v|_{x=2} = 0 \\ v|_{t=0} = 3\sin\pi x \\ v_t|_{t=0} = 0 \end{cases}$$

$$\begin{cases} X_n(x) = \sin\left(\frac{\pi n x}{2}\right) \\ \lambda_n = \left(\frac{\pi n}{2}\right)^2, n \in \mathbb{Z}_{>0} \end{cases}$$

$$v(x,y) = \sum_{n=1}^{N} T_n(t)X_n(t)$$

$$\begin{cases} T_n'' + a^2\lambda_n T_n = 0 \\ T_n(0) = \hat{\varphi}_n \\ T_n'(0) = \hat{\psi}_n \end{cases}$$

$$\sum_{n=1}^{\infty} \hat{\varphi}_n X_n(x) \,\tilde{}\, \hat{\varphi}(x)$$

$$\sum_{n=1}^{\infty} \hat{\psi}_n X_n(x) \,\tilde{}\, \hat{\psi}(x)$$

Повторение разложения в ряд Фурье

$$3\sin\pi x = 3\cdot\sin\pi x + 0\sum_{n=1}^{N} 0$$

$$v_{t=0} = 3\sin\pi x$$

Разложить x на $\cos$. Коэффициенты будут как n^2

$$\begin{cases} T_n'' + a^2\left(\frac{\pi n}{2}\right)^2 T_n = 0 \\ T_n(0) = C_1 = 3\sin \pi x \; if \; n = 1, \; else \; 0 \\ T_n'(0) = C_2 = 0 \end{cases}$$

$$y = T_n$$

$$y = A\sin\left(\frac{a\pi n}{2}x\right) + B\cos\left(\frac{a\pi n}{2}x\right)$$

$$n = 1:$$

$$y(0) = B = 3$$

$$y' = \frac{a\pi n}{2}\left(A\cos\left(\frac{a\pi n}{2}x\right) - 3\sin\left(\frac{a\pi n}{2}x\right)\right)$$

$$y'(0) = \frac{a\pi n}{2}A = 0 \implies$$

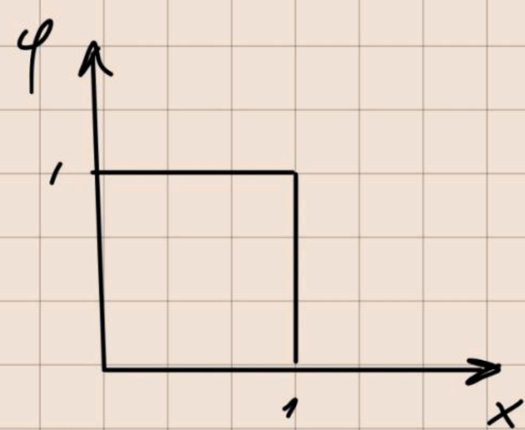
$$T_1 = 3\cos\left(\frac{a\pi}{2}x\right)$$

$$u_{tt} = a^2 u_{xx}, \quad 0 < x < \infty, \quad t > 0$$

$$u|_{t=0} = \varphi(x)$$

$$u_t|_{t=0} = \psi(x)$$

$$u|_{v=0} = 0$$



$$\psi = 0$$

φ_1 - нечёт. продолжение φ

$$\varphi = -\Theta(x+1) + 2\Theta(x) - \Theta(x-1)$$

$$\begin{cases} \xi = x - at \\ \eta = x + at \end{cases}$$

05/04/2025

Условие:

2 Вариант

\end{gather}$$

$$v=\sum_{n=1}^{\infty}\frac{15\pi}{\frac{25\pi^2}{16}-\frac{\pi^2n^2}{4}}\cdot\cos\left(\frac{\pi nx}{2}\right)+x+t$$

$$u=e^t\left(\sum_{n=1}^{\infty}\frac{15\pi}{\frac{25\pi^2}{16}-\frac{\pi^2n^2}{4}}\cdot\cos\left(\frac{\pi nx}{2}\right)+x+t\right)$$