Задание 1

Условие:

$$(T^{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}, \ (B^{ij}) = \begin{pmatrix} -2 & -1 \\ -2 & 1 \end{pmatrix}, \ \left(Q^j{}_i\right) = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

Найти:
$(C = (T \cdot B))_{ij}$ в базисе e_i
Решение:

$$\text{Замена } \overline{e}_i = r_i \Rightarrow$$
$$T = T^{ij} r_i \otimes r_j, B = B^{ij} r_i \otimes r_j, e_i = Q_i^j r_j, T \cdot B = (T \cdot B)_{ij} e^{ij}$$

2 пути:
1: Переводим всё в базис e_i и там считаем:

$$\begin{matrix} e_i = Q_i^j r_j \\ r_j = K_j^k e_k \end{matrix} \Rightarrow e_i = Q_i^j K_j^k e_k = \delta_i^k e_k \Rightarrow$$
$$K : Q_i^j K_j^k = \delta_i^k$$
$$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$
$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1 a_2 + b_1 c_2 & a_1 b_2 + b_1 d_2 \\ c_1 a_2 + d_1 c_2 & c_1 b_2 + d_1 d_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\begin{cases} b_2 = -\frac{b_1}{a_1} d_2 \\ c_2 = -\frac{c_1}{d_1} a_2 \\ a_1 a_2 - \frac{b_1 c_1}{d_1} a_2 = 1 \\ d_1 d_2 - \frac{c_1 b_1}{a_1} d_2 = 1 \end{cases} \Rightarrow \begin{cases} a_2 = \frac{d_1}{\Delta} \\ d_2 = \frac{a_1}{\Delta} \\ b_2 = -\frac{b_1}{\Delta} \\ c_2 = -\frac{c_1}{\Delta} \end{cases}$$

$$T = T^{ij} r_i \otimes r_j = T^{ij} K_i^k e_k \otimes K_j^l e_l = D^{kl} e_k \otimes e_l \Rightarrow$$
$$D^{kl} = T^{ij} K_i^k K_j^l$$

$$\begin{cases} D^{11} = T^{ij} K_i^1 K_j^1 = 1 \cdot 1 \cdot 1 + -3 \cdot 0 = 1 \\ D^{12} = T^{ij} K_i^1 K_j^2 = -\frac{1}{2} + -3 \cdot 0 = -\frac{1}{2} \\ D^{21} = T^{ij} K_i^2 K_j^1 = -\frac{1}{2} \\ D^{22} = T^{ij} K_i^2 K_j^2 = \frac{1}{4} + -3 \cdot \frac{1}{4} = -\frac{1}{2} \end{cases}$$
$$B = P^{kl} e_k \otimes e_l = B^{ij} K_i^k K_j^l e_k \otimes e_l$$
$$\begin{cases} P^{11} = -2 \\ P^{12} = \frac{1}{2} \\ P^{21} = B^{ij} K_i^2 K_j^1 = -2 \cdot -0.5 \cdot 1 = 0 \\ P^{22} = 0.5 \end{cases}$$

$$C = T \cdot B = (D^{kl} e_k \otimes e_l) \cdot (P^{mn} e_m \otimes e_n)$$

Из определения скалярного произведения:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$C = D_m^k P^{mn} e_k \otimes e_n$$
$$D_m^k = g_{lm} D^{kl}$$

2: Переводим всё в базис r_i:

```
def basis_change_contr(A:list[list], P:list[list], nameA:str, nameAnew:str, nameP:str) ->
list[list]:
    C: list[list] = [
```

```
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    for k in range(2):
        for l in range(2):
            print_output += f"{nameAnew}^{k+1}{l+1}={nameA}^ij*{nameP}^{k+1}_i*{nameP}^{l+1}_j="
            args: list[str] = []
            for i in range(2):
                for j in range(2):
                    args.append(f"{A[i][j]}*{P[k][i]}*{P[l][j]}")
                    C[k][l] += A[i][j]*P[k][i]*P[l][j]
            print_output += "+".join(args)+f"={C[k][l]}"+"\n"
    print(print_output)
    return C

def basis_change_co(A:list[list], Q:list[list], nameA:str, nameAnew:str, nameQ:str) ->
list[list]:
    C: list[list] = [
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    for k in range(2):
        for l in range(2):
            print_output += f"{nameAnew}_{k+1}{l+1}={nameA}_ij*{nameQ}^i_{k+1}*{nameQ}^j_{l+1}="
            args: list[str] = []
            for i in range(2):
                for j in range(2):
                    args.append(f"{A[i][j]}*{Q[i][k]}*{Q[j][l]}")
                    C[k][l] += A[i][j]*Q[i][k]*Q[j][l]
            print_output += "+".join(args)+f"={C[k][l]}"+"\n"
    print(print_output)
    return C



def basis_change_mix_l(A:list[list], P:list[list], Q:list[list], nameA:str, nameAnew:str,
nameP:str, nameQ:str) -> list[list]:
    C: list[list] = [
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    for k in range(2):
        for l in range(2):
            print_output += f"{nameAnew}^{k+1}_{l+1}={nameA}^i_j*{nameP}^{k+1}_i*
{nameQ}^j_{l+1}="
            args: list[str] = []
            for i in range(2):
                for j in range(2):
                    args.append(f"{A[i][j]}*{P[k][i]}*{Q[j][l]}")
                    C[k][l] += A[i][j]*P[k][i]*Q[j][l]
            print_output += "+".join(args)+f"={C[k][l]}"+"\n"
    print(print_output)
    return C

def contract_mixmix(A: list[list], B: list[list], nameA: str, nameB: str, nameC: str)->
list[list]:
    C: list[list] = [
        [0,0],
```

```
        [0,0]
    ]
    print_output: str = ""
    # C^i_j=A^i_k B^k_j
    for i in range(2):
        for j in range(2):
            print_output += f"{nameC}^{i+1}_{j+1}={nameA}^{i+1}_k {nameB}^k_{j+1}="
            args: list[str] = []
            for k in range(2):
                args.append(f"{A[i][k]}*{B[k][j]}")
                C[i][j] += A[i][k]*B[k][j]
            print_output += "+".join(args)+f"={C[i][j]}"+"\n"
    print(print_output)
    return C

def contract_contrco(A: list[list], B: list[list], nameA: str, nameB: str, nameC: str)->
list[list]:
    C: list[list] = [
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    # C^i_j=A^ik B_kj
    for i in range(2):
        for j in range(2):
            print_output += f"{nameC}^{i+1}_{j+1}={nameA}^{i+1}k {nameB}_k{j+1}="
            args: list[str] = []
            for k in range(2):
                args.append(f"{A[i][k]}*{B[k][j]}")
                C[i][j] += A[i][k]*B[k][j]
            print_output += "+".join(args)+f"={C[i][j]}"+"\n"
    print(print_output)
    return C

def contract_mixco(A: list[list], B: list[list], nameA: str, nameB: str, nameC: str)->
list[list]:
    C: list[list] = [
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    # C_ij=A^k_j B_ik
    for i in range(2):
        for j in range(2):
            print_output += f"{nameC}_{i+1}{j+1}={nameA}^k_{j+1} {nameB}^{i+1}_k="
            args: list[str] = []
            for k in range(2):
                args.append(f"{A[k][j]}*{B[i][k]}")
                C[i][j] += A[k][j]*B[i][k]
            print_output += "+".join(args)+f"={C[i][j]}"+"\n"
    print(print_output)
    return C

def contract_contrmix(A: list[list], B: list[list], nameA: str, nameB: str, nameC: str)->
list[list]:
    C: list[list] = [
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    # C^ij=A^ik B^j_k
```

```
    for i in range(2):
        for j in range(2):
            print_output += f"{nameC}^{i+1}{j+1}={nameA}^{i+1}k {nameB}^{j+1}_k="
            args: list[str] = []
            for k in range(2):
                args.append(f"{A[i][k]}*{B[j][k]}")
                C[i][j] += A[i][k]*B[j][k]
            print_output += "+".join(args)+f"={C[i][j]}"+"\n"
    print(print_output)
    return C

def contract2_contrcoco(A: list[list], B: list[list], nameA: str, nameB: str, nameC: str)->
list[list]:
    C: list[list] = [
        [0,0],
        [0,0]
    ]
    print_output: str = ""
    # C_ij=A^kl B_ik B_jl
    for i in range(2):
        for j in range(2):
            print_output += f"{nameC}_{i+1}{j+1}={nameA}^kl {nameB}_{i+1}k {nameB}_{j+1}l="
            args: list[str] = []
            for k in range(2):
                for l in range(2):
                    args.append(f"{A[k][l]}*{B[i][k]}*{B[j][l]}")
                    C[i][j] += A[k][l]*B[i][k]*B[j][l]
            print_output += "+".join(args)+f"={C[i][j]}"+"\n"
    print(print_output)
    return C

def inverse(A: list[list]) -> list[list]:
    det: float = A[0][0]*A[1][1]-A[0][1]*A[1][0]
    C = [
        [A[1][1]/det, -A[0][1]/det],
        [-A[1][0]/det, A[0][0]/det]
    ]
    print(C)
    return C

E = [
    [1, 0],
    [0,1]
]

T_r = [
    [1,0],
    [0,-3]
]

B_r=[
    [-2, -1],
    [-2, 1]
]

Q = [
    [1,0],
    [1,2]
]

"""
```

```
1 задание с проверкой
P = inverse(Q)
T_e = basis_change_contr(T_r,P,"T","D","P")
B_e = basis_change_contr(B_r,P,"B","P","K")
g = basis_change_co(E, Q, "δ", "g", "Q")
T_mix_e = contract_contrco(T_e, g, "D", "g", "D")
C_contr = contract_contrmix(B_e, T_mix_e, "P", "D", "C")
C_co = contract2_contrcoco(C_contr, g, "C", "g", "C")


T_mix_r = contract_contrco(T_r, E, "T", "δ", "T")
C_r = contract_contrmix(B_r, T_mix_r, "B", "T", "S")
C_r_co = contract2_contrcoco(C_r, E, "S", "δ", "S")
C_co_e = basis_change_co(C_r_co, Q, "S", "C", "Q")
"""

#A[i][j][k]
A_contrcontrcontr=[
    [
        [-3, -4, -4],
        [3, 3, 1],
        [-1, 3, 2]
    ],
    [
        [-4, 4, -4],
        [4, -2, 4],
        [-2, 3, 2]
    ],
    [
        [0, 3, 0],
        [3, -2, 4],
        [2, 4, -1]
    ]
]

def my_print3(A: list[list[list[float]]]):
    for i in range(3):
        for k in range(3):
            for j in range(3):
                print("\t"+f"{A[i][j][k]}", end="")
        print("")

#my_print3(A_contrcontrcontr)
def simm3(A: list[list[list[float]]]) -> list[list[list[float]]]:
    C = [[[0]*3]*3]*3
    for i in range(3):
        for k in range(3):
            for j in range(3):
                C[i][j][k] = 1/6 * (A[i][j][k]+A[j][k][i]+A[k][i][j]+A[j][i][k]+A[i][k][j]+A[k]
[j][i])
                print("\t"+f"{C[i][j][k]}", end="")
        print("")
    return C

def alter3(A: list[list[list[float]]]) -> list[list[list[float]]]:
    C = [[[0]*3]*3]*3
    for i in range(3):
        for k in range(3):
            for j in range(3):
                C[i][j][k] = 1/6 * (A[i][j][k]+A[j][k][i]+A[k][i][j]-A[j][i][k]-A[i][k][j]-A[k]
[j][i])
```

```
                print("\t"+f"{C[i][j][k]}", end="")
        print("")
    return C

"""
2 задание
simm3(A_contrcontrcontr)
alter3(A_contrcontrcontr)
"""
```

Задание 2

Не понял, зачем оно было нужно

Задание 3

$$A_j^i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$x^i = A_j^i \tilde{x}^j$$

$$\begin{cases} \tilde{x}^1 = r\cos\varphi \\ \tilde{x}^2 = r\sin\varphi \\ \tilde{x}^3 = z \end{cases} \Leftrightarrow \begin{cases} \tilde{x}^1 = X^1 \cos X^2 \\ \tilde{x}^2 = X^1 \sin X^2 \\ \tilde{x}^3 = X^3 \end{cases}$$

$$\tilde{x}_j^i = \begin{pmatrix} \cos X^2 & -X^1 \sin X^2 & 0 \\ \sin X^2 & X^1 \cos X^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x_j^i = A_k^i \tilde{x}_j^k$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \cos X^2 & -X^1 \sin X^2 & 0 \\ \sin X^2 & X^1 \cos X^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos X^2 & -X^1 \sin X^2 & 0 \\ -\frac{1}{2}\sin X^2 & -\frac{1}{2}X^1\cos X^2 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2}\sin X^2 & -\frac{\sqrt{3}}{2}X^1\cos X^2 & -\frac{1}{2} \end{pmatrix}$$

$$g_{ij} = r_i r_j :$$

$$\begin{cases} g_{11} = \cos^2 X^2 + \sin^2 X^2 = 1 \\ g_{12} = X^1 \left(-\sin X^2 \cos X^2 + \left(\frac{1}{4} + \frac{3}{4}\right) \sin X^2 \cos X^2\right) = 0 \\ g_{13} = 0 \\ g_{22} = (X^1)^2 (\sin^2 X^2 + \cos^2 X^2) = (X^1)^2 \\ g_{23} = 0 \\ g_{33} = 1 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & (X^1)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow (g^{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{(X^1)^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow Q^{ik} = g^{jk} Q_j^i$$

$$\begin{cases} Q^{11} = g^{j1} Q_j^1 = \cos X^2 \\ Q^{12} = g^{j2} Q_j^1 = (X^1)^{-2} \cdot -X^1 \sin X^2 \\ Q^{13} = g^{j3} Q_j^1 = 0 \\ Q^{21} = g^{j1} Q_j^2 = -\frac{1}{2}\sin X^2 \\ Q^{22} = g^{j2} Q_j^2 = (X^1)^{-2} \cdot -\frac{1}{2} X^1 \cos X^2 \\ Q^{23} = g^{j3} Q_j^2 = \frac{\sqrt{3}}{2} \\ Q^{31} = g^{j1} Q_j^3 = -\frac{\sqrt{3}}{2}\sin X^2 \\ Q^{32} = g^{j2} Q_j^3 = (X^1)^{-2} \cdot -\frac{\sqrt{3}}{2} X^1 \cos X^2 \\ Q^{33} = g^{j3} Q_j^3 = -\frac{1}{2} \end{cases} \Rightarrow \begin{pmatrix} \cos X^2 & -\frac{1}{X^1}\sin X^2 & 0 \\ -\frac{1}{2}\sin X^2 & -\frac{1}{2}\frac{1}{X^1}\cos X^2 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2}\sin X^2 & -\frac{\sqrt{3}}{2}\frac{1}{X^1}\cos X^2 & -\frac{1}{2} \end{pmatrix}$$

$$\Gamma_{ijk} = \frac{1}{2}\left(\frac{\partial g_{ik}}{\partial X^j} + \frac{\partial g_{jk}}{\partial X^i} - \frac{\partial g_{ij}}{\partial X^k}\right)$$

$$k = 1:$$

$$\frac{\partial g_{i1}}{\partial X^j} = 0, \frac{\partial g_{j1}}{\partial X^i} = 0, \frac{\partial g_{ij}}{\partial X^1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2X^1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$k = 2:$$

$$\frac{\partial g_{i2}}{\partial X^j} = \begin{pmatrix} 0 & 0 & 0 \\ 2X^1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \frac{\partial g_{j2}}{\partial X^i} = \begin{pmatrix} 0 & 2X^1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \frac{\partial g_{ij}}{\partial X^2} = 0$$

$$k = 3:$$

$$\frac{\partial g_{i3}}{\partial X^j} = 0 = \frac{\partial g_{j3}}{\partial X^i}, \frac{\partial g_{ij}}{\partial X^3} = 0 \Rightarrow$$

$$\Gamma_{ij1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -X^1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_{ij2} = \begin{pmatrix} 0 & X^1 & 0 \\ X^1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_{ij3} = 0$$

$$\Gamma_{ij}^l = g^{kl}\Gamma_{ijk} \Rightarrow$$

$$\begin{cases} \Gamma_{ij}^1 = g^{k1}\Gamma_{ijk} = g^{11}\Gamma_{ij1} + g^{21}\Gamma_{ij2} + g^{31}\Gamma_{ij3} = \Gamma_{ij1} \\ \Gamma_{ij}^2 = g^{k2}\Gamma_{ijk} = \frac{1}{(X^1)^2}\Gamma_{ij2} \\ \Gamma_{ij}^3 = g^{k3}\Gamma_{ijk} = \Gamma_{ij3} \end{cases}$$

$$H_\alpha = \sqrt{g_{\alpha\alpha}} = \begin{pmatrix} 1 & |X^1| & 1 \end{pmatrix}$$

Задание 4

Дано:

X^i - цилиндрические координаты

$T = T^{ij} e_i \otimes e_j$, где $T^{ij} = \begin{pmatrix} 0 & -X^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix}$ и e_i - цилиндрические координаты,

r_i - ортогональный локальный базис цилиндрической системы координат

Найти:

T_{r_i}, $(\nabla T)_{r_i}$

Решение:

$$X^i - \text{цилиндрические координаты} \Rightarrow \begin{cases} x^1 = X^1 \cos X^2 \\ x^2 = X^1 \sin X^2 \\ x^3 = X^3 \end{cases} \Rightarrow \frac{\partial x^i}{\partial X^j} = \begin{pmatrix} \cos X^2 & -X^1 \sin X^2 & 0 \\ \sin X^2 & X^1 \cos X^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = x_j^i$$

$$g_{ij} = \delta_{kl} x_i^k x_j^l$$

$$\begin{cases} g_{11} = 1 \\ g_{12} = g_{23} = g_{13} = 0 \\ g_{22} = (X^1)^2 \\ g_{33} = 1 \end{cases} \Rightarrow g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (X^1)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{(X^1)^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^{i}{}_{j} = T^{ik} g_{kj}$$

$$T^{i}{}_{j} = \begin{pmatrix} \begin{pmatrix} 0 \\ X^1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} -X^2 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ X^3 \\ 0 \end{pmatrix} \end{pmatrix} \left(\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & (X^1)^2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right) =$$

$$= \left(\begin{pmatrix} 0 \\ X^1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -X^2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & (X^1)^2 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ X^3 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right) =$$

$$\left(\begin{pmatrix} 0 & 0 & 0 \\ X^1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_{i}{}^{j} = g_{ik} T^{kj}$$

$$T_{i}{}^{j} = \left(\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & (X^1)^2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right) \begin{pmatrix} \begin{pmatrix} 0 \\ -X^2 \\ 0 \end{pmatrix} \\ \begin{pmatrix} X^1 \\ 0 \\ X^3 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix} =$$

$$= \left(\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -X^2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & (X^1)^2 & 0 \end{pmatrix} \begin{pmatrix} X^1 \\ 0 \\ X^3 \end{pmatrix} + \Theta \right) =$$

$$= \left(\begin{pmatrix} 0 & 0 & 0 \\ -X^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & (X^1)^3 & 0 \\ 0 & 0 & 0 \\ 0 & (X^1)^2 X^3 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & (X^1)^3 & 0 \\ -X^2 & 0 & 0 \\ 0 & (X^1)^2 X^3 & 0 \end{pmatrix}$$

Почему-то Nomotex нумерует матрицу, соответствующую этим компонентам, как транспонированную

$$T_{ij} = T_{i}{}^{k} g_{kj}$$

$$T_{ij} = \begin{pmatrix} \begin{pmatrix} 0 & (X^1)^3 & 0 \end{pmatrix} \\ \begin{pmatrix} -X^2 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & X^3(X^1)^2 & 0 \end{pmatrix} \end{pmatrix} \left(\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & (X^1)^2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right) =$$

$$= \left(\begin{pmatrix} 0 & (X^1)^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -X^2(X^1)^2 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \right.$$
$$\left. \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & X^3(X^1)^2 & 0 \end{pmatrix} \right) =$$
$$= \begin{pmatrix} 0 & (X^1)^3 & 0 & -X^2(X^1)^2 & 0 & 0 & 0 & X^3(X^1)^2 & 0 \end{pmatrix}$$

$$T_{ij} = g_{ik} T^{k}_{j}$$

$$T_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & (X^1)^2 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix} =$$

$$= \left(\begin{pmatrix} 0 & 0 & 0 & -X^2(X^1)^2 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & (X^1)^3 & 0 & 0 & 0 & 0 & 0 & (X^1)^2 X^3 & 0 \end{pmatrix} + \Theta \right) =$$
$$= \begin{pmatrix} 0 & (X^1)^3 & 0 & -X^2(X^1)^2 & 0 & 0 & 0 & (X^1)^2 X^3 & 0 \end{pmatrix}$$

Если записывать по-обычному, то $T_{ij} = \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ (X^1)^3 & 0 & X^3(X^1)^2 \\ 0 & 0 & 0 \end{pmatrix}$

$$T^{ij}_{k} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \right)$$

$$\frac{\partial g_{ij}}{\partial X^k} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2X^1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \quad \Theta \right) = g_{ijk}$$

\begin{gather} \Gamma_{ij}^k = \frac{1}{2} g^{kl}\left(g_{jli}+g_{ilj}-g_{ijl} \right) \\ g_{i_{1}i_{2}i_{3}}\neq 0\Leftrightarrow \

$$k = 2:$$
$$T^{il}\Gamma^j_{l2} = (T^1)$$

$$\nabla_k T^{ij} = T^{ij}_k + T^{il}\Gamma^j_{lk} + T^{lj}\Gamma^i_{lk}$$

$$\nabla_k T^{ij} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \right) +$$
$$+ \left(\begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} X^1X^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right) +$$
$$+ \left(\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -(X^1)^2 & 0 & -X^1X^3 \\ 0 & -\frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right) =$$
$$= \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 & X^1X^2-(X^1)^2 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & \frac{X^3}{X^1} & 0 & 1-\frac{X^2}{X^1} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\nabla_2 T^{ij}$ неправильно

$$\nabla_k T^{ij} = T^{ij}_k + T^{il}\Gamma^j_{lk} + T^{lj}\Gamma^i_{lk}$$
$$k = 2:$$
$$T^{ij}_2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$T^{il}\Gamma^j_{l2} = T^{i1}\Gamma^j_{12} + T^{i2}\Gamma^j_{22} \cancelto{0}{+T^{i3}\Gamma^j_{32}}$$
$$\Gamma^j_{12} \neq 0 \Leftrightarrow j = 2$$
$$\Gamma^j_{22} \neq 0 \Leftrightarrow j = 1$$
$$T^{il}\Gamma^1_{l2} = T^{i2} \cdot -X^1 = \begin{pmatrix} X^1X^2 \\ 0 \\ 0 \end{pmatrix}$$
$$T^{il}\Gamma^2_{l2} = T^{i1} \cdot \frac{1}{X^1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$
$$T^{il}\Gamma^j_{l2} = \begin{pmatrix} X^1X^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$T^{lj}\Gamma^i_{l2} = T^{1j}\Gamma^i_{12} + T^{2j}\Gamma^i_{22}$$
$$T^{lj}\Gamma^1_{l2} = T^{2j} \cdot -X^1 = \begin{pmatrix} -(X^1)^2 & 0 & -X^1X^3 \end{pmatrix}$$
$$T^{lj}\Gamma^2_{l2} = T^{1j} \cdot \frac{1}{X^1} = \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 \end{pmatrix}$$
$$T^{lj}\Gamma^i_{l2} = \begin{pmatrix} -(X^1)^2 & 0 & -X^1X^3 \\ 0 & -\frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\nabla_2 T^{ij} = \begin{pmatrix} X^1X^2-(X^1)^2 & -1 & -X^1X^3 \\ 0 & 1-\frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_k T_{ij} = T_{ijk} - T_{lj}\Gamma^l_{ik} - T_{il}\Gamma^l_{jk}$$

$$T_{ijk} = \left(\begin{pmatrix} 0 & -2X^1X^2 & 0 \\ 3(X^1)^2 & 0 & 2X^1X^3 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -(X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 \end{pmatrix} \right)$$

$$\Gamma^l_{ik} T_{lj}:$$

$$\Gamma^l_{ik} \neq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} l = 1 \\ i = k = 2 \end{cases} \\ \begin{cases} l = 2 \\ \left[\begin{array}{l} \begin{cases} i = 1 \\ k = 2 \end{cases} \\ \begin{cases} i = 2 \\ k = 1 \end{cases} \end{array} \right. \end{cases} \end{array} \right.$$

$$T_{lj}\Gamma^l_{22} = T_{1j} \cdot -X^1 = \begin{pmatrix} 0 & (X^1)^3X^2 & 0 \end{pmatrix}$$

$$T_{lj}\Gamma^l_{12} = T_{2j} \cdot \frac{1}{X^1} = \begin{pmatrix} (X^1)^2 & 0 & X^1X^3 \end{pmatrix}$$

$$T_{lj}\Gamma^l_{21} = \begin{pmatrix} (X^1)^2 & 0 & X^1X^3 \end{pmatrix}$$

$$T_{lj}\Gamma^l_{ik} = \left(\begin{pmatrix} 0 & 0 & 0 \\ (X^1)^2 & 0 & X^1X^3 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} (X^1)^2 & 0 & X^1X^3 \\ 0 & (X^1)^3X^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right)$$

$$T_{il}\Gamma^l_{jk}:$$

$$T_{il}\Gamma^l_{22} = \begin{pmatrix} 0 \\ -(X^1)^4 \\ 0 \end{pmatrix}$$

$$T_{il}\Gamma^l_{12} = \begin{pmatrix} -X^1X^2 \\ 0 \\ 0 \end{pmatrix} = T_{il}\Gamma^l_{21}$$

$$T_{il}\Gamma^l_{jk} = \left(\begin{pmatrix} 0 & -X^1X^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -X^1X^2 & 0 & 0 \\ 0 & -(X^1)^4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right)$$

$$\begin{aligned} \nabla_k T_{ij} = &\left(\begin{pmatrix} 0 & -2X^1X^2 & 0 \\ 3(X^1)^2 & 0 & 2X^1X^3 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -(X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 \end{pmatrix} \right) - \\ &- \left(\begin{pmatrix} 0 & 0 & 0 \\ (X^1)^2 & 0 & X^1X^3 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} (X^1)^2 & 0 & X^1X^3 \\ 0 & (X^1)^3X^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right) - \\ &- \left(\begin{pmatrix} 0 & -X^1X^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -X^1X^2 & 0 & 0 \\ 0 & -(X^1)^4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right) = \end{aligned}$$

$$\begin{pmatrix} 0 & -X^1X^2 & 0 & X^1X^2 - (X^1)^2 & -(X^1)^2 & -X^1X^3 & 0 & 0 & 0 \\ 2(X^1)^2 & 0 & X^1X^3 & 0 & (X^1)^4 - (X^1)^3X^2 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_k T^i{}_j = T^i{}_{jk} + T^l{}_j \Gamma^i_{lk} - T^i{}_l \Gamma^l_{jk}$$

$$T^i{}_{jk} = \begin{pmatrix} 0 & -X^1X^2 & 0 & 0 & -(X^1)^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T^l_j \Gamma^1_{l2} = \begin{pmatrix} -(X^1)^2 & 0 & -X^1X^3 \end{pmatrix}$$

$$T^l_j \Gamma^2_{l1} = \begin{pmatrix} 1 & 0 & \frac{X^3}{X^1} \end{pmatrix}$$

$$T^l_j \Gamma^2_{l2} = \begin{pmatrix} 0 & -X^1X^2 & 0 \end{pmatrix}$$

$$T^l_j \Gamma^i_{lk} = \begin{pmatrix} 0 & 0 & 0 & 0 & -X^1X^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{X^3}{X^1} & -(X^1)^2 & 0 & -X^1X^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T^i{}_l \Gamma^l_{22} = \begin{pmatrix} -(X^1)^2 \\ 0 \\ 0 \end{pmatrix}$$

$$T^i{}_l \Gamma^l_{21} = \begin{pmatrix} 0 \\ -X^1X^2 \\ 0 \end{pmatrix} = T^i{}_l \Gamma^l_{12}$$

$$T^i{}_l \Gamma^l_{jk} = \begin{pmatrix} 0 & 0 & 0 & 0 & -(X^1)^2 & 0 & 0 & 0 & 0 \\ 0 & -X^1X^2 & 0 & -X^1X^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \nabla_k T^i{}_j = &\begin{pmatrix} 0 & -X^1X^2 & 0 & 0 & -(X^1)^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \\ &+ \begin{pmatrix} 0 & 0 & 0 & 0 & -X^1X^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{X^3}{X^1} & -(X^1)^2 & 0 & -X^1X^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} - \\ &- \begin{pmatrix} 0 & 0 & 0 & 0 & -(X^1)^2 & 0 & 0 & 0 & 0 \\ 0 & -X^1X^2 & 0 & -X^1X^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \\ &= \begin{pmatrix} 0 & -X^1X^2 & 0 & 0 & -X^1X^2 & 0 & 0 & 0 & 0 \\ 2 & X^1X^2 & \frac{X^3}{X^1} & X^1X^2 - (X^1)^2 & 0 & -X^1X^3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

$\nabla_k T_i{}^j = T_i{}^j{}_k - T_l{}^j \Gamma^l_{ik} + T_i{}^l \Gamma^j_{lk}$

(Записываю так, чтобы удобно было вбивать в номотех)

$$T_i{}^j{}_k = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 3(X^1)^2 & 0 & 3(X^1)X^3 & 0 & 0 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$T_l{}^j \Gamma^l_{ik}$:

$$T_l{}^j \Gamma^l_{22} = \begin{pmatrix} 0 \\ X^1X^2 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 & X^1X^2 & 0 \end{pmatrix}$$

$$T_l{}^j \Gamma^l_{21} = \begin{pmatrix} (X^1)^2 \\ 0 \\ X^1X^3 \end{pmatrix} \to \begin{pmatrix} (X^1)^2 & 0 & X^1X^3 \end{pmatrix} \leftarrow T_l{}^j \Gamma^l_{12}$$

$$T_l{}^j \Gamma^l_{ik} = \begin{pmatrix} 0 & 0 & 0 & (X^1)^2 & 0 & X^2X^3 & 0 & 0 & 0 \\ (X^1)^2 & 0 & X^2X^3 & 0 & X^1X^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$T_i{}^l \Gamma^j_{lk}$:

$$T_i{}^l \Gamma^1_{l2} = \begin{pmatrix} X^1X^2 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} X^1X^2 \\ 0 \\ 0 \end{pmatrix}$$

$$T_i^l \Gamma^2_{l1} = \begin{pmatrix} -\frac{X^2}{X^1} & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -\frac{X^2}{X^1} \\ 0 \\ 0 \end{pmatrix}$$

$$T_i{}^l \Gamma^2_{l2} = \begin{pmatrix} 0 & (X^1)^2 & 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ (X^1)^2 \\ 0 \end{pmatrix}$$

$$T_i{}^l \Gamma^j_{lk} = \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 & X^1X^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (X^1)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_k T_i{}^j = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 3(X^1)^2 & 0 & 3(X^1)X^3 & 0 & 0 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} -$$

$$- \begin{pmatrix} 0 & 0 & 0 & (X^1)^2 & 0 & X^2X^3 & 0 & 0 & 0 \\ (X^1)^2 & 0 & X^2X^3 & 0 & X^1X^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 & X^1X^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (X^1)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 & X^1X^2 - (X^1)^2 & -1 & -X^2X^3 & 0 & 0 & 0 \\ 2(X^1)^2 & 0 & 3(X^1)X^3 - X^2X^3 & 0 & (X^1)^2 - X^1X^2 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\nabla_2 T^i{}_j, \nabla_1 T_1{}^j$ неправильные...

$\nabla_2 T^i_j$:

$$T^i_{j2} = \begin{pmatrix} 0 & -(X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^l_j \Gamma^i_{l2} = \begin{pmatrix} 0 & -X_1 & 0 \\ \frac{1}{X^1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -(X^1)^2 & 0 & -X^1X^3 \\ 0 & -X_1X_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^i{}_l \Gamma^l_{j2} = \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -X_1 & 0 \\ \frac{1}{X^1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -X_1X_2 & 0 & 0 \\ 0 & -(X_1)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T^i{}_j = \begin{pmatrix} X_1X_2 - (X^1)^2 & -(X^1)^2 & -X^1X^3 \\ 0 & (X_1)^2 - X_1X_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Неправильно: $\nabla_1 T_i{}^j, \nabla_2 T_i{}^j, \nabla_1 T^i{}_j$

$$T^i{}_{j1} = \begin{pmatrix} 0 & -2X^1X^2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^l_j \Gamma^i_{l1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^i_l \Gamma^l_{j1} = \begin{pmatrix} 0 & -X^2(X^1)^2 & 0 \\ X^1 & 0 & X^3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -X_1X_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_1 T^i{}_j = \begin{pmatrix} 0 & -X^1X^2 & 0 \\ 2 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix}$$

Неправильно: $\nabla_1 T_i{}^j, \nabla_2 T_i{}^j$

$$T_i{}^j{}_k = \left(\begin{pmatrix} 0 & 3(X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & 2X^1X^3 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & (X^1)^2 & 0 \end{pmatrix} \right)$$

$$T_l{}^j = \begin{pmatrix} 0 & (X^1)^3 & 0 \\ -X^2 & 0 & 0 \\ 0 & (X^1)^2X^3 & 0 \end{pmatrix}$$

$$\Gamma^l_{ik} = \begin{pmatrix} \Gamma^1_{ik} \\ \Gamma^2_{ik} \\ \Theta \end{pmatrix}$$

$$\Gamma_{ik}{}^l T_l{}^j = \begin{pmatrix} \Gamma^1_{ik} \\ \Gamma^2_{ik} \\ \Theta \end{pmatrix} \begin{pmatrix} 0 & (X^1)^3 & 0 \\ -X^2 & 0 & 0 \\ 0 & (X^1)^2X^3 & 0 \end{pmatrix} = \left(\Gamma^1_{ik} \begin{pmatrix} 0 \\ -X^2 \\ 0 \end{pmatrix} + \Gamma^2_{ik} \begin{pmatrix} (X^1)^3 \\ 0 \\ (X^1)^2X^3 \end{pmatrix} \right) =$$

$$= \left(\begin{pmatrix} \Theta & \begin{pmatrix} 0 & -X^1 & 0 \end{pmatrix} & \Theta \end{pmatrix} \begin{pmatrix} 0 \\ -X^2 \\ 0 \end{pmatrix} + \left(\begin{pmatrix} 0 & \frac{1}{X^1} & 0 \end{pmatrix} \quad \begin{pmatrix} \frac{1}{X^1} & 0 & 0 \end{pmatrix} \quad \Theta \right) \begin{pmatrix} (X^1)^3 \\ 0 \\ (X^1)^2X^3 \end{pmatrix} \right) =$$

$$= \left(\Theta \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & X^1X^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Theta \right) + \left(\begin{pmatrix} 0 & (X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & X^1X^3 & 0 \end{pmatrix} \quad \begin{pmatrix} (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \\ X^1X^3 & 0 & 0 \end{pmatrix} \quad \Theta \right) =$$

$$= \left(\begin{pmatrix} 0 & (X^1)^2 & 0 \\ 0 & 0 & 0 \\ 0 & X^1X^3 & 0 \end{pmatrix} \quad \begin{pmatrix} (X^1)^2 & 0 & 0 \\ 0 & X^1X^2 & 0 \\ X^1X^3 & 0 & 0 \end{pmatrix} \quad \Theta \right)$$

Сравнив с более ранним значением $\Gamma^l_{ik} T_l{}^j$ я обнаружил ошибку (вместо X^2X^3 должно было быть X^1X^3, а также что $\frac{\partial}{\partial X^1}((X^1)^2X^3) = 2X^1X^3$, а не $3X^1X^3$), после исправления которой результат получился верным

$$\nabla_k T_i{}^j = \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 & X^1X^2-(X^1)^2 & -1 & -X^1X^3 & 0 & 0 & 0 \\ 2(X^1)^2 & 0 & 2(X^1)X^3 - X^1X^3 & 0 & (X^1)^2 - X^1X^2 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^k T = g^{kl}\nabla_k T$$

$$\nabla^l T^{ij} = g^{kl}\nabla_k T^{ij} = \left(\begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 \\ 2 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} X^1X^2-(X^1)^2 & -1 & -X^1X^3 \\ 0 & 1-\frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\right) \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{(X^1)^2} \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 \\ 2 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} X^1X^2-(X^1)^2 & -1 & -X^1X^3 \\ 0 & 1-\frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{(X^1)^2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 \\ 2 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} \frac{X^2}{X^1} - 1 & -\frac{1}{(X^1)^2} & -\frac{X^3}{X^1} \\ 0 & \frac{1}{(X^1)^2} - \frac{X^2}{(X^1)^3} & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{pmatrix}$$

$$\nabla^k T_{ij} = \begin{pmatrix} 0 & -X^1X^2 & 0 & X^1X^2-(X^1)^2 & -(X^1)^2 & -X^1X^3 & 0 & 0 & 0 \\ 2(X^1)^2 & 0 & X^1X^3 & 0 & (X^1)^4-(X^1)^3X^2 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{(X^1)^2} \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -X^1X^2 & 0 \\ 2(X^1)^2 & 0 & X^1X^3 \\ 0 & 0 & 0 \\ \frac{X^2}{X^1} - 1 & -1 & -\frac{X^3}{X^1} \\ 0 & (X^1)^2 - X^1X^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^k T^i{}_j = \left(\begin{pmatrix} 0 & -X^1X^2 & 0 \\ 2 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} X_1X_2-(X^1)^2 & -(X^1)^2 & -X^1X^3 \\ 0 & (X_1)^2 - X_1X_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \right) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{(X^1)^2} \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & -X^1X^2 & 0 \\ 2 & 0 & \frac{X^3}{X^1} \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} \frac{X_2}{X^1} - 1 & -1 & -\frac{X^3}{X^1} \\ 0 & 1 - \frac{X_2}{X^1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{pmatrix}$$

$$\nabla^k T_j{}^i =$$

$$\begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 & X^1X^2-(X^1)^2 & -1 & -X^1X^3 & 0 & 0 & 0 \\ 2(X^1)^2 & 0 & 2(X^1)X^3 - X^1X^3 & 0 & (X^1)^2 - X^1X^2 & 0 & 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{(X^1)^2} \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -\frac{X^2}{X^1} & 0 \\ 2(X^1)^2 & 0 & X^1X^3 \\ 0 & 0 & 0 \\ \frac{X^2}{X^1} - 1 & -\frac{1}{(X^1)^2} & -\frac{X^3}{X^1} \\ 0 & 1 - \frac{X^2}{X^1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (X^1)^2 \\ 0 & 0 & 0 \end{pmatrix}$$