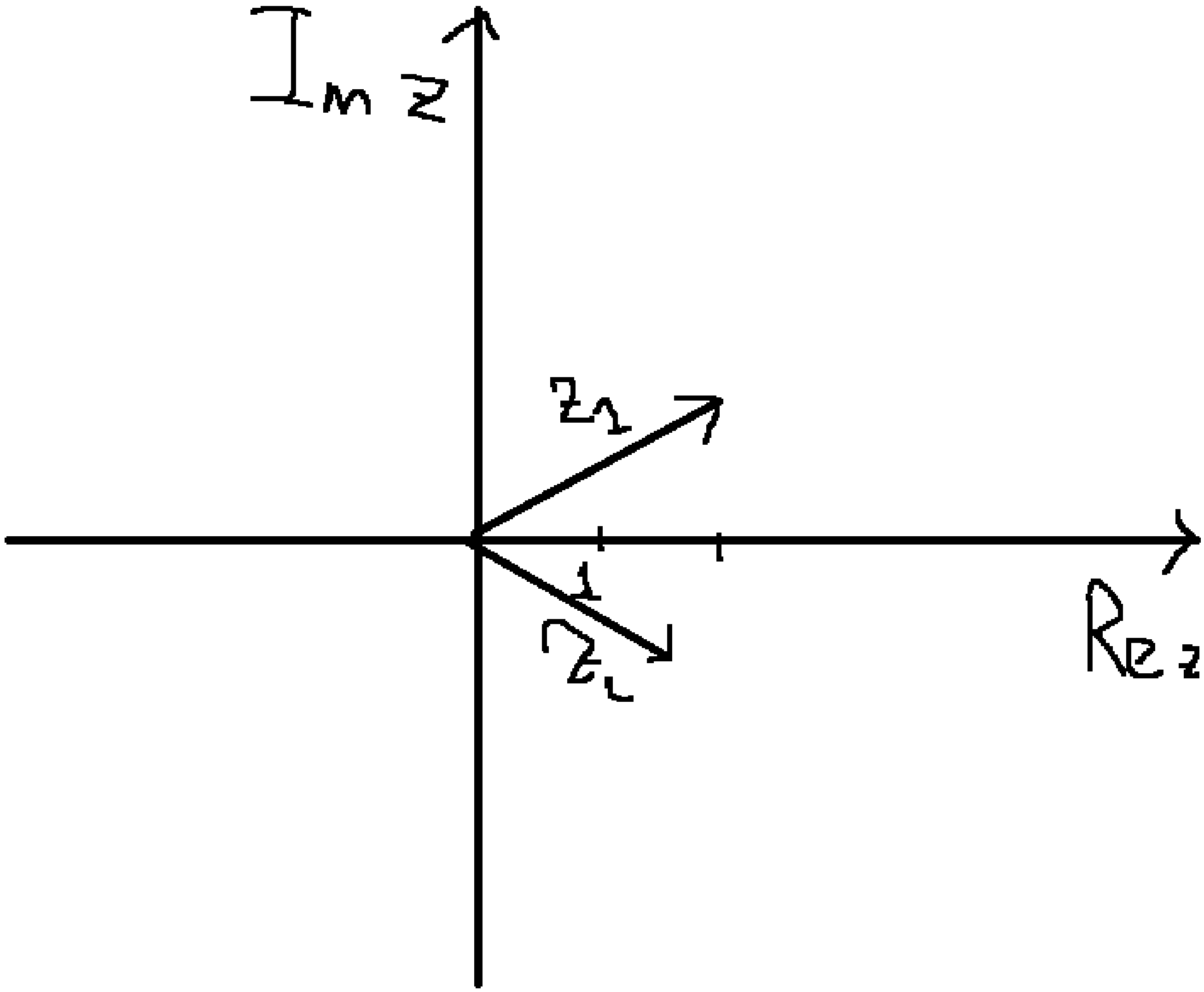
13/02/2025



$$z_1 = \sqrt{3} - i = 2e^{-i\frac{\pi}{6}}$$
$$z_2 = 2 + 2i = \sqrt{8}e^{i\frac{\pi}{4}}$$
$$z_1 \cdot z_2 = (\sqrt{3} - i)(2 + 2i) = 2\sqrt{3} + 2\sqrt{3}i - 2i + 2 = 2\sqrt{3} + 2 + i(2\sqrt{3} - 2)$$
$$z_1 \cdot z_2 = 2e^{-i\pi/8} \cdot \sqrt{8}e^{i\pi/4} = 2\sqrt{8}e^{i(\frac{\pi}{4} - \frac{\pi}{6})} = 4\sqrt{2}e^{i\frac{\pi}{12}}$$
$$4\sqrt{2}\cos\left(\frac{\pi}{12}\right) = 2\sqrt{3} + 2 \Rightarrow \cos\left(\frac{\pi}{12}\right) = \frac{2\sqrt{3} + 2}{4\sqrt{2}}$$

$$\frac{z_1^2}{\overline{z_2}} = \frac{(\sqrt{3}-i)^2}{2+2i} = \frac{3-2\sqrt{3}i-1}{2-2i} = \frac{(2-2\sqrt{3}i)(2+2i)}{(2-2i)(2+2i)} = \frac{4+4i+4\sqrt{3}i+4\sqrt{3}}{4+4} =$$

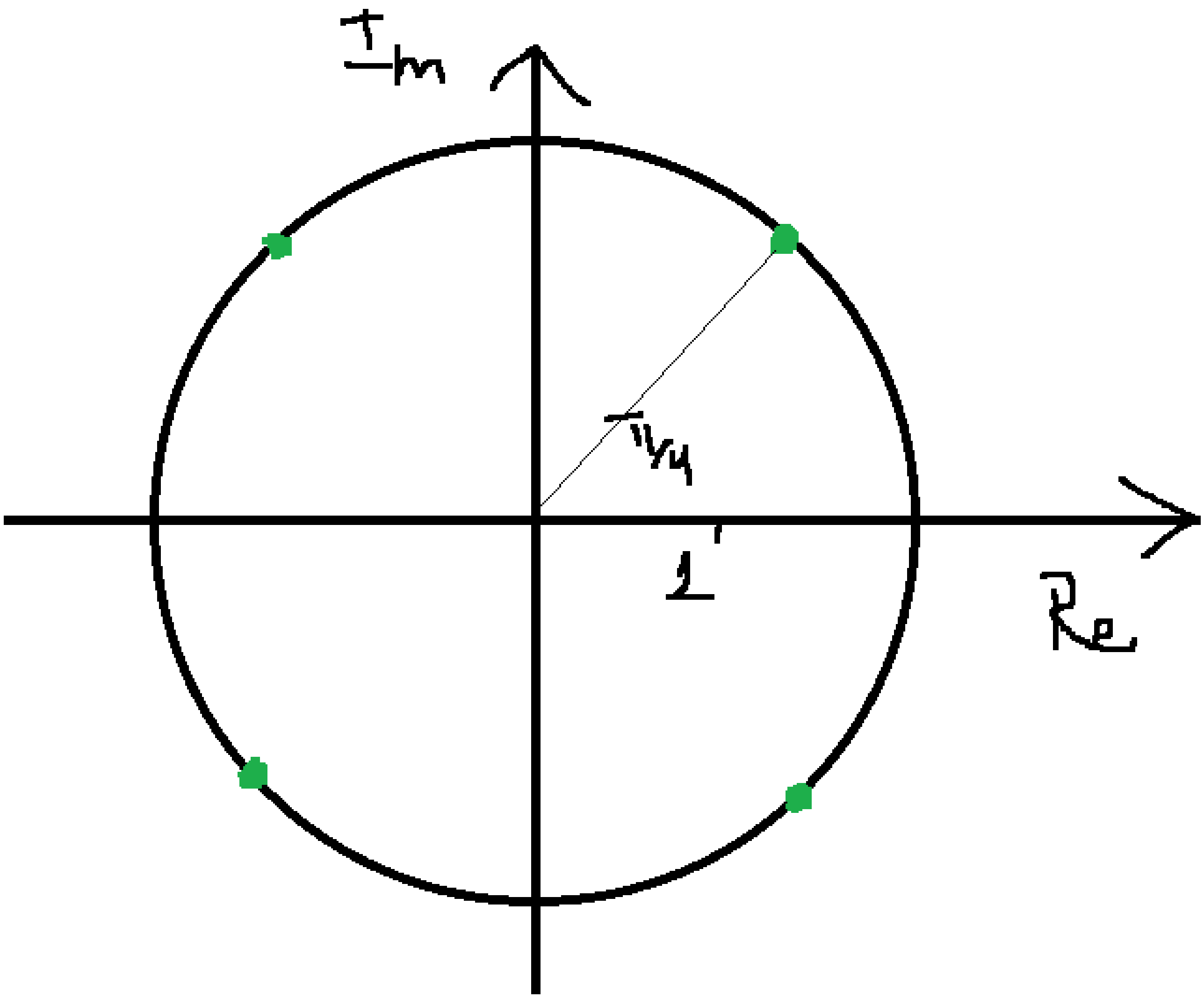
$$= \frac{1+\sqrt{3}+i(1-\sqrt{3})}{2}$$

$$\frac{z_1^2}{\overline{z_2}} = \frac{4e^{-i\pi/3}}{\sqrt{8}e^{-i\pi/4}} = \sqrt{2}e^{-i\frac{\pi}{12}}$$

$$z = -16 = 16e^{\pi i}$$

$$\sqrt[4]{z} = 2e^{\frac{1}{4}i(\pi/4+2\pi k)} = 2e^{\frac{1}{4}i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)},\ k \in \{0,1,2,3\}$$

$$x+iy = \pm\sqrt{2} \pm i\sqrt{2}$$

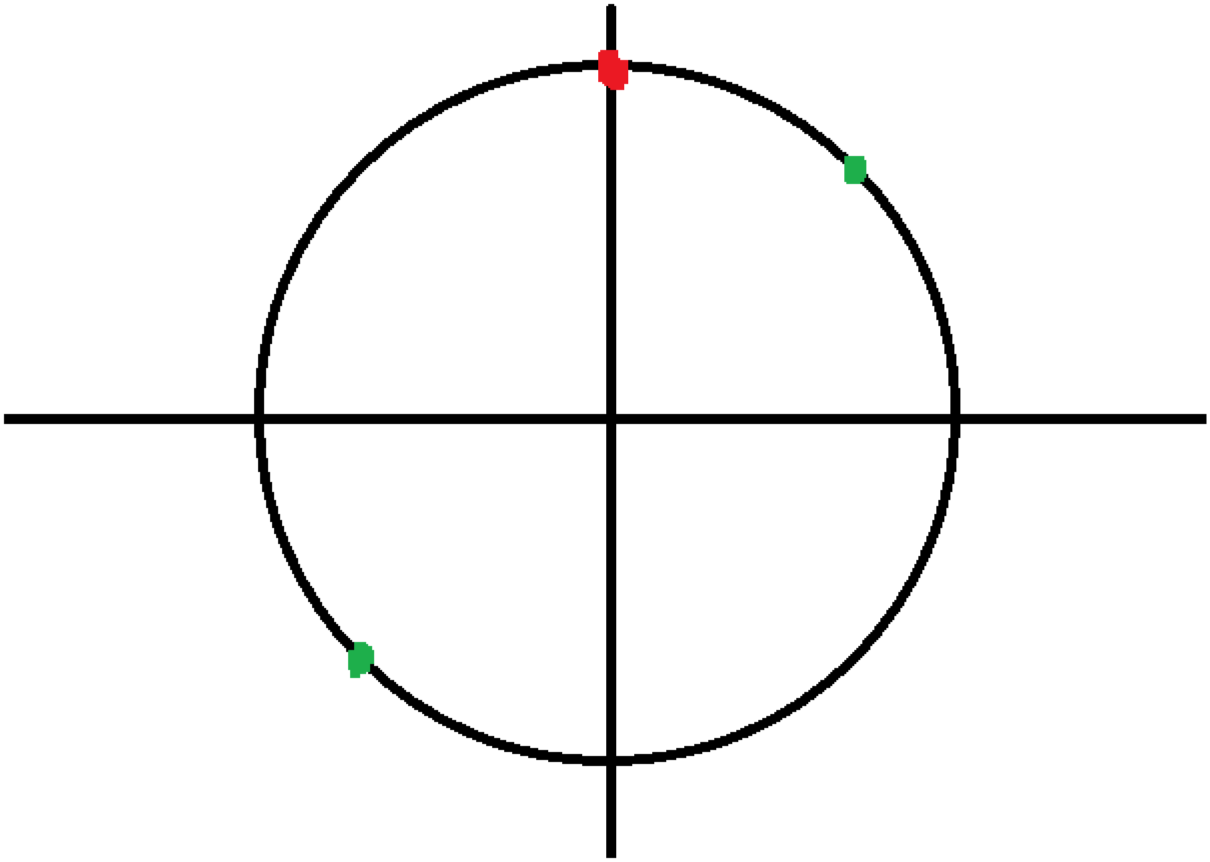


Корень n-ой степени - n значений

$$\sqrt{-1} = \pm 1$$

$$\sqrt{1} = \sqrt{e^{2\pi n i}} = e^{\pi n i} = \begin{cases} e^0 = 1 \\ e^{\pi i} = -1 \end{cases}$$

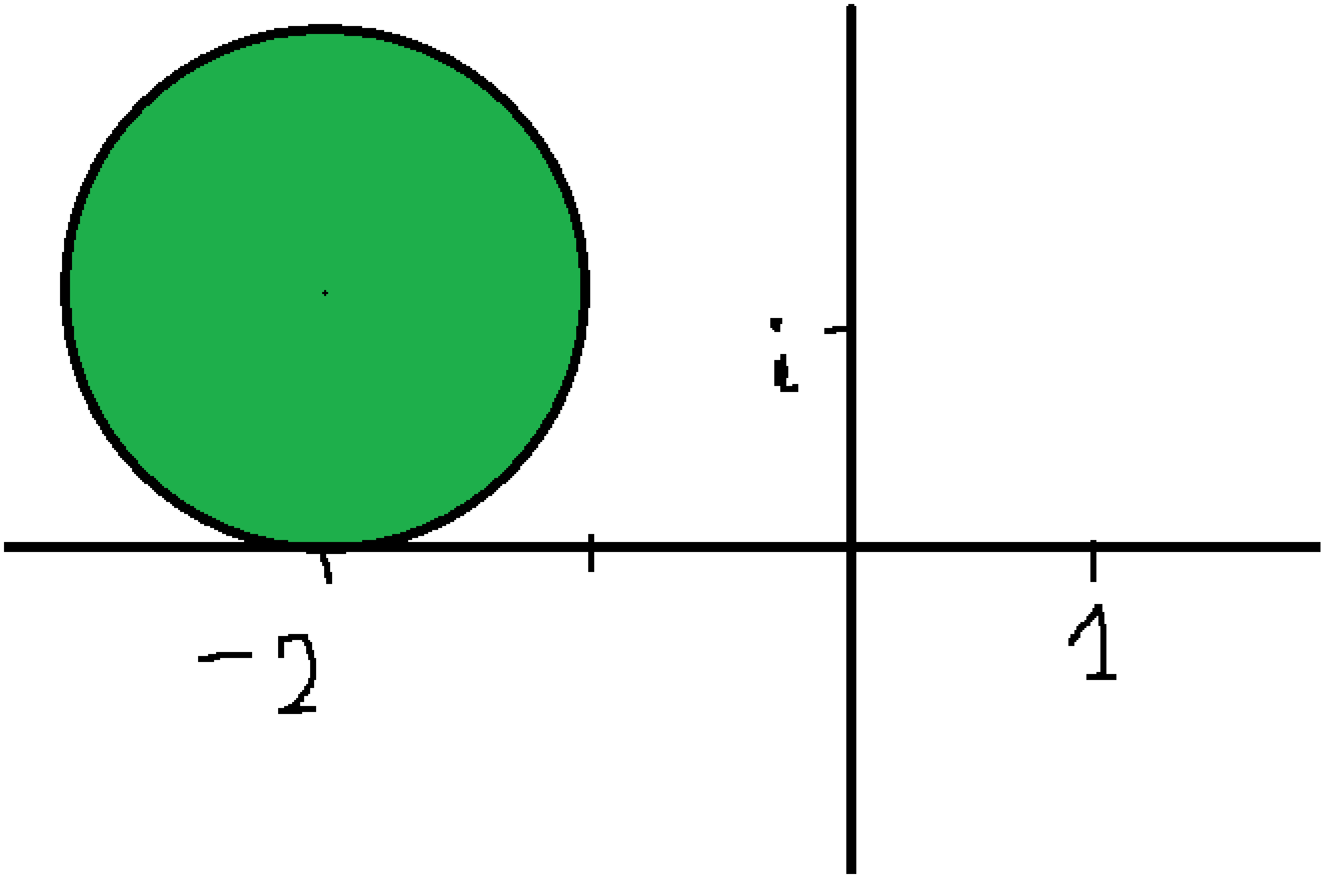
$$\sqrt{i} = e^{i\left(\frac{\pi}{2}+2\pi n\right)} = e^{i\left(\frac{\pi}{4}+\pi n\right)} = \begin{cases} e^{\frac{\pi i}{4}} = \frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2} \\ e^{\frac{5\pi i}{4}} = -\frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2} \end{cases}$$



$|z+2-i| \leq 1$

Можно свести задачу к школьной, перейдя к декартовым координатам,
но иногда можно решить проще

$|z-(-2+i)| \leq 1$ — Окружность с центром в (-2+i)



$$\begin{cases} |z+3|+|z+3i| \le 6 \ - \ \text{Эллипс} \\ |z+\frac{3}{2}+\frac{3}{2}i| > \frac{3\sqrt{2}}{2} \end{cases}$$

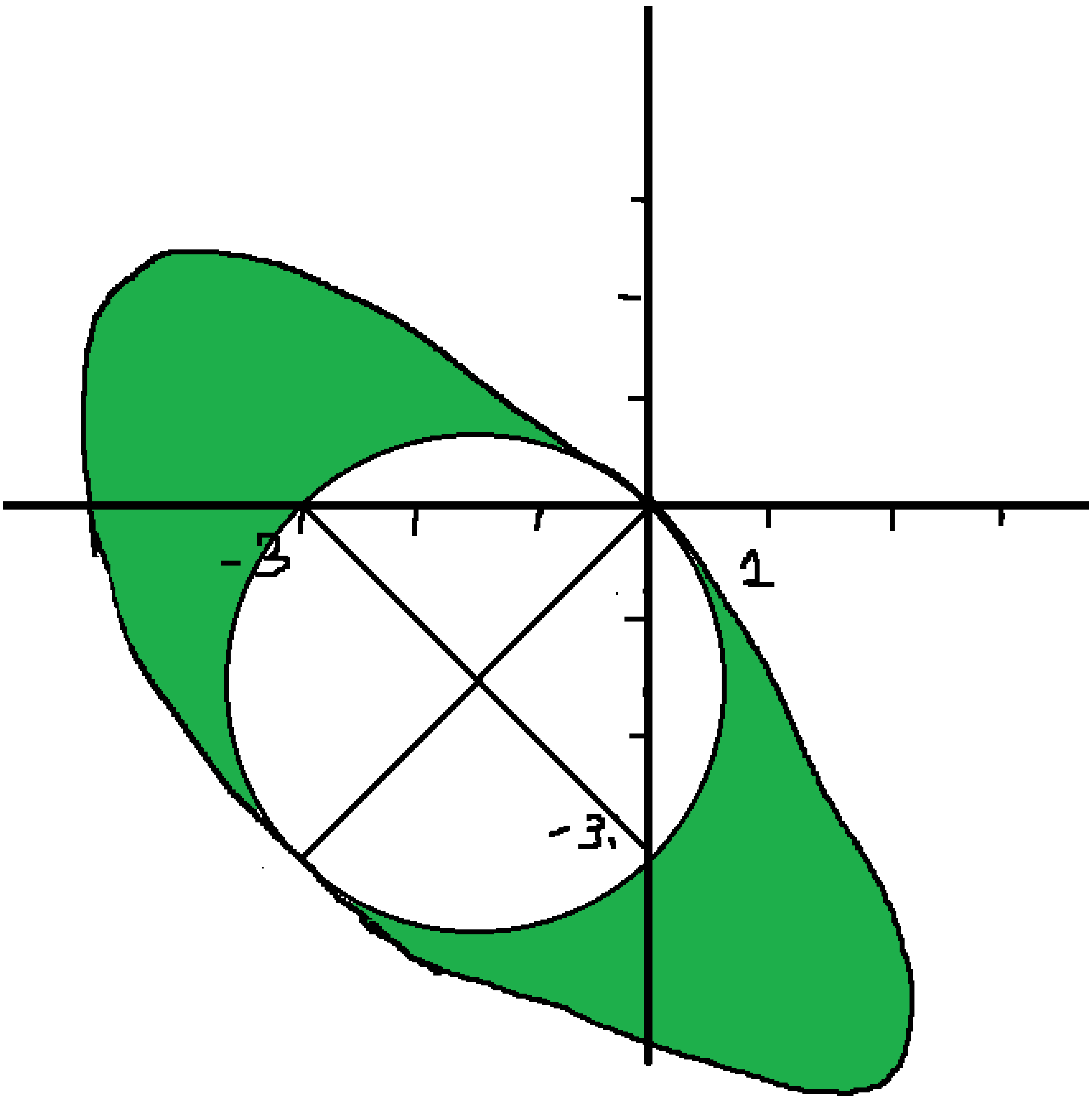
$$z_1 = -3$$

$$z_2 = -3i$$

$$a = \frac{6}{2} = 3$$

$$c = \frac{3\sqrt{2}}{2}$$

$$b = \sqrt{a^2 - c^2} = \frac{3\sqrt{2}}{2}$$



$$|z - z_1| + |z - z_2| = 2a$$

$$|z - z_1| + |z - z_2| = 2a \ - \ \text{гипербола}$$

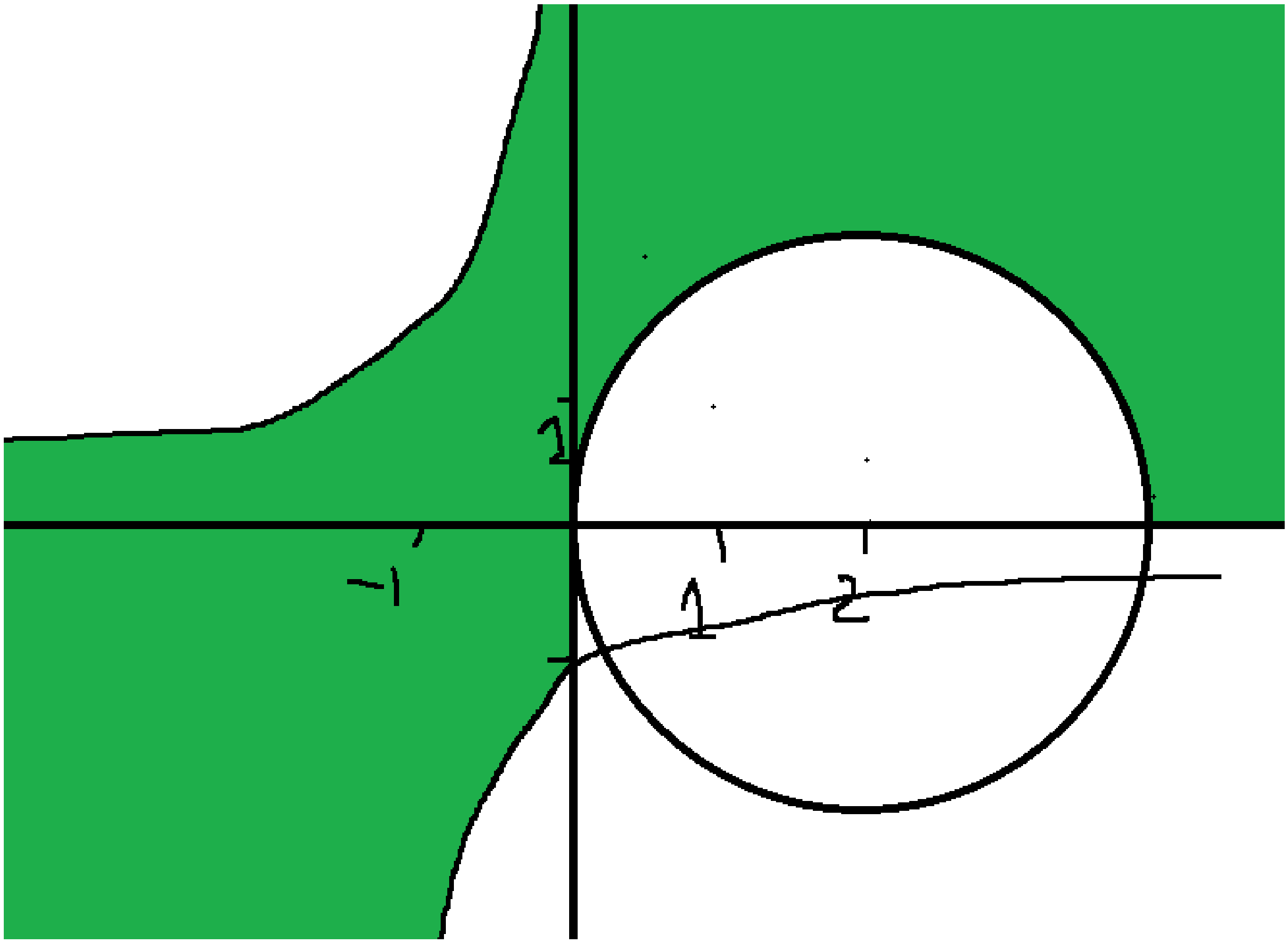
$$\begin{cases} \mathrm{Re}\left(\frac{1}{z}\right) \le \frac{1}{4} \\ \mathrm{Im}(\overline{z^2 - \overline{z}}) \le 2 + \mathrm{Im}(z) \end{cases}$$

$$z = x + iy$$

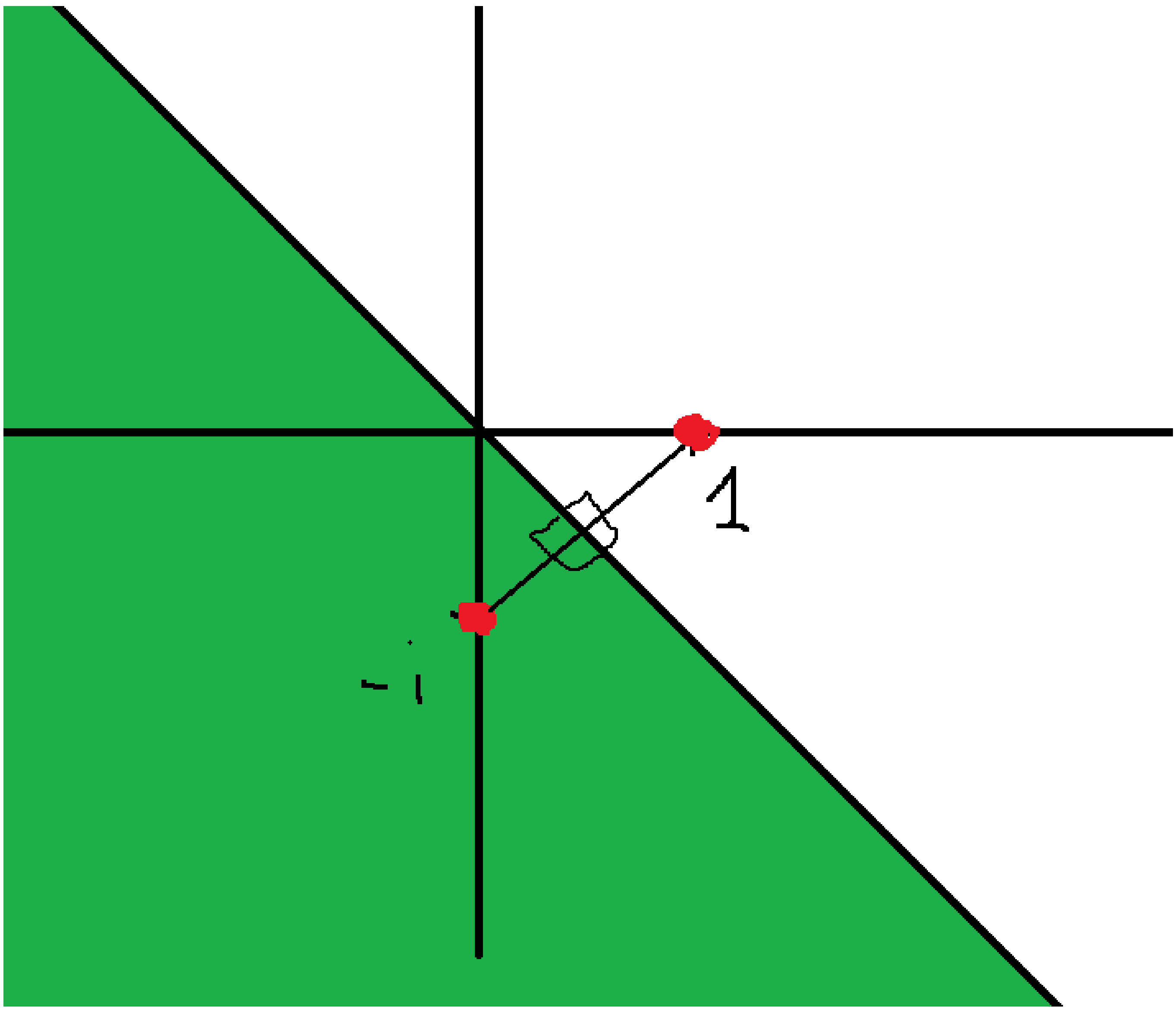
$$\mathrm{Re}\left(\frac{1}{x+iy}\right) = \mathrm{Re}\left(\frac{x-iy}{x^2+y^2}\right) = \frac{x}{x^2+y^2} \le \frac{1}{4}$$

$$\mathrm{Im}(\overline{(x+iy)^2 - \overline{x+iy}}) = \mathrm{Im}(\overline{x^2 - y^2 + 2xyi - (x - iy)}) = -2xy - y \le 2 + y$$

$$\begin{cases} \frac{x}{x^2+y^2} \le \frac{1}{4} \\ (x+1)y \ge -1 \end{cases} \Leftrightarrow \begin{cases} 4x \le x^2 + y^2 \\ (x+1)y \ge -1 \end{cases} \Leftrightarrow \begin{cases} (x-2)^2 + y^2 \ge 4 \\ (x+1)y \ge -1 \end{cases}$$



$|z+i| < |z-1|$



$$|x+iy+i| < |x+iy-1|$$
$$\sqrt{x^2+(y+1)^2} < \sqrt{(x-1)^2+y^2}$$
$$x^2+y^2+2y+1 < x^2-2x+1+y^2$$
$$2x+2y < 0$$
$$x+y < 0$$

$$\begin{cases} |z^2+4| \leq 4 \ - \ \text{Из номотеха: Лемниската} \\ \mathrm{Re}z < 0 \end{cases}$$

$$z = r(\cos(\varphi) + i\sin(\varphi))$$

$$z^2 = r^2(\cos(2\varphi) + i\sin(2\varphi)) \ - \ \text{Формула Муавра}$$

$$|z^2+4| \leq 4$$

$$|r^2(\cos(2\varphi) + i\sin(2\varphi)) + 4| \leq 4$$

$$\sqrt{(r^2\cos(2\varphi)+4)^2 + r^2\sin^2(2\varphi)} \leq 4$$

$$r^4\cos^2(2\varphi) + 16 + 8r^2\cos(2\varphi) + r^4\sin^2(2\varphi) \leq 16$$

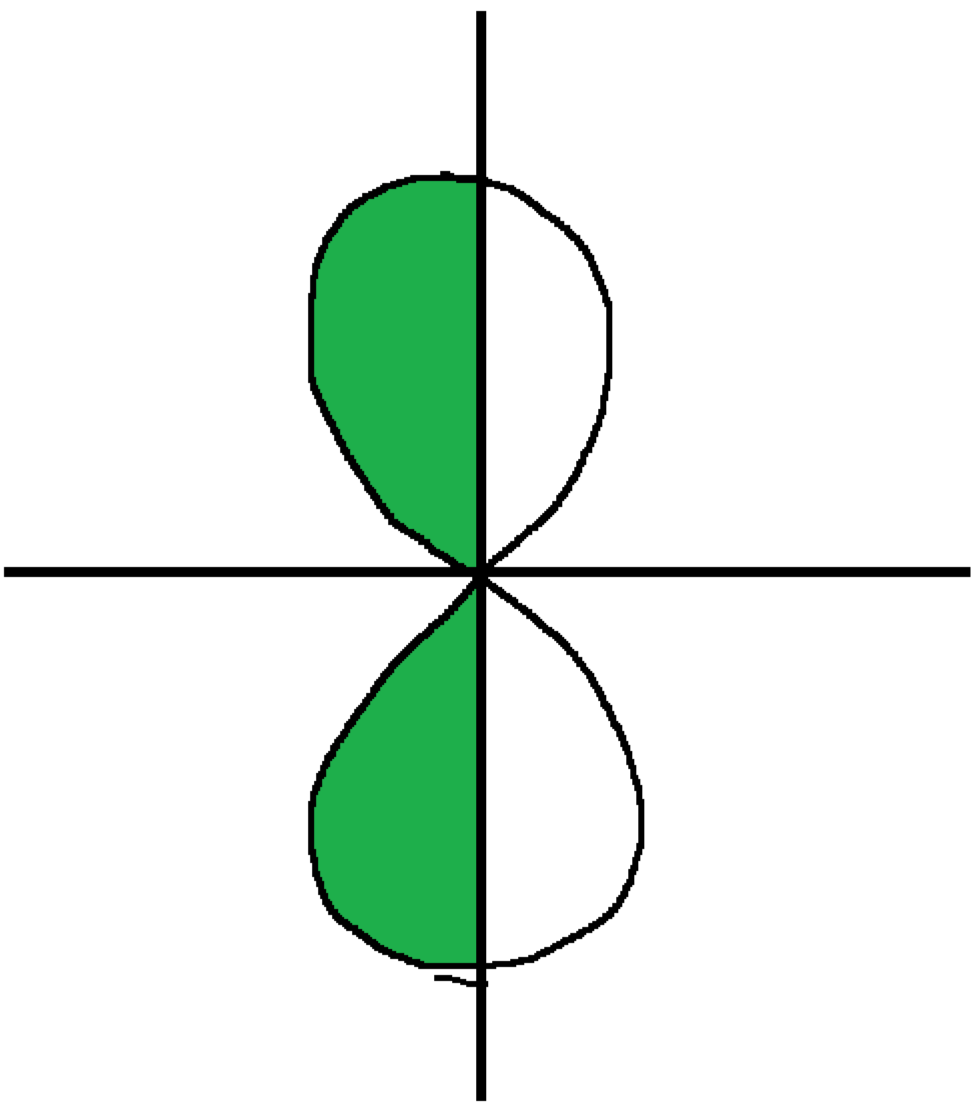
$$r^4\cos^2(2\varphi) + 8r^2\cos(2\varphi) + r^4\sin^2(2\varphi) \leq 0$$

$$r^4 + 8r^2\cos(2\varphi) \leq 0$$

$$r^2 + 8\cos(2\varphi) \leq 0$$

$$r^2 \leq -8\cos(2\varphi)$$

$$r < 2\sqrt{2}\sqrt{-\cos(2\varphi)}$$



20/02/2025

Пропустил

$$\sum_{n=1}^{N} \frac{\left(\frac{x}{|e-i|}\right)^n}{n!}$$

27/02/2025

$$\cos z = \cos x \operatorname{ch} y + i(-\sin x \operatorname{sh} y)$$
$$\operatorname{sh} iz = i \sin z$$
$$\operatorname{ch} iz = \cos z$$
$$\sin iz = i \operatorname{sh} z$$
$$\cos iz = \operatorname{ch} z$$

$$\operatorname{sh} \left(\ln 3 + \frac{i\pi}{4}\right) = \operatorname{sh} (\ln 3) \cos\left(\frac{\pi}{4}\right) + \operatorname{ch} (\ln 3) \cdot i \sin\left(\frac{\pi}{4}\right) =$$
$$\frac{1}{2\sqrt{2}}(e^{\ln 3} - e^{-\ln 3} + ie^{\ln 3} + ie^{-\ln 3})$$

Убедиться, что если подставить в $\alpha^z = e^{z \ln \alpha}$ $\alpha = e$, то полученные функции будут однозначными

$$i^i = e^{i \operatorname{Ln} i} = e^{i\left(i\frac{\pi}{2} + 2\pi k i\right)} = e^{-\left(\frac{\pi}{2} + 2\pi k\right)}, k \in \mathbb{Z}$$

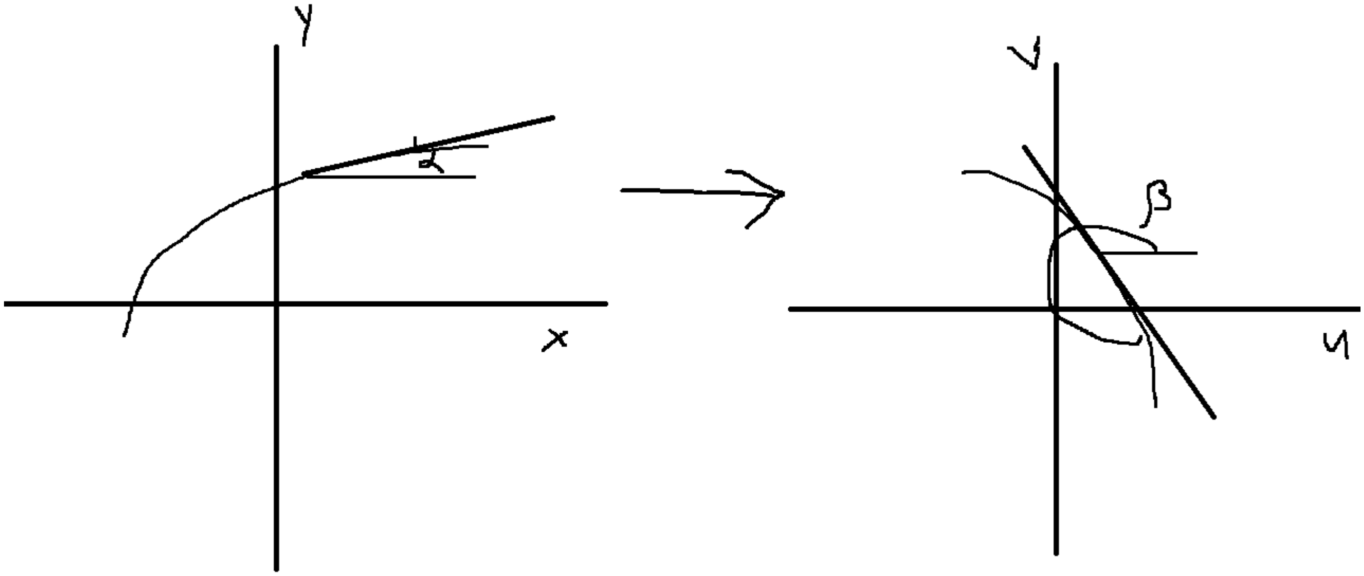
$$f(x) = \begin{cases} e^{-\frac{1}{|x|}}, x \neq 0 \\ 0, x = 0 \end{cases}$$

$$f(x, y) = f(z, \overline{z})$$
$$\text{C.R.} \Leftrightarrow \frac{\partial f}{\partial \overline{z}} = 0$$

$$\begin{cases} x = \frac{z + \overline{z}}{2} \\ y = \frac{z - \overline{z}}{2i} \end{cases}$$

$$f = |z| = \sqrt{z\overline{z}} = \psi(z, \overline{z}) \Rightarrow \text{ не дифференцируема}$$
$$z^2 = (x + iy)(x + iy) = x^2 - y^2 + 2xyi$$
$$\begin{pmatrix} 2x & 2y \\ -2y & 2x \end{pmatrix} = 2(x + iy) = 2z$$

Аргумент говорит о том, как поворачиваются касательные



$$\beta - \alpha = \arg f'(z)$$
$$|\Delta w| = |f'(z)||\Delta z| + |o(\Delta z)|$$

$$\begin{cases} u_{yx} = v_{yy} \\ u_{xy} = -v_{xx} \end{cases} \Rightarrow v_{yy} + v_{xx} = 0$$

u и v гармонические

$$v(x,y) = \frac{x^2 - y^2}{(x^2+y^2)^2}$$
$$v_{xx} = \left(\frac{2x(x^2+y^2)^2 - (x^2-y^2)(2(x^2+y^2)2x)}{(x^2+y^2)^4}\right)_x = \left(\frac{-2x^3+6xy^2}{(x^2+y^2)^3}\right)_x =$$
$$= \frac{2\left((3y^2-3x^2)(x^2+y^2)^3 - 3(x^2+y^2)^2 2x(3xy^2-x^3)\right)}{(x^2+y^2)^6} = \frac{2(-3x^4+3y^4-6(3x^2y^2-x^4))}{(x^2+y^2)^4} =$$
$$= \frac{6(x^4+y^4-6x^2y^2)}{(x^2+y^2)^4}$$
$$f(x,y) = -f(y,x)$$
$$f'(z) = u_x + iv_x = v_y - iu_y = u_x - iu_y = v_y + iv_x$$

$$f'(z) = \frac{2y^3-6yx^2}{(x^2+y^2)^3} + i\frac{-2x^3+6xy^2}{(x^2+y^2)^3} =$$
$$= \frac{2(y^3-3yx^2-ix^3+3ixy^2)}{(x^2+y^2)^3} = \frac{2(y+ix)^2}{(z\bar{z})^3} = 2\frac{i\cancel{\bar{z}^3}}{(z\cancel{\bar{z}})^3} = \frac{2i}{z^3} \Rightarrow$$
$$f(z) = -\frac{i}{z^2} + C$$

$u = \operatorname{ch} x \operatorname{sh} y$ - Очевидно, что не может быть действительной частью

Гармоническое векторное поле - это соленоидальное и потенциальное векторное поле

Соленоидальное векторное поле: $\operatorname{div} \vec{f} = 0$

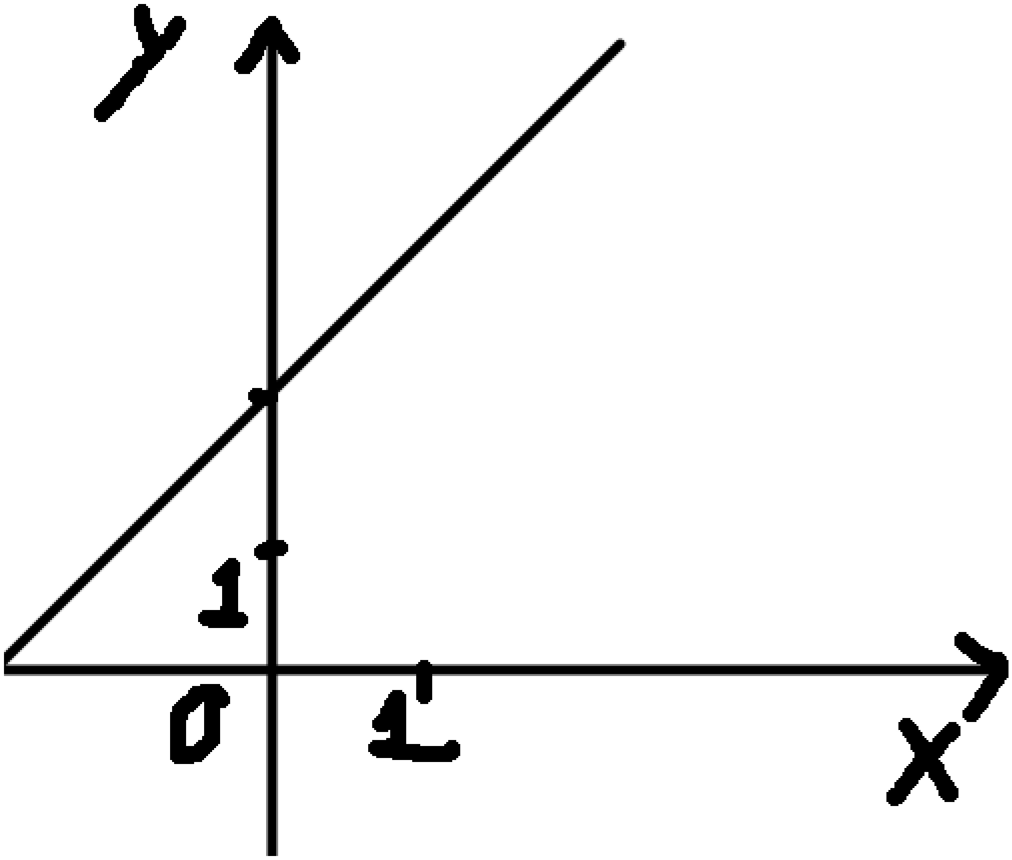
Потенциальное векторное поле: $\operatorname{rot} \vec{f} = 0$

06/03/2025

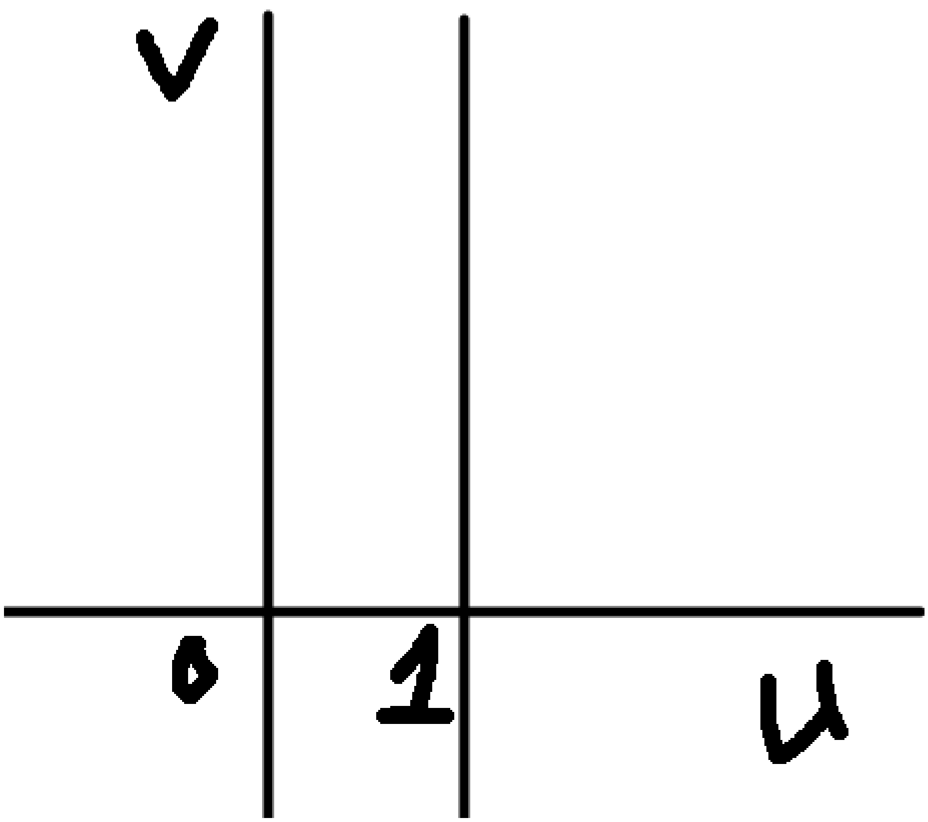
Конформные отображения.

Отображение, осуществляемое линейной функцией.

$$w = (1+i)z + (3-2i)$$
$$\text{б) } y = x + 2$$
$$w = (1+i)(x+iy) + (3-2i) = (x-y+3) + i(x+y-2) = u+iv$$
$$\begin{cases} u = x - y + 3 \\ v = x + y - 2 \end{cases} \Rightarrow \begin{cases} u + v = 2x + 1 \\ v - u = 2y - 5 \end{cases}$$
$$\begin{cases} x = \frac{u+v-1}{2} \\ y = \frac{v-u+5}{2} \end{cases}$$
$$\frac{v-u+5}{2} = \frac{u+v-1}{2} + 2$$
$$u = 1$$



y
1
0
1
x



v
0
1
u

$$y = \frac{3}{2}x$$
$$v - u + 5 = 3\frac{u + v - 1}{2}$$
$$2v - 2u + 10 = 3u + 3v - 3$$
$$13 = 5u + v$$
$$v = 13 - 5u$$

Дробно-линейные функции

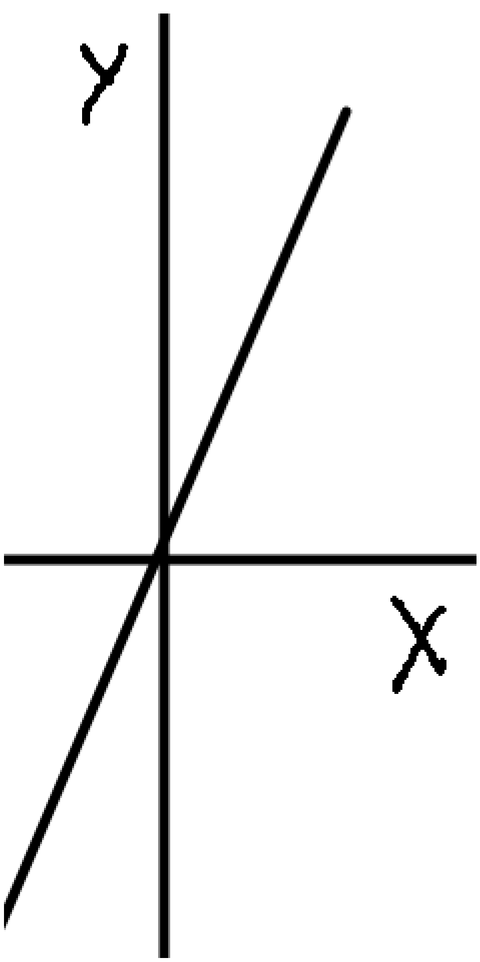
Сдвиг

Инверсия

Растяжение/сжатие, поворот

Круговое свойство: отображает окружности в окружности

$$w = \frac{1}{z}$$
а) $y = 3x$



$$w = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2}$$

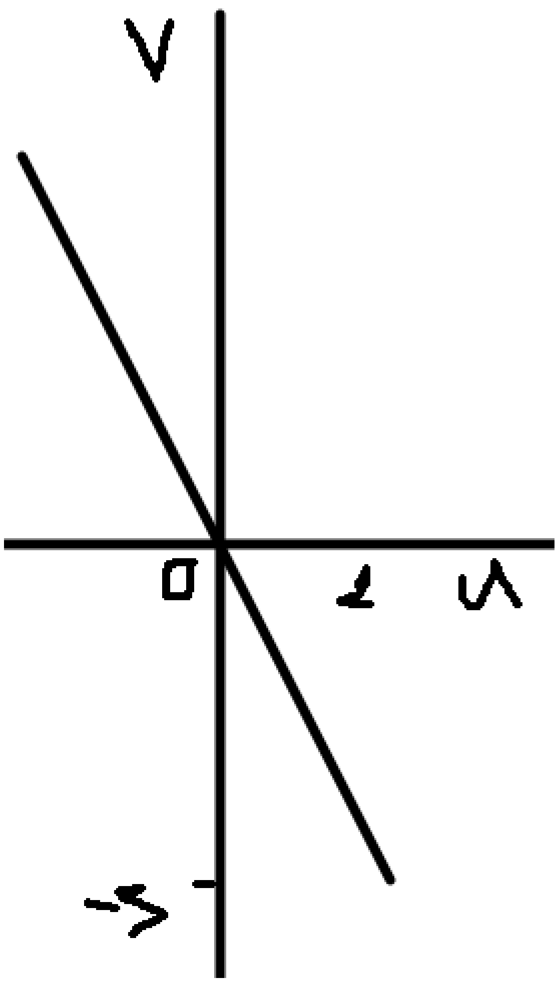
$$\begin{cases} u = \frac{x}{x^2+y^2} \\ v = -\frac{y}{x^2+y^2} \end{cases}$$

$$u^2+v^2 = \frac{1}{x^2+y^2} \Rightarrow$$

$$\begin{cases} x = \frac{u}{u^2+v^2} \\ y = -\frac{v}{u^2+v^2} \end{cases}$$

$$-\frac{v}{\cancel{u^2+v^2}} = 3\frac{u}{\cancel{u^2+v^2}}$$

$$v = -3u$$

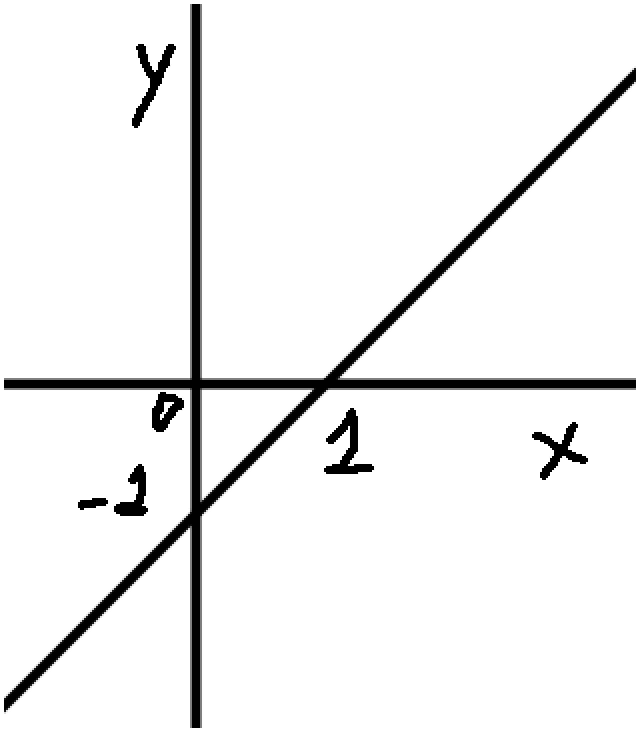


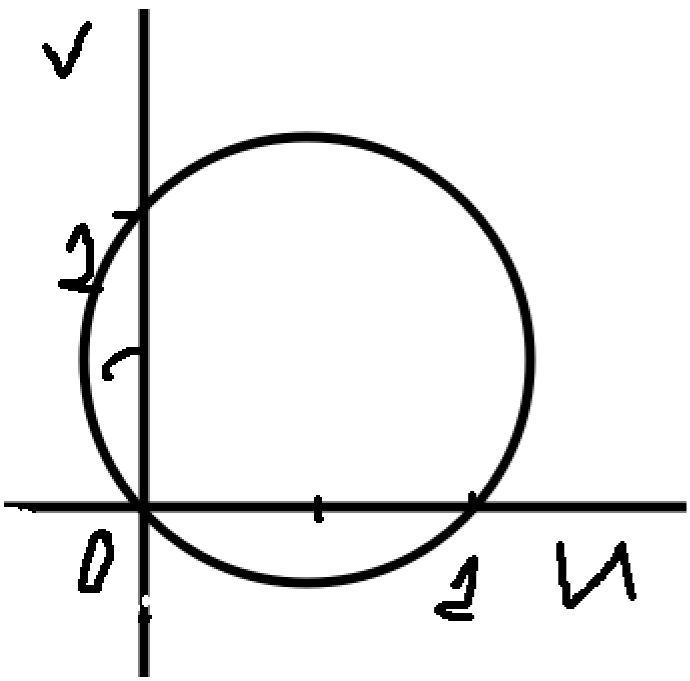
$$\text{б) } y = x - 1$$

$$-\frac{v}{u^2+v^2} = \frac{u}{u^2+v^2} - 1$$

$$0 = v + u - u^2 - v^2$$

$$\left(u - \frac{1}{2}\right)^2 + \left(v - \frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$$





$$в)\ |z-i-1|=\sqrt{2}$$

$$(x-1)^2+(y-1)^2=2$$

$$\frac{(u-u^2-v^2)^2}{(u^2+v^2)^2}+\frac{(-v-u^2-v^2)^2}{(u^2+v^2)^2}=2$$

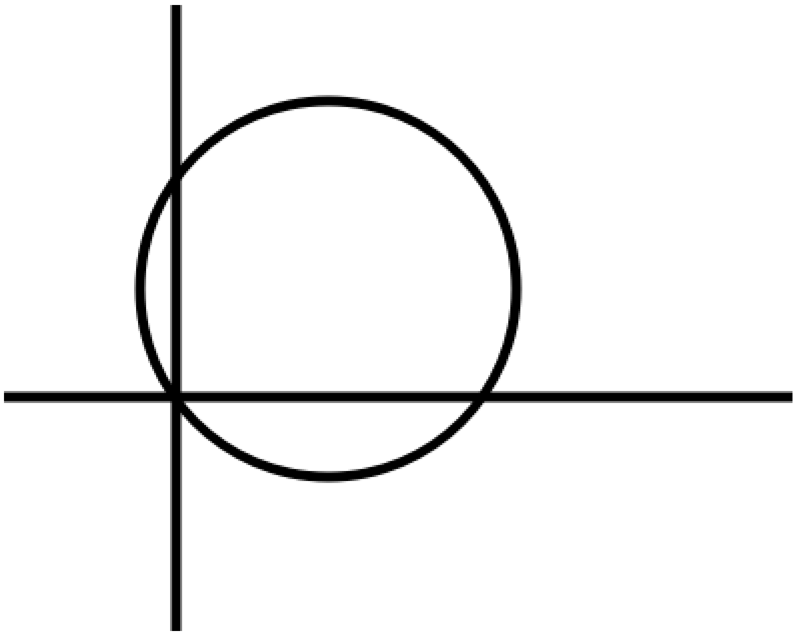
$$\begin{array}{l} u^2 \cancel{+u^4+v^4} - 2u^3 - 2uv^2 \cancel{+2u^2v^2} \\ v^2 \cancel{+u^4+v^4} + 2v^3 + 2vu^2 \cancel{+2u^2v^2} \end{array} + = \cancel{2u^4+2v^4+4u^2v^2}$$

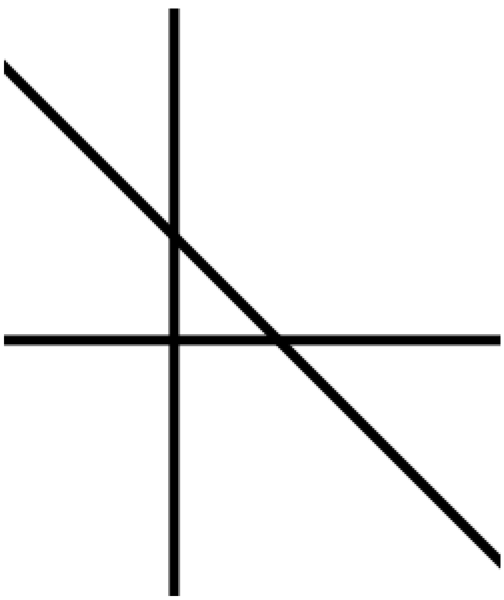
$$u^2-2u^3-2uv^2+2vu^2+2v^3+v^2=0$$

$$(u^2+v^2)-2u(u^2+v^2)+2v(u^2+v^2)=0$$

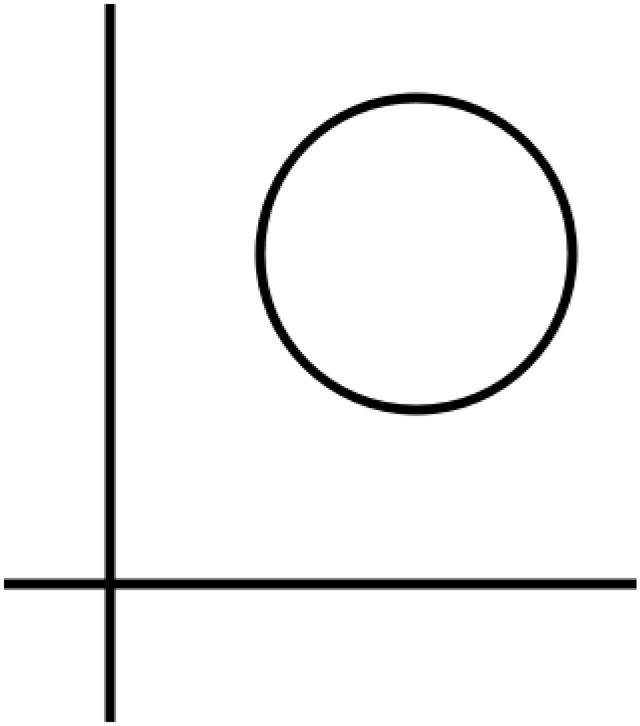
$$1-2u+2v=0$$

$$v=u-\frac{1}{2}$$

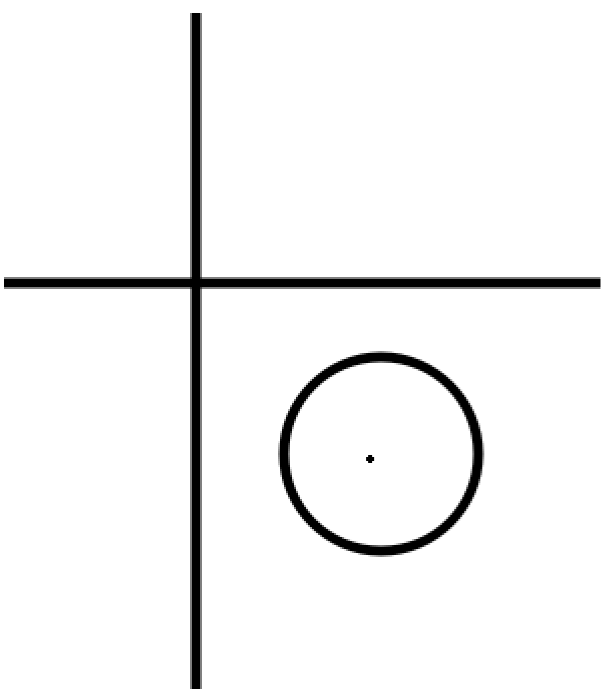




в) $|z-2-2i|=1$



$$(x-2)^2+(y-2)^2=1$$
$$\left(\frac{u}{u^2+v^2}-2\right)^2+\left(-\frac{v}{u^2+v^2}-2\right)^2=1$$
$$\frac{u^2}{(u^2+v^2)^2}+4-\frac{4u}{u^2+v^2}+\frac{v^2}{(u^2+v^2)^2}+4+\frac{4v}{u^2+v^2}=1$$
$$\frac{1-4u+4v}{u^2+v^2}=-7$$
$$\frac{1}{7}-\frac{4}{7}u+\frac{4}{7}v+u^2+v^2=0$$
$$\left(u-\frac{2}{7}\right)^2+\left(v+\frac{2}{7}\right)^2=\frac{1}{7^2}$$



$$f(z) = \frac{z-2i}{z+2}$$

a) $y = x + 2$

$$\frac{x+iy-2i}{x+iy+2} = \frac{(x+iy-2i)(x+2-iy)}{(x+2)^2+y^2} = \frac{x^2+y^2+\dots}{(x+2)^2+y^2}$$ - сложно

$$w = \frac{z-2i}{z+2}$$

$$(z+2)w = z - 2i$$

$$z(w-1) = -2i - 2w$$

$$z = -2\frac{w+i}{w-1}$$

$$x + iy = -2\frac{u+vi+i}{u+vi-1} = -2\frac{(u+(v+1)i)((u-1)-vi)}{(u-1)^2+v^2} =$$

$$= -2\frac{u^2-u+v^2+v+i(-uv+uv+u-v-1)}{(u-1)^2+v^2} =$$

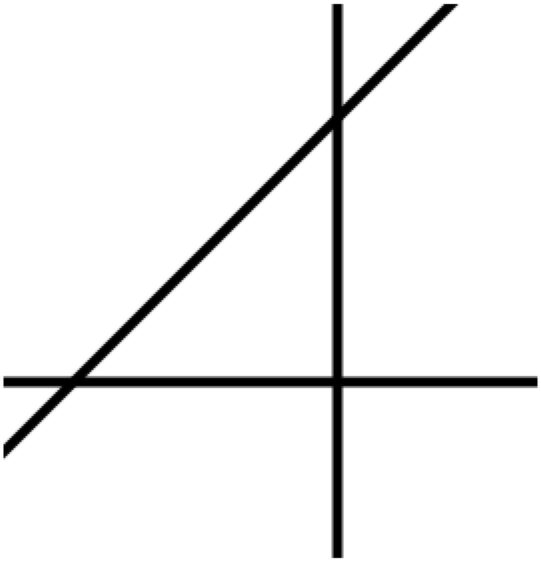
$$= -2\left(\frac{u^2-u+v^2+v}{(u-1)^2+v^2} + i\frac{u-v-1}{(u-1)^2+v^2}\right)$$

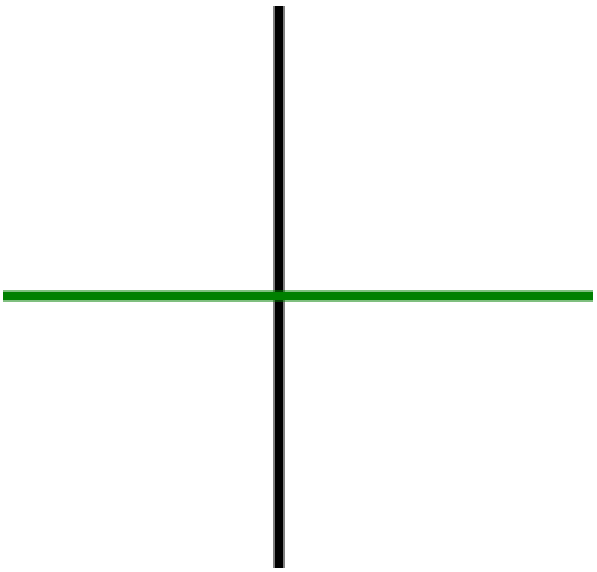
$$\begin{cases} x = -2\frac{u^2-u+v^2+v}{(u-1)^2+v^2} \\ y = -2\frac{u-v-1}{(u-1)^2+v^2} \end{cases}$$

$$-\cancel{2}(u-v-1) = -\cancel{2}(u^2-u+v^2+v) + \cancel{2}((u-1)^2+v^2)$$

$$\cancel{-u} + v \cancel{+1} = \cancel{-u^2} \cancel{+u} \cancel{-v^2} - v \cancel{+u^2} - \cancel{2}\cancel{u} \cancel{+1} \cancel{+v^2}$$

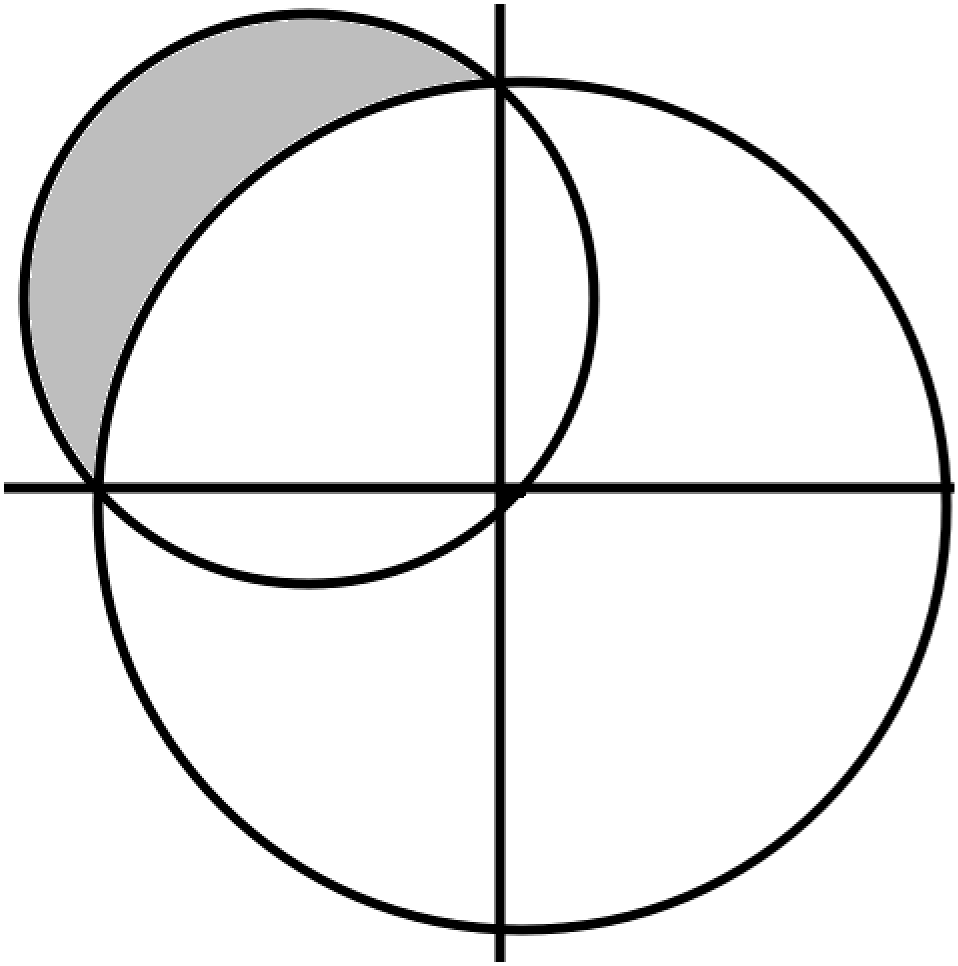
$$v = 0$$





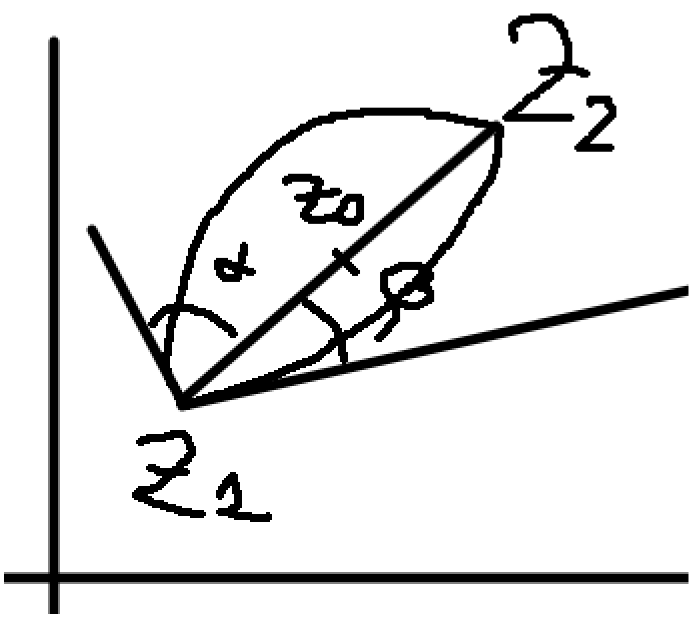
Лунка - область, ограниченная двумя окружностями

$$б)\ \begin{cases} |z| > 2 \\ |z+1-i| < \sqrt{2} \end{cases}$$



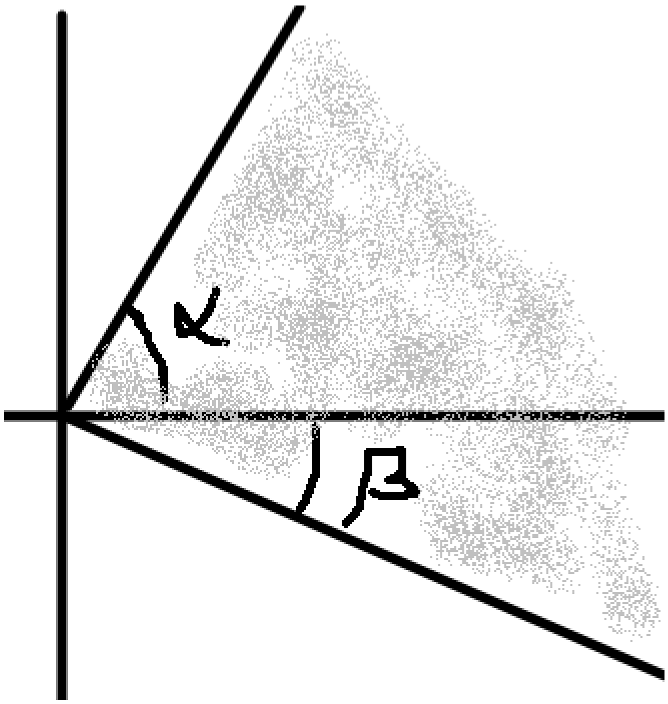
Следующие соображения

$$f(z) = -\frac{z - z_1}{z - z_2}$$



$$w_1 = f(z_1) = 0$$
$$w_2 = f(z_2) = \infty$$
$$z_0 = \frac{z_1 + z_2}{2}$$
$$f(z_0) = -\frac{\frac{z_1+z_2}{2} - z_1}{\frac{z_1+z_2}{2} - z_2} = -\frac{z_2 - z_1}{z_1 - z_2} = 1$$

Границы отображаются в прямые



ДОДЕЛАТЬ ЗАДАЧУ!!!