Candidate Number: BNTN3
Assignment in IRDR0010: Advanced Hazards London, 14th April 2023

Flood Coincidence Risk for Pairs of Stations Along the Severn

Data Preparation and Selection

The National River Flow Archive (NRFA) provides AMAX data in the NRFA Peak Flow Dataset Version 11.1 (released on 6th March 2023) for the UK river Severn at different gauging stations (National River Flow Archive, 2015a). AMAX is defined as "the largest observed flow (in cubic metres per second […]) in each water year" (National River Flow Archive, 2015b).

As not all stations have data available for the same timeframe, I opted to use data from five out of the seven available stations. These stations were chosen based on having the most recent recordings and the greatest degree of temporal overlap. This results in the assignment focussing on the stations Dolwen (ID: 54080), Abermule (ID: 54014), Montford (ID: 54005), Bewdley (ID: 54001) and Haw Bridge (ID: 54057). Data from the period spanning 2000 through 2020 has been retrieved.

Note that the measurements are not disturbed by tides, since gauging stations are installed above tidal limits of rivers (Severn Estuary Partnership a, n.d.).

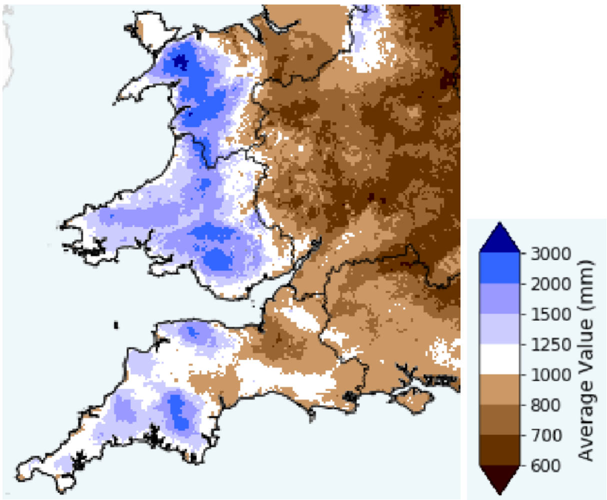
Introduction

The assignment investigates the question: "What is the probability of flooding occurring at selected gauging stations along the Severn River in the UK, in the event of flooding taking place at an upstream gauging station?"

Flooding Risk of the Severn

Flowing through mostly rural areas from North Wales to Bristol, the Severn is around 350 kilometres long (Douglas, 1988) and drains more than 10,000 square kilometres (Durand et al., 2014). Annual rainfall differs within the area of interest, with South Wales receiving more precipitation compared to regions further east such as Bristol and Gloucestershire (see figure 1) (Severn Estuary Partnership b, n.d.).

Figure 1: Annual rainfall patterns across the UK: Rainfall Amount Annual Average 1991-2020 (Met Office, n.d.)



The Severn has a long history of flooding. A flood in 2007 affecting 10,000 properties was the largest flood in the UK of the past decades. The term 'physiographic and climate variables' encompasses various factors that may contribute to an elevated risk of flooding (Howe et al., 1966). In this instance, some of these factors include rainfall, the river's large catchment area, (Environment Agency in England and Natural Resources Wales, 2018) and the soil type and land use (Environment Agency England, 2022). In the lower Severn region, the probability of flooding is linked to coastal risks. Compared to other areas in Europe, the Severn Estuary experiences an unusually high tidal range (Severn Estuary Partnership c, n.d.), which amplifies the risk of flooding during high tides, particularly when accompanied by stormy weather conditions (Lyddon et al., 2018).

Description of the Selected Area

The gauging stations of interest are located in the South-West of the UK (Wales and England). The square in figure 2 delimits the area in question. The waterlines in this area and the locations of the gauging stations, which measure the flow rate of the River Severn, are illustrated in Figure 3.

Figure 2: Area of Interest – South-West of the UK[1]

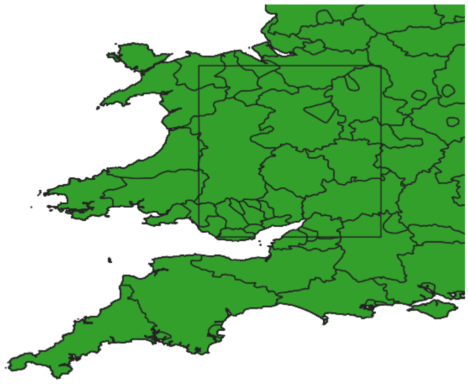
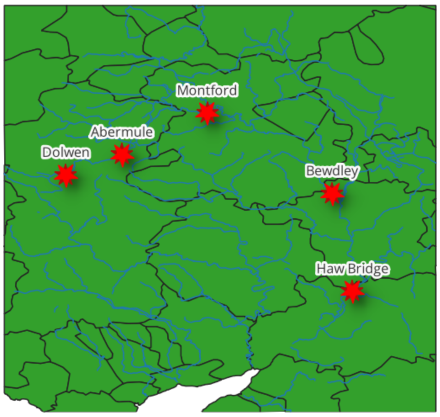


Figure 3: Gauging Stations Measuring Stream Flows at the River Severn[2]



[1] Author's depiction. The underlying shapefile is retrieved from DIVA-GIS (subject: administrative areas 2) (DIVA-GIS, n.d.).

[2] Author's depiction. The underlying shapefiles are retrieved from DIVA-GIS (subject: inland water and administrative areas 2) (DIVA-GIS, n.d.).

Justification of the Selected Study Area

Flood risk is particularly severe in Gloucester (Environment Agency England, 2019) which is close to the gauging station Haw Bridge. Gloucestershire is known to be particularly prone to pluvial and fluvial flooding amongst others due to a flat topography and mudstone and clay sediments that are easily saturated (Environment Agency England, 2022). Although it is crucial to examine flooding at Haw Bridge, it is also necessary to analyse the interplay between other upstream regions, which have historically experienced lower flood risks. This analysis is essential in developing a comprehensive flood management and risk mitigation plan for the entire River Severn watershed.

Figure 4: Kendall's Tau Correlation

	Dolwen	Abermule	Montford	Bewdley	Haw Bridge
Dolwen	1.0000000	0.5047619	0.3333333	0.2285714	0.1913898
Abermule	0.5047619	1.0000000	0.3904762	0.3428571	0.2296677
Montford	0.3333333	0.3904762	1.0000000	0.8000000	0.4401964
Bewdley	0.2285714	0.3428571	0.8000000	1.0000000	0.5454608
Haw Bridge	0.1913898	0.2296677	0.4401964	0.5454608	1.0000000

Kendall's Tau correlation, a non-parametric measure, (see figure 4) shows that each station's data is most strongly correlated with the data from the preceding station. As the distance between upstream gauging stations increases, the correlation between their data decreases. Even though correlation does not allow an inference regarding dependence structures, the decaying correlation pattern motivates an analysis of all five gauging stations. However, I limit the number of stations to five because as the dimensions of the copula increase, its ability to provide reliable information on bivariate dependence and pairwise conditional probability decreases.

Methodology

Conditional probabilities are employed to estimate the flood coincidence risk. To improve the accuracy of the outcome, it is important that the data used reflects the co-dependence between the variables. However, assuming a dependence $\neq 0$, the marginal distributions cannot be multiplied (Donov, 2023b). This is where copula comes into play: they account for co-dependence, in this case between AMAX at the gauging stations between 2000 and 2020.

Copula are "mapping functions that combine uniformly distributed marginals in order to represent the joint distribution and dependence structure of arbitrarily distributed dependent variables" (Chowdhary et al., 2011). They can account for marginal distributions from different families (Salleh et al., 2016; Karmakar and Simonovic, 2009) and can specify the dependence structure independent from individual marginal distributions (Jäckel, 2002). This facilitates the separation of "the effect of dependence from the effects of the marginal distributions" (Shiau et al., 2006). The enhanced flexibility and reliability of the approach leads to a reduced likelihood of under- or overestimating flood risk. (Latif and Mustafa, 2020). Lastly, different copula can capture tail dependence, hence extreme events, to a different degree (Chai et al., 2008; Poulin et al., 2007).

The two-dimensional copula is a function of $C: [0,1]^d \rightarrow [0,1]$ (Salleh et al., 2016; Latif and Mustafa, 2020), which is a joint cumulative distribution function (CDF) of a d-dimensional random vector with uniform marginals $U(0,1)$. It must fulfil three conditions (Salleh et al., 2016; Karmakar and Simonovic, 2009):

- $C(u_1, \dots, u_j, \dots, u_d) = 0$ if $u_j = 0$ for at least one $j \in \{1, \dots, d\}$
- $C(1, \dots, 1, u_j, 1, \dots, 1) = u_j$ for all u_j and $j \in \{1, \dots, d\}$
- C is d-increasing, that is, for all $a = (a_1, \dots, a_d) \epsilon [0,1]^d$ and $b = (b_1, \dots, b_d) \epsilon [0,1]^d$, where $a_i \leq b_i$:

$$V_C([a,b]) = \sum_{i_1=1}^{2} \dots \sum_{i_d}^{2} (-1)^{\sum_{j=1}^{d} i_j} C(u_{1i_1}, \dots, u_{di_d}) \geq 0$$

where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1, \dots, d\}$

The conditional probabilities of the occurrence of a flood a particular station is calculated as:

$$P(X_i > x_i | X_j > x_j) = \frac{P(X_i > x_i, X_j > x_j)}{P(X_j > x_j)}$$

where *i* denotes the station of interest and *j* denotes an upstream station (Donov, 2023a).

In the following, I estimate the first and second best-fitting copula for all five stations. Then, these copula are fitted to all station pairs' transformed uniform marginals. Subsequently, the pairwise conditional probability is computed based on the parameter of the fitted bivariate copula.

Note that the computation of the probabilities is not based on a range of bivariate copula because using different types of copula makes a direct comparison of conditional probabilities unfeasible. Therefore, for the sake of comparability, I accept the risk that the dependence parameter of the 5-dimensional copula does not reflect each pairwise co-dependence accurately (which subsequently might result in slightly skewed conditional probabilities).

There are several steps that are involved in answering the research question.

1. Identification of the need for copula modelling by applying Hoeffding's D independence test (see R, package *Hmisc*, function *hoeffd*).
2. Analysis and transformation of marginal distributions
   a. Visualize the time series data.
   b. Analyse the shape of the marginal distributions by analysing boxplots and descriptive statistics and by testing for normality (see R, package *stats*, function *shapiro.test*).
   c. Within the Pearson distribution system, determine the best fit for each marginal distribution by means of the Maximum Likelihood Estimator (details see R, package *PearsonDS*, function *pearsonFitML*).
   Plot the distribution function of the fitted Pearson distribution and of the kernel density estimate and test the goodness of fit of the former by means of QQ-plots and Anderson-Darling Tests (see R, package *ADGofTest*, function *ad.test*).
   d. Do the Probability Integral Transformation (PIT) of the fitted Pearson distributions (see package *PearsonDS*, function *pperasonI/II/…*) and plot the histograms. Analyse time-varying behaviour of the transformed uniform marginals by testing for stationarity (see R , package *tseries*, function *adf.test*) and autocorrelation (see R, package *stats*, function *Box.test*, type *Ljung-Box*) and by computing ARIMA (see R, package *forecast*, function *auto.arima*).
3. Copula Modelling
   a. For illustrative purposes, fit bivariate copula (see R, package *Vine Copula*, function *BiCopSelect*) and visualise the outcome.
   b. Estimate the parameter of the 5-dimensional Gumbel, Clayton, Frank, Normal and Student t copula by means of the Maximum Likelihood Estimator (see R, package

copula, function *fitCopula*). Test the goodness of fit of the copula by means of AIC and BIC.

   c. Fit the two best fitting copula to pairs of transformed uniform marginals.

4. Modelling of Flood Coincidence Risk
   a. Compute pairwise conditional probability of flooding (formula see above) based on the parameter of step 3c.

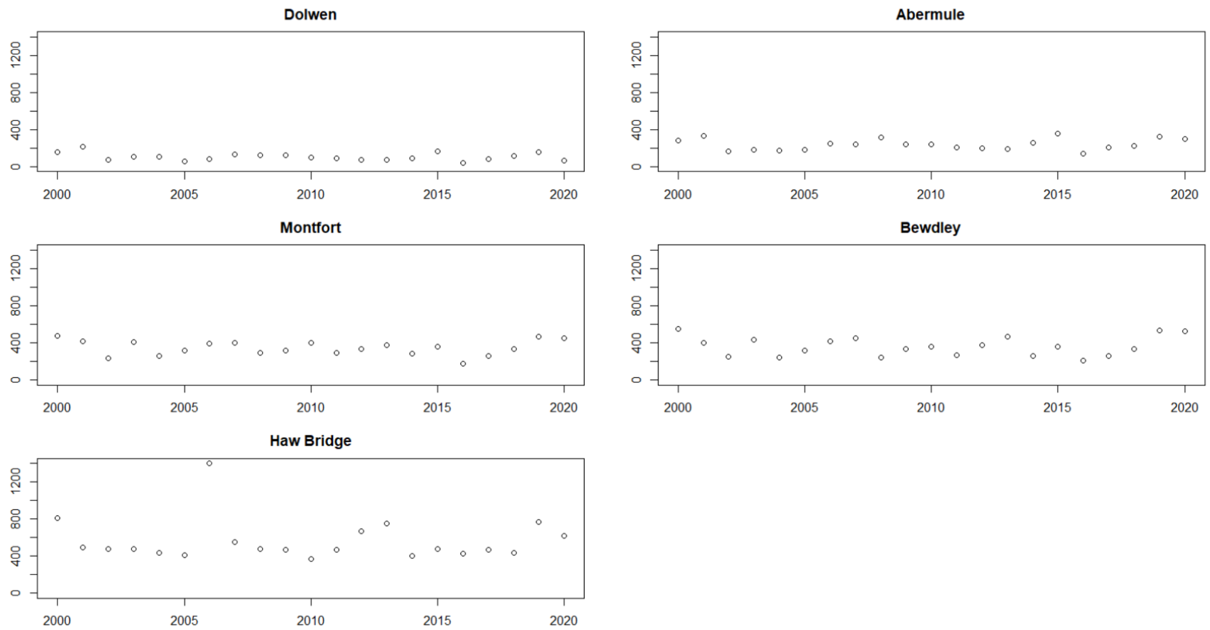
Results

1. Need for Copula Modelling

The Hoeffding's D test, which assess the independence of two variables in a bivariate distribution, rejects the null hypothesis of independence between seven of the ten bivariate pairs at significance level 0.05. This suggests that copula modelling is required.

2. Analysis and Transformation of Marginal Distributions

Figure 5: AMAX from 2000 to 2020 Recorded at Five Gauging Stations Along the Severn



a.) Figure 5 shows the historical data over time at each gauging station in m3s-1. The upper and lower boundaries increase the further you move down the Severn. In all time series, the data shows fluctuations in rather regular wave patterns. There is only one severe outlier at Haw Bridge station in 2006 (1400 m3s-1) that stands out.

b.) Similarly, the boxplots in figure 6 and descriptive statistics in figure 7 show that the both, the median and mean average AMAX, increase the further you go downstream. This suggests that water accumulates over time and space, which might have repercussions on conditional flooding risk at Haw Bridge.

Figure 6: Boxplot

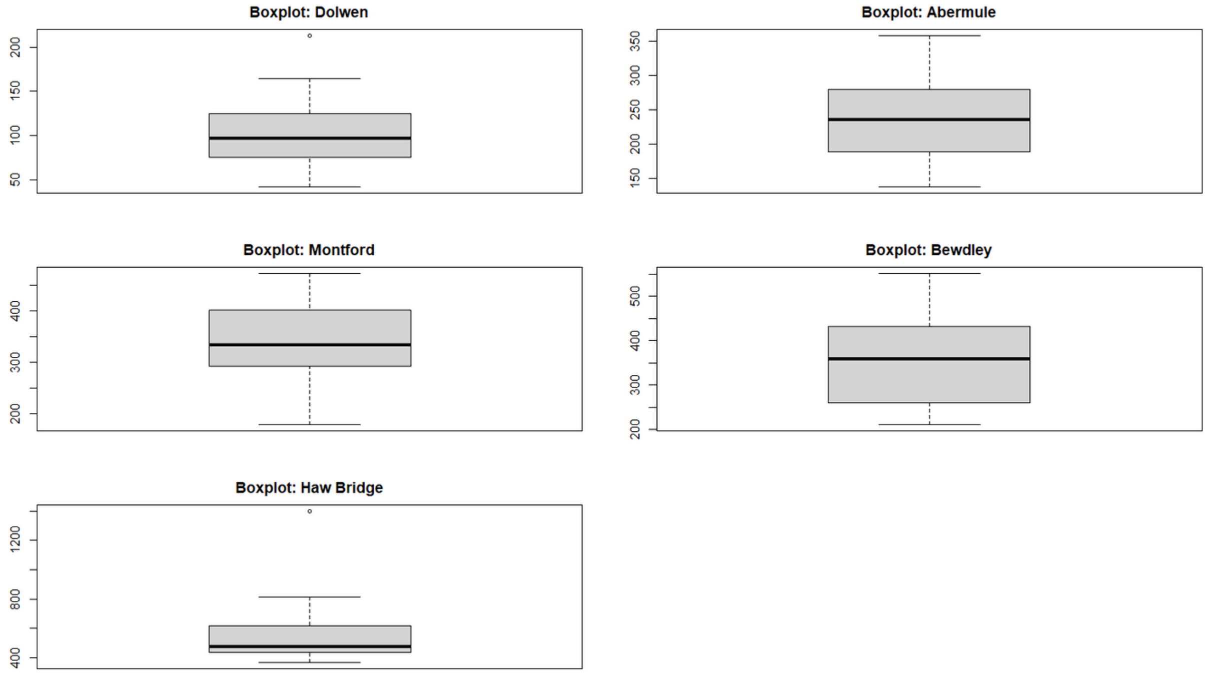
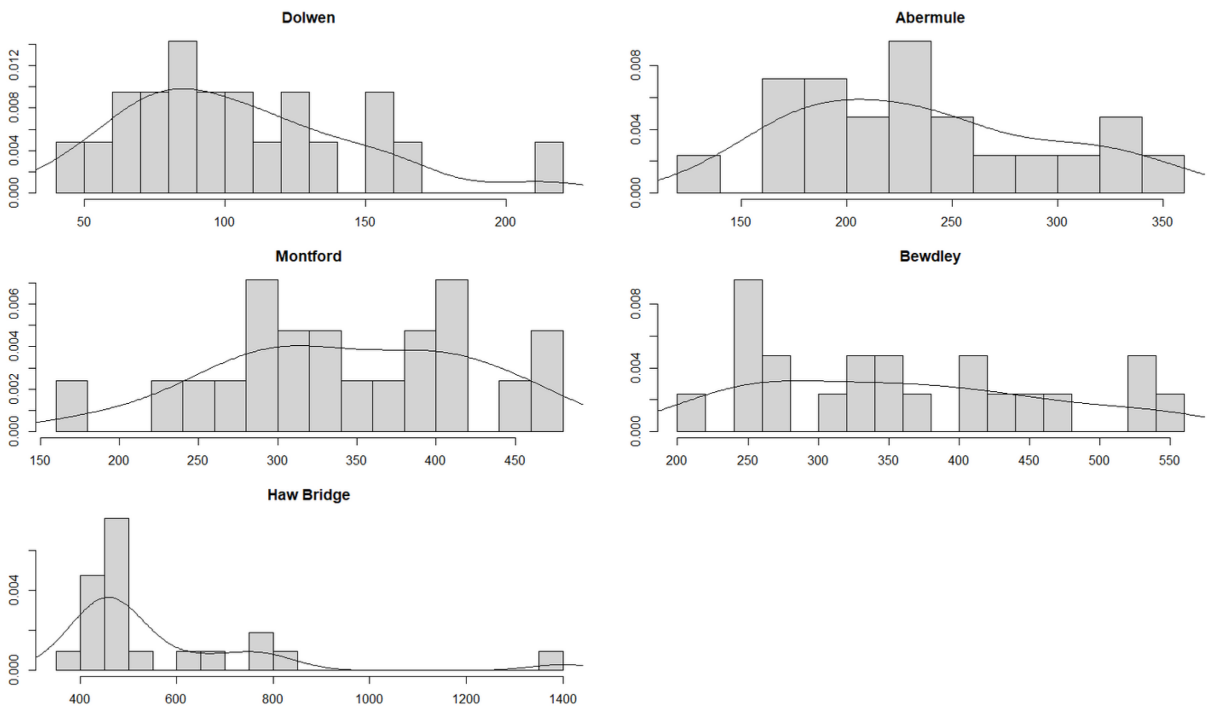


Figure 7: Descriptive Statistics

	n	Mean	Std.Dev	Median	Min	Max	25th	75th	Skew	Kurtosis
Dolwen	21	104.7720	41.68713	96.215	41.771	213.192	75.257	125.232	0.7555204	0.03782542
Abermule	21	235.7809	61.30593	235.456	137.357	357.965	188.094	279.700	0.3688649	-1.04195771
Montford	21	344.2213	80.01263	333.553	178.000	473.416	292.070	401.533	-0.1552689	-0.96900808
Bewdley	21	360.7290	104.21820	358.000	211.000	552.000	261.000	433.000	0.3493616	-1.17071101
Haw Bridge	21	564.7363	229.43678	475.000	366.000	1400.000	435.000	615.389	2.3093581	5.55316546

Figure 8: Kernel Density Estimation



All gauging stations except for the Montford station have positive skewness to the right (see figure 7). Positive skewness is highest for Haw Bridge (see figure 7), which becomes apparent in figure 8 as well. The values of kurtosis (see figure 7) indicate that only the distributions of the Dolwen and in particular Haw Bridge gauging stations are more peaked than a normal distribution and hence there are more values in the tails of the distributions. Except for Haw Bridge, all time series are normally distributed according to the Shapiro Test at significance level 0.05, which implies that the skewness and kurtosis are close enough to the normal distribution's values to pass the test. Note, however, that marginal normality does not imply joint normality.

c.) The determination of the best fit for each marginal distribution is limited to the Pearson distribution family which entails a wide range of distributions (Nair and Sankaran, 1991).

The Maximum Likelihood estimates suggest that the distributions belong to the Pearson families I and II which are the location-scale transformation of Beta and Symmetric Beta Distributions. The probability density functions are, respectively:

$$f(x) = \frac{1}{|s|}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\left(\frac{x-\lambda}{s}\right)^{a-1}\left(1-\frac{x-\lambda}{s}\right)^{b-1}$$

for a > 0, b >0, s ≠ 0, 0 < $\frac{x-\lambda}{s}$ < 1, with scale =s and location =λ (Becker et al., 2022)

and

$$f(x) = \frac{1}{|s|}\frac{\Gamma(2a)}{\Gamma(a)^2}\left(\frac{x-\lambda}{s} \times \left(1-\frac{x-\lambda}{s}\right)\right)^{a-1}$$

for a > 0, b >0, s ≠ 0, 0 < $\frac{x-\lambda}{s}$ < 1, with scale =s and location =λ (Becker et al., 2022).

Figure 9: Fitted Pearson Distributions

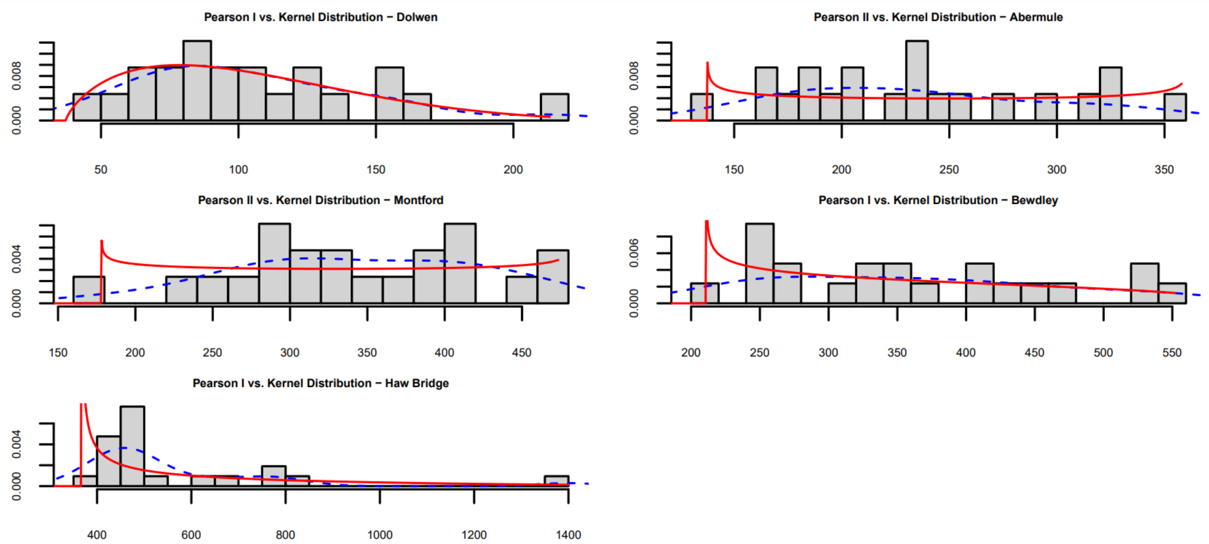
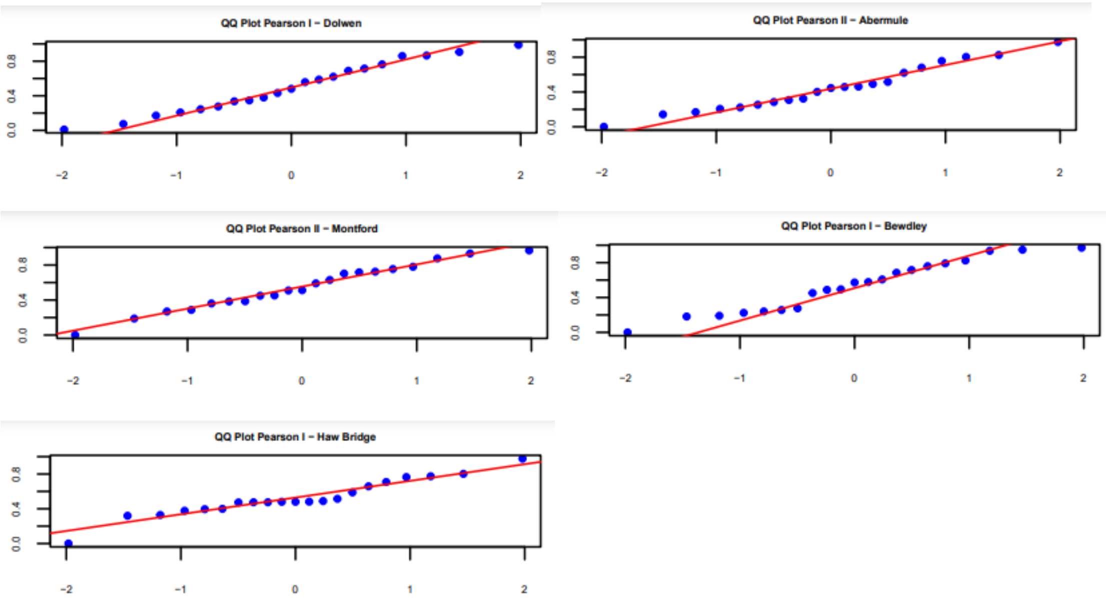


Figure 10: Parameters of Fitted Pearson Distribution and AD-Test Results

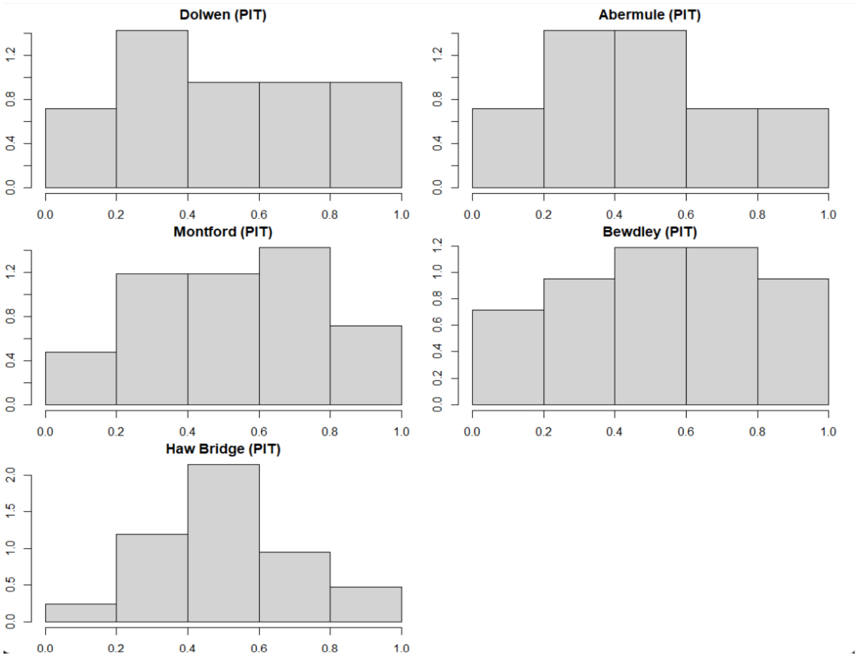
Station	Distribution	a	b	Location	Scale	AD-Test p-value	p-value > 0.05
Dolwen	Beta Distribution	1.7633412	5.090310	37.19145	262.6989	0.99941245	yes
Abermule	Symmetric Beta Distribution	0.8222571	NA	137.35700	223.7639	0.12351358	yes
Montford	Symmetric Beta Distribution	0.9002563	NA	178.00000	303.0519	0.10173063	yes
Bewdley	Beta Distribution	0.7849998	1.320747	211.00000	372.5859	0.24435775	yes
Haw Bridge	Beta Distribution	0.4238316	2.187523	366.00000	1473.8474	0.07997307	yes

Figure 11: QQ-Plot



As the fitted red lines in figure 9 show, except for Haw Bridge the fitted Pearson distributions do not follow the marginal distributions (blue lines) approached by the kernel densities closely at all times. The Anderson Darling Test (see figure 10), however, implies that the selected fitted Pearson distributions have an acceptable fit. The QQ-Plots confirm the goodness of fit, as the data points are located near the diagonal line (see figure 11).

Figure 12: Probability Integral Transformation (PIT) Defined Over The Interval [0,1]

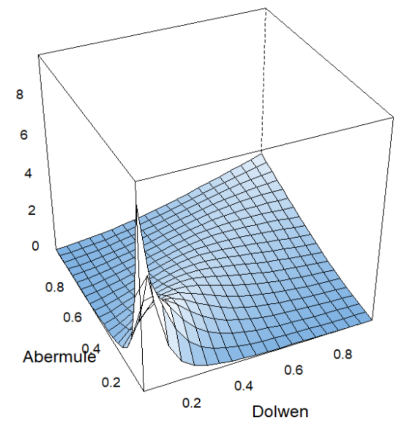


d.) The marginal distributions are transformed into uniform distributions by calculating the CDF of each time series and applying the inverse of this function to the original time series (see figure 12). According to the Augmented Dickey-Fuller (ADF) Test, except for Abermule, all time series are non-stationary despite their transformation. Hence they do not exhibit a high degree of persistence or trend-following behaviour over time. According to the Ljung-Box test, none of the time series displays autocorrelation. The best fit ARIMA-model for all time series is ARIMA(0,0,0), hence the different distributions do not have autoregressive or moving average terms.

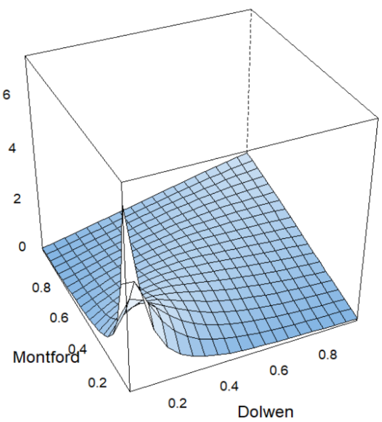
3. Copula Modelling

Figure 13: Dependence Between Independent Uniform Marginals

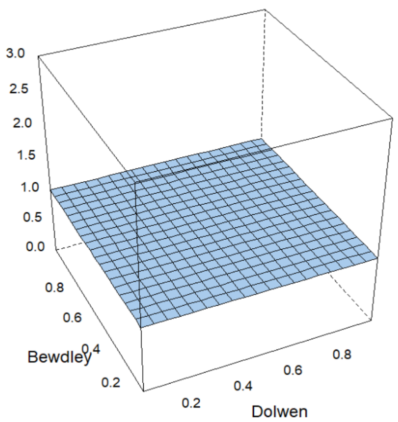
Survival BB8 Copula: Dolwen vs. Abermule



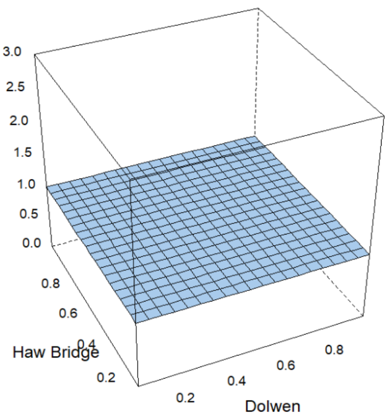
Survival BB8 Copula: Dolwen vs. Montford



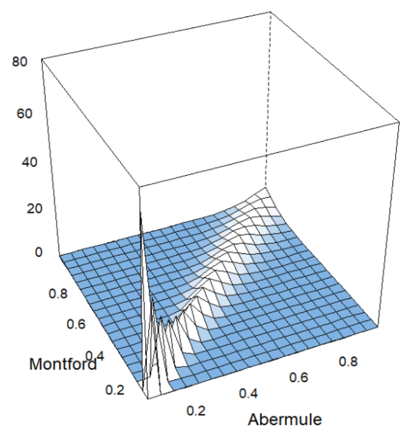
Independence Copula: Dolwen vs. Bewdley



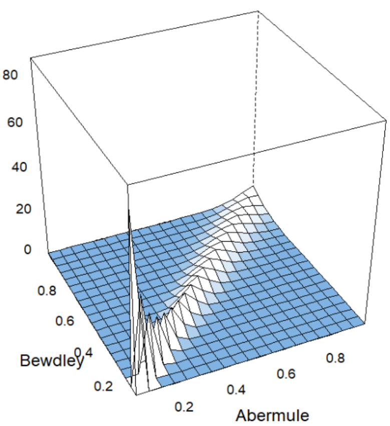
Independence Copula: Dolwen vs. Haw Bridge



Survival Joe Copula: Abermule vs. Montford



Survival Joe Copula: Abermule vs. Bewdley



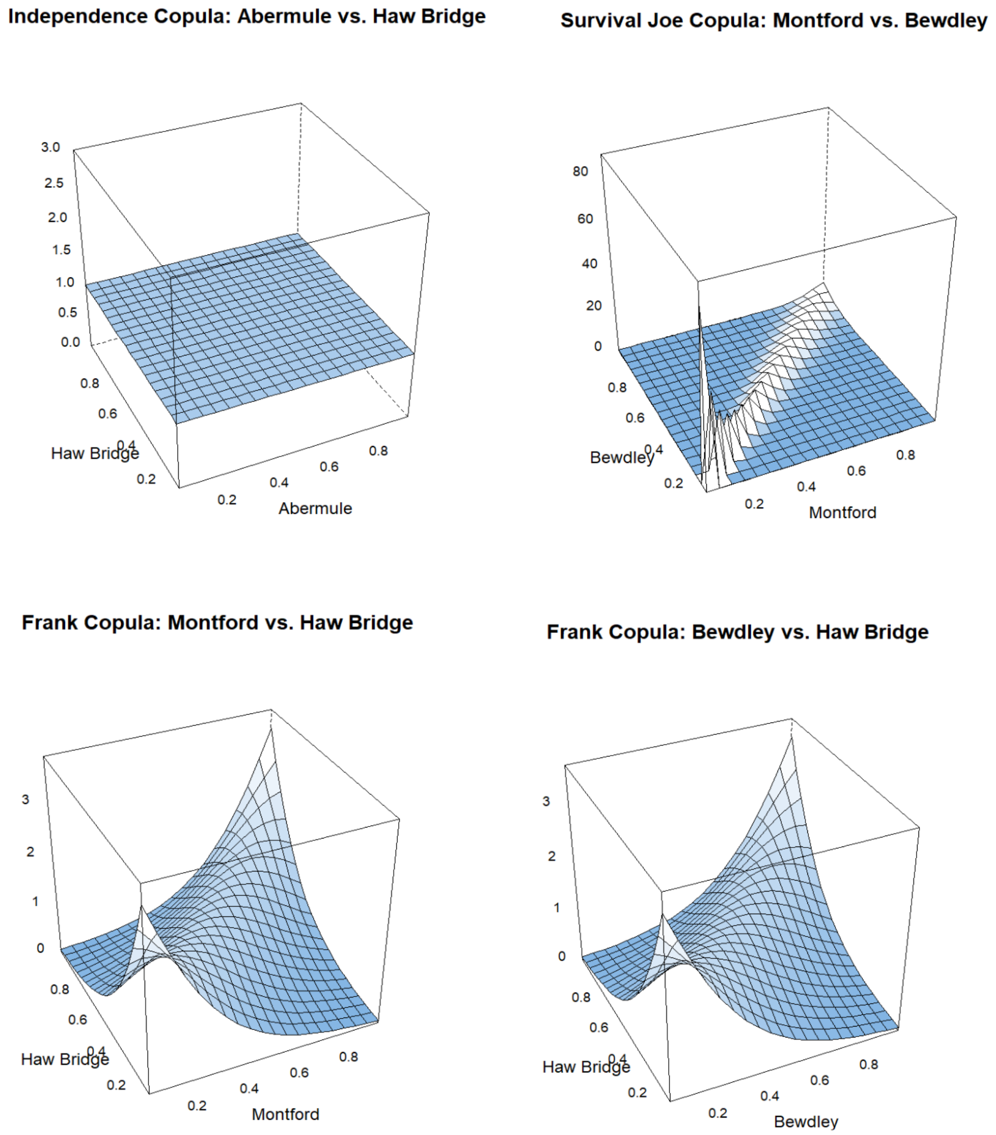


Figure 14: Copula Parameters Specifying the Dependence Between Independent Uniform Marginals

	Abermule		Montford		Bewdley		Haw Bridge	
Dolwen	*Survival BB8*		*Survival BB8*		*Independence*		*Independence*	
	Par 1	2.91	Par 1	2.11	Par 1	0	Par 1	0
	Par 2	0.98	Par 2	0.99				
	Tau	0.49	Tau	0.37	Tau	0	Tau	0
Abermule			*Survival Joe*		*Survival Joe*		*Independence*	
			Par 1	14.56	Par 1	15.62	Par 1	0
			Tau	0.87	Tau	0.88	Tau	0
Montford					*Survival Joe*		*Frank*	
					Par 1	15.62	Par 1	5.11
					Tau	0.88	Tau	0.46
Bewdley							*Frank*	
							Par 1	4.8
							Tau	0.44

a.) The pairs' dependence structure is displayed best by four different bivariate copula (see figure 13 and 14). The Independent Copula implies that the variables' joint distribution function is equivalent to the product of their marginal distribution functions (Donov, 2023b). The Frank copula is symmetric, has a range of $[-\infty, \infty]$ and allows negative dependence. The larger θ, the more upper tail dependence is exhibited (Donov, 2023b). The Survival Joe and Survival BB8 copula are rotated versions of the Joe and BB8 copula (by 180°). The latter's non-rotated version has a range of $[1, \infty]$ and approximates the independence copula the closer the first parameter is to the lower bound. The BB8 copula is flexible in modelling dependence due to having two parameters instead of one (Cheng, Du and Ji, 2020).

Figure 15: Goodness of Fit of the Fitted Copula

	Gumbel	Clayton	Frank	Normal	Student t
AIC	-62.10864	-75.29596	-25.35248	-54.41481	-88.94495
BIC	-64.10864	-77.29596	-27.35248	-56.41481	-92.94495

b.) Bivariate copula are not of importance for the remaining analysis. Rather, the specification of the best fit for the joint 5-dimensional copula is paramount, since it is the basis for the computation of conditional probability. After estimating the parameters of selected copula, the best-fitting copula is selected by means of AIC and BIC (see figure 15).

Due to a bug in the R function for computing the conditional probability that involves the student-t copula, I follow Professor Donov's advice and use the explicit Clayton copula instead, which is the second best copula. In the bivariate case it follows:

$$C(u_1, u_2)|\theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}}$$

where $\theta\epsilon(0, \infty)$ (Donov, 2023a). In the case at hand, θ = 0.1262.

For comparative reasons, the copula with the third lowest AIC and BIC, namely the explicit Gumbel copula, is analysed as well. In the bivariate case it follows:

$$C(u_1, u_2)|\theta) = \exp\left(-[(-log\ u_1)^{\theta} + (-\log u_2)^{\theta}]^{\frac{1}{\theta}}\right)$$

where $\theta\epsilon(1, \infty)$ (Donov, 2023a). In the case at hand, θ = 1.637.

The Clayton copula assumes lower (but no upper) tail dependence, while the Gumbel copula assumes the opposite. Both do not capture negative dependence. The former copula's parameter is close to zero and the latter copula's parameter is close to one which, mean that both approach the independence copula (Donov, 2023b). Hence, their variables' co-dependence is quite low.

4. Modelling of Flood Coincidence Risk

Figure 16: Conditional Probabilities of Flooding for a 100-Year Return Period – Clayton Copula Case

	Abermule	Montford	Bewdley	Haw Bridge
Dolwen	0.04878209	0.02941176	0.02159582	0.01928493
Abermule		0.03473512	0.03246784	0.02166716
Montford			0.1032434	0.0401901
Bewdley				0.05534746

Figure 17: Conditional Probabilities of Flooding for a 100-Year Return Period – Gumbel Copula Case

	Abermule	Montford	Bewdley	Haw Bridge
Dolwen	0.8126081	0.7694819	0.7357438	0.01000129
Abermule		0.8498603	0.8179759	0.5890911
Montford			0.9433641	0.7029353
Bewdley				0.7152124

When using the parameters of both copulas to calculate conditional probabilities, the likelihood of flooding decreases as the paired station is located further upstream. (see figure 16 and 17). Hence, it follows the same pattern as the Kendall's tau correlation (see above). This suggests that both, the correlation and the conditional probability of flooding, decrease the further away the gauging stations are from each other.

The probabilities deviate strongly between the copula. Since Clayton copula has a better overall fit, it is assumed that the values are relatively more co-dependent in the lower tail. The weight on the upper tail in the worse-fitting Gumbel copula case results in a skewed conditional flood risk estimation.

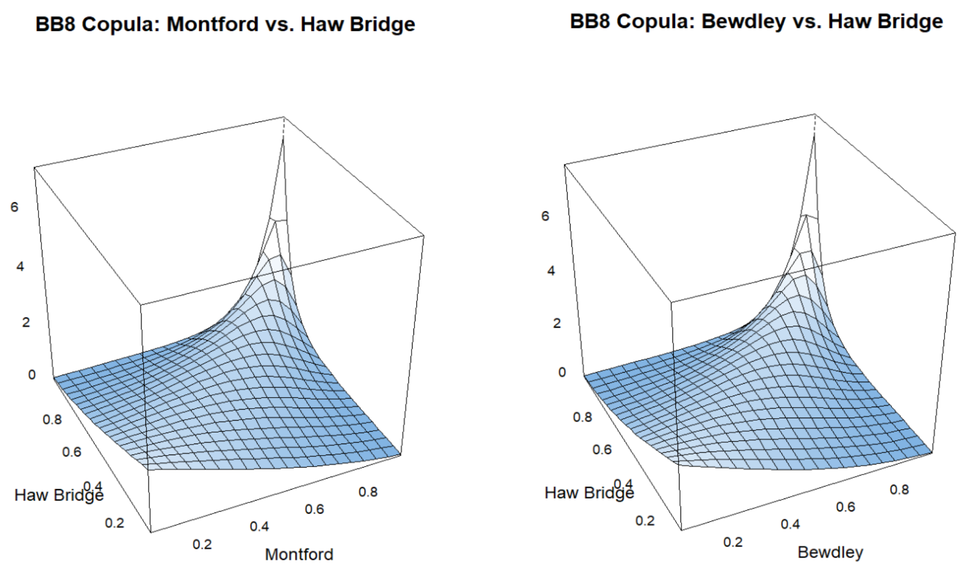
Discussion

The crux of this paper is to choose an appropriate copula which is accomplished by choosing the copula with the lowest AIC and BIC. Furthermore, upon comparing Kendall's tau correlations, it is evident that the fitted marginals accurately describe the empirical data: the correlation values of the empirical data (see figure 4) and the fitted uniform marginals correspond to each other. This is a good basis for modelling a well-fitting copula. To my knowledge, there are no built-in functions in R that evaluate the goodness of fit of copula with more than two dimensions (e.g. Kendall's tau). One option to assess the goodness of fit is to make use of the R function *rCopula*, run a Monte Carlo simulation and compare the simulated new data from the fitted copula model with the observed data. Due to the limited scope of this assignment, this is not done.

Note that copula in general and the chosen copula in this case in specific suffer from several limitations:

- Outliers can distort the modelling of tail dependence and the specification of copula. In this incident, if you remove the outlier recorded at Haw Bridge station in 2006 by replacing the value by the average of 2005 and 2007, the skewness and kurtosis decrease. The fitted Pearson family remains type I (Beta distribution), however, in particular the b-parameter and scale decrease. Subsequently, the bivariate copula for Haw Bridge and Montford and for Haw Bridge and Bewdley change to BB8 copulas with θ=2.68 and θ=3.08 (see figure 18).

Figure 18: Dependence Between Independent Uniform Marginals of Montford vs. Haw Bridge and Bewdley vs. Haw Bridge After Removing the Outlier



The correlation values of the uniform marginals of Haw Bridge increase slightly. The best, second-best and third-best fitting copula do not change.

Figure 19: Conditional Probabilities of Flooding for a 100-Year Return Period After Removing the Outlier

	Dolwen		Abermule		Montford		Bewdley	
Haw Bridge	Clayton	0.02044862	Clayton	0.02294454	Clayton	0.04253826	Clayton	0.05882231
	Gumbel	0.5695447	Gumbel	0.01000129	Gumbel	0.722534	Gumbel	0.7456461

However, the conditional probabilities of flooding change, even though in most cases only a little. However, the probabilities of flooding at Haw Bridge conditional on flooding at Dolwen and Abermule, respectively, change significantly when using the Gumbel copula parameter (see figure 19). The effect could have even been higher if there were more/more extreme outliers or if the sample was smaller.

- When modelling copula for time series, the assumption of stationarity is typically required for reliable outcomes. However, since four out of five transformed uniform marginals are non-stationary, this assumption is violated, and the results may not be trusted (Smith, 2013). Also, the orders of zero of the ARIMA model indicate that there is no pattern in time. Therefore, due to the random nature of the data the copula cannot be used to make a prediction of future risk without risking a significant under-or overestimation of risk (Latif and Mustafa, 2020).
- If an inappropriate copula is chosen, the spurious conditional probabilities can prompt adverse flood management decisions. Hence, conditional flood probabilities might not actually lie within the range of around 1 and 10 percent (for the 100 year return period). This might result in the implementation of inappropriate flood defence and preparedness measures. To prevent this from happening, a tail dependence test could be run or a tail concentration function could specify the tail dependence (Durante et al. 2015).
- In order to optimize flooding management, not only AMAX, but also the flood vectors volume and duration and their dependence structures should be considered (Salleh et al., 2016; Grimaldi and Serinaldi, 2006; Karmakar and Simonovic, 2009; Latif and Mustafa, 2020). Also, non-pluvial sources of flooding e.g. related to sea water dynamics must be considered when

aiming at a comprehensive flood risk assessment. This has not been done in this assignment, since it focusses on risk stemming from upstream rather than downstream sources.

Word Count: 3132 (without bibliography, headings and figure headings)

Bibliography

Becker, M, Klößner, S., Heinrich, J., 2022. Pearson Distribution System, R Package Version 1.2.3.

Chai, W.-Y., Zhu, Y. and Hou, Z.-Q., 2008. The Research of Copula Theory in Financial Risk Management, International Conference on Machine Learning and Cybernetics, Kumming, China, 1489-1493, doi: 10.1109/ICMLC.2008.4620641.

Cheng, Y., Du, J., Ji, H. 2020. Multivariate Joint Probability Function of Earthquake Ground Motion Prediction Equations Based on Vine Copula Approach, Mathematical Problems in Engineering 2020, Article ID 1697352, https://doi.org/10.1155/2020/1697352.

Chowdhary, H., Escobar, L.A., Singh, V.P., 2011. Identification of suitable copulas for bivariate frequency analysis of flood peak and flood volume data. Hydrology Research 42, 193–216. https://doi.org/10.2166/nh.2011.065

DIVA-GIS, n.d. Download data by country | DIVA-GIS [WWW Document]. URL https://www.diva-gis.org/gdata (accessed 3.27.23).

Donov, A., 2023a. Copula Theory for Multivariate Flood Risk Analysis, Lecture Slides, Guest Lecture in the Course IRDR0010 Advanced Hazards at University College London on 28th February 2023.

Donov, A., 2023b. Value-at-Risk Using Copulas, Lecture Notes, Course STAT0011/38 Decision and Risk at University College London, Last Updated on February 11, 2022.

Douglas, J.R., 1988. Regulating the River Severn. Regul. Rivers: Res. Mgmt. 2, 309–321. https://doi.org/10.1002/rrr.3450020309

Durand, M., Neal, J., Rodríguez, E., Andreadis, K.M., Smith, L.C., Yoon, Y., 2014. Estimating reach-averaged discharge for the River Severn from measurements of river water surface elevation and slope. Journal of Hydrology 511, 92–104. https://doi.org/10.1016/j.jhydrol.2013.12.050

Durante, F. , Fernández-Sánchez, J. and Papadà, R., 2015. Copulas, diagonals, and tail dependence, Fuzzy Sets and Systems, Vol. 264, pp. 22-41, https://doi.org/10.1016/j.fss.2014.03.014.

Environment Agency England, 2022. Severn River Basin District Flood Risk Management Plan 2021 to 2027.

Environment Agency England, 2019. Story Map Series [WWW Document]. URL https://environment.maps.arcgis.com/apps/MapSeries/index.html?appid=4d066e4a4373486e96dff8d3a86207ae (accessed 3.29.23).

Environment Agency in England and Natural Resources Wales, 2018. Severn preliminary flood risk assessment. Cardiff.

Grimaldi, S., Serinaldi, F., 2006. Asymmetric copula in multivariate flood frequency analysis. Advances in Water Resources 29, 1155–1167. https://doi.org/10.1016/j.advwatres.2005.09.005

Howe, G.M., Slaymaker, H.O., Harding, D.M., 1966. Flood Hazard in Mid-Wales. Nature 212, 584–585. https://doi.org/10.1038/212584a0

Jäckel, P., 2002. Monte Carlo Methods in Finance, Wiley Finance Series.

Karmakar, S., Simonovic, S.P., 2009. Bivariate flood frequency analysis. Part 2: a copula-based approach with mixed marginal distributions: Bivariate flood frequency analysis: Part 2. Journal of Flood Risk Management 2, 32–44. https://doi.org/10.1111/j.1753-318X.2009.01020.x

Latif, S., Mustafa, F., 2020. Copula-based multivariate flood probability construction: a review. Arab J Geosci 13, 132. https://doi.org/10.1007/s12517-020-5077-6

Lyddon, C., Brown, J.M., Leonardi, N., Plater, A.J., 2018. Flood Hazard Assessment for a Hyper-Tidal Estuary as a Function of Tide-Surge-Morphology Interaction. Estuaries and Coasts 41, 1565–1586. https://doi.org/10.1007/s12237-018-0384-9

Met Office, n.d. Cheltenham (Gloucestershire) UK climate averages [WWW Document]. URL https://www.metoffice.gov.uk/research/climate/maps-and-data/uk-climate-averages/gcnx0z9e5 (accessed 3.27.23).

Nair, N.U., Sankaran, P.G., 1991. Characterization of the Pearson family of distributions. IEEE Trans. Rel. 40, 75–77. https://doi.org/10.1109/24.75339

National River Flow Archive, 2015a. Peak Flow Dataset [WWW Document]. URL https://nrfa.ceh.ac.uk/peak-flow-dataset (accessed 3.27.23).

National River Flow Archive, 2015b. Annual Maximum (AMAX) [WWW Document]. URL https://nrfa.ceh.ac.uk/annual-maximum (accessed 3.27.23).

Poulin, A., Huard, D., Favre, Anne-Catherine, Pugin, S., 2007. Importance of Tail Dependence in Bivariate Frequency Analysis, Journal of Hydrological Engineering 12, 394—403.

Reynard, N.S., Prudhomme, C., Crooks, S.M., 2001. The Flood Characteristics of Large U.K. Rivers: Potential Effects of Changing Climate and Land Use. Climatic Change 48, 343–359. https://doi.org/10.1023/A:1010735726818

Salleh, N., Yusof, F., Yusop, Z., 2016. Bivariate copulas functions for flood frequency analysis. Presented at the ADVANCES IN INDUSTRIAL AND APPLIED MATHEMATICS: Proceedings of 23rd Malaysian National Symposium of Mathematical Sciences (SKSM23), Johor Bahru, Malaysia, p. 060007. https://doi.org/10.1063/1.4954612

Severn Estuary Partnership a, n.d. Freshwater Inputs. URL https://severnestuarypartnership.org.uk/the-estuary/physical-natural-environment/freshwater-inputs/ (accessed 3.27.23a).

Severn Estuary Partnership b, n.d. Weather. URL https://severnestuarypartnership.org.uk/the-estuary/environmental-quality/weather-and-climate-change/ (accessed 3.27.23b).

Severn Estuary Partnership c, n.d. Tidal Power Resource. URL https://severnestuarypartnership.org.uk/the-estuary/managing-the-estuary/tidal-power-resource/ (accessed 3.29.23c).

Shiau, J.-T., Wang, H.-Y., Tsai, C.-T., 2006. BIVARIATE FREQUENCY ANALYSIS OF FLOODS USING COPULAS [1]. J Am Water Resources Assoc 42, 1549–1564. https://doi.org/10.1111/j.1752-1688.2006.tb06020.x

Smith, M.S., 2013. Copula Modelling of Dependence in Multivariate Time Series. SSRN Journal. https://doi.org/10.2139/ssrn.2586849