Experiments on the Influences of Parameters in Kalman Filters

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Abstract

Even though calculating Kalman Filters according to formulas is not a difficult problem, understanding the impacts of different parameters within the formula is not intuitive to me. In this work, I look at the influences of parameters in Kalman Filters on the filtered result, by applying the information form of Kalman Filter on stock price data of Amazon.

1 Introduction

Consider a simplified version of discrete dynamic system, defined as

$$\begin{aligned} x(t+1) &= A(t)x(t) + G(t)w(t) && (1)\\ y(t) &= C(t)x(t) + v(t), && (2) \end{aligned}$$

where $x(t) \in R$, $y(t) \in R$, and $w(t)$, $v(t)$ are independent zero-mean, Gaussian white noise processes, with

$$\begin{aligned} E[w(t)w(s)] &= Q(t)\delta(t-s) && (3)\\ E[v(t)v(s)] &= R(t)\delta(t-s) && (4)\\ E[w(t)v(s)] &= 0. && (5) \end{aligned}$$

And think about the information form

$$\begin{aligned} \hat{z}(t|t-1) &= P^{-1}(t|t-1)\hat{x}(t|t-1) && (6)\\ \hat{z}(t|t) &= P^{-1}(t|t)\hat{x}(t|t). && (7) \end{aligned}$$

The Kalman Filter gives the update step

$$\begin{aligned} P^{-1}(t|t) &= P^{-1}(t|t-1) + C(t)R(t)^{-1}C(t) && (8)\\ \hat{z}(t|t) &= \hat{z}(t|t-1) + C(t)R(t)^{-1}y(t), && (9) \end{aligned}$$

and the prediction step

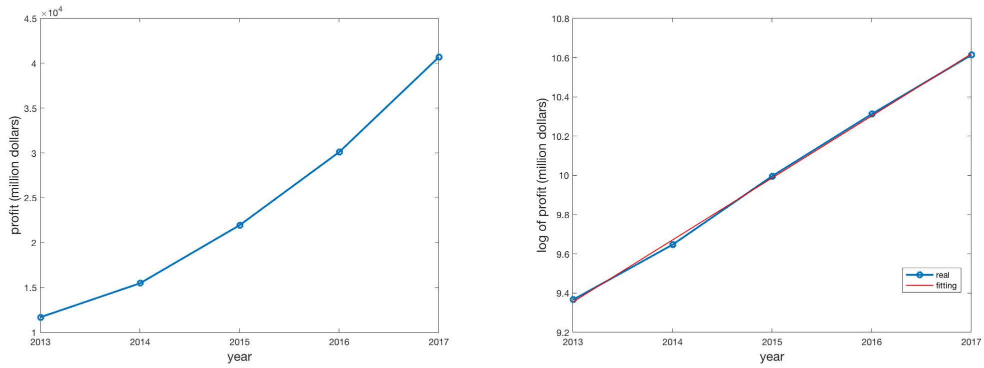
$$\begin{aligned} P^{-1}(t+1|t) &= M(t) - N(t)G(t)M(t) && (10)\\ \hat{z}(t+1|t) &= [I + M(t)G(t)Q(t)G(t)]^{-1}A(t)^{-1}\hat{z}(t|t), && (11) \end{aligned}$$

where

$$\begin{aligned} M(t) &= A(t)^{-1}P^{-1}(t|t)A(t)^{-1} && (12)\\ N(t) &= M(t)G(t)[Q(t)^{-1} + G(t)M(t)G(t)]^{-1}. && (13) \end{aligned}$$

But it's not intuitive what role each parameter plays in the filter, i.e. what the filtered result would look like with different parameters.

So in this report, I'm going to test this simplified version of Kalman Filter on real world data, and see how different parameters influence the filtered result.



(a) The annual profits of Amazon from 2013 to 2017.
(b) The log of annual profits of Amazon from 2013 to 2017, and its linear fitting.

Figure 1

2 Data and Experiments

2.1 Data and Preparations

The data I use is the daily stock close prices of Amazon for 10 years, i.e. from April 21, 2008 to April 20, 2018. That is to say, $y(t) \in R, t = 1, \ldots, 2519$. Also, to approximate the growth rate of Amazon, I use its annual profits from its financial statements from 2013 to 2017.

The annual profits of Amazon from 2013 to 2017 are shown in Fig.1a. It looks like the profit is increasing exponentially. So if I take the log of profits as shown in Fig.1b, and fit it linearly, I'll get the slope of 0.3161. Considering that there are around 252 trading days per year, the daily growth rate in profit is given by $\exp^{0.3161/252} = 1.0013$.

Alternatively, if I take the log of daily stock close prices and fit it linearly, as shown in Fig.2, the slope is 0.0012. That means the daily growth rate in stock price is given by $\exp^{0.0012} = 1.0012$. Thus, the growth rate in profit is a good estimation of the growth rate in stock prices.

2.2 Experiments

Let $x(t)$ in Eq.1 be the real value of the stock. $x(t+1)$ should be $x(t)$ times the average growth rate, plus a Gaussian white noise, corresponding to some sudden good or bad news, causing the stock value to increase or decrease. That is to say, we should set $A(t) = A = 1.0013$, $G(t) = G = 1$, and $w(t) \sim \mathcal{N}(0, Q(t))$. $Q(t)$, which is the variance of the stock value that can be caused by sudden good or bad news, is set to be the square of some factor $q(t)$ multiplied by $x(t-1)$.

Let $y(t)$ in Eq.2 be the stock price. $y(t)$ should be around the real stock value, with a difference of another Gaussian white noise, corresponding to peoples' irrationality or speculations, causing the stock price to be off its real value. That is to say, we should set $C(t) = C = 1$, and $v(t) \sim \mathcal{N}(0, R(t))$. $R(t)$, which is the variance of the deviation of the stock price from its real value, is set to be the square of some factor $r(t)$ multiplied by $x(t-1)$.

Since no information is given at the beginning, i.e. when the stock was listed, we are going to use the information form of Kalman Filter, with $P^{-1}(1|0)$ and $\hat{z}(1|0)$ both initialized as zeros.

Setting $q(t) = 3\%$ or $w(t) \sim \mathcal{N}(0, (3\% \cdot x(t-1))^2)$, and also $r(t) = 3\%$ or $v(t) \sim \mathcal{N}(0, (3\% \cdot x(t-1))^2)$, and applying Eq.8,9,10,11,12,13, the normalized daily stock price residuals are shown in Fig.3. The $y-$axis shows the normalized amount the stock price is above its estimated value, i.e. $\frac{y(t)}{x(t)} - 1$. The gray area gives the normalized uncertainty of the filtering, i.e. $\frac{\sqrt{P(t)}}{x(t)}$. We can see that the normalized uncertainty is relatively higher for just the first few days, and then drops down to a stable level ≈ 0.0381.

Changing $q(t)$ to 10% and keep $r(t) = 3\%$ gives Fig.4a, while changing $r(t)$ to 10% and keep $q(t) = 3\%$ gives Fig.4b. We can see that the uncertainty of $x(t+1)$ given $x(t)$ has a larger impact on the stable level of filtering uncertainty than the uncertainty of $y(t)$ given $x(t)$. And the approximate $\frac{\sqrt{P(t)}}{x(t)}$ can be calculated by combining Eq.8,10,12,13 and solving for the fixed point of the problem,

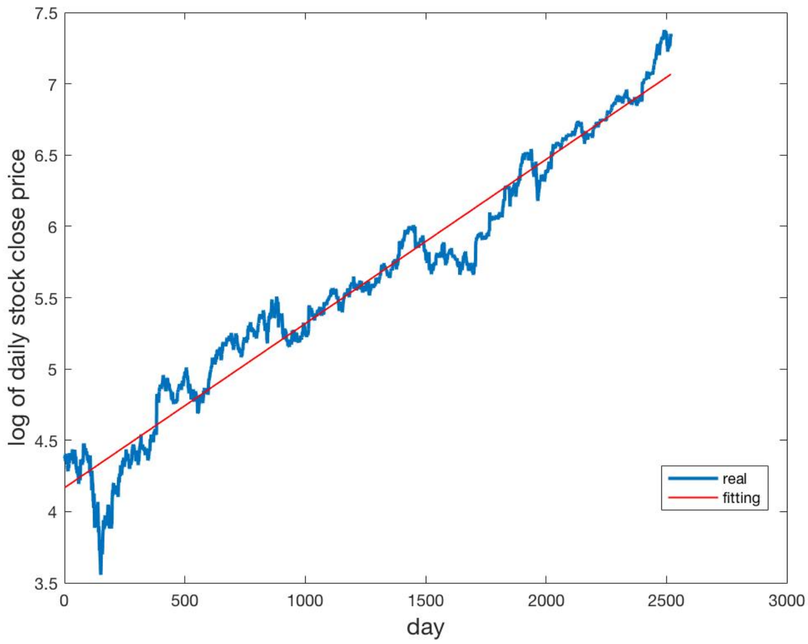


Figure 2: The log of daily stock close prices of Amazon from April 21, 2008 to April 20, 2018, and the linear fitting.

i.e.

$$\begin{aligned} P^{-1} = A^{-1}(P^{-1} + CR^{-1}C)A^{-1} - A^{-1}(P^{-1} + CR^{-1}C)A^{-1}G[Q^{-1} \\ + GA^{-1}(P^{-1} + CR^{-1}C)A^{-1}G]^{-1}GA^{-1}(P^{-1} + CR^{-1}C)A^{-1}. \end{aligned} \tag{14}$$

The solution is given by

$$\frac{\sqrt{P(t)}}{x(t)} = \left(\frac{\sqrt{[(A^2-1)r^2 + q^2]^2 + 4q^2r^2} - (A^2-1)r^2 - q^2}{2q^2r^2} \right)^{-\frac{1}{2}}. \tag{15}$$

And it's clear that when $1 < A < 2$, $q(t)$ has a larger impact on $\frac{\sqrt{P(t)}}{x(t)}$ than $r(t)$. Also, the normalized daily stock price residuals are less in Fig.4a than in Fig.4b since the smaller $r(t) = 3\%$ in Fig.4a allows less difference between the real stock price and its filtered result.

Changing $q(t)$ to 0.01% and keep $r(t) = 3\%$, we get the real and filtered daily stock prices of Amazon, shown in Fig.5. Allowing the stock value to be less impacted by sudden good or bad news, we can see the filtered result is more like a running average of the real prices with a longer window, with peoples' irrationality or speculations contribute more to the fluctuations in prices.

3 Discussion

In this report, I apply a simplified version of Kalman Filter on Amazon stock price data, and discuss the differences in filtered results given different parameters. This helps me to gain a more intuitive understanding of the mechanism of Kalman Filters, as an implementation of the pure calculations we've done so far.

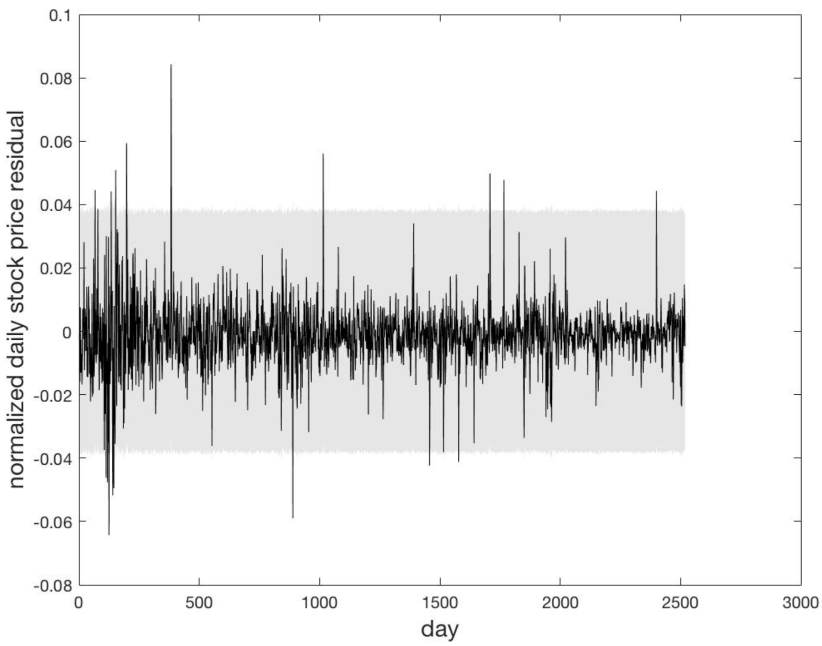
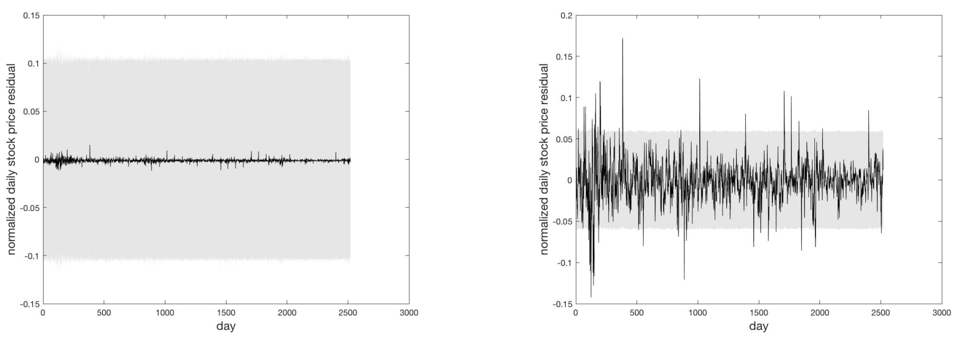


Figure 3: The normalized daily stock price residuals, i.e. the amount the stock price is above its estimated value, normalized by the uncertainty $P(t)$, of Amazon from April 21, 2008 to April 20, 2018, given $q(t) = 3\%$ and $r(t) = 3\%$.



(a) The normalized daily stock price residuals, i.e. the amount the stock price is above its estimated value, normalized by the uncertainty $P(t)$, of Amazon from April 21, 2008 to April 20, 2018, given $q(t) = 10\%$ and $r(t) = 3\%$.

(b) The normalized daily stock price residuals, i.e. the amount the stock price is above its estimated value, normalized by the uncertainty $P(t)$, of Amazon from April 21, 2008 to April 20, 2018, given $q(t) = 3\%$ and $r(t) = 10\%$.

Figure 4

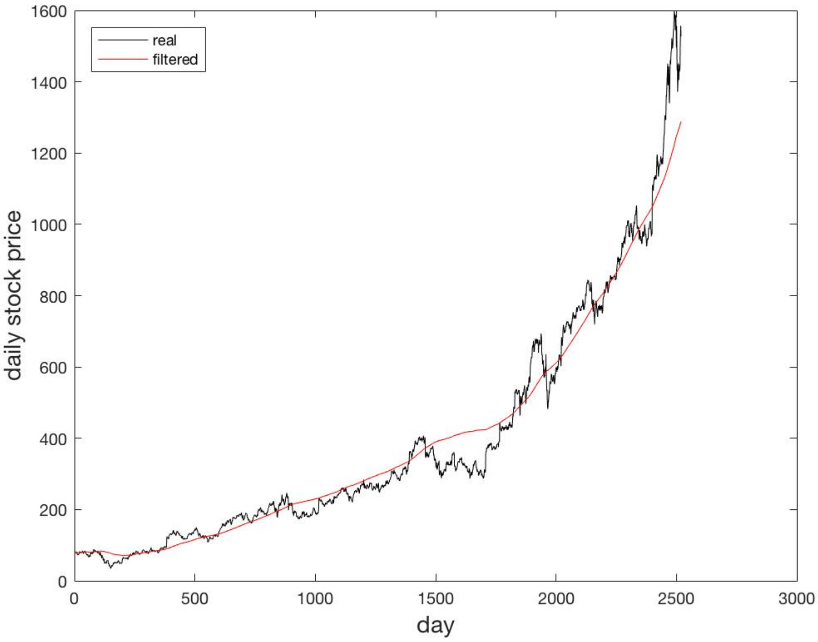


Figure 5: The real and filtered daily stock prices of Amazon from April 21, 2008 to April 20, 2018, given $q(t) = 0.01\%$ and $r(t) = 3\%$.