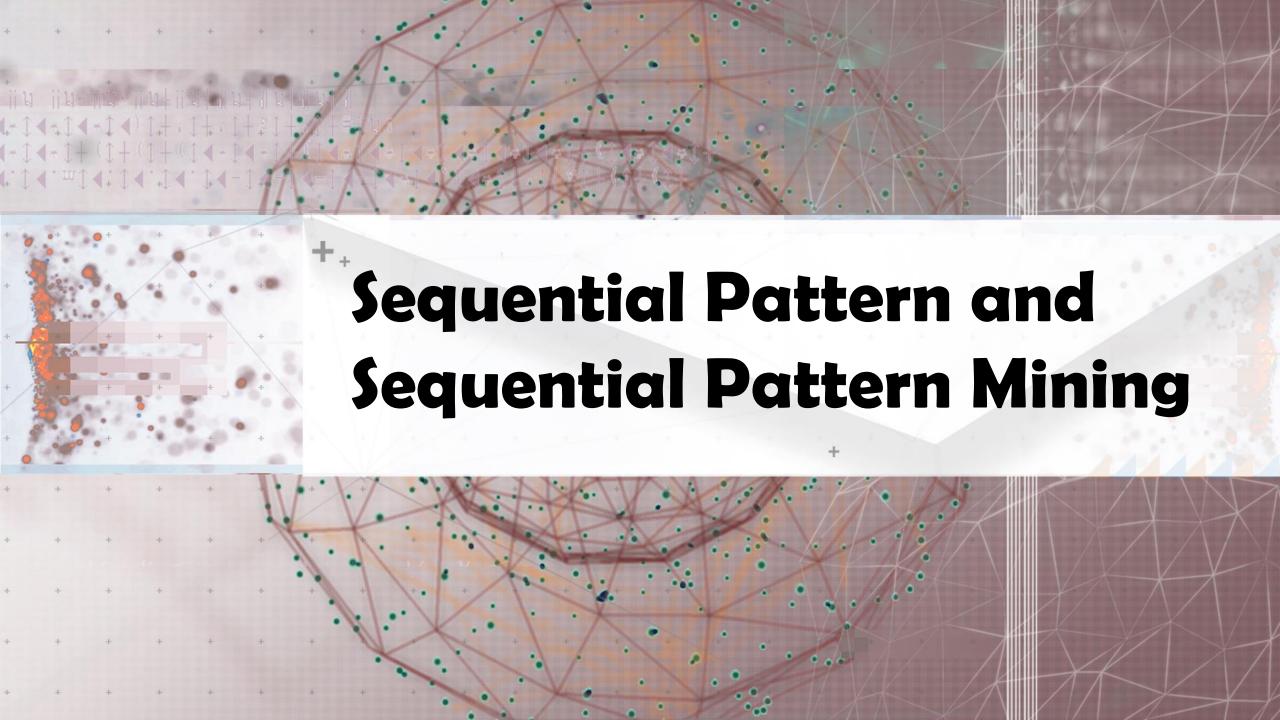


Sequential Pattern Mining

- Sequential Pattern and Sequential Pattern Mining
- GSP: Apriori-Based Sequential Pattern Mining
- □ SPADE: Sequential Pattern Mining in Vertical Data Format
- PrefixSpan: Sequential Pattern Mining by Pattern-Growth
- CloSpan: Mining Closed Sequential Patterns



Sequence Databases & Sequential Patterns

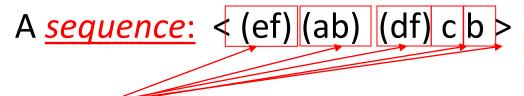
- Sequential pattern mining has broad applications
 - Customer shopping sequences
 - Purchase a laptop first, then a digital camera, and then a smartphone,
 within 6 months
 - Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, ...
 - Weblog click streams, calling patterns, ...
 - Software engineering: Program execution sequences, ...
 - Biological sequences: DNA, protein, ...
- Transaction DB, sequence DB vs. time-series DB
- Gapped vs. non-gapped sequential patterns
 - Shopping sequences, clicking streams vs. biological sequences

Sequential Pattern and Sequential Pattern Mining

Sequential pattern mining: Given a set of sequences, find the complete set of frequent subsequences (i.e., satisfying the min_sup threshold)

A <u>sequence database</u>

SID	Sequence
10	<a(<u>abc)(a<u>c</u>)d(cf)></a(<u>
20	<(ad)c(bc)(ae)>
30	<(ef)(<u>ab</u>)(df) <u>c</u> b>
40	<eg(af)cbc></eg(af)cbc>



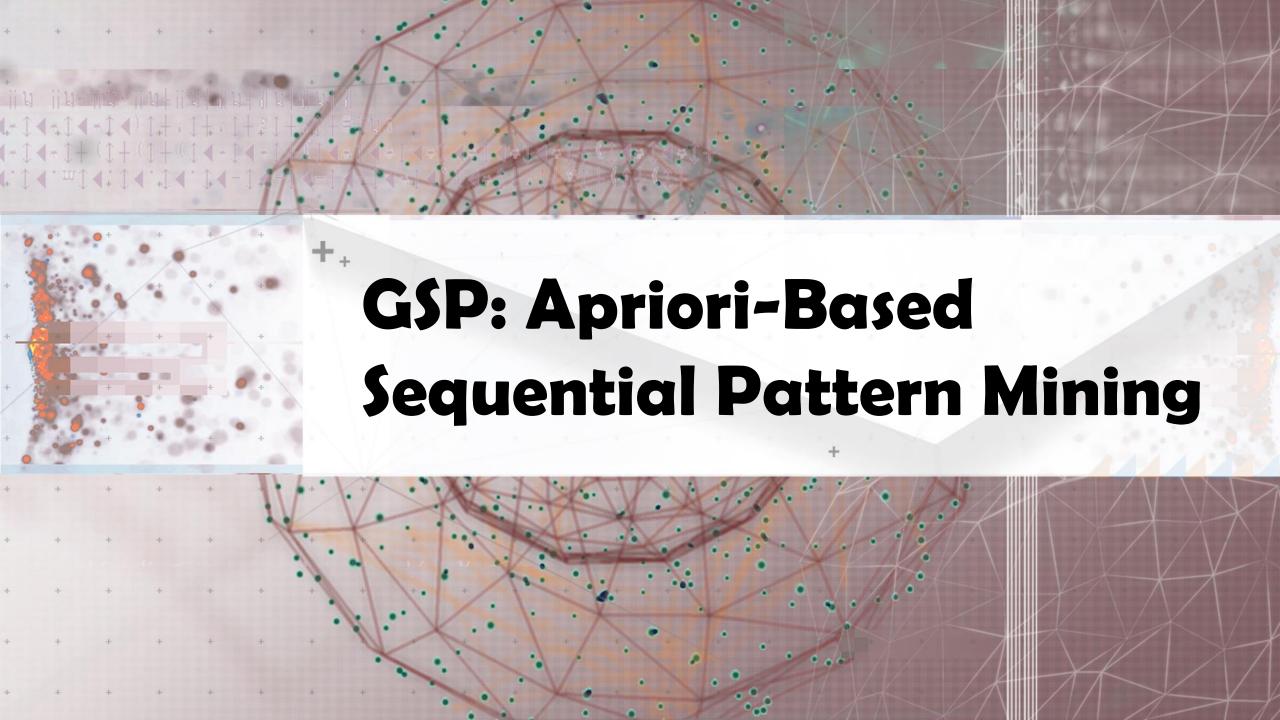
- An <u>element</u> may contain a set of *items* (also called *events*)
- ☐ Items within an element are unordered and we list them alphabetically

 $<a(bc)dc>is a <u>subsequence</u> of <math><\underline{a(abc)(ac)\underline{d(cf)}}>$

Given <u>support threshold</u> min_sup = 2, <(ab)c> is a <u>sequential pattern</u>

Sequential Pattern Mining Algorithms

- Algorithm requirement: Efficient, scalable, finding complete set, incorporating various kinds of user-specific constraints
- The Apriori property still holds: If a subsequence s_1 is infrequent, none of s_1 's super-sequences can be frequent
- Representative algorithms
 - GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)
 - Vertical format-based mining: SPADE (Zaki@Machine Leanining'00)
 - Pattern-growth methods: PrefixSpan (Pei, et al. @TKDE'04)
- Mining closed sequential patterns: CloSpan (Yan, et al. @SDM'03)
- Constraint-based sequential pattern mining (to be covered in the constraint mining section)



GSP: Apriori-Based Sequential Pattern Mining

- Initial candidates: All 8-singleton sequences
 - <a>, , <c>, <d>, <e>, <f>, <g>, <h>
- Scan DB once, count support for each candidate
- ☐ Generate length-2 candidate sequences

 $min_sup = 2$

$\overline{}$	
Cand.	sup
<a>	3
	5
<c></c>	4
<d></d>	3
<e></e>	3
<f></f>	2
285	
	(1)

	<a>		<c></c>	<d></d>	<e></e>	<f></f>
<a>	<aa></aa>	<ab></ab>	<ac></ac>	<ad></ad>	<ae></ae>	<af></af>
	<ba></ba>	>	<bc></bc>	<bd></bd>	<be></be>	<bf></bf>
<c></c>	<ca></ca>	<cb></cb>	<cc></cc>	<cd></cd>	<ce></ce>	<cf></cf>
<d></d>	<da></da>	<db></db>	<dc></dc>	<dd></dd>	<de></de>	<df></df>
<e></e>	<ea></ea>	<eb></eb>	<ec></ec>	<ed></ed>	<ee></ee>	<ef></ef>
<f></f>	<fa></fa>	<fb></fb>	<fc></fc>	<fd></fd>	<fe></fe>	<ff></ff>

	<a>		<c></c>	<d></d>	<e></e>	<f></f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c></c>				<(cd)>	<(ce)>	<(cf)>
<d></d>					<(de)>	<(df)>
<e></e>						<(ef)>
<f></f>						

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)></a(bd)bcb(ade)>

- □ Without Apriori pruning:(8 singletons) 8*8+8*7/2 =92 length-2 candidates
- With pruning, length-2 candidates: 36 + 15= 51

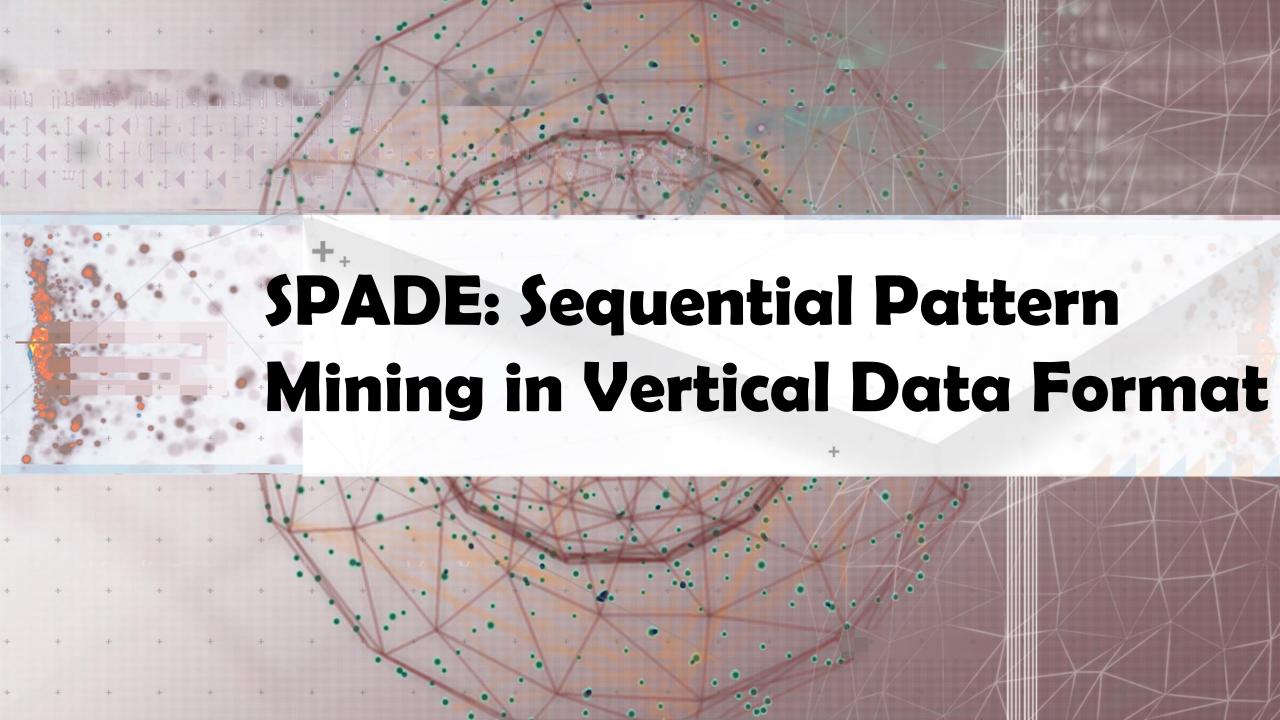
GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)

GSP Mining and Pruning

Candidates cannot pass min_sup 5th scan: 1 cand. 1 length-5 seq. pat. <(bd)cba> threshold 4th scan: 8 cand. 7 length-4 seq. pat. Candidates not in DB <abba> <(bd)bc> ... 3rd scan: 46 cand. 20 length-3 seq. pat. 20 <abb> <aab> <aba> <bab> ... cand, not in DB at all 2nd scan: 51 cand. 19 length-2 seq. pat. <aa> <ab> ... <af> <ba> ... <ff> <(ab)> ... <(ef)> 10 cand. not in DB at all <a> <c> <d> <e> <f> <g> <h> 1st scan: 8 cand. 6 length-1 seq. pat. min sup = 2

- Repeat (for each level (i.e., length-k))
- Scan DB to find length-k frequent sequences
- □ Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
- \Box set k = k+1
- Until no frequent sequence or no candidate can be found

	— · ·
SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)></a(bd)bcb(ade)>



Sequential Pattern Mining in Vertical Data Format: The SPADE Algorithm

- A sequence database is mapped to: <SID, EID>
- ☐ Grow the subsequences (patterns) one item at a time by Apriori candidate generation

SID	Sequence				
1	<a(<u>abc)(a<u>c</u>)d(cf)></a(<u>				
2	<(ad)c(bc)(ae)>				
3	<(ef)(<u>ab</u>)(df) <u>c</u> b>				
4	4 <eg(af)cbc></eg(af)cbc>				
	min_sup = 2				

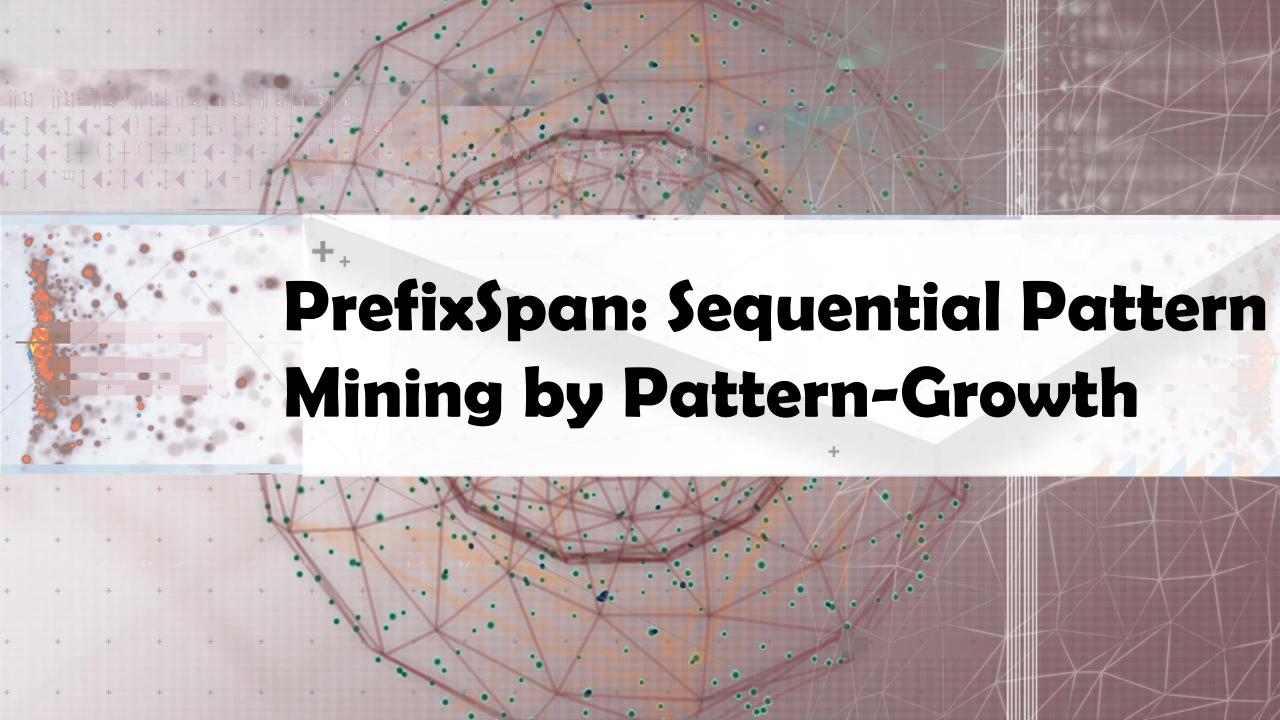
Ref: SPADE (<u>Sequential</u>
<u>PAttern Discovery</u>
using <u>Equivalent Class</u>)
[M. Zaki 2001]

SID	EID	Items
1	1	a
1	2	abc
1	3	ac
1	4	d
1	15	cf
2	$\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$	ad
2 2	2	\mathbf{c}
2	3	$_{\mathrm{bc}}$
2	4	ae
3	1	ef
3	$\frac{1}{2}$	ab
3	3	$\mathrm{d}\mathrm{f}$
3	4	\mathbf{c}
3	Э	b
4	$\frac{1}{2}$	\mathbf{e}
4	2	g
4	3	af
4	4	\mathbf{c}
4	5	b
4	6	\mathbf{c}

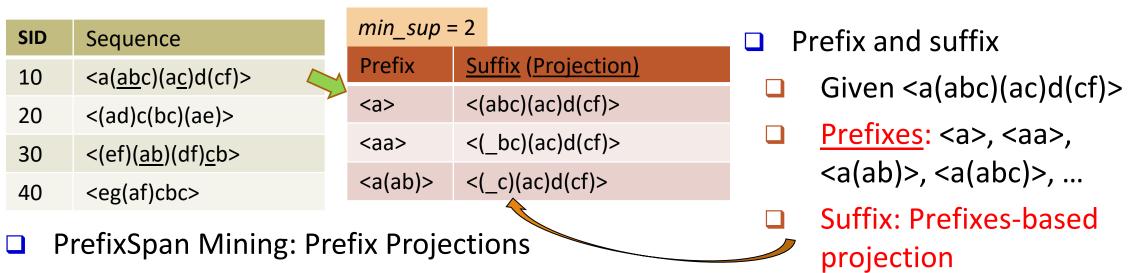
	\mathbf{a}	1	b	
SID	EID	SID	EID	
1	1	1	2	
1	2	2	3	
1	3	3	2	
2	1	3	5	
2	4	4	5	
3	2			
4	3			

	$^{\mathrm{ab}}$			ba		
SID	EID (a)	EID(b)	SID	EID (b)	EID(a)	
1	1	2	1	2	3	
2	1	3	2	3	4	
3	2	5				
4	3	5				

aba				
EID (a)	EID(b)	EID(a)		
1	2	3		
1	3	4		
		TORIVENIE		



PrefixSpan: A Pattern-Growth Approach

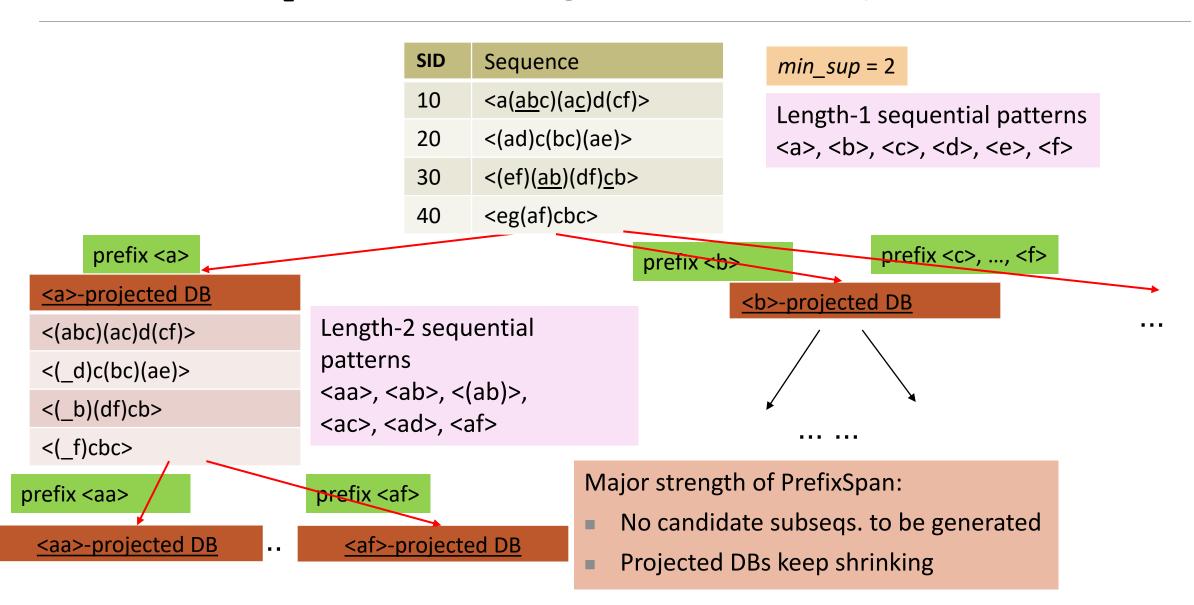


- Step 1: Find length-1 sequential patterns
 - □ <a>, , <c>, <d>, <e>, <f>
- Step 2: Divide search space and mine each projected DB
 - <a>-projected DB,
 - -projected DB,

 - <f>-projected DB, ...

PrefixSpan (Prefix-projected Sequential pattern mining) Pei, et al. @TKDE'04

PrefixSpan: Mining Prefix-Projected DBs

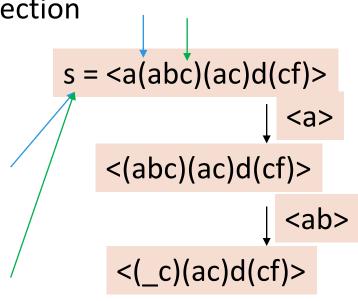


Implementation Consideration: Pseudo-Projection vs. **Physical Projection**

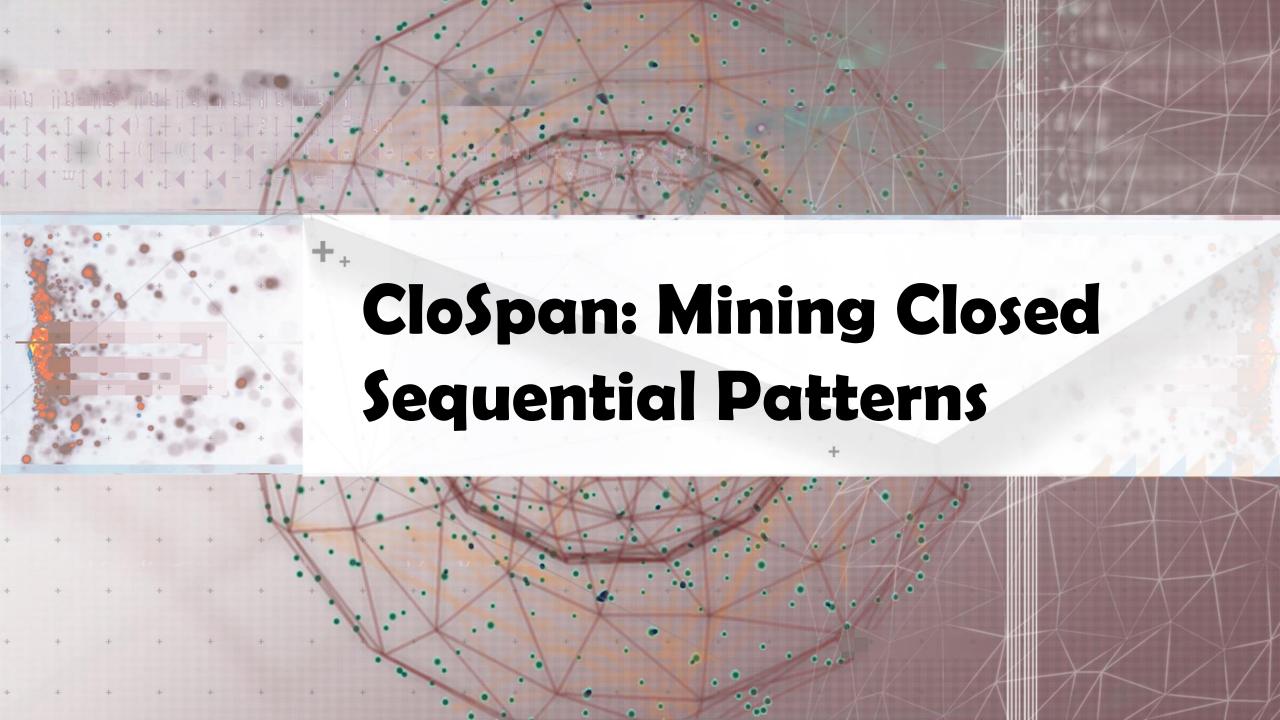
s | <a>: (, 2)

s|<ab>: (,5)

- Major cost of PrefixSpan: Constructing projected DBs
 - Suffixes largely repeating in recursive projected DBs
- When DB can be held in main memory, use pseudo projection
 - No physically copying suffixes
 - Pointer to the sequence
 - Offset of the suffix
- But if it does not fit in memory
 - Physical projection
- Suggested approach:
 - Integration of physical and pseudo-projection
 - Swapping to pseudo-projection when the data fits in memory

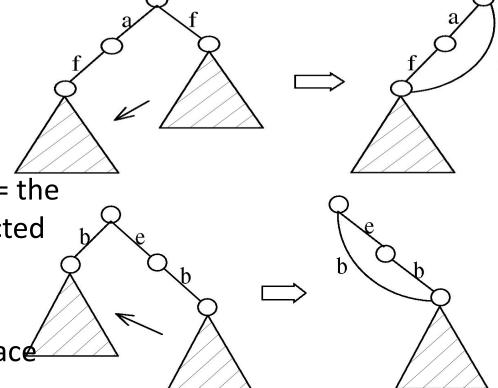


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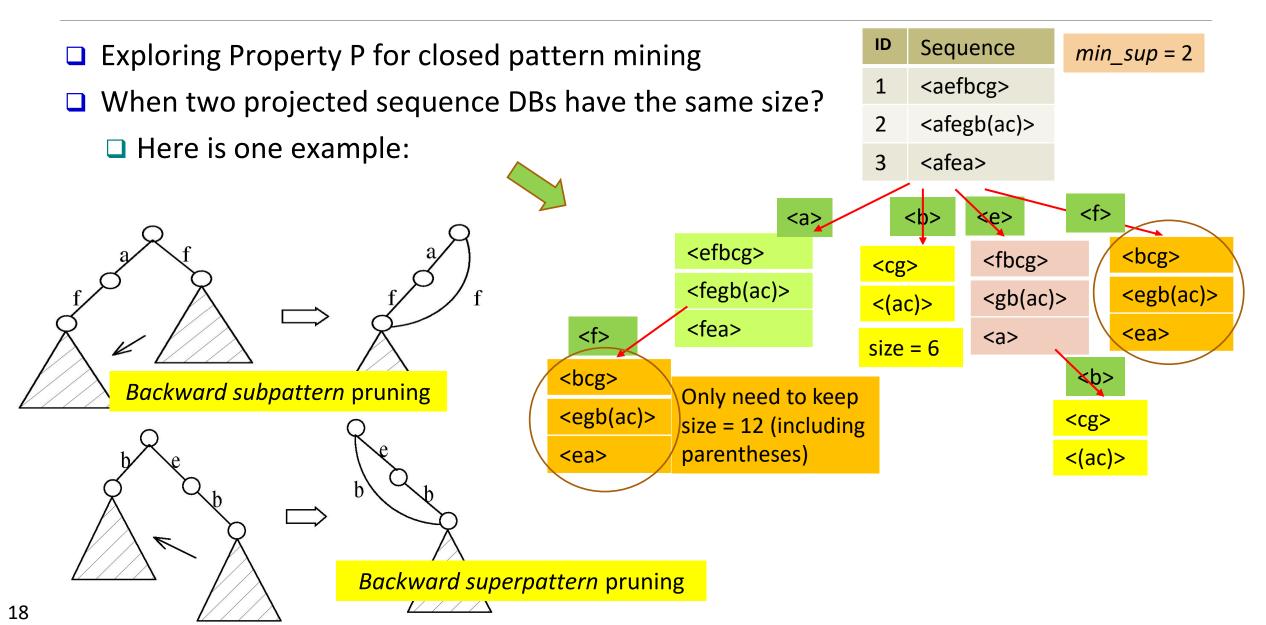


CloSpan: Mining Closed Sequential Patterns

- \square A closed sequential pattern s: There exists no superpattern s' such that $s' \supset s$, and s' and s have the same support
- □ Which ones are closed? <abc>: 20, <abcd>:20, <abcd>: 15
- Why directly mine closed sequential patterns?
 - Reduce # of (redundant) patterns
 - Attain the same expressive power
- Property P: Given two sequences s and s', if s is a subsequence of s', then the projected database of s = the projected database of s' iff the size of the two projected databases are the same.
- Explore Backward Subpattern and Backward Superpattern pruning to prune redundant search space
- Greatly enhances efficiency (Yan, et al., SDM'03)



CloSpan: When Two Projected DBs Have the Same Size





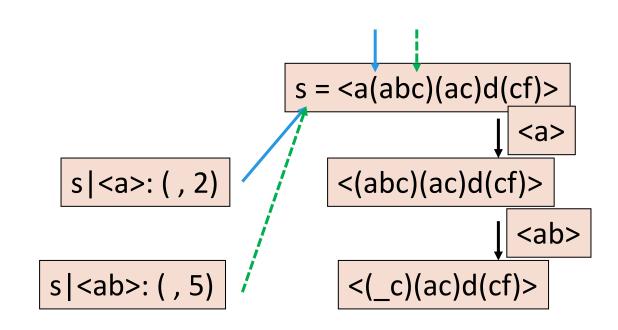
Summary: Sequential Pattern Mining

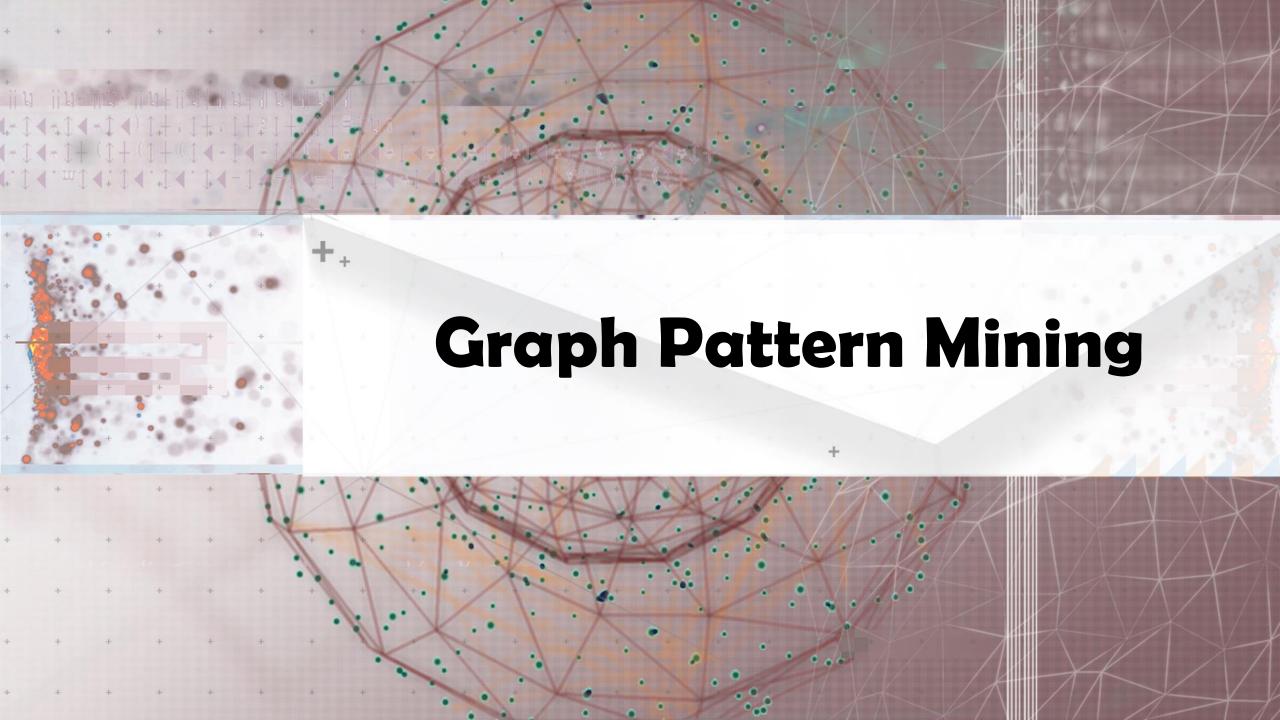
- Concepts of Sequential Pattern Mining
- Sequential Pattern Mining Algorithms
 - GSP (Generalized Sequential Patterns)
 - Vertical Format-Based Mining: SPADE
 - Pattern-Growth Methods: PrefixSpan
- Mining Closed Sequential Patterns: CloSpan

Recommended Readings

- R. Srikant and R. Agrawal, "Mining sequential patterns: Generalizations and performance improvements", EDBT'96
- M. Zaki, "SPADE: An Efficient Algorithm for Mining Frequent Sequences", Machine Learning, 2001
- J. Pei, J. Han, B. Mortazavi-Asl, J. Wang, H. Pinto, Q. Chen, U. Dayal, and M.-C. Hsu, "Mining Sequential Patterns by Pattern-Growth: The PrefixSpan Approach", IEEE TKDE, 16(10), 2004
- X. Yan, J. Han, and R. Afshar, "CloSpan: Mining Closed Sequential Patterns in Large Datasets", SDM'03

Pseudo-Projection vs. Physical Projection

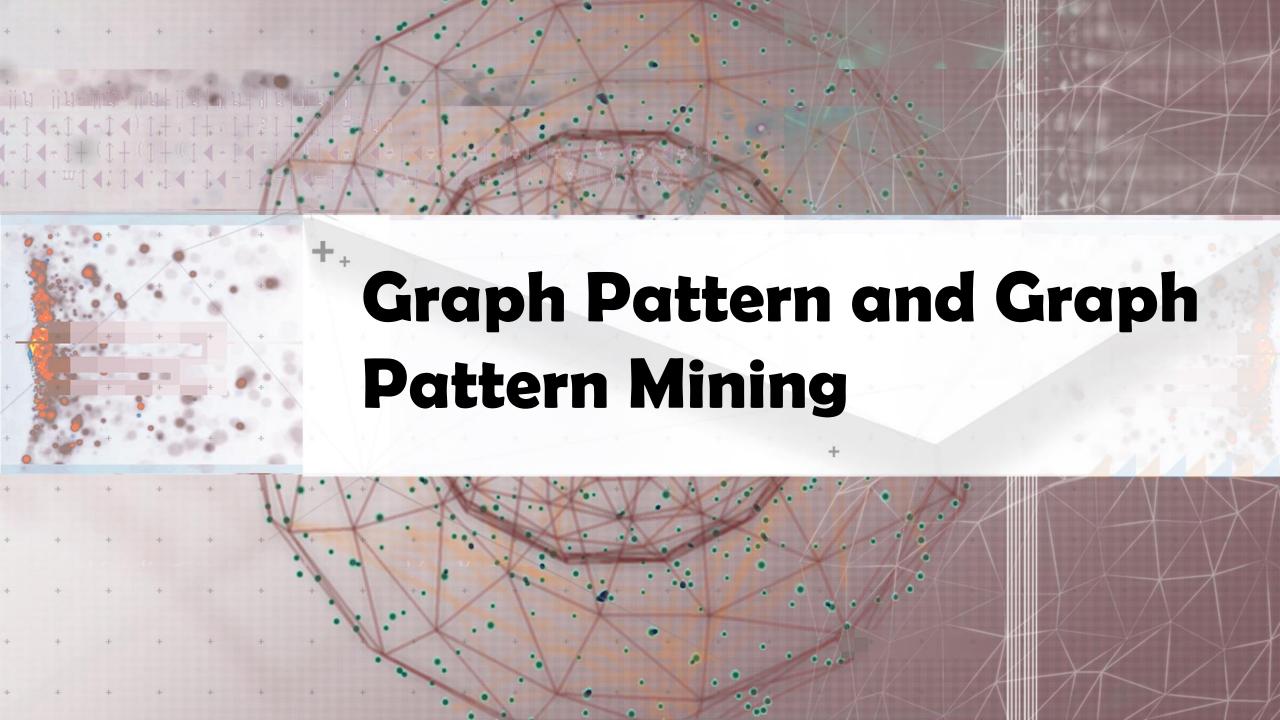




Graph Pattern Mining

- Graph Pattern and Graph Pattern Mining
- Apriori-Based Graph Pattern Mining Methods
- gSpan: A Pattern-Growth-Based Method
- CloseGraph: Mining Closed Graph Patterns
- Graph Pattern Mining Application I: Graph Indexing
- Graph Pattern Mining Application II: Graph Similarity Search

Thanks to Xifeng Yan@UCSB and Feida Zhu@SMU.SG for their contributions

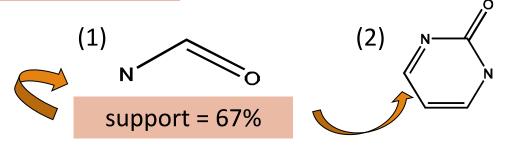


Frequent (Sub)Graph Patterns

- Given a labeled graph dataset D = $\{G_1, G_2, ..., G_n\}$, the supporting graph set of a subgraph g is $D_g = \{G_i \mid g \subseteq G_i, G_i \in D\}$
 - \square support(g) = $|D_g|/|D|$
- □ A (sub)graph g is **frequent** if support(g) \geq min_sup
- Ex.: Chemical structures
- Alternative:
 - Mining frequent subgraph patterns from a single large graph or network

 $min_sup = 2$

Frequent Graph Patterns

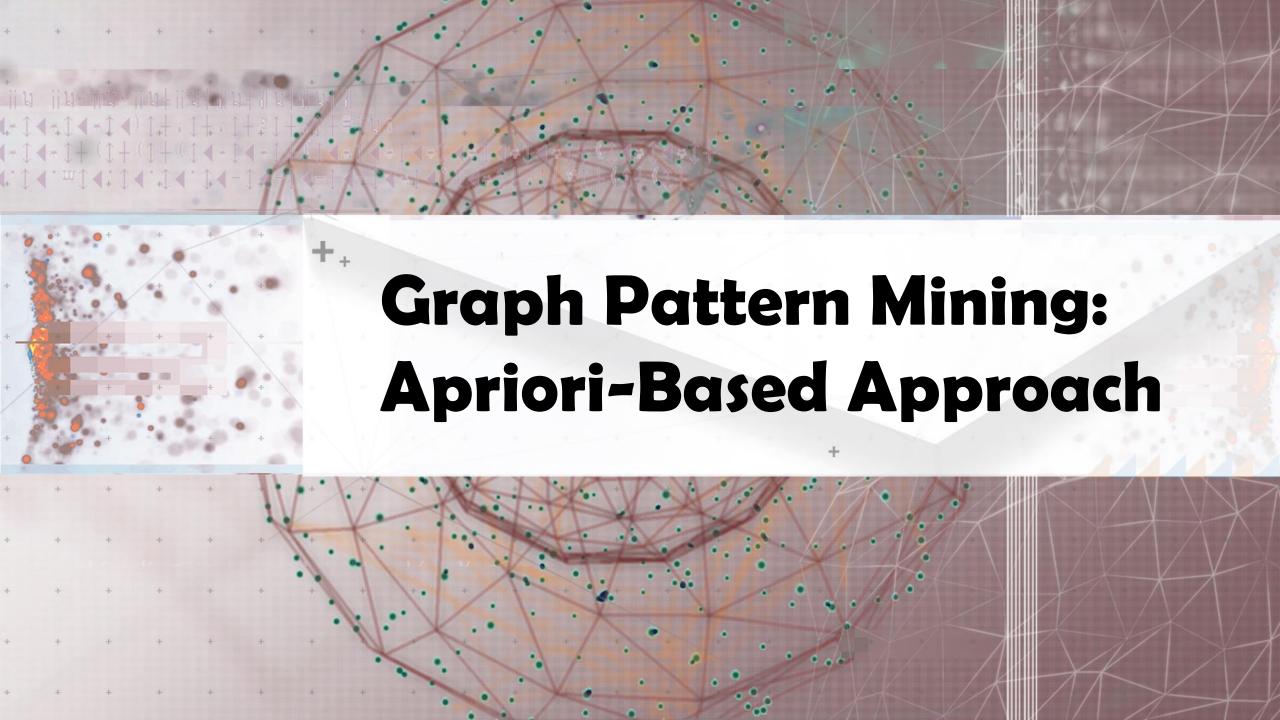


Applications of Graph Pattern Mining

- Bioinformatics
 - Gene networks, protein interactions, metabolic pathways
- Chem-informatics: Mining chemical compound structures
- Social networks, web communities, tweets, ...
- Cell phone networks, computer networks, ...
- Web graphs, XML structures, Semantic Web, information networks
- Software engineering: Program execution flow analysis
- Building blocks for graph classification, clustering, compression, comparison, and correlation analysis
- Graph indexing and graph similarity search

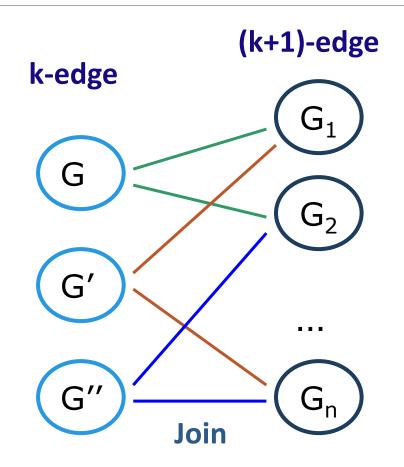
Graph Pattern Mining Algorithms: Different Methodologies

- Generation of candidate subgraphs
 - Apriori vs. pattern growth (e.g., FSG vs. gSpan)
- Search order
 - Breadth vs. depth
- Elimination of duplicate subgraphs
 - Passive vs. active (e.g., gSpan [Yan & Han, 2002])
- Support calculation
 - Store embeddings (e.g., GASTON [Nijssen & Kok, 2004], FFSM [Huan, Wang, & Prins, 2003], MoFa [Borgelt & Berthold, ICDM'02])
- Order of pattern discovery
 - □ Path → tree → graph (e.g., GASTON [Nijssen & Kok, 2004])



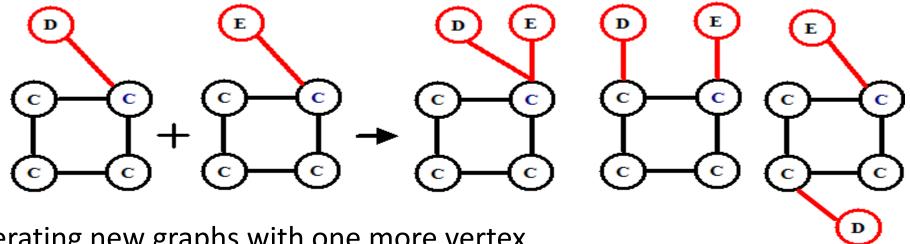
Apriori-Based Approach

- ☐ The Apriori property (anti-monotonicity): A size-*k* subgraph is frequent only if all of its subgraphs are frequent
- □ A candidate size-(k+1) edge/vertex subgraph is generated if its corresponding two k-edge/vertex subgraphs are frequent
- Iterative mining process:
 - □ Candidate-generation → candidate pruning → support counting → candidate elimination

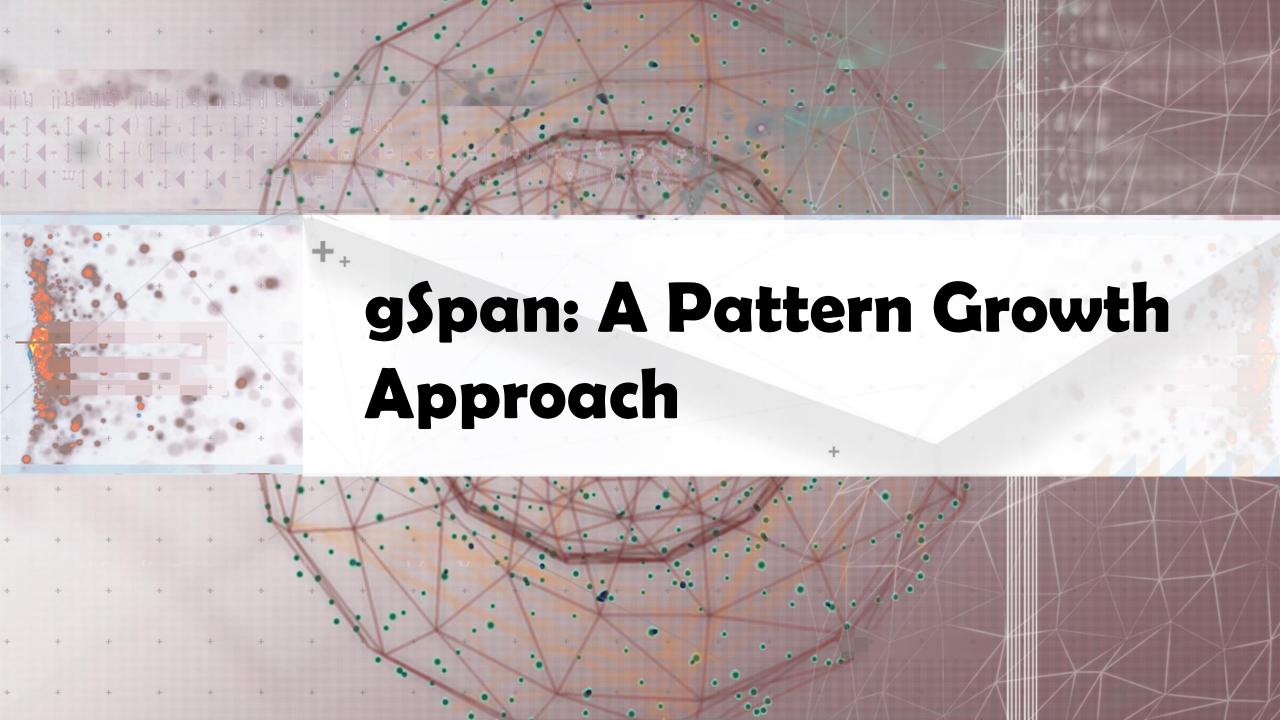


Candidate Generation: Vertex Growing vs. Edge Growing

- ☐ Methodology: Breadth-search, Apriori joining two size-k graphs
 - Many possibilities at generating size-(k+1) candidate graphs

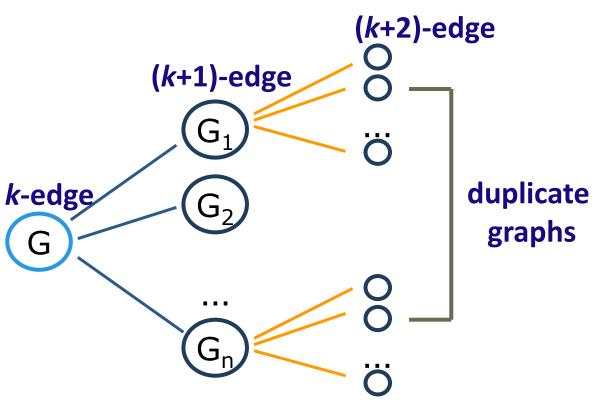


- Generating new graphs with one more vertex
 - AGM (Inokuchi, Washio, & Motoda, PKDD'00)
- Generating new graphs with one more edge
 - FSG (Kuramochi & Karypis, ICDM'01)
- ☐ Performance shows *via edge growing* is more efficient



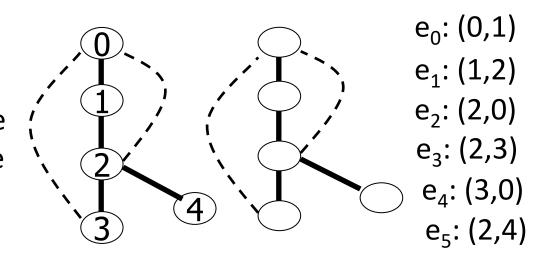
Pattern-Growth Approach

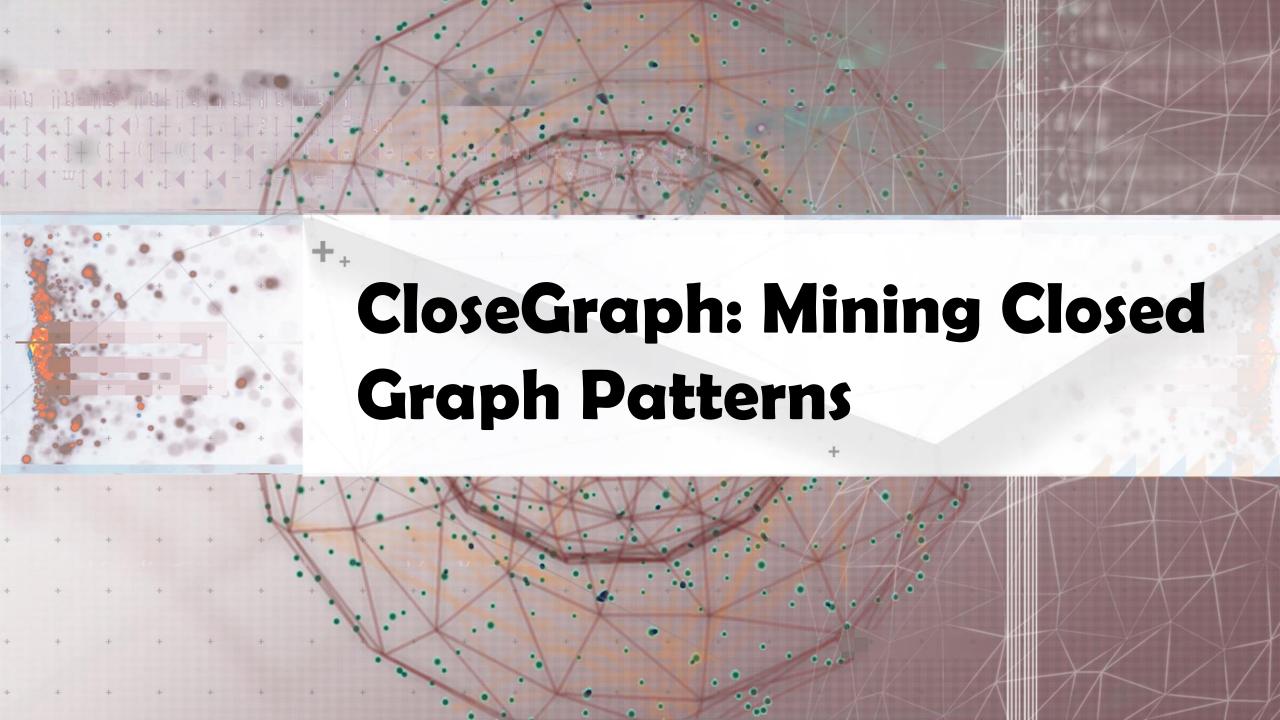
- □ Depth-first growth of subgraphs from k-edge to (k+1)-edge, then (k+2)-edge subgraphs
- Major challenge
 - Generating many duplicate subgraphs
- Major idea to solve the problem
 - Define an order to generate subgraphs
 - DFS spanning tree: Flatten a graph into a sequence using depth-first search
 - gSpan (Yan & Han, ICDM'02)



gSPAN: Graph Pattern Growth in Order

- Right-most path extension in subgraph pattern growth
 - Right-most path: The path from root to the right-most leaf (choose the vertex with the smallest index at each step)
 - Reduce generation of duplicate subgraphs
- Completeness: The enumeration of graphs using right-most path extension is <u>complete</u>
- DFS code: Flatten a graph into a sequence using depth-first search





Why Mine Closed Graph Patterns?

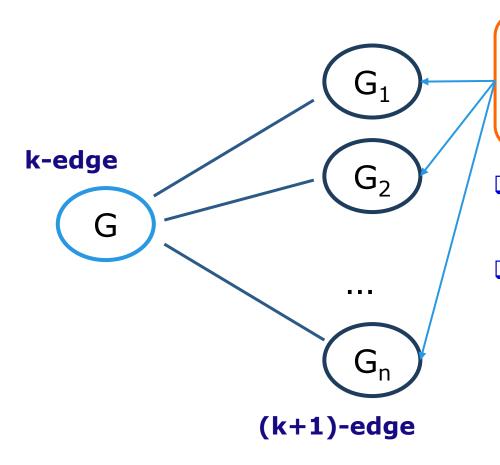
- □ Challenge: An **n**-edge frequent graph may have 2ⁿ subgraphs
- Motivation: Explore closed frequent subgraphs to handle graph pattern explosion problem
- □ A frequent graph G is *closed* if there exists no supergraph of G that carries the same support as G

If this subgraph is *closed* in the graph dataset, it implies that none of its frequent super-graphs carries the same support

- Lossless compression: Does not contain non-closed graphs, but still ensures that the mining result is complete
- Algorithm CloseGraph: Mines closed graph patterns directly

CloseGraph: Directly Mining Closed Graph Patterns

CloseGraph: Mining closed graph patterns by extending gSpan (Yan & Han, KDD'03)



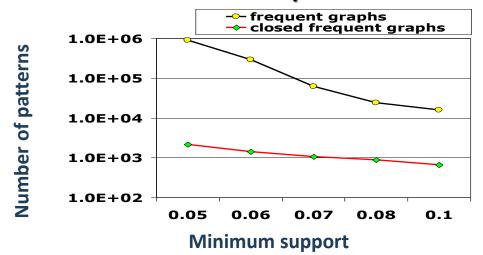
At what condition can we stop searching their children, i.e., early termination?

- Suppose G and G_1 are frequent, and G is a subgraph of G_1
 - If in any part of the graph in the dataset where G occurs, G_1 also occurs, then we need not grow G (except some special, subtle cases), since none of G's children will be closed except those of G_1

Experiment and Performance Comparison

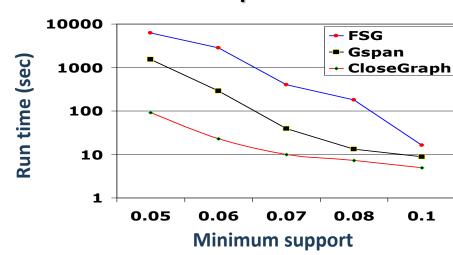
- ☐ The AIDS antiviral screen compound dataset from NCI/NIH
- The dataset contains 43,905 chemical compounds
- Discovered patterns: The smaller minimum support, the bigger and more interesting subgraph patterns discovered

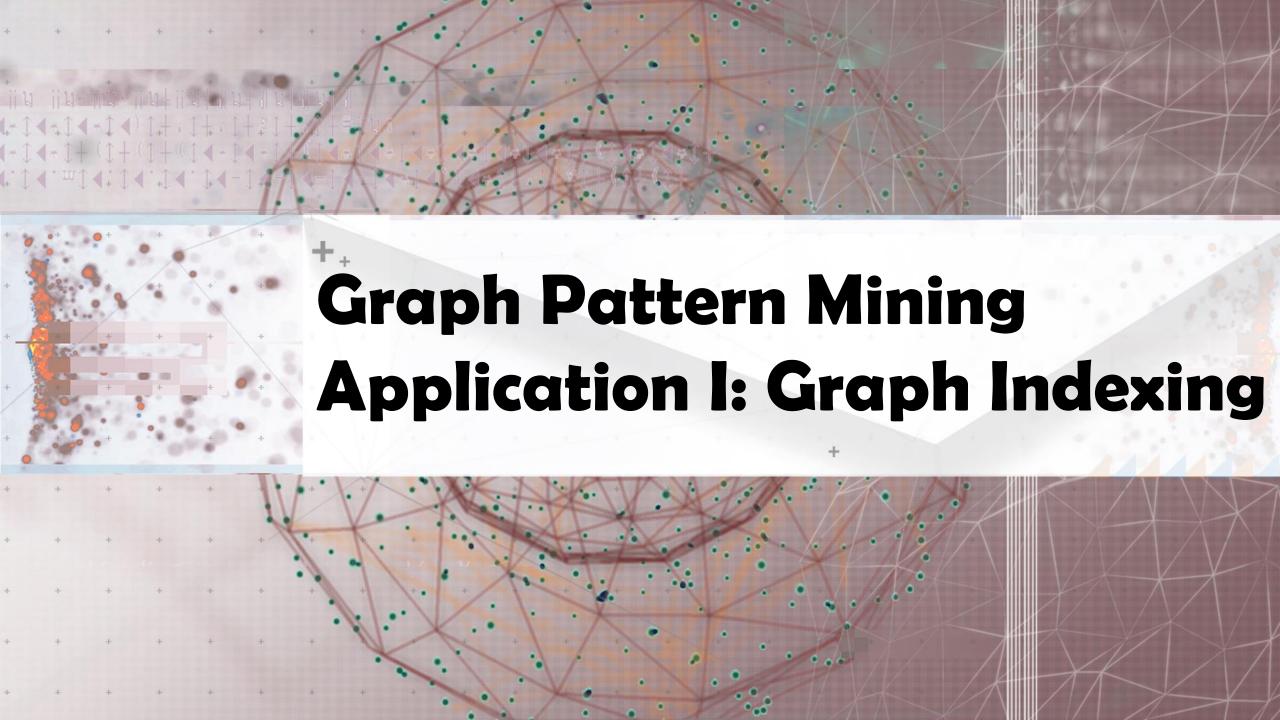
of Patterns: Frequent vs. Closed



Runtime: Frequent vs. Closed

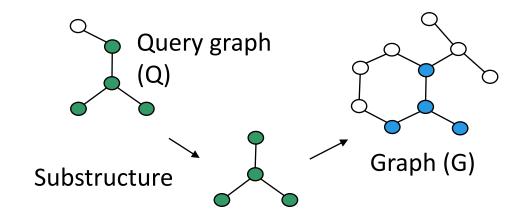
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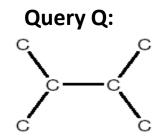


Application of Pattern Mining I: Graph Indexing

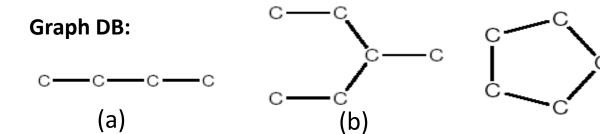
- Graph query: Find all the graphs in a graph DB containing a given query graph
- Index should be a powerful tool
- Path-index may not work well
- Solution: Index directly on substructures (i.e., graphs)



(c)



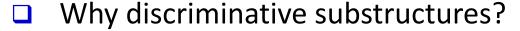
Only graph (c) contains Q



Path-indices: C, C-C, C-C-C, C-C-C cannot prune (a) & (b)

glndex: Indexing Frequent and Discriminative Substructures

- Why index frequent substructures?
 - Too many substructures to index
 - Size-increasing support threshold
 - Large structures will likely be indexed well by their substructures

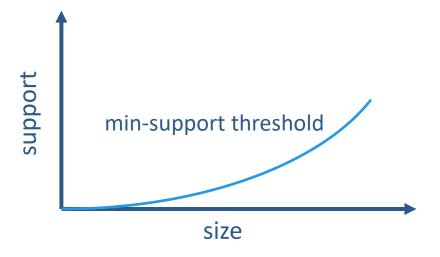


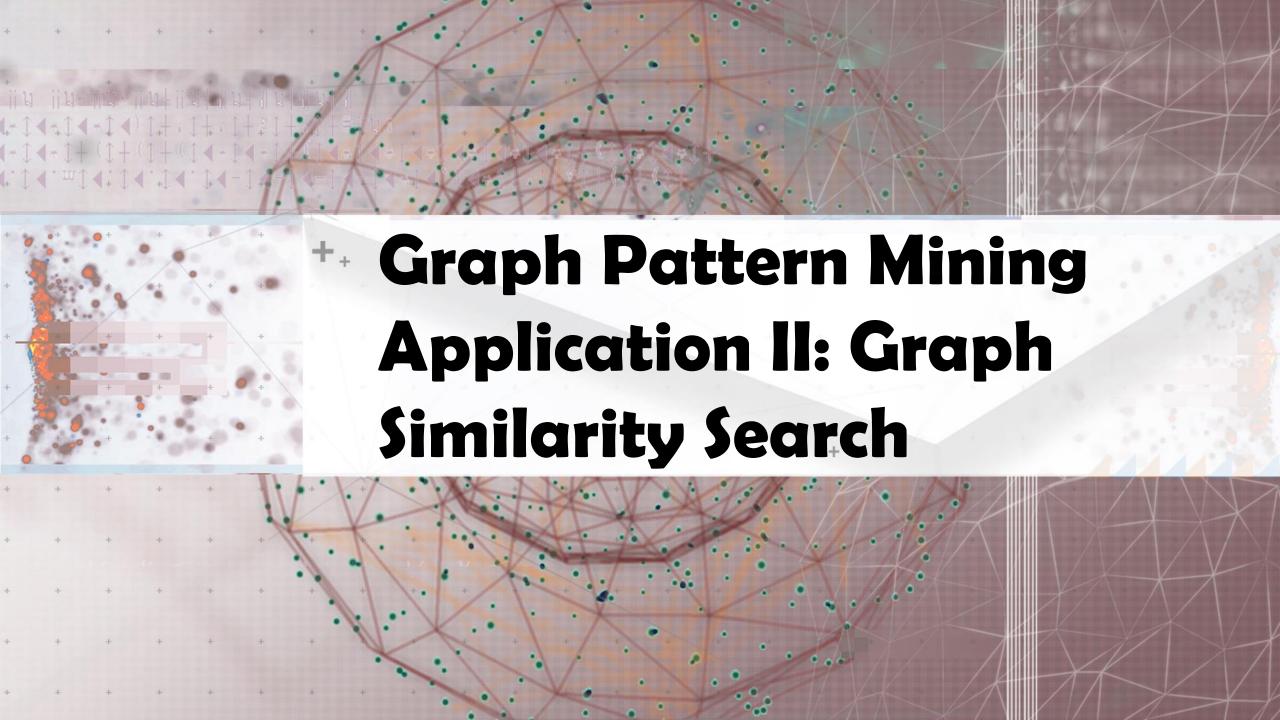
- Reduce the index size by an order of magnitude
- Selection: Given a set of selected structures f_1 , f_2 , ... f_n , and a new structure x, the extra indexing power is measured by

$$\Pr(x|f_1, f_2, \dots f_n), f_i \subset x$$

when $Pr(x|f_1, f_2, ..., f_n)$ is small enough, x is a discriminative structure and should be included in the index

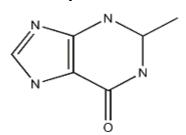
Experiments show that glndex is small, effective, and stable

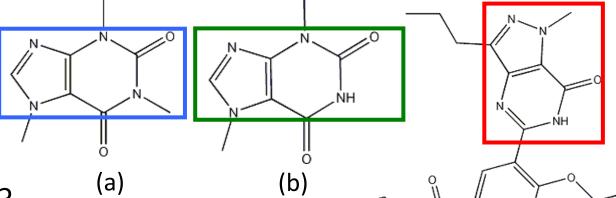




Application II: Support Substructure Similarity Search

- ☐ Find graphs in a graph DB containing substructures similar to a given query graph
- Ex. Data: A chemical compound DB
 - A query graph q:





(c)

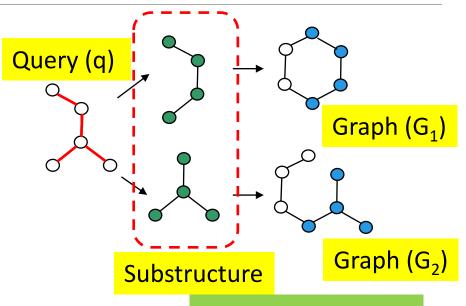
- How to do similarity search efficiently?
 - No indexing? Sequential scan + computing subgraph similarity too costly!
 - Build graph indices to support approximate search?
 - Need an explosive number of subgraphs to cover all the *similar* subgraphs!
- An elegant solution (Yan, Yu, & Han, SIGMOD'05):
 - Keep the graph index structure, but select features in the query space

Feature-Based Similarity Search

- Decompose a query graph into a set of features
- Feature-based similarity measure
 - Each graph is represented as a feature vector $X = \{x_1, x_2, ..., x_n\}$
 - □ Similarity is defined by the distance of their corresponding vectors
- If graph G contains the major part of a query graph q,
 G should share a number of common features with q
 - ☐ Given a relaxation ratio, one can calculate the maximal number of features that can be missed!

Assume: Query graph has 5 features
Relaxation threshold: Can miss at most 2 features

Then: G₁, G₂, G₃ are pruned



Graphs in database

	$G_{\scriptscriptstyle{1}}$	G_2	G_3	$G_{\scriptscriptstyle{4}}$	G_5
f_1	0	1	0	1	1
f ₂	0	1	0	0	1
f ₃	1	0	1	1	1
f_4	1	0	0	0	1
f_5	0	0	1	1	0

A feature-graph matrix



Summary: Graph Pattern Mining

- Graph Pattern and Graph Pattern Mining
- Apriori-Based Graph Pattern Mining Methods
- gSpan: A Pattern-Growth-Based Method
- CloseGraph: Mining Closed Graph Patterns
- Graph Pattern Mining Application I: Graph Indexing
- ☐ Graph Pattern Mining Application II: Graph Similarity Search

Recommended Readings

- Kuramochi, M., & Karypis, G. (2001). Frequent subgraph discovery. ICDM'01.
- Yan, X. & Han, J. (2002). gSpan: Graph-based substructure pattern mining. ICDM'02.
- Yan, X., & Han, J. (2003). CloseGraph: Mining closed frequent graph patterns. KDD'03.
- Yan, X., Yu, P. S., & Han, J. (2004). Graph indexing: A frequent structure-based approach. SIGMOD'04.
- Yan, X., Yu, P. S., & Han, J. (2005). Substructure similarity search in graph databases. *SIGMOD'05*.

Additional References

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- □ Huan, J., Wang, W., & Prins, J. (2003). Efficient mining of frequent subgraph in the presence of isomorphism. *ICDM'03*.
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Pattern Mining Application: Software Bug Detection

- Mining rules from source code
 - Bugs as deviant behavior (e.g., by statistical analysis)
 - Mining programming rules (e.g., by frequent itemset mining)
 - Mining function precedence protocols (e.g., by frequent subsequence mining)
 - Revealing neglected conditions (e.g., by frequent itemset/subgraph mining)
- Mining rules from revision histories
 - By frequent itemset mining
- Mining copy-paste patterns from source code
 - ☐ Find copy-paste bugs (e.g., CP-Miner [Li et al., OSDI'04]) (to be discussed here)
 - Reference: Z. Li, S. Lu, S. Myagmar, Y. Zhou, "<u>CP-Miner</u>: A Tool for Finding Copy-paste and Related Bugs in Operating System Code", OSDI'04

Application Example: Mining Copy-and-Paste Bugs

- Copy-pasting is common
 - □ 12% in Linux file system
 - 19% in X Window system
- Copy-pasted code is error-prone
- Mine "forget-to-change" bugs by sequential pattern mining
 - Build a sequence database from source code
 - Mining sequential patterns
 - Finding mismatched identifier names & bugs

```
void init prom meminit(void)
  for (i=0; i<n; i++) {
    total[i].adr = list[i].addr;
    total[i].bytes = list[i].size;
    total[i].more = &total[i+1];
                                    Code copy-and-
                                    pasted but forget
for (i=0; i<n; i++) {
                                    to change "id"!
     taken[i].adr = list[i].addr;
     taken[i].bytes = list[i].size,
     taken[i].more = &total[i+1];
```

Courtesy of Yuanyuan Zhou@UCSD

(Simplified example from *linux*-

Building Sequence Database from Source Code

- (mapped to)

 ☐ Statement → number
- ☐ Tokenize each component
 - □ Different operators, constants, key words
 → different tokens
 - □ Same type of identifiers → same token
- □ Program → A long sequence
 - Cut the long sequence by blocks

Map a statement to a number

```
old = 3; new = 3;

Tokenize

5 61 20

Hash
16

16
```

Hash values

```
for (i=0; i<n; i++) {
65
16
            total[i].adr = list[i].addr;
            total[i].bytes = list[i].size;
16
            total[i].more = &total[i+1];
71
          for (i=0; i<n; i++) {
65
            taken[i].adr = list[i].addr;
16
            taken[i].bytes = list[i].size;
16
            taken[i].more = &total[i+1];
71
```

```
Final sequence DB: (65) (16, 16, 71) ... (65) (16, 16, 71)
```

Sequential Pattern Mining & Detecting "Forget-to-Change" Bugs

- Modification to the *sequence pattern mining algorithm*
 - Constrain the max gap

- (16, 16, 71)
 Allow a maximal gap: inserting statements in copy-and-paste
- Composing Larger Copy-Pasted Segments
 - Combine the neighboring copy-pasted segments repeatedly
- ☐ Find conflicts: Identify names that cannot be mapped to the corresponding ones
 - E.g., 1 out of 4 "total" is unchanged, unchanged ratio = 0.25
 - ☐ If 0 < unchanged ratio < threshold, then report it as a bug
- CP-Miner reported many C-P bugs in Linux, Apache, ... out of millions of LOC (lines of code)

