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Multivariate refined composite multiscale entropy analysis

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ABSTRACT

Multiscale entropy (MSE) has become a prevailing method to quantify signals complexity. MSE relies on sample entropy. However, MSE may yield imprecise complexity estimation at large scales, because sample entropy does not give precise estimation of entropy when short signals are processed. A refined composite multiscale entropy (RCMSE) has therefore recently been proposed. Nevertheless, RCMSE is for univariate signals only. The simultaneous analysis of multi-channel (multivariate) data often over-performs studies based on univariate signals. We therefore introduce an extension of RCMSE to multivariate data. Applications of multivariate RCMSE to simulated processes reveal its better performances over the standard multivariate MSE.

Several entropy measures have been proposed to assess the regularity of times series. Among them, we can cite the sample entropy [1]. Sample entropy is equal to the negative of the natural logarithm of the conditional probability that sequences close to each other for m consecutive data points will also be close to each other when one more point is added to each sequence [1]. However, sample entropy operates on a single scale. Real world data, as physiological data, exhibit high degree of structural richness. Studies based on a single scale are therefore not adapted for real world signals. Analyses on multiple time scales have become necessary. In the 2000s, Costa et al. proposed the multiscale entropy (MSE) to quantify complexity over multiple scales [2,3]. The MSE algorithm is composed of two steps [2,3]: (i) a coarse-graining procedure to derive a set of time series representing the system dynamics on different time scales. The coarse-graining procedure for scale τ is obtained by averaging the samples of the time series inside consecutive but non-overlapping windows of length τ; (ii) the computation of the sample entropy for each coarse-grained time series. MSE has become a prevailing method to quantify the complexity of signals. It has been shown through several studies that MSE is able to underline the general loss of complexity behavior when a living system changes from a healthy state to a pathological state [2,3].

Nevertheless, the coarse-graining procedure used in the MSE algorithm shortens the length of the data that are processed: for an original time series of N samples, the length of the coarse-grained time series at a scale factor τ is N/τ. It has been reported that for an embedding dimension $m = 2$, the sample entropy is significantly independent of the time series length when the number of data points is larger than 750 [4]. For shorter time series, the variance of the entropy estimator may grow very fast with the reduction of the number of data points. Therefore, at large scales, the coarse-grained time series may not be adequately long to obtain an accurate value for the sample entropy. Moreover, for some cases, the sample entropy value may not be defined because no template vectors are matched to one another. These two drawbacks (inaccurate or undefined sample entropy values) lead to problems of accuracy and validity of MSE at large scales. In order to overcome the accuracy concern of MSE, Wu et al. proposed the composite MSE (CMSE) [5]. In the CMSE algorithm, all coarse-grained time series for a scale factor τ are processed to compute their sample entropy (each of the τ coarse-grained time series corresponding to different starting points of the coarse-graining process is used in the CMSE algorithm whereas, in the conventional MSE algorithm, for each scale, only the first coarse-grained time series is taken into account). The CMSE value for a given scale is therefore defined as the mean of several entropy values [5]. Therefore, CMSE estimates entropy more accurately than MSE. Unfortunately, CMSE increases the probability of inducing undefined entropy. This is why a refined CMSE (RCMSE) algorithm has been proposed in 2014, see below [6].

However, MSE, CMSE, and RCMSE are able to process univariate data only. For multivariate time series, the three algorithms treat individual time series separately. This may be satisfactory only if the individual signals are statistically independent or at

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least uncorrelated, which is often not the case when real world signals from a given system are registered simultaneously. To overcome this shortcoming, an extension of the MSE algorithm to multivariate data has been proposed in 2011: the multivariate MSE (MMSE) [8,9]. MMSE is able to operate on any number of data channels and provides a robust relative complexity measure for multivariate data [8,9]. MMSE has been used in studies from different fields [10–13]. However, the same concerns as MSE are found in MMSE. This is why in this work we propose an extension of the RCMSE algorithm to a more general case. To this end, we introduce the multivariate RCMSE (MRCMSE), and evaluate its performances on synthetic multivariate processes.

1. Multivariate refined composite multiscale entropy

1.1. Refined composite multiscale entropy

RCMSE aims at improving the CMSE algorithm because, as mentioned previously, CMSE estimates entropy more accurately than MSE but increases the probability of inducing undefined entropy [6,7].

For a discrete time series $x = \{x_i\}_{i=1}^{N}$, the RCMSE algorithm is based on the following three steps [6]

1. the kth coarse-grained time series for a scale factor τ is defined as $y_k^{(\tau)} = \{y_{k,j}\}_{j=1}^{N/\tau}$ where [5]

$$y_{k,j}^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+k}^{j\tau+k-1} x_i, \qquad 1 \le j \le \frac{N}{\tau}, 1 \le k \le \tau \tag{1}$$

2. for each scale factor τ, and for all τ coarse-grained time series, the number of matched vector pairs $n_{k,\tau}^{m+1}$ and $n_{k,\tau}^{m}$ is computed, where $n_{k,\tau}^{m}$ represents the total number of m-dimensional matched vector pairs and is computed from the kth coarse-grained time series at a scale factor τ
3. RCMSE is then defined as [6]

$$RCMSE(x, \tau, m, r) = -\ln\left(\frac{\sum_{k=1}^{\tau} n_{k,\tau}^{m+1}}{\sum_{k=1}^{\tau} n_{k,\tau}^{m}}\right). \tag{2}$$

Using the same notation, CMSE is defined as [6]

$$CMSE(x, \tau, m, r) = \frac{1}{\tau} \sum_{k=1}^{\tau} \left(-\ln \frac{n_{k,\tau}^{m+1}}{n_{k,\tau}^{m}}\right). \tag{3}$$

The CMSE value is therefore undefined when one of the values $n_{k,\tau}^{m+1}$ or $n_{k,\tau}^{m}$ is zero. By opposition, RCMSE value is undefined only when all $n_{k,\tau}^{m+1}$ or $n_{k,\tau}^{m}$ are zeros. It has been reported that RCMSE outperforms CMSE in validity, accuracy of entropy estimation, independence of data length, and computational efficiency [6]. RCMSE has been used in recent studies [14].

1.2. Multivariate multiscale entropy

MMSE is an extension of the MSE algorithm to multivariate data. MMSE relies on the same steps as MSE [8,9]: (i) a coarse-graining procedure; (ii) a sample entropy computation for each coarse-grained time series. However, due to the multivariate nature of the data processed by MMSE, these two steps are adapted to multivariate signals. Thus, for the coarse-graining procedure, temporal scales are defined by averaging a p-variate time series $\{x_{l,i}\}_{i=1}^{N}$ ($l = 1, \dots, p$ is the channel index and N is the number of samples in every channel) over non-overlapping time segments of increasing length. Thus, for a scale factor τ, a coarse-grained multivariate time series is computed as $y_{l,j}^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x_{l,i}$ where $1 \le j \le N/\tau$, and the channel index l goes from 1 to p. For the entropy computation, the multivariate sample entropy (MSampEn) is used for each coarse-grained multivariate. The MSampEn algorithm is an extension of the univariate sample entropy [1]. For a tolerance level r, MSampEn is calculated as the negative of the natural logarithm of the conditional probability that two composite delay vectors close to each other in a m dimensional space will also be close to each other when the dimensionality is increased by one. The detailed MSampEn algorithm can be found in [8,9].

1.3. Multivariate refined composite multiscale entropy

Based on RCMSE and MSampEn, we define the MRCMSE algorithm as follows:

1. for a p-variate time series $\{x_{l,i}\}_{i=1}^{N}$, $l = 1, \dots, p$, where p denotes the number of variates (channels) and N is the number of samples in each variate, and for a scale factor τ, determine the coarse-grained multivariate time series $\{y_{l,k,j}^{(\tau)}\}_{j=1}^{N/\tau}$ as

$$y_{l,k,j}^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+k}^{j\tau+k-1} x_{l,i}, \tag{4}$$

where $1 \le j \le N/\tau$, $1 \le k \le \tau$, $l = 1, \dots, p$
2. for each coarse-grained multivariate compute $B^m(r)$ and $B^{m+1}(r)$ as defined in Table 1 [8,9]. For the coarse-grained multivariate $\{y_{l,k,j}^{(\tau)}\}_{j=1}^{N/\tau}$, $l = 1, \dots, p$, these two quantities are denoted as $B_{k,\tau}^{m}(r)$ and $B_{k,\tau}^{m+1}(r)$, respectively
3. compute $MRCMSE(\tau, M, r, \epsilon, N) = -\ln\left(\frac{\sum_{k=1}^{\tau} B_{k,\tau}^{m+1}(r)}{\sum_{k=1}^{\tau} B_{k,\tau}^{m}(r)}\right)$.

2. Results and discussion

In order to analyze the behavior of MRCMSE on multivariate data, we generated a trivariate time series, where originally all the data channels were realizations of mutually independent white noise [8]. We then gradually decreased the number of variates representing white noise (from 3 to 0) and simultaneously increased the number of data channels representing independent $1/f$ noise (from 0 to 3), as already proposed in [8,9]. The total number of variates was always three. For each kind of trivariate data, 50 independent realizations were simulated and, for each realization, 10 000 samples were generated in each variate. Scales were chosen between 1 and 20. Therefore, the shortest coarse-grained time series had a length of 500 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to 0.15 × (standard deviation of the normalized time series) for each data channel. We recall that a multivariate time series is considered more structurally complex than another if, for most of the scale factors τ, its multivariate entropy values are higher than those of the other time series. When the multivariate entropy values decrease with the scale factor τ, the time series that is processed only contains information at the smallest scales. It is thus not structurally complex. This is the same as what is observed for the univariate MSE where sample entropy values of random white noise (uncorrelated) decrease with the scale factor whereas for $1/f$ noise (long-range correlated), the sample entropy values are constant over multiple scales.

For each above-mentioned simulated trivariate data, MRCMSE and MMSE were determined. The results are presented in Fig. 1. We observe that MRCMSE and MMSE curves are close to each other. Moreover, the higher the number of variates representing $1/f$ noise, the higher the multivariate entropy value, for a given scale factor τ. This is in accordance with what was expected. This behavior is the same for MRCMSE and MMSE.

Table 1
Computation of $B^m(r)$ and $B^{m+1}(r)$ (from [8,9]).

1.	Form $(N-n)$ composite delay vectors $X_m(i) \in \mathbf{R}^m$, where $i = 1, 2, \ldots, N-n$ and $n = \max\{M\} \times \max\{\epsilon\}$. For a p-variate time series $\{x_{l,i}\}_{i=1}^{N}$, $l = 1, \ldots, p$, the composite delay vector $X_m(i) \in \mathbf{R}^m$ is defined as $X_m(i) = [x_{1,i}, x_{1,i+\epsilon_1}, \ldots, x_{1,i+(m_1-1)\epsilon_1}, x_{2,i}, x_{2,i+\epsilon_2}, \ldots, x_{2,i+(m_2-1)\epsilon_2}, \ldots, x_{p,i}, x_{p,i+\epsilon_p}, \ldots, x_{p,i+(m_p-1)\epsilon_p}]$ where $M = [m_1, \ldots, m_p] \in \mathbf{R}^p$ is the embedding vector, $\epsilon = [\epsilon_1, \ldots, \epsilon_p]$ the time lag vector and $m = \sum_{l=1}^{p} m_l$
2.	Define the distance between any two vectors $X_m(i)$, $X_m(j)$ as the maximum norm, that is, $d[X_m(i), X_m(j)] = \max_{l=1,\ldots,m}\{\lvert x(i+l-1) - x(j+l-1)\rvert\}$
3.	For a given composite delay vector $X_m(i)$ and a threshold r, count the number of instances P_i for which $d[X_m(i), X_m(j)] \le r$, $j \ne i$, then calculate the frequency of occurrence, $B_i^m(r) = \frac{1}{N-n-1} P_i$, and define $B^m(r) = \frac{1}{N-n} \sum_{i=1}^{N-n} B_i^m(r)$
4.	Extend the dimensionality of the multivariate delay vector defined in step 1. from m to $(m+1)$. A total of $p \times (N-n)$ vectors $X_{m+1}(i)$ in $\mathbf{R}^{m+1}$ are obtained, where $X_{m+1}(i)$ corresponds to any embedded vector upon increasing the embedding dimension from m_α to $m_\alpha + 1$ for a specific variable α
5.	For a given $X_{m+1}(i)$, calculate the number of vectors Q_i, such that $d[X_{m+1}(i), X_{m+1}(j)] \le r$, where $j \ne i$, then calculate the frequency of occurrence $B_i^{m+1}(r) = \frac{1}{p\times(N-n)-1} Q_i$ and define $B^{m+1}(r) = \frac{1}{p\times(N-n)} \sum_{i=1}^{p\times(N-n)} B_i^{m+1}(r)$

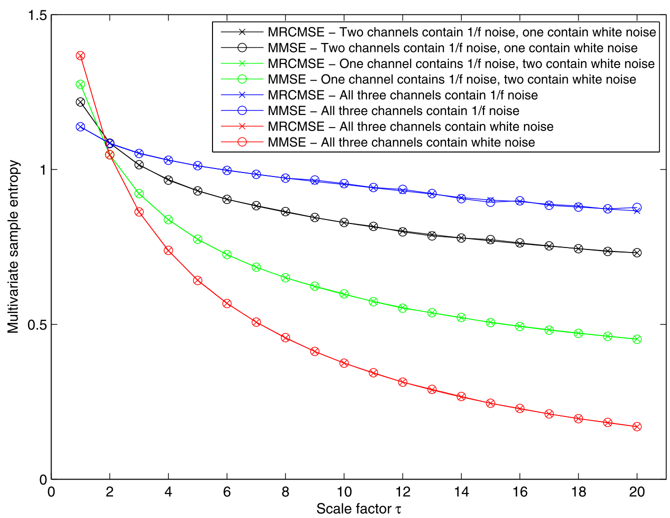

Fig. 1. MRCMSE and MMSE average values for trivariate data containing white and $1/f$ noise, each with 10 000 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to 0.15 × (standard deviation of the normalized time series). The average values have been obtained with 50 independent realizations.

We also analyzed the behavior of MRCMSE for *correlated* bivariate white noise and for *correlated* bivariate $1/f$ noise, each with 10 000 samples. The results are shown in Fig. 2a. We observe that MRCMSE is able to deal with within- and cross-channel correlations. Thus, at large scales, the highest complexity is observed for correlated bivariate $1/f$ noise, followed by the uncorrelated $1/f$ noise, and correlated and uncorrelated white noise. This is in accordance with the expected results: for white noise and $1/f$ noise, the complexity of multivariate processes with within- and cross-channel correlations is higher than the complexity of uncorrelated multivariate processes. Moreover, these results are also in accordance with those given by MMSE, as previously shown by others [8].

In order to compare the precision of MRCMSE with the one of MMSE, four kinds of trivariate signals were considered (as above). However, the length of the data has been reduced: previously the data had 10 000 samples whereas now they have 1000 samples. Moreover, as above, for each kind of trivariate signal, 50 independent realizations were simulated. Scales were chosen between 1 and 20. Therefore, the shortest coarse-grained time series had a length of 50 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to 0.15 × (standard deviation of the normalized time series). For each above-mentioned trivariate simulated signal, MRCMSE and MMSE were determined. The results are presented in Figs. 3 and 4. We observe that, for time series containing at least one channel with white noise, the means of the entropy values given by MRCMSE and MMSE are similar. However, for the time series containing channels with $1/f$ noises only, some entropy values computed with

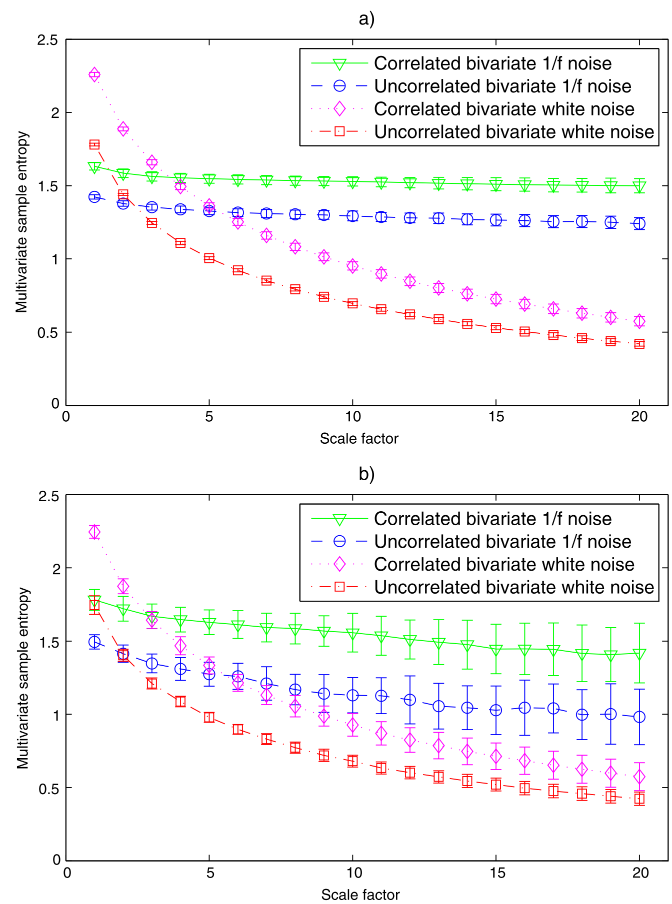

Fig. 2. MRCMSE average values for bivariate data containing white and $1/f$ noise, (a) each with 10 000 samples; (b) each with 1000 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to 0.15 × (standard deviation of the normalized time series). The average values have been obtained with 50 independent realizations. The error bars correspond to the standard deviation values.

MMSE are undefined (for large scale factors). By opposition, the entropy values given by the MRCMSE algorithm are all defined. Moreover, we note that the standard deviation values given by MRCMSE are lower than those given by MMSE, for all the trivariate signals processed (see Fig. 4). This result indicates that the entropy values given by the MRCMSE algorithm are more precise than those given by the MMSE algorithm. We also computed the MRCMSE and MMSE standard deviation values for the trivariate data with 10 000 samples. The results show that, even with long time series, the standard deviation values given by MRCMSE are lower than those given by MMSE, for all the trivariate signals processed (see Fig. 5).

When comparing Fig. 1 (where data with 10 000 samples are processed) and Fig. 3 (where data with 1000 samples are processed), we note that the order of the curves are different. We also

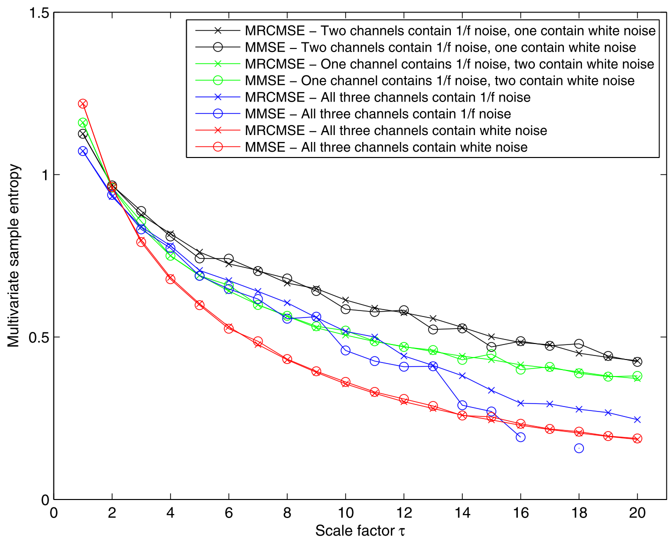

Fig. 3. MRCMSE and MMSE average values for trivariate data containing white and $1/f$ noise, each with 1000 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to $0.15\times$ (standard deviation of the normalized time series). The average values have been obtained with 50 independent realizations.

observe from Figs. 1 and 3 that the analysis has more difficulty for $1/f$ noises than for white noises. The explanations of these differences are the following:

1. for the univariate case, it has been shown that RCMSE is dependent on data length [6]
2. the full multivariate approach for sample entropy is sensitive to data length. As shown by Ahmed and Mandic in their paper [9], the naive approach of the multivariate sample entropy shows an increasing trend as the data length decreases. Moreover, the full multivariate approach leads to a decrease of the multivariate sample entropy as the data length decreases (see Fig. 6). Multivariate sample entropy is a relative estimate not an absolute parameter. Moreover, MRCMSE also leads to a decrease of entropy as the data length decreases (see Fig. 6)
3. for the univariate case, it has been reported that the mean value of sample entropy diverges as the number of data points decreases. Moreover, since $1/f$ noise time series are not stationary, as the number of data points decreases, the discrepancy between the sample entropy value calculated numerically and the mean value for simulated time series increases faster for $1/f$ noise than for white noise time series [3]. MSE presents the same limitation and MRCMSE too.

From several tests, we conclude that for original trivariate time series longer than 3000 samples, the order of the curves is similar to the one obtained for long time series (10 000 samples), for scale factors between 1 and 20 (see Figs. 1 and 7). For trivariate time series shorter than 3000 samples, the order of the curves changes (see Figs. 3 and 7a). For original time series of 2000 samples, this is observed from scale factor $\tau = 6$ where the trivariate time series having three channels with $1/f$ noise show lower entropy values than the trivariate time series having two channels with $1/f$ noise and one channel with white noise. These results are rather similar for MRCMSE and MMSE. Moreover, as for MMSE, MRCMSE leads to a decrease of entropy as the data length decreases (see Fig. 6). Therefore, for short multivariate time series, the entropy estimate with MRCMSE may be erroneous (as it is for MMSE). From Fig. 6 we observe that the decrease of entropy as data length decreases begins at similar lengths for MRCMSE and full MMSE, and is in the same order of magnitude for MRCMSE and full MMSE. Therefore, and as suggested by Ahmed et al. for MMSE [9,15], for an embedding dimension $m_k = 2$, MRCMSE estimates are consistent for data length $N \geq 300$ (the highest scale should have at least 300 points). For shorter time series, MRCMSE may not be adequate. However, for short and long multivariate time series, the standard deviation values given by MRCMSE are lower than those given by MMSE. Moreover, no undefined entropy values are observed with MRCMSE when there are with MMSE. The entropy values given by the MRCMSE algorithm are therefore more consistent than those given by the MMSE algorithm. MRCMSE shows better precision than MMSE.

When the behavior of MRCMSE for correlated bivariate white noise and for correlated bivariate $1/f$ noise is analyzed for short time series (1000 samples and not 10 000 samples), we observe that the expected results are still obtained (see Fig. 2b): at large scales, the highest complexity is observed for correlated bivariate $1/f$ noise, followed by the uncorrelated $1/f$ noise, and correlated and uncorrelated white noise. However, we note that the standard

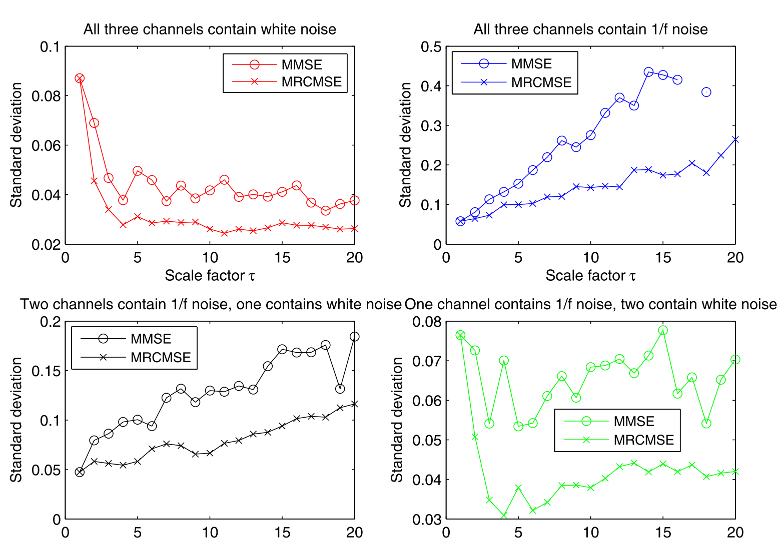

Fig. 4. MRCMSE and MMSE standard deviation values for trivariate data containing white and $1/f$ noise, each with 1000 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to $0.15\times$ (standard deviation of the normalized time series). The standard deviation values have been obtained with 50 independent realizations.

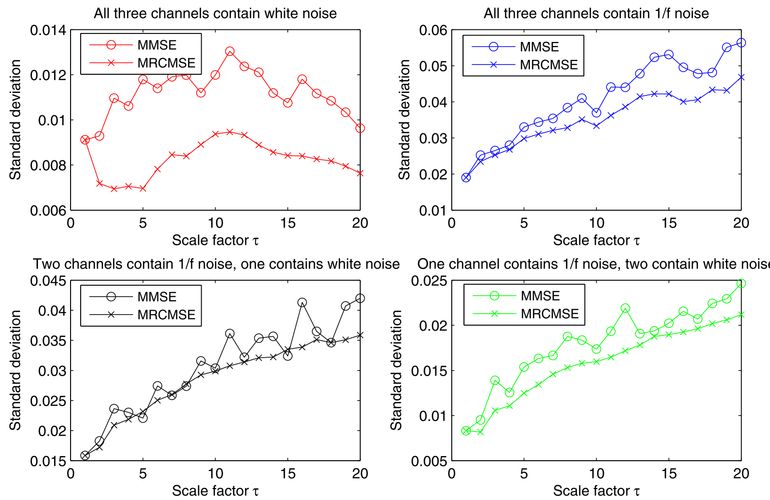

Fig. 5. MRCMSE and MMSE standard deviation values for trivariate data containing white and $1/f$ noise, each with 10 000 samples. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to $0.15\times$ (standard deviation of the normalized time series). The standard deviation values have been obtained with 50 independent realizations.

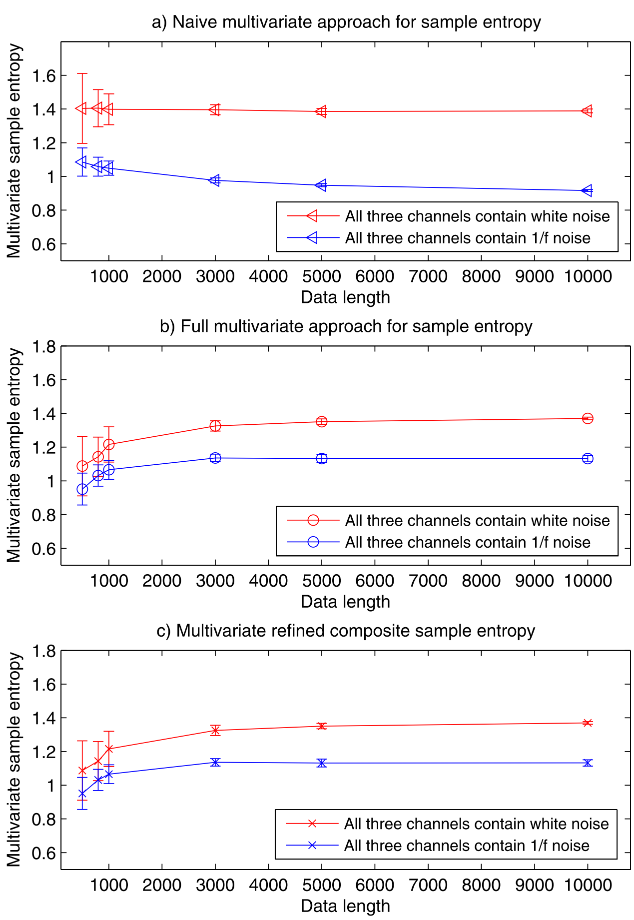

Fig. 6. (a) Naive multivariate approach for sample entropy; (b) full multivariate approach for sample entropy; (c) multivariate refined composite sample entropy, for trivariate data containing white and $1/f$ noise. Results are computed from data with different lengths. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to $0.15\times$ (standard deviation of the normalized time series). The average values have been obtained with 50 independent realizations. The error bars correspond to the standard deviation values.

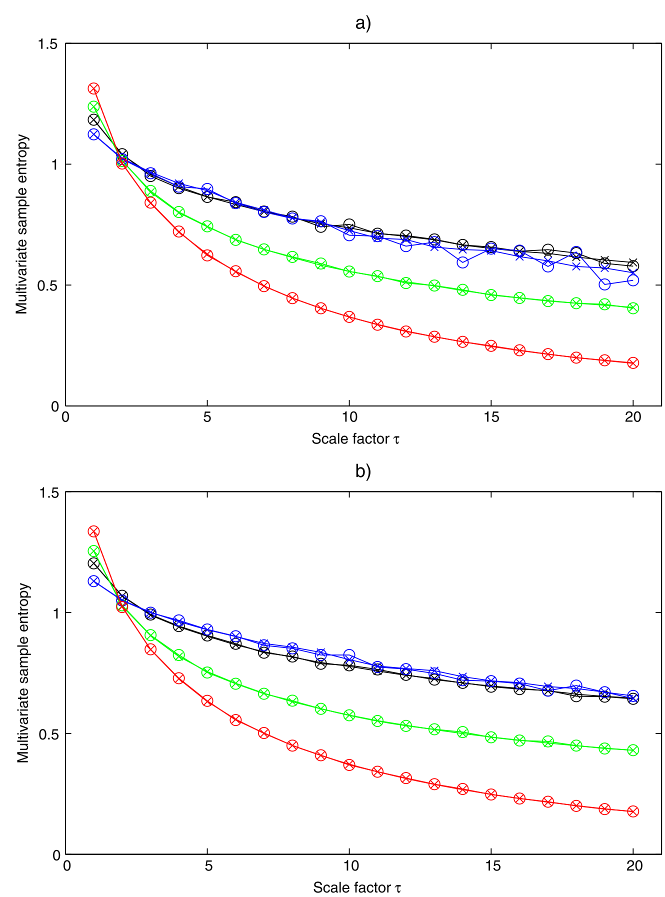

Fig. 7. MRCMSE and MMSE average values for trivariate data containing white and $1/f$ noise, each with a) 2000 samples; b) 3000 samples. The symbols are the same as for Figs. 1 and 3. For each channel the embedding dimension m_k was chosen equal to 2 and the threshold r was fixed to $0.15\times$ (standard deviation of the normalized time series). The average values have been obtained with 50 independent realizations.

deviations obtained with short time series are higher than those obtained with longer time series (comparison of Figs. 2a and b).

For the univariate case, several papers mentioned arguments regarding the selection of the parameter r value. Thus, Nikulin and Brismar indicated that the coarse-graining procedure is like a smoothing and decimation of the original sequences [16]. If r is constant with scales, changes in MSE on each scale depend on both the regularity and variation of the coarse-grained time series [16]. By opposition, Costa et al. reported that the degree of irregularity of a complex signal is a property measured by entropy that cannot be entirely captured by the standard deviation or correlation measures, individually or in combination [17]: following the normalization, modifications in the variance generated by the coarse-graining procedure are due to the temporal structure of the original signal. Therefore, it should be accounted for by the entropy measure [3]. For the multivariate case, and as suggested for MMSE [8,9], MRCMSE uses the multivariate generalization of the threshold parameter r: we take r as a percentage of the total variation of the covariance matrix. To have the same total variation for all the multivariate signals, we normalized each channel to unit variance (taking r as a percentage of the total variation of the covariance matrix is thus similar to take r at the same value for each channel). Therefore, the differences in the variance among the multivariate signals do not play a role in the computation of multivariate sample entropy [8,9].

Finally, we pointed out above the problems of accuracy and validity of MSE at large scales. Let us note that some authors also reported that the sample time of the time series, the correlation time, and the period of possible nonlinear oscillations may also play a role in the signature of MSE [18].

3. Conclusion

MSE has been proposed in the 2000s to quantify the complexity of time series over multiple scales. However, the coarse-graining procedure used in the MSE algorithm shortens the length of the data processed; the larger the scale factor, the shorter the data processed. This may lead to inaccurate or undefined sample entropy values. CMSE and RCMSE have been proposed to overcome these drawbacks. Unfortunately, these two algorithms are dedicated to univariate data only. MMSE has been proposed recently to take into account multivariate data. However, MMSE presents the same disadvantages as MSE. We therefore proposed an extension of RCMSE to multivariate data and we show that it leads to lower standard deviation values than MMSE. In this sense, MRCMSE outperforms MMSE.

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