Aufgabe 1

$$t_1 = \sigma(\sigma(x_1, \quad \alpha), \sigma(\gamma(x_3), x_3))$$
$$t_2 = \sigma(\sigma(\gamma(x_2), \alpha), \sigma(x_2, \quad x_3))$$

Aufgabe 3

$$\left\{ \begin{pmatrix} \sigma(\sigma(x_1, \alpha), & \sigma(\gamma(x_3), x_3)) \\ \sigma(\sigma(\gamma(x_2), \alpha), & \sigma(x_2, x_3)) \end{pmatrix} \right\}$$

Aufgabe 3

$$\left\{ \begin{pmatrix} \sigma(\sigma(x_1, \alpha), & \sigma(\gamma(x_3), x_3)) \\ \sigma(\sigma(\gamma(x_2), \alpha), & \sigma(x_2, x_3)) \end{pmatrix} \right\}$$

$$\overset{\text{Dekomp.}}{\Rightarrow} \left\{ \begin{pmatrix} \sigma(x_1, \alpha) \\ \sigma(\gamma(x_2), \alpha) \end{pmatrix}, \begin{pmatrix} \sigma(\gamma(x_3), x_3) \\ \sigma(x_2, x_3) \end{pmatrix} \right\}$$

Aufgabe 3

$$\left\{ \begin{pmatrix} \sigma(\sigma(x_1, \alpha), & \sigma(\gamma(x_3), x_3)) \\ \sigma(\sigma(\gamma(x_2), \alpha), & \sigma(x_2, x_3)) \end{pmatrix} \right\}$$

$$\overset{\text{Dekomp.}}{\Rightarrow} \left\{ \begin{pmatrix} \sigma(x_1, \alpha) \\ \sigma(\gamma(x_2), \alpha) \end{pmatrix}, \begin{pmatrix} \sigma(\gamma(x_3), x_3) \\ \sigma(x_2, x_3) \end{pmatrix} \right\}$$

$$\overset{\text{Dekomp.}}{\Rightarrow} \left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ x_3 \end{pmatrix} \right\}$$

Aufgabe 3

$$\left\{ \begin{pmatrix} \sigma(\sigma(x_1, \alpha), & \sigma(\gamma(x_3), x_3)) \\ \sigma(\sigma(\gamma(x_2), \alpha), & \sigma(x_2, x_3)) \end{pmatrix} \right\}$$

$$\overset{\text{Dekomp.}}{\Rightarrow} \left\{ \begin{pmatrix} \sigma(x_1, \alpha) \\ \sigma(\gamma(x_2), \alpha) \end{pmatrix}, \begin{pmatrix} \sigma(\gamma(x_3), x_3) \\ \sigma(x_2, x_3) \end{pmatrix} \right\}$$

$$\overset{\text{Dekomp.}}{\Rightarrow} \left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ x_3 \end{pmatrix} \right\}$$

$$\overset{\text{Elim.}}{\Rightarrow} \left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix} \right\}$$

Aufgabe 3

$$\left\{ \begin{pmatrix} \sigma(\sigma(x_1, \alpha), & \sigma(\gamma(x_3), x_3)) \\ \sigma(\sigma(\gamma(x_2), \alpha), & \sigma(x_2, x_3)) \end{pmatrix} \right\}$$

Dekomp. $\Rightarrow$

$$\left\{ \begin{pmatrix} \sigma(x_1, \alpha) \\ \sigma(\gamma(x_2), \alpha) \end{pmatrix}, \begin{pmatrix} \sigma(\gamma(x_3), x_3) \\ \sigma(x_2, x_3) \end{pmatrix} \right\}$$

Dekomp. $\Rightarrow$

$$\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ x_3 \end{pmatrix} \right\}$$

Elim. $\Rightarrow$

$$\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix} \right\}$$

Dekomp. $\Rightarrow$

$$\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix} \right\}$$

Aufgabe 3

$$\left\{ \begin{pmatrix} \sigma(\sigma(x_1, \alpha), & \sigma(\gamma(x_3), x_3)) \\ \sigma(\sigma(\gamma(x_2), \alpha), & \sigma(x_2, x_3)) \end{pmatrix} \right\}$$

Dekomp. $\Rightarrow$ $\left\{ \begin{pmatrix} \sigma(x_1, \alpha) \\ \sigma(\gamma(x_2), \alpha) \end{pmatrix}, \begin{pmatrix} \sigma(\gamma(x_3), x_3) \\ \sigma(x_2, x_3) \end{pmatrix} \right\}$

Dekomp. $\Rightarrow$ $\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ x_3 \end{pmatrix} \right\}$

Elim. $\Rightarrow$ $\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix} \right\}$

Dekomp. $\Rightarrow$ $\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} \gamma(x_3) \\ x_2 \end{pmatrix} \right\}$

Vert. $\Rightarrow$ $\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} x_2 \\ \gamma(x_3) \end{pmatrix} \right\}$

Aufgabe 3 (cont.)

$$\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} x_2 \\ \gamma(x_3) \end{pmatrix} \right\}$$

Aufgabe 3 (cont.)

$$\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} x_2 \\ \gamma(x_3) \end{pmatrix} \right\}$$

$$\overset{\text{Subst.}}{\Rightarrow} \quad \left\{ \begin{pmatrix} x_1 \\ \gamma(\gamma(x_3)) \end{pmatrix}, \begin{pmatrix} x_2 \\ \gamma(x_3) \end{pmatrix} \right\}$$

Aufgabe 3 (cont.)

$$\left\{ \begin{pmatrix} x_1 \\ \gamma(x_2) \end{pmatrix}, \begin{pmatrix} x_2 \\ \gamma(x_3) \end{pmatrix} \right\}$$

$$\overset{\text{Subst.}}{\Rightarrow} \quad \left\{ \begin{pmatrix} x_1 \\ \gamma(\gamma(x_3)) \end{pmatrix}, \begin{pmatrix} x_2 \\ \gamma(x_3) \end{pmatrix} \right\}$$

$$x_1 \mapsto \gamma(\gamma(x_3)) \qquad x_2 \mapsto \gamma(x_3) \qquad x_3 \mapsto x_3$$