

Aufgabe 02a

$$(\lambda f x. \underbrace{fx}_{GV=\{f,x\}})(\underbrace{\lambda zy.xy}_{FV=\{x\}})x$$

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$$\begin{aligned} & (\lambda f x. \underbrace{fx}_{GV=\{f,x\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\alpha} & (\lambda f x'. \underbrace{fx'}_{GV=\{f,x'\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \end{aligned}$$

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$$\begin{aligned} & (\lambda f x. \underbrace{fx}_{GV=\{f,x\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\alpha} & (\lambda f x'. \underbrace{fx'}_{GV=\{f,x'\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\beta} & (\lambda x'. \underbrace{(\lambda zy. xy)x'}_{GV=\{x'\}}) \underbrace{x}_{FV=\{x\}} \end{aligned}$$

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$$\begin{aligned} & (\lambda f x. \underbrace{fx}_{GV=\{f,x\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\alpha} & (\lambda f x'. \underbrace{fx'}_{GV=\{f,x'\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\beta} & (\lambda x'. \underbrace{(\lambda zy. xy) x'}_{GV=\{x'\}}) \underbrace{x}_{FV=\{x\}} \\ \Rightarrow_{\beta} & (\lambda z x. \underbrace{xy}_{GV=\{y\}}) \underbrace{x}_{FV=\{x\}} \end{aligned}$$

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$$\begin{aligned} & (\lambda f x. \underbrace{fx}_{GV=\{f,x\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\alpha} & (\lambda f x'. \underbrace{fx'}_{GV=\{f,x'\}}) (\underbrace{\lambda zy. xy}_{FV=\{x\}}) x \\ \Rightarrow_{\beta} & (\lambda x'. \underbrace{(\lambda zy. xy)x'}_{GV=\{x'\}}) \underbrace{x}_{FV=\{x\}} \\ \Rightarrow_{\beta} & (\lambda zx. \underbrace{xy}_{GV=\{y\}}) \underbrace{x}_{FV=\{x\}} \\ \Rightarrow_{\beta} & (\lambda y. xy) \end{aligned}$$

Aufgabe 2b

$g :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$g\ a\ 0 = a$

$g\ a\ b$

$\mid b == 1 = g\ (a + 1)\ (b - 1)$

$\mid \text{otherwise} = g\ (a + 2)\ (b - 2)$

$\langle G \rangle = ((\lambda gxy. \langle ite \rangle (\langle iszero \rangle y)$

$(\langle ite \rangle (\langle iszero \rangle (\langle pred \rangle y)))$

$(g(\langle succ \rangle x)(\langle pred \rangle y))$

$(g(\langle succ \rangle (\langle succ \rangle x))(\langle pred \rangle (\langle pred \rangle y)))$

$))$