$(\lambda x.xy)(\lambda y.y)$

$$\begin{aligned} (\lambda x.x \textcolor{red}{y})(\lambda y.y) \\ FV &= \{y\} \end{aligned}$$

$$(\lambda \mathbf{x}.xy)(\lambda \mathbf{y}.y)$$

$$FV = \{y\}$$

$$GV = \{x, y\}$$

 $(\lambda x.(\lambda y.z(\lambda z.z(\lambda x.y))))$

$$(\lambda x.(\lambda y. \textcolor{red}{z}(\lambda z. z(\lambda x. y))))$$

$$FV = \{z\}$$

$$(\lambda x.(\lambda y.z(\lambda z.z(\lambda x.y))))$$

$$FV = \{z\}$$

$$GV = \{x, y, z\}$$

 $(\lambda x.(\lambda y.xz(yz)))(\lambda x.y(\lambda y.y))$

$$\begin{split} (\lambda x.(\lambda y.x \pmb{z}(y \pmb{z})))(\lambda x. \pmb{y}(\lambda y.y)) \\ FV &= \{y,z\} \end{split}$$

$$\begin{split} (\lambda \pmb{x}.(\lambda \pmb{y}.xz(yz)))(\lambda \pmb{x}.y(\lambda \pmb{y}.y)) \\ FV &= \{y,z\} \\ GV &= \{x,y\} \end{split}$$

$$(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}}$$

$$(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}}$$

$$\Rightarrow_{\alpha}(\lambda x.(\lambda y'.xz(y'z)))(\lambda x.y(\lambda y.y))$$

$$\begin{split} &(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}} \\ \Rightarrow_{\alpha} &(\lambda x.(\lambda y'.xz(y'z)))(\lambda x.y(\lambda y.y)) \\ \Rightarrow_{\beta} &(\lambda y'.(\lambda x.\underbrace{y(\lambda y.y)}_{GV=\{y\}})\underbrace{z}_{FV=\{z\}}) \end{split}$$

$$\begin{split} &(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}} \\ \Rightarrow_{\alpha} &(\lambda x.(\lambda y'.xz(y'z)))(\lambda x.y(\lambda y.y)) \\ \Rightarrow_{\beta} &(\lambda y'.(\lambda x.\underbrace{y(\lambda y.y)}_{GV=\{y\}})\underbrace{z}_{FV=\{z\}} \\ \Rightarrow_{\beta} &(\lambda y'.y(\lambda y.y)(y'z)) \end{split}$$

Aufgabe 4b (restliche Lösungen)

$$(\lambda x.(\lambda y.(\lambda z.z)))x(+y1) \Rightarrow^* (\lambda z.z)$$

$$(\lambda x.(\lambda y.x(\lambda z.yz)))(((\lambda x.(\lambda y.y))8)(\lambda x.(\lambda y.y)x)) \Rightarrow^* (\lambda yz.yz)$$

$$(\lambda h.(\lambda x.h(xx))(\lambda x.h(xx)))((\lambda x.x)(+15))$$

$$\Rightarrow^* (\lambda x.(+15)(xx))(\lambda x.(+15)(xx))$$

$$\Rightarrow^* (\lambda x.(+15)((\lambda x.(+15)(xx))(\lambda x.(+15)(xx))))$$

$$\Rightarrow^* ... \text{ (endlose Rekursion)}$$

$$(\lambda f.(\lambda a.(\lambda b.fab)))(\lambda x.(\lambda y.x)) \Rightarrow^* (\lambda a.(\lambda b.a))$$