$(\lambda x.xy)(\lambda y.y)$ 

$$\begin{aligned} (\lambda x.x \textcolor{red}{y})(\lambda y.y) \\ FV &= \{y\} \end{aligned}$$

$$(\lambda \mathbf{x}.xy)(\lambda \mathbf{y}.y)$$
 
$$FV = \{y\}$$
 
$$GV = \{x, y\}$$

 $(\lambda x.(\lambda y.z(\lambda z.z(\lambda x.y))))$ 

$$(\lambda x.(\lambda y. \textcolor{red}{z}(\lambda z. z(\lambda x. y))))$$
 
$$FV = \{z\}$$

$$\begin{split} (\lambda \pmb{x}.(\lambda \pmb{y}.z(\lambda \pmb{z}.z(\lambda \pmb{x}.y)))) \\ FV &= \{z\} \\ GV &= \{x,y,z\} \end{split}$$

 $(\lambda x.(\lambda y.xz(yz)))(\lambda x.y(\lambda y.y))$ 

$$\begin{split} (\lambda x.(\lambda y.x \pmb{z}(y \pmb{z})))(\lambda x. \pmb{y}(\lambda y.y)) \\ FV &= \{y,z\} \end{split}$$

$$\begin{split} (\lambda \pmb{x}.(\lambda \pmb{y}.xz(yz)))(\lambda \pmb{x}.y(\lambda \pmb{y}.y)) \\ FV &= \{y,z\} \\ GV &= \{x,y\} \end{split}$$

$$(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}}$$

$$\begin{split} &(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}} \\ \Rightarrow_{\alpha} &(\lambda x.(\lambda y'.xz(y'z)))(\lambda x.y(\lambda y.y)) \end{split}$$

$$\begin{split} &(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}} \\ \Rightarrow_{\alpha} &(\lambda x.(\lambda y'.xz(y'z)))(\lambda x.y(\lambda y.y)) \\ \Rightarrow_{\beta} &(\lambda y'.(\lambda x.\underbrace{y(\lambda y.y)}_{GV=\{y\}})\underbrace{z}_{FV=\{z\}} (y'z)) \end{split}$$

$$(\lambda x.\underbrace{(\lambda y.xz(yz))}_{GV=\{y\}})\underbrace{(\lambda x.y(\lambda y.y))}_{FV=\{y\}}$$
 
$$\Rightarrow_{\alpha}(\lambda x.(\lambda y'.xz(y'z)))(\lambda x.y(\lambda y.y))$$
 
$$\Rightarrow_{\beta}(\lambda y'.(\lambda x.\underbrace{y(\lambda y.y)}_{GV=\{y\}})\underbrace{z}_{FV=\{z\}}$$
 
$$\Rightarrow_{\beta}(\lambda y'.y(\lambda y.y)(y'z))$$