Zu zeigen

Für alle Listen `xs :: [Int]` gilt:

```
sum (foo xs) = 2 * sum xs - length xs
```

Induktionsanfang (IA)

Sei `xs = []`, dann:

```
sum (foo [])
```

Induktionsanfang (IA)

Sei `xs = []`, dann:

$$
\begin{aligned}
& \texttt{sum (foo [])} \\
\overset{(2)}{=}\ & \texttt{sum []}
\end{aligned}
$$

Induktionsanfang (IA)

Sei `xs = []`, dann:

$$
\begin{aligned}
& \texttt{sum (foo [])} \\
\overset{(2)}{=} \; & \texttt{sum []} \\
\overset{(6)}{=} \; & 0
\end{aligned}
$$

Induktionsanfang (IA)

Sei `xs = []`, dann:

$$
\begin{aligned}
& \texttt{sum (foo [])} \\
\overset{(2)}{=}\; & \texttt{sum []} \\
\overset{(6)}{=}\; & 0 \\
=\; & \texttt{2 * 0 - 0}
\end{aligned}
$$

Induktionsanfang (IA)

Sei `xs = []`, dann:

$$
\begin{aligned}
& \texttt{sum (foo [])} \\
\overset{(2)}{=}\ & \texttt{sum []} \\
\overset{(6)}{=}\ & \texttt{0} \\
=\ & \texttt{2 * 0 - 0} \\
\overset{(6)}{=}\ & \texttt{2 * sum [] - 0}
\end{aligned}
$$

Induktionsanfang (IA)

Sei `xs = []`, dann:

$$
\begin{aligned}
& \texttt{sum (foo [])} \\
\overset{(2)}{=}\; & \texttt{sum []} \\
\overset{(6)}{=}\; & \texttt{0} \\
=\; & \texttt{2 * 0 - 0} \\
\overset{(6)}{=}\; & \texttt{2 * sum [] - 0} \\
\overset{(2)}{=}\; & \texttt{2 * sum [] - length []}
\end{aligned}
$$

Induktionsbehauptung (IB)

Es gibt eine beliebige, aber feste Liste `xs'` für die gilt:

```
sum (foo xs') = 2 * sum xs' - length xs'
```

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

```
sum (foo (x:xs'))
```

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

```
   sum (foo (x:xs'))
(3)
 =  sum (x : x : (-1) : foo (xs'))
```

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
\overset{(3)}{=}\ & \texttt{sum (x : x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + sum (x : (-1) : foo (xs'))}
\end{aligned}
$$

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
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\overset{(7)}{=}\; & \texttt{x + sum (x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\; & \texttt{x + x + sum ((-1) : foo (xs'))}
\end{aligned}
$$

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

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\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
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\stackrel{(7)}{=}\ & \texttt{x + sum (x : (-1) : foo (xs'))} \\
\stackrel{(7)}{=}\ & \texttt{x + x + sum ((-1) : foo (xs'))} \\
\stackrel{(7)}{=}\ & \texttt{x + x + (-1) + sum (foo (xs'))}
\end{aligned}
$$

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

```
   sum (foo (x:xs'))
(3)
 =  sum (x : x : (-1) : foo (xs'))
(7)
 =  x + sum (x : (-1) : foo (xs'))
(7)
 =  x + x + sum ((-1) : foo (xs'))
(7)
 =  x + x + (-1) + sum (foo (xs'))
IV
 =  x + x + (-1) + 2 * sum xs' - length xs'
```

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
\overset{(3)}{=}\ & \texttt{sum (x : x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + sum (x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + sum ((-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + (-1) + sum (foo (xs'))} \\
\overset{\mathrm{IV}}{=}\ & \texttt{x + x + (-1) + 2 * sum xs' - length xs'} \\
=\ & \texttt{2 * x + 2 * sum xs' - 1 - length xs'}
\end{aligned}
$$

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
\overset{(3)}{=}\ & \texttt{sum (x : x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + sum (x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + sum ((-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + (-1) + sum (foo (xs'))} \\
\overset{\mathrm{IV}}{=}\ & \texttt{x + x + (-1) + 2 * sum xs' - length xs'} \\
=\ & \texttt{2 * x + 2 * sum xs' - 1 - length xs'} \\
=\ & \texttt{2 * (x + sum xs') - (1 + length xs')}
\end{aligned}
$$

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
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\overset{(7)}{=}\ & \texttt{x + sum (x : (-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + sum ((-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + (-1) + sum (foo (xs'))} \\
\overset{\text{IV}}{=}\ & \texttt{x + x + (-1) + 2 * sum xs' - length xs'} \\
=\ & \texttt{2 * x + 2 * sum xs' - 1 - length xs'} \\
=\ & \texttt{2 * (x + sum xs') - (1 + length xs')} \\
\overset{(7)}{=}\ & \texttt{2 * sum (x:xs') - (1 + length xs')}
\end{aligned}
$$

Induktionsschritt (IS)

Für alle `x::Int` gilt: Sei `xs = (x:xs')`:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs'))} \\
\overset{(3)}{=}\ & \texttt{sum (x : x : (-1) : foo (xs'))} \\
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\overset{(7)}{=}\ & \texttt{x + x + sum ((-1) : foo (xs'))} \\
\overset{(7)}{=}\ & \texttt{x + x + (-1) + sum (foo (xs'))} \\
\overset{\mathrm{IV}}{=}\ & \texttt{x + x + (-1) + 2 * sum xs' - length xs'} \\
=\ & \texttt{2 * x + 2 * sum xs' - 1 - length xs'} \\
=\ & \texttt{2 * (x + sum xs') - (1 + length xs')} \\
\overset{(7)}{=}\ & \texttt{2 * sum (x:xs') - (1 + length xs')} \\
\overset{(11)}{=}\ & \texttt{2 * sum (x:xs') - length (x:xs')} \quad \square
\end{aligned}
$$