

AM 1 - Resolução Ficha 5

Limites e Continuidade de funções

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Questão 1

$$1 - f) \quad \lim_{x \rightarrow \infty} e^{\cos(x)} / x$$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} e^{\cos(x)} * \lim_{x \rightarrow \infty} 1/x = \lim_{x \rightarrow \infty} e^{\cos(x)} * 0; \quad e^{\cos(x)} \text{ é uma função limitada } \implies \\ &\implies \lim_{x \rightarrow \infty} e^{\cos(x)} / x = 0 \end{aligned}$$

$$1 - g) \quad \lim_{x \rightarrow 0^+} e^{1/x}$$

$$= e^{\lim_{x \rightarrow 0^+} 1/x} = e^{+\infty} = +\infty$$

$$1 - h) \quad \lim_{x \rightarrow 0^-} e^{1/x}$$

$$= e^{\lim_{x \rightarrow 0^-} 1/x} = e^{-\infty} = 0$$

Questão 2 Prove $\nexists f(x)$

$$2 - a) \quad \nexists a(x) = x/|x| : x = 0$$

$$\iff \lim_{x \rightarrow 0^+} a(x) = 1 \neq \lim_{x \rightarrow 0^-} a(x) = -1$$

$$2 - b) \quad \nexists b(x) = \frac{x^2-1}{|x-1|} : x = 1$$

$$\begin{aligned} \iff \lim_{x \rightarrow 1^+} b(x) &= \lim_{x \rightarrow 1^+} \frac{(x+1)(x-1)}{x-1} = 2 \neq \lim_{x \rightarrow 1^-} b(x) = \\ &= \lim_{x \rightarrow 1^-} \frac{(x+1)(x-1)}{1-x} = \lim_{x \rightarrow 1^-} \frac{-(x+1)}{1} = -2 \end{aligned}$$

$$2 - c) \quad \nexists c(x) = \arctan(e^{1/x}) : x = 0$$

$$\begin{aligned} \iff \lim_{x \rightarrow 0^+} c(x) &= \arctan(e^{\lim_{x \rightarrow 0^+} 1/x}) = \arctan(e^{\infty}) = \arctan(\infty) = \pi/2 \neq \\ &\neq \lim_{x \rightarrow 0^-} c(x) = \arctan(e^{\lim_{x \rightarrow 0^-} 1/x}) = \arctan(e^{-\infty}) = \arctan(0) = 0 \end{aligned}$$

2 - d) $\nexists d(x) = e^{\cos(x)} : x = +\infty$ **Incompleta**

$$\iff \lim_{x \rightarrow \infty} d(x); x_n = 2n\pi \quad \forall n \in \mathbb{N}$$

...

Questão 3

3 - a) $\lim_{x \rightarrow 0} \tan(x)/x$ **Incompleto**

$$= \lim_{x \rightarrow 0} \cos^{-1}(x) \frac{\sin(x)}{x} = \lim_{x \rightarrow 0} \dots$$

3 - b) $\lim_{x \rightarrow 0} (1 - e^{2x})/x$ **Incompleto**

= ...

3 - c) $\lim_{x \rightarrow 0} (1 - e^{2x})/\sin(3x)$ **Incompleto**

$$= \lim_{x \rightarrow 0} \frac{1 - e^{2x}}{2x} \frac{3x}{\sin(3x)} \frac{2x}{3x} = \dots$$

3 - d) $\lim_{x \rightarrow 0} \ln(x+1)/x$ **Incompleto**

$$x+1 = e^y \implies \lim_{x \rightarrow 0} \ln(x+1)/x = \lim_{x \rightarrow 0} \frac{y}{e^y - 1} = \dots$$

3 - e) $\lim_{x \rightarrow 0} (1 - \cos(3x))/x^2$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{1 - \cos^2(3x)}{x^2(1 + \cos(3x))} = \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{x^2(1 + \cos(3x))} = \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{(3x)^2} \frac{9x^2}{x^2(1 + \cos(3x))} = \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{3x} \right)^2 \lim_{x \rightarrow 0} \frac{9}{1 + \cos(3x)} = 9/2 \end{aligned}$$