SECTION 5

SEDIMENTATION

Problem 5.1

A slurry containing 5 kg of water per kg of solids is to be thickened to a sludge containing 1·5 kg of water per kg of solids in a continuous operation. Laboratory tests using five different concentrations of the slurry yielded the following results:

Concentration (kg water/kg solid):	5·0	4·2	3·7	3·1	2·5
Rate of sedimentation (mm/s):	0·17	0·10	0·08	0·06	0·042

Calculate the minimum area of a thickener to effect the separation of 0·6 kg of solids per second.

Solution

Basis: 1 kg of solids: 1·5 kg water is carried away in underflow, balance in overflow, $V = 1{\cdot}5$.

Concentration U	Water to overflow $(U - V)$	Sedimentation rate u_c (mm/s)	$(U - V)/u_c$ (s/mm)
5·0	3·5	0·17	20·56
4·2	2·7	0·10	27·0
3·7	2·2	0·08	27·5
3·1	1·6	0·06	26·67
2·5	1·0	0·042	23·81

Maximum value of $(U - V)/u_c = 27{\cdot}5$ s/mm or 27,500 s/m.
From equation 5.101:

$$A = [(U - V)/u_c](W/\rho)$$
$$= 27{,}500(0{\cdot}6/1000)$$
$$= \underline{\underline{16{\cdot}5\ \mathrm{m}^2}}$$

Problem 5.2

If a centrifuge is 0·9 m diameter and rotates at 20 Hz, at what speed should a laboratory centrifuge of 150 mm diameter run if it is to duplicate plant conditions?

Solution

If a particle of mass m is rotating at radius x with an angular velocity w, it is subjected to a centrifugal force $m \times w^2$ in a radial direction and a gravitational force mg in a vertical direction. The ratio of the centrifugal to gravitational forces, xw^2/g, is a measure of the separating power of the machine, and for duplicate conditions this must be the same in both machines

In this case:

$$x_1 = 0.45 \text{ m}$$

$$w_1 = 20 \times 2\pi = 40\pi \text{ rad/s}$$

$$x_2 = 0.075 \text{ m}$$

$$0.45(40\pi)^2/g = 0.075w_2^2/g$$

$$w_2 = \sqrt{[6(40\pi)^2]}$$

$$= 2.45 \times 40\pi = 98\pi \text{ rad/s}$$

and the speed of rotation $= 98\pi/2\pi$

$$= \underline{\underline{49 \text{ Hz}}}$$

Problem 5.3

What is the maximum safe speed of rotation of a phosphor-bronze centrifuge basket, 0.3 m diameter and 5 mm thick, when it contains a liquid of density 1000 kg/m^3 forming a layer 75 mm thick at the walls? Take the density of phosphor-bronze as 8900 kg/m^3 and the safe working stress as 55 MN/m^2.

Solution

The centrifugal pressure due to the liquid is given by (the nomenclature is defined in Problem 5.4):

$$P_c = 0.5\rho w^2(b^2 - x^2)$$

$$= 0.5 \times 1000 \times w^2(0.15^2 - 0.075^2)$$

$$= 8.438w^2 \text{ N/m}^2$$

The stress in the walls of the basket is given by:

$$f = (b/\delta)(P_c + \rho_m \delta b \omega^2)$$

$$= (0.15/0.005)(8.438w^2 + 8900 \times 0.005 \times 0.15w^2)$$

$$= 453w^2 \text{ N/m}^2$$

The maximum speed of rotation is therefore

$$w = \sqrt{(55 \times 10^6/453)}$$

$$= 348 \text{ rad/s} \quad \text{or} \quad \underline{\underline{55.5 \text{ Hz}}} \quad (3327 \text{ rev/min})$$

Problem 5.4

A centrifuge with a phosphor-bronze basket 375 mm diameter is to be run at 30 Hz with a 100 mm layer of solids of bulk density 2000 kg/m^3 at the walls. What should be the thickness of the walls of the basket if the perforations are so small that they have a negligible effect on strength?

Density of phosphor-bronze = 8900 kg/m^3.

Maximum safe stress for phosphor-bronze = 55 MN/m^2.

Solution

The pressure exerted by the solids on the wall of the basket is given by:

$$P_c = 0{\cdot}5\rho w^2(b^2 - x^2)$$

where b is the basket radius (0·1875 m), x is the radius of inner surface of solids (0·1875 − 0·10) = 0·0875 m, w is the angular velocity (30 × 2π) = 60π rad/s, and ρ is the density of solids (2000 kg/m^3).

$$\therefore \quad P_c = 0{\cdot}5 \times 2000(60\pi)^2(0{\cdot}1875^2 - 0{\cdot}0875^2)$$

$$= 3{\cdot}55 \times 10^7(0{\cdot}275)(0{\cdot}10)$$

$$= 9{\cdot}76 \times 10^5 \text{ N/m}^2$$

or $$P_c = 0{\cdot}98 \text{ MN/m}^2$$

The stress in the walls is given by:

$$f = (b/\delta)(P_c + \rho_m \delta b w^2)$$

f will be taken as the maximum safe stress of phosphor-bronze, 55 × 10^6 N/m^2:

$$\rho_m = 8900 \text{ kg/m}^3$$

$$\therefore \quad 55 \times 10^6 = (0{\cdot}1875/\delta)[9{\cdot}75 \times 10^5 + 8900\delta \times 0{\cdot}1875(60\pi)^2]$$

$$\therefore \quad \delta = 3{\cdot}409 \times 10^{-9}(9{\cdot}75 \times 10^5 + 5{\cdot}929 \times 10^7\delta)$$

$$= 3{\cdot}323 \times 10^{-3} + 0{\cdot}202\delta$$

$$\therefore \quad \delta = 4{\cdot}16 \times 10^{-3} \text{ m} \quad \text{or} \quad 4{\cdot}16 \text{ mm}$$

In practice some safety margin would be allowed and a wall thickness of 5 mm would be specified.

Problem 5.5

An aqueous suspension consisting of particles of specific gravity 2·5 m in the size range 1–10 μm is introduced into a centrifuge with a basket 450 mm diameter rotating at 80 Hz. If the suspension forms a layer 75 mm thick in the basket, approximately how long will it take to cause the smallest particle to settle out?

Solution

Where the motion of the fluid with respect to the particle is turbulent, the time, taken for a particle to settle from h_1 to distance h_2 from the surface in a radial direction, is given by:

$$t = \frac{2}{a'}[(x+h_2)^{0.5} - (x+h_1)^{0.5}]$$

where $a' = \sqrt{[3dw^2(\rho_s-\rho)/\rho]}$, d is the diameter of the smallest particle $= 1 \times 10^{-6}$ m, w is the angular velocity of the basket $= (80 \times 2\pi) = 502{\cdot}7$ rad/s, ρ_s is the density of the solid $= 2500$ kg/m^3, ρ is the density of the fluid $= 1000$ kg/m^3, and x is the radius of the inner surface of the liquid $= 0{\cdot}150$ m.

$$a' = \sqrt{[3 \times 1 \times 10^{-6} \times 502{\cdot}7^2(2500-1000)/1000]}$$

$$= 1{\cdot}066$$

and

$$t = (2/1{\cdot}066)[(0{\cdot}150+0{\cdot}075)^{0.5} - (0{\cdot}150+0)^{0.5}]$$

$$= 1{\cdot}876(0{\cdot}474 - 0{\cdot}387)$$

$$= 0{\cdot}163 \text{ s}$$

This is a very low value, equivalent to a velocity of $(0{\cdot}075/0{\cdot}163) = 0{\cdot}46$ m/s. Because of the very small diameter of the particle, it is more than likely that the conditions are streamline, even at this particle velocity.

For water, taking $\mu = 0{\cdot}001$ N s/m^2,

$$Re = 1 \times 10^{-6} \times 0{\cdot}46 \times 1000/0{\cdot}001 = 0{\cdot}46$$

and hence the following may be applied:

$$t = \{18\mu/[d^2w^2(\rho_s-\rho)]\}\ln[(x+h_2)/(x+h_1)]$$

$$= \{18 \times 0{\cdot}001/[10^{-12} \times 502{\cdot}7^2(2500-1000)]\}\ln[(0{\cdot}150+0{\cdot}075)/(0{\cdot}150+0)]$$

$$= 47{\cdot}5\ln(0{\cdot}225/0{\cdot}150)$$

$$= \underline{\underline{19{\cdot}3 \text{ s}}}$$

Problem 5.6

A centrifuge with a phosphor-bronze basket 375 mm diameter is to be run at 60 Hz with a 75 mm layer of liquid of specific gravity 1·2 in the basket. What thickness of walls is required in the basket?

Density of phosphor-bronze = 8900 kg/m^3

Maximum safe working stress for phosphor-bronze = 55 MN/m^2.

Solution

The stress in the walls is given by:

$$f = (b/\delta)(P_c + \rho_m \delta b w^2)$$

where b is the radius of the basket (0·1875 m), δ is the thickness of the basket (m), ρ_m is the density of the basket material (8900 kg/m^3), w is the angular velocity of the basket = $(60 \times 2\pi) = 377$ rad/s, P_c is the centrifugal pressure given by:

$$P_c = 0{\cdot}5\rho w^2(b^2 - x^2)$$

where ρ is the liquid density (1200 kg/m^3), and x is the radius of the inner surface of fluid (0·1125 m).

$$\therefore \quad P_c = 0{\cdot}5 \times 1200 \times 377^2(0{\cdot}1875^2 - 0{\cdot}1125^2)$$

$$= 8{\cdot}527 \times 10^7(0{\cdot}3)(0{\cdot}075)$$

$$= 1{\cdot}92 \times 10^6 \text{ N/m}^2$$

Taking f as the maximum safe stress for phosphor-bronze, 55 MN/m^2 = 55×10^6 N/m^2:

$$55 \times 10^6 = (0{\cdot}1875/\delta)(1{\cdot}92 \times 10^6 + 8900\delta \times 0{\cdot}1875 \times 377^2)$$

$$\therefore \quad 2{\cdot}93 \times 10^8\delta = 1{\cdot}92 \times 10^6 + 2{\cdot}37 \times 10^8\delta$$

$$\therefore \quad \delta = 0{\cdot}034 \text{ m} \quad \text{or} \quad 34 \text{ mm}$$

and the thickness to be specified allowing a reasonable margin would be

$$\underline{\underline{38{\cdot}1 \text{ mm}}} \quad (1{\cdot}5 \text{ in})$$

Problem 5.7

A centrifuge basket 600 mm long and 100 mm internal diameter has a discharge weir 25 mm diameter. What is the maximum volumetric flow of liquid through the centrifuge such that when the basket is rotated at 200 Hz all particles of diameter greater than 1 μm are retained on the centrifuge wall? The retarding force on a particle moving in a liquid can be taken as equal to $3\pi\mu du$, where u is the particle velocity relative to the liquid, μ is the liquid viscosity, and d is the particle diameter.

Sp. gr. of liquid = 1·0.
Sp. gr. of solid = 2·0.
Viscosity of liquid (μ) = 1·0 mN s/m^2.
The inertia of the particle can be neglected.

Solution

With a basket radius of b m, the radius of the inner surface of liquid x m and h m the distance radially from the surface of the liquid, the equation of motion of a spherical particle of diameter d m under streamline conditions in the radial direction is:

$$(\pi d^3/6)(\rho_s - \rho)(x + h)w^2 - 3\pi\mu du - (\pi d^3/6)\rho_s \, du/dt = 0$$

Replacing u by dh/dt and neglecting the acceleration term:

$$(dh/dt) = d^2(\rho_s - \rho)w^2(x + h)/18\mu$$

The time any element of material remains in the basket is V'/Q, where Q is the volumetric rate of feed to the centrifuge and V' is the volume of liquid retained in the basket at any time. If the flow rate is so adjusted that a particle of diameter d is just retained when it has to travel through the maximum distance $h = (b - x)$ before reaching the wall,

$$h = d^2(\rho_s - \rho)bw^2V'/(18\mu Q)$$

or

$$Q = d^2(\rho_s - \rho)bw^2V'/(18\mu h)$$

In this case,

$$V' = (\pi/4)(0.1^2 - 0.025^2) \times 0.6 = 0.0044\ \mathrm{m}^3$$

$$h = (0.10 - 0.025/2)$$

$$\therefore Q = (1 \times 10^{-6})^2(2000 - 1000) \times 0.1 \times (200 \times 2\pi)^2 \times 0.0044/(18 \times 0.001 \times 0.0375)$$

$$= \underline{\underline{1.03 \times 10^{-3}\ \mathrm{m}^3/\mathrm{s}}} \quad \text{(approximately 1 cm}^3\text{/s)}$$

Problem 5.8

Calculate the minimum area and diameter of a thickener with a circular basin to treat 0.1 m^3/s of a slurry of solids concentration of 150 kg/m^3. The results of batch settling tests are as follows:

Solids concentration (kg/m^3)	Settling velocity (μm/s)
100	148
200	91
300	55.33
400	33.25
500	21.40
600	14.50
700	10.29
800	7.38
900	5.56
1000	4.20
1100	3.27

A value of 1290 kg/m^3 for underflow concentration was selected from a retention time test. Estimate the underflow volumetric flow rate assuming total separation of all solids and that a clear overflow is obtained.

Solution

The settling velocity of the solids, u kg/m^2 s, is calculated as

$$u = u_s c$$

where u_s is the settling velocity (m/s) and c the concentration of solids (kg/m^3) and the data are plotted in Fig. 5a. From the point $u = 0$, $c = 1290$ kg/m^2, a line is drawn which is just below the curve. This intercepts the axis at $u = 0.0154$ kg/m^2 s.

The area of the thickener is then,

$$A = 0.1 \times 150/0.0154 = \underline{\underline{974\ \mathrm{m}^2}}$$

and the diameter, $d = (4 \times 974/\pi)^{0.5}$

$= 35.2\,\text{m}$

The volumetric flow rate of underflow is obtained from a mass balance as

$= 0.1 \times 150/1290$

$= \underline{\underline{0.0116\,\text{m}^3/\text{s}}}$

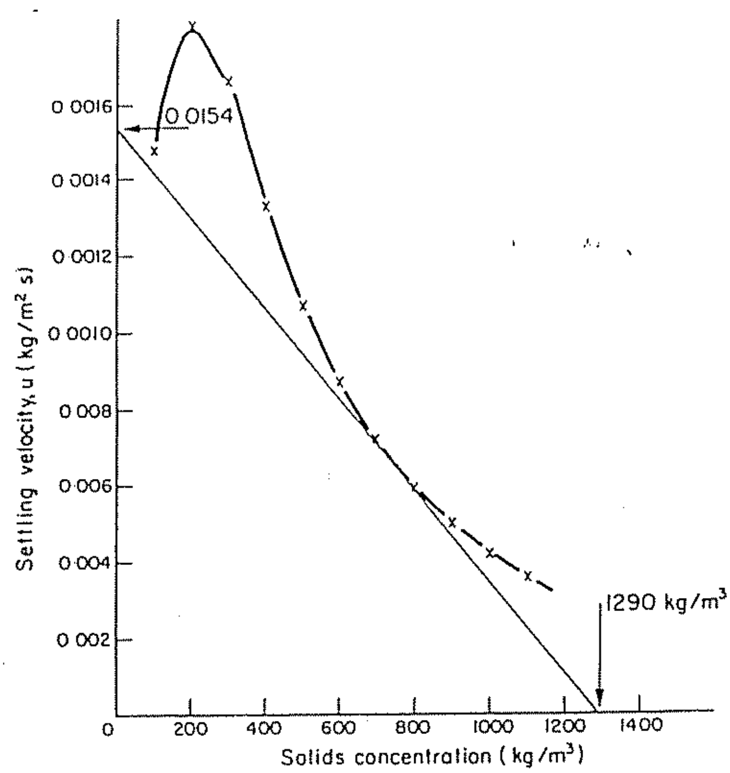

FIG. 5a