SECTION 9

FILTRATION

Problem 9.1

A slurry containing 0·2 kg solid (specific gravity 3·0) per kilogram of water is fed to a rotary drum filter 0·6 m long and 0·6 m diameter. The drum rotates at one revolution in 350 s and 20% of the filtering surface is in contact with the slurry at any instant. If filtrate is produced at the rate of 0·125 kg/s and the cake has a voidage of 0·5, what thickness of cake is produced when filtering at an absolute pressure of 35 kN/m^2?

The rotary filter breaks down and the operation has to be carried out temporarily in a plate and frame press with frames 0·3 m square. The press takes 100 s to dismantle and 100 s to reassemble and, in addition, 100 s is required to remove the cake from each frame. If filtration is to be carried out at the same overall rate as before, with an operating pressure of 275 kN/m^2, what is the minimum number of frames that need to be used and what is the thickness of each? Assume the cakes to be incompressible and neglect the resistance of the filter media.

Solution

Drum filter

Area of filtering surface $= 0{\cdot}6 \times 0{\cdot}6\pi = 0{\cdot}36\pi\ \text{m}^2$ $= 2\pi r l$

Rate of filtration $= 0{\cdot}125$ kg/s

$= 0{\cdot}125/1000 = 1{\cdot}25 \times 10^{-4}\ \text{m}^3/\text{s}$ of filtrate

Volumetric rate of deposition of solids (bulk)

20%

$= 1{\cdot}25 \times 10^{-4} \times 0{\cdot}2/(0{\cdot}5 \times 3{\cdot}0) = 1{\cdot}67 \times 10^{-5}\ \text{m}^3/\text{s}$

One revolution takes 350 s; therefore a given piece of filtering surface is immersed for $(350 \times 0{\cdot}2) = 70$ s.

Bulk volume of cake deposited per revolution

$= 1{\cdot}67 \times 10^{-5} \times 350 = 5{\cdot}85 \times 10^{-3}\ \text{m}^3$

Thickness of cake produced

$= 5{\cdot}85 \times 10^{-3}/0{\cdot}36\pi = 5{\cdot}17 \times 10^{-3}$ m

or 5·2 mm

Now
$$\frac{dV}{dt} = \frac{(-\Delta P)A^2}{r\mu l} = \frac{(-\Delta P)A^2}{r\mu V_v} \quad \text{(from equation 9.2)}$$

At constant pressure, $V^2 = (2/r\mu v)(-\Delta P)A^2 t = K(-\Delta P)A^2 t$ (say) (from equation 9.12)

Then, expressing pressures, areas, times, and volumes in kN/m^2, m^2, s, and m^3 respectively, for one revolution of the drum,

$$(1{\cdot}25 \times 10^{-4} \times 350)^2 = K(101{\cdot}3 - 35)(0{\cdot}36\pi)^2 \times 70$$

(since each element of area is immersed for one-fifth of cycle),

i.e. $$K = 3{\cdot}22 \times 10^{-7}$$

Filter press

Use a filter press with n frames of thickness d m.

Total time for one complete cycle of press $= t_f + 100n + 200$ s, where t_f is the time during which filtration is occurring.

Overall rate of filtration $= V_f/(t_f + 100n + 200) = 1{\cdot}25 \times 10^{-4}\ m^3/s$, where V_f is the total volume of filtrate per cycle.

Now V_f = volume of frames/volume of cake deposited by unit volume of filtrate (v)

$$= 0{\cdot}3^2 nd/[0{\cdot}2/(0{\cdot}5 \times 3{\cdot}0)] = 0{\cdot}675nd$$

But $$V_f = 3{\cdot}22 \times 10^{-7}(275 - 101{\cdot}3)(2n \times 0{\cdot}3 \times 0{\cdot}3)^2 \times t_f = (0{\cdot}675nd)^2$$

i.e. $$t_f = 2{\cdot}516 \times 10^5 d^2$$

Thus $$1{\cdot}25 \times 10^{-4} = 0{\cdot}675nd/(2{\cdot}516 \times 10^5 d^2 + 100n + 200)$$

i.e. $$31{\cdot}45d^2 + 0{\cdot}0125n + 0{\cdot}0250 = 0{\cdot}675nd$$

giving $$n = \frac{0{\cdot}0250 + 31{\cdot}45d^2}{0{\cdot}675d - 0{\cdot}0125}$$

n is a minimum when $dn/dd = 0$,

i.e. when $$(0{\cdot}675d - 0{\cdot}0125) \times 62{\cdot}9d - (0{\cdot}0250 + 31{\cdot}45d^2) \times 0{\cdot}675 = 0$$

$$d^2 - 0{\cdot}0370d - 0{\cdot}000\,796 = 0$$

$$\therefore\ d = 0{\cdot}0185 \pm \sqrt{(0{\cdot}000\,343 + 0{\cdot}000\,796)}$$

$$= 0{\cdot}0522\ \text{m} \quad \text{or} \quad \underline{\underline{52{\cdot}2\ \text{mm}}}$$

Hence $$n = (0{\cdot}0250 + 31{\cdot}45 \times 0{\cdot}0522^2)/(0{\cdot}675 \times 0{\cdot}0522 - 0{\cdot}0125)$$

$$= 4{\cdot}87$$

Thus a minimum of 5 frames must be used.

The size of frames which will give exactly the required rate of filtration when five are used are given by, $$0{\cdot}0250 + 31{\cdot}45d^2 = 3{\cdot}375d - 0{\cdot}0625$$

i.e. $$d^2 - 0{\cdot}107d + 0{\cdot}002\,78 = 0$$

$$d = 0{\cdot}0535 \pm \sqrt{(0{\cdot}002\,85 - 0{\cdot}002\,78)}$$

$$= 0{\cdot}044\ \text{m} \quad \text{or} \quad 0{\cdot}063\ \text{m}$$

i.e. 5 frames of thickness 44 mm or 63 mm will give exactly the required filtration rate; intermediate sizes give higher rates.

Thus any frame thickness between 44 and 63 mm will be satisfactory. In practice 2 in (50·4 mm) frames would be used

Problem 9.2

A slurry containing 100 kg of whiting, of specific gravity 3·0, per m^3 of water is filtered in a plate and frame press, which takes 900 s to dismantle, clean, and reassemble. If the filter cake is incompressible and has a voidage of 0·4, what is the optimum thickness of cake for a filtration pressure of 1000 kN/m^2? If the cake is washed at 500 kN/m^2 and the total volume of wash water employed is one-quarter of that of the filtrate, how is the optimum thickness of the cake affected? Neglect the resistance of the filter medium and take the viscosity of water as 1 mN s/m^2. In an experiment, a pressure of 165 kN/m^2 produced a flow of water of 0·02 cm^3/s through a centimetre cube of filter cake.

Solution

The basic filtration equation may be written:

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\mu l} \qquad \text{(equation 9.2)}$$

r is defined as the specific resistance of the cake, and using the data given may be calculated for the flow through a cube of cake.

$\Delta P = 165 - 101{\cdot}3 = 63{\cdot}7\ kN/m^2 = 63{\cdot}7 \times 10^3\ N/m^2$.

$A = 1\ cm^2$.

$l = 1\ cm$.

$\mu = 1 \times 10^{-3}\ N\,s/m^2$.

$dV/dt = 0{\cdot}02\ cm^3/s$.

Hence
$$r = \frac{63{\cdot}7 \times 10^3 \times 1}{1 \times 10^{-3} \times 1 \times 0{\cdot}02} = 3185 \times 10^6/cm^2$$
$$= 3185 \times 10^{10}/m^2$$

The slurry contains 100 kg whiting/m^3 of water.

Volume of 100 kg whiting $= 100/3000 = 0{\cdot}0333\ m^3$.

Volume of cake $= 0{\cdot}0333/(1 - 0{\cdot}4) = 0{\cdot}0556\ m^3$.

Volume of liquid in cake $= 0{\cdot}0333 \times 0{\cdot}4/0{\cdot}6 = 0{\cdot}0222\ m^3$

Volume of filtrate $= 1 - 0{\cdot}0222 = 0{\cdot}978\ m^3$.

$$v = \text{volume of cake/volume of filtrate} = 0{\cdot}056$$

From equation 9.2,
$$V^2 = \frac{2A^2(-\Delta P)t}{r\mu v}$$

But
$$L = \text{half frame thickness} = Vv/A \qquad \text{(equation 9.6)}$$
$$V^2 = L^2A^2/v^2$$

and
$$L^2 = \frac{2A(-\Delta P)vt}{r\mu}$$

$$= \frac{2 \times (1000 - 101\cdot3) \times 10^3 \times 0\cdot056t}{3185 \times 10^{10} \times 1 \times 10^{-3}}$$

$$L^2 = 3\cdot16 \times 10^{-6}t$$

It is shown in section 9.4.3 that if the resistance of the filter medium is neglected, the optimum cake thickness occurs when the filtration time is equal to the downtime,

i.e. for L_{opt}, $t = 900$

$\therefore$ $$L^2_{opt} = 3\cdot16 \times 10^{-6} \times 900 = 2\cdot84 \times 10^{-3}$$

and $$L = 0\cdot053\,\text{m}$$

$\therefore$ optimum frame thickness = 106 mm

For the washing process, if the filtration pressure is halved, the rate of washing is halved. The wash water has twice the thickness to penetrate and half the area for flow that is available to the filtrate, so that, considering these factors, the washing rate is one-eighth of the final filtration rate.

The final filtrate $$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu vV}$$

$$= \frac{898\cdot7 \times 10^3 A^2}{3185 \times 10^{10} \times 10^{-3} \times 0\cdot056V} = \frac{5\cdot04 \times 10^{-4}A^2}{V}$$

$\therefore$ washing rate = final rate/8 = $63 \times 10^{-5}A^2/V$

The volume of wash water = $V/4$.

Hence washing time = $(V/4)/(6\cdot3 \times 10^{-5}A^2/V)$

i.e. $$t_w = 3\cdot97 \times 10^3 V^2/A^2$$

Now $$V^2 = L^2A^2/v^2$$

$\therefore$ $$t_w = \frac{L^2A^2}{(0\cdot056)^2} \times \frac{3\cdot97 \times 10^3}{A^2} = 1\cdot27 \times 10^6 L^2$$

The filtration time t_f was shown earlier to be:

$$t_f = L^2/3\cdot16 \times 10^{-6} = 3\cdot16 \times 10^5 L^2$$

$\therefore$ total cycle time = $L^2(1\cdot27 \times 10^6 + 3\cdot16 \times 10^5) + 900$

$$= 1\cdot58 \times 10^6 L^2 + 900$$

The rate of cake production

$$= \frac{L}{1\cdot58 \times 10^6 L^2 + 900} = R$$

For $dR/dL = 0$, $\quad 1.58 \times 10^6 L^2 + 900 - 3.16 \times 10^6 L^2 = 0$

and $\quad L = 0.024$ m

$$\text{frame thickness} = 0.048 \text{ m} = \underline{\underline{48 \text{ mm}}}$$

Problem 9.3

A plate and frame press, filtering a slurry, gave a total of 8 m^3 of filtrate in 1800 s and 11 m^3 in 3600 s, when filtration was stopped. Estimate the washing time in seconds if 3 m^3 of wash water are used. The resistance of the cloth can be neglected and a constant pressure is used throughout

Solution

For constant pressure filtration with no cloth resistance,

$$t = \frac{r\mu v}{2A^2(-\Delta P)} V^2 \qquad \text{(equation 9.12)}$$

At $t_1 = 1800$ s, $V_1 = 8$ m^3, and when $t_2 = 3600$ s, $V_2 = 11$ m^3

$$3600 - 1800 = \frac{r\mu v}{2A^2(-\Delta P)}(11^2 - 8^2)$$

$$\frac{r\mu v}{2A^2(-\Delta P)} = 31.6$$

As

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v V}$$

$$\frac{dV}{dt} = \frac{1}{2 \times 31.6 V} = \frac{0.0158}{V}$$

The final rate of filtration $= 0.0158/11 = 1.44 \times 10^{-3}$ m^3/s

For thorough washing in a plate and frame filter the wash water has twice the thickness of cake to penetrate and half the area for flow that is available to the filtrate. Thus the flow of wash water at the same pressure will be one-quarter of the filtration rate.

Hence $\quad$ rate of washing $= 1.44 \times 10^{-3}/4 = 3.6 \times 10^{-4}$ m^3/s

time of washing $= 3/(3.6 \times 10^{-4})$

$= \underline{\underline{8400 \text{ s}}}$ (2.3 h)

Problem 9.4

In the filtration of a certain sludge the initial period is effected at a constant rate with the feed pump at full capacity till the pressure reaches 400 kN/m^2. The pressure is then maintained at this value for the remainder of the filtration. The constant rate operation requires 900 s, and one-third of the total filtrate is obtained during this period.

Neglecting the resistance of the filter medium, determine (a) the total filtration time, and (b) the filtration cycle with the existing pump for the maximum daily capacity, if the time for removing the cake and reassembling the press is 1200 s The cake is not washed.

Solution

For filtration carried out at a constant filtration rate for time t_1 in which time a volume V_1 is collected and followed by a constant pressure period such that the total filtration time is t and the total volume of filtrate is V:

$$V^2 - V_1^2 = \frac{2A^2(-\Delta P)}{r\mu v}(t - t_1) \qquad \text{(equation 9.13)}$$

Assuming no cloth resistance,

For the constant rate period:

$$t_1 = \frac{r\mu v}{A^2(-\Delta P)} V_1^2 \qquad \text{(equation 9.10)}$$

Using the data given:

$$t_1 = 900\,\text{s}, \quad \text{volume} = V_1$$

$$\therefore \quad \frac{r\mu v}{A^2(-\Delta P)} = \frac{900}{V_1^2}$$

(a) For the constant pressure period,

$$V - V_1 = 2V_1 \quad \text{and} \quad t - t_1 = t_p$$

$$\therefore \quad (2V_1)^2 = \frac{2V_1^2}{900} t_p$$

$$\therefore \quad t_p = 1800\,\text{s}$$

$$\therefore \quad \text{total filtration time} = 900 + 1800 = \underline{\underline{2700\,\text{s}}}$$

$$\text{total cycle time} = 2700 + 1200 = \underline{\underline{3900\,\text{s}}}$$

(b) For the constant rate period,

$$t_1 = \frac{r\mu v}{A^2(-\Delta P)} V_1^2 = \frac{V_1^2}{K}$$

For the constant pressure period,

$$t - t_1 = \frac{r\mu v}{2A^2(-\Delta P)} (V^2 - V_1^2) = \frac{V^2 - V_1^2}{2K}$$

Total filtration time $= t = \frac{1}{K}\left(V_1^2 + \frac{V^2 - V_1^2}{2}\right)$

$$= \frac{(V^2 + V_1^2)}{2K}$$

Rate of filtration $= \frac{V}{t + t_d}$ where t_d = downtime

$$= \frac{2KV}{V^2 + V_1^2 + 2Kt_d}$$

$\frac{d(\text{rate})}{dV} = 0$ for a maximum gives.

$$V_1^2 - V^2 + 2Kt_d = 0$$

or $\qquad t_d = \frac{1}{2K}(V^2 - V_1^2) = t - t_1$

Now $\qquad t_d = 1200\,\text{s} = t - 900$

$\therefore \qquad t = 2100\,\text{s}$

and $\qquad$ cycle time $= 2100 + 1200 = \underline{\underline{3300\,\text{s}}}$

Problem 9.5

A rotary filter, operating at 0·03 Hz, filters 0·0075 m³/s. Operating under the same vacuum and neglecting the resistance of the filter cloth, at what speed must the filter be operated to give a filtration rate of 0·01~~0~~5 m³/s?

Solution

For constant pressure filtration in a rotary filter:

$$V^2 = \frac{A^2(-\Delta P)}{r\mu v}\,t$$

i.e. $\qquad V^2 \propto t \propto 1/N$

where N is the speed rotation.

As $V \propto 1/\sqrt{N}$ and the rate of filtration is V/t, then:

$$\frac{V}{t} \propto \left(\frac{1}{\sqrt{N}} \times \frac{1}{t}\right) \propto \left(\frac{1}{\sqrt{N}} \times N\right) \propto \sqrt{N}$$

$$\frac{(V/t)_1}{(V/t)_2} = \frac{\sqrt{N_1}}{\sqrt{N_2}}$$

$$\frac{0{\cdot}0075}{0{\cdot}0150} = \frac{\sqrt{0{\cdot}03}}{\sqrt{N_2}}$$

$$N_2 = \underline{\underline{0{\cdot}12\ \text{Hz}}} \quad (7{\cdot}2\ \text{rev/min})$$

Problem 9.6

A slurry is filtered in a plate and frame press containing 12 frames, each 0·3 m square and 25 mm thick. During the first 200 s, the filtration pressure is slowly raised to the final value of 500 kN/m^2, and during this period the rate of filtration is maintained constant. After the initial period, filtration is carried out at constant pressure and the cakes are completely formed after a further 900 s. The cakes are then washed at 375 kN/m^2 for 600 s, using "thorough washing". What is the volume of filtrate collected per cycle and how much wash water is used?

A sample of the slurry had previously been tested, using a vacuum leaf filter of 0·05 m^2 filtering surface and a vacuum equivalent to an absolute pressure of 30 kN/m^2. The volume of filtrate collected in the first 300 s was 250 cm^3 and, after a further 300 s, an additional 150 cm^3 was collected. Assume the cake to be incompressible and the cloth resistance to be the same in the leaf as in the filter press.

Solution

In the leaf filter, filtration is at constant pressure from the start.

Thus $$V^2 + 2(AL/v)V = 2(-\Delta PA^2/r\mu v)t \qquad \text{(from equation 9.18)}$$

In the filter press, a volume V_1 of filtrate is obtained under constant rate conditions in time t_1, and filtration is then carried out at constant pressure.

Thus $$V_1^2 + (AL/v)V_1 = (-\Delta PA^2/r\mu v)t_1 \qquad \text{(from equation 9.17)}$$

and $$(V^2 - V_1^2) + 2(AL/v)(V - V_1) = 2(-\Delta PA^2/r\mu v)(t - t_1)$$

(from equation 9.18)

For the leaf filter

When $t = 300\ \text{s}$, $V = 250\ \text{cm}^3$,

and when $t = 600\ \text{s}$, $V = 400\ \text{cm}^3$

$$A = 0{\cdot}05\ \text{m}^2, \quad \text{and} \quad -\Delta P = (101{\cdot}3 - 30{\cdot}0) = 71{\cdot}3\ \text{kN/m}^2$$

$$\therefore \quad 250^2 + 2(0{\cdot}05L/v)250 = 2(71{\cdot}3 \times 0{\cdot}05^2/r\mu v)300$$

and $$400^2 + 2(0{\cdot}05L/v)400 = 2(71{\cdot}3 \times 0{\cdot}05^2/r\mu v)600$$

i.e. $$62{,}500 + 25L/v = 106.95/r\mu v$$

$$160{,}000 + 40L/v = 213.9/r\mu v$$

Hence $$L/v = 3500 \quad \text{and} \quad r\mu v = 7.13 \times 10^{-4}$$

For the filter press

The volume of filtrate V_1 collected during the constant rate period is given by

$$[A = 2.16\ \text{m}^2, \quad -\Delta P = (500 - 101.3) = 398.7\ \text{kN/m}^2, \quad t = 200\ \text{s}]:$$

$$V_1^2 + 2.16 \times 3500 V_1 = (398.7 \times 2.16^2/7.13 \times 10^{-4})\,200$$

i.e. $$V_1^2 + 7560 V_1 - 5.218 \times 10^8 = 0$$

$\therefore$ $$V_1 = -3780 + \sqrt{(1.429 \times 10^7 + 5.218 \times 10^8)} = 1.937 \times 10^4\ \text{cm}^3$$

For the constant pressure period,

$$t - t_1 = 900\ \text{s}$$

The total volume of filtrate collected is therefore given by,

$$(V^2 - 3.75 \times 10^8) + 15{,}120(V - 1.937 \times 10^4) = 5.218 \times 10^6 \times 900$$

i.e. $$V^2 + 15{,}120V - 53.64 \times 10^8 = 0$$

$\therefore$ $$V = -7560 + \sqrt{(5.715 \times 10^7 + 53.64 \times 10^8)}$$

$$= 6.607 \times 10^4\ \text{cm}^3 \quad \text{or} \quad \underline{\underline{0.066\ \text{m}^3}}$$

Final rate of filtration $= -\Delta P A^2/[r\mu v(V + AL/v)]$ (from equation 9.16)

$$= (398.7 \times 2.16^2)/[7.13 \times 10^{-4}(6.607 \times 10^4 + 216 \times 3500)]$$

$$= 35.4\ \text{cm}^3/\text{s}$$

If the viscosity of the filtrate is the same as that of the wash water,

rate of washing at 500 kN/m^2 = 35.4 cm^3/s

rate of washing at 375 kN/m^2 = 35.4(375 − 101.3)/(500 − 101.3)

= 24.3 cm^3/s

Thus amount of wash water passing in 600 s

$$= 600 \times 24.3 = 1.458 \times 10^4\ \text{cm}^3 \quad \text{or} \quad \underline{\underline{0.0146\ \text{m}^3}}$$

Problem 9.7

A sludge is filtered in a plate and frame press fitted with ~~50~~ 25 mm frames. For the first ~~3~~600 s the slurry pump runs at maximum capacity. During this period the pressure rises to 500 kN/m^2 and a quarter of the total filtrate is obtained. The filtration takes a further 3600 s to complete at constant pressure and 900 s is required for emptying and resetting the press.

It is found that, if the cloths are precoated with filter aid to a depth of 1·6 mm, the cloth resistance is reduced to a quarter of its former value. What will be the increase in the overall throughput of the press if the precoat can be applied in 180 s?

Solution

The basic filtration equation is.

$$\frac{1}{A}\frac{dV}{dt} = \frac{-\Delta P}{r\mu(Vv/A + l)} \qquad \text{(equation 9.16)}$$

$$\therefore \quad \frac{dV}{dt} = \frac{A^2 - \Delta P}{r\mu v(V + Al/v)} = \frac{a}{V+b}$$

where $a = A^2 - \Delta P/r\mu v$ and $b = Al/v$.

At constant rate: $$\frac{V_0}{t_0} = \frac{a}{V_0 + b} \quad \text{or} \quad V_0^2 + bV_0 = at_0 \qquad (1)$$

At constant pressure: $$\tfrac{1}{2}(V^2 - V_0^2) + b(V - V_0) = a(t - t_0) \qquad (2)$$

In case 1: $t_0 = 600\,\text{s}$, $(t - t_0) = 3600\,\text{s}$, $V_0 = V/4$

$\therefore$ in (1) $$(V/4)^2 + bV/4 = 600a$$

in (2) $$\tfrac{1}{2}[V^2 - (V/4)^2] + b(V - V/4) = 3600a$$

from which $$a = 0.000\,156V^2 \quad \text{and} \quad b = 0.125V$$

Total cycle time $= 600 + 3600 + 900 = 5100\,\text{s}$
Filtration rate $= V/5100 = 0.000\,196V\ \text{m}^3/\text{s}$.
For case 2, the cloth resistance is a quarter of its previous value, i.e.

$$b' = b/4$$

During the constant rate period, the slurry pump operates at constant capacity so that:

$$V_1/t_1 = V_0/t_0$$

and therefore: $$\frac{a}{V_1 + b/4} = \frac{a}{V_0 + b}$$

Precoat thickness $= 2 \times 1.6 = 3.2$ mm.
$\therefore$ cake thickness $= 25 - 3.2 = 21.8$ mm
$\therefore$ volume of cake in case 2 $= (21.8/25) = 0.872 \times$ volume of cake in case 1.

$\therefore$ for case 2, total filtrate $V_2 = 0.872V$

For constant pressure:

$$\tfrac{1}{2}[(0.872V)^2 - V_1^2] + (b/4)(0.872V - V_1) = a(t - t_1) \qquad (3)$$

but $$V_0/t_0 = (V/4)/600 = 0.000\,417V$$

and
$$a = 0{\cdot}000\,156V^2$$
$$b' = b/4 = 0{\cdot}031\,25V$$
$$0{\cdot}000\,417V = \frac{0{\cdot}000\,156V^2}{V_1^2 + 0{\cdot}031\,25V}$$
or
$$V_1 = 0{\cdot}344V$$
Now
$$\frac{V_1}{t_1} = \frac{V_0}{t_0} \quad \therefore t_1 = V_1 t_0/V_0$$
but
$$V_0 = V/4, \quad t_0 = 600\,\text{s}, \quad V_1 = 0{\cdot}344V$$
$$t_1 = 825\,\text{s}$$

Substituting $V_1 = 0{\cdot}344V$ and $b' = 0{\cdot}031\,25V$ into equation (3) gives:
$$\tfrac{1}{2}[(0{\cdot}872V)^2 - (0{\cdot}344V)^2] + 0{\cdot}031\,25V(0{\cdot}872V - 0{\cdot}344V) = 0{\cdot}000\,156V^2(t - t_1)$$
and
$$t - t_1 = 2159\,\text{s}$$
$$\therefore \quad \text{New cycle time} = 180 + 900 + 825 + 2159 = 4064\,\text{s}$$
$$\text{New filtration rate} = 0{\cdot}872V/4064 = 0{\cdot}000\,215V\,\text{m}^3/\text{s}$$
$$\%\ \text{increase} = (0{\cdot}000\,215 - 0{\cdot}000\,196)V \times 100/0{\cdot}000\,196V$$
$$= \underline{\underline{9{\cdot}7\%}}$$

Problem 9.8

Filtration is carried out in a plate and frame filter press, with 20 frames 0·3 m square and 50 mm thick, and the rate of filtration is maintained constant for the first 300 s. During this period, the pressure is raised to 350 kN/m^2, and one-quarter of the total filtrate per cycle is obtained. At the end of the constant rate period, filtration is continued at a constant pressure of 350 kN/m^2 for a further 1800 s, after which the frames are full. The total volume of filtrate per cycle is 0·7 m^3 and dismantling and refitting of the press takes 500 s.

It is decided to use a rotary drum filter, 1·5 m long and 2·2 m in diameter, in place of the filter press. Assuming that the resistance of the cloth is the same in the two plants and that the filter cake is incompressible, calculate the speed of rotation of the drum which will result in the same overall rate of filtration as was obtained with the filter press. The filtration in the rotary filter is carried out at a constant pressure difference of 70 kN/m^2 and the filter operates with 25% of the drum submerged in the slurry at any instant.

Solution

Data from the plate and frame filter press are used to evaluate the cake and cloth resistance for use with the rotary drum filter.

For the constant rate period,

$$V_1^2 + \frac{LA}{v} V_1 = \frac{A^2 - \Delta P}{r\mu v} t_1 \qquad \text{(equation 9.17)}$$

For the subsequent constant pressure period,

$$(V^2 - V_1^2) + \frac{2LA}{v}(V - V_1) = \frac{2A^2 - \Delta P}{r\mu v}(t - t_1) \qquad \text{(equation 9.18)}$$

From the data given,

$$t_1 = 300 \text{ s}, \quad -\Delta P = 350 - 101{\cdot}3 = 248{\cdot}7 \text{ kN/m}^2,$$

$$V_1 = 0{\cdot}175 \text{ m}^3, \quad \text{and} \quad A = 2 \times 20 \times 0{\cdot}3 \times 0{\cdot}3 = 3{\cdot}6 \text{ m}^2$$

$$\therefore \quad (0{\cdot}175)^2 + \frac{L}{v} \times 3{\cdot}6 \times 0{\cdot}175 = \frac{(3{\cdot}6)^2 \times 248{\cdot}7 \times 10^3 \times 300}{r\mu v}$$

i.e.
$$0{\cdot}0306 + 0{\cdot}63(L/v) = 9{\cdot}68 \times 10^8/r\mu v \qquad \text{(i)}$$

For the constant pressure period,

$$V = 0{\cdot}7 \text{ m}^3, \quad V_1 = 0{\cdot}175 \text{ m}^3,$$

$$t - t_1 = 1800 \text{ s}, \quad A = 3{\cdot}6 \text{ m}^2$$

$$\therefore \quad (0{\cdot}7^2 - 0{\cdot}175^2) + 2(L/v) \times 3{\cdot}6(0{\cdot}7 - 0{\cdot}175) = \frac{(2 \times 3{\cdot}6)^2 \times 248{\cdot}7 \times 10^3}{r\mu v} \times 1800$$

i.e.
$$0{\cdot}459 + 3{\cdot}78(L/v) = 116{\cdot}08 \times 10^8/r\mu v \ ✓ \qquad \text{(ii)}$$

Solving (i) and (ii) simultaneously gives:

$$r\mu v = 210{\cdot}9 \times 10^8 \quad \text{and} \quad L/v = 0{\cdot}0243$$

For the rotary drum filter,

$$D = 2{\cdot}2 \text{ m}, \quad L = 1{\cdot}5 \text{ m}, \quad -\Delta P = 70 \text{ kN/m}^2$$

$$A = 2{\cdot}2\pi \times 1{\cdot}5 = 10{\cdot}37 \text{ m}^2$$

$$\Delta P = 70 \times 10^3 \text{ N/m}^2$$

Let θ be the time of one revolution, then as the time of filtration is $0{\cdot}25\theta$,

$$V^2 + 2A\frac{L}{v}V = \frac{2A^2(-\Delta P)}{r\mu v} \times 0{\cdot}25\theta$$

$$V^2 + 2 \times 10{\cdot}37 \times 0{\cdot}0243V = \frac{2(10{\cdot}37)^2 \times 70 \times 10^3 \times 0{\cdot}25\theta}{210{\cdot}9 \times 10^8}$$

$$V^2 + 0{\cdot}504V = 1{\cdot}785 \times 10^{-4}\theta$$

Now the rate of filtration $= V/t = 0{\cdot}7/(300 + 1800 + 500)$

$$= 2{\cdot}7 \times 10^{-4} \text{ m}^3/\text{s}$$

$$\therefore \quad V = 2{\cdot}7 \times 10^{-4}t$$

$$\therefore \quad (2{\cdot}7 \times 10^{-4}t)^2 + 0{\cdot}504 \times 2{\cdot}7 \times 10^{-4}t = 1{\cdot}785 \times 10^{-4}t$$

from which $t = 580 \text{ s}$

Hence speed $= 1/580 = \underline{\underline{0.002 \text{ Hz}}}$

Problem 9.9

It is required to filter a certain slurry to produce 2.25 m^3 filtrate per working day of 8 h. The process is carried out in a plate and frame filter press with 0.45 m square frames and a working pressure of 450 kN/m^2. The pressure is built up slowly over a period of 300 s, and during this period the rate of filtration is maintained constant.

When a sample of the slurry was filtered, using a pressure of 35 kN/m^2 on a single leaf filter of filtering area 0.05 m^2, 400 cm^3 of filtrate was collected in the first 300 s of filtration and a further 400 cm^3 was collected during the following 600 s. Assuming that the dismantling of the filter press, the removal of the cakes and the setting up again of the press takes an overall time of 300 s, plus an additional 180 s for each cake produced, what is the minimum number of frames that need be employed? Take the resistance of the filter cloth to be the same in the laboratory tests as on the plant.

Solution

For constant pressure filtration on the leaf filter, equation 9.18 applies.

$$V^2 + 2\frac{L}{v}AV = \frac{2A^2(-\Delta P)t}{r\mu v}$$

When $t = 300 \text{ s}$, $V = 0.0004 \text{ m}^3$, $A = 0.05 \text{ m}^2$, $-\Delta P = 66.3 \text{ kN/m}^2$

$$\therefore \quad (0.0004)^2 + 2(L/v) \times 0.05 \times 0.0004 = \frac{2 \times (0.05)^2 \times 66.3 \times 300}{r\mu v}$$

or
$$1.6 \times 10^{-7} + 4 \times 10^{-5}(L/v) = 99.4/r\mu v$$

When $t = 900 \text{ s}$, $V = 800 \text{ cm}^3 = 0.0008 \text{ m}^3$

and substitution gives:

$$6.4 \times 10^{-7} + 8 \times 10^{-5}(L/v) = 298.4/r\mu v$$

Hence
$$L/v = 4 \times 10^{-3} \quad \text{and} \quad r\mu v = 3.1 \times 10^8$$

In the filter press

For the constant rate period: $$V_1^2 + \frac{LA}{v}V_1 = \frac{A^2(-\Delta P)t_1}{r\mu v} \qquad \text{(equation 9.17)}$$

$A = 2 \times 0.45n = 0.9n$ where n is the number of frames

$t_1 = 300 \text{ s}$

$$V_1^2 + 4 \times 10^{-3} \times 0.9nV_1 = 0.81n^2(450 - 101.3) \times 300/3.1 \times 10^8$$

or
$$V_1^2 + 3.6 \times 10^{-3}nV_1 = 2.73 \times 10^{-4}n^2$$

and $$V_1 = 0.0148n$$

For the constant pressure period:

$$\left(\frac{V^2 - V_1^2}{2}\right) + \frac{LA}{v}(V - V_1) = \frac{A^2(-\Delta P)}{r\mu v}(t - t_1)$$

Substituting for L/v, $r\mu v$, $t_1 = 300$ and $V_1 = 0.0148n$ gives:

$$\left(\frac{V^2 - 2.2 \times 10^{-4}n^2}{2}\right) + (V - 0.0148n)4 \times 10^{-3} \times 0.9n = \frac{0.81n^2 \times 348.7}{3.1 \times 10^8}(t_f - 300)$$

or $$0.5V^2 + 1.1 \times 10^{-4}n^2 + 3.6 \times 10^{-3}nV = 9.11 \times 10^{-7}n^2t \qquad \text{(i)}$$

Now the total cycle time $= (t_f + 300 + 180n)$ s.
Required filtration rate $= 2.25/(8 \times 3600) = 7.81 \times 10^{-5}$ m^3/s.
Volume of filtrate $= V$ m^3.

$$\therefore \qquad \frac{V}{t_f + 300 + 180n} = 7.81 \times 10^{-5}$$

and $$t_f = 1.28 \times 10^4 V - 300 - 180n \qquad \text{(ii)}$$

Thus value of t_f from (ii) may be substituted in equation (i) to give:

$$V^2 + V(7.2 \times 10^{-3}n - 2.34 \times 10^{-2}n^2) + (7.66 \times 10^{-4}n^2 + 3.28 \times 10^{-4}n^3) = 0 \qquad \text{(iii)}$$

This equation is of the form $V^2 + AV + B = 0$ and may be solved to give:

$$V = \frac{-A \pm (A^2 - 4B)}{2}$$

where A and B are the expressions in parentheses in equation (iii). In order to find the minimum number of frames, $\mathrm{d}V/\mathrm{d}n$ must be found and equated to zero. From above, $(V - a)(V - b) = 0$, where a and b are complex functions of n.

Thus $V = a$ or $V = b$ and $\mathrm{d}V/\mathrm{d}n$ can be evaluated for each root.

Putting $\mathrm{d}V/\mathrm{d}n = 0$ gives, for the positive value,

$$\underline{\underline{n = 13}}$$

Problem 9.10

The relation between flow and head for a certain slurry pump can be represented approximately by a straight line, the maximum flow at zero head being 0.0015 m^3/s and the maximum head at zero flow 760 m of liquid.

Using this pump to feed a particular slurry to a pressure leaf filter:

(a) How long will it take to produce 1 m^3 of filtrate?

(b) What will be the pressure across the filter after this time?

A sample of the slurry was filtered at a constant rate of 0.000 15 m^3/s through a leaf filter covered with a similar filter cloth but of one-tenth the area of the full-scale unit, and after 625 s the pressure across the filter was 360 m of liquid. After a further 480 s the pressure was 600 m.

Solution

For constant rate filtration through the filter leaf:

$$V^2 + \frac{LA}{v}V = \frac{A^2(-\Delta P)t}{r\mu v} \qquad \text{(equation 9.17)}$$

At a constan rate of 0·000 15 m³/s when time = 625 s,

$$V = 0{\cdot}094\,\mathrm{m}^3, \quad -\Delta P = 3530\,\mathrm{kN/m^2}$$

and at $t = 1105$ s, $\quad V = 0{\cdot}166\,\mathrm{m}^3 \quad$ and $\quad -\Delta P = 5890\,\mathrm{kN/m^2}$

Substituting these values into equation 9.17 gives:

$$(0{\cdot}094)^2 + LA/v \times 0{\cdot}094 = (A^2/r\mu v) \times 3530 \times 625$$

i.e.

$$0{\cdot}0088 + 0{\cdot}094LA/v = 2{\cdot}21 \times 10^6 A^2/r\mu v \qquad \text{(i)}$$

and

$$(0{\cdot}166)^2 + LA/v \times 0{\cdot}166 = (A^2/r\mu v) \times 5890 \times 1105$$

i.e.

$$0{\cdot}0276 + 0{\cdot}166LA/v = 6{\cdot}51 \times 10^6 A^2/r\mu v \qquad \text{(ii)}$$

Equations (i) and (ii) may be solved simultaneously to give:

$$LA/v = 0{\cdot}0154 \quad \text{and} \quad A^2/r\mu v = 4{\cdot}64 \times 10^{-9}$$

As the full-size plant is 10 times that of the leaf filter,

$$LA/v = 0{\cdot}154 \quad \text{and} \quad A^2/r\mu v = 4{\cdot}64 \times 10^{-7}$$

If the pump develops 760 m (7460 kN/m²) at zero flow and has zero head at $Q = 0{\cdot}0015\,\mathrm{m^3/s}$, its performance can be expressed as:

$$\Delta P = 7460 - (7460/0{\cdot}0015)Q$$

or

$$\Delta P = 7460 - 4{\cdot}97 \times 10^6 Q\,(\mathrm{kN/m^2})$$

Now

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v(V + LA/v)} \qquad \text{(equation 9.16)}$$

Substituting for $-\Delta P$ and the filtration constants gives:

$$\frac{dV}{dt} = \frac{A^2}{r\mu v}\,\frac{(7460 - 4{\cdot}97 \times 10^6\,dV/dt)}{(V + 0{\cdot}154)}$$

Since $Q = dV/dt$:

$$\frac{dV}{dt} = \frac{4{\cdot}67 \times 10^{-7}[7460 - 4{\cdot}97 \times 10^6(dV/dt)]}{(V + 0{\cdot}154)}$$

$$(V + 0{\cdot}154)\,dV = 3{\cdot}46 \times 10^{-3} - 2{\cdot}31\,dV/dt$$

The time to collect 1 m³ is then given by:

$$\int_0^1 (V + 0{\cdot}154 + 2{\cdot}31)\,dV = \int_0^t (3{\cdot}46 \times 10^{-3})\,dt$$

and $$t = \underline{\underline{857\,\text{s}}}$$

The pressure at this time is found by substitution in equation 9.17 with $V = 1\,\text{m}^3$ and $t = 857\,\text{s}$:

$$1^2 + 0{\cdot}154 \times 1 = 4{\cdot}64 \times 10^{-7} \times 857\Delta P$$

and $$-\Delta P = \underline{\underline{2902\,\text{kN/m}^2}}$$

Problem 9.11

A slurry containing 40% by weight solid is to be filtered on a rotary drum filter 2 m diameter and 2 m long which normally operates with 40% of its surface immersed in the slurry and under a pressure of $17\,\text{kN/m}^2$. A laboratory test on a sample of the slurry using a leaf filter of area $200\,\text{cm}^2$ and covered with a similar cloth to that on the drum produced $300\,\text{cm}^3$ of filtrate in the first 60 s and $140\,\text{cm}^3$ in the next 60 s, when the leaf was under an absolute pressure of $17\,\text{kN/m}^2$. The bulk density of the dry cake was $1500\,\text{kg/m}^3$ and the density of the filtrate $1000\,\text{kg/m}^3$. The minimum thickness of cake which could be readily removed from the cloth was 5 mm.

At what speed should the drum rotate for maximum throughput and what is this throughput in terms of the weight of the slurry fed to the unit per unit time?

Solution

For the leaf filter:

$$A = 0{\cdot}02\,\text{m}^2, \quad \Delta P = 101{\cdot}3 - 17 = 84{\cdot}3\,\text{kN/m}^2 = 84{,}300\,\text{N/m}^2$$

When $t = 60$, $\quad V = 0{\cdot}0003\,\text{m}^3$

When $t = 120$, $\quad V = 0{\cdot}000\,44\,\text{m}^3$

These figures are substituted into the constant pressure filtration equation 9.18.

$$V^2 + \frac{2LAV}{v} = \frac{2(-\Delta P)A^2 t}{r\mu v} \qquad \text{(equation 9.18)}$$

This enables the filtration constants to be determined as:

$$L/v = 2{\cdot}19 \times 10^{-3} \quad \text{and} \quad r\mu v = 3{\cdot}48 \times 10^{10}$$

For the rotary filter equation 9.18 applies as the whole operation is at constant pressure. The maximum throughput will be obtained when the cake thickness is a minimum, i.e. 5 mm = 0·005 m.

Area of filtering surface $= 2\pi \times 2 = 4\pi\,\text{m}^2$.

Bulk volume of cake deposited $= 4\pi \times 0{\cdot}005 = 0{\cdot}063\,\text{m}^3/\text{rev}$.

Let the rate of filtrate production $= w\,\text{kg/s} = 0{\cdot}001w\,\text{m}^3/\text{s}$.

For a 40% slurry: $\quad S/(S + w) = 0{\cdot}4$

and weight solids $= 0{\cdot}66w$.

$$\text{volume of solids deposited/s} = 0.66w/1500 = 4.4 \times 10^{-4} w \text{ m}^3/\text{s}$$

If one revolution takes t s,

$$4.4 \times 10^{-4} wt = 0.063$$

and weight $= 143$ kg

Rate of production of filtrate $= 0.001w \text{ m}^3/\text{s} = V/t$

$$\therefore \quad V^2 = 1 \times 10^{-6} w^2 t^2$$

$$= 1 \times 10^{-6} (143)^2$$

$$= 0.02 \text{ m}^6$$

$$V = 0.141 \text{ m}^3$$

Substituting $V = 0.141 \text{ m}^3$ and the constants into equation 9.18,

$$(0.141)^2 + 2 \times 2.19 \times 10^{-3} \times 0.141 = 2 \times 84{,}300 \times (4\pi)^2 t/3.48 \times 10^{10}$$

from which $t = 26.95$ s $=$ time of submergence/rev.

$$\therefore \quad \text{time for 1 rev} = 26.9/0.4 = 67.3 \text{ s}$$

$$\therefore \quad \text{speed} = 1/67.3 = \underline{\underline{0.015 \text{ Hz}}} \quad (0.9 \text{ rev/min})$$

$$w = 143/67.3 = 2.11 \text{ kg/s}$$

$$S = 0.66 \times 2.11 \text{ kg/s}$$

$$\therefore \quad \text{weight of slurry} = 1.66 \times 2.11 = \underline{\underline{3.5 \text{ kg/s}}}$$

Problem 9.12

A continuous rotary filter is required for an industrial process for the filtration of a suspension to produce $0.002 \text{ m}^3/\text{s}$ of filtrate. A sample was tested on a small laboratory filter of area 0.023 m^2 to which it was fed by means of a slurry pump to give filtrate at a constant rate of $12.5 \text{ cm}^3/\text{s}$. The pressure difference across the test filter increased from 14 kN/m^2 after 300 s filtration to 28 kN/m^2 after 500 s at which time the cake thickness had reached 38 mm. Suggest suitable dimensions and operating conditions for the rotary filter, assuming that the resistance of the cloth used is one-half that on the test filter, and that the vacuum system is capable of maintaining a constant pressure difference of 70 kN/m^2 across the filter.

Solution

Data from the laboratory filter may be used to find the cloth and cake resistance of the rotary filter. For the laboratory filter operating under constant rate conditions:

$$V_1^2 + \frac{LA}{v} V_1 = \frac{A^2(-\Delta P)t}{r\mu v} \qquad \text{(equation 9.17)}$$

$A = 0.023 \text{ m}^2$ and the filtration rate $= 12.5 \text{ cm}^3/\text{s}$.

At $t = 300$ s,

$$-\Delta P = 14\,\text{kN/m}^2 \quad \text{and} \quad V_1 = 3750\,\text{cm}^3 = 3{\cdot}75 \times 10^{-3}\,\text{m}^3$$

When $t = 900$ s,

$$-\Delta P = 28\,\text{kN/m}^2 \quad \text{and} \quad V_1 = 11{,}250\,\text{cm}^3 = 1{\cdot}125 \times 10^{-2}\,\text{m}^3$$

Hence $\quad (3{\cdot}75 \times 10^{-3})^2 + (L/v) \times 0{\cdot}023 \times 3{\cdot}75 \times 10^{-3} = \dfrac{14}{r\mu v} \times (0{\cdot}023)^2 \times 300$

$$1{\cdot}41 \times 10^{-5} + 8{\cdot}63 \times 10^{-5}(L/v) = 2{\cdot}22/r\mu v$$

and $\quad (1{\cdot}25 \times 10^{-2})^2 + (L/v) \times 0{\cdot}023 \times 1{\cdot}125 \times 10^{-3} = \dfrac{28 \times (0{\cdot}023)^2}{r\mu v} \times 900$

$$1{\cdot}27 \times 10^{-4} + 2{\cdot}59 \times 10^{-4}(L/v) = 13{\cdot}33/r\mu v$$

from which $\quad L/v = 0{\cdot}164; \quad r\mu v = 7{\cdot}86 \times 10^4$

If the cloth resistance is halved by using the rotary filter, $L/v = 0{\cdot}082$. As the filter operates at constant pressure, equation 9.18 is applicable:

$$V^2 \frac{2LA}{v} = \frac{2A^2(-\Delta P)t}{r\mu v} \qquad \text{(equation 9.18)}$$

Let θ = time for 1 rev × fraction submerged and V' = volume of filtrate/rev (given by equation 9.18).

Assume speed = 0·0167 Hz (1 rev/min) and 20% submergence.

Then $\quad \theta = 60 \times 0{\cdot}2 = 12\,\text{s}$

$$\therefore \quad V'^2 + 2 \times 0{\cdot}082AV' = \frac{2A^2 \times 70 \times 12}{7{\cdot}86 \times 10^4}$$

or $\quad V'^2 + 0{\cdot}164AV' = 0{\cdot}0214A^2$

from which $\quad A/V' = 11{\cdot}7$

The required rate of filtration = 0·002 m³/s.

$$\therefore \quad V' = \text{volume/rev} = 0{\cdot}002 \times 60 = 0{\cdot}12\,\text{m}^3$$

$$\therefore \quad A = 11{\cdot}7 \times 0{\cdot}12 = 1{\cdot}41\,\text{m}^2$$

If $L = D$, $\quad$ area of drum $= \pi D^2 = 1{\cdot}41\,\text{m}^2$

and $\quad D = L = \underline{\underline{0{\cdot}67\,\text{m}}}$

The cake thickness on the drum should now be checked.

$$v = AL/V$$

and from data on the laboratory filter,

$$v = 0{\cdot}023 \times 0{\cdot}038/1{\cdot}125 \times 10^{-2}$$

$$= 0{\cdot}078$$

Hence, cake thickness on drum $= vV'/A = 0.078/11.7$

$= 0.0067\text{ m} = \underline{\underline{6.7\text{ mm}}}$ which is acceptable

Problem 9.13

A rotary drum filter 1·2 m diameter and 1·2 m long can handle 6·0 kg/s of slurry containing 10% of solids when rotated at 0·005 Hz. By increasing the speed to 0·008 Hz, it is found that it can handle 7·2 kg/s. What will be the percentage change in the amount of wash water which can be applied to each kilogram of cake caused by this increase of speed? What are the limitations to increased production by increase in the speed of rotation of the drum, and what is the theoretical maximum quantity of slurry which can be handled?

Solution

For constant pressure filtration:

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v[V+(LA/v)]} = \frac{a}{V+b} \quad \text{(say)}$$

$$\therefore \quad V^2/2 + bV = at$$

For case 1: 1 rev. takes $1/0.005 = 200$ s and the rate $= V_1/200$.
For case 2: 1 rev. takes $1/0.008 = 125$ s and the rate $= V_2/125$.

But
$$\frac{V_1/200}{V_2/125} = \frac{6.0}{7.2}$$

$$\therefore \quad V_1/V_2 = 1.33 \quad \text{and} \quad V_2 = 0.75V_1$$

For case 1, using the filtration equation,

$$V_1^2 + 2bV_1 = 2a \times 200$$

and for case 2,

$$V_2^2 + 2bV_2 = 2a \times 125$$

Substituting $V_2 = 0.75V_1$ in these two equations allows the filtration constants to be found as

$$a = 0.00375V_1^2 \quad \text{and} \quad b = 0.25V_1$$

The rate of flow of wash water will equal the final rate of filtration so that for case 1:

Wash water rate $= a/(V_1 + b)$.

Wash water per rev. $\propto 200a/(V_1 + b)$.

Wash water/rev. per unit solids $\propto 200a/V_1(V_1 + b)$,

i.e. $\propto (200 \times 0.00375V_1^2)/V_1(V_1 + 0.25V_1)$

$\propto 0.6$

Similarly for case 2, the wash water per rev. per unit solids is proportional to:

$$125a/V^2(V_2+b)$$

$$\propto (125 \times 0.003\,75V_1^2)/0.75V_1(0.75V_1+0.25V_1)$$

$$\propto 0.625$$

Hence the % increase $= [(0.625-0.6)/0.6] \times 100$

$$= \underline{\underline{4.17\%}}$$

As $0.5V^2+bV=at$, the rate of filtration V/t is given by:

$$a/(0.5V+b)$$

The highest rate will be achieved when V tends to zero and $(V/t)_{max} = a/b$

$$= 0.003\,75V_1^2/0.25V_1$$

$$= 0.015V_1$$

For case 1, the rate $= V_1/200 = 0.005V_1$.

Hence the limiting rate is three times the original rate,

i.e. $\underline{\underline{18.0 \text{ kg/s}}}$

Problem 9.14

When an aqueous slurry is filtered in a plate and frame press, fitted with two 50 mm thick frames each 150 mm square at 450 kN/m^2, the frames are filled in 3·5 ks. The liquid in the slurry has the same density as water. The slurry is then filtered in a perforate basket centrifuge with a basket 300 mm diameter and 200 mm deep. If the radius of the inner surface of the slurry is maintained constant at 75 mm and the speed of rotation is 65 Hz, how long will it take to produce as much filtrate as was obtained from a single cycle of operations with the filter press?

Assume that the filter cake is incompressible and that the resistance of the cloth is equivalent to 3 mm of cake in both cases.

Solution

In the filter press

$$V^2 + 2(AL/v)V = 2(-\Delta P)A^2/r\mu v)t \quad (\because \quad V=0 \quad \text{when} \quad t=0)$$

(from equation 9.18)

Now $V = lA/v$

$$\therefore \quad l^2A^2/v^2 + 2(AL/v)(lA/v) = 2(-\Delta PA^2/r\mu v)t$$

$$\therefore \quad l^2 + 2Ll = 2(-\Delta Pv/r\mu)t$$

For one cycle:

$$l = 25\,\text{mm} = 0.025\,\text{m}, \quad L = 3\,\text{mm} = 0.003\,\text{m}$$

$$\Delta P = (450 - 101.3) = 348.7\,\text{kN/m}^2 \quad \text{or} \quad 3.49 \times 10^5\,\text{N/m}^2$$

$$t = 3500\,\text{s}$$

$$0.025^2 + (2 \times 0.003 \times 0.025) = 2 \times 3.49 \times 10^5 \times 3500(v/r\mu)$$

$$r\mu/v = 3.15 \times 10^{12}$$

In the centrifuge

$$(b^2 - b'^2)(1 + 2L/b) + 2b'^2 \ln(b'/b) = (2vt\rho\omega^2/r\mu)(b^2 - x^2) \quad \text{(equation } \underline{9.51}\text{)}$$

$$b = 0.15\,\text{m}, \quad H = 0.20\,\text{m}$$

Volume of cake $= 2 \times 0.050 \times 0.15^2 = 0.002\,25\,\text{m}^3$

$$\pi(b^2 - b'^2) \times 0.20 = 0.002\,25$$

$$(b^2 - b'^2) = 0.003\,58$$

$$b'^2 = 0.15^2 - 0.003\,58 = 0.0189\,\text{m}^2$$

$$b' = 0.138\,\text{m}$$

$$x = 75\,\text{mm} = 0.075\,\text{m}, \quad w = 65 \times 2\pi = 408.4\,\text{rad/s}$$

The time taken to produce the same volume of filtrate or cake as in one cycle of the filter press is therefore given by:

$$(0.15^2 - 0.138^2)(1 + 2 \times 0.003/0.15) + 2(0.0189)\ln(0.138/0.15)$$

$$= [2t \times 1000 \times 408.4^2/(3.15 \times 10^{12})](0.15^2 - 0.075^2)$$

$$0.003\,59 - 0.003\,15 = 1.787 \times 10^{-6}t$$

$$t = 4.4 \times 10^{-4}/1.787 \times 10^{-6}$$

$$= \underline{\underline{246\,\text{s}}} \quad (\simeq 4\,\text{min})$$

Problem 9.15

A rotary drum filter of area 3 m^2 operates with an internal pressure of 30 kN/m^2 and with 30% of its surface submerged in the slurry. Calculate the rate of production of filtrate and the thickness of cake when it rotates at 0.0083 Hz (0.5 rev/min), if the filter cake is incompressible and the filter cloth has a resistance equal to that of 1 mm of cake.

It is desired to increase the rate of filtration by raising the speed of rotation of the drum. If the thinnest cake that can be removed from the drum has a thickness of 5 mm, what is the maximum rate of filtration which can be achieved and what speed of rotation of the drum is required?

Voidage of cake = 0.4

Specific resistance of cake = 2×10^{12}/m^2

Density of solids = 2000 kg/m^3.
Density of filtrate = 1000 kg/m^3.
Viscosity of filtrate = 10^{-3} N s/m^2.
Slurry concentration = 20% by weight solids.

Solution

A 20% slurry contains 20 kg solids/80 kg solution.

Volume of cake = 20/2000(1 − 0·4) = 0·0167 m^3.

Volume of liquid in cake = 0·167 × 0·4 = 0·0067 m^3.

Volume of filtrate = (80/1000) − 0·0067 = 0·0733 m^3.

$$v = 0{\cdot}0167/0{\cdot}0733 = 0{\cdot}23$$

The rate of filtration is given by:

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v[V + (LA/v)]} \qquad \text{(equation 9.16)}$$

In this problem:

$$A = 3\,\mathrm{m}^2$$
$$\Delta P = 101{\cdot}3 - 30 = 71{\cdot}3\,\mathrm{kN/m^2} = 71{\cdot}3 \times 10^3\,\mathrm{N/m^2}$$
$$r = 2 \times 10^{12}/\mathrm{m}^2$$
$$\mu = 1 \times 10^{-3}\,\mathrm{N\,s/m^2}$$
$$v = 0{\cdot}23$$
$$L = 1\,\mathrm{mm} = 1 \times 10^{-3}\,\mathrm{m}$$

$$\frac{dV}{dt} = \frac{3^2 \times 71{\cdot}3 \times 10^3}{0{\cdot}23 \times 2 \times 10^{12} \times 1 \times 10^{-3}[V + (1 \times 10^{-3} \times 3/0{\cdot}23)]}$$

$$= \frac{1{\cdot}395 \times 10^3}{V + 0{\cdot}013}$$

From which $\quad V^2/2 + 0{\cdot}013V = 1{\cdot}395 \times 10^{-3}t$

If the speed = 0·0083 Hz, 1 revolution takes 120·5 s and a given piece is immersed for 120·5 × 0·3 = 36·2 s. When t = 36·2 s, V may be found by substitution to be 0·303 m^3.
Hence the rate of filtration = 0·303/120·5 = 0·0025 m^3/s.

Volume of filtrate for 1 rev = 0·303 m^3.

Volume of cake = 0·23 × 0·303 = 0·07 m^3.

$$\text{cake thickness} = 0{\cdot}07/3 = 0{\cdot}023\,\mathrm{m} = 23\,\mathrm{mm}$$

As the thinnest cake = 5 mm, volume of cake = 3 × 0·005 = 0·015 m^3.

As $v = 0.23$, volume of filtrate $= 3 \times 0.005/0.23 = 0.065\ m^3$

$$(0.065)^2/2 + 0.013 \times 0.065 = 1.395 \times 10^{-3} t$$

and

$$t = 2.12\ s$$

time for 1 rev $= 2.12/0.3 = 7.1$ s and speed $= \underline{\underline{0.14\ Hz}}$ (8.5 rev/min)

Maximum filtrate rate $= 0.065\ m^3$ in 7.1 s

$$= 0.065/7.1 = \underline{\underline{0.009\ m^3/s}}$$