AM 2C – Exame Epoca Especial 2023
Resolução

Felipe B. Pinto 61387 – MIEQB

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Conteúdo

Questão 1

$$\begin{cases} \cos(\pi + t_0) = 1 \\ \sin(\pi + t_0) = 0 \\ \sqrt{3}\, t_0 = 3\sqrt{3}\,\pi \implies t_0 = 3\,\pi \end{cases} \qquad r'(t) = \frac{\mathrm{d}\cos(\pi + t)}{\mathrm{d}x}\hat{i}\ldots$$

Questão 2

- Eq plano tg a sup

$$\boldsymbol{\nabla F x, y, z(x-a, y-b, z-c)=0}$$

Resposta

$$\nabla F(x,y,z)(x-a,y-b,z-c)=\begin{pmatrix} 3\,x^2+e^z, \\ 3\,y^2 \\ (x+1)\,e^z \end{pmatrix}(1,-1,0)=(4,3,2)\implies$$

$$\implies 4\,(x-1)+3\,(y+1)+2\,z=4\,x+3\,y+2\,z-1$$

Resposta D.

Questão 3

- Conjuntos
- Feixo, aderencia, pontos isolados

Questão 4

Integrais de curva: Paremtrizar do segimento e multiplicar pela derivada da parametrização

$$\int_{C,t} f(s)\,\mathrm{d}s = \int_a^b f(\phi(t))\,\|\phi'(t)\|\,\mathrm{d}t$$

$$\phi(t) = A + (B - A) = (0,0) + ((1,1) - (0,0))\,t = (1,1)\,t = (t,t) : t \in [0, 1$$

$$\implies \int_{C,t} f(s)\,\mathrm{d}s = \int_a^b f(\phi(t))\,\|\phi'(t)\|\,\mathrm{d}t = \int_0^1 f((t,t))\,\|(1,1)\|\,\mathrm{d}t = \int_0^1 ($$

Questão 5

- limites direcionais:

Resposta

Q5 a.

B

$$|f(x,y)-0| = \left|\frac{x\,y^2}{x^4+y^2}\right| = \frac{y^2\ |x|}{x^4+y^2} \leq \|(x,y)\|\,\sqrt{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x\,y^2}{x^4+y^2} = \lim_{r\to 0} \frac{r\ \cos(\theta)\ (r\ \sin(\theta))^2}{(r\ \cos(\theta))^4+(r\ \sin(\theta))^2} =$$

$$= \lim_{r\to 0} \frac{r\ \cos(\theta)\ \sin^2(\theta)}{r^2\ \cos^4(\theta)+\sin^2(\theta)} = \frac{0}{0+\sin^2(\theta)} = 0$$

$\therefore\ f$ é continua em $(0,0)$

Questão 6

$$\lim_{(x,y)\to(1,1)} \frac{3(y-1)\ \exp(-(x-1)^2)}{\sqrt{x-1}^2+(y-1)^2}$$

Resposta

$$\lim_{(x,y)\to(1,1)} \frac{3(y-1)\ \exp(-(x-1)^2)}{\sqrt{(x-1)^2+(y-1)^2}} =$$

$$= \lim_{x\to1, y=m(x-1)+1} \frac{3(m(x-1)+1-1)\ \exp(-(x-1)^2)}{\sqrt{(x-1)^2+(m(x-1)+1-1)^2}} =$$

$$= \lim_{x\to1, y=m(x-1)+1} \frac{3(m(x-1))\ \exp(-(x-1)^2)}{\sqrt{(m^2+1)(x-1)^2}} =$$

$$= \lim_{x\to1, y=m(x-1)+1} \frac{3\,m\ \exp(-(x-1)^2)}{\frac{|x-1|}{x-1}\sqrt{(m^2+1)}} =$$

$$= \lim_{x\to1, y=m(x-1)+1} \frac{3\,m\ \exp(-(x-1)^2)}{\frac{|x-1|}{x-1}\sqrt{(m^2+1)}} \implies$$

$$\implies \lim_{x\to1^+, y=m(x-1)+1} \frac{3\,m\ \exp(-(x-1)^2)}{\frac{|x-1|}{x-1}\sqrt{(m^2+1)}} = \frac{3\,m\ \exp(0)}{\sqrt{(m^2+1)}} = \frac{3\,m}{\sqrt{(m^2+1)}};$$

$$\lim_{x\to1^-, y=m(x-1)+1} \frac{3\,m\ \exp(-(x-1)^2)}{\frac{|x-1|}{x-1}\sqrt{(m^2+1)}} = \frac{3\,m\ \exp(0)}{-\sqrt{(m^2+1)}} = \frac{3\,m}{-\sqrt{(m^2+1)}}$$

Questão 7

$$\boldsymbol{f(x,y) = \begin{cases} \frac{x\,y^2}{x^4+y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{h*0^2}{h^4+0^2}}{h} = 0;$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = \lim_{k\to 0} \frac{\frac{0*k^2}{0^4+k^2}}{k} = 0;$$

$$D_{\vec{u}} f(0,0) = \lim_{t\to 0} \frac{f(0+t/\sqrt{2}, 0+t/\sqrt{2}) - f(0,0)}{t} =$$

$$= \lim_{t\to 0} \frac{\frac{(t/\sqrt{2})*(t/\sqrt{2})^2}{(t/\sqrt{2})^4+(t/\sqrt{2})^2}}{t} = \lim_{t\to 0} \frac{1/2\sqrt{2}}{t^2/4+1/2} = \frac{1/2\,\sqrt{2}}{1/2} = 1/\sqrt{2}$$

Questão 8

$$\boldsymbol{f : \mathbb{R}^2 \to \mathbb{R}; C^1 \in \mathbb{R}^2 : \nabla f(1/2, 0) = (-1, 1)}$$
$$\boldsymbol{H(x, y) = f\left(\frac{\sin y}{1 + x^2}, x + \cos(2\,y)\right)}$$

Resposta

$$H(x, y) = f\left(\frac{\sin y}{1 + x^2}, x + \cos(2\,y)\right) = f\left(\phi(x, y), \rho(x, y)\right);$$

$$\frac{\mathrm{d}H}{\mathrm{d}x}(1, \pi/2) = \frac{\mathrm{d}f}{\mathrm{d}\phi(x, y)}\left(\frac{\sin \pi/2}{1 + 1^2}, 1 + \cos(2\,\pi/2)\right)\frac{\mathrm{d}\phi}{\mathrm{d}x}(1, \pi/2) =$$
$$= -1\,\frac{\sin \pi/2}{(1 + 1^2)^2}\,2(1 + 1^2) = -1;$$

$$\frac{\mathrm{d}H}{\mathrm{d}x}(1, \pi/2) = \frac{\mathrm{d}f}{\mathrm{d}x}\left(\frac{\sin \pi/2}{1 + 1^2}, 1 + \cos(2\,\pi/2)\right)\frac{\mathrm{d}\frac{\sin y}{1+x^2}}{\mathrm{d}x}(1, \pi/2) =$$
$$= \frac{\mathrm{d}f}{\mathrm{d}x}\left(\frac{1}{2}, 0\right)\frac{\sin \pi/2}{(1 + 1^2)^2}\,2 * 1 = \frac{\mathrm{d}f}{\mathrm{d}x}\left(\frac{1}{2}, 0\right)\frac{\sin \pi/2}{(1 + 1^2)^2}\,2 * 1 = -2$$

Questão 9

Seja $\varphi : \mathbb{R} \to \mathbb{R}$ uma função de classe C^2 em $\mathbb{R}$ tal que $\varphi'(0) = 5, \varphi"(0) = -1$. Considere $u(x,t) = \varphi(x^2 - 2\,t)$. Tem-se:

Resposta

$$\frac{\partial^2 u}{\partial x^2}(2,2) = \frac{\partial^2(\varphi(x^2 - 2\,t))}{\partial x^2}(2,2) = \frac{\partial(\varphi'(x^2 - 2\,t)\,2\,x)}{\partial x}(2,2) =$$

$$= 2\,\left(\varphi"(2^2 - 2 * 2)\,2 + \varphi'(2^2 - 2 * 2)\right) =$$

$$= 2\,(-1 * 2 + 5) = 6$$

Questão 10

Seja

$$\int_0^2 \int_y^2 \exp(x^2)\ \mathrm{d}x\,\mathrm{d}y$$

Tem-se:

Sugestão: Troque a ordem de integração

Resposta

$$\begin{cases} y \in [0,2] \\ x \in [y,2] \end{cases}$$

$$\begin{cases} y \in [0,x] \\ x \in [0,2] \end{cases} \quad ;$$

$$\int_0^2 \int_y^2 \exp(x^2)\ \mathrm{d}x\,\mathrm{d}y = \int_0^2 \int_0^x \exp(x^2)\ \mathrm{d}y\,\mathrm{d}x = \int_0^2 \exp(x^2)\,(x-0)\,\mathrm{d}x =$$

$$= \int_0^2 \exp(x^2)\,2\,x\,\mathrm{d}x/2 = (\exp 2^2 - \exp(0^2))/2 = (\exp 4 - 1)/2$$

Questão 11

A equação

$$\exp(\boldsymbol{x}\,\boldsymbol{z}) + \boldsymbol{y}\ \sin \boldsymbol{x} - \boldsymbol{y}^{\mathbf{2}} + \boldsymbol{z}^{\mathbf{3}} + \mathbf{2}\,\boldsymbol{x} = \mathbf{2}\,\boldsymbol{\pi}$$

define implicitamente x como função de y e z numa vizinhança do ponto $(x_0, y_0, z_0) = (\pi, 1, 0)$. Para essa função tem-se:

Resposta

$$\frac{\partial x}{\partial y}(1,0) = -\frac{\frac{\partial f}{\partial y}(\pi,1,0)}{\frac{\partial f}{\partial x}(\pi,1,0)} = -\frac{\sin(\pi) - 2 * 1}{\exp(\pi * 0)\,0 + 1\ \cos\pi + 2} = 2$$

Questão 12

Considere a função

$$\boldsymbol{f(x,y) = y^4/2 - x\,y^2 + x^2 - 4\,x}$$

Escolha a affirmativa correta

Resposta

$$\det H_f = \begin{vmatrix} \frac{\partial^2 f}{\partial x\,\partial x} & \frac{\partial^2 f}{\partial x\,\partial y} \\ \frac{\partial^2 f}{\partial y\,\partial x} & \frac{\partial^2 f}{\partial y\,\partial y} \end{vmatrix} = \begin{vmatrix} 2 & 2\,y \\ -2\,y & 6\,y^2 - x\,2 \end{vmatrix} =$$
$$= 12\,y^2 - 4\,x + 4\,y^2 = 16\,y^2 - 4\,x;$$

$$\det H_f(4,-2) = 16\,(-2)^2 - 4 * 4 = 48$$
$\therefore$ Crítico mínimo Local;

$$\det H_f(2,0) = 16\,(0)^2 - 4 * 2 = -8$$
$\therefore$ Ponto de sela

Questão 13

Cnsidere a superfície

$$\rho = \left\{ (x,y,z) \in \mathbb{R}^3 : z = \sqrt{3}\,x, (x,y) \in [0,1] \times [0,2] \right\}$$

orientada com a terceira componente do campo vetorial normal não negativa, e o campo vetorial

$$\vec{F}(x,y,z) = -3\left(x^2 + \sqrt{z^2} 4\hat{\jmath} \right)$$

Seja $\mathcal{L}$ o bordo de σ orientado positivamente de acordo com σ. O valor do integral curvilíneo

$$\int_{\mathcal{L}} -\frac{z^3}{4}\,\mathrm{d}x + x^3\ \mathrm{d}z$$

body

Questão 14

Seja $f(x, y)$ uma função contínua em $\mathbb{R}^2$. Considere a igualdade

$$\iint_{\mathcal{R}} f(x,y)\ \mathrm{d}x\,\mathrm{d}y = \int_0^1 \int_{y^2}^{\sqrt{2-y^2}} f(x,y)\ \mathrm{d}x\,\mathrm{d}y$$

Tem se:

Resposta

$$\begin{cases} x = \sqrt{2-y^2} \implies |y| = \sqrt{2-x^2} \\ x = y^2 \implies \sqrt{x} = |y| \\ \text{Integra com} \end{cases}$$

$$\int_0^1 \int_{y^2}^{\sqrt{2-y^2}} f(x,y)\ \mathrm{d}x\,\mathrm{d}y =$$