AM 2C – Teste 2023

Felipe B. Pinto 61387 – MIEQB

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Conteúdo

Questão 1

Seja $\mathcal{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2, y \leq -1\}$ e r, θ as coordenadas polares. Tem se:

A. $\mathcal{D} = \{(x, y) = (r\ \cos\theta, r\ \sin\theta) : \pi\, 1/4 \leq \theta \leq \pi\, 3/4, -\sin^{-1}\theta \leq r \leq \sqrt{2}\}$

B. $\mathcal{D} = \{(x, y) = (r\ \cos\theta, r\ \sin\theta) : \pi\, 3/4 \leq \theta \leq \pi\, 5/4, +\sin^{-1}\theta \leq r \leq \sqrt{2}\}$

C. $\mathcal{D} = \{(x, y) = (r\ \cos\theta, r\ \sin\theta) : \pi\, 5/4 \leq \theta \leq \pi\, 7/4, -\sin^{-1}\theta \leq r \leq \sqrt{2}\}$

D. $\mathcal{D} = \{(x, y) = (r\ \cos\theta, r\ \sin\theta) : \pi\, 5/4 \leq \theta \leq \pi\, 7/4, +\sin^{-1}\theta \leq r \leq \sqrt{2}\}$

E. Nenhum dos casos anteriores

Resposta

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$x^2 + y^2 = (r\cos\theta)^2 + (r\sin\theta)^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta$$
$$\implies r \leq \pm\sqrt{2} \iff |r| \leq \sqrt{2}$$

$$y = r\sin\theta \leq \sqrt{2}\sin\theta \leq -1 \implies$$
$$\implies \theta \leq \sin^{-1}(-1/\sqrt{2}) = \sin^{-1}(-\sqrt{2}/2) = -\pi/4 = -\pi/4 + 2\pi = 7\pi/4$$

$$y = r\sin\theta \leq -1 \implies r \leq -1/\sin\theta$$

Questão 2

Plano tangente no ponto $0, 0, 1$

$$\boldsymbol{f(x, y) = e^{-y^2} + \sin(2\,x - y)}$$

Questão 3

Considere a função f. A curva de nível de valor -4 de f é

$$\boldsymbol{f(x,y) = \frac{y^2}{1-x^2}}$$

$$\frac{y^2}{1-x^2} = -4 \implies$$
$$\implies y^2 = -4 + (2\,x)^2 \implies$$
$$\implies -y^2/4 + x^2 = 1$$

$$|x| \neq 1$$

Questão 4

Seja $T(\rho, \theta, \phi)$ onde ρ, θ, ϕ são as c esfericas. a região do espaço limitada pelas superficies

$$\boldsymbol{T(\rho,\theta,\phi)} = \begin{pmatrix} \boldsymbol{\rho}\ \cos\boldsymbol{\theta}\ \sin\boldsymbol{\phi}, \\ \boldsymbol{\rho}\ \sin\boldsymbol{\theta}\ \sin\boldsymbol{\phi}, \\ \boldsymbol{\rho}\ \cos\boldsymbol{\phi} \end{pmatrix}$$

$$\boldsymbol{z = \sqrt{3 - x^2 - y^2}} \qquad \boldsymbol{z^2 = 3(x^2 + y^2)}$$

$$3 - x^2 - y^2 = \pm\left(3(x^2+y^2)\right) = \pm 3\,x^2 \pm 3\,y^2 \implies 3 = x^2(1 \pm 3) + y^2(1 \pm$$

$$\implies \begin{cases} 3 = +4\,(x^2+y^2) \\ 3 = -2\,(x^2+y^2) \end{cases}$$

$$3 = 4\,((x)^2 + y^2)$$

$$\rho\cos\phi = \sqrt{3 - (\rho\ \cos\theta\ \sin\phi)^2 - (\rho\ \sin\theta\ \sin\phi)^2} =$$

$$= \sqrt{3 - \rho^2\ \sin^2\phi\left(\cos^2\theta + \sin^2\theta\right)} = \sqrt{3 - \rho^2\ \sin^2\phi} \implies$$

$$\implies \rho^2\cos^2\phi = 3 - \rho^2\ \sin^2\phi \implies \rho^2(\cos^2\phi + \sin^2\phi)\rho^2 = 3 \implies \rho =$$

$$\rho^2\ \cos^2\theta = 3\left((\rho\ \cos\theta\ \sin\phi)^2 + (\rho\ \sin\theta\ \sin\phi)^2\right) = 3\left(\rho^2\ \cos^2\theta\ \sin^2\phi + \right.$$

$$3 - x^2 - y^2 = 3 - (\rho\ \cos\theta\ \sin\phi)^2 - (\rho\ \sin\theta\ \sin\phi)^2 = 3 - \rho^2\ \sin^2\theta \geq 0$$

$$\implies \rho \leq \sqrt{3}$$

$$3(x^2+y^2) = 3\left((\rho\ \cos\theta\ \sin\phi)^2 + (\rho\ \sin\theta\ \sin\phi)^2\right) = 3\left(\rho^2\ \cos^2\theta\ \sin^2\phi + \right.$$

$$\implies$$

Questão 5

Seja C a curva em $\mathbb{R}^3$ definida por

$$\boldsymbol{z = 8 - 4\,x^2 - y^2; \qquad z = 4}$$

tem-se:

$$4 = 8 - 4\,x^2 - y^2 \implies 4\,x^2 + y^2 = 4$$

$$4\,(2\,\cos t)^2 + \sin^2 t = 15\,\cos^2 t + \cos^2 t + \sin^2 t = 15\,\cos^2 t = 4 \implies$$

$$\implies t = \cos^{-1}\sqrt{(4/15)}$$

$$\nabla f(x,y)(1/2,\sqrt{3}) = \begin{pmatrix} -\,8\,x \\ -\,2\,y \end{pmatrix}(1/2,\sqrt{3}) = \begin{pmatrix} -\,4 \\ -\,2\sqrt{3} \end{pmatrix}$$

Questão 6

Comprimento da curva

$$\begin{cases} y = x^2 \\ x^2 + y^2 = 2 \end{cases}$$

Questão 7

Considere

$$\lim_{(x,y)\to(0,0)} \frac{-x^2 - y^2 - 2\,x^4}{x^2 + y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{-x^2 - y^2 - 2\,x^4}{x^2 + y^2} =$$

$$= \lim_{y\to 0}\lim_{x\to 0} \frac{-x^2 - y^2 - 2\,x^4}{x^2 + y^2} = \lim_{y\to 0} \frac{-y^2}{y^2} = \lim_{y\to 0} -1 = -1 =$$

$$= \lim_{x\to 0}\lim_{y\to 0} \frac{-x^2 - y^2 - 2\,x^4}{x^2 + y^2} = \lim_{x\to 0} \frac{-x^2 - 2\,x^4}{x^2} = \lim_{x\to 0} \frac{-x^2(1 + 2\,x^2)}{x^2} = \lim_{x\to 0}$$

$$= \lim_{x\to 0}\lim_{y\to x} \frac{-x^2 - y^2 - 2\,x^4}{x^2 + y^2} = \lim_{x\to 0} \frac{-x^2 - x^2 - 2\,x^4}{x^2 + x^2} = \lim_{x\to 0} -(1 - x^2) = -1$$

$$= \lim_{x\to 0}\lim_{y\to x^2} \frac{-x^2 - y^2 - 2\,x^4}{x^2 + y^2} = \lim_{x\to 0} \frac{-x^2 - x^4 - 2\,x^4}{x^2 + x^4} = \lim_{x\to 0} \frac{-x^2 - 3\,x^4}{x^2 + x^4} = \mathrm{li}$$

Questão 8

A eq do plano tg a sup no ponto $(1, -1, 0)$

$$\boldsymbol{y^3 + (x+1)\, e^z = 2 - x^3}$$

$(1, -1, 0) + \nabla(f(x))(1, -1, 0)$

Questão 9

Considere a função $f : \mathbb{R}^2 \to \mathbb{R}$ definia por

$$\boldsymbol{f(x,y) = \begin{cases} \frac{x^3 - y^5}{x^2 + 2\,y^4}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}}$$

$$\boldsymbol{\vec{u} = (2^{-1/2}, 2^{-1/2}) \qquad \nabla f(0,0) = (1, -1/2)}$$

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Questão 10

$\frac{\mathrm{d}f}{\mathrm{d}x}(0,0)$ e $\frac{\mathrm{d}f}{\mathrm{d}y}(0,1)$ de f

$$\boldsymbol{f(x,y) = \begin{cases} \frac{5\,x^3+3\,y^4}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}}$$

$$\frac{\mathrm{d}f}{\mathrm{d}x}(0,0) = \frac{15\,x^2\,(x^2+y^2) - (5\,x^3+3\,y^4)(2\,x)}{(x^2+y^2)^2}(0,0) =$$

$$= \frac{15\,x^4 - 10\,x^4 15\,x^2\,y^2 - 6\,y^4\,x}{x^4+2\,x^2\,y^2+y^4}(0,0) =$$

$$= \frac{5\,x^4 + 15\,x^2\,y^2 - 6\,x\,y^4}{x^4+2\,x^2\,y^2+y^4}(0,0) =$$

$$= \frac{5\,x^4}{x^4+2\,x^2\,y^2+y^4}(0,0) + \frac{15\,x^2\,y^2}{x^4+2\,x^2\,y^2+y^4}(0,0) + \frac{-6\,x\,y^4}{x^4+2\,x^2\,y^2+y^4}(0,0$$

$$= \frac{5}{1+2\,y^2/x^2+y^4/x^4}(0,0) + \frac{15}{x^2/y^2+2+y^2/x^2}(0,0) + \frac{-6\,x}{x^4/y^4+2\,x^2/y^2}$$

$$= \frac{5}{1+2+1}(0,0) + \frac{15}{1+2+1}(0,0) + \frac{-6\,x}{1+2+1}(0,0) =$$

$$= \frac{5}{4} + \frac{15}{4} = 5$$

$$\frac{\mathrm{d}f}{\mathrm{d}y}(0,1) = \frac{(12\,y^3)(x^2+y^2) - (5\,x^3+3\,y^4)(2\,y)}{x^4+2\,x^2\,y^2+y^4}(0,1) =$$

$$= \frac{12\,y^3\,x^2 + 12\,y^5 - 10\,x^3\,y - 6\,y^5}{x^4+2\,x^2\,y^2+y^4}(0,1) =$$

$$= \frac{+6\,y^5 12\,y^3\,x^2 - 10\,x^3\,y}{x^4+2\,x^2\,y^2+y^4}(0,1) =$$

$$= 6$$

Questão 15

$\frac{\mathrm{d}z}{\mathrm{d}x}$

$$\boldsymbol{F(x, y, z) = \frac{y}{z} - \frac{3\,x}{y} - \frac{2\,z}{x}}$$
$$\boldsymbol{z = z(x, y) : P(1, -1, 1) \land F(x, y, z) = 0}$$

$$0 = \frac{y}{z} - \frac{3\,x}{y} - \frac{2\,z}{x} = y^2\,x - 3\,x^2\,z - 2\,z^2\,y$$
$$0 = \frac{\mathrm{d}y^2\,x}{\mathrm{d}x}(1, -1, 1) - \frac{\mathrm{d}3\,x^2\,z}{\mathrm{d}x}(1, -1, 1) - \frac{\mathrm{d}2\,z^2\,y}{\mathrm{d}x}(1, -1, 1) = 1 - 6 - 3\frac{\mathrm{d}z}{\mathrm{d}x}($$
$$\implies \frac{\mathrm{d}z}{\mathrm{d}x}(1, -1) = 7$$