

# ALGA – Teste 1

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## Questão 1

$$\begin{bmatrix} 0 & 1 & 0 & 3 \\ -1 & 2 & 0 & 0 \\ 1 & -1 & 0 & 3 \end{bmatrix} \in \mathcal{M}_{3 \times 4}(\mathbb{R})$$

$$B : B_{(2)}^T = (4, 2, 3, -1) \wedge \exists BA$$

(i)

$$BA \in \mathcal{M}_{4 \times 4}$$

(ii)

$$(AB)_{(2,3)}^T = B_2^T A^{T(3)} = 1 * 4 + (-1) * 2 + 0 * 3 + 3 * (-1) = -1$$

(iii)

$$r(A) = 3$$

## Questão 2

$$\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow[l_2+ = -l_1]{l_1+ = -2l_2} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} -1 & 3 & -1 \\ 2 & -4 & 2 \end{bmatrix} \xrightarrow{l_2 < l_2/2l_2 < - > l_1} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

(v)

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{l_2+ = -l_1} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

### Questão 3

$$\begin{vmatrix} 0 & 1 & 2 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 1 & 4 & 0 & 0 \end{vmatrix}$$

$$= (-1)(-1)^{1+4} \begin{vmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \\ 1 & 4 & 0 \end{vmatrix} = (-2)(-1)^{2+3} \begin{vmatrix} -1 & 1 \\ 1 & 4 \end{vmatrix} = 2((-1) * 4 - 1 * 1) = -10$$

### Questão 4

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{\text{adj } B}{\det B} = (\det B)^{-1} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 2 \end{bmatrix}^T = -1 \begin{bmatrix} 2 & 1 & 2 \\ -1 & 0 & -1 \\ 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -2 \\ 1 & 0 & 1 \\ -3 & 1 & -2 \end{bmatrix}$$

(i)

$$\left( \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & 1 \\ -3 & 1 & -2 \end{bmatrix} \right)_{(2,2)} = 1 * 1 + 2 * 0 + 1 * 1 = 2 \neq 1$$

$$\begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & 1 \\ -3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

## Questão 5

$$\{A, B, C\} \in \mathcal{M}_{n \times n}(\mathbb{R})$$

(i)

$$AB = AC \wedge \det A \neq 0 \implies \exists A^{-1} \wedge A^{-1}AB = A^{-1}AC \implies B = C$$

(ii)

$$A^2 = AA = I \implies \det(A) \det(A) = \det(I) \implies \det(A)^2 = 1 \implies |\det(A)| = 1$$

(iii)

$$\implies \det B = \det(C^{-1}AC) = \det(C^{-1}) \det(A) \det(C) = \det(A)$$

## Questão 6

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \in \mathcal{M}_3(\mathbb{R}) \wedge \det A = k \neq 0$$

(i)

$$\det(A/2) = (1/2)^3 \det A = k/6$$

(ii)

$$\begin{aligned} \begin{vmatrix} -a & 2b & -c \\ d & -2e & f \\ -g & 2h & -i \end{vmatrix} &= (-1)(-1) \begin{vmatrix} a & 2bc \\ d & -2ef \\ g & -2hi \end{vmatrix} = \begin{vmatrix} a & d & g \\ -2b & -2e & -2h \\ c & f & i \end{vmatrix} = (-2) \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = \\ &= (-2) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -2k \end{aligned}$$

## Questão 7

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ -1 & -2 & \alpha + 1 & 1 & 1 \\ 2 & 3 & -1 & \alpha - 1 & \beta - 3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ -1 & -2 & \alpha+1 & 1 & 1 \\ 2 & 3 & -1 & \alpha-1 & \beta-3 \end{array} \right] \xrightarrow{\substack{l_3+ = l_1 \\ l_4+ = -l_1 \\ l_4+ = -l_2}} \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & \alpha & 1 & 0 \\ 1 & 0 & 0 & \alpha-3 & \beta-3 \end{array} \right]$$

(i)

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0-3 & 0-3 \end{array} \right] \xrightarrow{\substack{l_4+ = 3l_3 \\ l_2+ = -2l_3 \\ l_1+ = -l_4 \\ l_1+ = -2l_2 \\ l_1 < - - l_1 \\ l_1 < - > l_3 \\ l_1 < - > l_4}} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

(ii)

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -4 & -3 \end{array} \right] \xrightarrow{\substack{l_2+ = -2l_3 \\ l_4+ = 4l_3 \\ l_1+ = -2l_2 \\ l_4+ = -l_1 \\ l_3+ = l_4 \\ l_2+ = -2l_4 \\ l_1+ = 5l_4 \\ l_4 < - > l_3}} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

## Questão 8

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -3 & \beta - 3 \end{array} \right]$$

(i)

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -2 & -3 \end{array} \right] \xrightarrow{\begin{array}{l} l_2+ = l_4 \\ l_3+ = l_4/2 \\ l_1+ = l_3 \\ l_1+ = -2l_2 \\ l_1 < - - l_1 * 2 \\ l_2+ = -l_1 \\ l_3+ = -l_1/2 \\ l_4+ = -l_1 \\ l_4 < - - l_4/2 \end{array}} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$



## Questão 10

$$\begin{array}{ccccccc} A & \xrightarrow{l_2 < - > l_3} & A_1 & \xrightarrow{l_1 + = - 2 l_3} & A_2 & \xrightarrow{2 l_2} & B \\ & & C & \xrightarrow{2 l_2} & C_1 & \xrightarrow{l_2 + = - 2 l_3} & B \end{array}$$

(i)

$$E_3 E_2 E_1 A = E_3 E_2 E_1 B$$

(ii)

$$E_3 E_2 E_1 A = B = I \therefore A^{-1} = E_3 E_2 E_1$$

(iii)

$$A^T = (E_1^{-1} E_2^{-1} E_3^{-1} B)^T = (E_3^{-1})^T (E_2^{-1})^T (E_1^{-1})^T$$

(iv)