

Lista S1.Fisica I - Vectores

Felipe Pinto

April 1, 2021

Contents

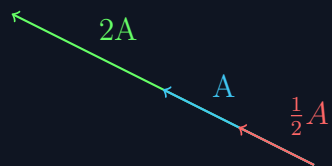
I	Questões	3
	Q1	3
	Q2	3
	a)	3
	b)	3
	c)	3
	Q3	3
	Q4	3
	a)	3
	b)	3
	c)	4
	Q5	4
	Q6	4
	Q7	4
	Q8	4

Q9		5
a)	5
b)	5
c)	5
II Problemas		6
P1		6
a)	6
b)	6
P2		6
a)	$a_x = 5; a_y = 4; a_z = -3$ $b_x = 3; b_y = -4; b_z = 5$	
	6
b)	6
c)	6
d)	6
P3		7
P4		7
a)	7
b)	7
c)	7
d)	7
e)	7
f)	7
P5		7
P6		8
P7		8

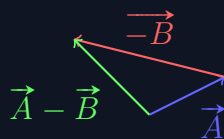
Part I

Questões

Q1

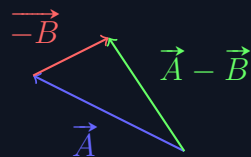


b)

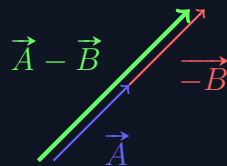


Q2

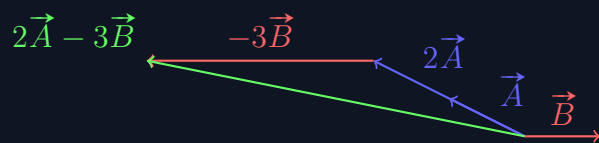
a)



c)



Q3



Q4

a)

$$\vec{A} = 5 \cos(130^\circ) \hat{i} + 5 \sin(130^\circ) \hat{j} \cong -3.2 \hat{i} + 3.8 \hat{j}$$

b)

$$\vec{B} = -4 \cos(30^\circ) \hat{i} - 4 \sin(30^\circ) \hat{j} = -2\sqrt{3} \hat{i} - 2 \hat{j}$$

c)

$$\vec{C} = 5 \cos(45^\circ) \hat{i} - 5 \sin(45^\circ) \hat{j} = \frac{5\sqrt{2}}{2} \hat{i} - \frac{5\sqrt{2}}{2} \hat{j}$$

Q5

$$\vec{D} = \vec{A} + \vec{B} + \vec{C} \cong (-3.2 - 2\sqrt{3} + 5\sqrt{2}/2) \hat{i} + (3.8 - 2 - 5\sqrt{2}/2) \hat{j} \cong -3.1 \hat{i} - 1.7 \hat{j}$$

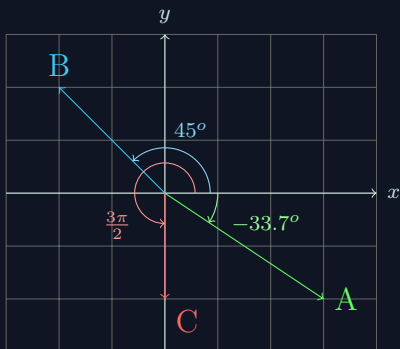
Q6

$$\text{Sim, } \{\forall V \subset \mathbb{R}^n : v_i = 0; i \in \mathbb{N}\} \implies \sum_{k=1}^n v_k^2 \geq 0 \implies \|\vec{V}\| \geq 0$$

Q7

$$\text{N\~ao, } \{\forall V \subset \mathbb{R}^n : v_i \neq 0; i \in \mathbb{N}\} \implies \sum_{k=1}^n v_k^2 > 0 \implies \|\vec{V}\| > 0$$

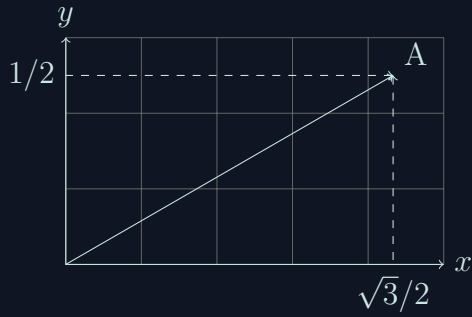
Q8



Q9

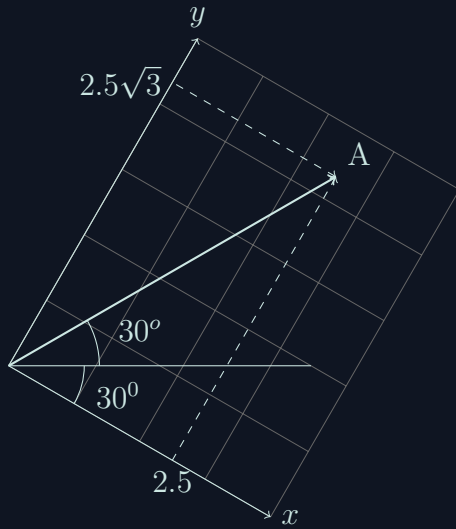
a)

$$\vec{A} = 5 \cos(30^\circ) \hat{i} + 5 \sin(30^\circ) \hat{j} = 2.5\sqrt{3} \hat{i} + 2.5 \hat{j}$$



b)

$$\vec{A} = 5 \cos(60^\circ) \hat{i} + 5 \sin(60^\circ) \hat{j} = 2.5 \hat{i} + 2.5\sqrt{3} \hat{j}$$



c)

$$\vec{A} = 5 \cos(-15^\circ) \hat{i} + 5 \sin(-15^\circ) \hat{j}$$

Part II

Problemas

P1

a)

$$\|\vec{a}\| = \sqrt{1 + 2^2 + 2^2} = 3$$

b)

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|} = 3^{-1} \hat{i} + 1.5^{-1} \hat{j} + 1.5^{-1} \hat{k}$$

P2

a)

$$a_x = 5; \quad a_y = 4; \quad a_z = -3$$

$$b_x = 3; \quad b_y = -4; \quad b_z = 5$$

$$\vec{c} = (30 - 9)\hat{i} + (24 + 12)\hat{j} + (-18 - 15)\hat{k} = 21\hat{i} + 36\hat{j} - 33\hat{k}$$

b)

$$\vec{a}^2 + \vec{b}^2 = 25 + 16 + 9 + 9 + 16 + 25 = 100$$

c)

$$\|\vec{a}\| \|\vec{b}\| \cos(\theta) = a_x b_x + a_y b_y + a_z b_z \implies \theta = \arccos\left(\frac{15 - 16 - 15}{\sqrt{50} \sqrt{50}}\right) \cong 108.66^\circ$$

d)

$$\|\vec{b}\| \cos(\theta) \hat{a} = \sqrt{50} \cos(108.66^\circ) \hat{a} \cong -2.26 \hat{a}$$

P3

$$\begin{aligned}\overrightarrow{PQ} &= \vec{Q} - \vec{P} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ \|\overrightarrow{PQ}\| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\end{aligned}$$

P4

a)

$$\begin{aligned}\vec{u} &= \sqrt{3} \cos(30^\circ) \hat{i} + \sqrt{3} \sin(30^\circ) \hat{j} + 0 \hat{k} &= 1.5 \hat{i} + \sqrt{3}/2 \hat{j} + 0 \hat{k} \\ \vec{v} &= \cos(60^\circ) \hat{i} + \sin(60^\circ) \hat{j} + 0 \hat{k} &= 0.5 \hat{i} + \sqrt{3}/2 \hat{j} + 0 \hat{k}\end{aligned}$$

b)

c)

$$\vec{u} + \vec{v} = 2\hat{i} + \sqrt{3}\hat{j} + 0\hat{k} \qquad \|\vec{u} + \vec{v}\| = \sqrt{2^2 + (\sqrt{3})^2 + 0^2} = \sqrt{7}$$

d)

e)

$$\vec{u} - \vec{v} = 1\hat{i} + 0\hat{j} + 0\hat{k} \qquad \|\vec{u} - \vec{v}\| = 1$$

f)

$$\vec{u} \cdot \vec{v} = 1.5 * 0.5 + \frac{\sqrt{3}}{2} * \frac{\sqrt{3}}{2} + 0 * 0 = 0.75 + 0.75 + 0 = 1.5$$

P5

$$\begin{aligned}\|\vec{r}\| &= \|\vec{a} + \vec{b} + \vec{c}\| = \|(37 \cos(30^\circ) + 25 \cos(60^\circ) + 30 \cos(135^\circ))\hat{i} + \\ &\quad + (37 \sin(30^\circ) + 25 \sin(60^\circ) + 30 \sin(135^\circ))\hat{j}\| = \|(18.5\sqrt{3} + 12.5 - 15\sqrt{2})\hat{i} \\ &\quad + (18.5 + 12.5\sqrt{3} + 15\sqrt{2})\hat{j}\| \cong \|23.33\hat{i} + 61.36\hat{j}\| \cong 65.65 \\ \theta_r &= \arccos(r_x/\|\vec{r}\|) \cong \arccos(23.33/65.65) \cong 69.18^\circ\end{aligned}$$

P6

$$\vec{d} = (80 \text{ Km}, -60^\circ) = 80 \text{ Km} \cos(60) \hat{e} - 80 \text{ Km} \sin(60) \hat{s} = 40 \text{ km} \hat{e} - 40 \sqrt{3} \hat{s}$$

P7

$$\|B_{ne} \widehat{ne}\| + \|B_{nw} \widehat{nw}\| - \|B_n \hat{n}\| = 160 (\cos(45^\circ) + \sin(45^\circ) - 1) \cong 66.27$$