SECTION 3

MOTION OF PARTICLES IN A FLUID

Problem 3.1

A finely ground mixture of galena and limestone in the proportion of 1 to 4 by weight is subjected to elutriation by an upwards current of water flowing at 5 mm/s. Assuming that the size distribution for each material is the same, and as shown by the following table, estimate the percentage of galena in the material carried away and in the material left behind. Take the absolute viscosity of water as 1 mN s/m^2 and use Stokes' equation.

Diameter (μm)	20	30	40	50	60	70	80	100
Percentage weight of undersize	15	28	48	54	64	72	78	88

Specific gravity of galena = 7·5; specific gravity of limestone = 2·7.

Solution

It is necessary to determine the size of particle which has a settling velocity equal to that of the upward flow of fluid, 5 mm/s.

Taking the largest particle, $d = 100 \times 10^{-6} = 10^{-4}$ m

$$\therefore \quad Re' = 5 \times 10^{-3} \times 10^{-4} \times 1000/1 \times 10^{-3} = 0{\cdot}5$$

Thus for the bulk of particles the flow will be within region a and the settling velocity is given by Stokes' equation (3.17):

$$u = (d^2 g/18\mu)(\rho_s - \rho)$$

For a particle of galena settling at 5 mm/s,

$$5 \times 10^{-3} = (d^2 \times 9{\cdot}81/18 \times 10^{-3})(7500 - 1000)$$

$$= 3{\cdot}54 \times 10^6 d^2$$

and $$d = 3{\cdot}76 \times 10^{-5}\ \text{m} = 37{\cdot}6\ \mu\text{m}$$

For a particle of limestone settling at 5 mm/s,

$$5 \times 10^{-3} = (d^2 \times 9{\cdot}81/18 \times 10^{-3})(2700 - 1000)$$

$$= 9{\cdot}27 \times 10^5 d^2$$

and $$d = 7{\cdot}35 \times 10^{-5}\ \text{m} = 73{\cdot}5\ \mu\text{m}$$

Thus particles of galena less than 37·6 μm and particles of limestone less than 73·5 μm will be removed in the water stream.

Interpolation of the data given shows that 43% of the galena and 74% of the limestone will be removed in this way.

In 100 kg feed, there is 20 kg galena and 80 kg limestone.

Therefore galena removed = (20 × 0·43) = 8·6 kg, leaving 11·4 kg, and limestone removed = (80 × 0·74) = 59·2 kg, leaving 20·8 kg.

Hence in *material removed*,

$$\text{percentage galena} = 8{\cdot}6 \times 100/(8{\cdot}6 + 59{\cdot}2) = \underline{\underline{12{\cdot}7\%}}$$

and in *material remaining*,

$$\text{percentage galena} = 11{\cdot}4 \times 100/(11{\cdot}4 + 20{\cdot}8) = \underline{\underline{35{\cdot}4\%}}$$

Problem 3.2

Calculate the terminal velocity of a steel ball, 2 mm diameter (density 7870 kg/m³) in oil (density 900 kg/m³, viscosity 50 mN s/m²).

Solution

For a sphere $(R'_0/\rho u_0^2)\,Re_0'^2 = (2d^3/3\mu^2)\,\rho(\rho_s - \rho)g$ (equation 3.28)

$$= (2 \times 0{\cdot}002^3/3 \times 0{\cdot}05^2)\,900(7870 - 900)\,9{\cdot}81$$

$$= 131{\cdot}3$$

$$\log_{10} 131{\cdot}3 = 2{\cdot}118$$

From Table 3.2, $\log_{10} Re'_0 = 0{\cdot}833$

$$\therefore \quad Re'_0 = 6{\cdot}80$$

$$\therefore \quad u_0 = 6{\cdot}80 \times 0{\cdot}05/(900 \times 0{\cdot}002)$$

$$= \underline{\underline{0{\cdot}189\ \text{m/s}}}$$

Problem 3.3

What will be the settling velocity of a spherical particle 0·40 mm diameter in an oil of specific gravity 0·82 and viscosity 10 mN s/m²? The specific gravity of steel is 7·87.

Solution

For a sphere

$$(R'_0/\rho u_0^2)\,Re_0'^2 = (2d^3\rho/3\mu^2)(\rho_s - \rho)g \qquad \text{(equation 3.28)}$$

$$= [2 \times 0{\cdot}0004^3 \times 820/3(10 \times 10^{-3})^2](7870 - 820)\,9{\cdot}81$$
$$= 24{\cdot}2$$
$$\log_{10} 24{\cdot}2 = 1{\cdot}384$$
$$\log_{10} Re_0' = 0{\cdot}222 \quad \text{(from Table 3.2)}$$
$$Re_0' = 1{\cdot}667$$
$$u_0 = 1{\cdot}667 \times 10 \times 10^{-3}/(0{\cdot}0004 \times 820)$$
$$= \underline{0{\cdot}051\ \text{m/s}} \quad (51\ \text{mm/s})$$

Problem 3.4

What will be the settling velocities of mica plates 1 mm thick and ranging in area from 6 to 600 mm^2 in an oil of specific gravity 0·82 and viscosity 10 mN s/m^2? The specific gravity of mica is 3·0.

Solution

	Smallest particles	Largest particles
A'	$= 6 \times 10^{-6}\ \text{m}^2$	$6 \times 10^{-4}\ \text{m}^2$
d_p	$= \sqrt{(4 \times 6 \times 10^{-6}/\pi)} = 2{\cdot}76 \times 10^{-3}\ \text{m}$	$\sqrt{(4 \times 6 \times 10^{-4}/\pi)} = 2{\cdot}76 \times 10^{-2}\ \text{m}$
d_p^3	$2{\cdot}103 \times 10^{-8}\ \text{m}^3$	$2{\cdot}103 \times 10^{-5}\ \text{m}^3$
volume	$6 \times 10^{-9}\ \text{m}^3$	$6 \times 10^{-7}\ \text{m}^3$
k'	0·285	0·0285

$$(R_0'/\rho u^2)\,Re_0'^2 = (4k'/\mu^2\pi)(\rho_s - \rho)\,\rho d_p^3 g \qquad \text{(equation 3.36)}$$

$V_{sphere} = \frac{4}{3}\pi r^3$

$A_{sphere} = \pi r^2$

$= \frac{\pi d^2}{4}$

$$= (4 \times 0{\cdot}285/\pi \times 0{\cdot}01^2)(3000 - 820)\,820 \times 2{\cdot}103 \times 10^{-8} \times 9{\cdot}81$$
$$= 1340 \text{ for smallest particle and } 134{,}000 \text{ for largest particle}$$

	Smallest particles	Largest particles	
$\log_{10}(R_0'/\rho u^2)\,Re_0'^2$	3·127	5·127	
$\log_{10} Re_0'$	1·581	2·857	(from Table 3 2)
Correction from Table 3.4	−0·038	−0·300	(estimated)
Corrected $\log_{10} Re_0'$	1·543	2·557	
Re_0'	34·9	361	
u	<u>0·154 m/s</u>	<u>0·159 m/s</u>	

Thus it is seen that all the mica particles settle at approximately the same velocity.

Problem 3.5

A material of specific gravity 2·5 is fed to a size separation plant where the separating fluid is water which rises with a velocity of 1·2 m/s. The upward vertical component of the velocity of the particles is 6·0 m/s. How far will an approximately spherical particle, 6 mm diameter, rise relative to the walls of the plant before it comes to rest in the fluid?

Solution

Initial velocity of particle relative to fluid, $v = (6{\cdot}0 - 1{\cdot}2) = 4{\cdot}8$ m/s

$$Re' = (6 \times 10^{-3} \times 4{\cdot}8 \times 1000)/1 \times 10^{-3} = 28{,}800$$

When the particle has been retarded to such a velocity that $Re' = 500$, the minimum value for which equation 3.76 is applicable,

$$\dot{y} = (4{\cdot}8 \times 500/28{,}800) = 0{\cdot}083 \text{ m/s}$$

When Re' is greater than 500, the relation between the displacement of the particle y and time t is:

$$y = -(1/c)\ln(\cos fct - (v/f)\sin fct) \qquad \text{(equation 3.76)}$$

where

$$c = (0{\cdot}33/d)(\rho/\rho_s) = (0{\cdot}33/6 \times 10^{-3})(1000/2500) = 22{\cdot}0 \qquad \text{(equation 3.62)}$$

$$f = \sqrt{\{[d(\rho_s - \rho)g]/0{\cdot}33\rho\}} = \sqrt{[(6 \times 10^{-3} \times 1500 \times 9{\cdot}81)/0{\cdot}33 \times 1000]} = 0{\cdot}517 \qquad \text{(equation 3.75)}$$

$$v = -4{\cdot}8 \text{ m/s}$$

Thus

$$y = -(1/22{\cdot}0)\ln[\cos 0{\cdot}517 \times 22t + (4{\cdot}8/0{\cdot}517)\sin 0{\cdot}517 \times 22t]$$

$$= -0{\cdot}0455\ln(\cos 11{\cdot}37t + 9{\cdot}28\sin 11{\cdot}37t)$$

$$\therefore\ \dot{y} = -0{\cdot}0455\left(\frac{-11{\cdot}37\sin 11{\cdot}37t + 9{\cdot}28 \times 11{\cdot}37\cos 11{\cdot}37t}{\cos 11{\cdot}37t + 9{\cdot}28\sin 11{\cdot}37t}\right)$$

$$= -\frac{0{\cdot}517(9{\cdot}28\cos 11{\cdot}37t - \sin 11{\cdot}37t)}{\cos 11{\cdot}37t + 9{\cdot}28\sin 11{\cdot}37t}$$

The time taken for the velocity of the particle relative to the fluid to fall from 4·8 m/s to 0·083 m/s is given by:

$$-0{\cdot}083 = 0{\cdot}517(9{\cdot}28\cos 11{\cdot}37t - \sin 11{\cdot}37t)/(\cos 11{\cdot}37t + 9{\cdot}28\sin 11{\cdot}37t)$$

i.e. $\cos 11{\cdot}37t + 9{\cdot}28\sin 11{\cdot}37t = -6{\cdot}23\sin 11{\cdot}37t + 57{\cdot}8\cos 11{\cdot}37t$

i.e. $56{\cdot}8\cos 11{\cdot}37t = 15{\cdot}51\sin 11{\cdot}37t$

$\therefore$ $\sin 11{\cdot}37t = 3{\cdot}66\cos 11{\cdot}37t$

Squaring $$1 - \cos^2 11{\cdot}37t = 13{\cdot}4 \cos^2 11{\cdot}37t$$

$\therefore$ $$\cos 11{\cdot}37t = 0{\cdot}264$$

$\therefore$ $$\sin 11{\cdot}37t = \sqrt{(1 - 0{\cdot}264^2)} = 0{\cdot}965$$

The distance moved by the particle relative to the fluid during this period is therefore given by:

$$y = -0{\cdot}0455 \ln (0{\cdot}264 + 9{\cdot}28 \times 0{\cdot}965)$$
$$= -0{\cdot}101 \text{ m}$$

If equation 3.76 were applied for a relative velocity down to zero, the time taken for the particle to come to rest would be given by:

$$9{\cdot}28 \cos 11{\cdot}37t = \sin 11{\cdot}37t$$

Squaring, $$1 - \cos^2 11{\cdot}37t = 86{\cdot}1 \cos^2 11{\cdot}37t$$

$\therefore$ $$\cos 11{\cdot}37t = 0{\cdot}107$$

and $$\sin 11{\cdot}37t = \sqrt{(1 - 0{\cdot}107^2)} = 0{\cdot}994$$

The corresponding distance the particle moves relative to the fluid is then given by:

$$y = -0{\cdot}0455 \ln (0{\cdot}017 + 9{\cdot}28 \times 0{\cdot}994)$$
$$= -0{\cdot}102 \text{ m}$$

i.e. the particle moves only a very small distance with a velocity of less than 0·083 m/s.

If form drag were neglected for all velocities less than 0·083 m/s, the distance moved by the particle would be given by equation 3.53:

$$y = (b/a)t + (v/a) - (b/a^2) + \left(\frac{b}{a^2} - \frac{v}{a}\right) e^{-at}$$

and $$\dot{y} = \frac{b}{a} - \left(\frac{b}{a} - v\right) e^{-at}$$

where $$a = 18\mu/d^2\rho_s = (18 \times 0{\cdot}001)/(0{\cdot}006^2 \times 2500) \qquad \text{(equation 3.51)}$$
$$= 0{\cdot}20$$

$$b = (1 - \rho/\rho_s)g = (1 - 1000/2500)\, 9{\cdot}81 = 5{\cdot}89 \qquad \text{(equation 3.52)}$$

$\therefore$ $$b/a = 29{\cdot}43$$

and $$v = -0{\cdot}083 \text{ m/s}$$

Thus $$y = 29{\cdot}43t - \left(\frac{0{\cdot}083}{0{\cdot}20} + \frac{29{\cdot}43}{0{\cdot}20}\right)(1 - e^{-0{\cdot}20t})$$

$$= 29{\cdot}43t - \frac{29{\cdot}51}{0{\cdot}20}(1 - e^{-0{\cdot}20t})$$

$$\dot{y} = 29{\cdot}43 - 29{\cdot}51\, e^{-0{\cdot}20t}$$

When the particle comes to rest in the fluid, $y = 0$ and

$$e^{-0.20t} = 29.43/29.51$$

$$\therefore \quad t = 0.0141 \text{ s}$$

The corresponding distance moved by the particle is given by:

$$y = 29.43 \times 0.0141 - (29.51/0.20)(1 - e^{-0.20 \times 0.0141})$$
$$= 0.41442 - 0.41550 = -0.00108 \text{ m}$$

Thus, whether the resistance force is calculated by equation 3.9 or equation 3.13, the particle moves a negligible distance with a velocity relative to the fluid of less than 0.083 m/s. Further, the time is also negligible and thus the fluid also has moved only a very small distance.

It can therefore be taken that the particle moves through 0.102 m before it comes to rest in the fluid. The time taken for the particle to move this distance is given by equation 3.79, on the assumption that the drag force corresponds to that given by equation 3.13. The time is therefore given by:

$$\cos 11.37t = 0.264 \qquad \text{(equation 3.79)}$$

$$\therefore \quad 11.37t = 1.304$$

and

$$t = 0.115 \text{ s}$$

The distance travelled by the fluid in this time $= (1.2 \times 0.115) = 0.138$ m. Thus the total distance moved by the particle relative to the walls of the plant

$$= (0.102 + 0.138) = \underline{\underline{0.240 \text{ m}}}$$

Problem 3.6

A spherical glass particle is allowed to settle freely in water. If the particle starts initially from rest and if the value of the Reynolds number (Re') with respect to the particle is 0.1 when it has attained its terminal velocity, calculate:

(a) the distance travelled before the particle reaches 90% of its terminal velocity, and
(b) the time which has elapsed when the acceleration of the particle is one-hundredth of its initial value.

Solution

When $Re' < 0.2$, the terminal velocity is given by equation 3.17:

$$u_0 = (d^2 g/18\mu)(\rho_s - \rho)$$

Taking the densities of glass and water as 2750 and 1000 kg/m^3 respectively and the viscosity of water as 0.001 Ns/m^2,

$$u_0 = (9.81d^2/18 \times 0.001)(2750 - 1000)$$
$$= 9.54 \times 10^5 d^2 \text{ m/s}$$

The Reynolds number $Re' = 0{\cdot}1$ and substituting for u,

$$d(9{\cdot}54 \times 10^5 d^2) \times 1000/0{\cdot}001 = 0{\cdot}1$$

$$\therefore \quad d = 4{\cdot}76 \times 10^{-5}\ \text{m}$$

Now $\quad a = 18\mu/d^2\rho_s = (18 \times 0{\cdot}001)/(4{\cdot}76 \times 10^{-5})^2 \times 2750 = 2889/\text{s}$

and $\quad b = (1 - \rho/\rho_s)g = (1 - 1000/2750)\,9{\cdot}81 = 6{\cdot}24\ \text{m/s}^2$

In equation 3.53

$$y = \frac{b}{a}t + \cancel{\frac{v}{a}} - \frac{b}{a^2} + \left(\frac{b}{a^2} - \frac{v}{a}\right)e^{-at}$$

In this case $v = 0$ and on differentiating:

$$\dot{y} = \frac{b}{a}(1 - e^{-at})$$

$$y = u_0 t - \frac{u_0}{a} + \frac{u_0}{a} e^{-at}$$

$$y = u_0 t - \frac{u_0}{a}(1 - e^{-at})$$

or, since $b/a = u$, the terminal velocity,

$$\dot{y} = u(1 - e^{-at})$$

When $\dot{y} = 0{\cdot}9u$, $\quad 0{\cdot}9 = (1 - e^{-2889t})$

$$\therefore \quad 2889t = 2{\cdot}303$$

$$\therefore \quad t = 8{\cdot}0 \times 10^{-4}\ \text{s}$$

Thus in equation 3.53:

$$y = (6{\cdot}24 \times 8{\cdot}0 \times 10^{-4}/2889) - (6{\cdot}24/2889^2) + (6{\cdot}24/2889^2)\exp(-2889 \times 8{\cdot}0 \times 10^{-4})$$

$$= 1{\cdot}73 \times 10^{-6} - 7{\cdot}52 \times 10^{-7} + 7{\cdot}513 \times 10^{-8}$$

$$= 1{\cdot}053 \times 10^{-6}\ \text{m} \quad \text{or} \quad \underline{\underline{1{\cdot}05\ \text{mm}}}$$

From equation 3.50: $\quad \ddot{y} = b - a\dot{y}$

At the start of the fall, $\dot{y} = 0$ and the initial acceleration, $\ddot{y} = b$.

When $\ddot{y} = 0{\cdot}01b$,

$$0{\cdot}01b = b - a\dot{y}$$

or $\quad \dot{y} = (0{\cdot}89 \times 6{\cdot}24)/2889$

$$= 0{\cdot}00214\ \text{m/s}$$

$$\therefore \quad 0{\cdot}00214 = (6{\cdot}24/2889)(1 - e^{-2889t})$$

$$2889t = 4{\cdot}605$$

$$\therefore \quad \underline{\underline{t = 0{\cdot}0016\ \text{s}}}$$

Problem 3.7

In a hydraulic jig, a mixture of two solids is separated into its components by subjecting an aqueous slurry of the material to a pulsating motion and allowing the particles to settle for a series of short-time intervals such that they do not attain their terminal falling velocities. It is desired to separate materials of specific gravities 1·8 and 2·5 whose particle size ranges from 0·3 to 3 mm diameter. It may be assumed that the particles are approximately spherical and that Stokes' law is applicable. Calculate approximately the maximum time interval for which the particles may be allowed to settle so that no particle of the less dense material falls a greater distance than any particle of the denser material.

Viscosity of water = 1 mN s/m^2.

Solution

For Stokes' law to apply, $Re' < 0{\cdot}2$ and resistance is due to skin friction only. Equation 3.53 may be used:

$$y = \frac{b}{a}t + \frac{v}{a} - \frac{b}{a^2} + \left(\frac{b}{a^2} - \frac{v}{a}\right)e^{-at}$$

or, assuming the initial velocity $v = 0$,

$$y = \frac{b}{a}t - \frac{b}{a^2} + \frac{b}{a^2}e^{-at}$$

where $b = (1 - \rho/\rho_s)g$ and $a = 18\mu/d^2\rho_s$.

For small particles of the dense material,

$$b = (1 - 1000/2500)\,9{\cdot}81 = 5{\cdot}89\ \text{m/s}^2$$

$$a = (18 \times 0{\cdot}001)/(0{\cdot}3 \times 10^{-3})^2 2500 = 80/\text{s}$$

For large particles of the light material,

$$b = (1 - 1000/1800)\,9{\cdot}81 = 4{\cdot}36\ \text{m/s}^2$$

$$a = (18 \times 0{\cdot}001)/(3 \times 10^{-3})^2 1800 = 1{\cdot}11/\text{s}$$

In order that these particles should fall the same distance, in equation 3.53:

$$(5{\cdot}89/80)t - (5{\cdot}89/80^2)(1 - e^{-80t}) = (4{\cdot}36/1{\cdot}11)t - (4{\cdot}36/1{\cdot}11^2)(1 - e^{-1{\cdot}11t})$$

$$\therefore \quad 3{\cdot}8504t + 3{\cdot}5316\,e^{-1{\cdot}11t} - 0{\cdot}00092\,e^{-80t} = 3{\cdot}5307$$

and solving by trial and error,

$$\underline{\underline{t = 0{\cdot}01\ \text{s}}}$$

Problem 3.8

Two spheres of equal terminal velocity settle in water starting from rest at the same horizontal level. How far apart vertically will the particles be when they have both reached their terminal falling velocities? Assume Stokes' law is valid and then check the assumption.

Data

	Density (kg/m³)	Viscosity (mN s/m²)	Diameter (μm)
Particle 1	1500		40
Particle 2	3000		
Water	1000	1	

Solution

Assuming Stokes' law is valid, the terminal velocity is given by equation 3.17:

$$u_t = (d^2g/18\mu)(\rho_s - \rho)$$

For particle 1,

$$u_t = [(40 \times 10^{-6})^2 \times 9{\cdot}81/(18 \times 1 \times 10^{-3})](1500 - 1000)$$
$$= 4{\cdot}36 \times 10^{-4}\ \text{m/s}$$

Since particle 2 has an equal terminal velocity:

$$4{\cdot}36 \times 10^{-4} = [(d_2^2 \times 9{\cdot}81)/(18 \times 1 \times 10^{-3})](3000 - 1000)$$

From which,

$$d_2 = 2 \times 10^{-5}\ \text{m} \quad \text{or} \quad 20\ \mu\text{m}$$

From equation 3.51:

$$a = 18\mu/d^2\rho_s$$

and for particle 1,

$$a_1 = 18 \times 1 \times 10^{-3}/(40 \times 10^{-6})^2 \times 1500 = 7{\cdot}5 \times 10^3/\text{s}$$

and for particle 2,

$$a_2 = 18 \times 1 \times 10^{-3}/(20 \times 10^{-6})^2 \times 3000 = 1{\cdot}5 \times 10^4/\text{s}$$

From equation 3.52:

$$b = (1 - \rho/\rho_s)g$$

and for particle 1, $b_1 = (1 - 1000/1500)9{\cdot}81 = 3{\cdot}27\ \text{m/s}^2$

and for particle 2, $b_2 = (1 - 1000/3000)9{\cdot}81 = 6{\cdot}54\ \text{m/s}^2$

The initial velocity of both particles, $v = 0$ and from equation 3.53:

$$y = \frac{b}{a}t - \frac{b}{a^2} + \frac{b}{a^2}e^{-at}$$

Differentiating,

$$\dot{y} = \frac{b}{a}(1 - e^{-at})$$

or, from equation 3.17:

$$\dot{y} = u_t(1 - e^{-at})$$

When $\dot{y} = u_t$, the terminal velocity, it is not possible to solve for t and hence $\dot{y}$ will be taken as $0{\cdot}99u_t$.

For particle 1:

$$0{\cdot}99 \times 4{\cdot}36 \times 10^{-4} = (4{\cdot}36 \times 10^{-4})[1 - \exp(-7{\cdot}5 \times 10^3 t)]$$

$$\therefore \quad t = 6{\cdot}14 \times 10^{-4}\,\text{s}$$

The distance travelled in this time is given by equation 3.53:

$$y = (3{\cdot}27/7{\cdot}5 \times 10^3)\,6{\cdot}14 \times 10^{-4} - [3{\cdot}27/(7{\cdot}5 \times 10^3)^2]$$
$$\times [1 - \exp(-7{\cdot}5 \times 10^3 \times 6{\cdot}14 \times 10^{-4})]$$
$$= 2{\cdot}10 \times 10^{-7}\,\text{m}$$

For particle 2:

$$0{\cdot}99 \times 4{\cdot}36 \times 10^{-4} = (4{\cdot}36 \times 10^{-4})[1 - \exp(-1{\cdot}5 \times 10^4 t)]$$

$$\therefore \quad t = 3{\cdot}07 \times 10^{-4}\,\text{s}$$

and

$$y = (6{\cdot}54/1{\cdot}5 \times 10^4)\,3{\cdot}07 \times 10^{-4} - [6{\cdot}54/(1{\cdot}5 \times 10^4)^2]$$
$$\times [1 - \exp(-1{\cdot}5 \times 10^4 \times 3{\cdot}07 \times 10^{-4})]$$
$$= 1{\cdot}03 \times 10^{-7}\,\text{m}$$

Particle 2 reaches its terminal velocity after $3{\cdot}07 \times 10^{-4}$ s and it then travels at $4{\cdot}36 \times 10^{-4}$ m/s for a further $(6{\cdot}14 \times 10^{-4} - 3{\cdot}07 \times 10^{-4}) = 3{\cdot}07 \times 10^{-4}$ s during which time it travels a further $(3{\cdot}07 \times 10^{-4} \times 4{\cdot}36 \times 10^{-4}) = 1{\cdot}338 \times 10^{-7}$ m.

Thus the total distance moved by particle 1 $= 2{\cdot}10 \times 10^{-7}$ m

and the total distance moved by particle 2 $= (1{\cdot}03 \times 10^{-7} + 1{\cdot}338 \times 10^{-7})$

$$= 2{\cdot}368 \times 10^{-7}\,\text{m}$$

The distance apart when both particles have attained their terminal velocities

$$= (2{\cdot}368 \times 10^{-7} - 2{\cdot}10 \times 10^{-7}) = \underline{\underline{2{\cdot}68 \times 10^{-8}\,\text{m}}}$$

For Stokes' law to be valid, Re' must be less than 0·2 when the terminal velocities are attained:

for particle 1,

$$Re = (40 \times 10^{-6} \times 4{\cdot}36 \times 10^{-4} \times 1500)/(1 \times 10^{-3}) = 0{\cdot}026$$

and for particle 2,

$$Re = (20 \times 10^{-6} \times 4{\cdot}36 \times 10^{-4} \times 3000)/(1 \times 10^{-3}) = 0{\cdot}026$$

and the law does apply.

Problem 3.9

The size analysis of a powder is carried out by sedimentation in a vessel having the sampling point 180 mm below the liquid surface. If the viscosity of the liquid is $1{\cdot}2\,\text{mN s/m}^2$, and the densities of the powder and liquid are 2650 and $1000\,\text{kg/m}^3$ respectively, determine the time which must elapse before any sample will exclude particles larger than 20 μm.

If turbulent conditions occur when the Reynolds number is greater than 0·2, what is the approximate maximum size of particle to which Stokes' law can be applied under the above conditions?

Solution

The problem involves obtaining the time taken for a 20 μm particle to fall below the sampling point, i.e. 180 mm. Assuming that skin friction is the only resistance, equation 3.53 may be used, taking the initial velocity $v = 0$:

$$y = bt/a - b/a^2(1 - e^{-at})$$

where
$$b = g(1 - \rho/\rho_s) = 9{\cdot}81(1 - 1000/2650) = 6{\cdot}108\,\text{m/s}^2$$
$$a = 18\mu/d^2\rho_s = (18 \times 1{\cdot}2 \times 10^{-3})/(20 \times 10^{-6})^2 \times 2650$$
$$= 20{,}377/\text{s}$$

In this case $y = 180\,\text{mm} = 0{\cdot}180\,\text{m}$

$$\therefore \quad 0{\cdot}180 = (6{\cdot}108/20{,}377)t - (6{\cdot}108/20{,}377^2)(1 - e^{-20.377t})$$
$$= 0{\cdot}0003t + 1{\cdot}4071 \times 10^{-8} e^{-20.377t}$$

Ignoring the exponential term as being negligible,

$$t = 0{\cdot}180/0{\cdot}0003$$
$$= \underline{\underline{600\,\text{s}}}$$

The velocity is given by differentiating equation 3.53:

$$\dot{y} = \frac{b}{a}(1 - e^{-at})$$

When $t = 600$ s,

$$\dot{y} = [(6{\cdot}108d^2 \times 2650)/(18 \times 0{\cdot}0012)]\{1 - \exp[-(18 \times 0{\cdot}0012 \times 600)/d^2 \times 2650]\}$$
$$= 7{\cdot}49 \times 10^5 d^2[1 - \exp(-4{\cdot}89 \times 10^{-3} d^{-2})]$$

For $Re' = 0{\cdot}2$,

$$d(7{\cdot}49 \times 10^5 d^2)[1 - \exp(-4{\cdot}89 \times 10^{-3} d^{-2})] \times 2650/0{\cdot}0012 = 0{\cdot}2$$
$$\therefore \quad 1{\cdot}65 \times 10^{12} d^3[1 - \exp(-4{\cdot}89 \times 10^{-3} d^{-2})] = 0{\cdot}2$$

As d will be small, the exponential term is negligible and

$$d^3 = 1{\cdot}212 \times 10^{-13}$$
$$d = 5{\cdot}46 \times 10^{-5}\,\text{m} = \underline{\underline{54{\cdot}6\,\mu\text{m}}}$$

Problem 3.10

Calculate the distance a spherical particle of lead shot of diameter (d) 0·1 mm will settle in a glycerol/water mixture before it reaches 99% of its terminal falling velocity.

Density of lead = 11,400 kg/m³.

Density of liquid = 1000 kg/m³.

Viscosity of liquid (μ) = 10 mN s/m².

Assume that the resistance force can be calculated from Stokes' law and is equal to $3\pi\mu du$, where u is the velocity of the particle relative to the liquid.

Solution

The terminal velocity, when Stokes' law applies, is given by:

$$\tfrac{1}{6}\pi d^3(\rho_s - \rho)g = 3\pi\mu du$$

$$\therefore \quad u_0 = \frac{d^2 g}{18\mu}(\rho_s - \rho) \qquad \text{(equation 3.17)}$$

$$= \frac{d^2\rho_s}{18\mu} g(1 - \rho/\rho_s)$$

$$= b/a$$

where $b = g(1 - \rho/\rho_s) = 9{\cdot}81(1 - 1000/11{,}400) = 8{\cdot}95\ \text{m/s}^2$

and $a = 18\mu/d^2\rho_s = (18 \times 10 \times 10^{-3})/(0{\cdot}1 \times 10^{-3})^2\, 11{,}400$

$$= 1579/\text{s}$$

$$\therefore \quad u_0 = 8{\cdot}95/1579 = 5{\cdot}67 \times 10^{-3}\ \text{m/s}$$

When 99% of this velocity is attained,

$$\dot{y} = 0{\cdot}99 \times 5{\cdot}67 \times 10^{-3}$$

$$= 5{\cdot}61 \times 10^{-3}\ \text{m/s}$$

Assuming the initial velocity v is zero, equation 3.53 may be differentiated to give:

$$\dot{y} = (b/a)(1 - e^{-at})$$

$$\therefore \quad 5{\cdot}61 \times 10^{-3} = 5{\cdot}67 \times 10^{-3}(1 - e^{-1579t})$$

and $t = 0{\cdot}0029$ s

Substituting in equation 3.53:

$$y = (b/a)t - (b/a^2)(1 - e^{-at})$$

$$= (5{\cdot}67 \times 10^{-3} \times 0{\cdot}0029) - (5{\cdot}67 \times 10^{-3}/1579)(1 - e^{-1579 \times 0{\cdot}0029})$$

$$= 1{\cdot}644 \times 10^{-5} - 3{\cdot}59 \times 10^{-6} \times 9{\cdot}89 \times 10^{-1}$$

$$= 1{\cdot}29 \times 10^{-5}\ \text{m} \quad \text{or} \quad \underline{\underline{0{\cdot}013\ \text{mm}}}$$

Problem 3.11

Find the weight of a sphere of material of specific gravity 7·5 which falls with a steady velocity of 0·6 m/s in a large deep tank of water.

Solution

In equation 3.33:

$$\frac{R'_0}{\rho u_0^2} Re_0'^{-1} = \frac{2\mu g}{3\rho^2 u_0^3}(\rho_s - \rho)$$

Taking the density and viscosity of water as 1000 kg/m³ and 0·001 N s/m² respectively,

$$(R'_0/\rho u_0^2)/Re'_0 = [(2 \times 0{\cdot}001 \times 9{\cdot}81)/(3 \times 1000^2 \times 0{\cdot}6^3)](7500 - 1000)$$

$$= 0{\cdot}000\,197$$

$\therefore$ $$\log_{10}(R'_0/\rho u_0^2)/Re'_0 = \bar{4}{\cdot}296$$

From Table 3.3, $$\log_{10} Re'_0 = 3{\cdot}068$$

$\therefore$ $$Re'_0 = 1169{\cdot}5$$

$\therefore$ $$d = (1169{\cdot}5 \times 0{\cdot}001)/(0{\cdot}6 \times 1000)$$

$$= 0{\cdot}001\,95\text{ m} \equiv 1{\cdot}95\text{ mm}$$

and the weight $$= \pi d^3 \rho_s/6$$

$$= \pi \times 0{\cdot}001\,95^3 \times 7500/6$$

$$= 2{\cdot}908 \times 10^{-5}\text{ kg} \quad \text{or} \quad \underline{\underline{0{\cdot}029\text{ g}}}$$

Problem 3.12

Two ores, of specific gravities 3·7 and 9·8, are to be separated in water by a hydraulic classification method. If the particles are all of approximately the same shape and each is sufficiently large for the drag force to be proportional to the square of the velocity in the fluid, calculate the maximum ratio of sizes which can be separated if the particles attain their terminal velocities. Explain why a wider range of sizes can be separated if the time of settling is so small that the particles do not reach their terminal velocities.

Obtain an explicit expression for the distance through which a particle will settle in a given time if it starts from rest and if the resistance force is proportional to the square of the velocity. The acceleration period is to be taken into account.

Solution

If the total drag force is proportional to the square of the velocity, when the terminal velocity u is attained:

$$F = k_1 u^2 d_p^2$$

since the area is proportional to d_p^2 and the accelerating force $= (\rho_s - \rho) g k_2 d_p^3$ where k_2 is a constant depending on the shape of the particle and d_p is a mean projected area.

When the terminal velocity is reached,

$$k_1 u^2 d_p^2 = (\rho_s - \rho) g k_2 d_p^3$$

$$\therefore \quad u = [(\rho_s - \rho) g k_3 d_p]^{0.5}$$

In order to achieve separation, the terminal velocity of the smallest particle (diameter d_1) of the dense material must be at least equal to that of the largest particle (diameter d_2) of the light material. That is:

$$[(9800 - 1000) 9.81 k_3 d_1]^{0.5} = [(3700 - 1000) 9.81 k_3 d_2]^{0.5}$$

$$\therefore \quad (d_2/d_1) = 8800/2700$$

$$\therefore \quad = \underline{\underline{3.26}}$$

which is the maximum range of sizes which can be separated if the terminal velocities are attained.

If the particles are allowed to settle in the fluid for only a very short time, they will not attain their terminal falling velocities and a better degree of separation can be obtained. A particle of the denser material will have an initial acceleration $g(1 - \rho/\rho_s)$ because there is no fluid friction when the relative velocity is zero. Thus the initial velocity is a function of density only and is unaffected by size and shape. A very small particle of the denser material will therefore always commence settling at a greater rate than a large particle of the less dense material. Theoretically it should be possible to separate materials completely irrespective of the size range provided that the periods of settling are sufficiently short. In practice the required periods will often be so short that it is impossible to make use of this principle alone. As the time of settling increases the larger particles of the less dense material catch up and overtake the smaller particles of the denser material.

If the total drag force is proportional to the velocity squared, i.e. $\dot{y}^2$, then the equation of motion for a particle falling downwards under the influence of gravity may be written as:

$$m\ddot{y} = mg(1 - \rho/\rho_s) - k_1 \dot{y}^2$$

$$\therefore \quad \ddot{y} = g(1 - \rho/\rho_s) - (k_1/m) \dot{y}^2$$

or

$$\ddot{y} = b - c\dot{y}^2$$

where $b = g(1 - \rho/\rho_s)$, $c = k_1/m$, and k_1 is a proportionality constant.

$$\therefore \quad d\dot{y}/(b - c\dot{y}^2) = dt$$

or

$$d\dot{y}/(f^2 - \dot{y}^2) = c\,dt$$

where $f = \sqrt{(b/c)}$.

Integrating, $(1/2f) \ln[(f + \dot{y})/(f - \dot{y})] = ct + k_4$

When $t = 0$, $\quad \dot{y} = 0$ and $k_4 = 0$

$$\therefore \quad (1/2f) \ln[(f + \dot{y})/(f - \dot{y})] = ct$$

$$(f + \dot{y})/(f - \dot{y}) = e^{2fct}$$

$$f - \dot{y} = 2f/(1 + e^{2fct})$$

$$y = ft - 2f \int dt/(1 - e^{2fct})$$

$$y = ft - (1/c)\ln[e^{2fct}/(1 + e^{2fct})] + k_5$$

When $t = 0$, $\quad y = 0$ and $k_5 = (1/a)\ln 0{\cdot}5$

$$\therefore \quad \underline{\underline{y = ft - (1/c)\ln(0{\cdot}5\,e^{2fct})/(1 + e^{2fct})}}$$

where $f = \sqrt{(b/c)}$, $b = g(1 - \rho/\rho_s)$, and $c = k_1/m$.

Problem 3.13

Salt, of specific gravity 2·35, is charged to the top of a reactor containing a 3 m depth of aqueous liquid (specific gravity 1·1 and viscosity 2 mN s/m^2) and the crystals must dissolve completely before reaching the bottom. If the rate of dissolution of the crystals is given by the relation:

$$-\mathrm{d}d/\mathrm{d}t = 3 \times 10^{-4} + 2 \times 10^{-4}u$$

where d is the size of the crystal (cm) at time t (s) and u its velocity in the fluid (cm/s); calculate the maximum size of crystal which can be charged. The inertia of the particles can be neglected and the resistance force can be taken as given by Stokes' law ($3\pi\mu du$), d being taken as the equivalent spherical diameter of the particle.

Solution

Assuming the salt always travels at its terminal velocity, then for the Stokes' law region, this is given by equation 3.17:

$$u_0 = (d^2 g/18\mu)(\rho_s - \rho)$$

or, in this case,

$$u_0 = (d^2 \times 9{\cdot}81/18 \times 2 \times 10^{-3})(2350 - 1100)$$

$$= 3{\cdot}46 \times 10^5 d^2 \text{ m/s}$$

The rate of dissolution,

$$-\mathrm{d}d/\mathrm{d}t = 3 \times 10^{-6} + 2 \times 10^{-4}u \text{ m/s}$$

and substituting,

$$\mathrm{d}d/\mathrm{d}t = -3 \times 10^{-6} - (2 \times 10^{-4} \times 3{\cdot}406 \times 10^5 d^2)$$

$$= -3 \times 10^{-6} - 68{\cdot}1 d^2$$

The velocity at any point h from the top of the reactor is $u = \mathrm{d}h/\mathrm{d}t$.

$$\frac{\mathrm{d}h}{\mathrm{d}d} = \frac{\mathrm{d}h}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}d} = 3{\cdot}406 \times 10^5 d^2/(-3 \times 10^{-6} - 68{\cdot}1 d^2)$$

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and
$$\int_0^3 \mathrm{d}h = -\int_D^0 \frac{3{\cdot}406 \times 10^5 d^2\,\mathrm{d}d}{3 \times 10^{-6} + 68{\cdot}1 d^2}$$

$$\therefore \qquad 3 = 3{\cdot}406 \times 10^5 \left(\int_0^D \frac{\mathrm{d}d}{B} - \frac{A}{B^2} \int_0^D \frac{\mathrm{d}d}{(A/B) + d^2} \right)$$

where $A = 3 \times 10^{-6}$ and $B = 68{\cdot}1$.

$$\therefore \qquad 3 = 3{\cdot}406 \times 10^5 \left\langle \left[\frac{d}{B} \right]_0^D - \left[\frac{A}{B^2} \frac{1}{\sqrt{(A/B)}} \tan^{-1}\left(\frac{d}{\sqrt{(A/B)}} \right) \right]_0^D \right\rangle$$

$$\therefore \qquad 3 = (3{\cdot}406 \times 10^5/B)[D - (A/B)^{1/2} \tan^{-1} D(A/B)^{-1/2}]$$

Substituting for A and B,

$$D = 6 \times 10^{-4} + 2{\cdot}1 \times 10^{-4} \tan^{-1}(4{\cdot}76 \times 10^3 D)$$

and solving by trial and error,

$$D = 8{\cdot}8 \times 10^{-4}\ \mathrm{m} \quad \text{or} \quad \underline{\underline{0{\cdot}88\ \mathrm{mm}}}$$

The integration may also be carried out numerically and the working is as follows:

d	d^2	$\left(\dfrac{3{\cdot}406 \times 10^5 d^2}{3 \times 10^{-6} + 68{\cdot}1 d^2}\right)$	Interval of d	Mean value of function in interval	Integral in interval	Total integral
0	0	0				
			1×10^{-4}	$4{\cdot}63 \times 10^2$	0·0463	0·0463
1×10^{-4}	1×10^{-8}	$9{\cdot}25 \times 10^2$				
			1×10^{-4}	$1{\cdot}65 \times 10^3$	0·1653	0·2116
2×10^{-4}	4×10^{-8}	$2{\cdot}38 \times 10^3$				
			1×10^{-4}	$2{\cdot}86 \times 10^3$	0·2869	0·4985
3×10^{-4}	9×10^{-8}	$3{\cdot}358 \times 10^3$				
			1×10^{-4}	$3{\cdot}64 \times 10^3$	0·364	0·8625
4×10^{-4}	$1{\cdot}6 \times 10^{-7}$	$3{\cdot}922 \times 10^3$				
			1×10^{-4}	$4{\cdot}09 \times 10^3$	0·409	1·2715
5×10^{-4}	$2{\cdot}5 \times 10^{-7}$	$4{\cdot}25 \times 10^3$				
			1×10^{-4}	$4{\cdot}35 \times 10^3$	0·435	1·706
6×10^{-4}	$3{\cdot}6 \times 10^{-7}$	$4{\cdot}46 \times 10^3$				
			1×10^{-4}	$4{\cdot}52 \times 10^3$	0·452	2·158
7×10^{-4}	$4{\cdot}9 \times 10^{-7}$	$4{\cdot}589 \times 10^3$				
			1×10^{-4}	$4{\cdot}634 \times 10^3$	0·463	2·621
8×10^{-4}	$6{\cdot}4 \times 10^{-7}$	$4{\cdot}679 \times 10^3$				
			1×10^{-4}	$4{\cdot}709 \times 10^3$	0·471	3·09
9×10^{-4}	$8{\cdot}1 \times 10^{-7}$	$4{\cdot}74 \times 10^3$				

From which $D = \underline{\underline{0{\cdot}9\ \mathrm{mm}}}$.

The acceleration of the particle to its terminal velocity has been neglected, and in practice the time to reach the bottom of the reactor would be slightly longer, allowing a larger crystal to dissolve completely.

Problem 3.14

A balloon weighing 7 g is charged with hydrogen to a pressure of 104 kN/m^2. The balloon is released from ground level and, as it rises, hydrogen escapes in order to maintain a constant differential pressure of 2·7 kN/m^2 under which condition the diameter of the balloon is 0·3 m. If conditions are assumed to remain isothermal at 273 K as the balloon rises, what is the ultimate height reached and how long does it take to rise through the first 3000 m?

It may be assumed that the value of the Reynolds number with respect to the balloon exceeds 500 throughout, so that the resistance coefficient is constant at 0·22. Neglect the inertia of the balloon, i.e. assume that it is rising at its equilibrium velocity at any moment.

Solution

Volume of balloon = $(4/3)\pi(0{\cdot}15)^3 = 0{\cdot}0142\ \text{m}^3$.

Mass of balloon = 7 g or 0·007 kg.

The upthrust = weight of air at p N/m^2 − weight of hydrogen at $(p + 2700)$ N/m^2.

If ρ_a is the density of air at 101,300 N/m^2 and 273 K (28·9/22·4) = 1·29 kg/m^3, where the mean molecular weight of air is taken as 28·9 kg/mol, then the net upthrust force W is given by:

$$W = 9{\cdot}81\,\{0{\cdot}0142[(\rho_a p/101{,}300) - \rho_a(2/28{\cdot}9)(p + 2700)/101{,}300] - 0{\cdot}007\}$$

$$= 0{\cdot}139[0{\cdot}000\,012\,7p - 0{\cdot}000\,000\,881(p + 2700)] - 0{\cdot}0687$$

$$= 0{\cdot}000\,001\,64p - 0{\cdot}0690\ \text{N}$$

The balloon will stop when $W = 0$, that is when

$$p = 0{\cdot}0690/0{\cdot}000\,001\,64 = 42{,}092\ \text{N/m}^2$$

The variation of pressure with height is given by:

$$g\,dz + v\,dp = 0$$

But $\quad v = (1/\rho_a)(101{,}300/p)\ \text{m}^3$ for isothermal conditions

$\therefore \quad dz + [101{,}300/(9{\cdot}81 \times 1{\cdot}29p)]\,dp = 0$

$\therefore \quad z_2 - z_1 = 8005 \ln(101{,}300/p)$

When $p = 42{,}092$ N/m^2,

$$z_2 - z_1 = 8005 \ln(101{,}300/42{,}092)$$

$$= \underline{\underline{7030\ \text{m}}}$$

The resistance force R on the balloon is given by:

$$(R/\rho_a u^2) = 0{\cdot}22$$

or $\quad R = 0{\cdot}22\rho_a(p/101{,}300)(\pi \times 0{\cdot}3^2/4)(dz/dt)^2\ \text{N/m}^2$

$$= 1{\cdot}98 \times 10^{-7} p(dz/dt)^2$$

This must be equal to the net upthrust force W, or:

$$0.000\,001\,64p - 0.0690 = 1.98 \times 10^{-7}p(\mathrm{d}z/\mathrm{d}t)^2$$

$$\therefore \quad (\mathrm{d}z/\mathrm{d}t)^2 = 8.28 - 3.49 \times 10^5/p$$

But
$$z = 8005 \ln(101{,}300/p)$$

$$\therefore \quad (\mathrm{d}z/\mathrm{d}t)^2 = 8.28 - (3.49 \times 10^5\,\mathrm{e}^{z/8005}/101{,}300)$$

$$\therefore \quad (\mathrm{d}z/\mathrm{d}t) = 1.89(2.41 - \mathrm{e}^{1.25 \times 10^{-4}z})^{0.5}$$

The time taken to rise 3000 m is therefore given by:

$$t = (1/1.89)\int_0^{3000} \mathrm{d}z/(2.41 - \mathrm{e}^{1.25 \times 10^{-4}z})^{0.5}$$

Writing the integral as
$$I = \int_0^{3000} \mathrm{d}z/(a - \mathrm{e}^{bz})^{0.5}$$

and putting
$$(a - \mathrm{e}^{bz}) = x^2$$

$$\therefore \quad \mathrm{d}z = 2x\,\mathrm{d}x/[b(a - x^2)]$$

and
$$I = (-2/b)\int \mathrm{d}x/(a - x^2)$$

$$= (-2/b)(1/2\sqrt{a})\left[\ln \frac{\sqrt{a} - \sqrt{(a - \mathrm{e}^{bz})}}{\sqrt{a} + \sqrt{(a - \mathrm{e}^{bz})}}\right]_0^{3000}$$

$$= (1/b\sqrt{a}) \ln \frac{[\sqrt{a} - \sqrt{(a - \mathrm{e}^{3000b})}][\sqrt{a} + \sqrt{(a-1)}]}{[\sqrt{a} + \sqrt{(a - \mathrm{e}^{3000b})}][\sqrt{a} - \sqrt{(a-1)}]}$$

Now,
$$a = 2.41 \quad \text{and} \quad b = 1.25 \times 10^{-4}$$

$$\therefore \quad I = 5161 \ln[(1.55 - 0.977)/(1.55 + 0.977)][(1.55 - 1.19)/(1.55 + 1.19)]$$

$$= 2816$$

$$\therefore \quad t = [2816(1/1.89)]$$

$$= \underline{\underline{1490\,\mathrm{s}}} \quad (25\,\mathrm{min})$$

Problem 3.15

A mixture of quartz of specific gravity 3·7 and galena of specific gravity 9·8 whose size range is 0·3 to 1 mm is to be separated by a sedimentation process. If Stokes' law is assumed to be applicable, what is the minimum density required for the liquid if the particles all settle at their terminal velocities?

Consideration was given to devising a separating system using water as the liquid. In this case the particles were to be allowed to settle for a series of short-time intervals so that the smallest particle of galena settled a larger distance than the largest particle of quartz. What approximately is the maximum permissible settling period?

According to Stokes' law the resistance force F acting on a particle of diameter d settling at a velocity u in a fluid of viscosity μ is given by:

$$F = 3\pi\mu d u$$

Viscosity of water = 1 mN s/m².

Solution

For streamline conditions, equation 1.31 applies:

$$d_B/d_A = [(\rho_A - \rho)/(\rho_B - \rho)]^{0.5}$$

For separation it is necessary that a large particle of the less dense material does not overtake a small particle of the dense material, i.e.

$$(1/0{\cdot}3) = [(9800 - \rho)/(3700 - \rho)]^{0.5}$$

$$\therefore \quad \rho = \underline{\underline{3097\ \text{kg/m}^3}}$$

Assuming Stokes' law is valid, the distance travelled including the period of acceleration is given by equation 3.53:

$$y = (b/a)t + (v/a) - (b/a^2) + [(b/a^2) - (v/a)]\,e^{-at}$$

When the initial velocity $v = 0$,

$$y = (b/a)t + (b/a^2)(e^{-at} - 1)$$

where $\quad b = g(1 - \rho/\rho_s) \quad$ (equation 3.52)

and $\quad a = 18\mu/d^2\rho_s$

For a small particle of galena:

$$b = 9{\cdot}81(1 - 1000/9800) = 8{\cdot}81\ \text{m/s}^2$$

$$a = (18 \times 1 \times 10^{-3})/[(0{\cdot}3 \times 10^{-3})^2 \times 9800] = 20{\cdot}4/\text{s}$$

For a large particle of quartz:

$$b = 9{\cdot}81(1 - 1000/3700) = 7{\cdot}15\ \text{m/s}^2$$

$$a = (18 \times 1 \times 10^{-3})/[(1 \times 10^{-3})^2 \times 3700] = 4{\cdot}86/\text{s}$$

In order to achieve separation, these particles must travel at least the same distance in time t or:

$$(8{\cdot}81/20{\cdot}4)t + (8{\cdot}81/20{\cdot}4^2)(e^{-20{\cdot}4t} - 1) = (7{\cdot}15/4{\cdot}86)t + (7{\cdot}15/4{\cdot}86^2)(e^{-4{\cdot}86t} - 1)$$

$$\therefore \quad (0{\cdot}0212\,e^{-20{\cdot}4t} - 0{\cdot}303\,e^{-4{\cdot}86t}) = 1{\cdot}039t - 0{\cdot}282$$

Solving by trial and error, $\quad \underline{\underline{t = 0{\cdot}05\ \text{s}}}$