OSF – Particle Size Reduction and Enlargement

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Conteúdo

1 Introduction

Our objective is to take a feed spend energy applying unit operations to it so that we get a product with smaller particles

Unit operations

Normally a crusher or a mill is an unitary operation that receives a feed and spends energy to reduce to a product

The whole process takes several stages of size reduction each specifying on reducing its specific feed particle sizes:

1. Coarse reduction
2. intermediate reduction
3. Fine reduction

Particle properties

- **Size of the feed:** energy needed is very different, caracterizes reduction in:
  - coarse
  - interediate
  - fine
- **Compressive strength:** minimum compressive strength that causes solid fracture,
- **Brittleness (fragilidade):** Poor capacity to resistst impact and vibration of load
  Most of inorganic non-metalic materials are brittle materials (ex: glass)
  - High compressive strength
  - low tensile strength
- **Stickiness:** adherence to equipment
  causes considerable difficulty in size reduction
- **Soapiness:** how low is the friction coefficient of the material surface
- **Humidity:** how wet (water) is the solid
  between $(5 \rightarrow 50)$ % solids tend to cake
- **Friability:** Measures the tendency for a solid to break when handling
  - A **crystaline** material will break along well defined planes
  - Energy to break $\propto$ Size^{-1}

Types of forces

Prasher suggest that four **basic patterns** may be identified.

Impact: Particle concussion by a single rigid force.

Compression: Particle disintegration by two rigid forces.

Shear: Produced by a fluid or by particle-particle interaction.

Attrition: Arising from particles scrapping against one another or against a rigid surface.

2 Size reduction of Solids

2.3 Energy Requirement for Size Reduction

Although its impossible to estimate the accurate amount of energy required in order to effect size reduction of a given material, a number of empirical laws have been proposed:

Law	Accurate for type of grinding	p	L_0	L
Kick's	Coarse	-1.0	$(50 \to 1500)\,\mathrm{mm}$	$(5 \to 50)\,\mathrm{mm}$
Bonds	Intermediate	-1.5	$(2 \to 50)\,\mathrm{mm}$	$(0.1 \to 5)\,\mathrm{mm}$
Rittinger's	Fine	-2.0	$(2 \to 5)\,\mathrm{mm}$	$< 0.1\,\mathrm{mm}$

These laws may all derive from the basic differential equation:

$$\frac{\mathrm{d}\boldsymbol{E}}{\mathrm{d}\boldsymbol{L}} = -\boldsymbol{C}\,\boldsymbol{L}^{\boldsymbol{p}}$$

Which states that the **energy** $\mathrm{d}E$ required to effect a small change in **size** $\mathrm{d}L$ of unit mass of material is a simple power function of the size $(-C\,L^p)$

Rittinger's Law

$$\boldsymbol{E = C\ \Delta\left(L^{-1}\right) = k_r\ f_c\ \Delta\left(L^{-1}\right)}$$

Since **surface** of unit mass of material $\propto L^{-1}$, the interpretation of this law is that $E \propto$ **increase of surface area**

Rittinger's law is applicable mainly to that part of the process where a new surface is being created and holds most accurately for **fine grinding** ($L_0 = (2 \to 5)\ \mathrm{mm} \wedge L < 0.1\ \mathrm{mm}$) where increase in surface per unit mass of material is large.

$$\frac{\mathrm{d}E}{\mathrm{d}L} = -C\,L^p \wedge p = -2.0 \implies$$

$$\implies \int \mathrm{d}E = E = \int -C\,L^{-2}\ \mathrm{d}L = C\ \Delta(L^{-1}) = K_R\,f_c\ \Delta(L^{-1})$$

Kick's Law

$$\boldsymbol{E = -C \;\Delta \ln L = -K_K \, f_C \;\Delta \ln L}$$

This supposes that energy required is directly related to the reduction ratio L_0/L

Kick's law more closely relates to the energy required to effect elatic deformation before fracure occurs, and is more accurate than Rittinger's law for coarse crushing where the amount of surface produced is considerably less.

$$\frac{\mathrm{d}E}{\mathrm{d}L} = -C\,L^p \wedge p = -1.0 \implies$$
$$\implies \int \mathrm{d}E = E = \int -C\,L^{-1}\,\mathrm{d}L = -C\;\Delta \ln L = -K_K\,f_c\;\Delta \ln L$$

Bond's Law

$$\boldsymbol{E = 2\,C\ \Delta(1/\sqrt{L}) = E_i\sqrt{\frac{100}{L}}\left(1-\frac{1}{\sqrt{q}}\right);\quad q=\frac{L_0}{L}}$$

Bond calls E_i **the work index**, and express it as the amoun t of **energy required to reduce** the unit mass of material from size $\infty \to 100\ \mu\mathrm{m}$, that is $q=\infty$. the size of the material is take as the size of the square hole through 80% if the material will pass.

$$\frac{\mathrm{d}E}{\mathrm{d}L} = -C\,L^p \wedge p = -1.5 \implies$$

$$\implies \int \mathrm{d}E = E = \int -C\,L^{-1.5}\,\mathrm{d}L = 2\,C\ \Delta\left(\frac{1}{\sqrt{L}}\right);\quad C = 5\,E_i \wedge q = \frac{L_0}{L} \implies$$

$$\implies E = 2 * 5\,E_i\left(\frac{1}{\sqrt{L}} - \frac{1}{\sqrt{q}\,L}\right) = E_i\sqrt{\frac{100}{L}}\left(1-\frac{1}{\sqrt{q}}\right)$$

Exemplo 1

A material is crushed in a blake jaw cruser such that the average size of particle is reduced from $(50.000 \to 10.000)$ mm with the consumption of energy of $13.0\,\mathrm{kW/(kg/s)}$. What would be the consumption of energy needed to crush the same material of an average size of 75 mm to an average size of 25.000 mm

which of these results would be regarded as more reliable and why?

Resposta

Kick's law for the average sizes being grinded are common for fine reduction which's kick's law is more accurate

E1 a)

Assuming Rittinger's law applies?

Resposta

$$E = K_R \, f_c \, \Delta L^{-1} =$$
$$= 162.5 * (25^{-1} - 75^{-1}) \, \mathrm{kW/(kg/s)} = 4.333 \, \mathrm{kW/(kg/s)};$$

$$K_R \, f_c = \frac{E}{L^{-1} - L_0^{-1}} = \frac{13.0}{10^{-1} - 50^{-1}} \frac{\mathrm{kW/(kg/s)}}{\mathrm{mm}} = 162.5 \frac{\mathrm{kW/(kg/s)}}{\mathrm{mm}}$$

E1 b)

Assuming Kick's law applies?

Resposta

$$E = -K_K\, f_c\, \Delta \ln L \cong$$
$$\cong -8.077\ \ln 25/75\, \mathrm{kW/(kg/s)} \cong 8.874\, \mathrm{kW/(kg/s)};$$

$$-\,K_K\, f_c = \frac{E}{\ln L/L_0} = \frac{13.0}{\ln 10/50}\, \mathrm{kW/(kg/s)} \cong -8.077\, \mathrm{kW/(kg/s)}$$

3 Types of Crushing Equipment

Coarse Crushers	Intermediate Crushers	Fine Crushers
		Buhrstone mill
	Crushing rolls	Roller mill
	Disc crusher	NEI pendulum mill
Stag Jaw crusher	Edge runner mill	Griffin mill
Dodge jaw crusher	Hammer mill	Ring roller mill (lopulco)
Gyratory crusher	Single roll crusher	Ball mill
...	Pin mill	Tube mill
	symons disc crusher	Hardinge mill
	...	Babcock mill
		...

3.1 Coarse Crushers

Jaw Crushers

The Stag jaw crusher has a fixed jaw and a moving jaw pivoted at the top with the crushing faces formed of manganese steel.

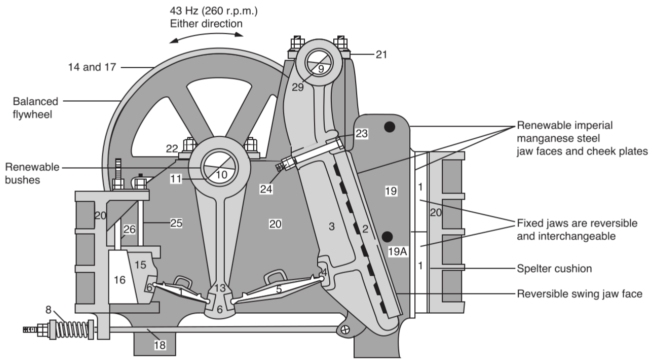


Fig. 5.4

Typical cross-section of Stag jaw crusher. 1: Fixed jaw face, 2: swing jaw face, 3: swing jaw stock, 4: toggle seating, 5: front toggle plate, 6: toggle seating, 7: back toggle plate, 8: springs and cups, 9: swing jaw shaft, 10: eccentric shaft, 11: Pitman bush, 13: Pitman, 14: flywheel grooved for V rope drive, 15: toggle block, 16: wedge block, 17: flywheel, 18: tension rods, 19: cheek plates (top), 19A: cheek plates (bottom), 20: body, 21: swing jaw shaft bearing caps, 22: eccentric shaft bearing caps, 23: wedge for swing jaw face, 24: bolts of wedge, 25: bolts for toggle block, 26: bolts for wedge block, 27: eccentric shaft bearing bush (bottom), 28: eccentric shaft bearing bush (top), 29: swing stock bush. *The author and editor thank Vaba Process Plant Ltd, Rotherham, Yorks for this figure.*

3.2 Intermediate Crushers

Crushing rolls

Two rolls, one in adjustable bearings, rotate in opposite directions, and the clearance between them can be adjusted according to the size of feed and the required size of product. Main forces: compressive/attrition

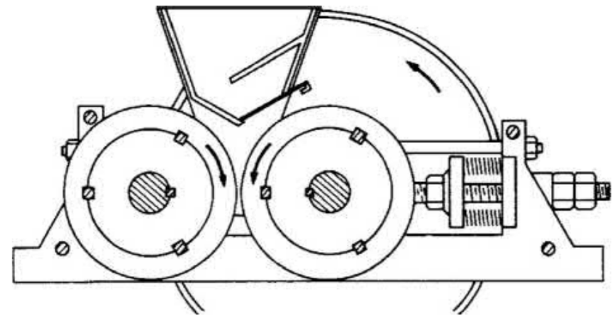


Fig. 5.15
Crushing rolls.

Nip angle

$$\cos \alpha/2 = \frac{r_{roll} + b/2}{r_{roll} + r_0} \leq 31°$$

r_{roll}: Roll radius

r_0: Feed particle size

b: Distance between rolls

$\alpha \leq 31°$: Maximum value for nip angle

$\tan \alpha = \mu$ The coefficient of friction

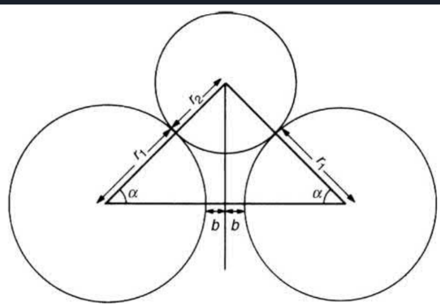
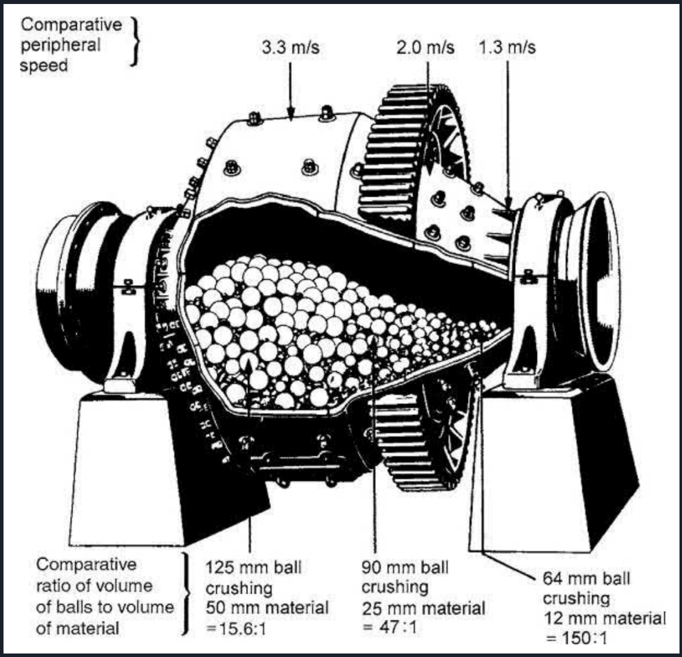


Fig. 5.16
Particle fed to crushing rolls.

3.3 Fine Crushers

Ball Mill

In its simplest form, the ball mill consists of a rotating hollow cylinder, partially filled with balls, with its axis either horizontal or at a small angle to the horizontal. The material to be ground may be fed in throughhja hollow trunnion at one end, and the product leaves through a similar trunnion at the other end. The gutlet is normally covered with a coarse screen to prevent the escape of the balls.



During grinding, the balls wear and are constantly replaced by new ones so that the mill contains balls of various ages, and, hence, of various sizes. This is advantageous because the large balls deal effectively with the feed, and the small ones are responsible for giving a fine product.

Rotation speed

A ball mill has a critical rotation speed ($[w_c]$ = rad/s) that must be avoided. At the critical point, the ball (with mass m) is subject to a centrifugal force ($m\,u/r^2$) equal to the gravitational force ($m\,g$)

$$w_c = g/r$$

The optimal rotation speed ($[w_o]$ = rad/s) is choosen below w_c for optimal efficiency

$$w_o \sim \alpha\, w_c; \quad \alpha \in [0.50, 0.75]$$

Exemplo 2

A ball mill, $1.2\,\mathrm{m}$ in diameter, is operated at $0.80\,\mathrm{Hz}$, and it is found that the mill is not working properly. Should any modification in the conditions of operation be suggested

Resposta

$$w_o \in [0.50\,w_c, 0.75\,w_c] \cong$$
$$\cong [0.321, 0.482]\ \mathrm{Hz};$$

$$w_c = \sqrt{\frac{g}{r}} \cong \sqrt{\frac{9.780}{0.6}} \cong 4.037\,\mathrm{rad/s} \cong 0.643\,\mathrm{Hz}$$

rotation speed should be reduced to optimal w_o values