Convecção – Análise Dimensional e Correlações

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Fenómenos de Transferência II

Coeficiente de transferência de massa

$$N_A = k_c \left(C_{As} - C_A\right)$$

Coeficiente de transferência de massa

Avaliação de Kc:

- ❑ Análise Dimensional
- ❑ Correlações experimentais
- ❑ Analogias entre tansferência de massa, calor e quantidadede movimento
- ❑ Modelos
- ❑ Camada limite

Análise Dimensional

Variável	Símbolo	Dimensão
diâmetro	D	L
massa esp. fluido	ρ	$M L^{-3}$
viscosidade fluido	μ	$M L^{-1} T^{-1}$
velocidade fluido	v	$L T^{-1}$
coeficiente difusão	$\mathcal{D}_{AB}$	$L^2 T^{-1}$
coef. transfª massa	k_c	$L T^{-1}$

Teorema π de Buckingham

$$i = n - K$$

i: nº grupos adimensionais

n: nº variáveis

K: nº grandezas fundamentais

Números adimensionais: 3

Análise Dimensional

$$\pi_1 = D_{AB}^{a} \rho^{b} D^{c} k_c$$

$$\pi_1 = \frac{k_c D}{D_{AB}}$$

Nº Sherwood

Razão entre a resistência à transferência de massa por convecção e por difusão

Para o 1º Número adimensional

$$1 = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \left(\frac{L}{t}\right)$$

L : 0 = 2a – 3b + c + 1

t : 0 = – a – 1

M : 0 = b

a = –1, b = 0 c = 1

Análise Dimensional

Para os restantes números adimensionais

$$\pi_2 = D_{AB}^{d}\, \rho^{e}\, D^{f}\, v$$

$$\pi_3 = D_{AB}^{g}\, \rho^{h}\, D^{i}\, \mu$$

$$\pi_2 = \frac{D\,v}{D_{AB}}$$

$$\pi_3 = \frac{\mu}{\rho\, D_{AB}} = Sc$$

$$\frac{\pi_2}{\pi_3} = \left(\frac{D\,v}{D_{AB}}\right) \Bigg/ \left(\frac{\mu}{\rho\, D_{AB}}\right) = \frac{D\,v\,\rho}{\mu} = \mathrm{Re}$$

Nº Reynolds

Nº Schmidt

Razão entre as forças cinéticas e as forças viscosas

Razão entre a difusão molecular de quantidade de movimento e de massa

Correlações experimentais

Transferência de massa:

$$Sh = \psi\left(\mathrm{Re},\ Sc\right)$$

Transferência de calor:

$$Nu = \psi\left(\mathrm{Re},\ \mathrm{Pr}\right)$$

Correlações - condutas

Regime turbulento

Gilliland and Sherwood

$$\boxed{Sh \frac{p_{B,lm}}{P} = 0.023 \operatorname{Re}^{0.83} Sc^{0.44}}$$

2000 < Re < 35000
0.6 < Sc < 2.5

Linton and Sherwood

$$\boxed{Sh = 0.023 \operatorname{Re}^{0.83} Sc^{1/3}}$$

2000 < Re < 70000
1000 < Sc < 2260

Regime laminar

$$\boxed{Sh = 1.86\left(\operatorname{Re} Sc \frac{d}{L}\right)^{1/3}}$$

Correlações

Colunas com enchimento

a = área interfacial / vol. leito

$k_c a$ = coef. de capacidade

Sherwood e Holloway (absorção)

$$\frac{k_c a}{\mathcal{D}_{AB}} = \alpha \left(\frac{v_L}{\mu}\right)^{1-n} \left(\frac{\mu}{\rho \mathcal{D}_{AB}}\right)^{0.5}$$

		α	n
anéis Raschig	2"	80	0.22
Selas de Berl	1"	170	0.28
espiralóides	3"	110	0.28

Table 8.3-2 *Selected mass transfer correlations for fluid–fluid interfaces*[a]

Physical situation	Basic equation[b]	Key variables	Remarks
Liquid in a packed tower	$k\left(\frac{1}{\nu g}\right)^{1/3} = 0.0051\left(\frac{v^0}{a\nu}\right)^{0.67}\left(\frac{D}{\nu}\right)^{0.50}(ad)^{0.4}$	a = packing area per bed volume d = nominal packing size	Probably the best available correlation for liquids; tends to give lower values than other correlations.
	$\frac{kd}{D} = 25\left(\frac{dv^0}{\nu}\right)^{0.45}\left(\frac{\nu}{D}\right)^{0.5}$	d = nominal packing size	The classical result, widely quoted; probably less successful than above.
	$\frac{k}{v^0} = \alpha\left(\frac{dv^0}{\nu}\right)^{-0.3}\left(\frac{D}{\nu}\right)^{0.5}$	d = nominal packing size	Based on older measurements of height of transfer units (HTU's); α is of order one.
Gas in a packed tower	$\frac{k}{aD} = 3.6\left(\frac{v^0}{a\nu}\right)^{0.70}\left(\frac{\nu}{D}\right)^{1/3}(ad)^{-2.0}$	a = packing area per bed volume d = nominal packing size	Probably the best available correlation for gases.
	$\frac{kd}{D} = 1.2(1-\epsilon)^{0.36}\left(\frac{dv^0}{\nu}\right)^{0.64}\left(\frac{\nu}{D}\right)^{1/3}$	d = nominal packing size ε = bed void fraction	Again, the most widely quoted classical result.
Pure gas bubbles in a stirred tank	$\frac{kd}{D} = 0.13\left(\frac{(P/V)d^4}{\rho\nu^3}\right)^{1/4}\left(\frac{\nu}{D}\right)^{1/3}$	d = bubble diameter P/V = stirrer power per volume	Note that k does not depend on bubble size.
Pure gas bubbles in an unstirred liquid	$\frac{kd}{D} = 0.31\left(\frac{d^3 g\Delta\rho/\rho}{\nu^2}\right)^{1/3}\left(\frac{\nu}{D}\right)^{1/3}$	d = bubble diameter $\Delta\rho$ = density difference between gas and liquid	For small swarms of bubbles rising in a liquid.
Large liquid drops rising in unstirred solution	$\frac{kd}{D} = 0.42\left(\frac{d^3\Delta\rho g}{\rho\nu^2}\right)^{1/3}\left(\frac{\nu}{D}\right)^{0.5}$	d = bubble diameter $\Delta\rho$ = density difference between bubbles and surrounding fluid	Drops 0.3-cm diameter or larger.
Small liquid drops rising in unstirred solution	$\frac{kd}{D} = 1.13\left(\frac{dv^0}{D}\right)^{0.8}$	d = drop diameter v^0 = drop velocity	These small drops behave like rigid spheres.
Falling films	$\frac{kz}{D} = 0.69\left(\frac{zv^0}{D}\right)^{0.5}$	z = position along film v^0 = average film velocity	Frequently embroidered and embellished.

Notes: [a]The symbols used include the following: D is the diffusion coefficient; g is the acceleration due to gravity; k is the local mass transfer coefficient; v^0 is the superficial fluid velocity; and ν is the kinematic viscosity.

[b]Dimensionless groups are as follows: dv/ν and $v/a\nu$ are Reynolds numbers; ν/D is the Schmidt number; $d^3g(\Delta\rho/\rho)/\nu^2$ is the Grashoff number, kd/D is the Sherwood number; and $k/(\nu g)^{1/3}$ is an unusual form of Stanton number.

Table 8.3-3 *Selected mass transfer correlations for fluid–solid interfaces*[a]

Physical situation	Basic equation[b]	Key variables	Remarks
Membrane	$\frac{kl}{D} = 1$	l = membrane thickness	Often applied even where membrane is hypothetical.
Laminar flow along flat plate[c]	$\frac{kL}{D} = 0.646\left(\frac{Lv^0}{\nu}\right)^{1/2}\left(\frac{\nu}{D}\right)^{1/3}$	L = plate length v^0 = bulk velocity	Solid theoretical foundation, which is unusual.
Turbulent flow through horizontal slit	$\frac{kd}{D} = 0.026\left(\frac{dv^0}{\nu}\right)^{0.8}\left(\frac{\nu}{D}\right)^{1/3}$	v^0 = average velocity in slit $d = [2/\pi]$ (slit width)	Mass transfer here is identical with that in a pipe of equal wetted perimeter.
Turbulent flow through circular tube	$\frac{kd}{D} = 0.026\left(\frac{dv^0}{\nu}\right)^{0.8}\left(\frac{\nu}{D}\right)^{1/3}$	v^0 = average velocity in tube d = pipe diameter	Same as slit, because only wall regime is involved.
Laminar flow through circular tube	$\frac{kd}{D} = 1.62\left(\frac{d^2v^0}{LD}\right)^{1/3}$	d = pipe diameter L = pipe length v^0 = average velocity in tube	Very strong theory and experiment
Flow outside and parallel to a capillary bed	$\frac{kd}{D} = 1.25\left(\frac{d_e^2v^0}{\nu l}\right)^{0.93}\left(\frac{\nu}{D}\right)^{1/3}$	d_e = 4 area/wetted perimeter v^0 = superficial velocity	Not reliable because of channeling in bed.
Flow outside and perpendicular to a capillary bed	$\frac{kd}{D} = 0.80\left(\frac{dv^0}{\nu}\right)^{0.47}\left(\frac{\nu}{D}\right)^{1/3}$	d = capillary diameter v^0 = velocity approaching bed	Reliable if capillaries evenly spaced.
Forced convection around a solid sphere	$\frac{kd}{D} = 2.0 + 0.6\left(\frac{dv^0}{\nu}\right)^{1/2}\left(\frac{\nu}{D}\right)^{1/3}$	d = sphere diameter v^0 = velocity of sphere	Very difficult to reach $(kd/D) = 2$ experimentally; no sudden laminar-turbulent transition.
Free convection around a solid sphere	$\frac{kd}{D} = 2.0 + 0.6\left(\frac{d^3\Delta\rho g}{\rho\nu^2}\right)^{1/4}\left(\frac{\nu}{D}\right)^{1/3}$	d = sphere diameter g = gravitational acceleration	For a 1-cm sphere in water, free convection is important when $\Delta\rho = 10^{-9}\,g/cm^3$.
Packed beds	$\frac{k}{v^0} = 1.17\left(\frac{dv^0}{\nu}\right)^{-0.42}\left(\frac{D}{\nu}\right)^{2/3}$	d = particle diameter v^0 = superficial velocity	The superficial velocity is that which would exist without packing.
Spinning disc	$\frac{kd}{D} = 0.62\left(\frac{d^2\omega}{\nu}\right)^{1/2}\left(\frac{\nu}{D}\right)^{1/3}$	d = disc diameter ω = disc rotation (radians/time)	Valid for Reynolds numbers between 100 and 20,000.

Note: [a]The symbols used include the following: D is the diffusion coefficient of the material being transferred; k is the local mass transfer coefficient; ρ is the fluid density; ν is the kinematic viscosity. Other symbols are defined for the specific situation.
[b]The dimensionless groups are defined as follows: (dv^0/ν) and $(d^2\omega/\nu)$ are the Reynolds number; ν/D is the Schmidt number; $(d^3\Delta\rho g/\rho\nu^2)$ is the Grashöf number; kd/D is the Sherwood number; k/v is the Stanton number.
[c]The mass transfer coefficient given here is the value averaged over the length L.

Correlações

Faz-se escoar ar a 10ºC e à pressão de 1 atm ao longo de uma conduta feita em naftaleno com diâmetro interno igual a 2.5 cm e 183 cm de comprimento. Supondo que a variação de pressão ao longo do tubo é desprezável e que a superfície do naftaleno está a 10ºC, determine o teor de naftaleno do ar que sai da conduta e a velocidade de sublimação, se a velocidade média do ar for:

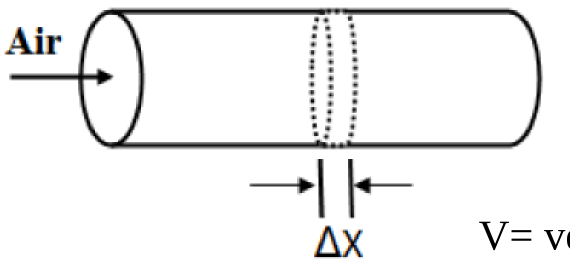
a) 61 cm/s
b) 15.25 m/s

Propriedades do ar: $\nu = 1.5\times10^{-5}$ m²/s

Propriedades do naftaleno: Pressão de vapor = 0.0209 mmHg
Coeficiente de difusão no ar = 5.16×10^{-2} cm²/s
Massa molecular = 128.2 g/mol

$$Sh = 1.86\left(Re\ Sc\ \frac{d}{L}\right)^{1/3}$$ Regime laminar

$$Sh = 0.023\,Re^{0.83}\,Sc^{0.44}$$ Regime turbulento



V= velocidade

$$c_A V \frac{\pi d^2}{4}\Big|_x + k_c (c_{A_s} - c_A)\pi d \Delta x = c_A V \frac{\pi d^2}{4}\Big|_{x+\Delta x}$$

$$\div \frac{\pi d^2}{4}\Delta x$$

$$\frac{c_A\big|_{x+\Delta x} - c_A\big|_x}{\Delta x} = \frac{4}{d}\frac{k_c}{V}(c_{A_s} - c_A)$$

Take the limits as Δx approaches zero

$$\frac{dc_A}{dx} = \frac{4}{d}\frac{k_c}{V}(c_{A_s} - c_A)$$

$$-\int_{c_{A_o}}^{c_{A_L}} \frac{-dc_A}{(c_{A_s} - c_A)} = \frac{4}{d}\frac{k_c}{V}\int_0^L dx$$

$$\boxed{ln\left(\frac{c_{A_s} - c_{A_o}}{c_{A_s} - c_{A_L}}\right) = \frac{4}{d}\frac{k_c}{V}L}$$

$$\boxed{W = V\ \frac{\pi d^2}{4}\ (C_{AL} - C_{A0})}$$

a)

Re= 1017

Sc= 2.91

k_C= $1.32x10^{-3}$ m/s

C_{As}=C*=P*/RT= $1.17x10^{-3}$ mol/m^3

C_{AL}= $5.5x10^{-4}$ mol/m^3

W= $1.65x10^{-7}$ mol/s

b)

Re= 25417

Sc= 2.91

kc= $3.44x10^{-2}$ m/s

C_{As}=C*=P*/RT= $1.17x10^{-3}$ mol/m^3

C_{AL}= $5.7x10^{-4}$ mol/m^3

W= $4.21x10^{-6}$ mol/s

Correlações

Uma esfera de glucose com 0.5 cm de diâmetro é dissolvida numa corrente de água, que se desloca a uma velocidade de 15 cm/s, à temperatura de 25°C. Calcule o coeficiente de transferência de massa e o tempo necessário para que o volume da esfera se reduza a metade.

Dados: $D_{glucose\text{-}água} = 6.0\times10^{-6}\ cm^2/s$

$C^*_{glucose\ \text{-}água} = 3\ mol/L$

$\rho_{glucose} = 1.4\times10^3\ kg/m^3$

M (glucose) = 180.16 g/mol

$\rho_{água} = 1\times10^3\ kg/m^3$

$\mu_{água} = 1\times10^{-3}\ Ns/m^2$

$$\mathrm{Sh} = 2 + 0.6\,\mathrm{Re}^{1/2}\,\mathrm{Sc}^{1/3}$$

$Re = 750$

$Sc = 1667$ $k_C = 2.36 \times 10^{-5}$ m/s

Esfera glucose

Massa dissolvida = ½ Volume*ρ/M= 2.54×10^{-4} mol

$W = K_C\, A\, C_{As} = 5.56 \times 10^{-6}$ mol/s

$t = 45.7$ s