Chapter 5.
Centrifugal separation

5.1 Introduction: Centrifugal force; Classification of centrifuges; examples of centrifuges;

5.2 Sedimentation under centrifugal acceleration: settling velocity; settling time

5.3 Tubular centrifuge design: Batch and continuous operation; the SIGMA Factor; Separation efficiency;

5.4 Generalization to ANY centrifuge design: example of the Disk stack centrifuge design;

5.5 Mechanical design: Centrifugal pressure and wall tension;

5.6 Centrifugation of immiscible liquids;

5.7 Gas cleaning with cyclones

J.M. Coulson and J.F. Richardson pp 475 - 501

Introduction

- Many Chemical Engineering unit use the **centrifugal force** (instead of the gravitational force) in order to effect separations based on the size and density of materials.
- Many different configurations for the separation of solids, liquids, molecules, atoms (e.g. solid-liquid centrifuge, liquid centrifuge, gas centrifuge, cyclone, hydrocyclone, etc,…)
- Centrifugal separation operate at significantly higher forces than gravity settlers. **Colloidal suspensions may be separated**, which would be impossible in the gravitational field.
- Even atoms in a gas may be separated in an ultra-centrifuge (with acceleration higher than 100 000 g) for e.g. the enrichment of U-235 from a mixture U-235/U-238.

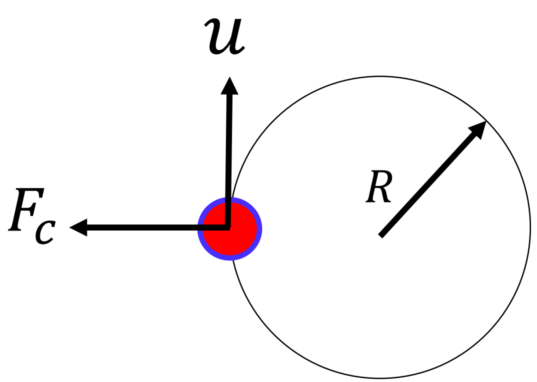
Introduction

A **centrifuge** may be classified according to the working principle and mode of operation:

- **Settling centrifuge**: the solids settle against the surface of the bowl at a much higher velocity than in gravity settlers with significantly higher efficiency (faster operation in a smaller volume).
- **Filtration centrifuge**: the perforated centrifuge bowl works as a filter that retains the solids inside while the liquid is discharged. The very high centrifugal forces increase the filtration rate in comparison to gravity filtration.
- **Batch**: intermittent operation; the solids and the liquids are discharged manually from the centrifugal device
- **Continuous**: solids and liquids are continuously discharged as separate streams. Many different designs

Centrifugal force

The **centrifugal force, F_c,** is used (instead of the gravitational force) to separate different materials based on their size and density. Let's consider a solid particle of mass, m, moving with a circular trajectory of radius, R, and tangential velocity, u



R – radius of circular trajectory, m
u – particle tangential velocity, m/s
w = **u**/**R** – particle angular velocity, rad/s
F_c – Centrifugal force, N

Centrifugal force

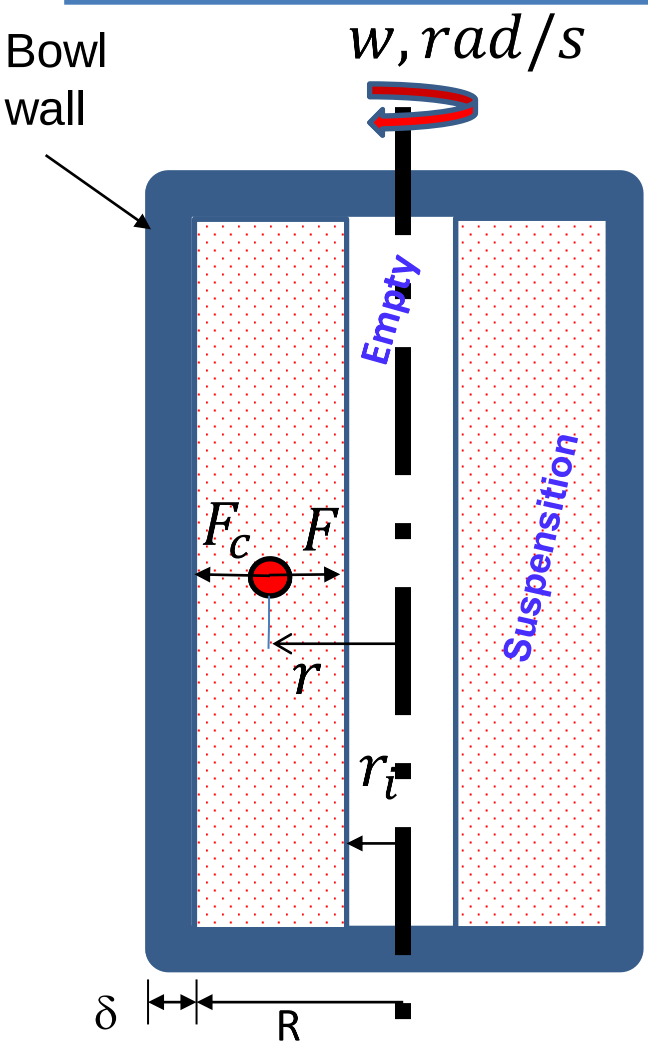
$$F_c = m\frac{u^2}{R} \Leftrightarrow \boxed{\boldsymbol{F_c = mR\omega^2}}$$

Centrifugal separation power

$$\frac{F_c}{F_g} = \frac{mR\omega^2}{mg} = \frac{R\omega^2}{g}$$

>> Exercise V.1

Settling under centrifugal field



Let's consider the motion of a spherical particle with diameter, d, in a fluid, under a centrifugal acceleration field, $a_c \gg g$. Which are the forces acting on the particle in radial direction ?

<u>F_c – apparent centrifugal force, N</u>

$$F_{c,app} = m_{app} \mathrm{r} w^2 = \frac{\pi d^3}{6}(\rho_s - \rho)\mathrm{r} w^2$$

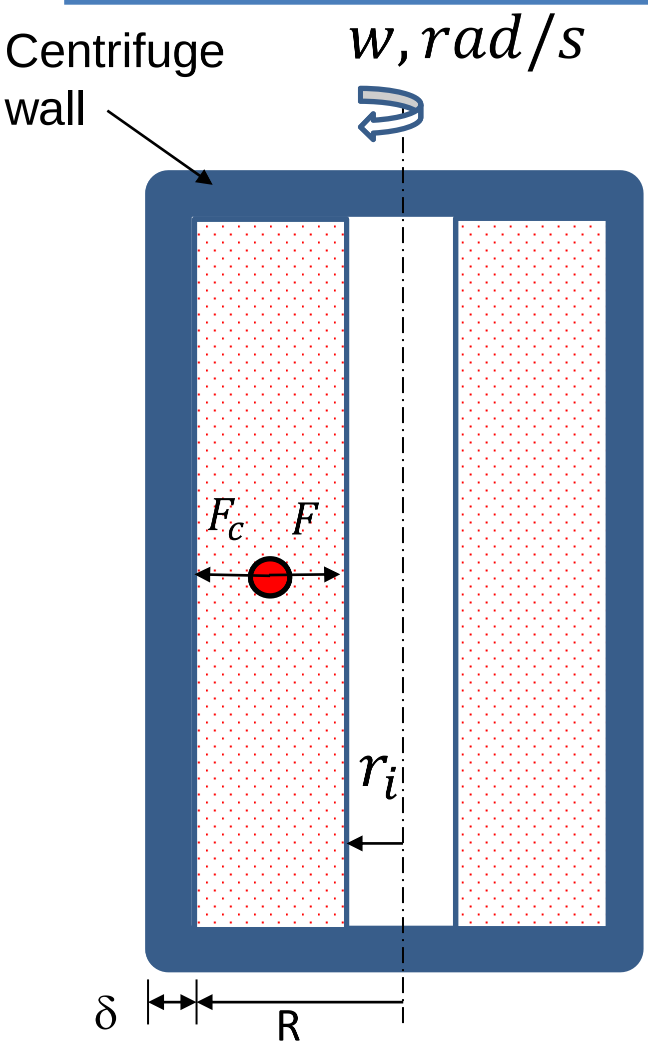
<u>F – drag force, N</u>

$$F = 3\pi\mu u d$$ (assuming laminar flow, $Re' < 0{,}2$)

$u = \frac{dr}{dt}$ is the particle settling velocity, m/s

Question: is it constant?

Nonuniform movement



In a centrifugation field, the acceleration is not constant ($a_c =$ rw^2, m²/s) , thus the movement of the sphere is always nonuniform. The 2nd law of Newton holds:

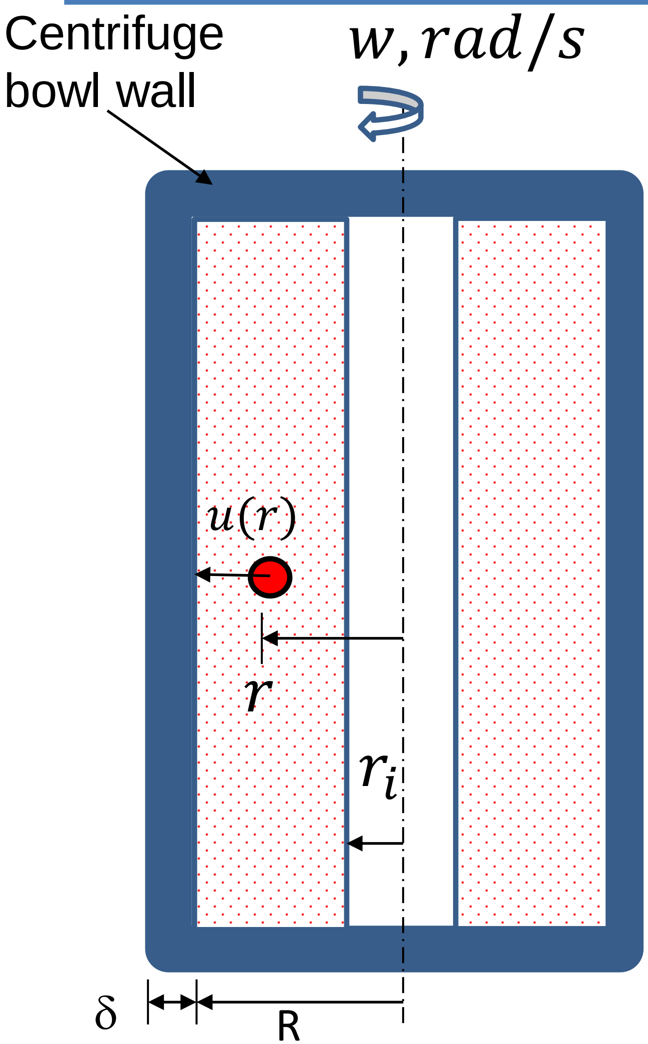
$$F_r = m a_c$$

$$F_c - F = \frac{\pi d^3}{6} \rho_s \frac{d^2 r}{dt^2}$$

$$\boxed{\frac{\pi d^3}{6} (\rho_s - \rho) r w^2 - 3\pi\mu d \frac{dr}{dt} = \frac{\pi d^3}{6} \rho_s \frac{d^2 r}{dt^2}}$$

2nd order ODE of particle position, *r*, in time, *t*

Nonuniform movement

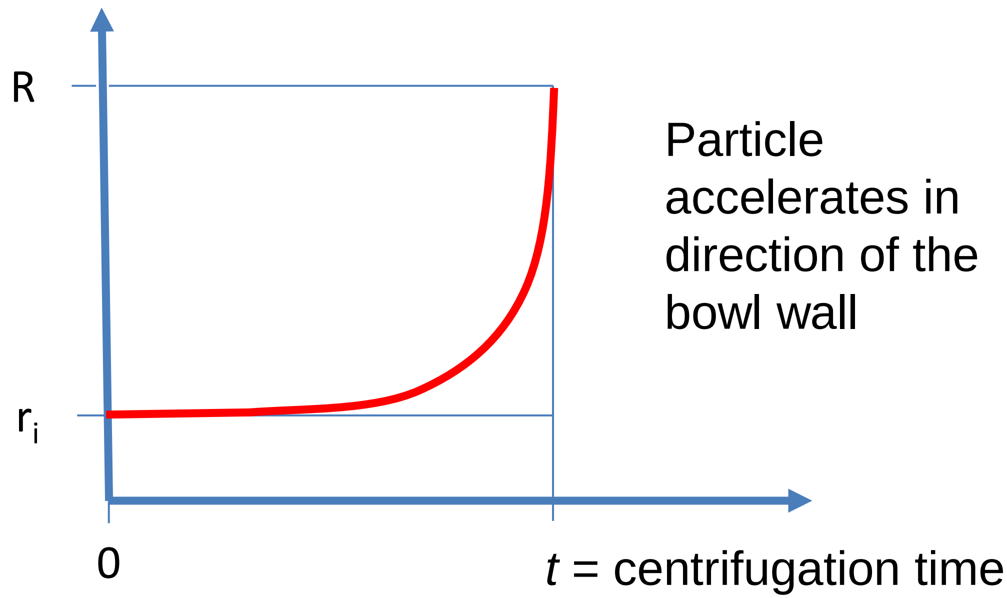


$$\frac{\pi d^3}{6}(\rho_s-\rho) r w^2 - 3\pi\mu d \frac{dr}{dt} = \frac{\pi d^3}{6}\rho_s \frac{d^2 r}{dt^2}$$

Boundary conditions:

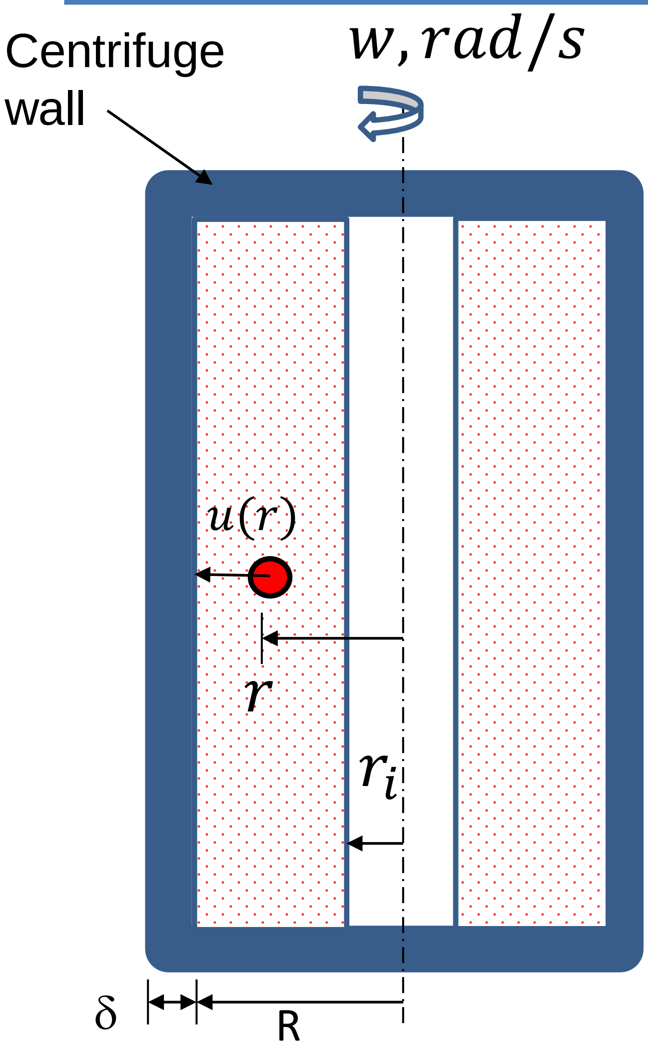
$t = 0, \quad r = r_i$

$t = 0, \quad u = \frac{dr}{dt} = 0$



Particle accelerates in direction of the bowl wall

Neglecting inertia



Neglecting inertia, then *Fc* = *F* at all times, implying:

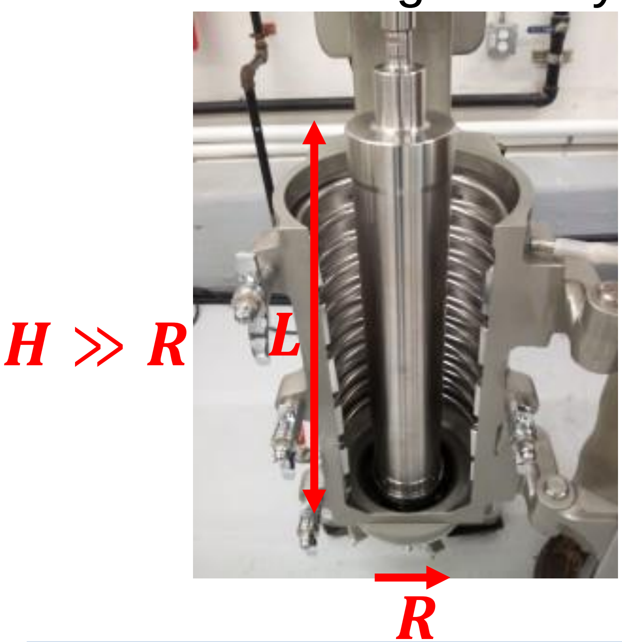
$$\frac{\pi d^3}{6}(\rho_s - \rho) r w^2 - 3\pi\mu d \frac{dr}{dt} = \frac{\pi d^3}{6}\rho_s \cancelto{0}{\frac{d^2 r}{dt^2}}$$

Valid only in laminar flow!!!! $\Rightarrow$

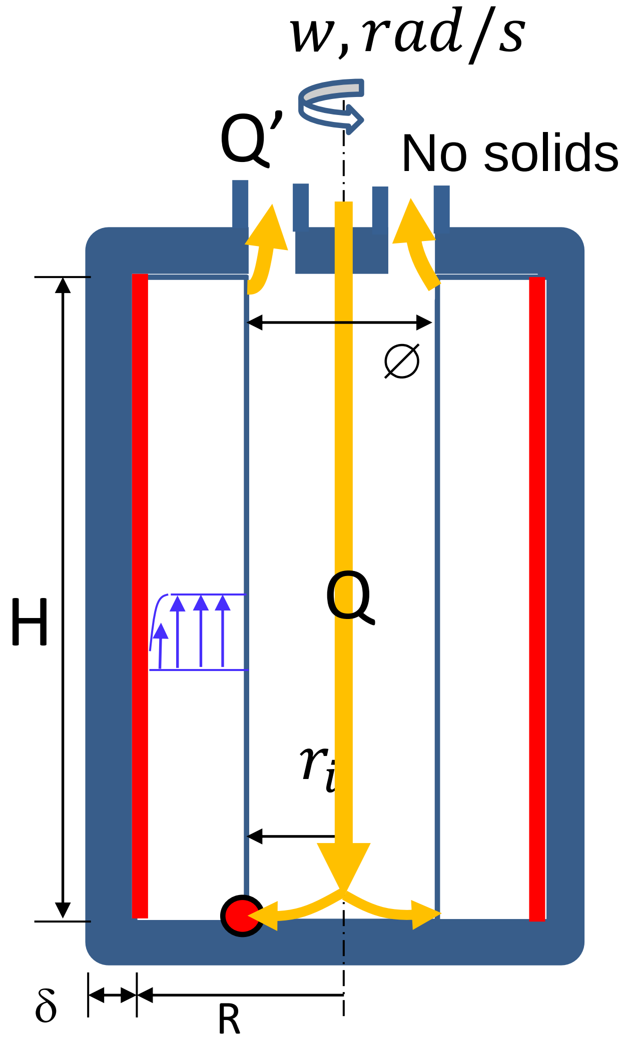
$$\begin{cases} u(r) = u_0 \dfrac{r\, w^2}{g} & \text{Settling velocity, m/s} \\[2ex] t = \dfrac{g}{u_0 w^2} \ln\left(\dfrac{R}{r_i}\right) & \text{Sedimentation time, s} \end{cases}$$

Tubular centrifuge

<u>Tubular centrifuges</u> are sedimentation centrifuges (batch or continuous). They are used when high separation power is needed (small particles or colloidal suspensions). Because the stress in the wall is $\boldsymbol{f \propto R^3 w^2 \delta^{-1}}$, machines with high separation power (high Rw^2) generally use very tall bowls (high H) of small diameter (small R) and high thickness (δ), i.e. they have a tubular geometry with thick wall.



Continuous tubular centrifuge



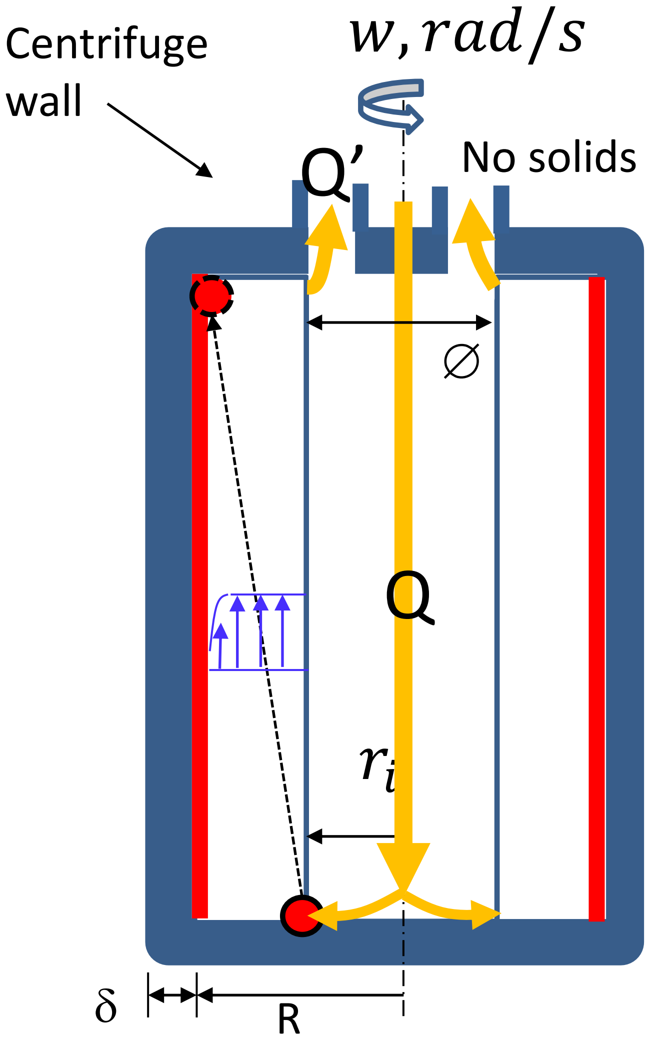
Design problem: for a given inflow stream (*Q*, *C*, solids size distribution) what's the required centrifuge size (**H**, **R**, **ri**) and angular speed (**w**) for complete separation?

∅ - outflow weird diameter [m]

Q - inflow at the central axis [m^3/s]

Q' - outflow at the weird [m^3/s]

Design condition



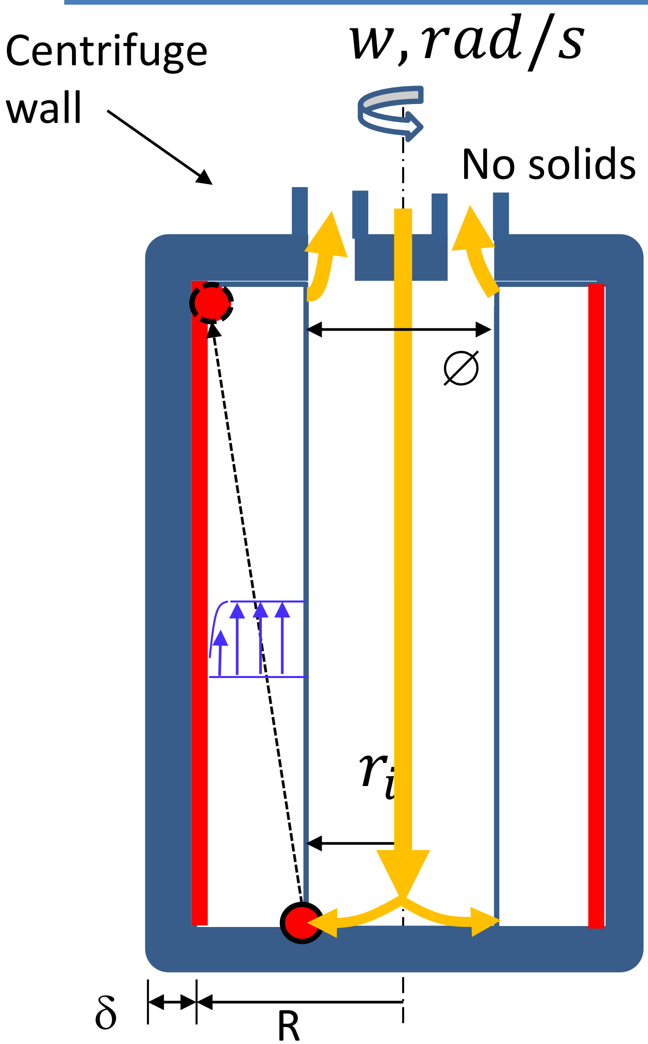
Residence time of smallest sphere, τ **=** Settling time of smallest sphere, t

$$\frac{V}{Q} = \frac{g}{u_0 w^2} \ln\left(\frac{R}{r_i}\right)$$

$$\frac{\pi\left(R^2 - r_i^2\right)H}{Q} = \frac{g}{u_0 w^2} \ln\left(\frac{R}{r_i}\right)$$

$$\Leftrightarrow (\ldots)$$

Universal design equation



m3/s m/s m2

$$\boxed{Q = u_0 \Sigma}$$

Maximum inflow to centrifuge

Suspension properties

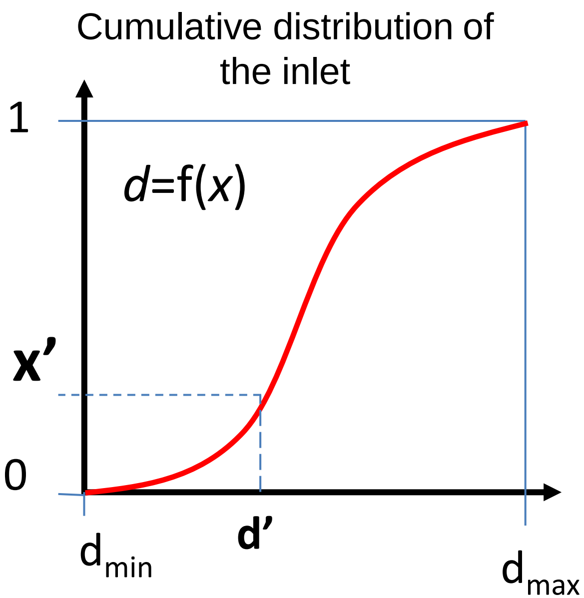
Centrifuge characteristics (geometry, size, angular velocity)

The **sigma factor [m2]** of a tubular centrifuge is given by:

$$\Sigma = \frac{\pi\left(R^2 - r_i^2\right)Hw^2}{g\ln\left(\frac{R}{r_i}\right)}$$

Separation efficiency, η

A centrifugal separation equipment will typically not achieve 100% of materials separation. Given a well defined size distribution of the inlet stream, the separation efficiency **is determined by the smallest particle that the centrifuge is able to separate**.



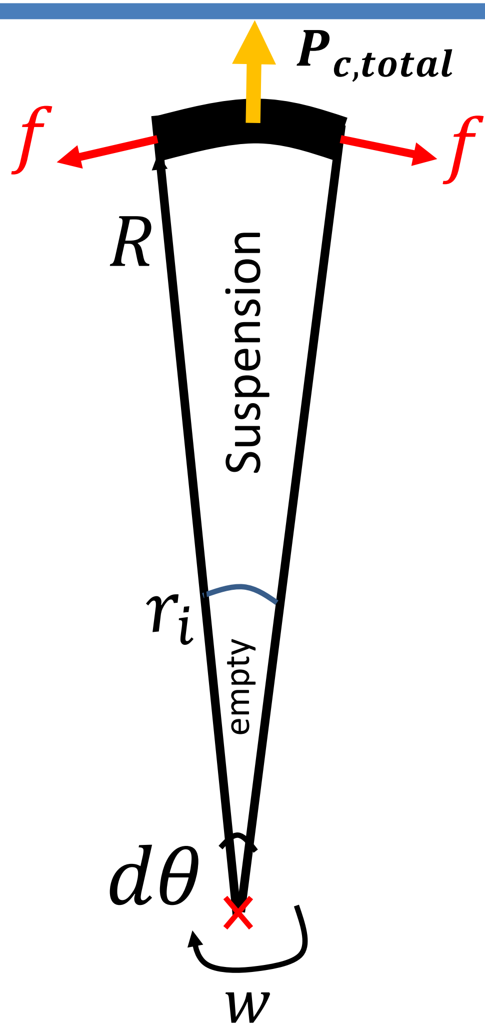
$$\boxed{\eta = 1 - x'}$$

Mass fraction of solids retained by the equipment

d' **- size of the smallest particle captured in the centrifuge**

x' – mass fraction with size $d \leq d'$

Mechanical design



Problem: is the tension, f **[N/m2]**, developed internally in the bowl wall material with thickness δ, below the critical rupture tension, f_{cri} **[N/m2]?**

$$\boxed{f = \frac{R}{\delta} P_{c,total} \le f_{cri}}$$

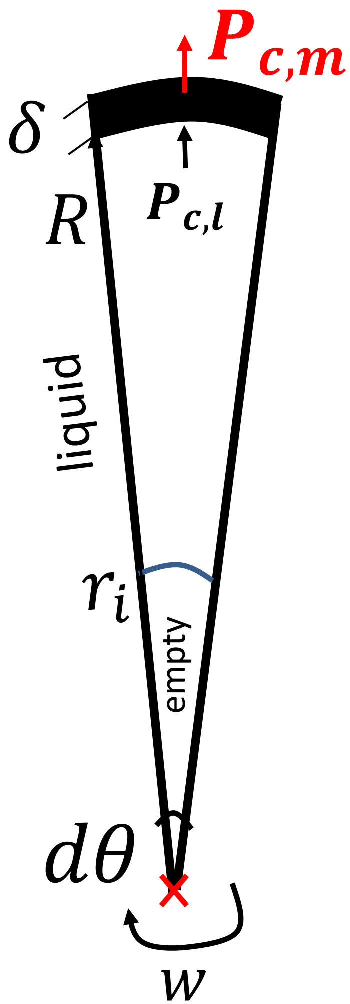
$$f = \frac{R}{\delta}\left(P_{c,s} + P_{c,l} + P_{c,m} \right) \le f_{cri}$$

Pressure due to settled solids

Pressure due to column of liquid

Pressure due to bowl wall

Pressure due to the bowl material



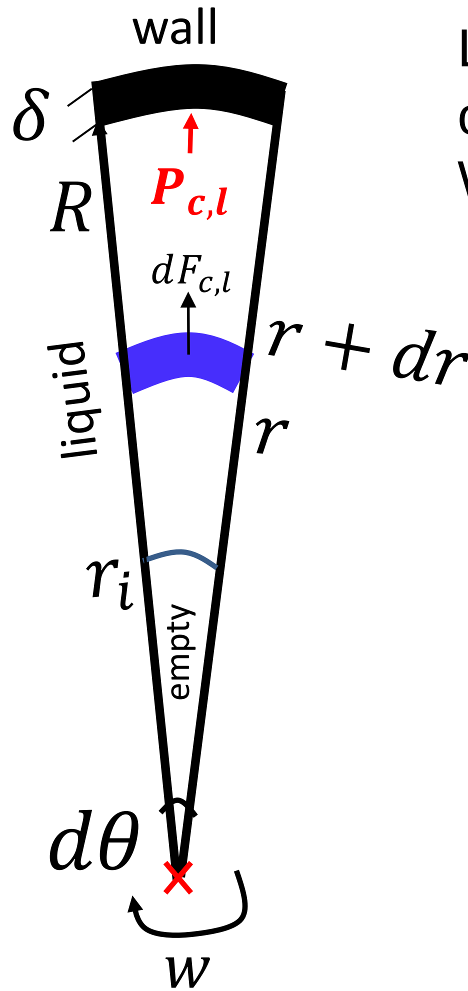
Consider the centrifuge bowl wall with thickness, $\delta\ [m]$. The centrifugal pressure due to the wall material, $P_{c,m}$ [N/m2], in radial direction, is given by:

$$F_{c,m} = A\delta\rho_m R w^2$$

$$P_{c,m} = \frac{F_{c,m}}{A} = \delta\rho_m R w^2$$

$$\boxed{P_{c,m} = \delta\rho_m R w^2}$$

Pressure due to the liquid column



Liquid pressure, $P_{c,l}$ [N/m2], of the column of liquid $[r_i, R]$ exerting on the centrifuge Wall:

$$dF_{c,l} = dV\rho r w^2$$

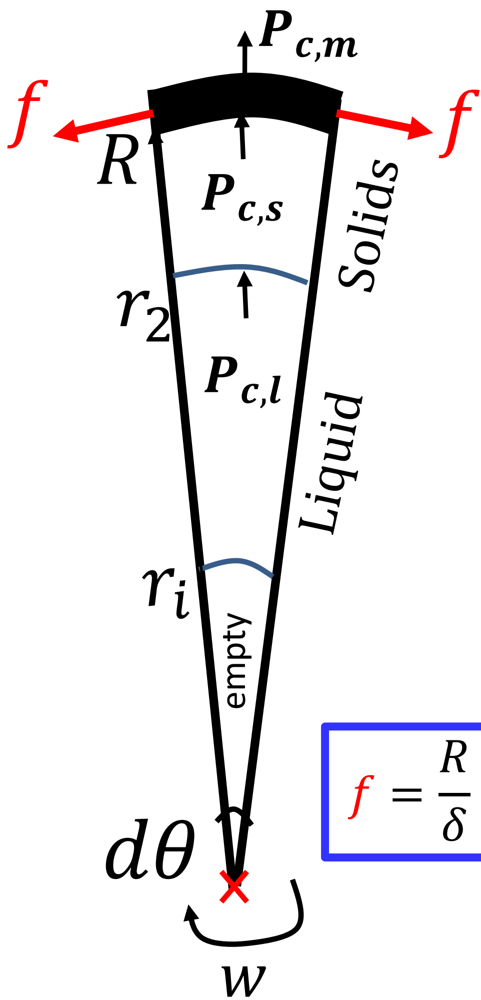
$$dF_{c,l} = A dr \rho r w^2$$

$$dP_{c,l} = \frac{dF_l}{A} = dr\rho r w^2$$

$$P_{c,l} = \int_{r_i}^{R} \rho r w^2 dr$$

$$\boxed{P_{c,l} = \frac{1}{2}\rho\left(R^2 - r_i^2\right)w^2}$$

Overall tension, f [N/m2]



Problem: is the wall tension, f [N/m2], developed internally in the centrifugal wall material with thickness δ, below the critical rupture tension, f_{cri} [N/m2]?

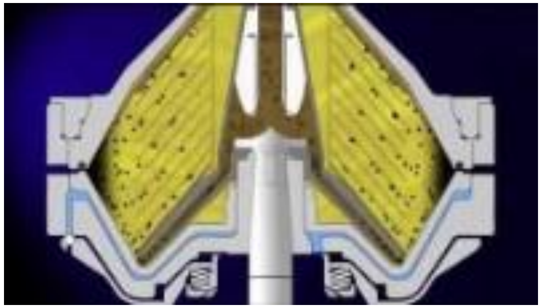
$$f = \frac{R}{\delta} P_{c,total} \leq f_{cri}$$

$$f = \frac{R}{\delta}\left(P_{c,s} + P_{c,l} + P_{c,m}\right) \leq f_{cri}$$

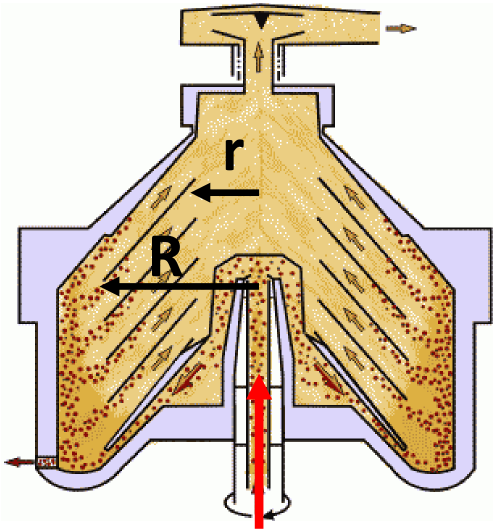
$$\boxed{f = \frac{R}{\delta}\left(\underbrace{\frac{1}{2}\rho s\left(R^2 - r_2^2\right)w^2}_{\text{Pressure due to settled solids}} + \underbrace{\frac{1}{2}\rho\left(r_2^2 - r_i^2\right)w^2}_{\text{Pressure due to column of liquid}} + \underbrace{\delta \rho_m R w^2}_{\text{Pressure due to centrifuge wall}}\right) \leq f_{cri}}$$

Disk stack centrifuge

<u>Disk stack centrifuge</u> are continuous sedimentation centrifuges. The bowl contains conical discs enabling the liquid stream to be split into a large number of very thin layers. A disc bowl, although still a high speed machine, can be run at lower speeds than the tubular bowl. As such, the disk bowl diameter can be higher than the tubular centrifuge. Also, the inclusion of many disks allows to decrease the overall size of the equipment.



Design of a disk stack centrifuge



Q

Maximum inflow to centrifuge

$$\underset{\text{m3/s}}{Q} = \underset{\text{m/s}}{u_0}\,\underset{\text{m2}}{\Sigma}$$

Suspension properties

Centrifuge characteristics (geometry, size, angular velocity)

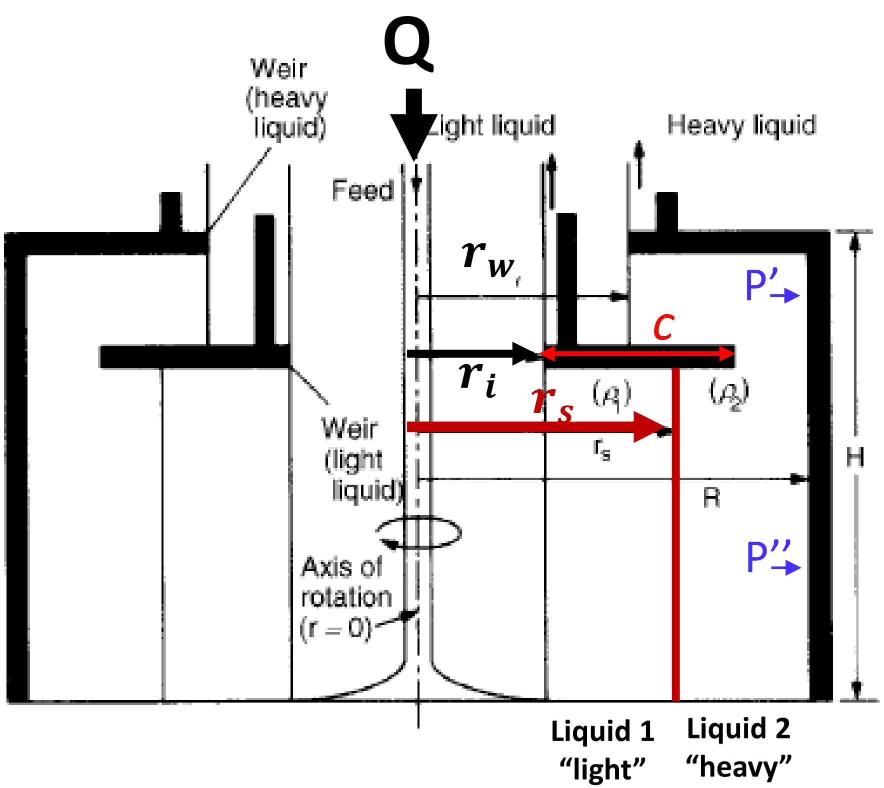
Sigma factor of a disk stack centrifuge

ω is the angular velocity (rad/s)
n is the number of discs
R is the outer radius of the discs (m)
r is the inner radius of the discs (m)
θ is the angle between disc and vertical (rad)
g is the acceleration due to gravity (m/s^2)

$$\boxed{\Sigma = \frac{2}{3}\frac{\pi\omega^2 n\left(R^3 - r^3\right)}{g\tan\theta}}$$

Liquid centrifugation

Problem statement: how to position the "heavy", r_w, and "light" liquid weirs, r_i, so that continuous separation of two immiscible liquids is possible?



Balance of forces:

$$P' = P''$$

$$\frac{1}{2}\rho_2(R^2 - r_w^2)w^2 = \frac{1}{2}\rho_1(r_s^2 - r_i^2)w^2 + \frac{1}{2}\rho_2(R^2 - r_s^2)w^2$$

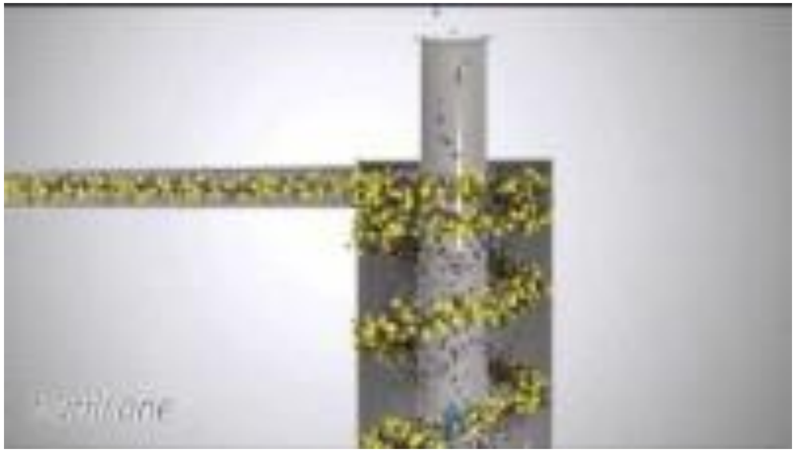
$$\boxed{\frac{r_s^2 - r_i^2}{r_s^2 - r_w^2} = \frac{\rho_2}{\rho_1}} \quad r_s = ?$$

For correct operation:

if $r_i < r_s < r_i + c$ than the continuous separation of both liquids is possible

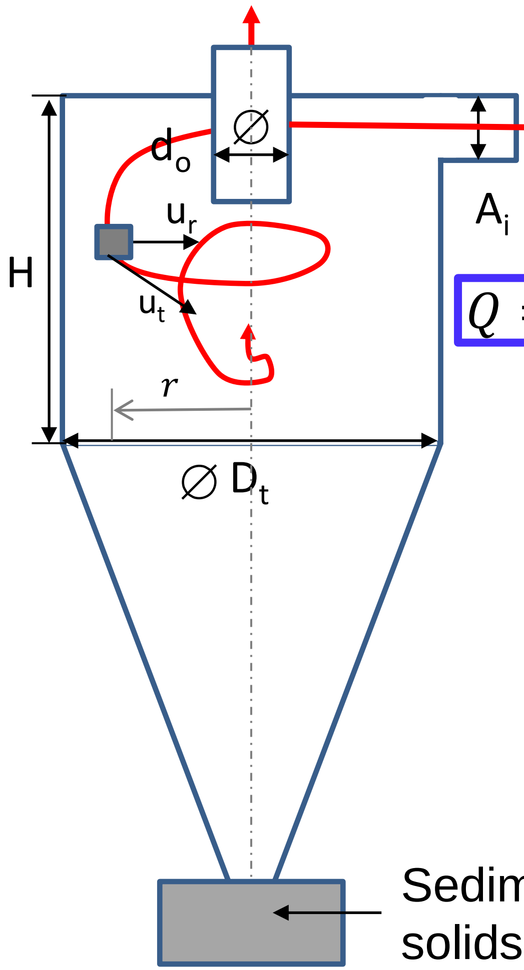
Cyclone design

Cyclones are unit operations for gas cleaning, namely to remove/recover solid particles from a gas stream; the separation principle is the centrifugal force caused by the circular flow trajectory of the gas; cyclones do not contain moving mechanical components.



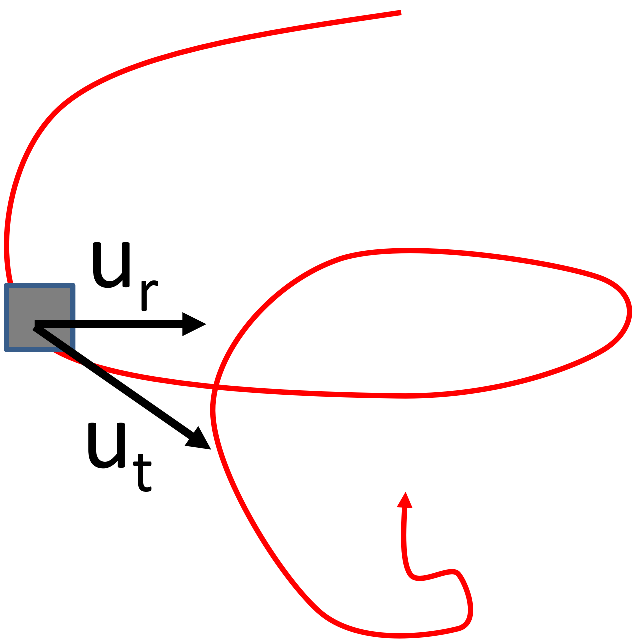
https://www.advancedcyclonesystems.com/pt/hurricane/

Gas field flow



$$\boxed{Q = u_i A_i \; [m^3/s]}$$

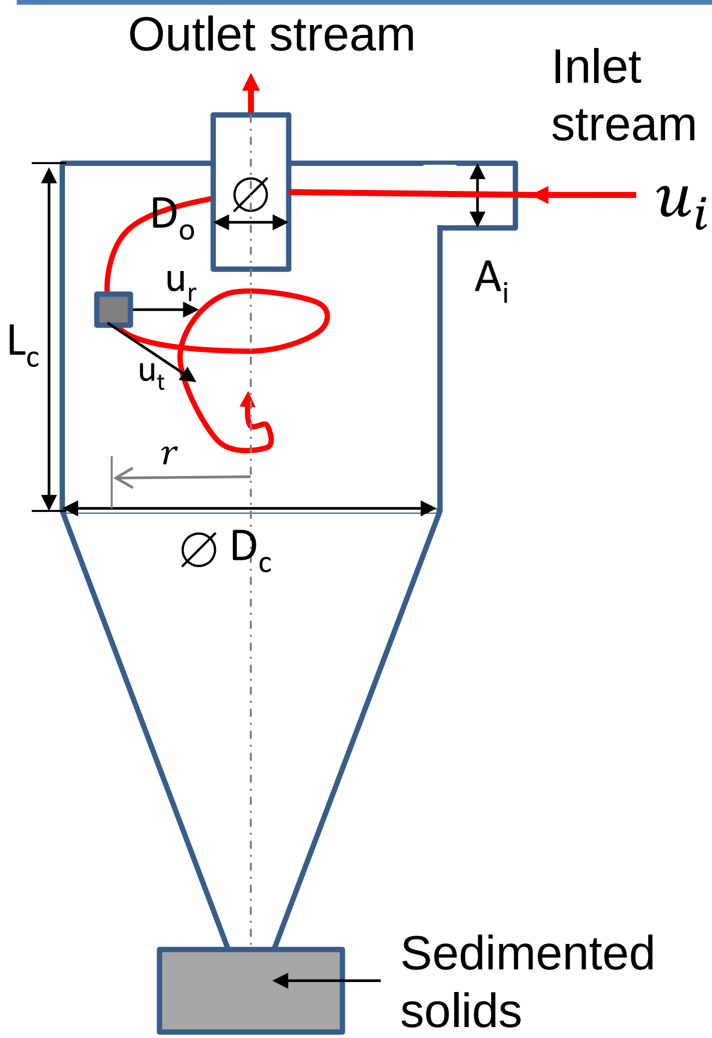
Motion of a gas element inside the cyclone:



u_r – gas velocity radial component, m/s

u_t – gas velocity tangencial component, m/s

Gas field flow



Radial flow component, $u_r(r)$

Gas material balance in steady-state:

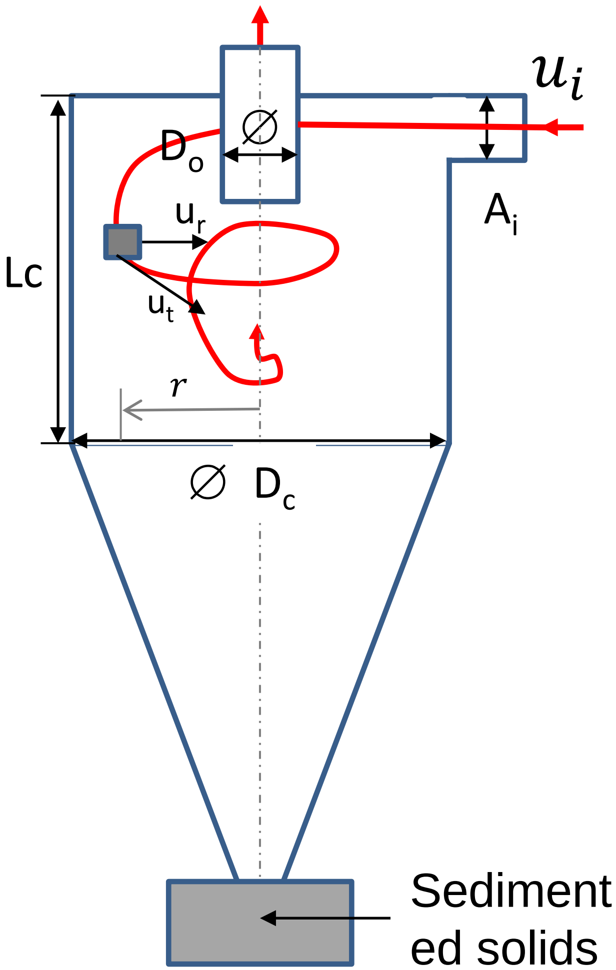
$$Q = u_i A_i = (2\pi r L_c) u_r(r) \quad \forall_r$$

$$\boxed{u_r(r) = u_i \frac{A_i}{2\pi r L_c} \quad \forall_r}$$

Tangencial flow component $u_t(r)$

$$\boxed{u_t(r) = u_i \sqrt{\frac{D_c}{2r}} \quad \forall_r}$$

Balance of forces



Balance of forces on a spherical particle with diameter, d, at position, r

$$\frac{\pi d^3}{6}(\rho_s - \rho)\frac{u_t^2}{r} = 3\pi\mu u_r d$$

Apparent Centrifugal force at position *r*

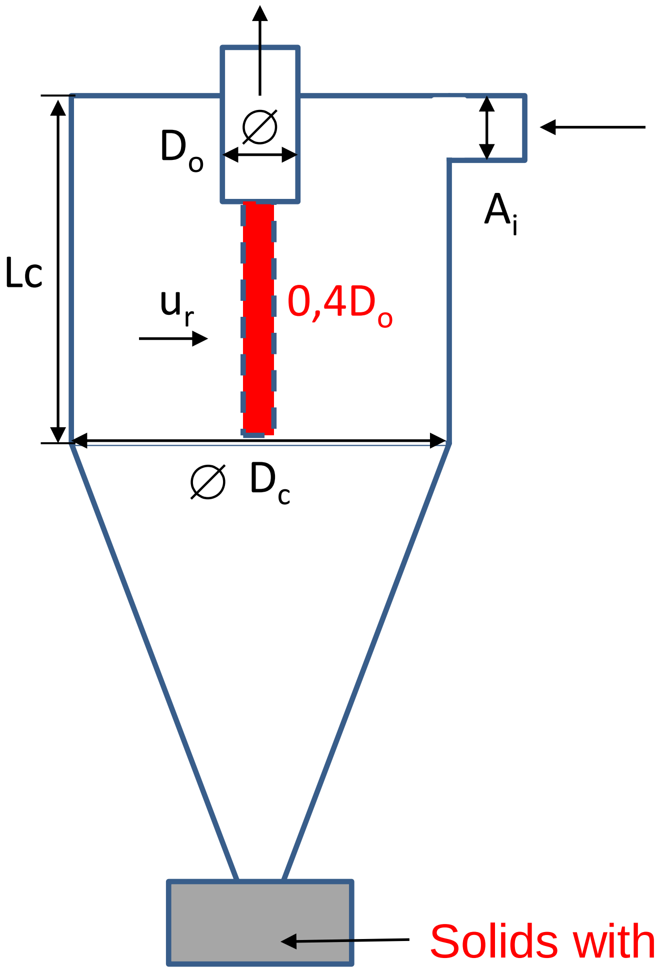
Drag force at position *r*

Particles will acquire an equilibrium circular trajectory, $r(d)$, which depends on their size, d, and specific mass ρ_S

$$\boxed{r = \frac{u_o u_i \pi D_c L_c}{g A_i} = f(d)}$$

Theoretical cut-off model

Outflow: gas + solids with **d≤d***



Solids with **d>d***

Assumption: All particles with equilibrium position inside an imaginary cylinder of diameter 0,4Do (e.g. r=0,2D_o) will be dragged out in the outflow stream. All other particles will sediment.

$$\mathbf{0,2D_0} = \frac{d^2(\rho_s - \rho)\pi D_c L_c u_i}{18\mu A_i}$$

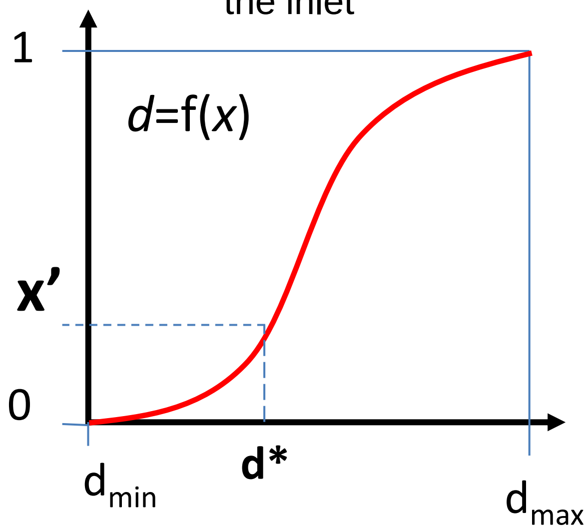
Critical cut-off particle size, *d**

$$\boxed{d^* = \sqrt{\frac{3,6\mu D_0 A_i}{(\rho_s - \rho)\pi D_c L_c u_i}}}$$

PROOF

Cyclone separation efficiency

The cyclone will not have a separation efficiency of 100%. Given a well-defined size distribution of the inlet stream, the separation efficiency **is determined by the smallest particle that the cyclone is able to separate**.



$$\boxed{\eta = 1 - x'}$$

Mass fraction of solids retained by the cyclone

d* **- size of the smallest particle captured in the cyclone**

x' – mass fraction with size $d \leq d*$

$$\boxed{d^* = \sqrt{\frac{3{,}6\mu D_0 A_i}{(\rho_s - \rho)\pi D_c L_c u_i}}}$$

Exercises V.1-V.6

V – CENTRÍFUGA

1. Se uma centrífuga industrial tiver 0.9 m de diâmetro e rodar a 1200 rpm, a que velocidade deve rodar uma centrífuga laboratorial de 150 mm de diâmetro se se pretender que ela reproduza as condições na fábrica?

2. Qual é a máxima velocidade de rotação segura de um cesto de centrífuga em bronze fosforoso, com 0.3 m de diâmetro e 5 mm de espessura, quando contém um líquido (densidade=1) que forma uma camada de 75 mm de espessura nas paredes? Considerar a densidade do bronze fosforoso igual a 8.9 e a tensão de segurança igual a 55×10^6 N/m^2.

3. Pretende rodar-se uma centrífuga com um cesto de bronze fosforoso com 375 mm de diâmetro a 1800 rpm, com uma camada de 100 mm de sólidos nas paredes, cuja massa específica a granel é de 2000 kg/m^3. Qual deve ser a espessura das paredes do cesto se as perfurações forem tão pequenas que tenham um efeito desprezável sobre a resistência?

Massa específica do bronze fosforoso = 8900 kg/m^3
Máxima tensão segura para o bronze fosforoso = 55×10^6 N/m^2

4. Introduz-se uma suspensão aquosa constituída por partículas de densidade 2.5 na gama de tamanhos 1-10 mícrons numa centrífuga com um cesto de 450 mm de diâmetro, que roda a 4800 rpm. Se a suspensão formar uma camada de 75 mm de espessura, quanto tempo levará aproximadamente para que a partícula mais pequena sedimente?

5. Pretende rodar-se uma centrífuga com um cesto de bronze fosforoso com 375 mm de diâmetro a 3600 rpm, com uma camada de 75 mm de líquido de densidade 1.2 no cesto. Qual é a espessura de parede necessária para o cesto?

Massa específica do bronze fosforoso = 8900 kg/m^3
Máxima tensão segura para o bronze fosforoso = 55×10^6 N/m^2

6. Um cesto de centrífuga com 600 mm de comprimento e 100 mm de diâmetro interno tem uma represa de descarga com 25 mm de diâmetro. Qual é o caudal volumétrico máximo de líquido através da centrífuga, de tal modo que quando o cesto rodar a 12000 rpm todas as partículas de diâmetro superior a 1 mícron fiquem retidas na parede da centrífuga? Considere que a força de retardação sobre uma partícula que se move num líquido pode ser calculada pela lei de Stokes.

<u>Dados:</u>
Lei de Stokes: $F=3\pi\mu du$
u = velocidade da partícula em relação ao fluido
μ = viscosidade do líquido, e