Chapter 4. Sedimentation

4.1 *Solids suspensions*

- Definition
- Concentration of solids
- Viscosity of suspensions

4.2 *Settling kinetics*

- Selective settling,
- Sludge line settling,
- Coarse settling

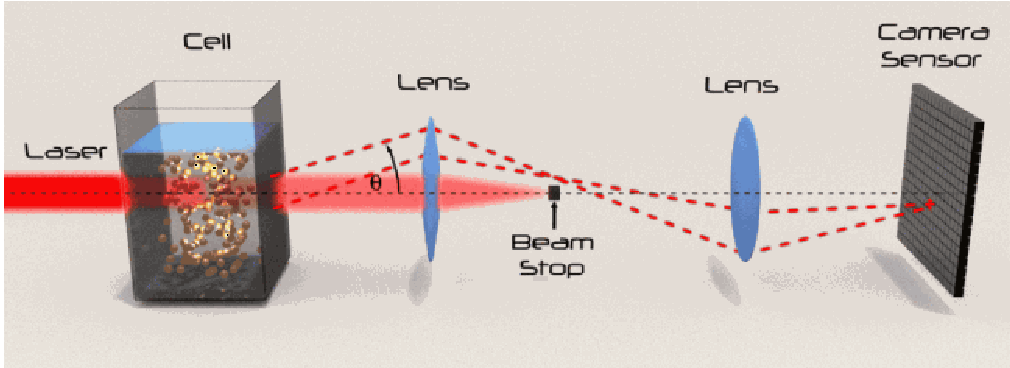
4.3 *Design of a settler*

- Settling operation,
- Material balances
- Kynch method,
- Clarification method

J.M. Coulson and J.F. Richardson pp 237 - 290

4.1 Solids suspensions

- A **solids suspension** is a heterogeneous mixture of materials consisting of two phases:
  (1) **<u>Dispersed phase ("fase dispersa")</u>**: an insoluble solid phase composed by a large number of small particles
  (2) **<u>Dispersion phase ("fase dispersante")</u>**: A continuous dispersion phase, gas or liquid, in which the solid is dispersed
- Opposed to **solutions**, which are transparent homogeneous mixtures, solid suspensions present **turbidity**, i.e. a light beam is scattered by a given angle, θ, due to the presence of undissolved solid particles



Solid suspensions

Solid suspensions may be classified according to the particle size and physicochemical stability as **colloidal**, **fine** and **coarse** suspension

Coarse: d > 100 μm (macroscopic solids e.g. sand)

Fine: 1 μm < d < 100μm (e.g. bacterial or eucariotic cells)

Less stable; **Settle freely**; Are filterable

$\approx 1\ \mu m$

Colloid: 1 nm d < 1 μm (e.g. macromolecules such as polysaccharides, lipids and proteins)

Very stable; **don't settle freely**; are not filterable

Concentration of solids

Different ways to define concentration of solids in a suspension (***C***, ***X***, ***e***, ***1-e***, ***V***). Are these variables related?

C : mass concentration, kg-solid/m^3-suspension

X: mass fraction, kg-solid/kg-suspension

e: void fraction (dimensionless)

$$\boldsymbol{e} = \frac{void\ volume}{void\ volume + solids\ volume} = \frac{void\ volume}{total\ volume}$$

1-*e:* volumetric concentration of solids (v/v)

$$(\mathbf{1} - \boldsymbol{e}) = \frac{solids\ volume}{total\ volume}$$

V: mass ratio, e.g. (kg water/kg solids)

Suspension viscosity, μ_c

- The presence of solids typically increases the viscosity of the suspension, $\mu_c > \mu$
- Gentle stirring typically decreases the viscosity of the suspension

Diluted suspensions

Einstein Eq: $1 - e < 0.02$

$$\boxed{\frac{\mu_c}{\mu} = [1 + k''(1 - e)]}$$

Concentrated suspensions

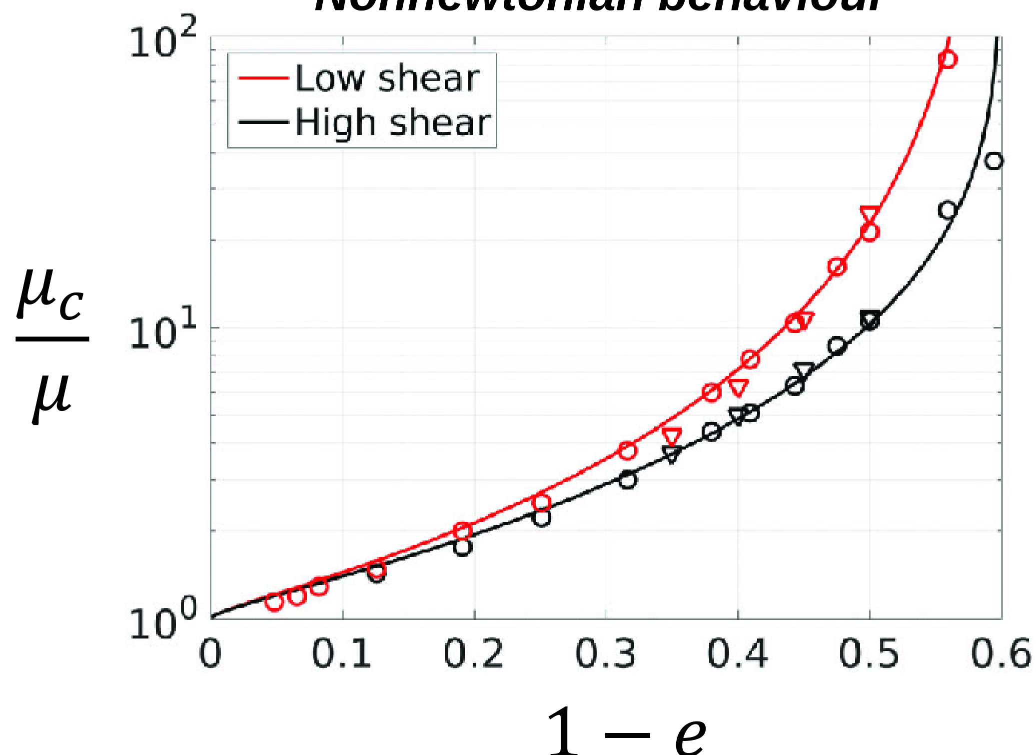
Vand Eq: $1 - e > 0.02$

$$\boxed{\frac{\mu_c}{\mu} = \cdot\, e^{\frac{k''(1-e)}{1-a'(1-e)}}}$$

$a' = 0.609\ (spheres)$

$k'' = 2.5\ (spheres)$

Nonnewtonian behaviour



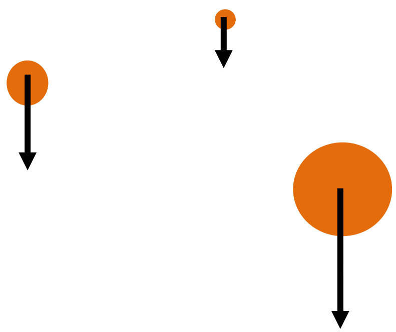
4.2 Settling kinetics

What is the rate of settling, u_c (m/s), that particles will acquire in a solids suspension under the gravity field?

- **Selective settling:** diluted solids suspensions with low concentration of solids
- **Sludge line settling:** solids suspensions with high concentration of fine particles typically in range 1 μm < d < 100μm (e.g. bacterial or eucariotic cells)
- **Coarse settling:** solids suspensions with high concentration of coarse particles typically in range d > 0,1 mm (macroscopic solids e.g. sand)

Selective settling

<u>Selective settling:</u> In diluted suspensions ($C \rightarrow 0$) , particles settle independently of each other according to their size, geometry and specific mass. Different particle sizes/specific mass settle at different velocities.



If $C \rightarrow 0$, particles are very distant of each other thus particle interactions are negligible.

$$\boxed{u_c \cong u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu}}$$

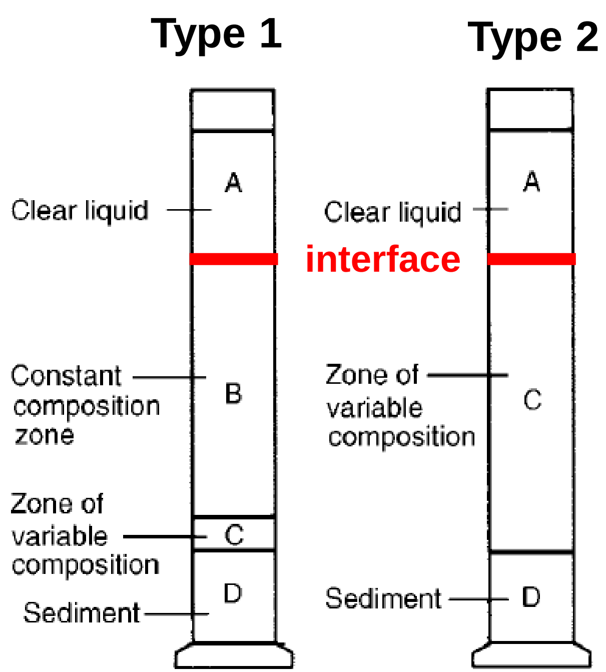
>>Exercise III.4

Sludge line settling (SLS)

<u>Sludge line settling</u>: In concentrated solids suspension (C>>0) with particle size differences less than 6:1, all particles settle at the same velocity and a very **well-defined interface** between clear liquid (zone A) and solids region (zones B,C,D) is formed.

∴ Larger particles are retarded while small particles are accelerated

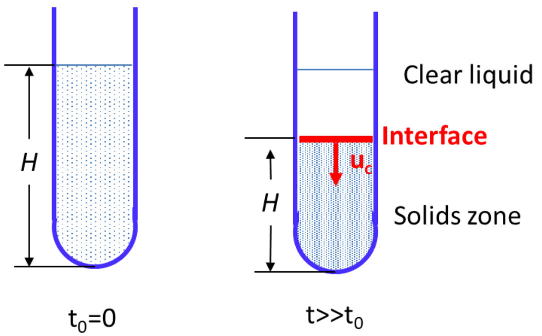
Type 1 SLS is the most frequent case, characterized by a pronounced zone with constant solids concentration (zone B) followed by a less marked zone where the solids concentration rapidly increases (zone C)



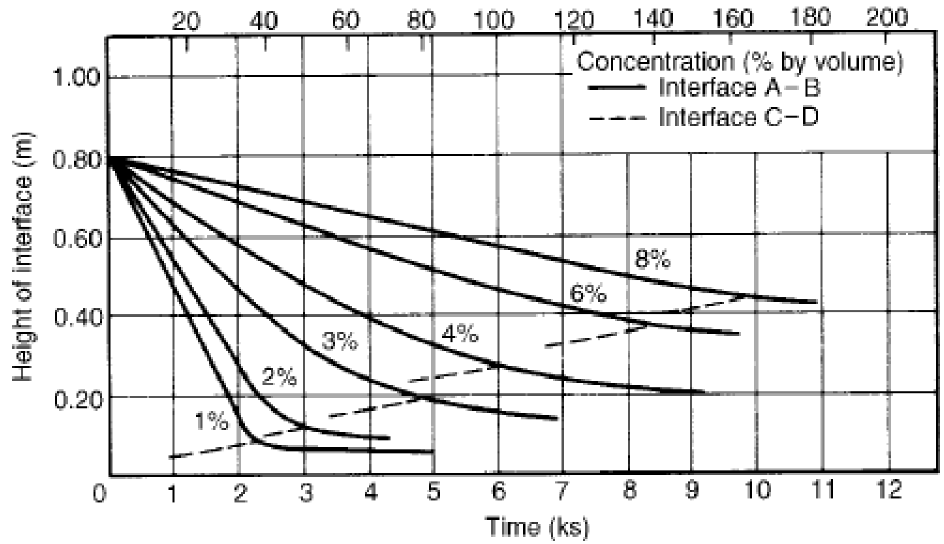
Type 2 SLS is a less frequent case, that does not develop a constant solids zone (zone B), instead showing a very pronounced zone with variable solids concentration (zone C). Type 2 SLS has typically very different particle sizes in zone C

Effect of solids concentration in SLS

Experiment with a sedimentation tube



$$u_c = \frac{dH}{dt} \approx \frac{\Delta H}{\Delta t}$$



Effect of volumetric concentration of solids, (1-e)x100 (%), on interface high, H (m) in calcium carbonate suspensions (**M. Coulson and J.F. Richardson pp 242**)

Observations:

(1) linear decay of interface A-C height, *H* (constant $\boldsymbol{u_c}$)

(2) the higher the solids concentration the lower is the slope (**u_c** decreases with *C*)

(3) after some time the interface height, *H*, slows down and stabilizes at the final value H_∞ (**u_c** ~ 0)

SLS kinetic models, u_c

Question: *I*s the settling velocity, u_c, in SLS smaller or greater than the terminal fall velocity, u_o?

Robinson Eq.:

$$\boxed{u_c = \frac{K'' d^2(\rho_s - \rho_c)g}{\mu_c}}$$

Steinour Eq.: small uniform particles;

$$\boxed{u_c = \frac{e^2 d^2(\rho_s - \rho_c)g}{18\mu} f(e)}$$

Case dependente tapioca in oil $f(e) = 10^{-1.82(1-e)}$

Hawsksley Eq.:

$$\boxed{u_c = \frac{e d^2(\rho_s - \rho_c)g}{18\mu_c}}$$

Note: ρ_c and μ_c are properties of the suspension. Also note that these equations show a dependency of u_c on solids concentration.

$\rho_c = e\rho + (1-e)\rho_s$ (specific mass of suspension)

Coarse suspensions settling kinetics, u_c

Settling velocity for coarse suspensions with d>0,1 mm (macroscopic kinetics, e.g. sand) and high concentration of solids

$$\boxed{\frac{u_c}{u_i} = e^n}$$

$$\log(u_0) = \log(u_i) + \frac{d}{d_t}$$

u_c – settling velocity, m/s
u_i – settling velocity for infinit dilution, m/s
e – void faction
n = coeficiente = $f(Re'_0, d/d_t)$
d – particle diameter
d_t – tube diameter
$Re'_0 = \frac{\rho u_0 d}{\mu}$ - particle Reynolds at u_0

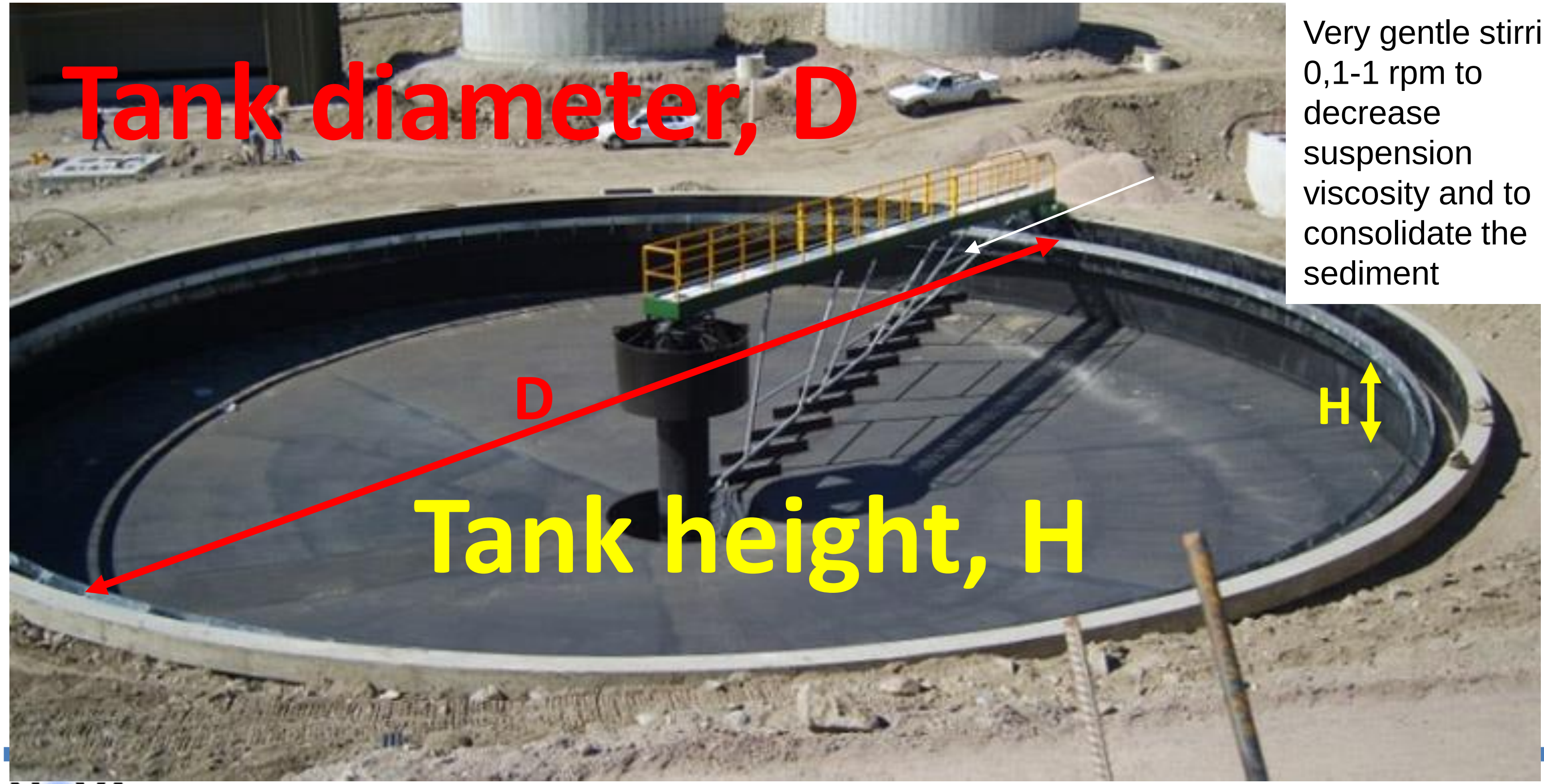
Table 5.1. n as a function of Ga or Re'_0 and d/d_t

Range of Ga	Range of Re'_0	n as function of $Ga, d/d_t$	n as function of $Re'_0, d/d_t$
0–3.6	0–0.2	$4.6 + 20\, d/d_t$	$4.6 + 20\, d/d_t$
3.6–21	0.2–1	$(4.8 + 20\, d/d_t)\, Ga^{-0.03}$	$(4.4 + 18\, d/d_t)\, Re_0'^{-0.03}$
21–2.4 $\times 10^4$	1–200	$(5.5 + 23\, d/d_t)\, Ga^{-0.075}$	$(4.4 + 18\, d/d_t)\, Re_0'^{-0.1}$
2.4×10^4–8.3×10^4	200–500	$5.5 Ga^{-0.075}$	$4.4 Re_0'^{-0.1}$
$>8.3 \times 10^4$	>500	2.4	2.4

J.M. Coulson and J.F. Richardson, pp 272

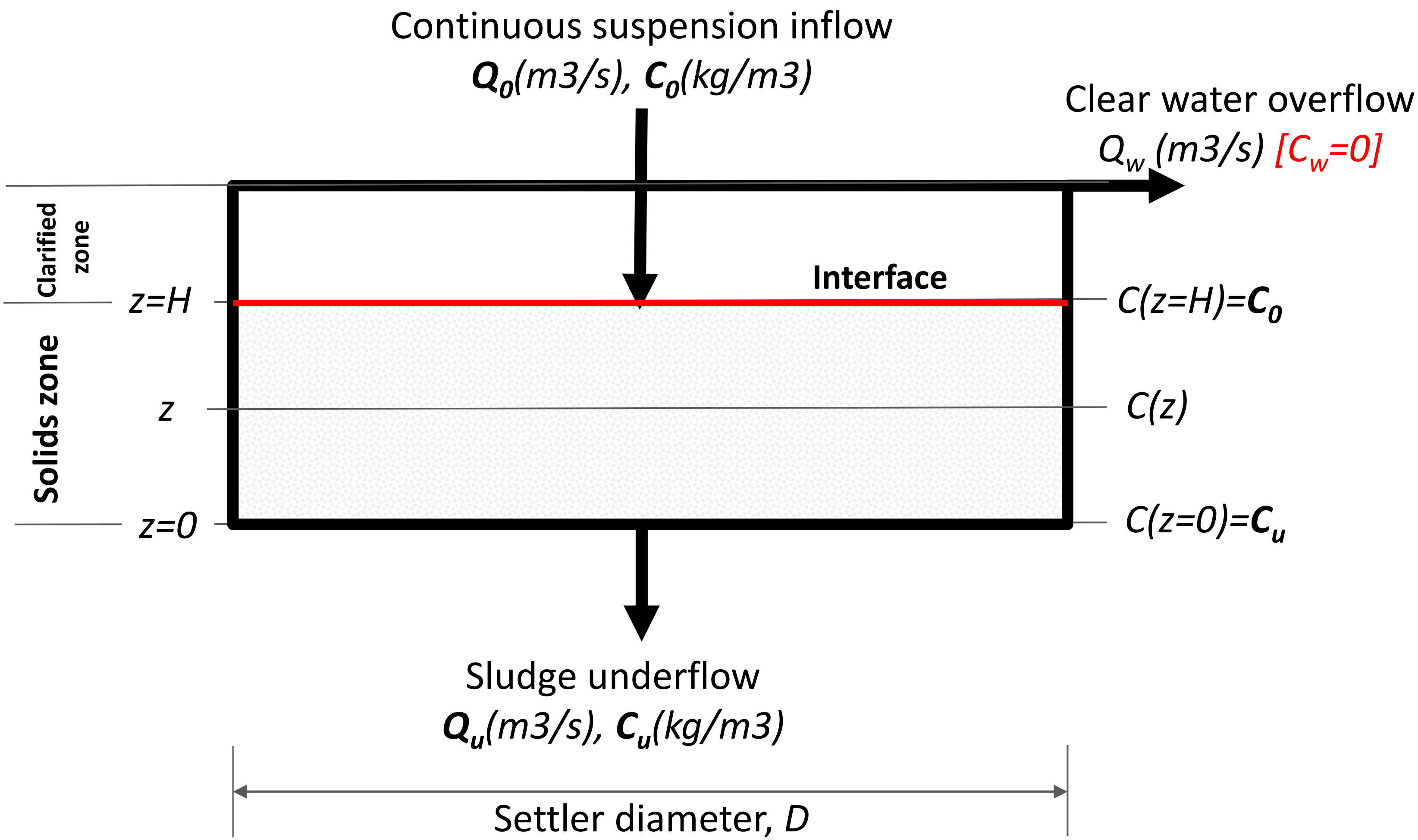
4.3 Design of a settler

Given a liquid-solid suspension inflow stream $\mathbf{Q_0}$, $\mathbf{C_0}$, what's the size of the settler (***H***, ***D***=?) such as to ensure 1) a solids free (clarified) outflow stream and 2) solids outflow stream with desired concentration $\mathbf{C_u}$

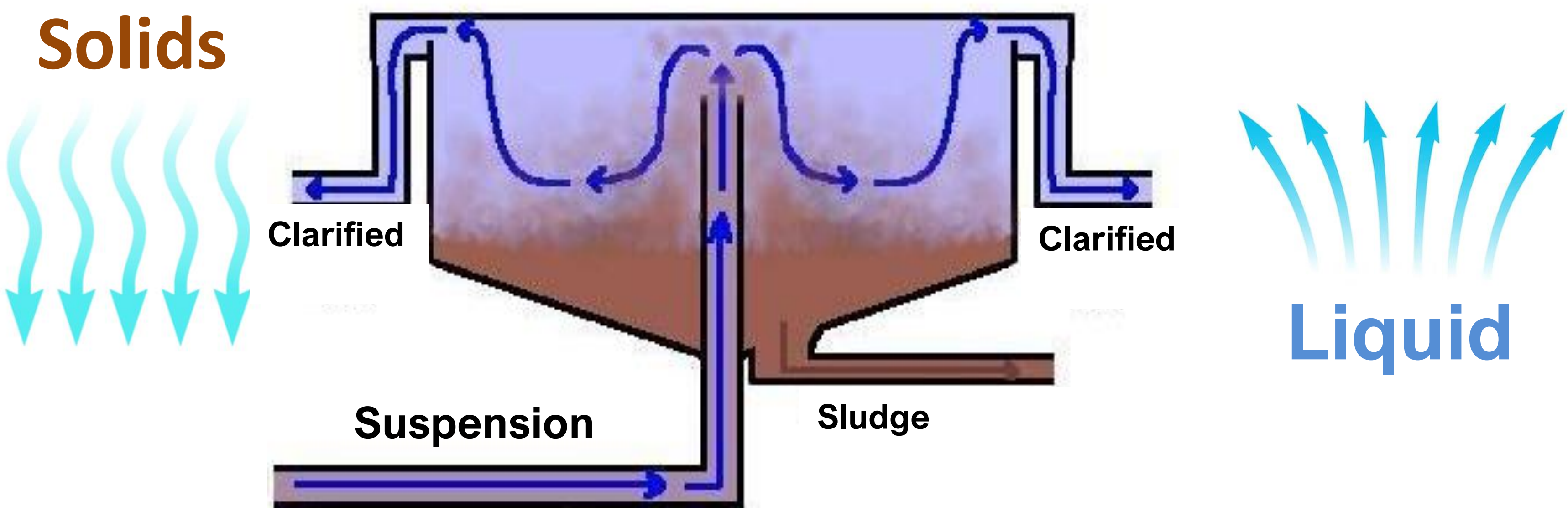


Very gentle stirring 0,1-1 rpm to decrease suspension viscosity and to consolidate the sediment

Design of a settler



Flow of liquid versus flow of solids



The solids are moving down due to gravity, taking the space of the liquid, which is impelled to move up.

Material balances in steady-state

Hypotheses: no solids in the clear water overflow stream

Solids:

$$Q_0 C_0 \quad = \quad Q_u C_u$$

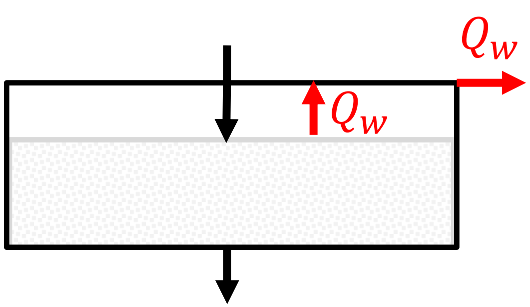
Solids in in the inflow stream, kg/s

Solids in in the underflow stream, kg/s

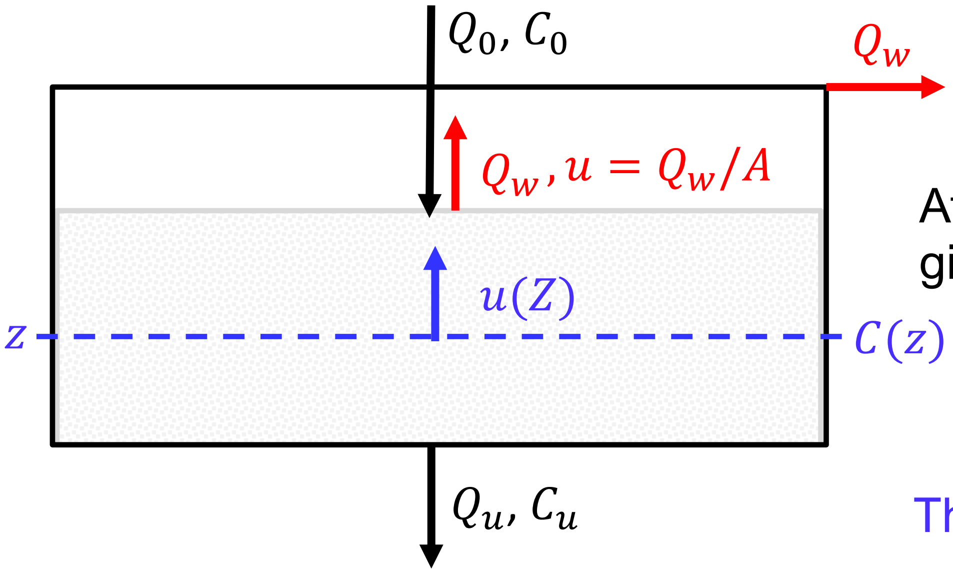
$$\boxed{Q_u = \frac{Q_0 C_0}{C_u} \ [\mathrm{m^3/s}]}$$

Liquid:

$$\color{red}{Q_w} = Q_0 - Q_u \ [\mathrm{m^3/s}]$$



Upflow of liquid, u [m/s]



Material balance equations in steady state:

$$\begin{cases} Q_u = \dfrac{Q_0 C_0}{C_u} \\ Q_w = Q_0 - Q_u \end{cases}$$

At the interface, the upflow liquid velocity is given by:

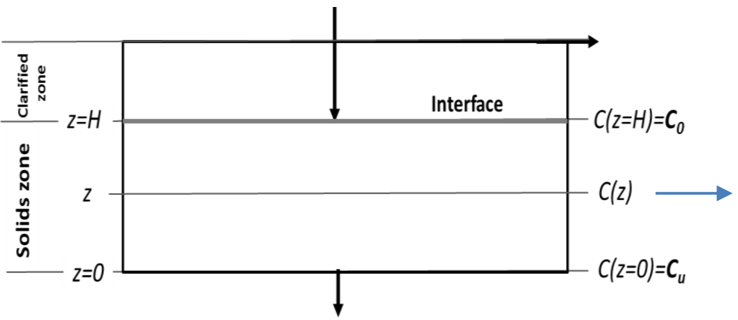
$$u = \frac{Q_u C_u}{A}\left(\frac{1}{C_0} - \frac{1}{C_u}\right) \quad \text{[m/s].}$$

Therefore, at any plan z:

$$\boxed{u(z) = \frac{Q_u C_u}{A}\left(\frac{1}{C(z)} - \frac{1}{C_u}\right)}$$

Downflow of solids, ψ [kg/m2 s]

The flux of solids, ψ, applies only to the solids zone and corresponds to the mass flow of solids normalized by the cross-section area of the tank. It has units of kg/[s m2]. It has two terms: (1) the flux due to settling, and (2) the flux due to advective transport.



(1) Flux of solids due to settling

(2) Flux of solids due to advective transport

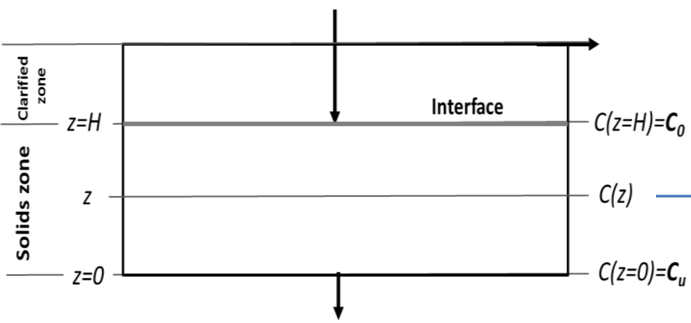
$$\psi(z) = u_c(C(z))C(z) + u_u C(z)$$

$$u_u = \frac{Q_u}{A}$$

∴ Average linear velocity [m/s] of suspension (downward direction) in the solids zone(it is a constant)

Downflow of solids, ψ [kg/m2 s]

The flux of solids, ψ, applies only to the solids zone and corresponds to the mass flow of solids normalized by the cross-section area of the tank. It has units of kg/[s m2]. It has two terms: (1) the flux due to settling, and (2) the flux due to transport.



$$\psi_0 = \quad u_c(C_0)C_0 \ + u_u C_0$$

$$\psi(z) = u_c\big(C(z)\big)C(z) + u_u C(z)$$

$$\psi_u = \qquad 0 \qquad + u_u C_u$$

$$u_u = \frac{Q_u}{A}$$

At z=0 there is no sedimentation (tank wall physical barrier), thus flux of solids is caused solely by transport (i.e. underflow discharge stream)

Design by the Kynch method

Kynch condition: The flux of solids kg/(s m^2) in the underflow, ψ_u, must not exceed the flux of solids, $\psi(z)$, at any plane z of the continuous settler

Flux of solids due to settling

Flux of solids due to advective transport

$$\psi_u \leq \psi(z) = u_c(z)C(z) + u_u C(z) \quad \forall_z$$

Interpretation: to ensure a solids free overflow stream, there is a minimum tank area, A, and a maximum flux of solids in the underflow, ψ_u, that must be obeyed

Design by the Kynch method - interpretation

Condition for a solids-free overflow stream:

$$u(z) \leq u_c\big(C(z)\big) \quad \forall_z$$

the upflow velocity of liquid, **u(z)**, must be smaller than the settling velocity at any plane z, otherwise the drag force will drag solids upwards

since: $$u(z) = \frac{Q_u C_u}{A}\left(\frac{1}{C(z)} - \frac{1}{C_u}\right)$$

$$\Rightarrow \quad A \geq \frac{Q_o C_o}{u_c(C(z))}\left(\frac{1}{C(z)} - \frac{1}{C_u}\right) \quad \forall_z$$

large enough cross-section tank area (A) to ensure a solids free overflow stream

$$\Rightarrow \quad \boxed{\psi_u \leq u_c(C(z))C(z) + u_u C(z) \quad \forall_z}$$

low enough flux of solids in the underflow stream (ψ_u) - Kynch condition

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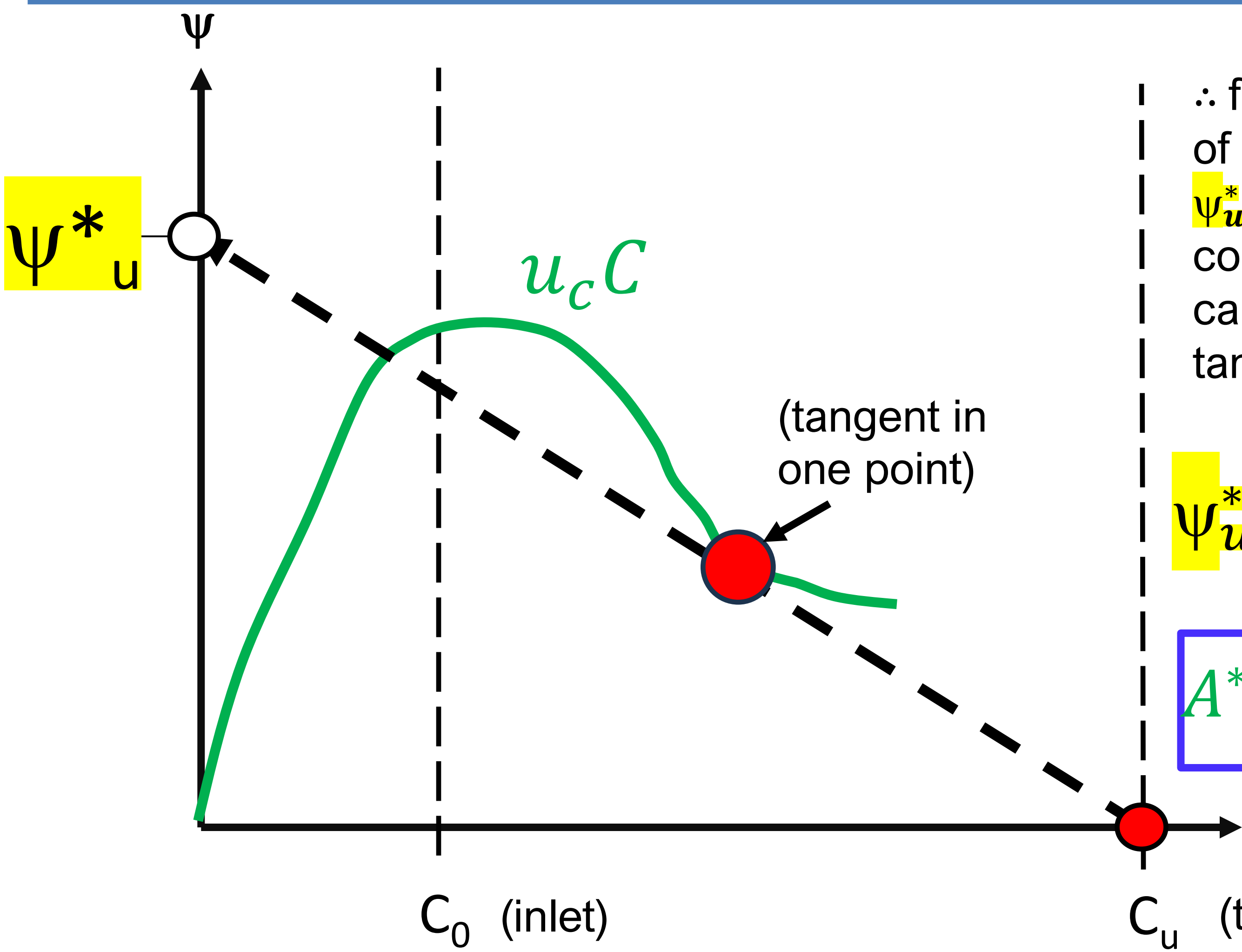
Determination of minium tank area, A^*

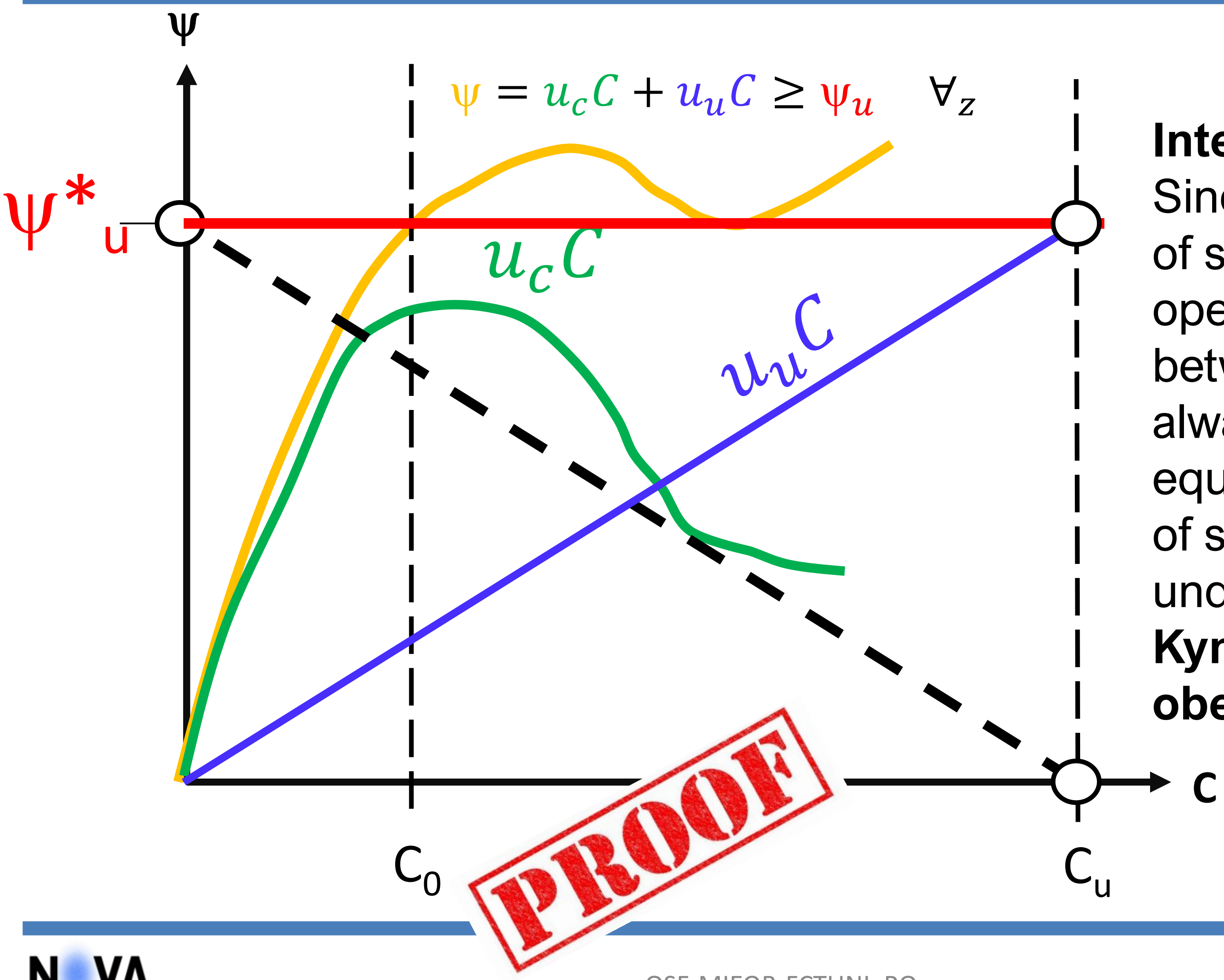


∴ find the maximum flux of solids in the underflow, ψ^*_u, that obeys to Kynch condition and then calculate the minimum tank area, **A***

$$\psi^*_u = \frac{Q_u C_u}{A^*} = \frac{Q_0 C_0}{A^*}$$

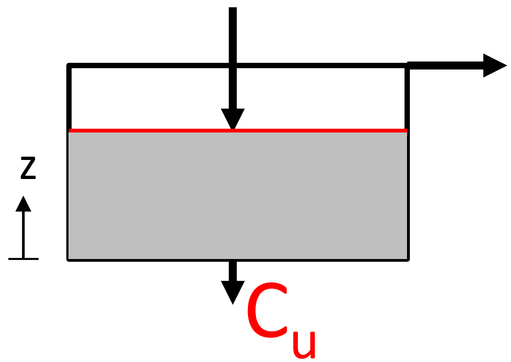
$$\boxed{A^* = \frac{Q_u C_u}{\psi^*_u} = \frac{Q_0 C_0}{\psi^*_u}}$$

Determination of minium tank area, A^*



Interpretation: Since the total flux of solids, ψ, at any operation point between [C0,Cu], is always greater or equal than the flux of solids in the underflow, ψ_u, **the Kynch criterion is obeyed!**

Height (H) of the settling tank



How to ensure the desired solids concentration , C_u, in the underflow stream?

Sludge residence time in the continuous settler [h] = Settling time needed to reach Cu in the batch sedimentation tube [h]

$$\frac{AH}{Q_0} = t_{batch,C_u}$$

$$\boxed{H = \frac{Q_0 \times t_{batch,C_u}}{A}}$$

∴ Height of the settling tank [m] that sets the sludge residence time sufficiently long for the solids to reach the desired concentration, Cu

Exercises IV.1 – IV-7

IV – SEDIMENTAÇÃO E ESPESSAMENTO

1. Um ensaio de decantação em tubo de ensaio foi realizado com uma suspensão de carbonato de cálcio (massa específica = 2710 kg/m^3) em água, cuja concentração inicial é de 236 g/l. Os resultados do ensaio vêm expressos na seguinte tabela.

Tempo (h)	Altura de interface (cm)
0	36.0
0.25	32.4
0.5	28.6
1.0	21.0
1.75	14.7
3.0	12.3
4.75	11.6
12.0	9.8
20.0	8.0

a) Determine a concentração de sólidos na zona de espessado em função do tempo
b) Determine a porosidade na zona de espessado em função do tempo.
c) Determine a velocidade de sedimentação em função da concentração de sólidos (represente graficamente).
d) Determine o fluxo (médio) de sólidos em função da concentração de sólidos.
e) Estime a velocidade de sedimentação usando uma lei apropriada assumindo que o diâmetro médio das partículas é 5 μm.

2. Pretende-se projectar um sedimentador a operar em contínuo para concentrar uma polpa com 5 kg de água por kg de sólidos numa lama que contenha 1.5 kg de água por kg de sólidos. O caudal de entrada ao sedimentador é 0.6 kg-sólido/s e a massa específica do sólido é 2500 kg/m3. Ensaios laboratoriais de decantação determinaram que o tempo necessário para concentrar de 5 kg-água/kg-sólido a 1,5 kg-água/kg-sólido é de 3 horas de acordo com o seguinte perfil de velocidade de sedimentação:

Concentração (kg de água/kg de sólidos)	5.0	4.2	3.7	3.1	2.5
Velocidade de sedimentação (mm/s)	0.17	0.10	0.08	0.06	0.042

a) Calcule o caudal de água em kg/s na corrente do clarificado.
b) Calcule a área mínima por forma a assegurar que não passa sólido para a corrente de clarificado.
c) Calcule o fluxo de sedimentação (em kg s^{-1} m^{-2})
d) Calcule a área mínima por forma a assegurar o fluxo de sólidos pretendido na corrente de espessado pelo método de Kynch.
e) Determine a altura e o diâmetro do sedimentador

3. Pretende-se dimensionar um sedimentador para tratar 0.1 m^3/s duma suspensão de sólidos com uma concentração de 150 kg/m^3. A concentração pretendida no espessado é de 1290 kg/m^3. Um ensaio laboratorial de decantação com a mesma suspensão forneceu os resultados indicados na tabela. O ensaio de decantação demonstrou que são necessárias 19 horas para a suspensão decantar duma concentração inicial de 150 kg/m3 a uma concentração final de 1290 kg/m3.

Concentração de sólidos (kg/m^3)	Velocidade de sedimentação (μm/s)
100	148
200	91
300	55.33
400	33.25
500	21.40
600	14.50
700	10.29
800	7.38
900	5.56
1000	4.20
1100	3.27

4. Um ensaio de decantação foi realizado com uma suspensão de carbonato de cálcio, cuja concentração inicial é de 236 g/l. Deseja-se calcular o diâmetro de um decantador com capacidade para processar 8 t/h de suspensão, alimentada ao decantador contendo 236 kg/m^3 de $CaCO_3$. A lama espessada deverá conter 550 kg/m^3 de $CaCO_3$. Os resultados do ensaio de decantação encontram-se na tabela abaixo.

Tempo (h)	Altura da interface (cm)
0	36.0
0.25	32.4
0.5	28.6
1.0	21.0
1.75	14.7
3.0	12.3
4.75	11.6
12.0	9.8
20.0	8.0