

# AM 1 - PO

## Resolução Lista 4

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### Conteúdo

<b>Questão 4</b>	$u_{n+1} = \sqrt{2u_n}$	<b>2</b>
4 - a)	$u_n \in [1, 3] \quad \forall n \in \mathbb{N}$	2
4 - b)	$ u_{n+2} - u_{n+1}  \leq \frac{\sqrt{2}}{2}  u_{n+1} - u_n $	2
<b>Questão 5</b>	$u_{n+1} = 1.5u_n + 1$	<b>2</b>
5 - a)	Prove que $u_n$ é convergente	2

**Questão 4**  $u_1 = 1; \quad u_{n+1} = \sqrt{2 u_n}$

**4 - a)**  $u_n \in [1, 3] \quad \forall n \in \mathbb{N}$

$$\Longleftrightarrow 1 \leq u_n \leq 3 \Longleftrightarrow 2 \leq 2 u_n \leq 6 \Longleftrightarrow 1 \leq \sqrt{2} \leq \sqrt{2 u_n} \leq \sqrt{6} \leq 3$$

**4 - b)**  $|u_{n+2} - u_{n+1}| \leq \frac{\sqrt{2}}{2} |u_{n+1} - u_n|$

$$\begin{aligned} \Longleftrightarrow & \left| \sqrt{2 u_{n+1}} - \sqrt{2 u_n} \right| = \left| \frac{2 u_{n+1} - 2 u_n}{\sqrt{2 u_{n+1}} + \sqrt{2 u_n}} \right|; u_n \geq 1 \quad \forall n \in \mathbb{N} \Rightarrow \\ \Rightarrow & \frac{2}{\sqrt{2 u_{n+1}} + \sqrt{2 u_n}} |u_{n+1} - u_n| \leq \frac{2}{2\sqrt{2}} |u_{n+1} - u_n| = \frac{\sqrt{2}}{2} |u_{n+1} - u_n| \end{aligned}$$

**Questão 5**  $u_1 = 0; \quad u_{n+1} = 1.5 u_n + 1$

**5 - a)** Prove que  $u_n$  é convergente

$$\Longleftrightarrow 1 \leq \frac{|u_{n+2} - u_{n+1}|}{|u_{n+1} - u_n|} = \frac{|1.5 u_{n+1} + 1 - 1.5 u_n + 1|}{|u_{n+1} - u_n|} = 1.5 \frac{|u_{n+1} - u_n + 4/3|}{|u_{n+1} - u_n|}$$