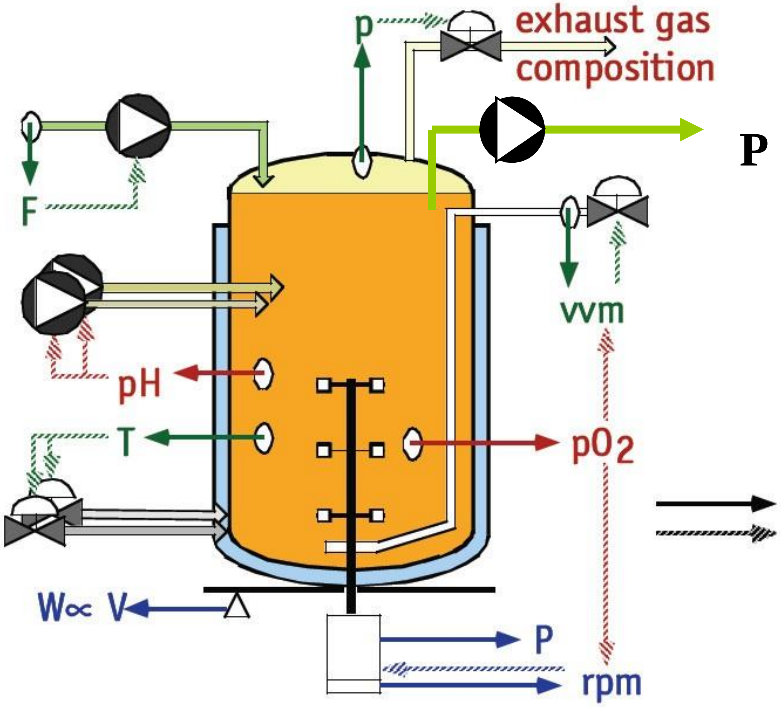
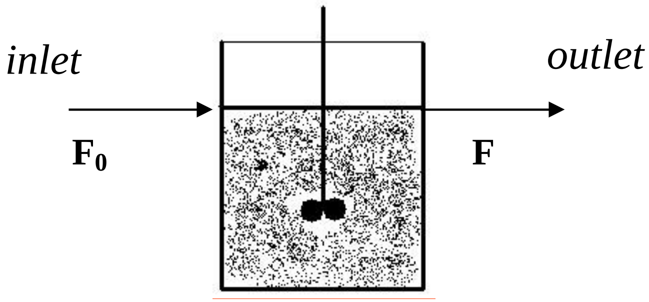
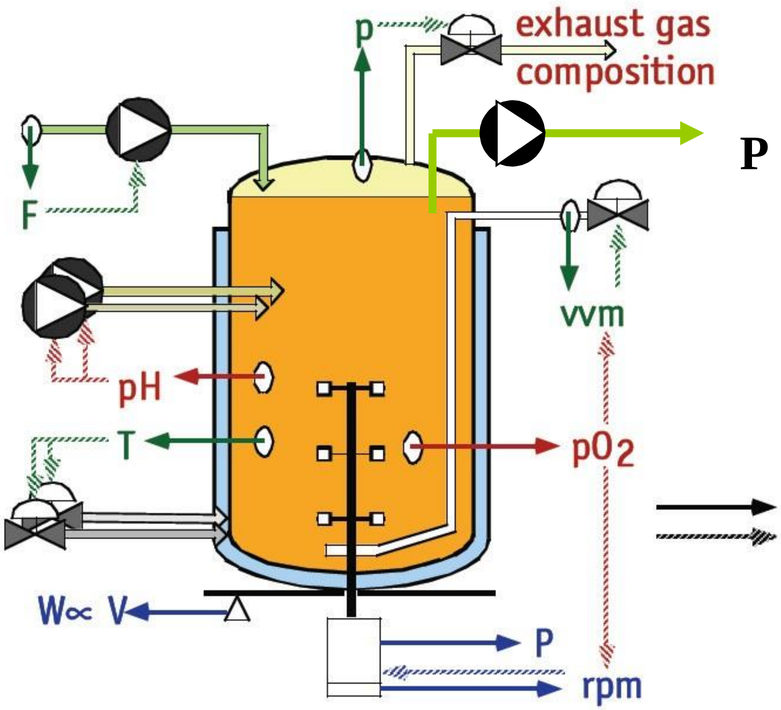
Continuous Stirred Tank Reactor - CSTR



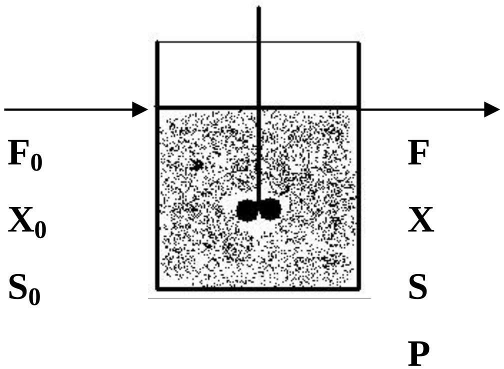
Continuous Stirred Tank Reactor - CSTR

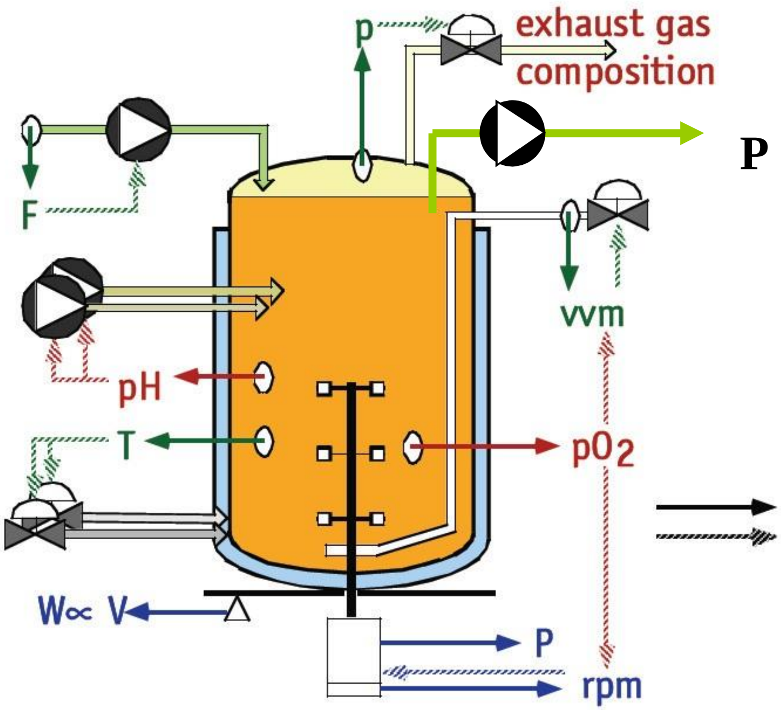




F – flow rate (l/h)

Continuous Stirred Tank Reactor - CSTR





F – flow rate (l/h)
x – cell concentration (gX/l)
S – substrate concentration (gS/l)
P – product concentration (gP/l)

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2.1 – Mass balance to the cell concentration

$$\frac{dx}{dt} = \frac{F_0}{V} x_0 - \frac{F}{V} x + \mu x - k_d x$$

F_0 – inlet flow rate (l/h)
V - volume of the reactor (l)
k_d – specific cell death rate (h^{-1})
x_0 - inlet cell concentration (gX/l)
with $F_0 = F$

2.1 – Mass balance to the cell concentration

$$\frac{dx}{dt} = \frac{F_0}{V} x_0 - \frac{F}{V} x + \mu x - k_d x$$

Cells entering the reactor

Cells exiting the reactor

Cell growth

Cell death

2.1 – Mass balance to the cell concentration

$$\frac{dx}{dt} = \cancel{\frac{F_0}{V}x_0} - \frac{F}{V}x + \mu x - \cancel{k_d x}$$

F_0 – inlet flow rate (l/h)
V - volume of the reactor (l)
k_d – specific cell death rate (h^{-1})
x_0 - inlet cell concentration (gX/l)
with $F_0 = F$

In most fermentations ($x_0 = 0$) and ($\mu \gg k_d$):

$$\frac{dx}{dt} = -\frac{F}{V}x + \mu x$$

$$\frac{F}{V} = D = \frac{1}{TRH}$$

D - dilution rate (h^{-1})
TRH - hydraulic retention time (h)

2 – Continuous Reactor

2.1 – Mass balance to the cell concentration

$$\frac{dx}{dt} = -\frac{F}{V}x + \mu x \quad \longrightarrow \quad \frac{dx}{dt} = \mu x - Dx$$

$$\frac{F}{V} = D$$

2 – Continuous Reactor

2.1 – Mass balance to the cell concentration

$$\frac{dx}{dt} = -\frac{F}{V}x + \mu x \quad \Rightarrow \quad \frac{dx}{dt} = \mu x - Dx$$

In steady state: $$\frac{dx}{dt} = 0$$

$$D\,x = \mu\,x \quad \Rightarrow \quad \mu = D$$

In steady state, without cell death and sterile feeding:

$$\boxed{\mu = D}$$

2.2 – Mass balance to the substrate

$$\frac{dS}{dt} = \frac{F\,S_0}{V} - \frac{F\,S}{V} - \frac{1}{Y_{x/s}}\mu x - mx$$

2.2 – Mass balance to the substrate

$$\frac{dS}{dt} = \frac{F\,S_0}{V} - \frac{F\,S}{V} - \frac{1}{Y_{x/s}}\mu x - mx$$

Substrate entering the reactor

Substrate exiting the reactor

Substrate used for cell growth

Substrate used for maintenance

2.2 – Mass balance to the substrate

$$\frac{dS}{dt} = \frac{F\,S_0}{V} - \frac{F\,S}{V} - \frac{1}{Y_{x/s}}\mu x - mx$$

$$\frac{F}{V} = D$$

$$\frac{dS}{dt} = D\,S_0 - D\,S - \frac{1}{Y_{x/s}}\mu x - \cancel{mx}$$

if m is negligible (mx « µx):

$$\frac{dS}{dt} = D\,(S_0 - S) - \frac{1}{Y_{x/s}}\mu x$$

2.2 – Mass balance to the substrate

In steady state

$$\frac{dS}{dt} = 0$$

$$\mathrm{D}\ (\mathrm{S}_0\ - \mathrm{S}) = \frac{1}{Y_{x/s}}\mu x$$

$\mu = D$

$$\mathrm{D}\ (\mathrm{S}_0\ - \mathrm{S}) = \frac{1}{Y_{x/s}}\mathrm{D}\ x$$

$$Y_{x/s}\ (\mathrm{S}_0\ - \mathrm{S}) = x$$

2.3 – Relationship between substrate concentration and cell concentration with dilution rate

$$\mu = \frac{\mu_{max}\, S}{K_s + S}$$

In a continuous reactor under steady state

$$D = \frac{\mu_{max}\, S}{K_s + S}$$

Rearranging for S

$$S = \frac{K_s\, D}{\mu_{max} - D}$$

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2.3 – Relationship between substrate concentration and cell concentration with dilution rate

$$S = \frac{K_s\, D}{\mu_{max} - D}$$

$$Y_{x/s}\,(S_0 - S) = x$$

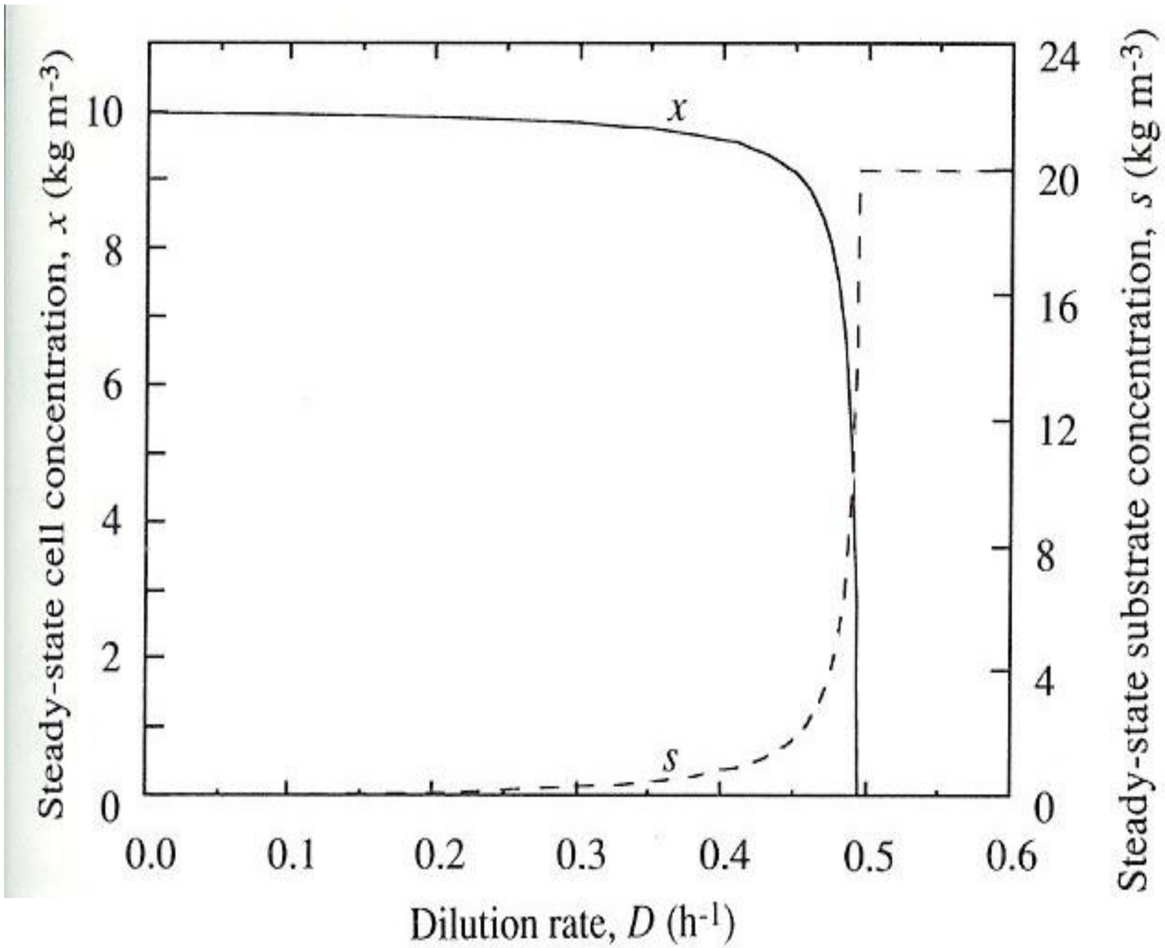
$$x = Y_{x/s}\left(S_0 - \frac{K_s\, D}{\mu_{max} - D}\right)$$

when $D \rightarrow 0$

$S \rightarrow 0$

$$x = Y_{x/s}\, S_0$$

2.3 – Relationship between substrate concentration and cell concentration with dilution rate



For high dilution rates:

- X decreases sharply
- S is not consumed
  → S_0

2.3 – Relationship between substrate concentration and cell concentration with dilution rate

The value at which x=0 and D ~ μ_{max} is named the critical washout rate D_c

At this D value we have an "washout" of the reactor

$$D_c = \frac{\mu_{max} S_0}{K_s + S_0}$$

If $S_0 \gg K_s$ then $D_c = \mu_{max}$

This point is very sensitive: a small variation of D gives a large variation of X or S

2.4 – Cell Productivity

$$\underbrace{DX}_{\text{productivity}}$$

2.4 – Cell Productivity

$$DX = Y_{x/s} D\left(S_0 - \frac{K_s D}{\mu_{max} - D}\right)$$

Maximum productivity is obtained for:

$$\frac{dDx}{dD} = 0$$

Then:

$$D_{max} = \mu_{max}\left[1 - \left(\frac{K_s}{K_s + S_0}\right)^{1/2}\right]$$

When $S_0 > Ks \rightarrow \mu = \mu_{max}$, , near washout.

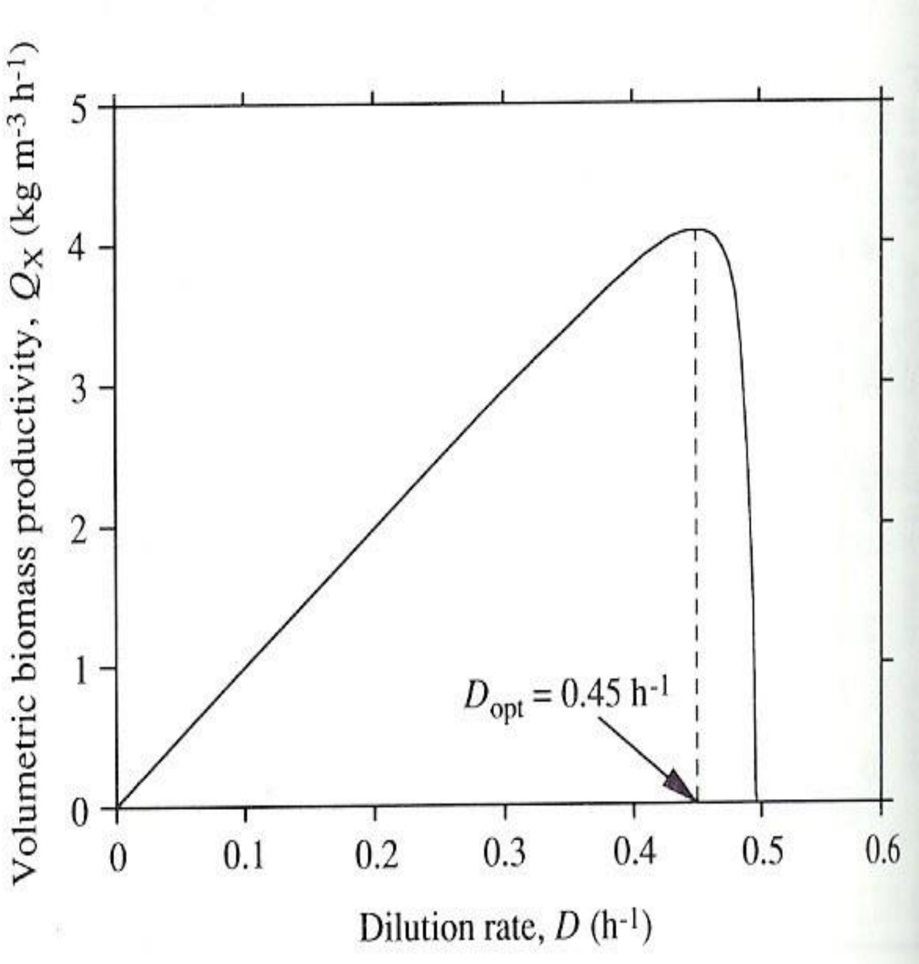
2.4 – Cell Productivity

The cell concentration corresponding to the point of maximum productivity is given by:

$$X_m = Y_{x/s}\,(S_0 + K_s - [K_s\,(S_0 + K_s)\,]^{1/2})$$

For $S_0 \gg K_s$ $$D_{max}\,X_m = D_{max}\,Y_{x/s}\,S_0$$

2.4 – Cell Productivity



The productivity:

- Increases with the dilution rate until D_{opt}
- Decreases sharply after D_{opt} → zero

2.5 – Effect of the maintenance coefficient

$$\frac{ds}{dt} = D(S_0 - S) - \frac{1}{Y'_{x/s}}\mu x - mx$$

Substrate entering the reactor

Substrate exiting the reactor

Substrate used for cell growth

Substrate used for maintenance

2.5 – Effect of the maintenance coefficient

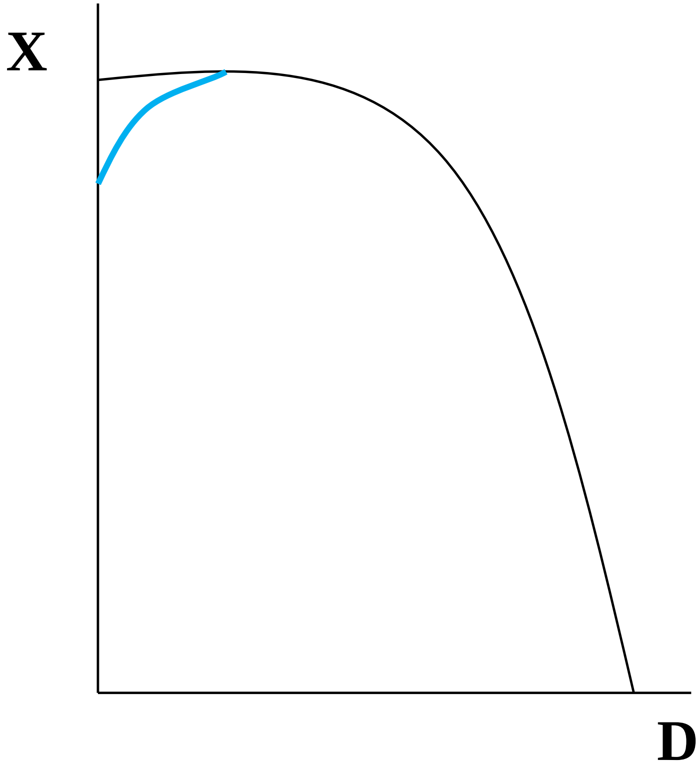
$$\frac{ds}{dt} = D(S_0 - S) - \frac{1}{Y'_{x/s}}\mu x - mx$$

In steady state: $\frac{ds}{dt} = 0 \Rightarrow D(S_0 - S) = \frac{1}{Y'_{x/s}} Dx + mx$

Rearranging: $x = \frac{D(S_0 - S)}{\frac{1}{Y'_{x/s}} D + m}$ or $x = \frac{S_0 - S}{\frac{1}{Y'_{x/s}} + \frac{m}{D}}$

2.5 – Effect of the maintenance coefficient

$$x = \frac{S_0 - S}{\frac{1}{Y'_{x/s}} + \frac{m}{D}}$$

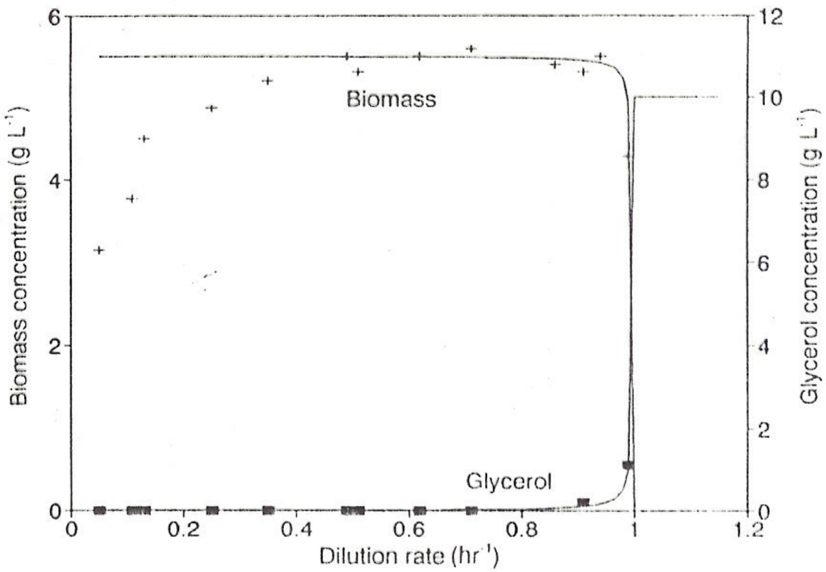


For low D values: X decreases due to m

For high D values: X does not depend on m

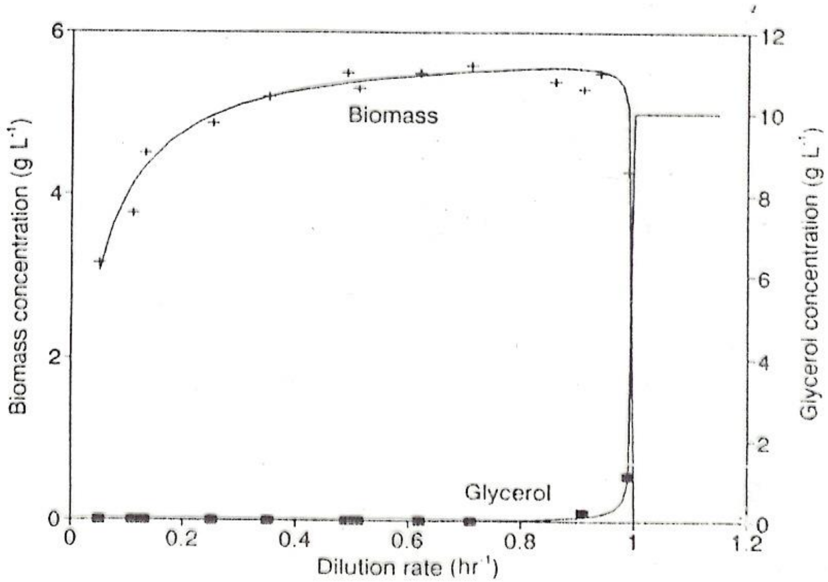
$$x = \frac{S_0 - S}{\frac{1}{Y'_{x/s}} + \frac{m}{D}} \longrightarrow x = \frac{S_0 - S}{\frac{1}{Y'_{x/s}}}$$

2.5 – Effect of the maintenance coefficient



Monod model aplied to CSTR

$$x = Y_{x/s}\left(S_0 - \frac{K_s D}{\mu_{max} - D}\right)$$

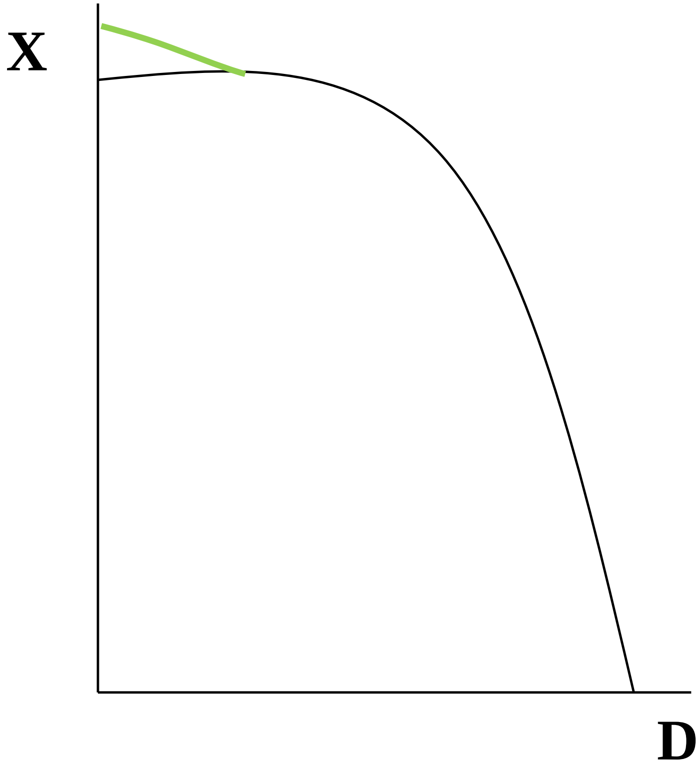


Monod model aplied to CSTR considering the cell maintenance

$$x = \frac{D(S_0 - S)}{\frac{1}{Y'_{x/s}} D + m}$$

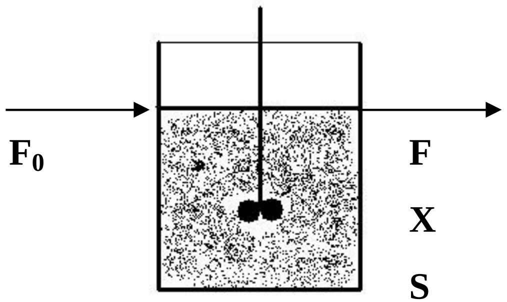
2.5 – Effect of the maintenance coefficient

increase in biomass weight due to the production of intracellular reserves



2.6 – Product production

2.6.1- Product associated to growth



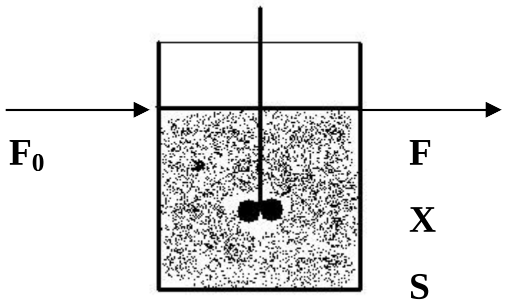
$$\frac{dP}{dt} = -DP + Y_{p/x}\mu X$$

Product exiting the reactor

Product formed

2.6 – Product production

2.6.1- Product associated to growth



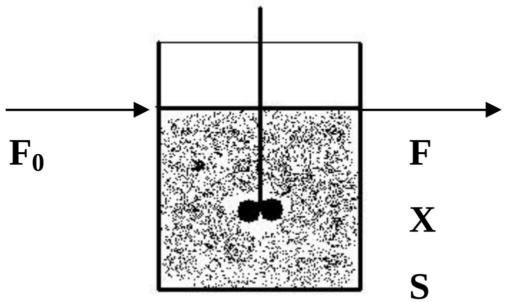
$$\frac{dP}{dt} = -DP + Y_{p/x}\mu X$$

At steady-state

$$\frac{dP}{dt} = 0$$

2.6 – Product production

2.6.1- Product associated to growth



$$\frac{dP}{dt} = -DP + Y_{p/x}\mu X$$

At steady-state

$$\frac{dP}{dt} = 0 \Rightarrow DP = Y_{p/x}\mu X$$ (Volumetric productivity (gP/l.h))

$$\Leftrightarrow \frac{D.P}{X} = Y_{p/x}\,\mu$$

$$\Leftrightarrow V_p = Y_{p/x}\,\mu$$ (Specific productivity (gP/gX.h))

2.6 – Product production

2.6.1- Product associated to growth

$$\left.\begin{aligned} X &= Y_{x/s}(S_0 - S) \\ S &= \frac{K_s\,D}{\mu_{max} - D} \end{aligned}\right\} \quad X = Y_{x/s}\left(S_0 - \frac{K_s\,D}{\mu_{max} - D}\right)$$

$$DP = Y_{p/x}\mu X \;\Leftrightarrow DP = Y_{p/x}DX$$

At steady-state

$$D = \mu$$

2.6 – Product production

2.6.1- Product associated to growth

$$\left.\begin{aligned} X &= Y_{x/s}(S_0 - S) \\ S &= \frac{K_s\, D}{\mu_{max} - D} \end{aligned}\right\} \quad X = Y_{x/s}\left(S_0 - \frac{K_s\, D}{\mu_{max} - D}\right)$$

$$DP = Y_{p/x}\mu X \;\Leftrightarrow DP = Y_{p/x} D X$$

$$\Leftrightarrow DP = Y_{p/x} D Y_{x/s}\left(S_0 - \frac{K_s\, D}{\mu_{max} - D}\right)$$

2.6 – Product production

2.6.1- Product associated to growth

$$DP = Y_{p/x} D Y_{x/s} \left(S_0 - \frac{K_s\, D}{\mu_{max} - D} \right)$$

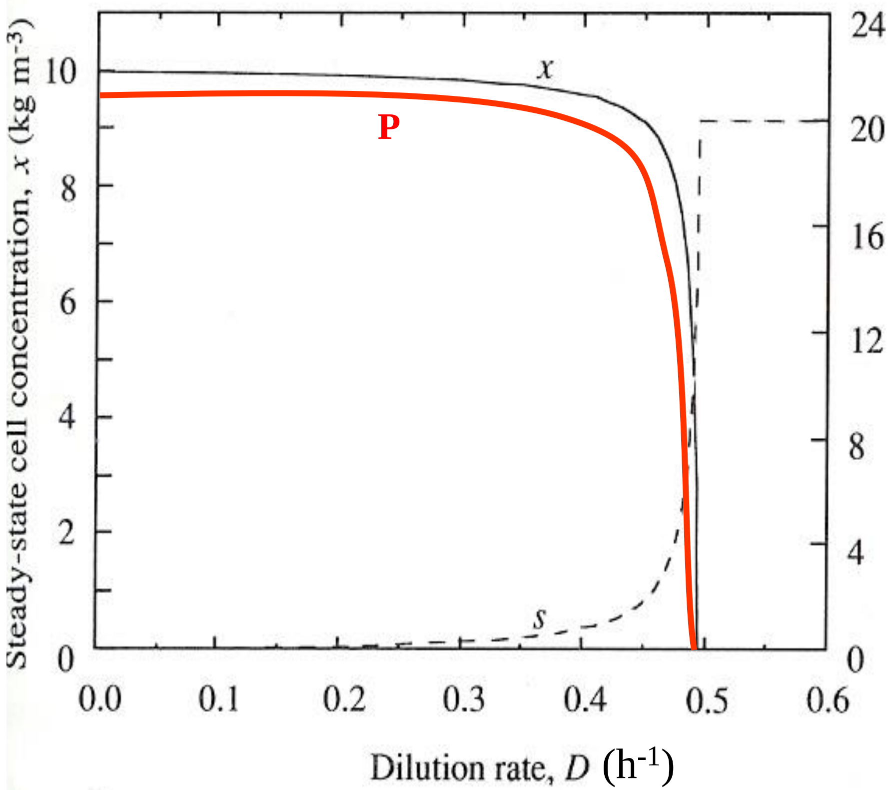
$$Y_{p/x} Y_{x/s} = Y_{p/s}$$

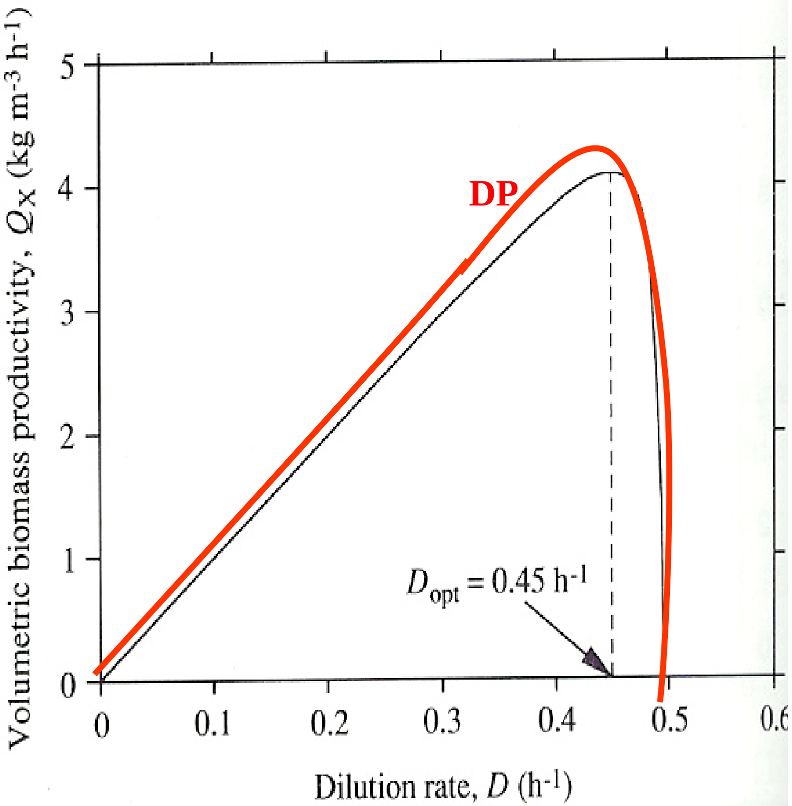
$$\Leftrightarrow DP = Y_{p/s} D \left(S_0 - \frac{K_s\, D}{\mu_{max} - D} \right)$$

Product concentration (P) and volumetric productivity (DP) follow the same trend of X and DX as a function of D (see figure below, lines in red).

2.6 – Product production

2.6.1- Product associated to growth





2.6 – Product production

2.6.1- Product associated to growth

$$DP = Y_{p/s} D \left(S_0 - \frac{K_s\ D}{\mu_{max} - D} \right) \Leftrightarrow P = Y_{p/s} \left(S_0 - \frac{K_s\ D}{\mu_{max} - D} \right)$$

2.6 – Product production

2.6.1- Product associated to growth

$$DP = Y_{p/s} D \left(S_0 - \frac{K_s\, D}{\mu_{max} - D} \right) \Leftrightarrow P = Y_{p/s} \left(S_0 - \frac{K_s\, D}{\mu_{max} - D} \right)$$

The maximum productivity is obtained for D_{max}

$$D_{max} = \mu_{max} \left[1 - \left(\frac{K_s}{K_s + S_0} \right)^{1/2} \right]$$

At this point the product concentration is:

$$P_m = Y_{p/s} \left(S_0 + K_s - (K_s (S_0 + K_s))^{1/2} \right)$$

2.6 – Product production

2.6.1- Product partially associated to growth

Mass balance to the substrate:

$$\frac{dS}{dt} = DS_0 - DS - \frac{1}{Y'_{x/s}}\mu X - \frac{1}{Y'_{p/s}}r_p - mX$$

2.6 – Product production

2.6.1- Product partially associated to growth

Mass balance to the substrate:

$$\frac{dS}{dt} = DS_0 - DS - \frac{1}{Y'_{x/s}}\mu X - \frac{1}{Y'_{p/s}} r_p - mX$$

Substrate entering the reactor

Substrate exiting the reactor

Substrate spent for cell growth

Substrate spent for product formation

Substrate spent for maintenance

2.6 – Product production

2.6.1- Product partially associated to growth

Mass balance to the substrate:

$$\frac{dS}{dt} = DS_0 - DS - \frac{1}{Y'_{x/s}}\mu X - \frac{1}{Y'_{p/s}} r_p - mX$$

$$V_p = \frac{r_p}{X} \Leftrightarrow r_p = V_p\, X$$

$$\frac{dS}{dt} = DS_0 - DS - \frac{1}{Y'_{x/s}}\mu X - \frac{1}{Y'_{p/s}} V_p X - mX$$

$$\Leftrightarrow \frac{dS}{dt} = D(S_0 - S) - (\frac{1}{Y'_{x/s}}\mu + \frac{1}{Y'_{p/s}} V_p + m)\, X$$

2.6 – Product production

2.6.1- Product partially associated to growth

$$\frac{dS}{dt} = D(S_0 - S) - \left(\frac{1}{Y'_{x/s}}\mu + \frac{1}{Y'_{p/s}}V_p + m\right) X$$

At steady-state

$$D = \mu$$

$$\frac{dS}{dt} = 0$$

$$D(S_0 - S) = \left(\frac{1}{Y'_{x/s}}D + \frac{1}{Y'_{p/s}}V_p + m\right) X$$

$$\Leftrightarrow \boxed{X = \frac{D(S_0 - S)}{\frac{D}{Y'_{x/s}} + \frac{1}{Y'_{p/s}}V_p + m}}$$

2.6 – Product production

2.6.1- Product partially associated to growth

$$\frac{dP}{dt} = -DP + V_p X$$

$$V_p = Y'_{p/x}\,\mu + m_p$$

$$\Leftrightarrow \frac{dP}{dt} = -DP + \left(Y'_{p/x}\,\mu + m_p\right) X$$

$$\Leftrightarrow \boxed{DP = \left(Y'_{p/x}\,\mu + m_p\right) X}$$

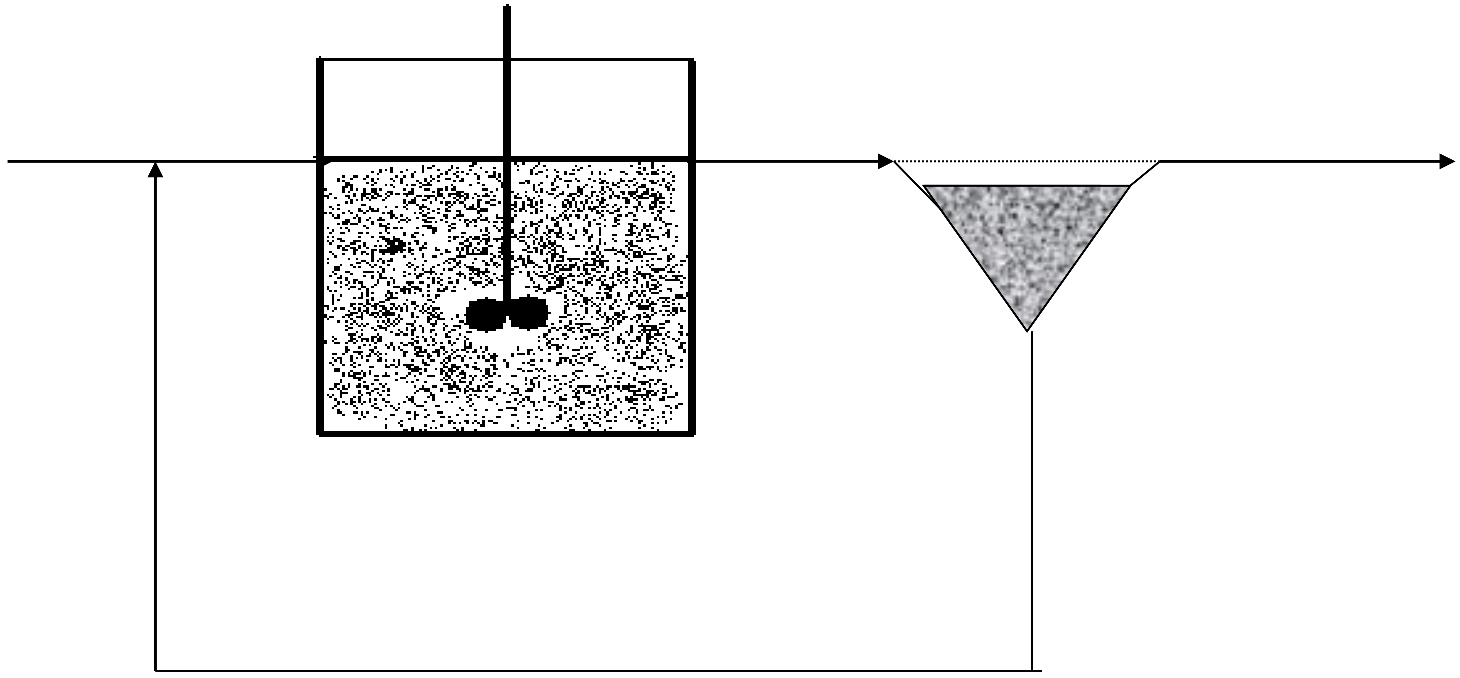
At steady-state

$$D = \mu$$

$$\frac{dS}{dt} = 0$$

2.7 – Cell recirculation reactors

With a decanter



2.7 – Cell recirculation reactors

Membrane bioreactors

Type of membranes

- Dense
- Microporous

Configurations

- External circuit membrane
- Submerged membrane

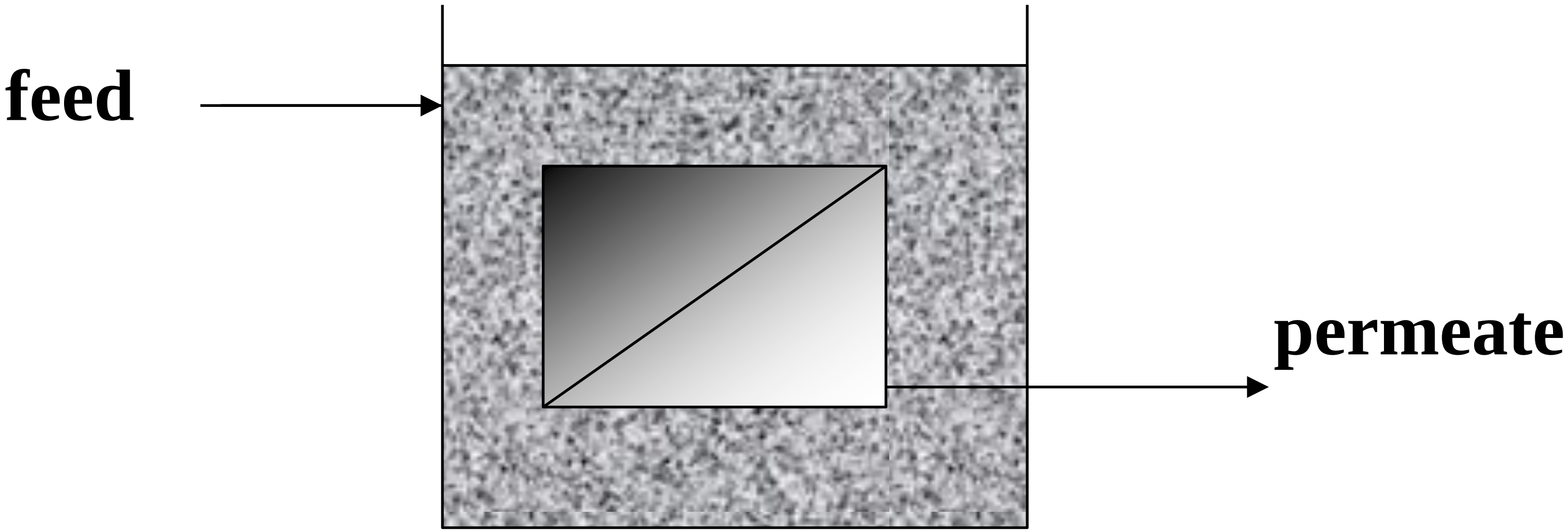
Applications

- Chemicals production
- Treatment of gaseous or liquid effluents

2.7 – Cell recirculation reactors

Membrane bioreactors

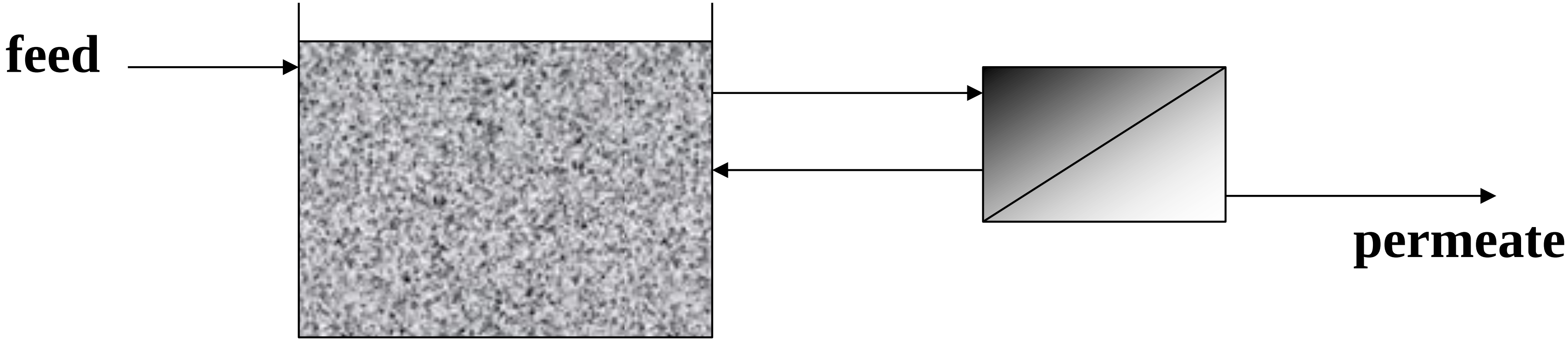
Submerged membrane bioreactor



2.7 – Cell recirculation reactors

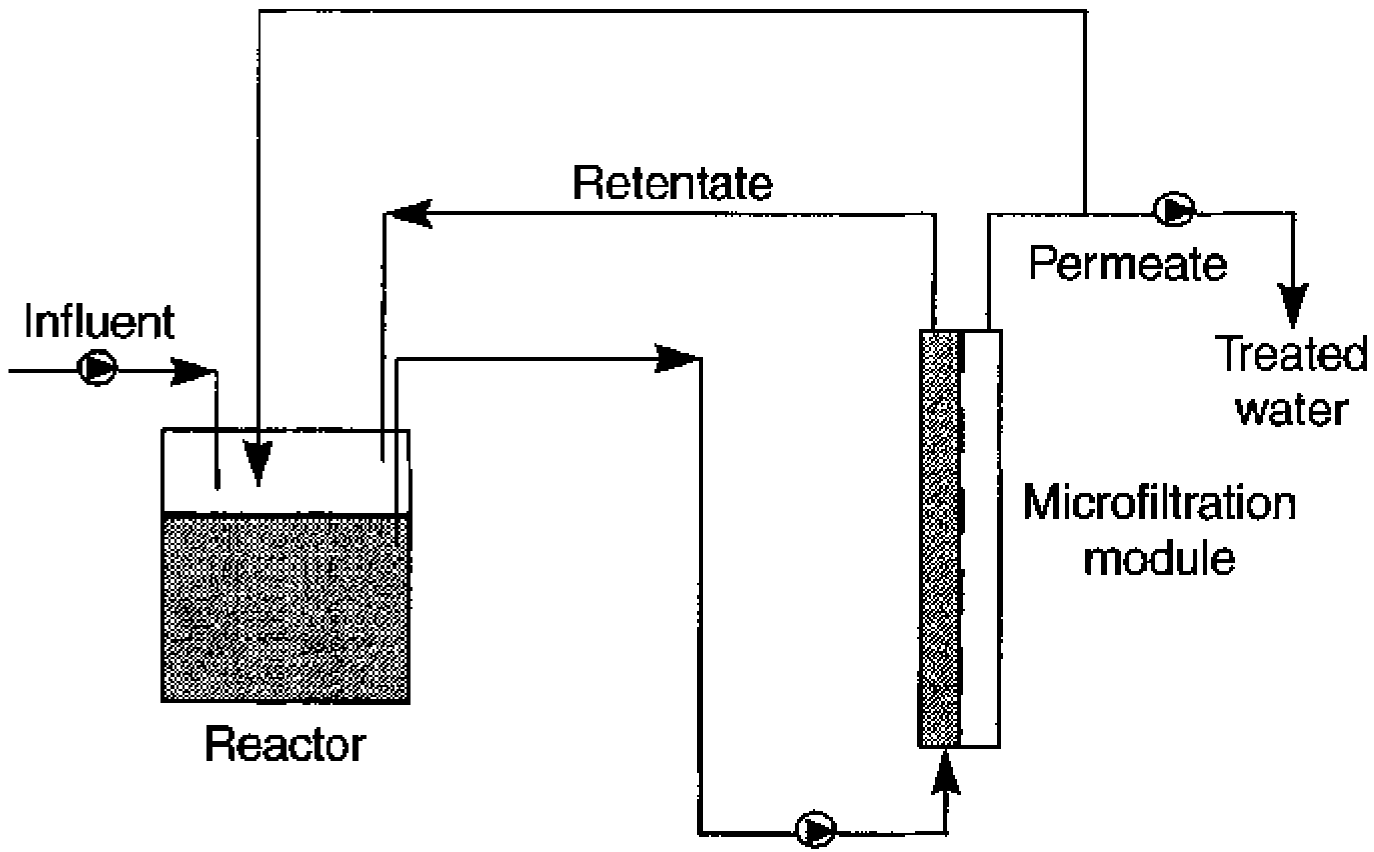
Membrane bioreactors

Membrane bioreactor with cell recirculation



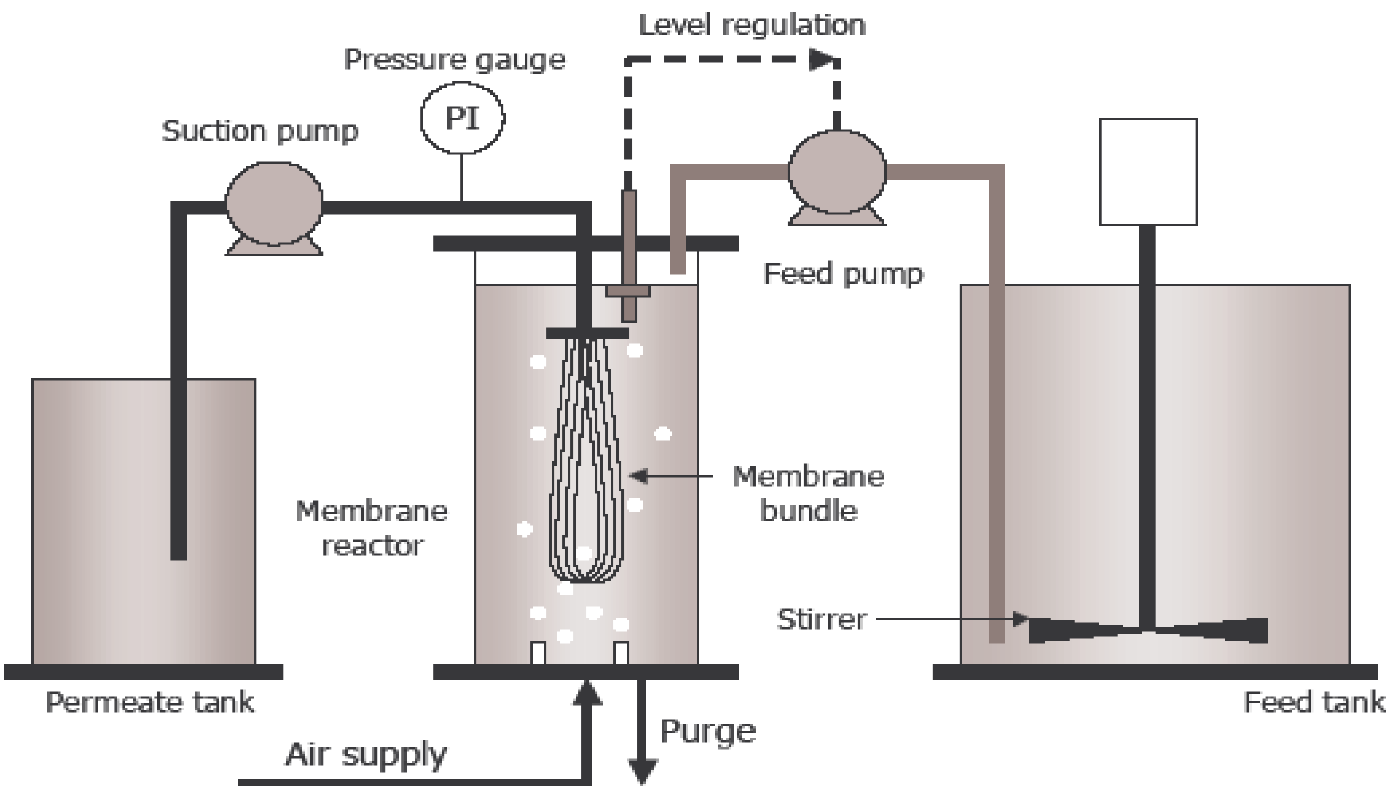
2.7 – Cell recirculation reactors

Membrane bioreactors

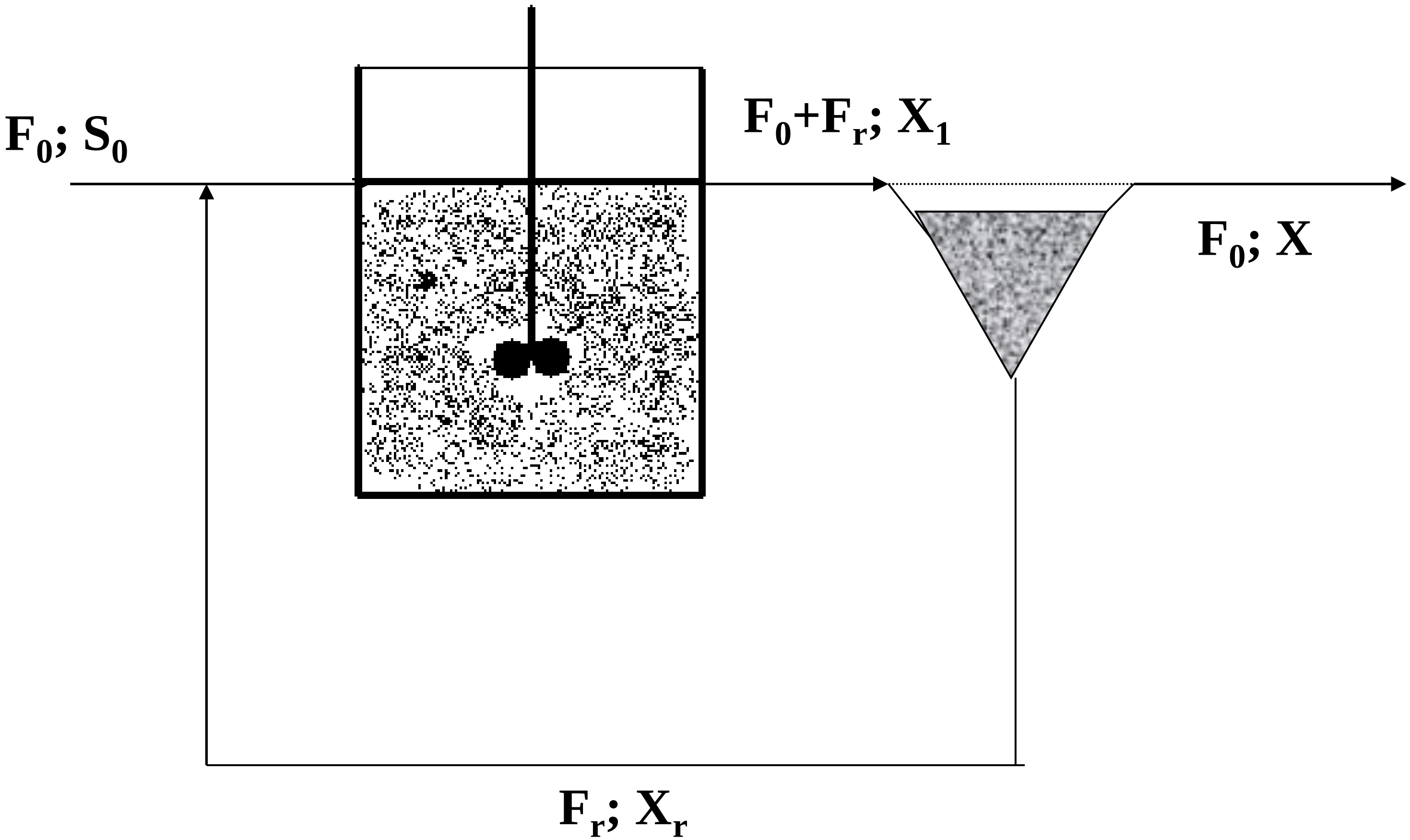


2.7 – Cell recirculation reactors

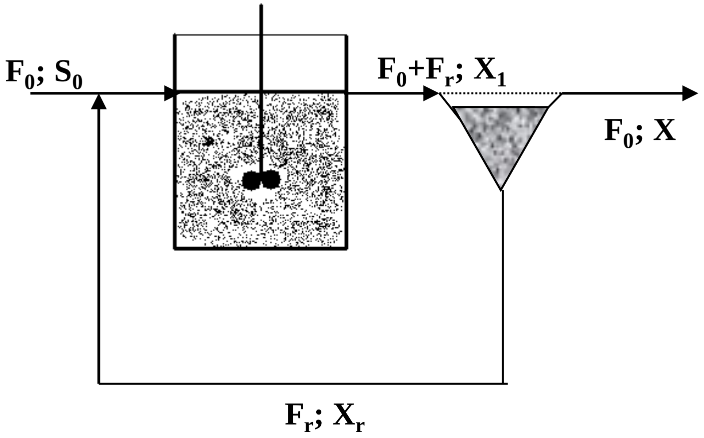
Submerged membrane bioreactors



2.7 – Cell recirculation reactors



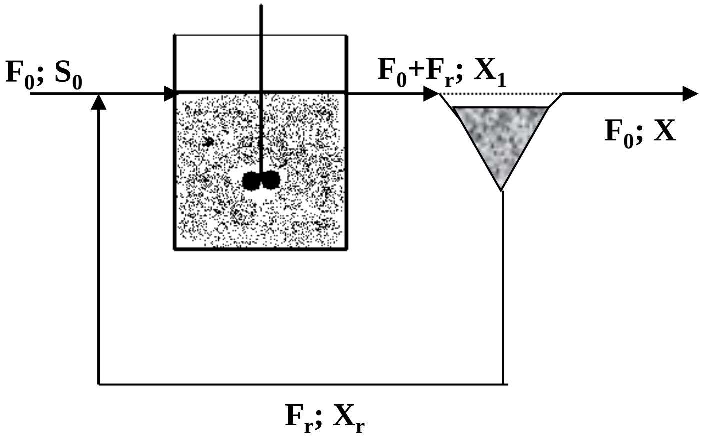
2.7 – Cell recirculation reactors



Balance to the biomass:

$$\frac{dX}{dt} = \frac{F_r}{V} X_r - \frac{F_0 + F_r}{V} X_1 + \mu X_1$$

2.7 – Cell recirculation reactors



Balance to the biomass:

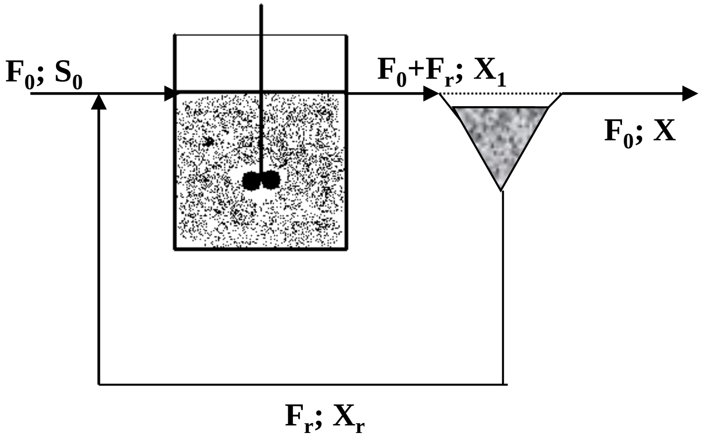
$$\frac{dX}{dt} = \frac{F_r}{V} X_r - \frac{F_0 + F_r}{V} X_1 + \mu X_1$$

Biomass entering the reactor

Biomass exiting the reactor

Cell growh

2.7 – Cell recirculation reactors



Balance to the biomass:

$$\frac{dX}{dt} = \frac{F_r}{V} X_r - \frac{F_0 + F_r}{V} X_1 + \mu X_1$$

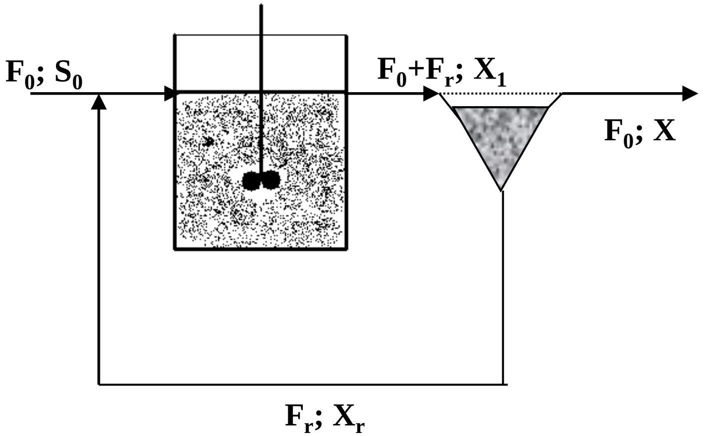
At steady-state $\frac{dX}{dt} = 0$

$$0 = \frac{F_r}{V} X_r - \frac{F_0 + F_r}{V} X_1 + \mu X_1$$

Multiplying by V

$$\Leftrightarrow 0 = F_r X_r - (F_0 + F_r) X_1 + \mu X_1 V$$

2.7 – Cell recirculation reactors



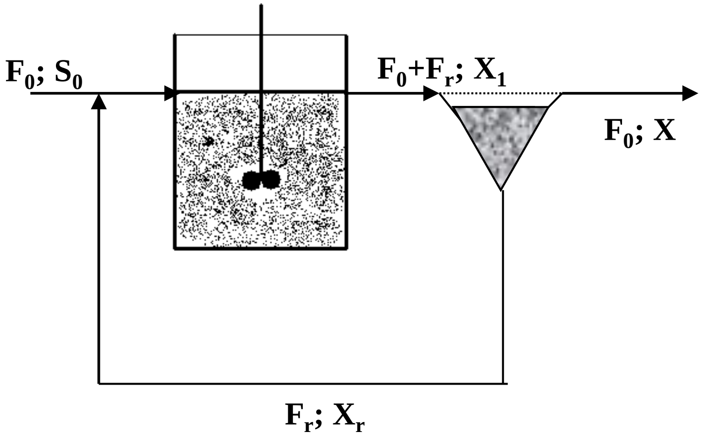
Balance to the biomass:

$$0 = F_r\, X_r - (F_0 + F_r)\, X_1 + \mu\, X_1 V$$

If $\quad a = \frac{F_r}{F_0} \qquad b = \frac{x_r}{x_1} \qquad D = \frac{F_0}{V}$

Then: $\quad D = \frac{\mu}{1 - a(b-1)}$

2.7 – Cell recirculation reactors



Balance to the substrate:

$$\frac{dS}{dt} = D(S_0 - S) - \frac{\mu X_1}{Y_{x/s}} = 0$$

$$a = \frac{F_r}{F_0} \qquad b = \frac{x_r}{x_1} \qquad D = \frac{F_0}{V} \longrightarrow \mu x_1 = \frac{\mu Y_{x/s}(S_0 - S)}{1 - a(b-1)}$$

The rate of biomass production increases with a factor : $[1\text{-}a\ (b\text{-}1)]^{-1}$

2.7 – Cell recirculation reactors

Comparison of reactors with and without cell recirculation