AM 2C – Exame Resolução

Felipe B. Pinto 61387 – MIEQB

7 de julho de 2023

Conteúdo

Questão 1

A equação do plano angente à superfície de equação

$$e^{x,y} + y\ \sin x - y^2 + z^2 + 2\,x = 2\,\pi$$

Resposta B.

Questão 2

Considere f, a curva de nível em 1/2 de f é

$$\boldsymbol{f(x, y) = \frac{y^2 - 4}{4\,x^2 + y^2 - 4}}$$

Resposta D.

$$f(x, y) = \frac{y^2 - 4}{4\,x^2 + y^2 - 4} = 1/2 \implies y^2 = 4\,x^2 + 4;$$

$$0 \neq 4\,x^2 + y^2 - 4 \implies y^2 \neq 4 - 4\,x^2 \implies$$
$$\implies 4 - 4\,x^2 \neq 4 + 4\,x^2 \implies 0 \neq x \implies y \neq \pm 2;$$

$$\therefore \left\{(x, y) \in \mathbb{R}^2 \backslash \{(0, 2), (0, -2)\} : y^2/4 - x^2 = 1\right\}$$

Questão 3

Resposta A.

Questão 4

Considere a curva C em $\mathbb{R}^2$ no sentido horario

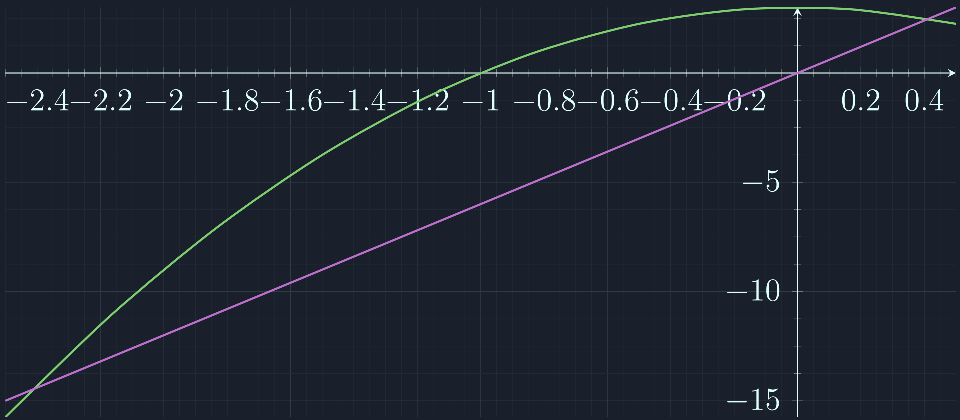
$$\boldsymbol{C = \left\{ (x, y, z) \in \mathbb{R}^3 : \begin{cases} z + x^2 + 3\,y^2 = 3 \\ z = 6\,y \end{cases} \right\}}$$

Resposta

$$x = 0 \implies \begin{cases} z = 3 - 3\,y^2 \\ z = 6\,y \end{cases}$$

$$6\,y = 3 - 3\,y^2 \implies 0 = 3\,y^2 + 6\,y - 3 \begin{cases} a = (4 * 3)^{-1} = 1/12 \\ y' = -6 * 2 * 1/12 = -1 \\ x' = -3 - (-1)^2/(4 * 1/12) = -6 \end{cases}$$

$$\therefore 0 = 3(y+1)^2 - 6 \implies y = \pm\sqrt{2} - 1$$



Questão 5

Resposta D.

Questão 6

O valor do limite é

$$\lim_{(x,y)\to(1,-1)} \frac{(x-1)(y+1)\,e^{(x-1)^2}}{\sqrt{(x-1)^2+(y+1)^2}}$$

Resposta D.

Questão 7

Considere a função real de duas variáveis reais, definida por

$$\boldsymbol{f(x,y)=\begin{cases}\frac{3\,x^2\,y+2\,x^5}{x^4+y^2} & \text{se } (x,y)\neq(0,0)\\ 0 & \text{se } (x,y)=(0,0)\end{cases}}$$

Relativamente a f temos:

Resposta C.

$$\frac{\partial f}{\partial x}(0,0)=\frac{\partial}{\partial x}(0,0)\left(\frac{3\,x^2\,y+2\,x^5}{x^4+y^2}\right)=$$

$$=\frac{\partial}{\partial x}(0,0)\left(\frac{3\,x^2\,y}{x^4+y^2}+\frac{2\,x^5}{x^4+y^2}\right)=$$

$$=3\,y\frac{\partial}{\partial x}(0,0)\left(\frac{x^2}{x^4+y^2}\right)+2\frac{\partial}{\partial x}(0,0)\left(\frac{x^5}{x^4+y^2}\right)=$$

$$=\left\{\begin{array}{l} 3\,y\dfrac{\left(\frac{\partial}{\partial x}(0,0)(x^2)(x^4+y^2)-\frac{\partial}{\partial x}(0,0)(x^4+y^2)(x^2)\right)}{(x^4+y^2)^2} \quad + \\ +2\dfrac{\partial}{\partial x}(0,0)\dfrac{\left(\frac{\partial}{\partial x}(0,0)(x^5)(x^4+y^2)-\frac{\partial}{\partial x}(0,0)(x^4+y^2)(x^5)\right)}{(x^4+y^2)^2}\end{array}\right\}=$$

$$=\left\{\begin{array}{l} 3\,y\dfrac{(2\,x\,(x^4+y^2)-4\,x^3\,(x^2))}{(x^4+y^2)^2} \quad + \\ +2\dfrac{\partial}{\partial x}(0,0)\dfrac{(5\,x^4(x^4+y^2)-5\,x^3(x^5))}{(x^4+y^2)^2}\end{array}\right\}=$$

$$=\left\{\begin{array}{l} 3\,y\dfrac{+2\,x\,y^2-2\,x^5}{(x^4+y^2)^2} \quad + \\ +\dfrac{10\,x^4\,y^2}{(x^4+y^2)^2}\end{array}\right\}=$$

$$=\frac{6\,x\,y^3}{x^8+2x^4\,y^2+y^4}-\frac{2\,x^5\,y}{x^8+2x^4\,y^2+y^4}+\frac{5\,x^4\,y^2}{x^8+2x^4\,y^2+y^4};$$

$$\lim_{(x\to 0),(y\to x^2)}\frac{6\,x\,y^3-2\,x^5\,y+5\,x^4\,y^2}{x^8+2x^4\,y^2+y^4}=$$

$$=\lim_{(x\to 0)}\frac{6\,x\,x^6-2\,x^5\,x^2+5\,x^4\,x^4}{x^8+2x^4\,x^4+x^8}=$$

$$=\lim_{(x\to 0)}\frac{6\,x^7-2\,x^7+5\,x^8}{4x^8}=$$

$$=\lim_{(x\to 0)}\frac{6-2+5\,x}{4x}$$

Questão 8 D.
Questão 9 C.
Questão 10 B.
Questão 11 D.

Questão 12

Considere a função f. Escolha a afirm correta

$$\boldsymbol{f(x, y) = x^3 + x^2\, y - y^2 - 4\, y}$$

Resposta E.

$$\begin{cases} \frac{\partial f}{\partial x} = 3\,x^2 + 2\,x\,y \\ \frac{\partial f}{\partial y} = x^2 - 2\,y - 4 \end{cases}$$

$$\det \mathrm{H}\, f(x, y) = \begin{pmatrix} \frac{\partial^2 f(x,y)}{\partial x\,\partial x} & \frac{\partial^2 f(x,y)}{\partial x\,\partial y} \\ \frac{\partial^2 f(x,y)}{\partial y\,\partial x} & \frac{\partial^2 f(x,y)}{\partial y\,\partial y} \end{pmatrix} = \begin{pmatrix} 6\,x + 2\,y & 2\,x \\ 2\,x & -2 \end{pmatrix} =$$

$$= -2(6\,x + 2\,y) - 4\,x^2 = -4\,x^2 - 12\,x + 4\,y;$$

$$\left\{ \begin{array}{cr} \det \mathrm{H}(f)(0, -2) = -8 & \text{Ponto de sela} \\ \det \mathrm{H}(f)(-4, 6) = 8 & \text{Critico local} \\ \det \mathrm{H}(f)(1, -3/2) = -10 & \text{Ponto de sela} \\ \det \mathrm{H}(f)(0, 0) = 0 & \text{Indeterminável} \end{array} \right\}$$

$$\frac{\partial^2 f}{\partial x^2}(1, -3/2) = 6 + 2\,(-3/2) = 6 - 3 = 3 \therefore \text{Mínimo local}$$

Questão 13

Seja D, calcule o integral

$$\boldsymbol{D = \left\{(x, y) \in \mathbb{R}^2 : y \leq x \leq 2 - y^2 \wedge y \geq 0\right\}}$$
$$\boldsymbol{\iint_D 4\,y\ \mathrm{d}x\,\mathrm{d}y}$$

Resposta B.

$$\begin{cases} y \leq x \\ y^2 \geq 2 - x \\ y \geq 0 \end{cases}$$

$$x^2 = 2 - x \implies x^2 + x - 2 = 0 \implies$$

$$\implies \left\{\begin{aligned} a &= 1/4 \\ x' &= -1/2 \\ y' &= -9/4 \end{aligned}\right\} \implies (x + 1/2)^2 = 9/4 \implies$$

$$\implies x = \pm(3/2) - 1/2; X = \{(1,1), (-2,-2)\}$$

$$\int_0^1 \int_y^{2-y^2} 4\,y\ \mathrm{d}x\,\mathrm{d}y = \int_0^1 4\,y((2 - y^2) - y)\ \mathrm{d}y =$$

$$= \Delta(8\,y^2/2 - 4\,y^4/4 - 4\,y^3/3)\Big|_0^1 = 4 - 1 - 4/3 = 5/3$$

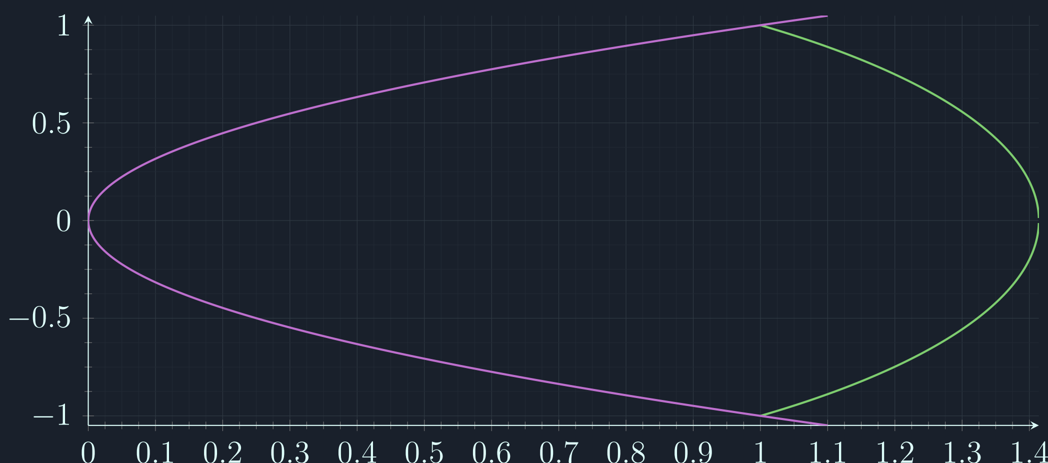
Questão 14

Seja f uma função contínua em $\mathbb{R}^2$. Considere a igualdade

$$\iint_{\mathcal{R}} f(x,y)\ \mathrm{d}x\,\mathrm{d}y = \left\{ \begin{array}{l} \displaystyle\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y)\ \mathrm{d}y\,\mathrm{d}x \qquad\qquad + \\ \displaystyle + \int_1^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} f(x,y)\ \mathrm{d}y\,\mathrm{d}x \end{array} \right\}$$

Resposta C.

$$\left\{ \begin{array}{rl} x \le y^2 & x \in [0,1] \\ 2 - x^2 \le y^2 & x \in \left[1, \sqrt{2}\right] \end{array} \right\} = \left\{ \begin{array}{rl} x \le y^2 & y \in [0,1] \\ x^2 \le 2 - y^2 & y \in [0,1] \end{array} \right\}$$



$$\therefore \int_{-1}^{1} \int_{y^2}^{\sqrt{2-y^2}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

Questão 15

Considere o domínio de $\mathbb{R}^2$ definido por

$$\boldsymbol{D = \left\{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, x + y \leq 1\right\}}$$

Usando transformação de variáveis:

$$\begin{cases} \boldsymbol{y - x = u} \\ \boldsymbol{x + y = v} \end{cases}$$

Determine:

$$\boldsymbol{I = \iint_D 3(y - x)^2 \sqrt{y + x}\ \mathrm{d}x\, \mathrm{d}y}$$

Resposta D.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 1 \end{cases}$$

$X = \{(0,0), (0,1), (1,0)\}$

$$\begin{Bmatrix} x - y = u \\ x + y = v \end{Bmatrix} = \begin{Bmatrix} x = (u+v)/2 \\ y = (v-u)/2 \end{Bmatrix}$$

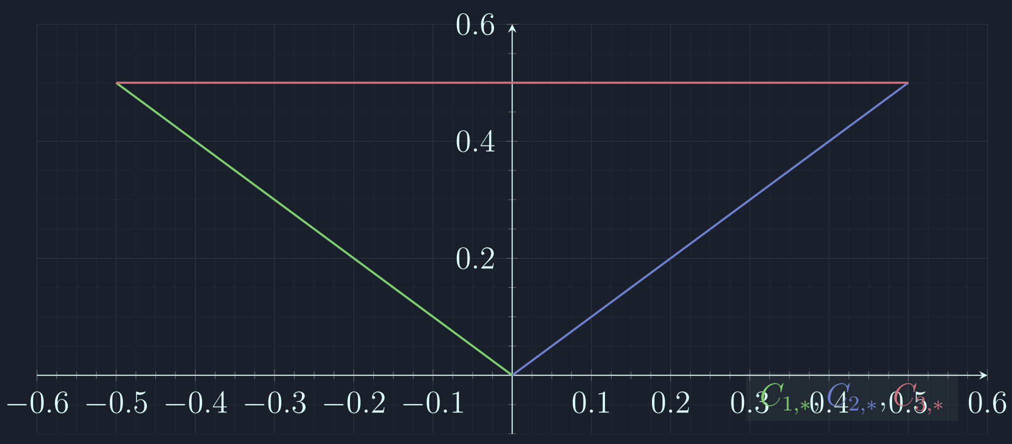
$$\begin{Bmatrix} C_1 = \{x = 0, y \in [0,1]\}, \\ C_2 = \{y = 0, x \in [0,1]\}, \\ C_3 = \{y = 1 - x, x \in [0,1]\}, \end{Bmatrix};$$

$$C_{1,*} = \{u + v = 0, u - v \in [0,1]\} = \{u = -v, v \in [-1/2, 0]\};$$

$$C_{2,*} = \{u - v = 0, u + v \in [0,1]\} = \{u = v, v \in [0, 1/2]\};$$

$$C_{3,*} = \{u - v = 1 - u - v, u + v \in [0,1]\} = \{u = 1/2, 1/2 + v \in [0,1]\} =$$
$$= \{u = 1/2, v \in [-1/2, 1/2]\}$$

$$\implies \begin{Bmatrix} C_{1,*} = \{u = -v, v \in [-1/2, 0]\}, \\ C_{2,*} = \{u = v, v \in [0, 1/2]\}, \\ C_{3,*} = \{u = 1/2, v \in [-1/2, 1/2]\} \end{Bmatrix}$$



$$\det \mathrm{J}\, f = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{pmatrix} = 1/4 + 1/4 = 1/2;$$

$$I = \iint_D 3(y - x)^2 \sqrt{y + x}\ \mathrm{d}x\, \mathrm{d}y = \int_0^{0.5} \int_{-v}^{v} 3\, u^2 \sqrt{v} (1/2)\, \mathrm{d}u\, \mathrm{d}v =$$
$$= \int_0^{0.5} (3/2)\, (v^3 + v^3)(1/3)\sqrt{v}\, \mathrm{d}v = (1/2) \int_0^{0.5} v^{7/2}\, \mathrm{d}v = \frac{(1/2)(0.5)^{9/2}}{9/2} = \frac{(0.5)^{9/2}}{9}$$

Questão 16

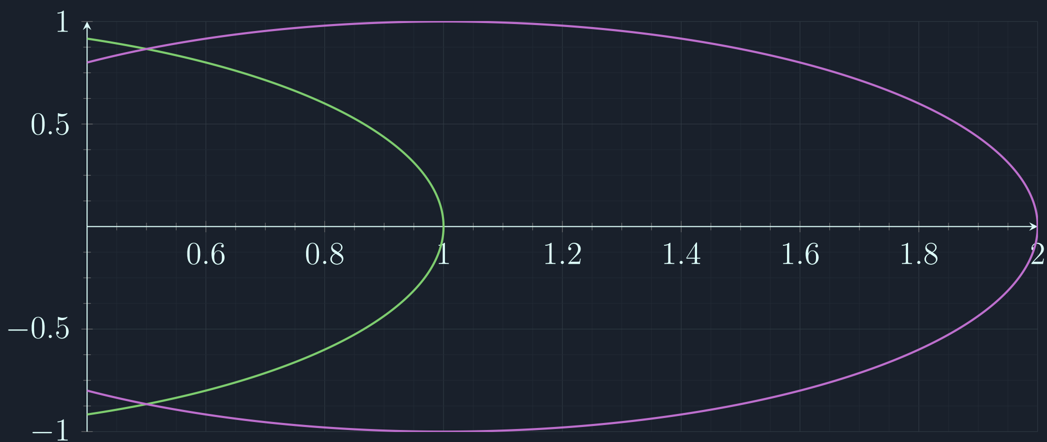
Denote por λ a área do domínio D no plano, Tem-se:

$$\boldsymbol{D = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \geq 1 \land x^2 + y^2 \leq 2\,x\right\}}$$

Resposta D.

$$\begin{Bmatrix} y^2 = 1 - x^2 \\ y^2 = 2\,x - x^2 \end{Bmatrix};$$

$$1 - x^2 = 2\,x - x^2 \implies 1/2 = x \implies X' = \left\{(1/2, \sqrt{3}/2), (1/2, -\sqrt{3}/2)\right\}$$



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1} y/x \end{cases}$$

$$r^2\,\cos^2\theta + r^2\,\sin^2\theta = 2\,r\,\cos\theta \implies$$

$$\implies r = 2\,\cos\theta$$

$$\left\{(x, y) = (1/2, \sqrt{3}/2) \iff (r, \theta) = (1, \pi/3)\right\};$$

$$\lambda = \int_0^{\pi/3}\int_1^{2\,\cos\theta} \mathrm{d}r\,\mathrm{d}\theta + \int_{\pi\,5/3}^{2\,\pi}\int_1^{2\,\cos\theta} \mathrm{d}r\,\mathrm{d}\theta$$

Questão 17

Considere o sólido $\mathcal{E}$ sabendo que tem por função de densidade $d(x, y, z)$. a massa desse sóldio é

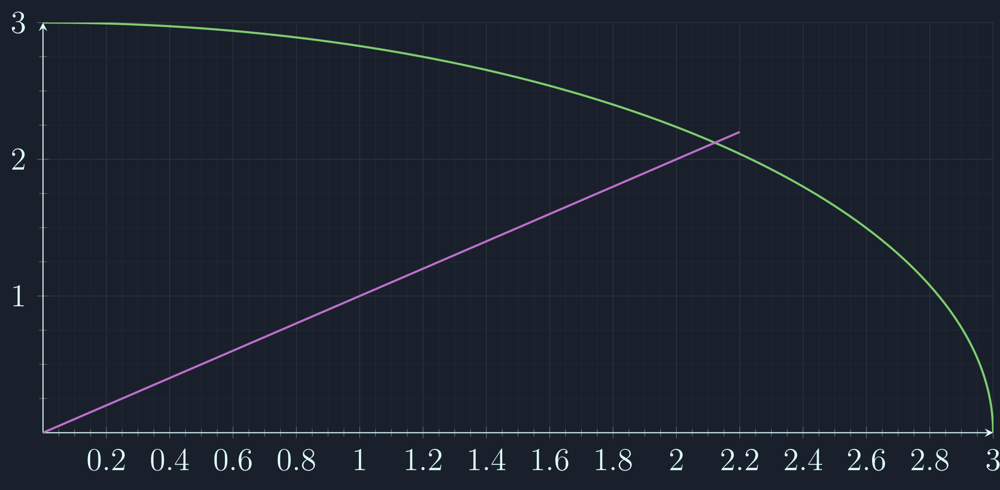
$$\boldsymbol{\mathcal{E} = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 9, z \leq \sqrt{x^2 + y^2}, z \leq 0\right\}}$$

$$\boldsymbol{d(x, y, z) = z}$$

Resposta A.

$$\left\{\begin{matrix} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{matrix}\right\} y = 0 \implies \begin{cases} z^2 \leq 9 - x^2 \\ z \leq x \end{cases}$$

$$x^2 = 9 - x^2 \implies X' = \left\{(3\sqrt{2}/2, 3\sqrt{2}/2), (-3\sqrt{2}/2, -3\sqrt{2}/2)\right\}$$



Questão 18	D.
Questão 19	C.
Questão 20	D.

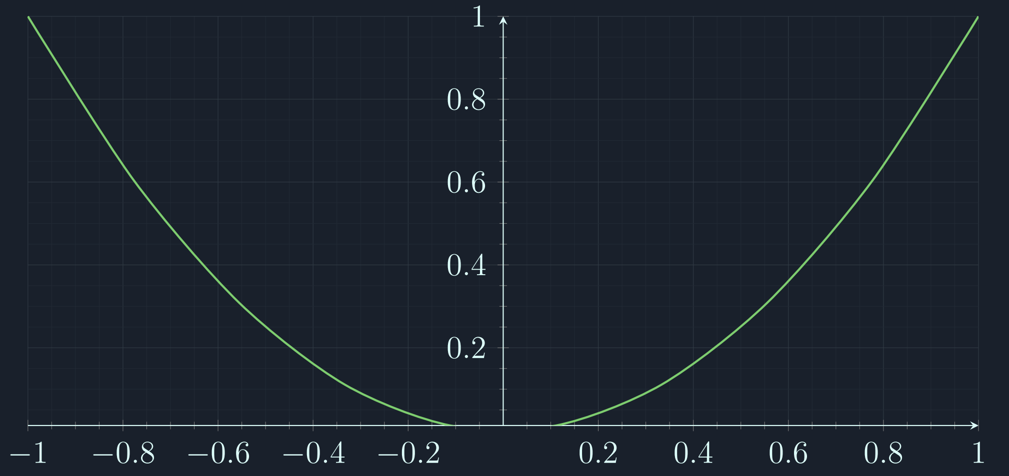
Questão 21

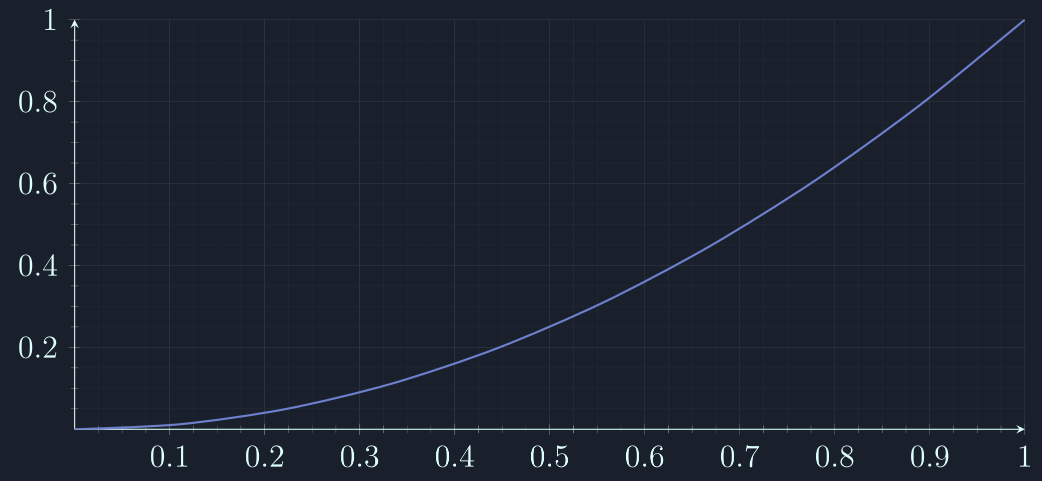
Volume do sólido

$$\boldsymbol{E = \left\{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq x^2 + y^2 \wedge x^2 + y^2 \leq 1 \wedge y \geq 0\right\}}$$

Resposta D.

$$y = 0 \implies \begin{cases} z \geq 0 \\ z \leq x^2 \\ x^2 \leq 1 \implies x \in [-1, 1] \\ y \geq 0 \end{cases}$$





$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{x^2+y^2}^1 \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y =$$

$$= \int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x^2 \, \mathrm{d}x \, \mathrm{d}y + \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (y^2 - 1) \, \mathrm{d}x \, \mathrm{d}y - \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \mathrm{d}x \, \mathrm{d}y =$$

$$= \int_0^1 (2/3)((\sqrt{1-y^2})^3) \, \mathrm{d}y + (y^2-1)(2\sqrt{1-y^2}) \, \mathrm{d}y - 2\sqrt{1-y^2} \, \mathrm{d}y$$

Questão 22	A.
Questão 23	A.
Questão 24	B.
Questão 25	C.