SECTION 2

SIZE REDUCTION OF SOLIDS

Problem 2.1

A material is crushed in a Blake jaw crusher and the average size of particle reduced from 50 mm to 10 mm with the consumption of energy at the rate of 13·0 kW/(kg/s). What will be the consumption of energy needed to crush the same material of average size 75 mm to an average size of 25 mm:

(a) Assuming Rittinger's law applies?
(b) Assuming Kick's law applies?

Which of these results would be regarded as being more reliable and why?

Solution

(a) *Rittinger's Law*

This is given by equation 2.2:

$$E = K_R f_c\left(\frac{1}{L_2} - \frac{1}{L_1}\right)$$

or

$$13{\cdot}0 = K_R f_c\left(\tfrac{1}{10} - \tfrac{1}{50}\right)$$

$\therefore$

$$K_R f_c = 13{\cdot}0 \times 50/4 = 162{\cdot}5\ \text{kW s/kg mm}$$

Thus the energy required to crush 75 mm material to 25 mm is given by:

$$E = 162{\cdot}5\left(\tfrac{1}{25} - \tfrac{1}{75}\right) = \underline{\underline{4{\cdot}33\ \text{kW/(kg/s)}}}$$

(b) *Kick's Law*

This is given by equation 2.3:

$$E = K_K f_c \ln (L_1/L_2)$$

or

$$13{\cdot}0 = K_K f_c \ln (50/10)$$

$\therefore$

$$K_K f_c = 13{\cdot}0/1{\cdot}609 = 8{\cdot}08\ \text{kW/(kg/s)}$$

Thus the energy required to crush 75 mm material to 25 mm is given by:

$$E = 8{\cdot}08 \ln (75/25) = \underline{\underline{8{\cdot}88\ \text{kW/(kg/s)}}}$$

The size range involved may be classified as coarse crushing, and because Kick's law more closely relates to the energy required to effect elastic deformation before fracture occurs, this would be taken as giving the more accurate result.

Problem 2.2

A crusher was used to crush a material whose compressive strength was $22 \cdot 5\ MN/m^2$. The size of the feed was *minus* 50 mm, *plus* 40 mm, and the power required was 13·0 kW/(kg/s). The screen analysis of the product was as follows:

Size of aperture (mm)	Per cent of product
Through 6·00	100
On 4·00	26
On 2·00	18
On 0·75	23
On 0·50	8
On 0·25	17
On 0·125	3
Through 0·125	5

What would be the power required to crush 1 kg/s of a material of compressive strength $45\ MN/m^2$ from a feed *minus* 45 mm, *plus* 40 mm to a product of average size 0·50 mm?

Solution

The first part of the working is to obtain a dimension representing the mean size of the product. Using Bond's method of taking the size of opening through which 80% of the material will pass, a value of just over 4·00 mm is indicated by the data. Alternatively, calculations may be made as follows:

Size of aperture (mm)	Mean d_1 (mm)	n_1	nd_1	nd_1^2	nd_1^3	nd_1^4
6·00						
	5·00	0·26	1·3	6·5	32·5	162.5
4·00						
	3·00	0·18	0·54	1·62	4·86	14·58
2·00						
	1·375	0·23	0·316	0·435	0·598	0·822
0·75						
	0·67	0·08	0·0536	0·0359	0·0241	0·0161
0·50						
	0·37	0·17	0·0629	0·0233	0·0086	0·003 19
0·25						
	0·1875	0·03	0·0056	0·001 05	0·000 20	0·000 037
0·125						
	0·125	0·05	0·006 25	0·000 78	0·000 098	0·000 012
			2·284	8·616	37·991	177·92

From equation 1.11, the weight mean diameter,

$$d_w = \Sigma n_1 d_1^4 / \Sigma n_1 d_1^3$$

$$= 177.92/37.991 = 4.683\,\text{mm}$$

From equation 1.14, the surface mean diameter,

$$d_s = \Sigma n_1 d_1^3 / \Sigma n_1 d_1^2$$

$$= 37.991/8.616 = 4.409\,\text{mm}$$

From equation 1.18, the linear mean diameter,

$$d_1 = \Sigma n_1 d_1^2 / \Sigma n_1 d_1$$

$$= 8.616/2.284 = 3.772\,\text{mm}$$

From equation 1.19, the mean linear diameter,

$$d_1' = \Sigma n_1 d_1 / \Sigma n_1$$

$$= 2.284/1.0 = 2.284\,\text{mm}$$

In the present situation, which is concerned with power consumption per unit mass, the weight mean diameter is probably of the greatest relevance. For the purposes of calculation, a mean value of 4·0 mm will be used, which agrees with the value obtained by Bond's method. For coarse crushing, Kick's law may be used as follows:

Case 1. mean diameter of feed = 45 mm
mean diameter of product = 4 mm
energy consumption = 13·0 kW/(kg/s)
compressive strength = 22·5 MN/m^2

In equation 2.3,

$$13.0 = K_K \times 22.5 \ln(45/4)$$

and $$K_K = 13.0/54.4 = 0.239\,\text{kW/(kg/s)(MN/m}^2)$$

Case 2. mean diameter of feed = 42·5 mm
mean diameter of product = 0·50 mm
compressive strength = 45 MN/m^2

$$E = 0.239 \times 45 \ln(42.5/0.50) = 0.239 \times 199.9$$

$$= 47.8\,\text{kW/(kg/s)}$$

or, for a feed of 1 kg/s, the energy required = $\underline{\underline{47.8\,\text{kW}}}$

Problem 2.3

A crusher in reducing limestone of crushing strength 70 MN/m^2 from 6 mm diameter average size to 0·1 mm diameter average size requires 9 kW. The same machine is used to crush dolomite at the same rate of output from 6 mm diameter average size to a product which consists of 20% with an average diameter of 0·25 mm, 60% with an average diameter of 0·125 mm, the balance having an average diameter of 0·085 mm. Estimate the power required to drive the crusher, assuming that the crushing strength of the dolomite is 100 MN/m² and that crushing follows Rittinger's law.

Solution

The weight mean diameter of the crushed dolomite is calculated as:

n_1	d_1	$n_1 d_1^3$	$n_1 d_1^4$
0·20	0·250	0·003 125	0·000 78
0·60	0·125	0·001 172	0·000 146
0·20	0·085	0·000 123	0·000 011
		0·004 42	0·000 937

and from equation 1.11:

$$d_v = \Sigma n_1 d_1^4 / \Sigma n_1 d_1^3 = 0{\cdot}000\,937/0{\cdot}004\,42$$
$$= 0{\cdot}212\ \text{mm}$$

For Case 1:
$$E = 9{\cdot}0\ \text{kW}$$
$$f_c = 70{\cdot}0\ \text{MN/m}^2$$
$$L_1 = 6{\cdot}0\ \text{mm}$$
$$L_2 = 0{\cdot}1\ \text{mm}$$

and in equation 2.2:

$$9{\cdot}0 = K_R \times 70{\cdot}0\left(\frac{1}{0{\cdot}1} - \frac{1}{6{\cdot}0}\right)$$

or
$$K_R = 0{\cdot}013\ \text{kW mm/(MN/m}^2)$$

For Case 2:
$$f_c = 100{\cdot}0\ \text{MN/m}^2$$
$$L_1 = 6{\cdot}0\ \text{mm}$$
$$L_2 = 0{\cdot}212\ \text{mm}$$

Hence
$$E = 0{\cdot}013 \times 100{\cdot}0\left(\frac{1}{0{\cdot}212} - \frac{1}{6{\cdot}0}\right)$$
$$= \underline{\underline{5{\cdot}9\ \text{kW}}}$$

Problem 2.4

If crushing rolls 1 m in diameter are set so that the crushing surfaces are 12·5 m apart and the angle of nip is 31°, what is the maximum size of particle which should be fed to the rolls?

If the actual capacity of the machine is 12% of the theoretical, calculate the throughput in kg/s when running at 2·0 Hz if the working face of the rolls is 0·4 m long and the feed weighs 2500 kg/m^3.

Solution

The particle size may be obtained from equation 2.6:

$$\cos\alpha = (r_1 + b)/(r_1 + r_2)$$

In this case, $2\alpha = 31°$ and $\cos\alpha = 0{\cdot}964$,

$$b = 12{\cdot}5/2 = 6{\cdot}25\text{ mm} \quad \text{or} \quad 0{\cdot}00625\text{ m}$$

$$r_1 = 1{\cdot}0/2 = 0{\cdot}5\text{ m}$$

$$\therefore \quad 0{\cdot}964 = (0{\cdot}5 + 0{\cdot}00625)/(0{\cdot}5 + r_2)$$

$$\therefore \quad r_2 = 0{\cdot}025\text{ m} \quad \text{or} \quad \underline{\underline{25\text{ mm}}}$$

Cross-sectional area for flow = $(0{\cdot}0125 \times 0{\cdot}4) = 0{\cdot}005$ m^2 and the volumetric flow rate = $(2{\cdot}0 \times 0{\cdot}005) = 0{\cdot}010$ m^3/s.

The actual throughput = $(0{\cdot}010 \times 12/100) = 0{\cdot}0012$ m^3/s or

$$(0{\cdot}0012 \times 2500) = \underline{\underline{3{\cdot}0\text{ kg/s}}}$$

Problem 2.5

A crushing mill reduces limestone from a mean particle size of 45 mm to a product:

Size (mm)	Per cent
12·5	0·5
7·5	7·5
5·0	45·0
2·5	19·0
1·5	16·0
0·75	8·0
0·40	3·0
0·20	1·0

and in so doing requires 21 kJ/kg of material crushed.

Calculate the power required to crush the same material at the same rate, from a feed having a mean size of 25 mm to a product with a mean size of 1 mm.

Solution

The mean size of the product may be obtained as:

n_1	d_1	$n_1 d_1^3$	$n_1 d_1^4$
0·5	12·5	3906	48,828
7·5	7·5	3164	23,731
45·0	5·0	5625	28,125
19·0	2·5	296·9	742·2
16·0	1·5	54·0	81·0
8·0	0·75	3·375	2·531
3·0	0·40	0·192	0·0768
1·0	0·20	0·008	0·0016
		13,049	101,510

and from equation 1.11, the weight mean diameter,

$$d_v = \Sigma n_1 d_1^4 / \Sigma n_1 d_1^3 = 101{,}510/13{,}049 = 7{\cdot}78 \text{ mm}$$

Kick's law will be used as the present case may be regarded as coarse crushing.

Case 1: $E = 21$ kJ/kg

$L_1 = 45$ mm

$L_2 = 7{\cdot}8$ mm

In equation 2.3:

$$21 = K_K f_c \ln (45/7{\cdot}8)$$

and

$$K_K f_c = 11{\cdot}98 \text{ kJ/kg}$$

Case 2: $L_1 = 25$ mm

$L_2 = 1{\cdot}0$ mm

$$\therefore \quad E = 11{\cdot}98 \ln (25/1{\cdot}0)$$
$$= \underline{\underline{38{\cdot}6 \text{ kJ/kg}}}$$

Problem 2.6

A ball mill 1·2 m in diameter is being run at 0·80 Hz; it is found that the mill is not working satisfactorily. Would you suggest any modification in the conditions of operation?

Solution

The critical angular velocity is given by equation 2.8:

$$w_c = \sqrt{(g/r)}$$

In this equation, r is the radius of the mill less that of the particle. For small particles, $r = 0{\cdot}6$ m and hence:

$$w_c = \sqrt{(9{\cdot}81/0{\cdot}6)} = 4{\cdot}04 \text{ rad/s}$$

$$\text{The actual speed} = (2\pi \times 0{\cdot}80) = 5{\cdot}02 \text{ rad/s}$$

and hence it may be concluded that the speed of rotation is too high and the balls are being carried round in contact with the sides of the mill with little relative movement or grinding taking place.

The optimum speed of rotation is $0{\cdot}5w_c$ to $0{\cdot}75w_c$, say $0{\cdot}6w_c$ or

$$(0{\cdot}6 \times 4{\cdot}04) = 2{\cdot}42 \text{ rad/s}$$

This is equivalent to $(2{\cdot}42/2\pi) = 0{\cdot}39$ Hz or, in simple terms, the speed of rotation should be halved.

Problem 2.7

3 kW has to be supplied to a machine crushing material at the rate of 0·3 kg/s from 12·5 mm cubes to a product having the following sizes:

80%	3·175 mm
10%	2·5 mm
10%	2·25 mm

What would be the power which would have to be supplied to this machine to crush 0·3 kg/s of the same material from 7·5 mm cube to 2·0 mm cube?

Solution

The weight mean diameter may be calculated as:

n_1	d_1	$n_1 d_1^3$	$n_1 d_1^4$
0·8	3·175	25·605	81·295
0·1	2·5	1·563	3·906
0·1	2·25	1·139	2·563
		28·307	87·763

and from equation 1.11:

$$d_v = \Sigma n_1 d_1^4/\Sigma n_1 d_1^3 = 87{\cdot}763/28{\cdot}307 = 3{\cdot}100 \text{ mm}$$

(Using Bond's approach, the mean diameter is clearly 3·175 mm.)

For the size ranges involved, the crushing may be considered as intermediate and Bond's law will be used.

Case 1: $E = 3/0{\cdot}3 = 10\,\text{kW/(kg/s)}$

$L_1 = 12{\cdot}5\,\text{mm}$

$L_2 = 3{\cdot}1\,\text{mm}$

$\therefore$ In equation 2.4: $q = L_1/L_2 = 4{\cdot}03$

and $E = 2C\sqrt{(1/L_2)}(1 - 1/q^{0{\cdot}5})$

$$10 = 2C\sqrt{(1/3{\cdot}1)}(1 - 1/4{\cdot}03^{0{\cdot}5})$$

$$= 2C \times 0{\cdot}568 \times 0{\cdot}502$$

$\therefore$ $C = 17{\cdot}54\,\text{kW mm}^{0{\cdot}5}\text{/(kg/s)}$

Case 2: $L_1 = 7{\cdot}5\,\text{mm}$

$L_2 = 2{\cdot}0\,\text{mm}$

$q = (7{\cdot}5/2{\cdot}0) = 3{\cdot}75$

$\therefore$ $E = 2 \times 17{\cdot}54(1/2{\cdot}0)(1 - 1/3{\cdot}75^{0{\cdot}5})$

$$= (35{\cdot}08 \times 0{\cdot}707 \times 0{\cdot}484)$$

$$= 12{\cdot}0\,\text{kW/(kg/s)}$$

For a feed of 0·3 kg/s, the power required $= (12{\cdot}0 \times 0{\cdot}3)$

$$= \underline{\underline{3{\cdot}60\,\text{kW}}}$$