AM 3C – Teste 2024.1 Resolucão

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Conteúdo

Grupo I

Questão 1 edo ord 1

A equação dif lin ord 1

$$\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}\boldsymbol{x}} + \frac{\cos(\boldsymbol{x})}{\sin(\boldsymbol{x})}\,\boldsymbol{y} = -\boldsymbol{x}; \qquad\qquad \boldsymbol{x} \in \,]0, \boldsymbol{\pi}[$$

Resposta

$$\boldsymbol{y} = \frac{\boldsymbol{c}}{\sin(\boldsymbol{x})} - \frac{\cos(\boldsymbol{x})}{\sin(\boldsymbol{x})}$$

$$\left(\mathrm{D}_x + \frac{\cos(x)}{\sin(x)}\right) y = -x$$

$$y = \frac{c_0}{\varphi(x)} + \frac{1}{\varphi(x)} \int -1\,\varphi(x)\ \mathrm{d}x =$$

$$= \frac{c_0}{c_2} + \frac{1}{c_2} \int -1\,c_2\ \mathrm{d}x;$$

$$\varphi(x) = \exp\left(\int \frac{\cos(x)}{\sin(x)}\ \mathrm{d}x\right) = e^{1/2+c_1} = c_2$$

$$\mathrm{d}\cos(x) = -\sin(x)\ \mathrm{d}x$$

$$\mathrm{d}\left(\frac{1}{\sin(x)}\right) = \frac{1}{-2\,\sin^2(x)}(\cos(x))\ \mathrm{d}x \implies$$

$$\implies \mathrm{d}(x) = \frac{-2\,\sin^2(x)}{\cos(x)}\ \mathrm{d}\left(\frac{1}{\sin(x)}\right)$$

$$\int u\,v'\ \mathrm{d}x = u\,v + \int u'\,v\ \mathrm{d}x$$

$$\int \frac{1}{\sin(x)}\ \cos(x)\ \mathrm{d}x = \int \frac{1}{\sin(x)}\ \frac{\mathrm{d}}{\mathrm{d}x}(\sin(x))\ \mathrm{d}x =$$

$$= \frac{\sin(x)}{\sin(x)} + \int \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\sin(x)}\right)\ \sin(x)\ \mathrm{d}x = 1 + \int \left(-\frac{\cos(x)}{\sin^2(x)}\right)\ \sin(x)\ \mathrm{d}x \implies$$

$$\implies \int \frac{\cos(x)}{\sin(x)}\ \mathrm{d}x = 1/2$$

Questão 2 bernoulli

A solução da eq de bernoulli

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y = \frac{1}{y'}$$

que satisfaz a condição $y(0) = 2$ é

Resposta

$$y = \sqrt{e^{-2\,x}\,3 + 1}$$

$y' + y = y^{-1}$

$y = \sqrt{z} = \sqrt{e^{-2\,x}\,3 + 1};$

$c : y(0) = \sqrt{z(0)} = \sqrt{e^{-2*0}\,c + 1} = \sqrt{c+1} = 2 \implies c = 4 - 1 = 3;$

$z = y^2 \implies$
$\implies z' + (1+1)1\,z = z' + 2\,z = (1+1) = 2$

$$z = \frac{c_0}{\varphi(x)} + \frac{1}{\varphi(x)} \int 2\,\varphi(x)\ \mathrm{d}x =$$

$$= \frac{c_0}{c_2\,e^{2\,x}} + \frac{1}{c_2\,e^{2\,x}} \int 2\,c_2\,e^{2\,x}\ \mathrm{d}x =$$
$$= e^{-2\,x}\,\frac{c_0}{c_2} + \frac{2\,c_2}{c_2\,e^{2\,x}} \left(\frac{e^{2\,x}}{2} + c_3 \right) = e^{-2\,x}\,\frac{c_0}{c_2} + 1 + \frac{2}{e^{2\,x}}\,c_3 = e^{-2\,x}\,c + 1;$$

$$\varphi(x) = \exp\left(\int 2\ \mathrm{d}x \right) = \exp\left(2\,x + c_1\right) = c_2\,e^{2\,x}$$

Questão 3 fator int

A equação differencial

$$(5\,x\,y^2 - 2\,y)\;\mathrm{d}x + (3\,x^2\,y - x) = 0$$

Admite um fator integrante na forma $\phi(x,y) = x^m\,y^n$, com $m, n \in \mathbb{N}$. então

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Resposta

$$m = 3, n = 2$$

Questão 4 met var const arb

A eq dif hom

$$x\,y'' + x^2\,y' + 4\,y = x^3$$

Tem como solução geral a função $y(x) = c_1(x)\,y_1(x) + c_1(x)\,y_2(x)$. ...

Resposta

$$y : \begin{pmatrix} 4 \\ +x^2\ \mathrm{D}_x \\ +x\ \mathrm{D}_x^2 \end{pmatrix}\ y = x^3$$

$$\begin{Bmatrix} c_1'(x)\ \mathrm{D}_x^0 y_1(x) + c_2'(x)\ \mathrm{D}_x^0 y_2(x) = 0 \\ c_1'(x)\ \mathrm{D}_x\, y_1(x) + c_2'(x)\ \mathrm{D}_x\, y_2(x) = \frac{x^3}{x} = x^2 \end{Bmatrix}$$

Questão 5 Transf laplace

- $f(t)$ def em ord ate 2 em $\mathbb{R}_0^+$

Resposta

$$(s+1)^2\,F(s+1) - s + 1 - s\,F'(s) - F(s)$$

Grupo II

Det sol ger da eq lin hom de coef const.

$$\frac{\mathrm{d}^{2} y}{\mathrm{d} x^{2}} + \frac{\mathrm{d} y}{\mathrm{d} x} - y\, 6 = 0$$

Resposta

$P\, y = \left(\mathrm{D}_x^2 + \mathrm{D}_x - 6\right)\, y = 0;$

$y = \varphi(x) \int z(x)\ \mathrm{d}x = \varphi(x) \int z(x)\ \mathrm{d}x;$

$P\, y = \left(\mathrm{D}_x^2 + \mathrm{D}_x - 6\right) \left(\varphi(x) \int z(x)\ \mathrm{d}x\right) = 0;$

$\mathrm{D}_x y = \mathrm{D}_x \left(\varphi(x) \int z(x)\ \mathrm{d}x\right);$

$\mathrm{D}_x^2 y = \mathrm{D}_x^2 \left(\varphi(x) \int z(x)\ \mathrm{d}x\right)$

met ver const arb

util o met da var das const arb det a sol ger da eq n homog

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} - y\,6 = -5\,e^{2\,x}\,\cos(x)$$

Resposta

$$y : \begin{pmatrix} -6 \\ +1\ \mathrm{D}_x \\ +1\ \mathrm{D}_x^2 \end{pmatrix}\ y = \left(-5\,e^{2\,x}\,\cos(x)\right)$$

$$y = c_1(x)\,y_1(x) + c_2(x)\,y_2(x);$$

$$c_1(x) = \int c_1'(x)\ \mathrm{d}x;$$

$$c_2(x) = \int c_2'(x)\ \mathrm{d}x$$

$$c_1'(x) = \frac{1}{W(y_1(x), y_2(x))} \begin{vmatrix} 0 & \mathrm{D}_x^0 y_2(x) \\ -5\,e^{2\,x}\,\cos(x) & \mathrm{D}_x y_2(x) \end{vmatrix}$$

$$c_2'(x) = \frac{1}{W(y_1(x), y_2(x))} \begin{vmatrix} \mathrm{D}_x^0 y_1(x) & 0 \\ \mathrm{D}_x y_1(x) & -5\,e^{2\,x}\,\cos(x) \end{vmatrix}$$

$$W(y_1(x), y_2(x)) = \det \begin{bmatrix} \mathrm{D}_x^0\,y_1(x) & \mathrm{D}_x^0\,y_2(x) \\ \mathrm{D}_x\,y_1(x) & \mathrm{D}_x\,y_2(x) \end{bmatrix}$$

$$\left\{ \begin{matrix} c_1'(x)\ \mathrm{D}_x^0 y_1(x) + c_2'(x)\ \mathrm{D}_x^0 y_2(x) & = & 0 \\ c_1'(x)\ \mathrm{D}_x\,y_1(x) + c_2'(x)\ \mathrm{D}_x\,y_2(x) & = & -5\,e^{2\,x}\,\cos(x) \end{matrix} \right\};$$

$$\mathrm{D}_x\,y_1(x) = \mathrm{D}_x\,y_1(x);$$

$$\mathrm{D}_x\,y_2(x) = \mathrm{D}_x\,y_2(x)$$

Grupo III

Q1 a.

Det todas as sol de cliraut

$$y = x\frac{\mathrm{d}y}{\mathrm{d}x} - \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

Q1 b. question

mud de var $x = 1/t$ resolva

$$y = -x\,\frac{\mathrm{d}y}{\mathrm{d}x} + x^6\,\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3 \qquad x > 0$$

Grupo V

Questão 1