ALGA - Resolução das Listas

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Conteúdo

Parte 1 - Matrizes

Questão 1.1

Q1.1 - (a)

B, E, F, H, I

Q1.1 - (b)

B, E, F, H, I

Q1.1 - (c)

B, E, F, I

Q1.1 - (d)

B, E

Questão 1.2

Q1.2 - (a)

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Q1.2 - (b)

$$\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

Q1.2 - (c)

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Questão 1.3

Q1.3 - (a)

$$= \begin{bmatrix} 4 & 1 & 5 \\ -2 & 1 & -1 \end{bmatrix}$$

Q1.3 - (c)

$$= \begin{bmatrix} 2 & 1 & -4 \\ 2 & -1 & 0 \end{bmatrix}$$

Q1.3 - (b)

$$= \begin{bmatrix} 8 & 2 & 10 \\ -2 & 2 & -2 \end{bmatrix}$$

Q1.3 - (d)

$$= \begin{bmatrix} 0 & 2 & -15 \\ 11 & 2 & -2 \end{bmatrix}$$

Questão 1.4

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Questão 1.10

Q1.10 - (c)

$$\boldsymbol{(AB)^{(i)} = (AB)^{(j)} \because A^{(j)} = A^{(i)} \quad \forall\, i \neq j}$$

$$A_i = A_j \land AB_{k_1,k_2} = \sum_{k=1}^{n} a_{k_1,k}\, b_{k,k_2} \implies$$
$$\implies (AB)_{i,k_2} = \sum_{k=1}^{n} a_{i,k}\, b_{k,k_2} =$$
$$= \sum_{k=1}^{n} a_{j,k}\, b_{k,k_2} = (AB)_{j,k_2} \implies$$
$$\implies (AB)_i = (AB)_j$$

Q1.10 - (d)

$$\boldsymbol{B^k = B^l : k \neq l \implies}$$
$$\boldsymbol{\implies (AB)^k = (AB)^l}$$

Questão 1.9

$$\boldsymbol{\{D, D'\} \in \mathcal{M}_{n\times n}(\mathrm{K}) :}$$
$$\boldsymbol{d_{i,j} = 0 \wedge d'_{i,j} \forall\, i \neq j \implies}$$
$$\boldsymbol{\implies (DD')_{i,j} = 0 \quad \forall\, i \neq j}$$

$$\{D, D'\} \in \mathcal{M}_{n\times n}(\mathrm{K}) : d_{i,j} = 0 \wedge d'_{i,j} \;\forall\, i \neq j;$$
$$(DD')_{i,j} = \sum_{k=1}^{n} d_{i,k}\, d'_{k,j} \implies$$
$$\implies (DD')_{i,j} = 0 \;\forall\, \{i, j\} \in \mathbb{K} : i \neq j$$

Questão 1.10 Indique...

Q1.10 - (a) Uma Condição para que uma matriz diag. seja invert.

$$A \in \mathcal{M}_{n\times n} : a_{i,j} = 0 \quad \forall\, i \neq j \wedge$$
$$\wedge\, \exists\, A^{-1} : AA^{-1} = I_n \impliedby$$
$$\impliedby a_{i,j} \neq 0 \quad \forall\, i = j$$

Q1.10 - (b)

Questão 1.108

$$\boldsymbol{J_n \in \mathcal{M}_{n\times s}(\mathbb{K}) : (J_n)_{i,j} = 1}$$
$$\forall \{\boldsymbol{i, j}\} \in \mathbb{K}$$

Questão 1.22

$$\boldsymbol{A} \in \boldsymbol{\mathcal{M}}_{n\times n}(\mathbb{K})$$

Q1.22 - (a)

$$\boldsymbol{A}^3 = \boldsymbol{I}_n$$

Q1.22 - (b)

$$\boldsymbol{A}^2 + 2\,\boldsymbol{A} = \boldsymbol{I}_n$$

Q1.22 - (c)

$$\boldsymbol{A}^2 + \alpha\,\boldsymbol{A} + \beta\,\boldsymbol{I}_n = \mathbf{0}$$
$$: \boldsymbol{\alpha} \in \mathbb{K} \wedge \boldsymbol{\beta} \in \mathbb{K}\backslash\{\mathbf{0}\}$$

Questão 1.34

Q1.34 - (a)

A e C

Q1.34 - (b)

E

Questão 1.37

Q1.37 - (a)

$$A \in \mathcal{M}_{n\times n}(\mathbb{K}) : a_{i,j} = 0 \quad \forall \{i, j\}$$

Q1.37 - (b)

...

Questão 1.129

Q1.129 - (a)

(i)

$$(A + A^T) = ((A + A^T)^T)^T = (A^T + A)^T = (A + A^T)^T$$
$$\therefore\ (A + A^T) \text{ é simétrica}$$

(ii)

Q1.129 - (b)

Questão 1.42

a = s, III b = s, II c = s, I d = n e = s, II

Questão 1.43

Q1.43 - (a)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Q1.43 - (b)

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q1.43 - (c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/5 & 1 \end{bmatrix}$$

Questão 1.45

Q1.45 - (a)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Questão 1.48

A = s B = n C = s D = n

Questão 1.51

A = s B = s C = n D = s E = s

Questão 1.49

Q1.49 - (a)

$$A' = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 0 & 2 & 2 \end{bmatrix} \xleftarrow[l_2 \to l_2 - 2\,l_1]{} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 0 & 2 & 2 \end{bmatrix} \xleftarrow[\substack{l_2 \to l_2 - 2\,l_1 \\ l_3 \to l_3 + l_1}]{}$$

$$\leftarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Questão 1.171

$$[A|I] = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[l2+=-2\,l1]{}$$

Parte 2 - Sistemas de Equações Lineares

Questão 2.2

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{2} & \mathbf{2} & \mathbf{-2} & \mathbf{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{6} & \mathbf{-4} \end{bmatrix} \in \boldsymbol{\mathcal{M}}_{\mathbf{3\times 4}}(\mathbb{R})$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{-1} \\ \mathbf{4} \\ \mathbf{-6} \end{bmatrix} \in \boldsymbol{\mathcal{M}}_{\mathbf{3\times 1}}(\mathbb{R})$$

Parte 3 - Determinantes

Questão 3.72

$$\boldsymbol{A \in \mathcal{M}_{n\times n} : A^2 = -A}$$

$\det(-A) = \det A(-1)^n = \det(A^2) \implies \det A(\det A - (-1)^n) = 0 \implies$
$\implies \det A = 0 \vee \det A = (-1)^n$

Questão 3.73

$$\boldsymbol{A \in \mathcal{M}_{n\times n} : A\,A^* = I_n}$$

$|\det A| = 1 = \cdots = \det(A)\ \det \overline{A}^T = \det(A)\,\overline{\det A^T} = \det(A\,A^*) = \det(I_n) = 1$

Questão 3.28

Q3.28 - (b)

$V_\alpha = \begin{vmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) - (-\sin^2(x)) = 1 \neq 0$
$\therefore \exists\, V_\alpha^{-1}$

$$\boldsymbol{\widehat{a_{i\,j}} = (-1)^{i+j}\,\mathbf{det}(A - A_i - A_j)}$$

$$V_\alpha^{-1} = \frac{\operatorname{adj} V_\alpha}{\det V_\alpha} = \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix}$$
$$\operatorname{adj} V_\alpha = \widehat{V_\alpha^T} = \begin{bmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{bmatrix}^T = \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix}^T$$

Questão 3.29

$$\begin{bmatrix} \boldsymbol{m} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \boldsymbol{m} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \boldsymbol{m} \end{bmatrix}$$

Q3.29 - (a)

$$\operatorname{adj} M = \begin{bmatrix} m^2-1 & 1-m & 1-m \\ 1-m & m^2-1 & 1-m \\ 1-m & 1-m & m^2-1 \end{bmatrix}$$

Q3.29 - (b)

$$\begin{aligned} &\det M = m(m^2-1) + 1(1-m) + 1(1-m) = \\ &= m(m+1)(m-1) - 2(m-1) = (m-1)(m^2+m-2) = \\ &= (m-1)(m(m-1) + 2(m-1)) = (m-1)^2(m+2) \\ &\therefore \exists M^{-1} \forall\, M : m \in \mathbb{R}\backslash\{1,-2\} \end{aligned}$$

Q3.29 - (c)

$$\begin{aligned} &M^{-1} = \frac{\operatorname{adj} M}{\det M} = \frac{1}{(m-1)^2(m+2)} \begin{bmatrix} m^2-1 & 1-m & 1-m \\ 1-m & m^2-1 & 1-m \\ 1-m & 1-m & m^2-1 \end{bmatrix} = \\ &= \frac{1}{(m-1)^2(m+2)} \begin{bmatrix} m+1 & -1 & -1 \\ -1 & m+1 & -1 \\ -1 & -1 & m+1 \end{bmatrix} \end{aligned}$$

Questão 3.31

Q3.31 - (a)

$$\exists (\operatorname{adj} A)^{-1} \because (\frac{A}{\det A}) \operatorname{adj} A = I_n : \det A \neq 0 \land \exists A^{-1}$$

Q3.31 - (c)

$$\det(\operatorname{adj} A) = \det(\det A\, A^{-1}) = (\det A)^n \det A^{-1} = (\det A)^n / \det A =$$
$$= (\det A)^{n-1})$$

$$\mathbf{adj(\boldsymbol{AB}) = (adj\ \boldsymbol{A})(adj\ \boldsymbol{B})}$$

$$A\,B \operatorname{adj}(A\,B) = \det(A\,B)\, I_n \implies \operatorname{adj}(A\,B) = (A\,B)^{-1} \det(A\,B)\, I_n =$$
$$= \det A \det B\, B^{-1} A^{-1} I_n = (\operatorname{adj} A)(\operatorname{adj} B)$$

Questão 3.25

$$\boldsymbol{A = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 4 & 4 \\ 1 & 3 & 1 & 1 \\ 0 & 0 & -2 & 0 \end{bmatrix}}$$

$$\det A = \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 4 & 4 \\ 1 & 3 & 1 & 1 \\ 0 & 0 & -2 & 0 \end{pmatrix} = 2 \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 4 \\ 1 & 3 & 1 \end{pmatrix} =$$
$$= 2 * 1 * (2 * 1 - 3 * 4) + 2 * 1 * (-1 * 4 - 2 * 1) = -32$$

Questão 3.33

$$\boldsymbol{A_k = \begin{bmatrix} 1 & -k & 10 & k & kk & k & -k \end{bmatrix}}$$