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# Chapter 7. Fluidization

*7.1 Introduction (definition; types of fluidization)*

*7.2 Pressure drop (Fixed bed; Fluidized bed; Bed transport)*

*7.3 Fluidized bed (condition; expansion; minimum fluidizing velocity)*

*7.3.1 Laminar flow*

*7.3.2 General case*

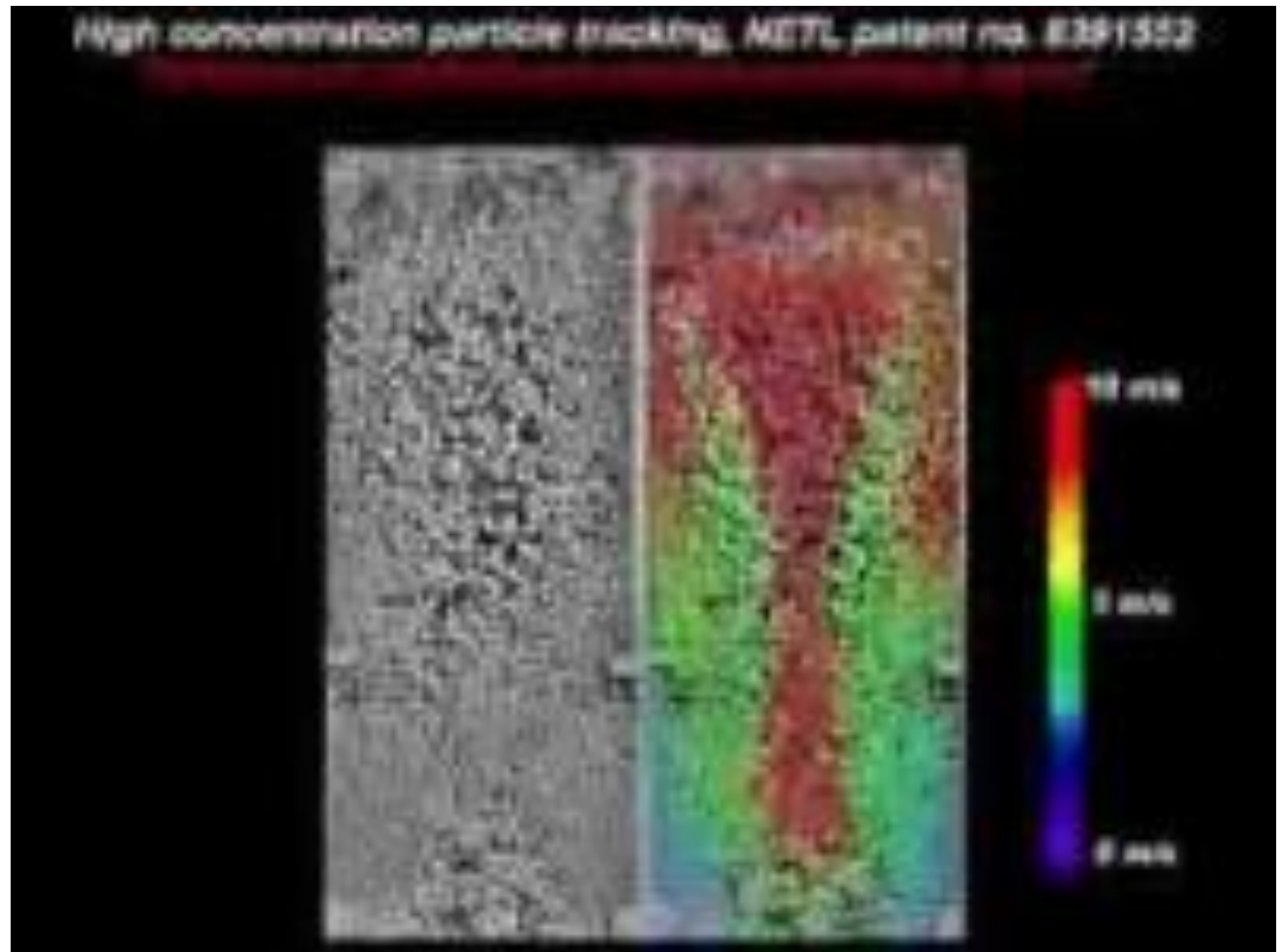
*7.4 Particulate fluidized beds*

*(7.5 Fluidized bed heat transfer enhancement)*

*Coulson & Richardson pp. 291-334*

# Introduction

A **fluidized bed** is a solids suspension (gas or liquid + solid particles) that exhibits properties of a normal fluid. The (typically) small solid particles follow the movement of elements of the fluid. A fluidized bed is considered as a single bulk phase with characteristic properties, such as density and viscosity, of a normal fluid. **Fluidization may be viewed as the opposite operation to coarse sedimentation (chapter 5).**



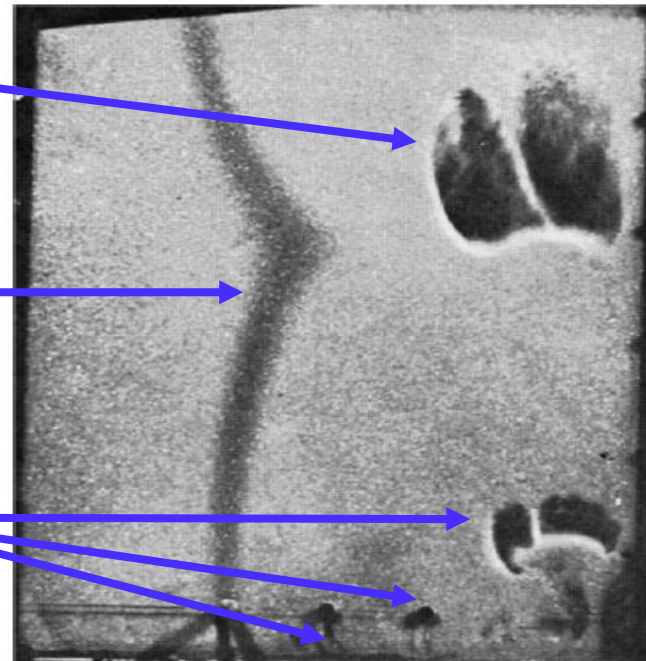
# Gas-solid fluidization

A fluidized bed may be very heterogeneous and unstable, specially in the case of a gas fluidized bed

Large bubbles on top

Preferable gas flow channel

Small bubbles in the bottom



Taken from Coulson pp. 322

# Types of fluidization

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## Particulate fluidization:

uniform bed expansion with increasing liquid velocity

**Low**  $(\rho_s - \rho)$

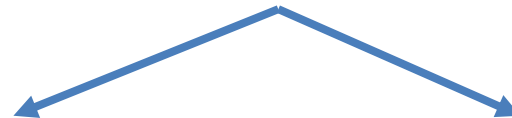
**(Liquid-Solid)**

## Aggregative fluidization:

two phases are formed, namely the dense phase / bubble phase

**High**  $(\rho_s - \rho)$

**(Gas-solid)**



**Quiescent bed**  
(low gas flowrate)

**Boiling bed**  
(high gas flowrate; very unstable dense/bubble phases)

# Froude number, $Fr$

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The **Froude number**,  $Fr$ , is a dimensionless number defined by the ratio between inertial forces and gravitational forces. For a fluidized bed with minimum fluidization velocity,  $u_{mf}$ :

$$Fr = \frac{u_{mf}^2}{dg}$$

$$Fr \leq 1$$

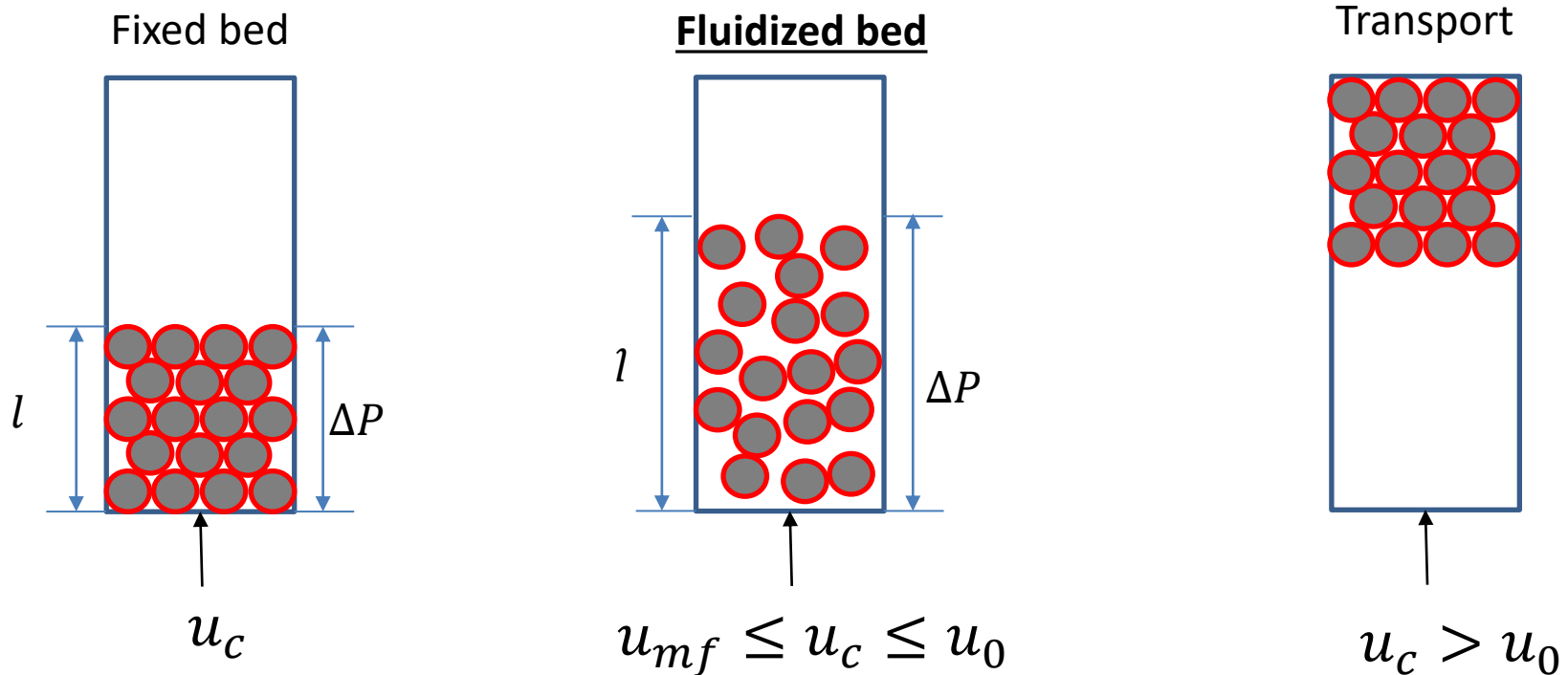
**Particulate fluidization:**  
uniform bed expansion  
with increasing liquid  
velocity

$$Fr > 1$$

**Aggregative fluidization:**  
two phases are formed,  
namely the dense phase /  
bubble phase

# Fluidized bed condition

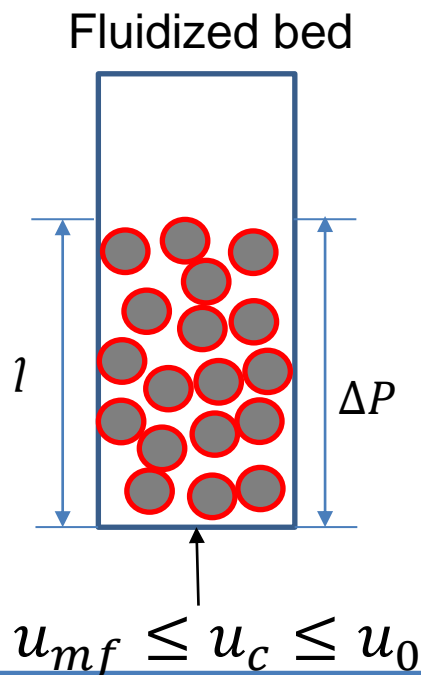
Consider a fluid (liquid or gas) flowing through a bed of solid particles. For sufficiently large fluid velocities,  $u_c > u_{mf}$ , the individual bed particles separate from one another and become freely supported in the fluid **due to the friction drag force**. At this stage, the bed is described as a **fluidized bed**.



$u_{mf}$  - minimum fluidization velocity, m/s,  $u_0$  - terminal fall velocity, m/s

# Fluidized bed condition

In a fluidized bed, the pressure drop ( $-\Delta P$ ) is always constant. More specifically, the pressure drop ( $-\Delta P$ ) must balance the apparent weight of bed of particle solids



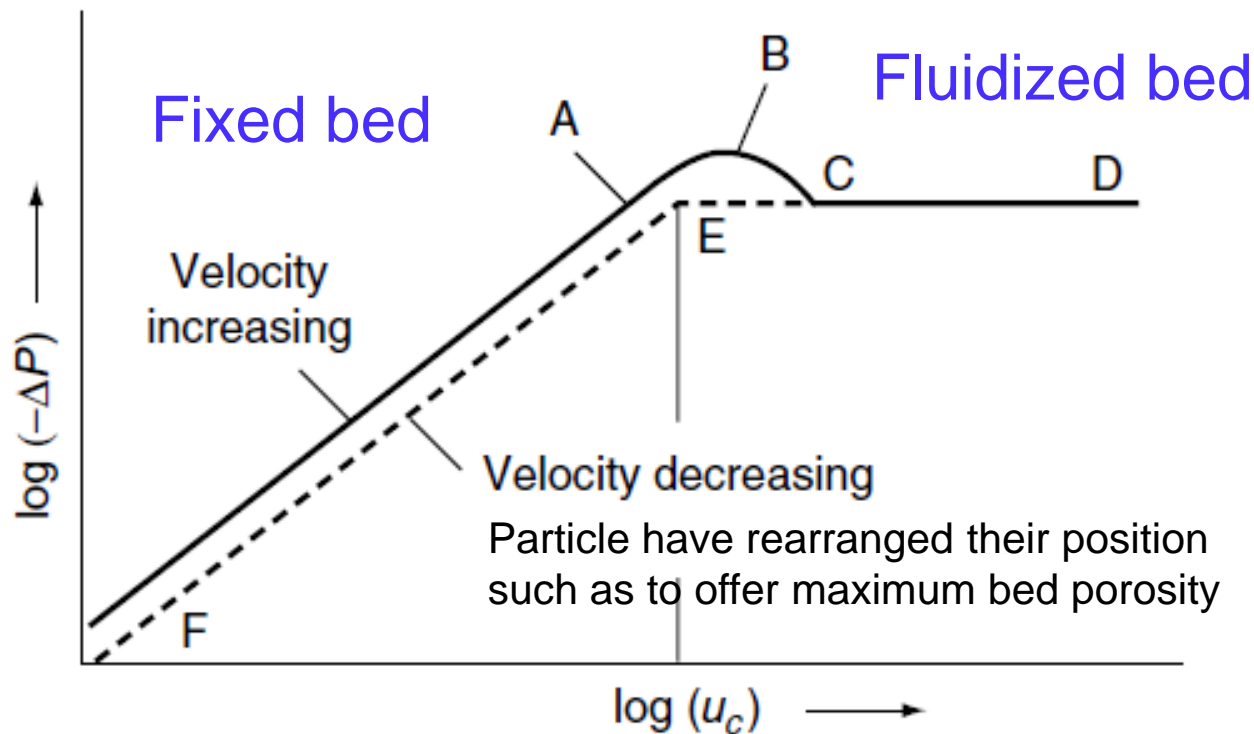
$$A(-\Delta P) = Al(1 - e)(\rho_s - \rho)g$$

Pressure drop  
due to friction  
drag force = Apparent weight of  
bed

$$\boxed{\frac{-\Delta P}{l} = (1 - e)(\rho_s - \rho)g}$$

# Pressure drop ( $-\Delta P$ )

A fluid flowing through a bed, be it fixed or fluidized, experiences a pressure drop,  $-\Delta P$ , due to the drag force

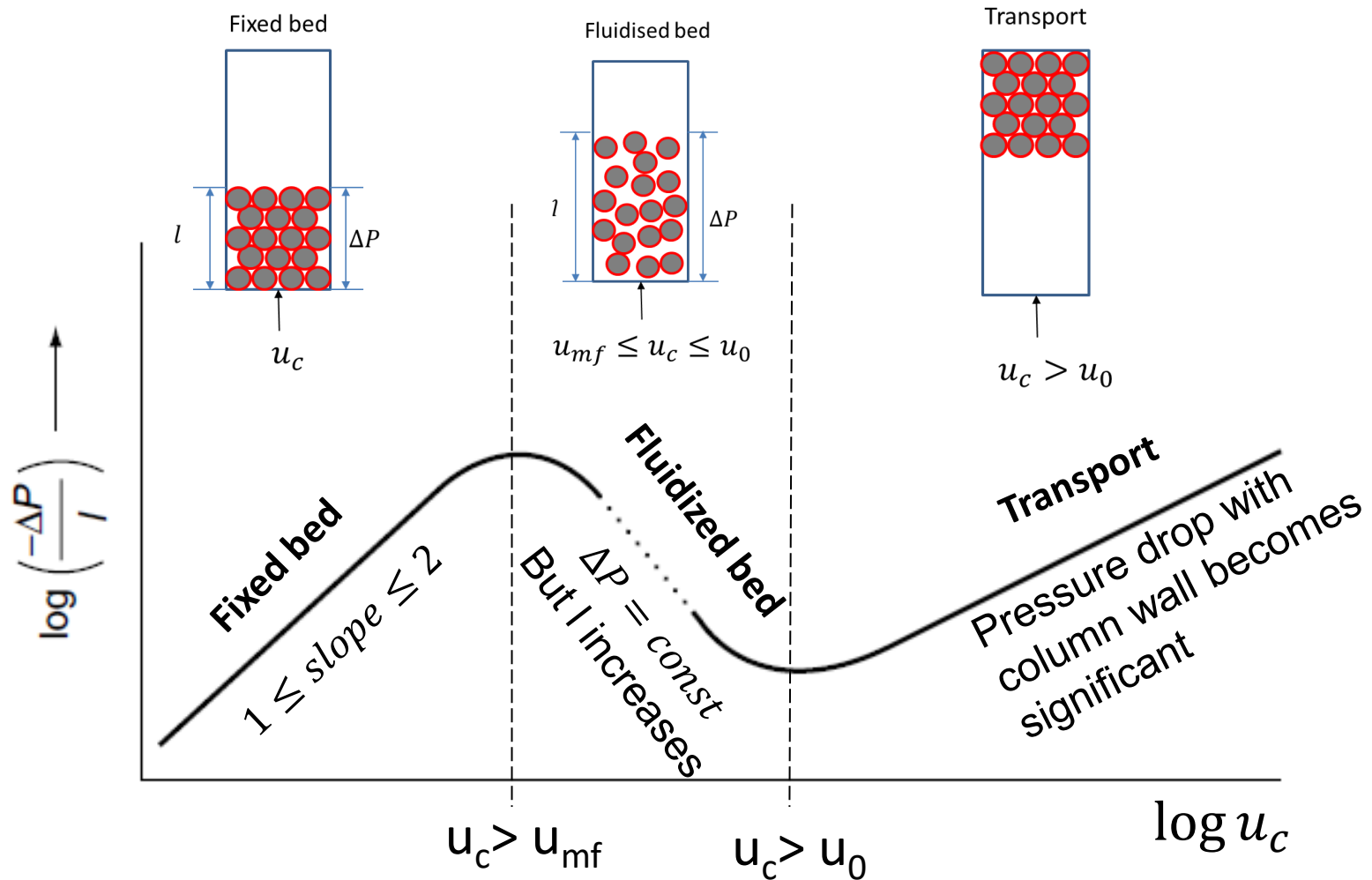


**Where fluidized bed?**

**What's the cause of the differences between the full and dashed line?**



# Pressure drop per unit length ( $-\Delta P/l$ )



# Fluidized bed – laminar flow

**Laminar flow:** combining the fluidization condition with the pressure drop as given by the Kozeny equation (valid for laminar flow, Chapter 6),

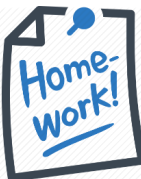
$$\frac{-\Delta P}{l} = (1 - e)(\rho_s - \rho)g \quad (\text{Fluidization condition})$$

$$u_c = \frac{1}{K''} \frac{e^3}{S_B^2} \frac{1}{\mu} \frac{(-\Delta P)}{l} \quad (\text{Kozeny equation})$$

The **Laminar flow fluidization Equation** is obtained:

$$u_c = \frac{e^3(\rho_s - \rho)g}{K''S^2(1 - e)\mu} = f(e)$$

$$u_{mf} \leq u_c \leq u_0$$



# Minimum fluidizing velocity – laminar flow

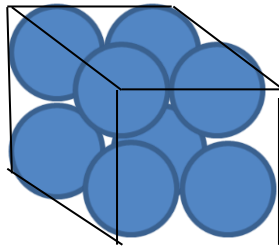
The **Minimum fluidizing velocity**,  $u_{mf}$ , is obtained by applying the porosity at the beginning of fluidization to the fluidization equation. In the case of laminar flow:

$$u_{mf} = \frac{e_{mf}^3 (\rho_s - \rho) g}{K'' S^2 (1 - e_{mf}) \mu}$$



$e_{mf}$  – **porosity at the beginning of fluidization** depends on particle size, geometry and orientation

Cube with 8 spheres of diameter  $d$



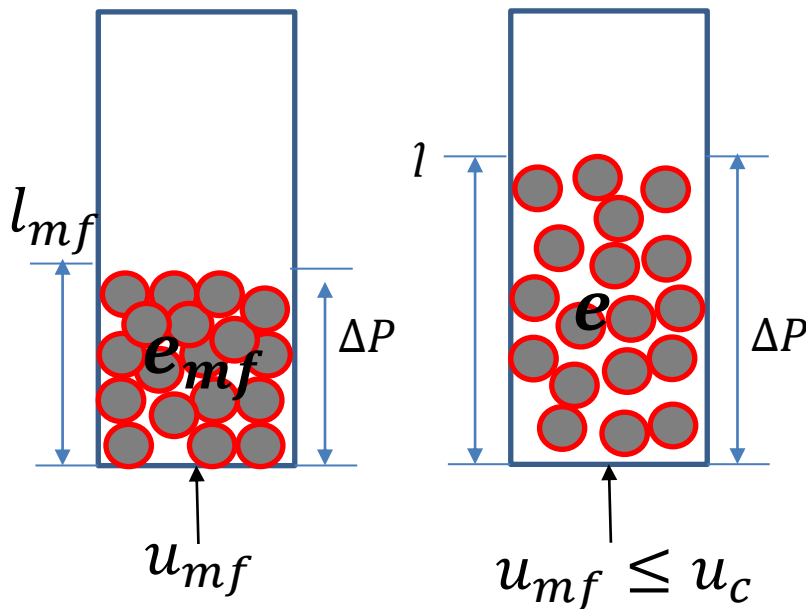
$$1 - e_{mf} = \frac{8 \frac{\pi d^3}{6}}{(2d)^3}$$

Spheres

$$e_{mf} = 0,48$$

# Bed expansion – laminar flow

A fluidized bed with length,  $l$ , and porosity,  $e$ , expands with increasing fluid velocity,  $u_c \geq u_{mf}$ . This means that bed length,  $l$ , and porosity,  $e$ , increase with fluid velocity,  $u_c$ .



$$Al_{mf}(1 - e_{mf})(\rho_s - \rho)g =$$

$$Al(1 - e)(\rho_s - \rho)g$$

(Weight of bed is always the same)

Thus:

$$l = \frac{1 - e_{mf}}{1 - e} l_{mf}$$

$l_{mf}$  and  $e_{mf}$  are conditions at the beginning of fluidization

# Fluidized bed equation – general case

Combining the fluidization condition with the Carman-Kozeny Eqs (Chap 6), the fluidization relationship,  $u_c = f(e)$ , is obtained, which is valid for both laminar and turbulent flow.

$$u_c = f(e)$$

$$\frac{-\Delta P}{l} = (1 - e)(\rho_s - \rho)g \quad (\text{Fluidization condition})$$

Carman-Kozeny Eqs.

$$\frac{R_1}{\rho u_1^2} \frac{S_B u_c^2}{e^3} = \frac{(-\Delta P)}{l}$$

CARMAN

$$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{0,4}{Re_1^{0,1}}$$

OR

SAWITOWSKI

$$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{1}{Re_1^{0,1}}$$

OR

ERGUN

$$\frac{R_1}{\rho u_1^2} = \frac{4,17}{Re_1} + 0,29$$

$$Re_1 = \frac{\rho u_c}{S_B \mu}$$

# Minimum fluidizing velocity – general case

The minimum fluidizing velocity,  $u_{mf}$ , for spheres and assuming the Ergun correlation, valid for laminar and/or turbulent flow:

$$Ga = 150 \frac{1 - e_{mf}}{e_{mf}} Re'_{mf} + \frac{1,75}{e_{mf}^3} Re'^2_{mf}$$



$$Ga = \frac{d^3 \rho (\rho_s - \rho) g}{\mu^2}$$

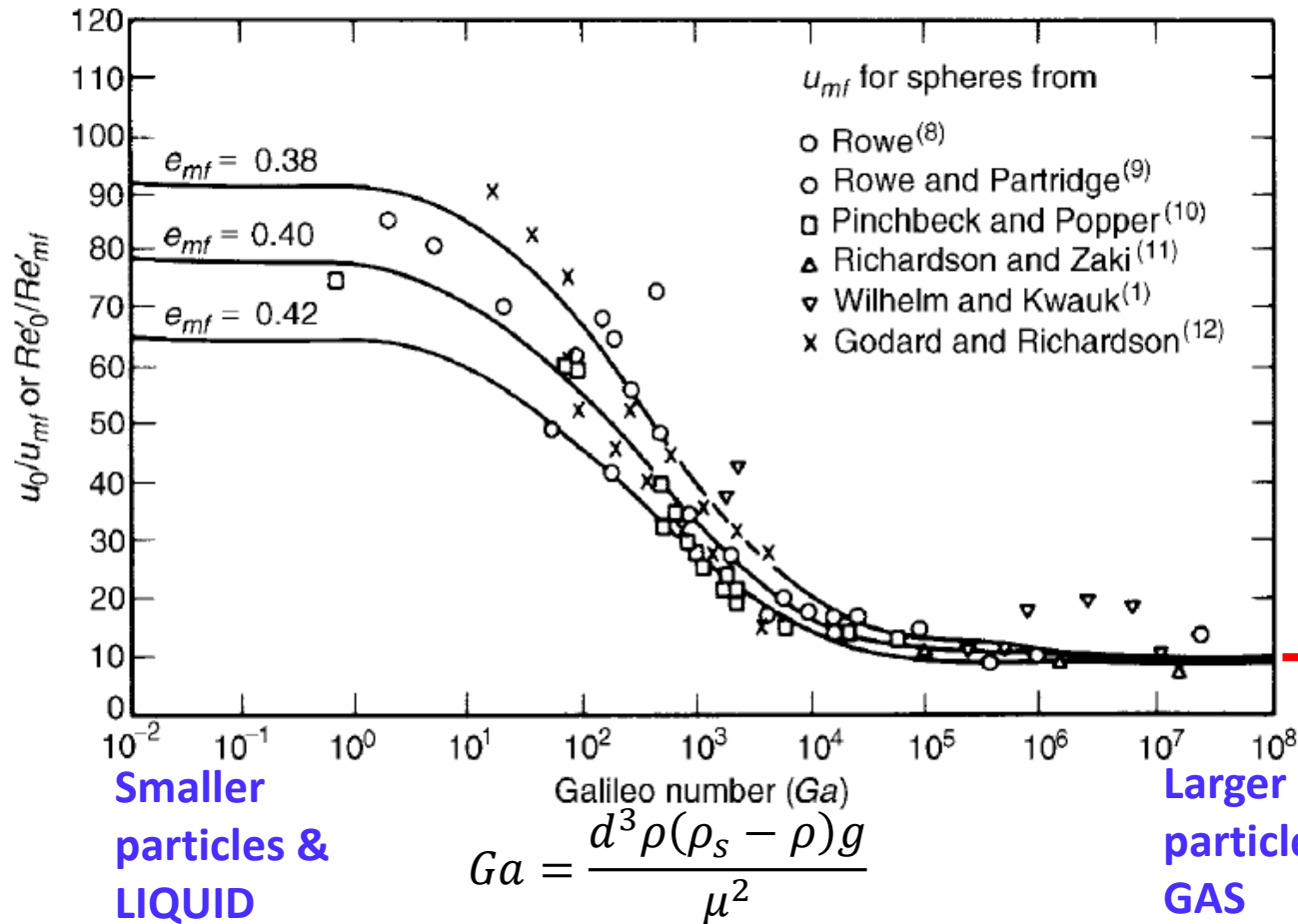
Galileo number =  $\frac{\text{gravity forces}}{\text{viscous forces}}$

$$Re'_{mf} = \frac{\rho u_{mf} d}{\mu}$$

Particle Reynolds number in conditions of initial fluidization

# Fluidized bed –general case

$u_o/u_{mf}$  as function of  $Ga$  for a bed of spherical particles



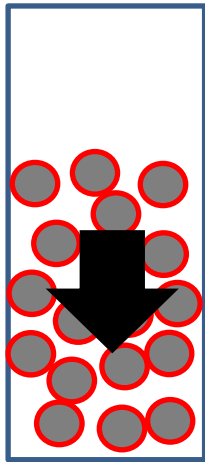
Independent of geometry and size

$$\frac{Re'_0}{Re'_{mf}} = \frac{u_o}{u_{mf}} = 9.1$$

# Particulate fluidization

Particulate fluidization, typical of solid-liquid systems, is in its essence the opposed operation (bed expansion) to coarse sedimentation (bed contraction). Thus the mathematical formalism of coarse sedimentation (Chapter 5) also applies to particulate fluidization (Chapter 7).

## Coarse sedimentation

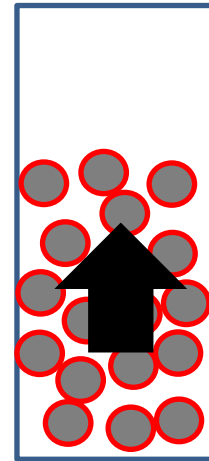


*Bed contraction*

$$\frac{u_c}{u_i} = e^n$$

$$\log(u_0) = \log(u_i) + \frac{d}{d_t}$$

## Fluidization



*Bed expansion*



# Coarse sedimentation

$$\frac{u_c}{u_i} = e^n$$

$$\log(u_0) = \log(u_i) + \frac{d}{d_t}$$

$u_c$  – settling velocity or fluidisation velocity, m/s

$u_i$  – settling velocity for infinit dilution, m/s

$e$  – void faction;

$n$  = coeficiente =  $f(Re'_0, d/d_t)$

$d$  – particle diameter

$d_t$  – tube diameter

$Re'_0 = \frac{\rho u_0 d}{\mu}$  - particle Reynolds at  $u_0$

Table 5.1.  $n$  as a function of  $Ga$  or  $Re'_0$  and  $d/d_t$

Range of $Ga$	Range of $Re'_0$	$n$ as function of $Ga, d/d_t$	$n$ as function of $Re'_0, d/d_t$
0–3.6	0–0.2	$4.6 + 20 d/d_t$	$4.6 + 20 d/d_t$ (Independent of Re)
3.6–21	0.2–1	$(4.8 + 20 d/d_t) Ga^{-0.03}$	$(4.4 + 18 d/d_t) Re'^{-0.03}_0$
$21-2.4 \times 10^4$	1–200	$(5.5 + 23 d/d_t) Ga^{-0.075}$	$(4.4 + 18 d/d_t) Re'^{-0.1}_0$
$2.4 \times 10^4 - 8.3 \times 10^4$	200–500	$5.5 Ga^{-0.075}$	$4.4 Re'^{-0.1}_0$
$> 8.3 \times 10^4$	$> 500$	2.4	2.4 (Independent of Re)

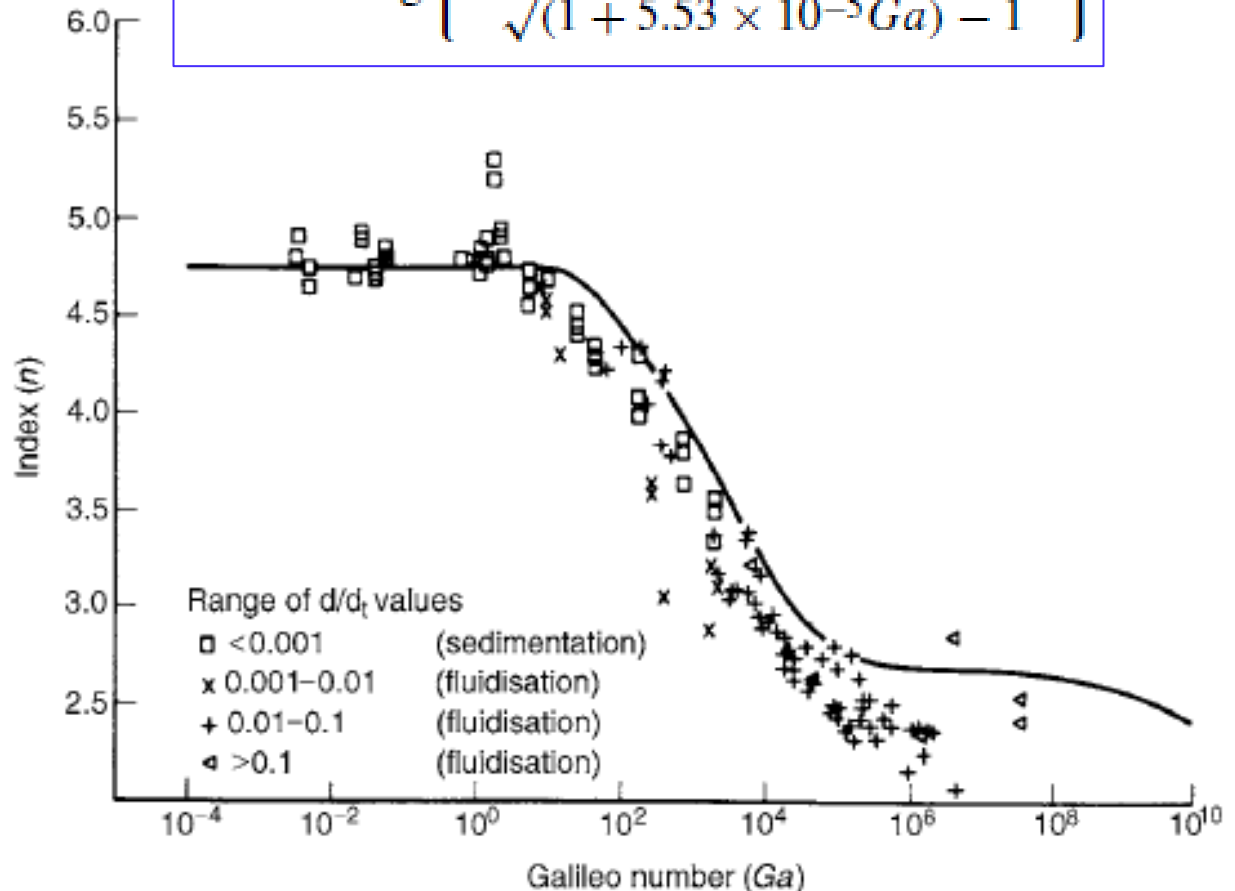
J.M. Coulson and J.F. Richardson, pp 272

# Particulate fluidization

$$\frac{u_c}{u_0} = e^n$$

Small particles compared to vessel diameter (e.g.  $d/d_t$  is small thus  $u_i = u_0$ )

$$n = 2.51 \log \left\{ \frac{(1.83 Ga^{0.018} - 1.2 Ga^{-0.016})^{13.3}}{\sqrt{(1 + 5.53 \times 10^{-5} Ga) - 1}} \right\}$$



# Particulate fluidization

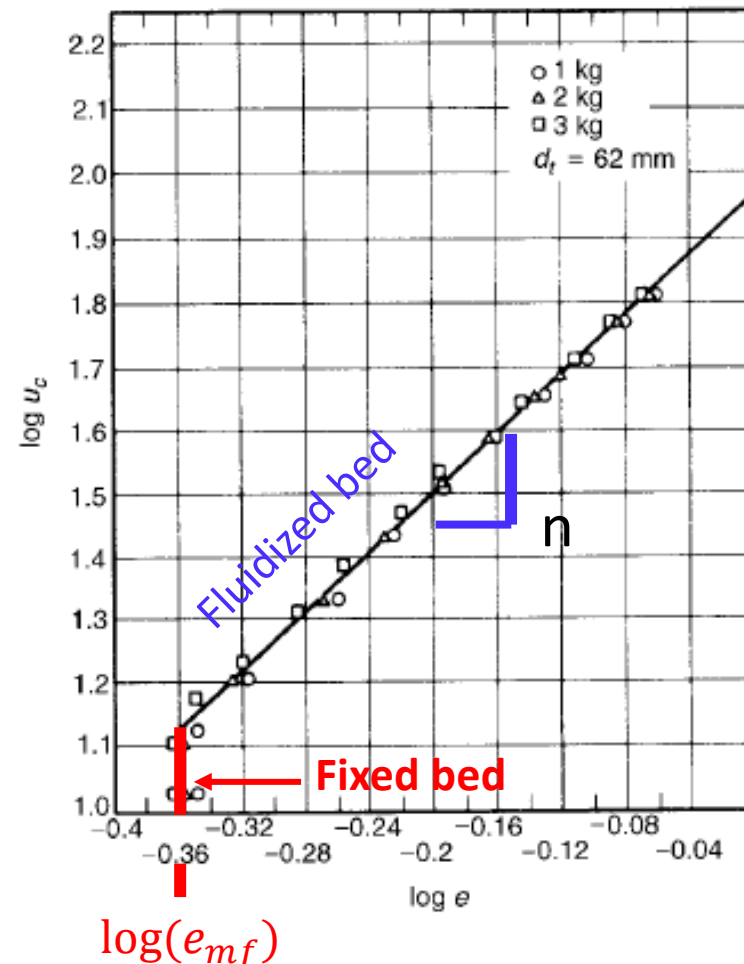
$$\frac{u_c}{u_0} = e^n$$

Small particles compared to vessel diameter (e.g.  $d/d_t$  is small thus  $u_i = u_0$ )

$$\log(u_c) = \log(u_0) + n \log(e)$$

$$n = 2.51 \log \left\{ \frac{(1.83Ga^{0.018} - 1.2Ga^{-0.016})^{13.3}}{\sqrt{(1 + 5.53 \times 10^{-5}Ga) - 1}} \right\}$$

Example: 6,4 mm steel spheres in water



# Exercises

## VII – FLUIDIZAÇÃO

1- Passa óleo de densidade 0.9 e viscosidade 3 cP, ascendendo verticalmente através de um leito de catalisador constituído por partículas aproximadamente esféricas com 0.1 mm de diâmetro e densidade 2.6. Aproximadamente que caudal em massa por unidade de área é que se verificará

- (a) fluidização?
- (b) Transporte de partículas?

2- Calcular a velocidade mínima a que fluidizarão partículas esféricas (densidade 1.6) de 1.5 mm de diâmetro numa coluna de 1 cm de diâmetro. Discutir as incertezas deste cálculo (viscosidade da água= 1cP, constante de Kozeny=5)

3 – A relação entre a porosidade do leito  $e$  e a velocidade do fluido  $u_c$  para fluidização homogénea de partículas uniformes e pequenas em comparação com o diâmetro da coluna é dado por:

$$\frac{u_c}{u_0} = e^n$$

em que  $u_0$  é a velocidade terminal de queda.

- (a) Discutir a variação do expoente  $n$  com as condições de fluxo, indicando por que é independente do número de Reynolds  $Re$  relativo à partícula para valores muito elevados e muito baixos de  $Re$ .
- (b) Quando é que se observam apreciáveis desvios de esta relação no caso de sistemas fluidizados por líquidos?
- (c) Para partículas de *ballotini* de vidro com velocidades limites de queda de 10 e 20 mm/s o expoente  $n=2.39$ . Fluidizando uma mistura de volumes iguais das duas partículas, qual será a relação entre a porosidade e velocidade do fluido se se supuser que se obtém segregação completa?

4- Um fluido (massa específica=1000 kg/m<sup>3</sup>, viscosidade=1 cP) ascende verticalmente através de uma coluna de 0.2 m de diâmetro e 1 m de altura com um leito constituído por partículas esféricas com diâmetro de 0.15 mm e densidade 2.2. Se o caudal em massa por unidade de área de secção recta da coluna for 5 kg s<sup>-1</sup>m<sup>-2</sup>

- a) o leito encontra-se fluidizado?
- b) Calcule a porosidade do leito.

7 - Obtenha a relação para a razão da velocidade terminal de queda de uma partícula pela velocidade mínima de fluidização para um leito de partículas uniformes em regime laminar. Assuma que a lei de Stokes e a equação de Kozeny se aplicam. Qual é o valor dessa razão se a porosidade do leito à velocidade mínima de fluidização for 0.48?

8 - Um leito de partículas esféricas uniformes, de diâmetro 0.15 mm e densidade 2000kg/m<sup>3</sup>, é fluidizado por um líquido de viscosidade 0.001 Ns/m<sup>2</sup> e densidade 1.1. a) Usando a equação de Ergun que conhece para calcular a queda de pressão através de um leito de altura  $L$  e porosidade  $e$ , calcule a velocidade mínima para fluidização ( $e = 0.48$ ). b) Calcule ainda a velocidade terminal de queda assumindo que o regime de escoamento é laminar e a lei de Stokes se aplica. Se o regime de escoamento não for laminar, proponha um método para determinar a velocidade terminal de queda e calcule-a.