

# AM 1 - Resolução ficha 3

03/29

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## Exercício 1

1 - a)

$$1 - b) \quad \left| \frac{\cos n}{\sqrt{n+1}} \right| < \frac{1}{10^3}$$

$$\implies \left| \frac{\cos n}{\sqrt{n+1}} \right| < \left| \frac{1}{\sqrt{n+1}} \right| < \frac{1}{10^3}$$

$$1 - c) \quad \left| e^{-n} + \frac{1}{n} \right| < 10^{-2}$$

$$\left| e^{-n} + \frac{1}{n} \right| = \frac{1}{e^n} + \frac{1}{n} < \frac{1}{n} + \frac{1}{n} < 10^{-2}$$

## Exercício 2

$$2 - a) \quad |u_n| < \sqrt{\epsilon}; \quad u_n = 1/n^2$$

$$|u_n| < \sqrt{\epsilon} \implies \dots$$

**Nota:**  $\lfloor X \rfloor = \text{Max}(k) \in \mathbb{Z} : k \leq x$

$$2 - b) \quad |v_n - 1| < \epsilon; \quad v_n = \frac{n}{n+1}$$

$$|v_n - 1| < \epsilon \dots$$

$$2 - c) \quad |w_n - 2| < \epsilon/3; \quad w_n = \frac{2n}{n-1/2}$$

$$|w_n - 2| < \epsilon/3 \implies \left| \frac{2n}{n-1/2} - 2 \right| < \epsilon/3 \dots$$

**Nota:**

$$\exists L \in \mathbb{R} : \forall \epsilon > 0 \quad \exists p \in \mathbb{N} : n > p \implies |u_n - L| < \epsilon$$

### Exercício 3

**3 - a)**  $\lim_{n \rightarrow \infty} \frac{n^2 + n\sqrt{n}}{3n^2 + 3}$

$$\frac{n^2 + n\sqrt{n}}{3n^2 + 3} = \frac{1 + 1/\sqrt{n}}{3 + 3/n^2} = f_{a(n)} \implies \lim_{n \rightarrow \infty} f_{a(n)} = 1/3$$

**3 - b)**  $\lim_{n \rightarrow \infty} \frac{n^3 + n}{n^2 + \ln n}$

$$\frac{n^3 + n}{n^2 + \ln n} = \frac{1 + 1/n^2}{1/n + \ln n/n^3} = f_{b(n)} \implies \lim_{n \rightarrow \infty} f_{b(n)} = \infty$$

**3 - c)**  $\lim_{n \rightarrow \infty} \frac{n \sqrt[3]{8n+1}}{(n^{2/3}+1)^2}$

$$\frac{n \sqrt[3]{8n+1}}{(n^{2/3}+1)^2} = \frac{\sqrt[3]{8n^4+n^3}}{(n^{2/3}+1)^2} * \frac{n^{4/3}}{n^{4/3}} = \frac{\sqrt[3]{\frac{8n^4}{n^4} + \frac{n^3}{n^4}}}{\left(\frac{n^{2/3}}{n^{2/3}} + \frac{1}{n^{2/3}}\right)^2} = \frac{\sqrt[3]{8 + n^{-1}}}{(1 + n^{-2/3})} = f_{c(n)} \implies$$

$$\implies \lim_{n \rightarrow \infty} f_{c(n)} = \frac{\sqrt[3]{8}}{1} = 2$$

**3 - d)**

$$\lim_{n \rightarrow \infty} \frac{n \cos(n^2)}{n^2 + 1} = \lim_{n \rightarrow \infty} \frac{n^{-1} \cos(n^2)}{1 + 1/n^2} = 0$$

**3 - e)**

$$\lim_{n \rightarrow \infty} \frac{2n e^{1/n}}{\sqrt{n^2 + 5}} = \lim_{n \rightarrow \infty} \frac{2 e^{1/n}}{\sqrt{1 + 5/n^2}} = 2$$

**3 - f)**

$$\lim_{n \rightarrow \infty} \frac{5^n + 4^n}{2 \cdot 5^{n+1} + 1} = \lim_{n \rightarrow \infty} \frac{1/5 + (4/5)^n \cdot 1/5}{2 + 1/5^{n+1}} = \frac{1}{10}$$

**3 - g)**

$$\lim_{n \rightarrow \infty} \frac{3^n + e^n}{3^n + \pi^n} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{\pi^n} + \left(\frac{e}{\pi}\right)^n}{\frac{3^n}{\pi^n} + 1} = 0$$

**3 - h)**

$$\lim_{n \rightarrow \infty} \frac{n^2 \cdot 9^n + n^3}{(n+2)^2 \cdot 3^{2n+1}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot 9^n + n^3}{(n+2)^2 \cdot 9^n \cdot 3} = \lim_{n \rightarrow \infty} \frac{1 + n/9^n}{(1 + 2/n)^2 \cdot 3} = 1/3$$

## Exercício 4

## Exercício 5

**5 - a)**

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt{n+5} - \sqrt{2n-1} &= \lim_{n \rightarrow \infty} \frac{n+5 - 2n+1}{\sqrt{n+5} + \sqrt{2n-1}} = \\ &= \lim_{n \rightarrow \infty} \frac{-1 + 6/n}{\sqrt{1/n + 5/n^2} + \sqrt{2/n - 1/n^2}} = -\infty \end{aligned}$$

**5 - b)**

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt{n^2 + 3n} - \sqrt{n^2 - n + 1} &= \lim_{n \rightarrow \infty} \frac{n^2 + 3n - n^2 + n - 1}{\sqrt{n^2 + 3n} + \sqrt{n^2 - n + 1}} = \\ &= \lim_{n \rightarrow \infty} \frac{4n - 1}{\sqrt{n^2 + 3n} + \sqrt{n^2 - n + 1}} = \lim_{n \rightarrow \infty} \frac{4 - 1/n}{\sqrt{1 + 3/n} + \sqrt{1 - 1/n + 1/n^2}} = 2 \end{aligned}$$

5 - c) incompleto

$$\lim_{n \rightarrow \infty} (1.1)^{n+1} - (1.05)^{2n-1} = \lim_{n \rightarrow \infty} \left( \frac{(1.1)^n (1.1)}{1.05^2} - (1.05)^{-1} \right) (1.05)^{2n}$$

5 - d)

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt{\ln(e^4 n + 1)} - \sqrt{\ln(n + 2)} &= \lim_{n \rightarrow \infty} \frac{\ln(e^4 n + 1) - \ln(n + 1)}{\sqrt{\ln(e^4 n + 1)} + \sqrt{\ln(n + 2)}} = \\ &= \lim_{n \rightarrow \infty} \frac{\ln\left(\frac{e^4 n + 1}{n + 1}\right)}{\sqrt{\ln(e^4 n + 1)} + \sqrt{\ln(n + 2)}} = \lim_{n \rightarrow \infty} \frac{\ln\left(\frac{e^4 + 1/n}{1 + 1/n}\right)}{\sqrt{\ln(e^4 n + 1)} + \sqrt{\ln(n + 2)}} = 0 \end{aligned}$$

5 - e)

$$\lim_{n \rightarrow \infty} n \ln(n + 1) - n \ln(n) = \lim_{n \rightarrow \infty} n \ln\left(\frac{n + 1}{n}\right) = \lim_{n \rightarrow \infty} \ln(1 + 1/n)^n = \ln(e^1) = 1$$

5 - f)

5 - g)

5 - h)

5 - i)

5 - j)

$$\lim_{n \rightarrow \infty} (1 + 1/n)^{n^2} (1 - 1/n)^{n^2} = \lim_{n \rightarrow \infty} (1 - 1/n^2)^{n^2} = e^{-1}$$

## Exercício 6 Extras

$$6 - a) \quad \lim_{n \rightarrow \infty} \frac{n 3^n + e^n}{(n + \sqrt{n}) 3^{n+1} + n^{100}}$$

$$\lim_{n \rightarrow \infty} \frac{n 3^n + e^n}{(n + \sqrt{n}) 3^{n+1} + n^{100}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{e^n}{n 3^n}}{3 + \frac{3}{\sqrt{n}} + \frac{n^{99}}{3^n}} = \lim_{n \rightarrow \infty} \frac{1 + n^{-1} \left(\frac{e}{3}\right)^n}{3 + \frac{3}{\sqrt{n}} + \frac{n^{99}}{3^n}} = 1/3$$

**Nota:** funções exponenciais crescem sempre mais rápido que qualquer outra

$$6 - b) \quad \lim_{n \rightarrow \infty} \frac{2 n e^{1/n}}{(-1)^n + \sqrt{n^2 + 5}}$$

$$\lim_{n \rightarrow \infty} \frac{2 n e^{1/n}}{(-1)^n + \sqrt{n^2 + 5}} = \lim_{n \rightarrow \infty} \frac{2 e^{1/n}}{(-1)^n/n + \sqrt{1 + 5/n^2}} = 2$$

$$6 - c) \quad \lim_{n \rightarrow \infty} \frac{2^n + 3^{n+1} + (-7)^n}{3^{n-2} + 5^{n+1}}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{2^n + 3^{n+1} + (-7)^n}{3^{n-2} + 5^{n+1}} &= \lim_{n \rightarrow \infty} \frac{2^n + 3 \cdot 3^n + (-7)^n}{3^{-2} 3^n + 5 \cdot 5^n} \\ &= \lim_{n \rightarrow \infty} \frac{(2/7)^n + 3 (3/7)^n + (-7/7)^n}{(3/7)^n/3^2 + 5(5/7)^n} = \{\infty \forall n \text{ par}; -\infty \forall n \text{ ímpar}\} \end{aligned}$$

$$6 - d) \quad \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0$$

$$6 - e) \quad \lim_{n \rightarrow \infty} \sqrt{\log(n^2 + 1)} - \sqrt{\log(n^2)}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt{\log(n^2 + 1)} - \sqrt{\log(n^2)} &= \lim_{n \rightarrow \infty} \frac{\log(n^2 + 1) - \log(n^2)}{\sqrt{\log(n^2 + 1)} + \sqrt{\log(n^2)}} = \\ &= \lim_{n \rightarrow \infty} \frac{\log(1 + 1/n^2)}{\sqrt{\log(n^2 + 1)} + \sqrt{\log(n^2)}} = 0 \end{aligned}$$

**6 - f)**  $\lim_{n \rightarrow \infty} \left( \frac{n+3}{n+1} \right)^{2n}$

$$\lim_{n \rightarrow \infty} \left( \frac{n+3}{n+1} \right)^{2n} = \lim_{n \rightarrow \infty} \left( \frac{(1+3/n)^n}{(1+1/n)^n} \right)^2 = \lim_{n \rightarrow \infty} \left( \frac{e^3}{e^1} \right)^2 = e^4$$

**6 - g)**  $\lim_{n \rightarrow \infty} \left( \frac{n+5}{2n+1} \right)^n$  **refazer**

$$\lim_{n \rightarrow \infty} \left( \frac{n+5}{2n+1} \right)^n = \lim_{n \rightarrow \infty} \frac{(1+5/n)^n}{(1+1/(2n))^n} = \lim_{n \rightarrow \infty} \frac{(1+5/n)^n}{((1+1/(2n))^{2n})^{1/2}} = \frac{e^5}{\sqrt{e^1}}$$

**6 - h)**  $\lim_{n \rightarrow \infty} \left( 1 - 3/n^2 \right)^n$

$$\lim_{n \rightarrow \infty} \left( 1 - 3/n^2 \right)^n = \lim_{n \rightarrow \infty} \left( \left( 1 - 3/n^2 \right)^{n^2} \right)^{1/n} = (e^{-3})^{1/\infty} = 0$$

**6 - i)**  $\lim_{n \rightarrow \infty} \left( 1 + 1/\log(n) \right)^n$

$$\lim_{n \rightarrow \infty} \left( 1 + 1/\log(n) \right)^n = \lim_{n \rightarrow \infty} \left( \left( 1 + 1/\log(n) \right)^{\log(n)} \right)^{n/\log(n)} = (e^1)^\infty = \infty$$

**6 - j)**  $\lim_{n \rightarrow \infty} \left( \frac{n^6-2}{n^6} \right)^{n^3+3}$

$$\lim_{n \rightarrow \infty} \left( \frac{n^6-2}{n^6} \right)^{n^3+3} = \lim_{n \rightarrow \infty} \left( \left( 1 - 2/n^6 \right)^{n^6} \right)^{\frac{n^3+3}{n^6}} = (e^{-2})^0 = 1$$