SECTION 6

FLUIDISATION

Problem 6.1

Oil of specific gravity 0·9 and viscosity 3 mN s/m^2 passes vertically upwards through a bed of catalyst consisting of approximately spherical particles of diameter 0·1 mm and specific gravity 2·6. At approximately what mass rate of flow per unit area of bed will (a) fluidisation, and (b) transport of particles occur?

Solution

(a) Use may be made of equations 4.9 and 6.1 to find the fluidising velocity, u_f

$$u = \frac{1}{K''}\frac{e^3}{S^2(1-e)^2}\frac{1}{\mu}\frac{(-\Delta P)}{L} \qquad \text{(equation 4.9)}$$

$$-\Delta P = (1-e)(\rho_s-\rho)Lg \qquad \text{(equation 6.1)}$$

S = surface area/volume $= \pi d^2/(\pi d^3/6) = 6/d$ for a sphere.
Substituting $K'' = 5$, $S = 6/d$, and $-\Delta P/L$ from equation 6.1 into equation 4.9 gives:

$$u_f = 0\cdot0055\frac{e^3}{(1-e)}\frac{d^2(\rho_s-\rho)g}{\mu}$$

Hence
$$G'_f = \rho u = \frac{0\cdot0055e^3}{(1-e)}\frac{d^2\rho(\rho_s-\rho)g}{\mu}$$

In this problem
$$\rho_s = 2\cdot6\times1000 = 2600\ \text{kg/m}^3$$
$$\rho = 0\cdot9\times1000 = 900\ \text{kg/m}^3$$
$$\mu = 3\cdot0\times10^{-3}\ \text{N s/m}^2$$
$$d = 0\cdot1\ \text{mm} = 1\times10^{-4}\ \text{m}$$

As no value for the voidage has been given, e will be calculated by considering eight close packed spheres of diameter d in a cube of side $2d$. Then:

$$\text{volume of spheres} = 8(\pi/6)d^3$$

$$\text{volume of enclosure} = (2d)^3 = 8d^3$$

$$\text{voidage } e = [8d^3 - 8(\pi/6)d^3]/8d^3 = 1-(\pi/6) = 0\cdot478, \quad \text{say} \quad e = 0\cdot48$$

Then $G'_f = 0\cdot0055(0\cdot48)^3(10^{-4})^2\times900\times1700\times9\cdot81/(1-0\cdot48)\times3\times10^{-3}$

$$= \underline{\underline{0\cdot059\ \text{kg/m}^2\,\text{s}}}$$

(b) Transport of particles will occur when the fluid velocity is equal to the terminal falling velocity of the particle.

Using Stokes' law:

$$u_0 = d^2 g(\rho_s - \rho)/18\mu$$
$$= (10^{-4})^2 \times 9.81 \times 1700/18 \times 3 \times 10^{-3}$$
$$= 0.0031\ \text{m/s}$$

Check Reynolds number $= 10^{-4} \times 0.0031 \times 900/3 \times 10^{-3} = 0.093$ (Stokes' law applies).

Hence required mass flow $= 0.0031 \times 900$

$$= \underline{\underline{2.78\ \text{kg/m}^2\,\text{s}}}$$

Alternatively, use may be made of Fig. 3.6 and equation 3.28:

$$(R/\rho u^2)Re^2 = 2d^3\rho g(\rho_s - \rho)/3\mu^2 \qquad \text{(equation 3.28)}$$
$$= 2 \times (10^{-4})^3 \times 900 \times 9.81 \times 1700/3(3 \times 10^{-3})^2$$
$$= 1.11$$

From Fig. 3.6, $Re = 0.09$

Hence $u_0 = Re\mu/\rho d = 0.09 \times 3 \times 10^{-3}/900 \times 10^{-4}$

$$= 0.003\ \text{m/s}$$

and $G' = 0.003 \times 900$

$$= \underline{\underline{2.7\ \text{kg/m}^2\,\text{s}}}$$

Problem 6.2

Calculate the minimum velocity at which spherical particles (specific gravity 1.6) of diameter 1.5 mm will be fluidised by water in a tube of diameter 10 mm. Discuss the uncertainties in this calculation.

Viscosity of water = 1 mN s/m²; Kozeny's constant = 5.

Solution

As in Problem 6.1, the equation for the minimum fluidising velocity may be derived as:

$$u_f = 0.0055 \frac{e^3}{(1-e)} \frac{d^2(\rho_s - \rho)g}{\mu} \quad (\times f_w)$$

As a wall effect applies in this problem, use is made of equation 4.21 to find the correction factor f_w.

$$f_w = \left(1 + 0.5\frac{Sc}{S}\right)^2 \quad \text{eq. 1.19}$$

$$Sc = \text{surface of the container/volume of bed}$$
$$= (\pi \times 0.01 \times 1)/(\pi/4)(0.01)^2 \times 1 = 400\ \text{m}^2/\text{m}^3$$

$$S = 6/d \text{ for a spherical particle}$$
$$= 6/1.5 \times 10^{-3} = 4000\ \text{m}^2/\text{m}^3$$

Hence $$f_w = 1.05$$

The uncertainty in this problem lies in the chosen value of the voidage e. If e is taken as 0.48 as in Problem 6.1:

$$u_f = 0.0055 \times \frac{(0.48)^3}{0.52} \times (1.5 \times 10^{-3})^2 \times \frac{(1600 - 1000)}{1 \times 10^{-3}} \times 9.81$$
$$= 0.0155\ \text{m/s}$$

Allowing for wall effect, $u_f = 1.05 \times 0.0155 = \underline{\underline{0.0163\ \text{m/s}}}$

If Ergun's equation is used to calculate the minimum fluidising velocity as illustrated fully in Problem 6.7, the value of u_f is found to be $\underline{\underline{0.013\ \text{m/s}}}$.

Problem 6.3

In a fluidised bed, iso-octane vapour is adsorbed from an air stream on to the surface of alumina microspheres. The mol fraction of iso-octane in the inlet gas is 1.442×10^{-2} and the mol fraction in the outlet gas is found to vary with time in the following manner:

Time from start (s)	Mol fraction in outlet gas ($\times 10^2$)
250	0.223
500	0.601
750	0.857
1000	1.062
1250	1.207
1500	1.287
1750	1.338
2000	1.373

Show that the results can be interpreted on the assumptions that the solids are completely mixed, that the gas leaves in equilibrium with the solids, and that the adsorption isotherm is linear over the range considered. If the flow rate of gas is 0.679×10^{-6} kmol/s and the mass of solids in the bed is 4.66 g, calculate the slope of the adsorption isotherm. What evidence do the results provide concerning the flow pattern of the gas?

Solution

Work on mass transfer between fluid and particles is discussed in Chapter <u>6</u> of Vol. <u>2</u> where it is shown that by a mass balance over a bed of particles at any time t after the start of the experiment, the following equation is obtained:

fracção Molar do Vapor à entrada

$$G_m(y_0 - y) = \frac{d(WF)}{dt} \qquad \text{(equation 6.32)}$$

6.14

nº moles vapor adsorvidos sobre a unid. Massa do Sólido

Caudal Molar do gás

Massa do Sólidos no Leito

where G_m is the molar flow rate of gas, W is the mass of solids in the bed, F is the number of mols of vapour adsorbed on unit mass of solid, and y_0, y is the mol fraction of vapour in the inlet and outlet stream respectively.

If the adsorption isotherm is linear, and if equilibrium is reached between the outlet gas and the solids and if none of the gas bypasses the bed, then F is given by:

$$F = f + by$$

where f and b are the intercept and slope of the isotherm respectively.

Combining these equations and integrating gives:

$$\ln(1 - y/y_0) = -(G_m/Wb)\,t \qquad \text{(equation 6.36)}$$

6.18

If the assumptions outlined above are valid, a plot of $\ln(1 - y/y_0)$ against t should yield a straight line of slope $-G_m/Wb$. As $y_0 = 0{\cdot}014\,42$, the following table may be produced.

Time (s)	y	y/y_0	$1 - y/y_0$	$\ln(1 - y/y_0)$
250	0·002 23	0·155	0·845	−0·168
500	0·006 01	0·417	0·583	−0·539
750	0·008 57	0·594	0·406	−0·902
1000	0·0106	0·736	0·263	−1·33
1250	0·0121	0·837	0·163	−1·81
1500	0·0129	0·893	0·107	−2·23
1750	0·0134	0·928	0·072	−2·63
2000	0·0137	0·952	0·048	−3·04

These data are plotted in Fig. 6a where a straight line is obtained. The slope is measured as $-0{\cdot}001\,67$/s.

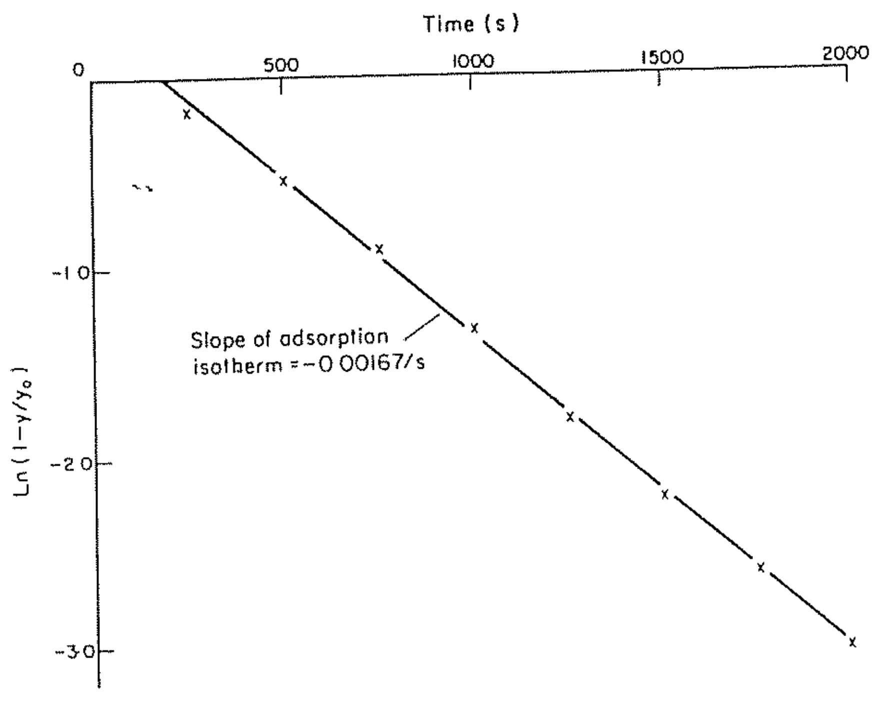

FIG. 6a

If $G_m = 0.679 \times 10^{-6}$ kmol/s and $W = 4.66$ g,

$$-0.00167 = -0.679 \times 10^{-6}/4.66b$$

from which

$$b = 87.3 \times 10^{-6}\ \text{kmol/g} \equiv \underline{0.0873\ \text{kmol/kg}}$$

Problem 6.4

Discuss the reasons for the good heat transfer properties of fluidised beds. Cold particles of glass ballotini are fluidised with heated air in a bed in which a constant flow of particles is maintained in a horizontal direction. When steady conditions have been reached, the temperatures recorded by a bare thermocouple immersed in the bed are as follows:

Distance above bed support (mm)	Temperature (K)
0	339.5
0.64	337.7
1.27	335.0
1.91	333.6
2.54	333.3
3.81	333.2

Calculate the coefficient for heat transfer between the gas and the particles, and the corresponding values of the particle Reynolds and Nusselt numbers. Comment on the results and on any assumptions made.

Gas flow rate = $0.2\ \text{kg/m}^2\,\text{s}$.

Specific heat of air = 0.88 kJ/kg K.

Viscosity of air = $0.015\ \text{mN s/m}^2$.

Particle diameter = 0.25 mm.

Thermal conductivity of air = 0.03 W/m K.

Solution

Aspects of heat transfer are discussed in Chapter 6 of Vol. 2. For the system described in this problem, equation 6.43 relates the rate of heat transfer between particles and fluid by:

$$dQ = ha'\Delta t\, dz \qquad \text{(equation 6.43)}$$

6.25

and on integration

$$Q = ha' \int_0^z \Delta t\, dz \qquad \text{(equation 6.44)}$$

6.26

where Q is the heat transferred, h is the heat transfer coefficient, a' is the area for transfer/unit height of bed, and Δt is the temperature difference at height z.

From the data given, Δt may be plotted against z as shown in Fig. 6b where the area under the curve gives the value of the integral as 8.82 mm K.

Heat transferred $= 0{\cdot}2 \times 0{\cdot}88(339{\cdot}5 - 332{\cdot}2) = 1{\cdot}11$ kW/m^2 of bed cross-section
$= 1100$ W/m^2

If bed voidage $= 0{\cdot}57$, consider a bed 1 m^2 × 1 m high, i.e. 1 m^3.
Volume of particles $= (1 - 0{\cdot}57) \times 1 = 0{\cdot}43$ m^3.
Volume of 1 particle $= (\pi/6)(0{\cdot}25 \times 10^{-3})^3 = 8{\cdot}18 \times 10^{-12}$ m^3.
Therefore number of particles $= 0{\cdot}43/8{\cdot}18 \times 10^{-12} = 5{\cdot}26 \times 10^{10}$/m^3.
Area of particles $= 5{\cdot}26 \times 10^{10} \times (\pi/4)(0{\cdot}25 \times 10^{-3})^2 = 1{\cdot}032 \times 10^4$ m^2/m^3 $= a'$.
Substituting in equation 6.44 gives:

$$1100 = h \times 1{\cdot}03 \times 10^4 \times 8{\cdot}82 \times 10^{-3}$$

$$h = \underline{\underline{12{\cdot}2\ \mathrm{W/m^2\,K}}}$$

From equation 6.46 (6.28), $Nu = 0{\cdot}11 Re^{1{\cdot}28}$

$$Re = G'd/\mu = 0{\cdot}2 \times 0{\cdot}25 \times 10^{-3}/0{\cdot}015 \times 10^{-3}$$

$$= \underline{\underline{3{\cdot}33}}$$

$$Nu = 0{\cdot}11 \times (3{\cdot}33)^{1{\cdot}28} = \underline{\underline{0{\cdot}513}}$$

$$h = 0{\cdot}513 \times 0{\cdot}03/0{\cdot}25 \times 10^{-3}$$

$$= \underline{\underline{61{\cdot}6\ \mathrm{W/m^2\,K}}}$$

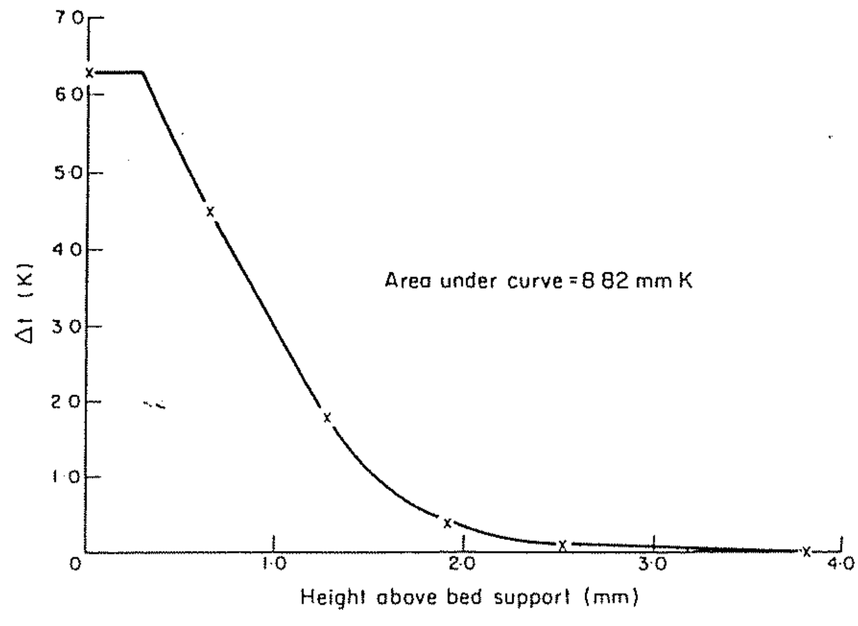

FIG. 6b

Problem 6.5

The relation between bed voidage e and fluid velocity u_c for particulate fluidisation of uniform particles, small compared with the diameter of the containing vessel, is given by:

$$\frac{u_c}{u_0} = e^n$$

where u_0 is the free-falling velocity

Discuss the variation of the index n with flow conditions, indicating why it is independent of the Reynolds number Re with respect to the particle at very low and very high values of Re. When are appreciable deviations from this relation observed with liquid fluidised systems?

For particles of glass ballotini with free-falling velocities of 10 and 20 mm/s the index n has a value of 2·4. If a mixture of equal volumes of the two particles is fluidised, what will be the relation between the voidage and fluid velocity if it is assumed that complete segregation is obtained?

Solution

The variation of the index n with flow conditions is fully discussed in Chapters 5 and 6 of Vol. 2 under sedimentation and thickening and fluidisation. The ratio u_c/u_0 is in general dependent on the Reynolds number, voidage, and the ratio of particle diameter to that of the containing vessel. At low velocities, i.e. when $Re < 0{\cdot}2$, the drag force is attributable entirely to skin friction, and at high velocities when $Re > 500$ skin friction becomes negligible and in these regions the ratio u_c/u_0 is independent of Re. For intermediate regions, equations 5.35 to 5.37 apply.

Consider unit volume of each particle, then:

Voidage of large particles $= e_1$, volume of liquid $= e_1/(1 - e_1)$.

Voidage of small particles $= e_2$, volume of liquid $= e_2/(1 - e_2)$.

Total volume of solids $= 2$.

Total volume of liquid $= e_1/(1 - e_1) + e_2/(1 - e_2)$

Total volume of system $= 2 + e_1/(1 - e_1) + e_2/(1 - e_2)$.

$$\therefore \quad \text{voidage} = \frac{e_1/(1 - e_1) + e_2/(1 - e_2)}{2 + e_1/(1 - e_1) + e_2/(1 - e_2)}$$

$$= \frac{e_1(1 - e_2) + e_2(1 - e_1)}{2(1 - e_1)(1 - e_2) + e_1(1 - e_2) + e_2(1 - e_1)}$$

i.e.

$$e = \frac{e_1 + e_2 - 2e_1 e_2}{2 - e_1 - e_2}$$

Now

$$e_1 = \left(\frac{u}{u_{01}}\right)^{1/2{\cdot}4} \quad \text{and} \quad e_2 = \left(\frac{u}{u_{01}/2}\right)^{1/2{\cdot}4}$$

(since the free-falling velocities are in the ratio 1:2)

$$e_2 = e_1\, 2^{1/2{\cdot}4}$$

$$e = \frac{e_1 + e_1 \times 2^{1/2{\cdot}4} - 2^{3{\cdot}4/2{\cdot}4} \times e_1^2}{2 - e_1 - 2^{1/2{\cdot}4} \times e_1}$$

$$e = \frac{(u/20)^{1/2{\cdot}4}(1 + 2^{1/2{\cdot}4}) - 2^{3{\cdot}4/2{\cdot}4}(u/20)^{1/1{\cdot}2}}{2 - (1 + 2^{1/2{\cdot}4})(u/20)^{1/2{\cdot}4}}$$

$$e = \frac{3u^{0.42} - u^{0.83}}{9 - 3u^{0.42}} \quad \text{(with } u \text{ in mm/s)}$$

$$9e = 3eu^{0.42} = 3u^{0.42} - u^{0.84}$$

or
$$u^{0.84} - 3(1+e)u^{0.42} + 9e = 0$$

and
$$\underline{\underline{u^{0.42} = 1.5(1+e) + [2.24(1+e)^2 - 9e]}}$$

This relationship is plotted in Fig. 6c.

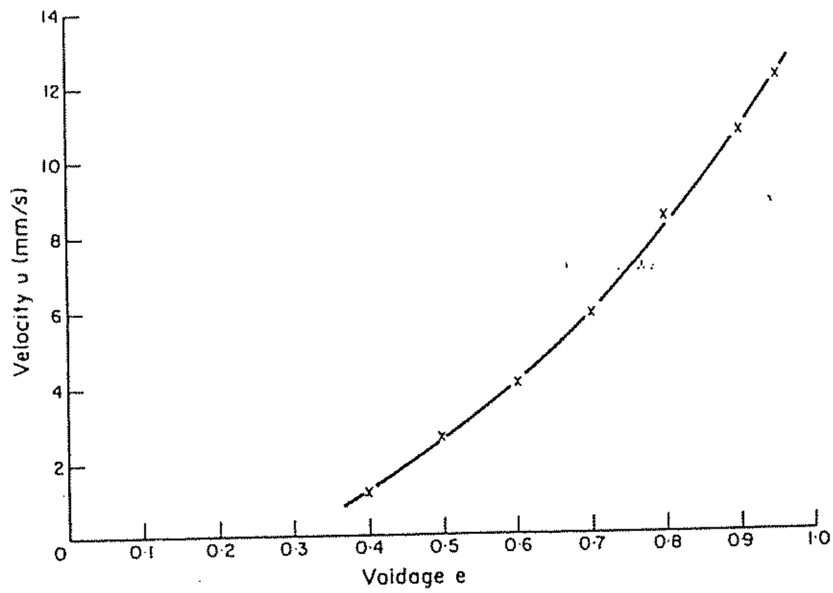

FIG. 6c

Problem 6.6

Obtain a relationship for the ratio of the terminal falling velocity of a particle to the minimum fluidising velocity for a bed of similar particles. Assume that Stokes' law and the Carman–Kozeny equation are applicable. What is the value of the ratio if the bed voidage at the minimum fluidising velocity is 0·4?

Discuss the validity of using the Carman–Kozeny equation for calculation of pressure drop through a fluidised bed.

Solution

Stokes' law (equation 3.17) and the Carman–Kozeny equation (equation 4.9) state respectively:

$$u_0 = d^2 g(\rho_s - \rho)/18\mu$$

and
$$u = \frac{1}{K''}\,\frac{e^3}{s^2(1-e)^2}\,\frac{1}{\mu}\,\frac{(-\Delta P)}{l}$$