

## Ficha 4 – Soluções

1.
  - a.  $\vec{r}(t) = (2t + 1)\vec{i} + t^2\vec{j}, \vec{r}'(t) = 2\vec{i} + 2t\vec{j}, \vec{r}''(t) = 2\vec{j}$ .
  - b.  $\vec{r}(t) = \sin(t)\vec{i} + (\cos(t) - 3)\vec{j}, \vec{r}'(t) = \cos(t)\vec{i} - \sin(t)\vec{j}, \vec{r}''(t) = -\sin(t)\vec{i} - \cos(t)\vec{j}, t \in [0, 2\pi]$ .
2.
  - a.  $\vec{r}(t) = 2\sin(t)\vec{i} + 4\cos(t)\vec{j} + \vec{k}, \vec{r}'(t) = 2\cos(t)\vec{i} - 4\sin(t)\vec{j}, \vec{r}''(t) = -2\sin(t)\vec{i} - 4\cos(t)\vec{j}, t \in [0, 2\pi]$ .
  - b.  $\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + \frac{\pi t}{2}\vec{k}, \vec{r}'(t) = -\sin(t)\vec{i} + \cos(t)\vec{j} + \frac{\pi}{2}\vec{k}, \vec{r}''(t) = -\cos(t)\vec{i} - \sin(t)\vec{j}, t \in [-2\pi, 2\pi]$ .
3.
  - a.  $\vec{r}(t) = \sin(3t)\vec{i} + \cos(3t)\vec{j} + 2t^{\frac{3}{2}}\vec{k}, \vec{r}'(t) = 3\cos(3t)\vec{i} - 3\sin(3t)\vec{j} + 3\sqrt{t}\vec{k}, \vec{r}''(t) = -9\sin(3t)\vec{i} - 9\cos(3t)\vec{j} + \frac{3}{2\sqrt{t}}\vec{k}$ .  
 Reta tangente em  $t = 1$ :  $(x, y, z) = (3\cos(3)\lambda + \sin(3), \cos(3) - 3\sin(3)\lambda, 3\lambda + 2), \lambda \in \mathbb{R}$ .
  - b.  $\vec{r}(t) = t\sin(t)\vec{i} + t\cos(t)\vec{j} + \sqrt{3}t\vec{k}, \vec{r}'(t) = (\sin(t) + t\cos(t))\vec{i} + (\cos(t) - t\sin(t))\vec{j} + \sqrt{3}\vec{k}, \vec{r}''(t) = (2\cos(t) - t\sin(t))\vec{i} - (2\sin(t) + t\cos(t))\vec{j}$ .  
 Reta tangente em  $t = 0$ :  $(x, y, z) = (0, \lambda, \sqrt{3}\lambda), \lambda \in \mathbb{R}$ .
4. Reta tangente em  $\theta_0 = \frac{\pi}{4}$ :  $(x, y, z) = \left((1 - \frac{1}{\sqrt{2}})\lambda + \frac{\pi}{4} - \frac{1}{\sqrt{2}}, \frac{\lambda}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1\right), \lambda \in \mathbb{R}$ .
5.
  - a.  $(x, y, z) = \left(2\cos(t), \frac{\sin(t)}{\sqrt{2}}, 2\right), t \in [0, 2\pi]$ .  
 Reta tangente em  $P_0$ :  $(x, y, z) = \left(\sqrt{2} - \sqrt{2}\lambda, \frac{\lambda}{2} + \frac{1}{2}, 2\right), \lambda \in \mathbb{R}$ .
  - b.  $(x, y, z) = (2(1 + \cos(t)), 4\sin(t), 8(1 + \cos(t))), t \in [0, 2\pi]$ .  
 Reta tangente em  $P_0$ :  $(x, y, z) = (4, 4\lambda, 16), \lambda \in \mathbb{R}$ .
6.
  - a. Reta tangente em  $P_0$ :  $(x, y) = \left(\lambda + \frac{2}{3}, 3\lambda\right), \lambda \in \mathbb{R}$ .
  - b. Reta tangente em  $P_0$ :  $(x, y, z) = (1, 1, \lambda), \lambda \in \mathbb{R}$ .
7.
  - a.  $\frac{16}{3}(-1 + 2\sqrt{2})$ .
  - b.  $(x, y, z) = \left(-2\lambda, \frac{\sqrt{\pi}\lambda}{2} + \frac{\pi^{\frac{3}{2}}}{12}, 1\right), \lambda \in \mathbb{R}$ .
8.
  - a.  $\sqrt{5}(e^{\frac{\pi}{4}} - 1)$ .
  - b.  $(x, y) = \frac{\sqrt{5}+s}{\sqrt{5}}\left(\cos\left(2\log\left(\frac{\sqrt{5}+s}{\sqrt{5}}\right)\right), \sin\left(2\log\left(\frac{\sqrt{5}+s}{\sqrt{5}}\right)\right)\right), s \in \left[0, \sqrt{5}\left(e^{\frac{\pi}{4}} - 1\right)\right]$ .
9.
  - a.  $]1, 2[ \cup ]2, +\infty[$ .
  - b.  $\vec{\sigma}'(t) = -\frac{1}{t^2}\vec{i} - \frac{1}{2(t-1)^{\frac{3}{2}}}\vec{j} - \frac{1}{(t-2)^2}\vec{k}, \vec{\sigma}''(t) = \frac{2}{t^3}\vec{i} + \frac{3}{4(t-1)^{\frac{5}{2}}}\vec{j} + \frac{2}{(t-2)^3}\vec{k}$ .
10.
  - a.  $\cos(2t)\vec{j} - 12t^5\vec{k}$ .
  - b.  $2te^{t^2}\vec{i} + 2t\cos(t^2)\vec{j} + 6t^5\vec{k}$ .

12.  $(3, -1, 1)$ .

13.

a.  $\frac{1}{3}\vec{i} - \frac{2\sqrt{2}}{3}\sin(\sqrt{2}t)\vec{j} + \frac{2\sqrt{2}}{3}\cos(\sqrt{2}t)\vec{k}, t \in [0, 2\pi]$ .

b.  $3\pi$ .

c.  $\frac{3t}{2}$ .

d.  $(x, y, z) = \left(\frac{s}{3}, \cos\left(\frac{2\sqrt{2}}{3}s\right), \sin\left(\frac{2\sqrt{2}}{3}s\right)\right), s \in [0, 3\pi]$ .

e. Posição inicial:  $(0, 1, 0)$ .

Posição final:  $(\pi, \cos(2\sqrt{2}\pi), \sin(2\sqrt{2}\pi))$ .

f.  $\frac{3\pi}{4}$ .

g.  $\left(\frac{\pi}{6\sqrt{2}}, \frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ .