CNA – Exam 2024.3 Resolution

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20 de janeiro de 2025

Conteúdo

Questão 1

Encotrar iesia iterada pelo met da bisexão

Resposta a)

Questão 2

Sucessõa jacobi

$$\boldsymbol{AX = B}$$
$$\boldsymbol{X_1^{(0)} = X_2^{(0)}}$$
$$\boldsymbol{\|G_1\|_\infty = 4/5}$$
$$\boldsymbol{\|G_1\|_1 = 7/6}$$
$$\boldsymbol{\|G_2\|_\infty = 6/5}$$
$$\boldsymbol{\|G_2\|_1 = 2}$$

Resposta a)

Convergencia

$$\|G_1\|_\infty = 4/5 < 1 \text{Converge}$$
$$\|G_2\|_\infty = 6/5 > 1 \text{não Converge}$$

Questão 3

Considere a tab

x_i	-1	0	2	5
$f(x_i)$	-6	2	5	8

Resposta d)

Encontrando $p_1(-1)$

Resposta eq:q3 L

Lagrange's polynom

$$p_1(x) = y_1\, l_1(x) + y_3\, l_3(x) = \qquad \text{using (1)}$$
$$= \dots$$

Finding singular l_i

$$\begin{cases} l_1 =; \\ l_2 = \end{cases} \tag{1}$$

Newton's polynom q_2

$$q_{2\,(x)} = f(x_0) + (x - x_0)\, f[x_0, x_1] + (x - x_0)(x - x_1)\, f[x_0, x_1, x_3] =$$
$$= -6 + (x - (-1))8 + (x - (-1))(x - 2)\, \frac{34}{30} =$$
$$= -48 + (x + 1) + (x + 1)(x - 2)\, \frac{34}{30}$$

Solving $f[\cdot, \cdot]$

$$f[x_0, x_1] = \frac{2 - (-6)}{0 - (-1)} = 8;$$
$$f[x_0, x_1, x_3] = \frac{6/5 - 8}{5 - (-1)} = \frac{34}{30};$$
$$f[x_1, x_3] = \frac{8 - 2}{5 - 0} = 6/5$$

Questão 4

$$\boldsymbol{I = \int_0^1 f(x)\ \mathrm{d}x; f(x) \in C^4([0,1])}$$

$$\boldsymbol{|f^{(k)}(x)| \leq \sqrt[k]{\cos(x) + 2^k - 1}, \forall\, x \in \mathbb{R}, k \in \mathbb{N}}$$

Qual o numero de app da regra de simp p garantir pelo menos 6 casas dec

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Resposta (4) b)

$$|\varepsilon_i| = \left| -\frac{h^5}{90} f^{(4)}(\xi) \right| =$$

$$h = (1-0)/n = 1/n$$

$$= \left| -\frac{(1/n)^5}{90} \right| |f^{(4)}(\xi)| = \left| -\frac{1}{90\,n^5} \right| \sqrt[4]{\cos(\xi) + 2^\xi - 1} =$$

$$n \in \mathbb{N}_0^+$$

$$\frac{1}{90\,n^5} \sqrt[4]{\cos(\xi) + 2^\xi - 1} \leq$$

$$k = 6 \text{ casas decimais}$$

$$\leq 5\,\mathrm{E}^{-1-6} = 5\,\mathrm{E}^{-7} \implies n \geq \left(\frac{\sqrt[4]{\cos(\xi) + 2^\xi - 1}}{90 * 5\,\mathrm{E}^{-7}} \right)^{1/5} \leq$$

$$\xi \in [0,1]$$

$$\leq \left(\frac{\sqrt[4]{\cos(1) + 2^1 - 1}}{90 * 5\,\mathrm{E}^{-7}} \right)^{1/5} \cong 7.564 \implies n = \lceil 7.564 \rceil = 8$$

Closest alternative

$$n = 7 \tag{4}$$

$$\cos(\xi) + 2^\xi \begin{cases} \cos(0) + 2^0 = 2 \\ \cos(1) + 2^1 \cong 0.540 + 2 = 2.540 \end{cases}$$

Questão 5

Resposta d)

$$I_{G,2} = \int_{-6}^{10} f(x)\ \mathrm{d}x =$$

Using simple gauss rule ($n = 2$)

$$= \frac{10 - (-6)}{2} \int_{-1}^{1} g(y)\ \mathrm{d}y = 8 \int_{-1}^{1} g(y)\ \mathrm{d}y =$$

Using (4)

$$g(y) = f\left(\frac{b-a}{2}\, y + \frac{b+a}{2}\right) = f\left(\frac{10-(-6)}{2}\, y + \frac{10+(-6)}{2}\right) = f(8\, y + 2) = \tag{4}$$

Questão 6

x_i	-2	-1	0	1
$f(x_i)$	19	1	1	1

Q6 a.

Obtenha o pol de newton e aproxime $f(0.01)$

Resposta

Resposta (6),(7)

Construindo tabela de diferencas divididas

x_i	$f(x_i)$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
-2	19			
		-18		
-1	1		9	
		0		-3
0	1		0	
		0		
1	1			

Tabela 1: Diferencas divididas Questão 6

Newton's polynom

$$p_{3(x)} = 19 + \begin{pmatrix} +(x-x_0)\,f[x_0,x_1] \\ +(x-x_0)(x-x_1)\,f[x_0,\ldots,x_2] \\ +(x-x_0)(x-x_1)(x-x_2)\,f[x_0,\ldots,x_3] \end{pmatrix} = \qquad \text{using tabela 1}$$

$$= 19 + \begin{pmatrix} +(x-(-2))\,18 \\ +(x-(-2))(x-(-1))\,9 \\ +(x-(-2))(x-(-1))(x-0)\,(-3) \end{pmatrix} =$$

$$= 19 + \begin{pmatrix} +(x+2)\,18 \\ +(x+2)(x+1)\,9 \\ +(x+2)(x+1)(x)\,(-3) \end{pmatrix} \tag{6}$$

approx para $f(0.01)$

$$f(0.01) \cong p_3(0.01) = \qquad \text{using (6)}$$

$$= 19 + \begin{pmatrix} +(0.01+2)\,18 \\ +(0.01+2)(0.01+1)\,9 \\ +(0.01+2)(0.01+1)(0.01)\,(-3) \end{pmatrix} \cong 73.390 \tag{7}$$

Q6 b.

f(x) é pol ord 4, coeff x^4 é 1, det maj erro abs

Resposta

$$|\varepsilon_p| = |f(x) - p(x)| = \qquad \text{using (6)}$$

$$= \left| a_0 + a_1\,x + a_2\,x^2 + a_3\,x^3 + x^4 - 19 - \begin{pmatrix} +(x+2)\,18 \\ +(x+2)(x+1)\,9 \\ +(x+2)(x+1)(x)\,(-3) \end{pmatrix} \right| \ldots$$

Q6 c.

$$\boldsymbol{S(x)} = \begin{cases} \frac{9}{2}\,x^3 + 27\,x^2 + \frac{63}{2}\,x + 10, & x \in [-2,1[\\ \frac{-9}{2}\,x^3 + \frac{9}{2}\,x + 1, & x \in [-1,0] \end{cases}$$

verif se é spline natural de f

Resposta

Pra ser spline natural

1. $S_k(x_{k+1}) = S_{k+1}x_{k+1}$
2. $S'_k(x_{k+1}) = S'_{k+1}x_{k+1}$
3. $S''_k(x_{k+1}) = S''_{k+1}x_{k+1}$
4. $S''_0(x_0) = S''_n x_n = 0$

$$S_1(-1) = \frac{9}{2}(-1)^3 + 27\,(-1)^2 + \frac{63}{2}(-1) + 10 = \frac{74-9-63}{2} = 1 =$$
$$= S_2(-1) = \frac{-9}{2}(-1)^3 + \frac{9}{2}(-1) + 1 = \frac{9}{2} - \frac{9}{2} + 1 = 1;$$
$$S'_1(-1) = \frac{3*9}{2}(-1)^2 + 2*27\,(-1) + \frac{63}{2} = \frac{-4*27+3*9+63}{2} = -9 =$$
$$= S'_2(-1) = \frac{-3*9}{2}(-1)^2 + \frac{9}{2} = \frac{-2*9}{2} = -9;$$
$$S''_1(-1) = \frac{9*3*2}{2}(-1) + 27*2*1 = 27 =$$
$$= S''_2(-1) = \frac{-9*3*2}{2}(-1) = 27;$$
$$S''(-2) = \frac{9*3*2}{2}(-2) + 27*2*1 = 0;$$
$$S''(0) = \frac{-9*3*2}{2}(0) = 0$$

Todas as condições são verificadas, é spline cubico natural de f

Questão 7

x_i	0	0.5	1	1.5	2	2.5	3
$f(x_i)$	5	6.1875	5	$f(1.5)$	-3	-5.3125	-1

Q7 a.

Sabendo a approx $\hat{I}_{PM}$ de $I = \int_0^3 f(x)\,\mathrm{d}x$ com $h = 1$ é 2.3125, obtenha $f(1.5)$

Resposta

$$\int_0^3 f_{(x)}\,\mathrm{d}x = 2.3125 \approx h\, f_{(\frac{0+3}{2})} = 1 * f(1.5) \implies f(1.5) = 2.3125$$

Q7 b.

Regra de simp c 3 app obtenha approx $\hat{I}_S$

Resposta

$$\int_0^3 f_{(x)}\,\mathrm{d}x \approx$$

$$\approx \hat{I} = \frac{h}{3}(f(x_0) + 4(f(x_1) + f(x_3) + f(x_5)) + 2(f(x_2) + f(x_4)) + f(x_6)) =$$

$$h = (3-0)/3 = 1$$

$$= \frac{1}{3}(5 + 4(6.1875 + 2.3125 + (-5.3125)) + 2(5 + (-3)) + (-1)) =$$

$$= \frac{1}{3}(5 + 4(6.1875 + 2.3125 + (-5.3125)) + 2(5 + (-3)) + (-1)) \cong 6.917 \tag{8}$$

Q7 c.

Supondo $f^{(4)} = 0, \forall\, x \in \mathbb{R}$ det o val exato de I

Resposta

$$I = \hat{I}_S + \varepsilon_i = \hat{I}_S - n\,\frac{h^5}{90}\,f^{(4)}(\sigma) =$$

using (8)

$$\cong 6.917 - 3 * \frac{1^5}{90} * 0 = 6.917$$

Questão 8

$$x_{n+1} = F(x_n) \quad y_{n+1} = G(y_n); \quad \{x_0, y_0\} \in [0.1, 1], n = 1, 2, \ldots$$
$$F(x) = e^{\frac{x-4}{2}} \quad G(x) = 4 + 2\ \ln(x)$$

Resposta

Q8 a.

Prove que α é raiz da eq $2 + \ln(x) - x/2$ se e so se é ponto fixo de $F(x), G(x)$

Resposta

$$G(\alpha) = 4 + 2\ \ln(\alpha) = 2 * (2 + \ln(\alpha) - \alpha/2) + \alpha =$$
$$2 + \ln(\alpha) - \alpha/2 = 0$$
$$= \alpha;$$
$$F(\alpha) = e^{\frac{\alpha-4}{2}} = e^{\frac{G(\alpha)-4}{2}} = e^{\frac{4+2\ \ln(\alpha)-4}{2}} = e^{\ln(\alpha)} = \alpha$$

Assim α é ponto médio de $G(x)$ e $f(x)$

Q8 b.

Convergencia de x_{i+1}, y_{i+1} para $\alpha \in I$, calss α rel as func F e G, just

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Resposta

Condições de convergencia do metodo do ponto fixo:

1. $\varphi(x), \varphi'(x)$ é continua no intervalo I
2. $\varphi(x) \in I, \forall x \in I$
3. $|\varphi'(x)| \leq \lambda < 1, \forall x \in I$

$$F'(x) = e^{\frac{x-4}{2}}(1/2);$$
$$G'(x) = 2/x;$$
$$|F'(x)| = e^{\frac{x-4}{2}}/2 \leq$$
$$x \in [0.1, 1]$$
$$\leq e^{\frac{1-4}{2}}/2 = e^{-3/2}/2 \cong -2.241 \leq -2.24 = \lambda < 1; \tag{9}$$
$$|G'(x)| = |2/x| = 2/x \leq$$
$$x \in [0.1, 1]$$
$$\leq 2/0.1 = 20 = \lambda > 1$$

F converge e G diverge para α

Q8 c.

$x_0 = 1$ quantas casas decimais garante para x_2

Resposta

Casas decimais

$$|\varepsilon_{x_2}| = |\alpha - x_2| \leq$$
erro a posteriori
$$\leq \left|\frac{\lambda}{1-\lambda}\right||x_2 - x_1| \cong$$
using (9)
$$\cong \frac{-2.24}{1-(-2.24)}|-69.466 - -4.482| \cong 44.928 < 5\,\mathrm{E}^{-1+3}$$

Não pode se garantir nenhuma casa decimal

x_2

$$x_2 = F(x_1) \cong$$
$$\cong F(-4.482) = e^{\frac{-4.482-4}{2}} \cong -69.466;$$
$$x_1 = F(x_0) = F(1) = e^{\frac{1-4}{2}} = e^{-3/2} \cong -4.482$$