# AM 2C – Aula

Felipe B. Pinto 61387 - MIEQB

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## Conteúdo

# 1 Proposição

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = a$$

$$|f(x_0 + r\cos\theta, y_0 + r\sin\theta) - a| \le g(r) \quad \forall\, r, \theta$$
$$g(r) \to 0 \quad \wedge \quad r \to 0$$

## Exemplo 1

$$f(x,y) = \frac{x^4 + y^4}{x^2 + y^2} \qquad D = \mathbb{R}^4 \backslash \{(0,0)\}$$

$$0 \le \frac{x^4 + y^4}{x^2 + y^2} = \frac{(R\cos\theta)^4 + (R\sin\theta)^4}{(R\cos\theta)^2 + (R\sin\theta)^2} = \frac{R^4(\cos^4\theta + \sin^4\theta)}{R^2(\cos^2\theta + \sin^2\theta)} =$$
$$= R^2(\cos^4\theta + \sin^4\theta) \le 2R^2 \to 0$$

$$\therefore \lim_{(x,y)\to(0,0)} f(x) = 0$$

Nota

$$\cos^4\theta + \sin^4\theta \le 1$$

$$\because 1^2 = (\cos^2\theta + \sin^2\theta)^2 = \cos^4\theta + \sin^4\theta + 2\sin^2\cos^2 \ge \cos^4\theta + \sin^4\theta$$

# Exemplo 2

$$f(x,y)=\frac{x^2\,y}{x^4+y^2}$$

$$f(R\cos\theta,R\sin\theta)=\frac{(R\cos\theta)^2\,(R\sin\theta)}{(R\cos\theta)^4+(R\sin\theta)^2}=\frac{R^3\,\cos^2\theta\,\sin\theta}{R^2(R^2\,\cos^4\theta+\sin^2\theta)}=$$
$$=\frac{R\,\cos^2\theta\,\sin\theta}{R^2\,\cos^4\theta+\sin^2\theta}$$
$$0\le\left|\frac{x^2\,y}{x^4+y^2}\right|=\left|\frac{R\,\cos^2\theta\,\sin\theta}{R^2\,\cos^4\theta+\sin^2\theta}\right|=\frac{R\,\cos^2\theta\,|\sin\theta|}{R^2\,\cos^4\theta+\sin^2\theta}\wedge R\to 0\implies$$
$$\implies\frac{0}{\sin^2\theta}\wedge(\theta=0\vee\theta=\pi)\implies\text{Indeterminação: }0/0$$

Teorema não é aplicável

**Buscando dois limites diferentes**

$$\lim_{(x,y)\to(0,0)}f(x,x)=\frac{x^2\,x}{x^4+x^2}=\frac{x^3}{x^2(x^2+1)}=\frac{x}{x^2+1}=\frac{0}{1}=0$$

$$\lim_{(x,y)\to(0,0)}f(x,x^2)=\frac{x^2\,x^2}{x^4+(x^2)^2}=\frac{x^4}{x^4+x^4}=\frac{1}{2}$$

$$\therefore\nexists\lim_{(x,y)\to(0,0)}f(x,y)$$

# Exemplo 3

$$\lim_{\|(x,y)\|\to\infty} \frac{1}{x^2+y^2+5} = 0$$

$\forall\, \delta > 0 \exists L > 0 : \|(x,y)\| > L \implies$
$\implies$ ...