AM 1 - Ficha 5 Resolução
Limites e Continuidade de funções

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21/04 - 2021.1

Conteúdo

Parte I
Questões

Questão 4

$$\boldsymbol{H_{(x)} = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}}$$

Q4 - a)

$$\iff \lim_{x \to 0^-} H_{(x)} = 0 \neq \lim_{x \to 0^+} H_{(x)} = 1$$

Q4 - b) Incompleta

(i) $H_{(x-1)}$

$$y \in \mathbb{R} : \lim_{x \to y^-} H_{(x-1)} \neq$$
$$\neq \lim_{x \to y^+} H_{(x-1)} \iff y - 1 = 0 \iff$$
$$\iff y = 1$$

(ii) $(H_{(x)} - H_{(x-1)})\, x$

$$y \in \mathbb{R} : \lim_{x \to y^+} (H_{(x)} - H_{(x-1)})\, x \neq$$
$$\neq \lim_{x \to y^-} (H_{(x)} - H_{(x-1)})\, x \implies$$
$$\implies y - 1 = 0 \implies y = 1$$

Questão 7

Q7 - a) $f_{(x)} = \sin(x^2)/x, \quad x \in \mathbb{R} \backslash \{0\}$

$$\lim_{x \to 0^-} f_{(x)} = 0 = \lim_{x \to 0^+} f_{(x)} = 0 \quad \therefore \bar{f}_{(x)} = \begin{cases} \sin(x^2)/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Q7 - b) $g_{(x)} = e^{-1/(1-x^2)}, \quad x \in (-1, 1)$

$$\lim_{x\to -1^+} g_{(x)} = e^{\lim_{x\to -1^+}\left(\frac{-1}{1-x^2}\right)} = e^{-\infty} = 0;$$

$$\lim_{x\to 1^-} g_{(x)} = e^{\lim_{x\to 1^-}\left(\frac{-1}{1-x^2}\right)} = e^{-\infty} = 0$$

$$\therefore \bar{g}_{(x)} = \begin{cases} e^{-1/(1-x^2)} & x \in (-1,1) \\ 0 & x = \{-1, 1\} \end{cases}$$

Q7 - c) $h_{(x)} = e^{\tan(x)}, \quad x \in (-\pi/2, \pi/2)$

$$\lim_{x\to(-\pi/2)^+} h_{(x)} = e^{\lim_{x\to(-\pi/2)^+}(\tan(x))} = e^{-\infty} = 0;$$

$$\lim_{x\to(\pi/2)^-} h_{(x)} = e^{\lim_{x\to(\pi/2)^-}(\tan(x))} = e^{\infty} = \infty$$

$\therefore \nexists\, \bar{h}_{(x)}$ pois função não é prolongavel por continuidade em $\pi/2$

Parte II

Extras

Extra 1 $\lim_{x\to 0} x/\tan(x)$

$$= \lim_{x\to 0} \frac{x}{\sin(x)} \lim_{x\to 0} \cos(x) = 1$$

Extra 2 $\lim_{x\to 0} \sin(x)/\sqrt{x^2}$

$$= \lim_{x\to 0} = \sin(x)/|x|; \ \lim_{x\to 0^+} \sin(x)/x = 1 \neq \lim_{x\to 0^+} \sin(x)/(-x) = -1$$

$$\therefore \nexists \lim_{x\to 0} \sin(x)/\sqrt{x^2}$$

Extra 3 $\lim_{x\to 0} \sin^2(x)/\sqrt{x^4}$

$$= \lim_{x\to 0} \left(\frac{\sin(x)}{x}\right)^2 = 1$$