ERQ II – Teste 2024.1 Resolução

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20 de abril de 2024

Conteúdo

Questão 1

- Reator CSTR com escoamento modelado em 2 CSTR iguais em serie
- Tempo de residencia max: $10\,\mathrm{min}$

Q1 a.

- Esquema de associação de reatores
- Equações
- Caldais, volumes, concentrações

Resposta

$$\longrightarrow \mathrm{R}_1 \longrightarrow \mathrm{R}_2 \longrightarrow$$

	Volume	Caldal in	Caldal out
1	$V/2$	ν	ν
2	$V/2$	ν	ν

R_1 :

$$\nu\, C_{in,1} = \nu\, C_{in} = \nu\, C_{out,1} + \frac{V}{2}\,\frac{\mathrm{d}C_{out,1}}{\mathrm{d}t};$$

R_2 :

$$\nu\, C_{in,2} = \nu\, C_{out,1} = \nu\, C_{out,2} + \frac{V}{2}\,\frac{\mathrm{d}C_{out,2}}{\mathrm{d}t} = \nu\, C_{out} + \frac{V}{2}\,\frac{\mathrm{d}C_{out}}{\mathrm{d}t}$$

Q1 b.

Tempo de residencia

Resposta

$$E(t) = \mathcal{L}\, g(s);$$

$g(s)$:$g(s) = \bar{C}_{out}/\bar{C}_{in};$

$\bar{C}_{out}/\bar{C}_{in}$

$$\nu\, C_{out,1} = \nu\, C_{out} + \frac{V}{2}\,\frac{\mathrm{d}C_{out}}{\mathrm{d}t} \implies$$
$$\implies C_{out,1} = C_{out} + \frac{\tau}{2}\,\frac{\mathrm{d}C_{out}}{\mathrm{d}t} \implies$$
$$\implies \mathcal{L}\, C_{out,1} = \bar{C}_{out,1} =$$
$$= \mathcal{L}\, C_{out} + \mathcal{L}\left(\frac{\tau}{2}\,\frac{\mathrm{d}C_{out}}{\mathrm{d}t}\right) = \bar{C}_{out} + \frac{\tau}{2}\, s\, \bar{C}_{out} \implies$$
$$\implies \frac{\bar{C}_{out}}{\bar{C}_{in}} = \frac{1}{1+\frac{\tau}{2}\, s}\,\frac{\bar{C}_{out,1}}{\bar{C}_{in}};$$

$\bar{C}_{out,1}/\bar{C}_{in}$

$$\nu\, C_{in} = \nu\, C_{out,1} + \frac{V}{2}\,\frac{\mathrm{d}C_{out,1}}{\mathrm{d}t} \implies$$
$$\implies C_{in} = C_{out,1} + \frac{\tau}{2}\,\frac{\mathrm{d}C_{out,1}}{\mathrm{d}t} \implies$$
$$\implies \mathcal{L}\, C_{in} = \bar{C}_{in} = \mathcal{L}\, C_{out,1} + \mathcal{L}\left(\frac{\tau}{2}\,\frac{\mathrm{d}C_{out,1}}{\mathrm{d}t}\right) =$$
$$= \bar{C}_{out,1} + \frac{\tau}{2}\, s\, \bar{C}_{out,1} \implies$$
$$\implies \frac{\bar{C}_{out,1}}{\bar{C}_{in}} = \frac{1}{1+\frac{\tau}{2}\, s};$$

$$\implies g(s) = \frac{1}{1+\frac{\tau}{2}\, s}\,\frac{1}{1+\frac{\tau}{2}\, s} = \frac{1}{(1+\frac{\tau}{2}\, s)^2} = \frac{1}{(2/\tau)^2}\,\frac{1}{(\frac{2}{\tau}+s)^2} =$$
$$= \frac{\tau^2}{4}\,\frac{1}{(\frac{2}{\tau}+s)^2} \implies$$

$$\implies E(t) = \mathcal{L}\, g(s) = \frac{\tau^2}{4}\, t\, \exp\left(-\frac{2}{\tau}\, t\right)$$

Q1 c.

Função Culmulativa

Resposta

$$F(t) = \int_0^t E(t)\ \mathrm{d}t = \int_0^t \left(\frac{\tau^2}{4}\, t\ \exp\left(-\frac{2}{\tau}\, t\right)\right)\ \mathrm{d}t =$$
$$= \frac{\tau^2}{4} \int_0^t \left(t\ \exp\left(-\frac{2}{\tau}\, t\right)\right)\ \mathrm{d}t;$$

Primitiva:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(t\ \exp\left(-\frac{2}{\tau}\, t\right)\right) =$$
$$= \exp\left(-\frac{2}{\tau}\, t\right) + t\ \left(-\frac{\tau}{2}\right)\ \exp\left(-\frac{2}{\tau}\, t\right) \implies$$
$$\implies \mathcal{P}\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(t\ \exp\left(-\frac{2}{\tau}\, t\right)\right)\right) = t\ \exp\left(-\frac{2}{\tau}\, t\right) =$$
$$= \mathcal{P}\left(\exp\left(-\frac{2}{\tau}\, t\right)\right) + \mathcal{P}\left(t\ \left(-\frac{\tau}{2}\right)\ \exp\left(-\frac{2}{\tau}\, t\right)\right) =$$
$$= -\frac{\tau}{2} \exp\left(-\frac{2}{\tau}\, t\right) - \frac{\tau}{2}\mathcal{P}\left(t\ \exp\left(-\frac{2}{\tau}\, t\right)\right) \implies$$
$$\implies -\left(t\,\frac{2}{\tau} + 1\right)\ \exp\left(-\frac{2}{\tau}\, t\right) = \mathcal{P}\left(t\ \exp\left(-\frac{2}{\tau}\, t\right)\right) \implies$$

$$\implies F(t) = \frac{\tau^2}{4}\,\Delta\left(-\left(t\,\frac{2}{\tau} + 1\right)\ \exp\left(-\frac{2}{\tau}\, t\right)\right)\Big|_0^t =$$
$$= \frac{\tau^2}{4}\left(-\left(t\,\frac{2}{\tau} + 1\right)\ \exp\left(-\frac{2}{\tau}\, t\right) + \left(0 * \frac{2}{\tau} + 1\right)\ \exp\left(-\frac{2}{\tau} * 0\right)\right) =$$
$$= \frac{\tau^2}{4}\left(-\left(t\,\frac{2}{\tau} + 1\right)\ \exp\left(-\frac{2}{\tau}\, t\right) + 1\right)$$

Q1 e.

Concentração do tracador de saída

- Traçador Impulso
- $t = 17\,\text{min}$
- $N = 8\,\text{min}$
- ν

Resposta

Traçador Impulso:

$$C = \frac{N}{\nu} E(t) = \frac{8}{\nu} E(17) =$$
$$= \frac{8}{\nu} \frac{(1000/\nu)^2}{4} 17 * \exp\left(-\frac{2}{1000/\nu} * 17\right);$$

$\nu :$

$$\frac{\mathrm{d}^2 E(t)}{\mathrm{d}t^2} = 0 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\tau^2}{4}\,\exp\left(-\frac{2}{\tau}t\right) - \frac{\tau^2}{4} t \frac{\tau}{2}\,\exp\left(-\frac{2}{\tau}t\right)\right) =$$
$$= -\frac{\tau^2}{4}\frac{\tau}{2}\,\exp\left(-\frac{2}{\tau}t\right) - \frac{\tau^2}{4}\frac{\tau}{2}\,\exp\left(-\frac{2}{\tau}t\right) + \frac{\tau^2}{4} t \frac{\tau}{2}\frac{\tau}{2}\,\exp\left(-\frac{2}{\tau}t\right) =$$
$$= -\frac{\tau^3}{8}\,\exp\left(-\frac{2}{\tau}t\right) - \frac{\tau^3}{8}\,\exp\left(-\frac{2}{\tau}t\right) + \frac{\tau^4}{16} t\,\exp\left(-\frac{2}{\tau}t\right) \implies$$
$$\implies \frac{\mathrm{d}^2 E}{\mathrm{d}t^2}(10) = -\frac{(1000/\nu)^3}{8}\,\exp\left(-\frac{2}{(1000/\nu)}10\right) +$$
$$- \frac{(1000/\nu)^3}{8}\,\exp\left(-\frac{2}{(1000/\nu)}10\right) +$$
$$+ \frac{(1000/\nu)^4}{16} 10\,\exp\left(-\frac{2}{(1000/\nu)}01\right) =$$
$$= \frac{1\,\mathrm{E}^9}{8\,\nu^3}\left(-2 + \frac{1\,\mathrm{E}^4}{2\,\nu}\right)\exp\left(-\frac{\nu}{50}\right);$$

Assumindo: $\nu = 10\,\mathrm{dm}^3/\mathrm{min};$

$$\implies C = \frac{8}{10}\frac{(1000/10)^2}{4} 17 * \exp\left(-\frac{2}{1000/10} * 17\right) =$$
$$= \frac{34\,\mathrm{E}^3}{1}\,\exp\left(-\frac{1}{85}\right) \cong$$
$$\cong 3.360\,\mathrm{E}^5\,\mathrm{M}$$

Concentração absurdamente alta, não faz sentido, um erro a apontar poderia ser ter assumido o caldal porem um caldal de $10\,\mathrm{dm}^3/\mathrm{min}$ é bastante normal.

Q1 f.

Traçador por degrau, Concentração de saída

Resposta

Traçador degral:

$$F(t) = \frac{C(t)}{C_0} \implies$$
$$\implies C(t) = F(t)\, C_0;$$

$$\begin{cases} C_0 = 0.2\,\mathrm{M} \\ t = 16\,\mathrm{min} \end{cases}$$
$$C(16) =$$
$$= \frac{(1000/10)^2}{4} \left(- \left(16\, \frac{2}{(1000/10)} + 1 \right) \exp\left(- \frac{2}{(1000/10)}\, 16 \right) + 1 \right) 0.2 \cong$$
$$\cong 9.793\,\mathrm{E}^2$$

Pelas contas $F(t)$ tem dado superior a 1 o que é impossível, falho em perceber a origem do erro