AM 1 PO1 - Análise Matemática 1 - PO

22/03/2021

$$\sum_{k=1}^{n} k^2 \leq (n+1)^3$$

$n = 1 \implies \sum_{k=1}^{1} k^2 < (1+1)^3 \implies 1^2 < 8$

$n = m+1 \implies \sum_{k=1}^{m+1} k^2 < (1+m+1)^3 \implies \sum_{k=1}^{m} (k^2) + (m+1)^2 < (2+m)^3 \implies$

$\implies \sum_{k=1}^{m} (k^2) + (m+1)^2 < (1+m)^3 + (m+1)^2 < (2+m)^3 \implies$

$\implies (1+m)^2(m+2) < (2+m)^3 \implies (1+m)^2 < (2+m)^2$

$$2^n \leq n! \ \{\forall\, n \in \mathbb{N};\ n \geq 4\}$$

$n = 4 \implies 2^4 \leq 4! \implies 16 \leq 24$

$n = m+1 \implies 2^{m+1} \leq (m+1)! \implies 2\ 2^m \leq (m+1)\,(m)! \implies$

$\implies 2\ 2^m \leq (m+1)\,2^m$

$$A = (\,]0,1[\cap Q) \cup \{2 - 1/n \ :\ n \in \mathbb{N}\}$$

$$a) A' = ?$$

$A' = [0,1] \cup \{2\}$

$$b) \sum_{k=1}^{n} 1/k^2 \leq 2 - 1/n$$

$$n = 1 \implies \sum_{k=1}^{1} 1/k^2 \leq 2 - 1/1 \implies 1 \leq 1$$

$$n = m + 1 \implies \sum_{k=1}^{m+1} 1/k^2 \leq 2 - \frac{1}{(m+1)} \implies \sum_{k=1}^{m} \left(\frac{1}{k^2}\right) + \frac{1}{(m+1)^2} \leq 2 - \frac{1}{(m+1)} \implies$$

$$\implies \sum_{k=1}^{m} \left(\frac{1}{k^2}\right) + \frac{1}{(m+1)^2} \leq 2 - \frac{1}{m} + \frac{1}{(m+1)^2} \leq 2 - \frac{1}{(m+1)} \implies$$

$$\implies \frac{1+m+1}{(m+1)^2} \leq \frac{1}{m} \implies (2+m)\, m \leq (m+1)^2 \implies m^2 + 2\, m \leq m^2 + 2\, m + 1$$