OSF – Particulate Solids

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Conteúdo

1 Introduction

The key characteristics of an individual particle include composition, size, shape, density, and hardness.

Composition determines properties such as density and conductivity if the particle is uniform. However, in many cases the particle is porous or it may consist of a continuous matrix in which small particles of a second material are distributed.

Particle size is important because it affects properties such as the surface per unit volume and the rate at which a particle will settle in a fluid

A Particle Shape A particle shape may be regular, such as spherical or cubic, or it may be irregular like a piece of broken glass.

Regular can be defined by mathematical equations

- Sphere
- Cylinder
- Cube...

Irregular and properties of irregular particles are usually found by comparison to specific characteristics of a regularly shaped particle.

Single particles:

- Shape
- hardness
- compressive resistence
- electrical charge
- (intraparticle) porosity

Bulk solids:

- Particle size distribution
- (interparticle) porosity
- humidity
- agglomeration
- flowability
- ...

Solids suspensions (heterogenius mixture in a fluid, gás or liquid)

- Particle size distribution
- Concentration of solids
- viscosity of suspension
- flocculation
- settleability
- ...

2 Particle Characterization

2.1 Single Particles

The simplest shape of a particle is the sphere. Because of its symmetry, the particle looks exactly the same from whatever direction it is viewed, and behaves in the same manner in fluid, regardless of its orientation. No other particle shape has this characteristic. However, perfect spheres are rarely found and, generally, a typical particle may not be of any regular known shape. In fact, irregular-shaped particles are common. They are typically observed in nature and regularly handled and processed in many industries. Frequently, the size of a particle of an irregular shape is defined in terms of the size of an equivalent sphere, although the particle is represented by a sphere of a different size according to the property that is selected. Some of the important sizes of equivalent spheres are that the spheres have the same:

- Volume as the particle
- surface area as the particle.
- surface area per unit volume (i.e. specific surface area) as the particle.
- area as the particle when projected onto a plane perpendicular to its direction of motion.
- projected area as the particle, as viewed from above, when lying in its position of
- maximum stability such as on a microscope slide, for example.
- ability to pass through the same size of square aperture as the particle, such as on a screen for example.
- same settling velocity as the particle in a specified fluid.

Depending on the process of interest, the relevant particle size is typically chosen as the method to define the particle size. A measure of particle shape that is frequently used is the sphericity, ψ, defined as:

$$\psi = \frac{surface area of sphere of same volume as particle}{surface area of particle}$$

Regular shapes

Sphere

$$\begin{aligned} \text{Volume:} \quad & \pi\, d^3/6 = 4\,\pi\, r^3/3 \\ \text{Surface Area:} \quad & \pi\, d^2 = 4\,\pi\, r^2 \\ \text{Projected area in a plane:} \quad & \pi\, d^2/2 = \pi\, r^2 \end{aligned}$$

Note: Spheres are special as its compleetely symmetrical whilist the others depend on the orientation

Irregular shapes

- Cannot be identified by math equations
- Characteristic dimension d
  - Sphere Diameter with same volume
    $V_{\text{particle}} = V_{\text{sphere}}$
  - Sphere Diameter with same Surface area
    $S_{\text{particle}} = S_{\text{sphere}}$
  - Sphere Diameter with same Surface per volume
    $\frac{S_{\text{particle}}}{V_{\text{particle}}} = \frac{S_{\text{Sphere}}}{V_{\text{Sphere}}}$

Derivando propriedades:

Length	$L = d$	Volume	$V = \ddot{k}\, d^3$
Surface area	$S = \dot{k}\, d^2$	Mass	$m = \rho_S\, V = \rho_S\, \ddot{k}\, d^3$

- Surface factor: $\dot{k}_{\text{sphere}} = \pi$
- Volume factor: $\ddot{k}_{\text{sphere}} = \pi/6$

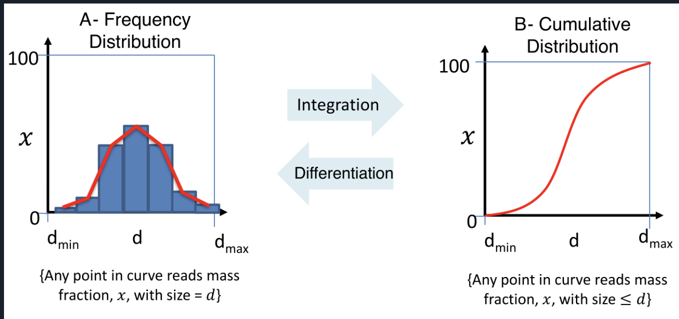
2.2 Measurement of Particle Size

Sieving > 50 µm

Sieve analysis may be carried out using a **nest of sieves, each lower sieve being of smaller aperture size**. Generally, sieve series is arranged so that the ratio of aperture sizes on consecutive sieves is 2, $2^{1/2}$, or $2^{1/4}$ according to the closeness of sizing that is required. **The sieves may either be hand-shaken or mounted on a vibrator**, which should be designed to give a degree of vertical movement and horizontal vibration.

2.3 Particle Size Distribution (PSD)

$$\text{Cumulative} \iff \text{Frequency}$$



Most particulate systems of practical interest consist of particles of a wide range of sizes and it is necessary to give a quantitative indication of the mean size and of the spread of sizes. The results of a size analysis can most conveniently be represented by means of a cumulative mass fraction curve, in which the proportion of particles (x) smaller than a certain size (d) is plotted against that size (d).

When the particle sizes are determined by image analysis, the results are represented by a cumulative number fraction curve, from which the mass fraction curve can be obtained

The distribution of particle sizes can be seen more readily by plotting a size frequency curve, in which the slope ($\frac{\mathrm{d}x}{\mathrm{d}d}$) of the cumulative curve is plotted against particle size (d). The most frequently occurring size is then shown by the maximum of the curve.

PSD Distribution base

Property	Whole	Fraction
Number	n_i, d_i	n_i
Length (m)	$l = n_i\, d_i$	$l_i = \frac{n_i\, d_i}{\sum n_j\, d_j}$
Surface (m^2)	$s = n_i\, \dot{k}\, d_i^2$	$s_i = \frac{n_i\, d_i^2}{\sum n_j\, d_j^2}$
Volume (m^3)	$v = n_i\, \ddot{k}\, d_i^3$	$v_i = \frac{n_i\, d_i^3}{\sum n_j\, d_j^3}$
Mass (kg)	$x = n_i\, \rho\, \ddot{k}\, d_i^3$	$x_i = \frac{n_i\, d_i^3}{\sum n_j\, d_j^3}$

Mean diameter:
Can be based on different properties of solid like weight, number or volume.

$$\bar{d_\alpha} = \frac{\int d\ \mathrm{d}\alpha}{\int \mathrm{d}\alpha} = \frac{\sum d_i\, \alpha_i}{\sum \alpha_i} : \quad \alpha \begin{cases} x: & \text{Weight} \\ n: & \text{Number} \\ v: & \text{Volume} \\ s: & \text{Surface} \\ l: & \text{Lenght} \end{cases}$$

Weight and Number

$$\begin{cases} \text{Measured in number } n \\ \bar{d_x} = \bar{d_v} = \frac{\int d\ \mathrm{d}x}{\int \mathrm{d}x} = \frac{\int d\ \mathrm{d}(n\, \rho\, \ddot{k}\, d^3)}{\int \mathrm{d}(n\, \rho\, \ddot{k}\, d^3)} = \frac{\int n\, d^3\, \mathrm{d}d}{\int n\, d^2\, \mathrm{d}d} = \\ = \frac{\sum d_i\, (n_i\, \rho\, \ddot{k}\, d_i^3)}{\sum (n_i\, \rho\, \ddot{k}\, d_i^3)} = \frac{\sum n_i\, d_i^4}{\sum n_i\, d_i^3} = \\ \text{Measured in Weight } x \\ = \frac{\sum x_i\, d_i}{\sum x_i} = \sum x_i\, d_i \end{cases}$$

Surface

$$\begin{cases} \text{Measured in number } n \\ \bar{d_S} = \frac{\int d\ \mathrm{d}s}{\int \mathrm{d}s} = \frac{\int d\ \mathrm{d}(n\, \dot{k}\, d^2)}{\int \mathrm{d}(n\, \dot{k}\, d^2)} = \frac{\int d^2\, n_i\, \mathrm{d}d}{\int d\, n_i\, \mathrm{d}d} = \\ = \frac{\sum d_i\, (n_i\, \dot{k}\, d_i^2)}{\sum (n_i\, \dot{k}\, d_i^2)} = \frac{\sum n_i\, d_i^3}{\sum n_i\, d_i^2} = \\ \text{Measured in Weight } x \\ = \frac{\sum \left(\frac{x_i}{\rho\, \ddot{k}\, d_i^3}\right)\, d_i^3}{\sum \left(\frac{x_i}{\rho\, \ddot{k}\, d_i^3}\right)\, d_i^2} = \frac{\sum x_i}{\sum x_i / d_i} = \frac{1}{\sum x_i / d_i} \end{cases}$$

Lenght

$$\begin{cases} \text{Measured in number } n \\ \bar{d_L} = \frac{\int d\ \mathrm{d}(n\, d)}{\int \mathrm{d}(n\, d)} = \frac{\int d\, n\ \mathrm{d}d}{\int n\ \mathrm{d}d} = \\ = \frac{\sum d_i\, (n_i\, d_i)}{\sum (n_i\, d_i)} = \frac{\sum n_i\, d_i^2}{\sum n_i\, d_i} = \\ \text{Measured in Weight } x \\ = \frac{\sum \left(\frac{x_i}{\rho\, \ddot{k}\, d_i^3}\right)\, d_i^2}{\sum \left(\frac{x_i}{\rho\, \ddot{k}\, d_i^3}\right)\, d_i} = \frac{\sum x_i / d_i}{\sum x_i / d_i^2} \end{cases}$$

Volume

$$\begin{cases} \bar{d_V} = \frac{\int d\ \mathrm{d}n\, \ddot{k}\, d^3}{\int \mathrm{d}n\, \ddot{k}\, d^3} = \frac{\int n\, d^3\, \mathrm{d}d}{\int n\, d^2\, \mathrm{d}d} = \\ = \frac{\sum d_i\, (n_i\, \ddot{k}\, d_i^3)}{\sum (n_i\, \ddot{k}\, d_i^3)} = \frac{\sum n_i\, d_i^4}{\sum n_i\, d_i^3} \\ \text{Assuming all particles have the same size} \\ \sum n_i\, \ddot{k}\, \bar{d}_V^3 = \ddot{k}\, \bar{d}_V^3\, \sum n_i = \sum n_i\, \ddot{k}\, d_i^3 \implies \\ \implies \bar{d_V} = \sqrt{\frac{\sum n_i\, d_i^3}{\sum n_i}} \end{cases}$$

Exemplo 1

The size analysis of a powdered material on a mass basis is represented by a straight line from 0% mass at $1\,\mu\text{m}$ particle size to 100% mass at $101\,\mu\text{m}$ particle size. Calculate the surface mean diameter of the particles constituting the system.

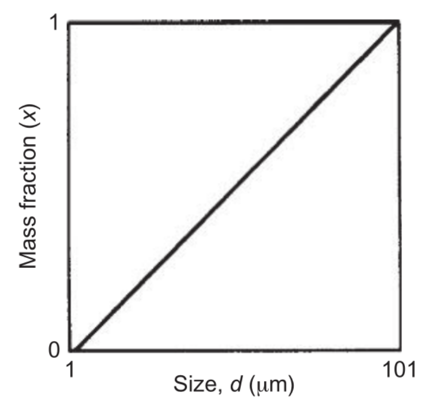


Fig. 2.8
Size analysis of powder.

Resposta

$$\bar{d}_s = \frac{\sum s\, d}{\sum s} =$$

$$= \frac{\sum x\, \frac{\dot{k}}{\rho\, \ddot{k}\, d}\, d}{\sum x\, \frac{\dot{k}}{\rho\, \ddot{k}\, d}} = \frac{\sum x}{\sum \frac{x}{d}} = \frac{1}{\sum \frac{x}{d}} = \frac{1}{\int \frac{\mathrm{d}x}{100\,x+1}} = \frac{100}{\int \frac{\mathrm{d}100\,x+1}{100\,x+1}} = \frac{100}{\ln 101/1} \cong 21.668\,\mu\text{m};$$

$$s = n\, \dot{k}\, d^2 = \left(\frac{x}{\rho\, \ddot{k}\, d^3}\right) \dot{k}\, d^2 = x\, \frac{\dot{k}}{\rho\, \ddot{k}\, d}$$