AM 1 - PO
Resolução Lista 4

Felipe Pinto - 61387

12/04 2021.1

Conteúdo

Questão 4 $\quad u_1 = 1; \quad u_{n+1} = \sqrt{2\,u_n}$

4 - a) $\quad u_n \in [1,3] \quad \forall\, n \in \mathbb{N}$

$$\iff 1 \leq u_n \leq 3 \iff 2 \leq 2\,u_n \leq 6 \iff 1 \leq \sqrt{2} \leq \sqrt{2\,u_n} \leq \sqrt{6} \leq 3$$

4 - b) $\quad |u_{n+2} - u_{n+1}| \leq \frac{\sqrt{2}}{2} |u_{n+1} - u_n|$

$$\iff \left|\sqrt{2\,u_{n+1}} - \sqrt{2\,u_n}\right| = \left|\frac{2\,u_{n+1} - 2\,u_n}{\sqrt{2\,u_{n+1}} + \sqrt{2\,u_n}}\right|; u_n \geq 1 \quad \forall\, n \in \mathbb{N} \implies$$

$$\implies \frac{2}{\sqrt{2\,u_{n+1}} + \sqrt{2\,u_n}}\,|u_{n+1} - u_n| \leq \frac{2}{2\sqrt{2}}\,|u_{n+1} - u_n| = \frac{\sqrt{2}}{2}\,|u_{n+1} - u_n|$$

Questão 5 $\quad u_1 = 0; \quad u_{n+1} = 1.5\,u_n + 1$

5 - a) **Prove que u_n é convergente**

$$\iff 1 \leq \frac{|u_{n+2} - u_{n+1}|}{|u_{n+1} - u_n|} = \frac{|1.5\,u_{n+1} + 1 - 1.5\,u_n + 1|}{|u_{n+1} - u_n|} = 1.5\frac{|\,u_{n+1} - \,u_n + 4/3|}{|u_{n+1} - u_n|}$$