03/21 – Limites

Felipe B. Pinto 61387 - MIEQB

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Conteúdo

Questão 1

$$f(x,y)=\begin{cases}5\,x-y & se\ x-y\leq 2\\ \frac{x^2-y^2+4\,x+8}{x-y} & se\ x-y>2\end{cases}$$

(i)

$$\lim_{\substack{(x,y)\to(a,b)\\(x,y)\in D_1}} f(x,y) : \left\{\begin{matrix}(a,b)\in \mathbf{fr}\,D_1=\mathbf{fr}\,D_2 & \wedge\\ \wedge\,(a-b=2) & \end{matrix}\right\}$$

$$=\lim_{\substack{(x,y)\to(a,b)\\(x,y)\in D_2}} \frac{x^2-y^2+4\,x+8}{x-y}=\frac{a^2-b^2+4\,a+8}{a-b}=$$

$$=\frac{(a+b)(a-b)+4\,a+8}{2}=3\,a+b+4=3\,a+(a-2)+4=4\,a+2$$

1 Testes para encontrar Limites

1. Iterados
2. Direcionais
3. Parabolas
4. Provar por definição

Questão 2

$$f(x,y)=\frac{x\,y}{\sqrt{x^2+y^2}}:\left\{\mathbf{D}=\mathbb{R}^2\backslash\{0,0\}\right.$$

(i)

$$\lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right)=0$$

(ii)

$$\lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right)=0$$

(iii) $\quad y = m\,x : m \in \mathbb{R}\backslash\{0\} \wedge x > 0$

$$\lim_{x \to 0} \frac{m\,x^2}{\sqrt{x^2 + m^2\,x^2}} = \lim_{x \to 0} \frac{m\,x^2}{x\sqrt{1 + m^2}} = 0$$

(iv) $\quad y = m\,x^2 : m \in \mathbb{R}\backslash\{0\}$

$$\lim \frac{x\,(m\,x^2)}{} = \lim \frac{m\,x^3}{} = \lim \frac{m\,x^2}{} = 0$$

Questão 3

$$f(x,y,z) = \frac{z}{\sqrt{(x-1)^2+(y+1)^2+z^2}} : \left\{\mathbf{D} = \mathbb{R}^3 \backslash \{1,-1,0\}\right.$$

Questão 4

$$f(x,y) = x^2 + y^2$$

(i)

$$\lim_{(x,y)\to(2,1)} f(x,y) = 2^2 + 1^2 = 5$$

(ii) Definição

$$\forall \delta > 0 \, \exists \, \varepsilon > 0 : (x,y) \in \mathbb{R}^2 \wedge \sqrt{(x-2)^2 + (y-1)^2} \leq \varepsilon$$

$$\begin{aligned}
&|x^2 + y^2 - 5| = |(x-2)^2 + (y-1)^2 - 4 + 4\,x - 1 + 2\,y - 5| = \\
&= |(x-2) + (y-1)^2 + 4\,x - 2\,y - 10| \leq \\
&\leq |(x-2) + (y-1)^2| + |4\,(x-2) + 2\,(y-1) + 8 + 2 - 10| \leq \\
&\leq (x-2) + (y-1)^2 + 4\,|x-2| + 2\,|y-1| < \varepsilon^2 + 6\,\varepsilon = \delta \implies \\
&\implies \varepsilon = \frac{-6 \pm \sqrt{36 + 4\,\delta}}{2} = -3 \pm \sqrt{9+\delta}\varepsilon = -3 + \sqrt{9+\delta} > 0
\end{aligned}$$

Questão 5

$$f(x,y) = \frac{x^3\,y}{2\,x^6 + y^2} : \begin{cases} \mathbf{D} = \mathbb{R}^2 \backslash \{(0,0)\} \\ (0,0) \in \overline{f(x,y)} \end{cases}$$

(i) $y = x^3$

$$\lim_{x \to 0} \frac{x^6}{2\,x^6 + x^6} = 1/3$$