Chapter 6. Flow of fluids in granular beds and packed columns

6.1 *Introduction to packed columns* (characterization of packings)

6.2 *Friction factor and pressure drop (Darcy, Kozeny, Carman-Kozeny)*

6.3 *The case of gas flow in vacuum columns (compressible fluid)*.

6.4 *Gas-liquid in counter-flow (Loading point, Flooding point, Pressure drop)*.

6.5 Economical design of packed bed columns.

J.M. Coulson and J.F. Richardson pp 191 - 234

Introduction

A packed column (or packed tower) consists of a (typically) cylindrical column **filled with a packing.** The objective of the packing is typically to create a large surface area per unit volume. Examples:

(1) **Packed bed reactor**: gas or liquid (or both) in contact with a solid surface. The catalyst is immobilized at the solid surface or inside the pores of the solid

(2) **Packed absorption column**: gas-liquid in counterflow. The liquid wets the packing and forms a thin film at the packing surface. The gas passes through the remaining free space.

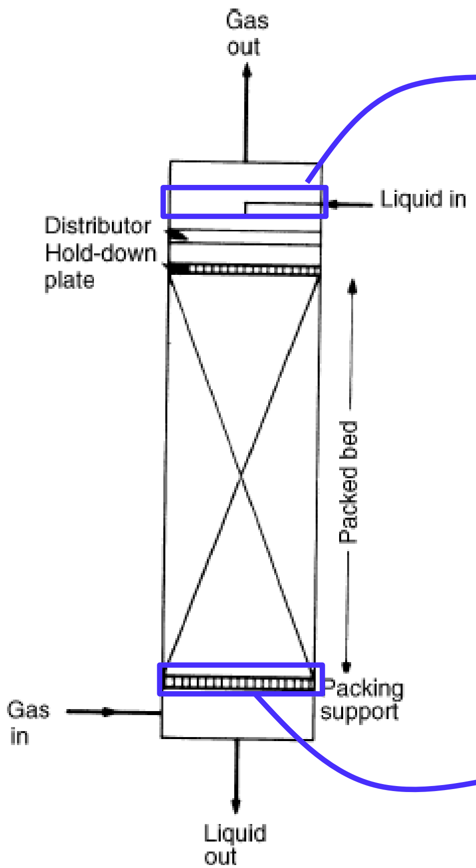
(3) **Contact distillation**: vapor-liquid in counterflow

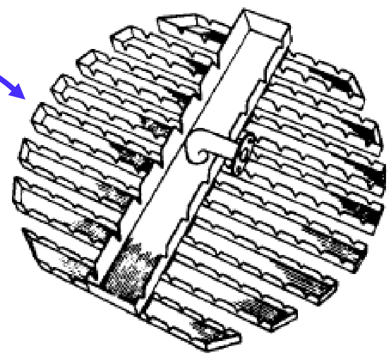
(4) **Liquid-liquid extraction**: “heavy” and “ligth” liquids in counterflow

(5) Etc…

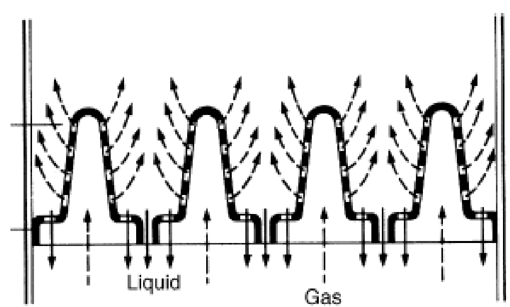
Introduction

<u>Example</u>: Packed absortion column with gas-liquid in counterflow, whereby a solute is tranferred from the gas to the liquid





<u>Liquid distributor</u> on top of the column ensures uniform radial distribution of liquid; the liquid will wet uniformly the bed particles on the radial direction



<u>Gas injection plate</u> on the bottom of the column; gas and liquid flow through separate openings; no hydrostatic head

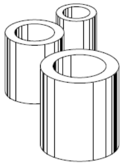
Types of packings

(1) broken solids
(2) Regular shapes (random or stacked)
(3) Grids (used in cooling towers)
(4) Structured packings

Broken solids: irregular particles (shape and size) disposed randomly (cheaper but less efficient)

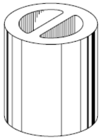
Regular shapes

Raschig Rings



(a)

Lessing Rings

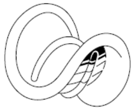


(b)

Nutter Ring

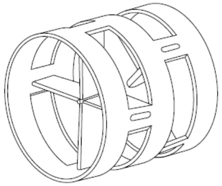
(f)

Berl Saddle



(c)

Pal Ring



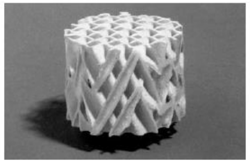
(d)

they can be arranged in the bed in a random fashion (dumped) or be stacked (one-by-one) in a regular packing

Structured packings: geometrically arranged packings

(a)

(b)



(c)

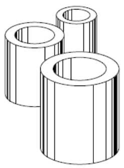
High efficiency
High capital cost

Figure 4.15. Structured packings (*a*) metal gauze (*b*) carbon (*c*) corrosion-resistant plastic

Parameters to characterize packings

Coulson, pp. 219

	Size (in.)	Size (mm)	Wall thickness (in.)	Wall thickness (mm)	Number (/ft^3)	Number (/m^3)	Bed density (lb/ft^3)	Bed density (kg/m^3)	Contact surface S_B (ft^2/ft^3)	Contact surface S_B (m^2/m^3)	Free space % (100 e)	Packing factor F (ft^2/ft^3)	Packing factor F (m^2/m^3)
Ceramic Raschig Rings	0.25	6	0.03	0.8	85,600	3,020,000	60	960	242	794	62	1600	5250
	0.38	9	0.05	1.3	24,700	872,000	61	970	157	575	67	1000	3280
	0.50	12	0.07	1.8	10,700	377,000	55	880	112	368	64	640	2100
	0.75	19	0.09	2.3	3090	109,000	50	800	73	240	72	255	840
	1.0	25	0.14	3.6	1350	47,600	42	670	58	190	71	160	525
	1.25	31			670	23,600	46	730			71	125	410
	1.5	38			387	13,600	43	680			73	95	310
	2.0	50	0.25	6.4	164	5790	41	650	29	95	74	65	210
	3.0	76			50	1765	35	560			78	36	120


(a)

e – void fraction

S – particle specific surface, m^{-1}

$$S_{sphere} = \frac{\pi d^2}{\pi d^3/6} = \frac{6}{d}$$

S_B – packing specific surface, m^{-1}

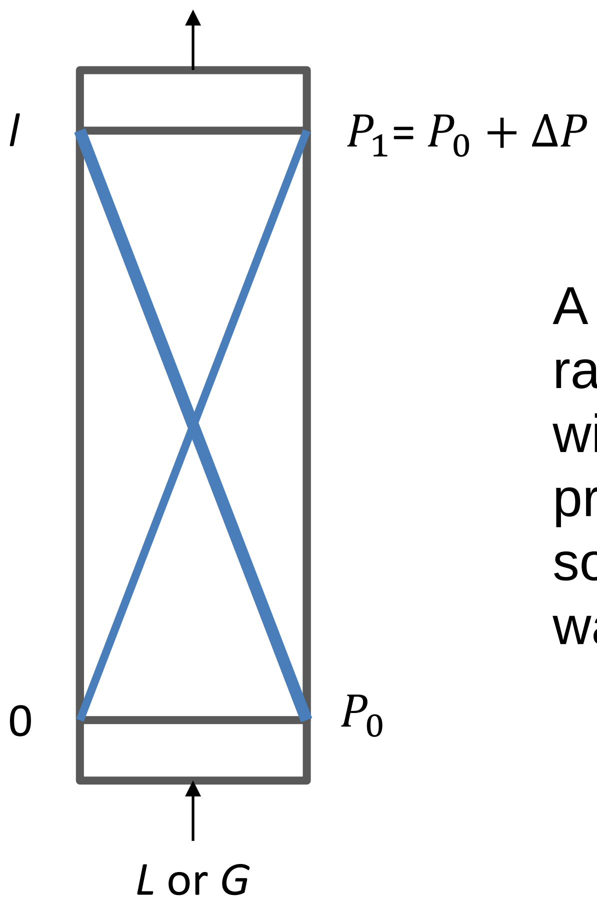
$$S_B = (1-e)S$$

F – packing factor, m^{-1}

$$F = \frac{effective\ S_B}{e^3} \sim \frac{S_B}{e^3}$$

(Generalised Pressure Drop Correlation (GPDC) experiment to evaluate F)

Pressure drop $(-\Delta P)$ in a packed bed

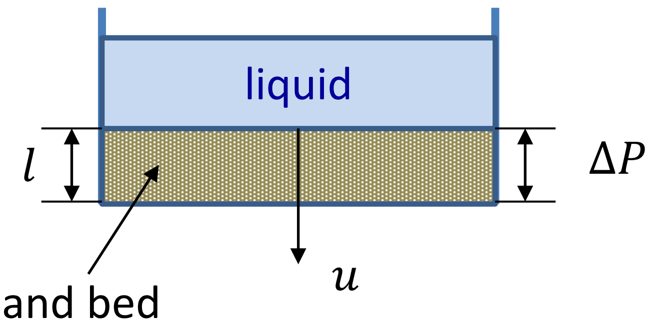


A liquid or a gas flows with a given mass flow rate (*L* for liquid and *G* for gas) in a packed bed with length, *l*. The fluid will experience a pressure drop $(-\Delta P)$ due to the friction with solid particles inside the bed and the column wall

Darcy's law

Darcy (1890) studied the flow of liquids through a sand bed and introduced the concept of bed permeability, **B [Darcy]**

$$\boxed{u = \frac{\boldsymbol{B}}{\mu}\frac{(-\Delta P)}{l}}$$



(-ΔP) - pressure drop across the bed, **atm**
l - thickness of the bed, **cm**
u - average fluid velocity across the bed, **cm/s**
μ - fluid viscosity, **cP**

B - bed permeability, Darcy: **1 Darcy** is the bed permeability that permits a flow of 1 cm/s of a fluid with viscosity 1 cP under a pressure gradient of 1 atm/cm

Darcy's law

Darcy (1890) studied the flow of liquids through a sand bed and introduced the concept of bed permeability, **B [Darcy]**

Table 4.1: properties of beds of some regular shaped materials

No.	Description	Specific surface area $S(m^2/m^3)$	Fractional voidage, e $(-)$	Permeability coefficient B (m^2)
	Spheres			
1	0.794 mm diam. ($\frac{1}{32}$ in.)	7600	0.393	6.2×10^{-10}
2	1.588 mm diam. ($\frac{1}{16}$ in.)	3759	0.405	2.8×10^{-9}
3	3.175 mm diam. ($\frac{1}{8}$ in.)	1895	0.393	9.4×10^{-9}
4	6.35 mm diam. ($\frac{1}{4}$ in.)	948	0.405	4.9×10^{-8}
5	7.94 mm diam. ($\frac{5}{16}$ in.)	756	0.416	9.4×10^{-8}
	Cubes			
6	3.175 mm ($\frac{1}{8}$ in.)	1860	0.190	4.6×10^{-10}
7	3.175 mm ($\frac{1}{8}$ in.)	1860	0.425	1.5×10^{-8}
8	6.35 mm ($\frac{1}{4}$ in.)	1078	0.318	1.4×10^{-8}
9	6.35 mm ($\frac{1}{4}$ in.)	1078	0.455	6.9×10^{-8}

Taken from Coulson, pp. 193

Kozeny correlation

Kozeny (1927) proposed a pressure drop correlation for laminar flow which is equivalent to Darcy equation, suggesting a definition for permeability, *B*.

$$\boxed{u = \frac{1}{K''}\frac{e^3}{S_B^2}\frac{1}{\mu}\frac{(-\Delta P)}{l}}$$

Reynolds number of the bed

$$Re_1 = \frac{\rho u}{S_B \mu} \leq 2 \quad \text{(laminar flow)}$$

K'' - dimensionless constant dependent on the structure of bed

Darcy

$$\Leftrightarrow \quad u = \frac{B}{\mu}\frac{(-\Delta P)}{l}, \qquad B = \frac{1}{K''}\frac{e^3}{S_B^2}$$

Permeability, m^2

Carman-Kozeny method

The Carman-Kozeny method is an extension of the Kozeny equation valid for both laminar and turbulent flow. It is a semi-empirical method based on an energy balance.

$$R_1 S_B A l \quad = \quad (-\Delta P) A e$$

(Energy loss due to friction) (Fluid pressure drop)

$$\boxed{\frac{R_1}{\rho u_1^2} \frac{S_B \rho u^2}{e^3} = \frac{(-\Delta P)}{l}}$$

$(-\Delta P)$ – Pressure drop, N/m2

R_1 – Drag force per unit surface area of bed, N/m2

S_B – specific surface of the bed, m^{-1}

A – Cross-section area of column, m2

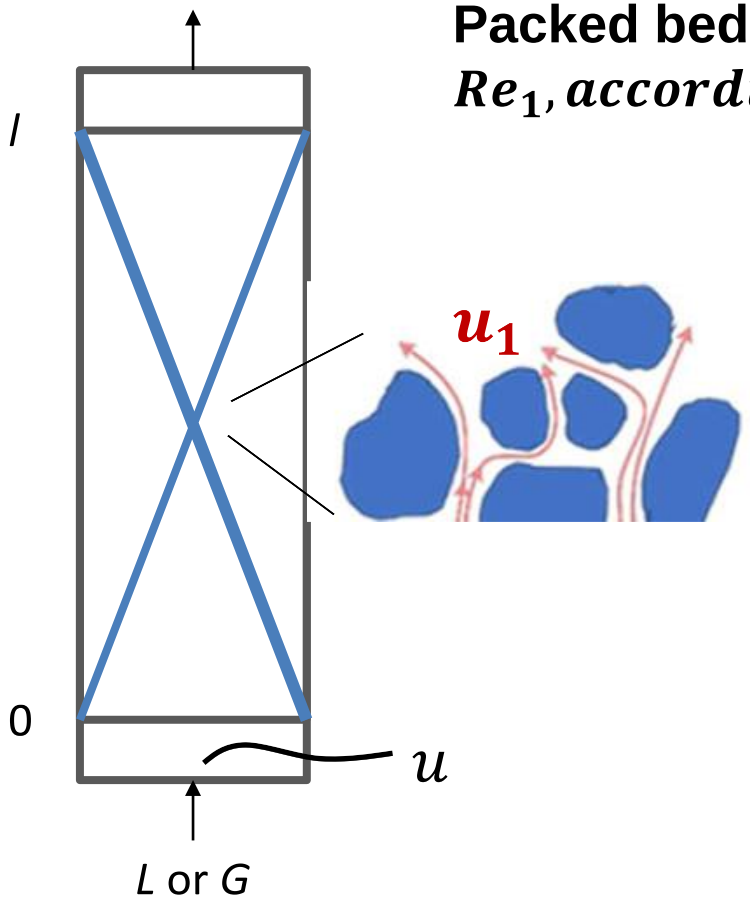
l – Column length, m

Ae – Free cross-section area for fluid flow, m2

$u_1 = \frac{u}{e}$ - Mean velocity in pore channel, m/s

$\frac{R_1}{\rho u_1^2}$ - Bed friction factor (dimensionless)

Packed bed Reynolds number, Re_1



Packed bed Reynolds number,
$\boldsymbol{Re_1, according\ to\ Carman - Kozeny}$

$$Re_1 = \frac{\rho u_1 d'_m}{\mu}$$

$u_1 = u/e$ - fluid velocity in the pores channel [m/s]

$d'_m = e/S_B$ - Equivalent diameter of pore space, m

$$\boxed{Re_1 = \frac{\rho u}{S_B \mu}}$$

Correlations of the bed friction factor

The Carman-Kozeny method applies correlations of the bed friction factor, $\frac{R_1}{\rho u_1^2}$, as function of bed Reynolds $\boldsymbol{Re_1}$

Laminar flow | Turbulent flow

$$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{0,4}{Re_1^{0,1}}$$

Carman correlation valid for full particles (e.g. spheres)

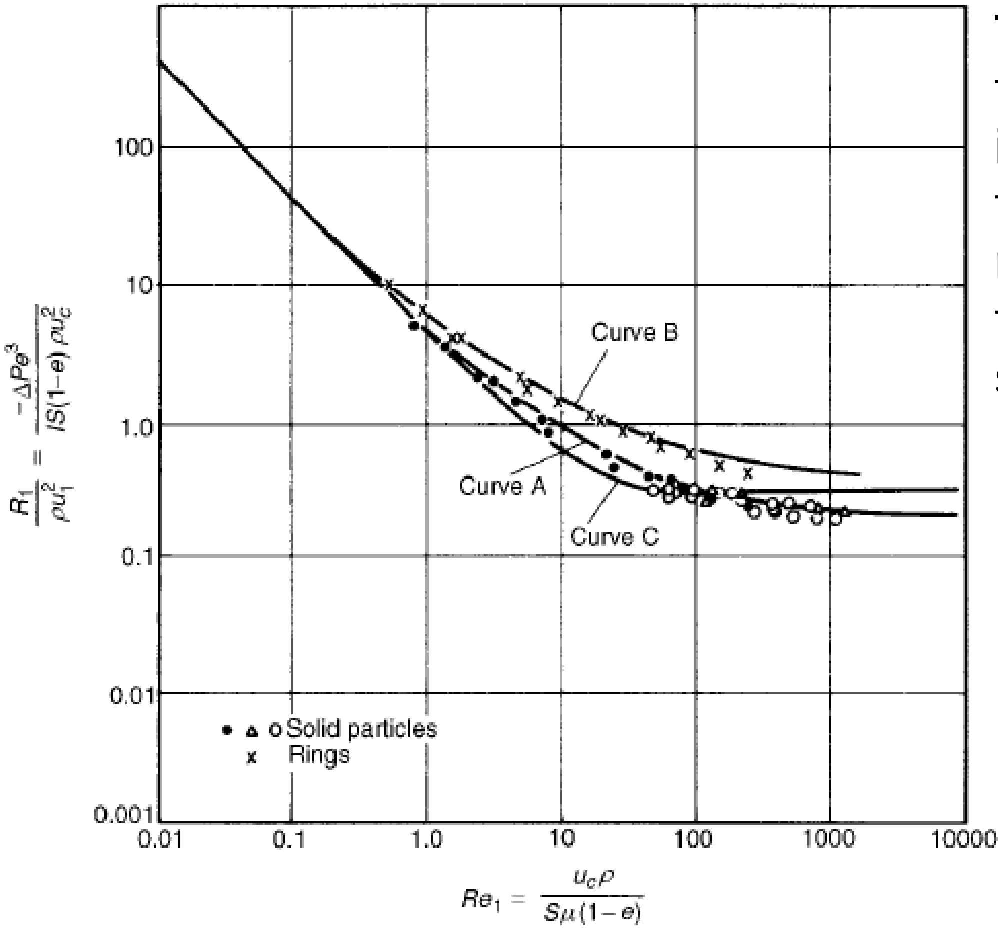
$$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{1}{Re_1^{0,1}}$$

Sawistowski correlation valid for hollow particles (e.g. Raschig rings)

$$\frac{R_1}{\rho u_1^2} = \frac{4,17}{Re_1} + 0,29$$

Ergun semi-empirical correlation also valid for hollow particles (e.g. Raschig rings)

Correlations of the bed friction factor



This figure shows that for laminar flow the friction factor is the same irrespective of type of bed particles; for turbulent flow, hollow particles (e.g. rings) have a significantly larger friction factor than solid particles (e.g. spheres)

***Curve A*: Carman** $$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{0{,}4}{Re_1^{0{,}1}}$$

***Curve B*: Sawitowski** $$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{1}{Re_1^{0{,}1}}$$

***Curve C*: Ergun** $$\frac{R_1}{\rho u_1^2} = \frac{4{,}17}{Re_1} + 0{,}29$$

(Note: Ergun is a semi-empirical correlation)

Homework

Show that there is a mathematical equivalence between the **Carman-Kozeny method** and the Kozeny Eq. for laminar flow

Carman-Kozeny method

$$\begin{cases} \dfrac{R_1}{\rho u_1^2}\dfrac{S_B \rho u^2}{e^3} = \dfrac{(-\Delta P)}{l} \\[2ex] \dfrac{R_1}{\rho u_1^2} = \dfrac{\underbrace{5}_{=K''}}{Re_1} + \cancel{\dfrac{0{,}4}{Re_1^{0,1}}} \\[2ex] Re_1 = \dfrac{\rho u}{S_B \mu} \end{cases}$$

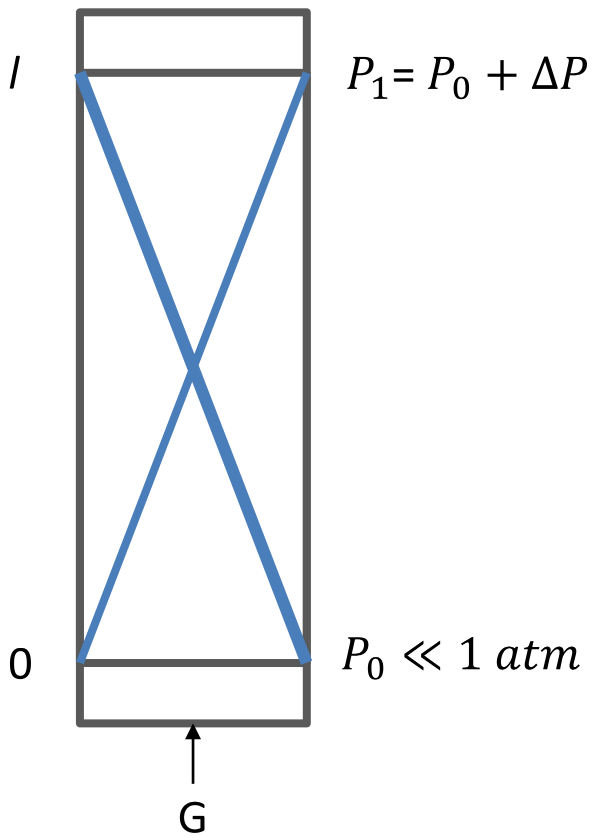
$$\Longleftrightarrow$$

Kozeny equation

$$\begin{cases} u = \dfrac{1}{K''}\dfrac{e^3}{S_B^2}\dfrac{1}{\mu}\dfrac{(-\Delta P)}{l} \\[2ex] K''=5 \end{cases}$$

Gas flow in packed vacuum columns

Vacuum columns operate at pressures $P_1 < P_0 \ll 1\ atm$



Is the gas specific mass, ρ, constant along the column length, *l*?

- No because a gas is a compressible fluid

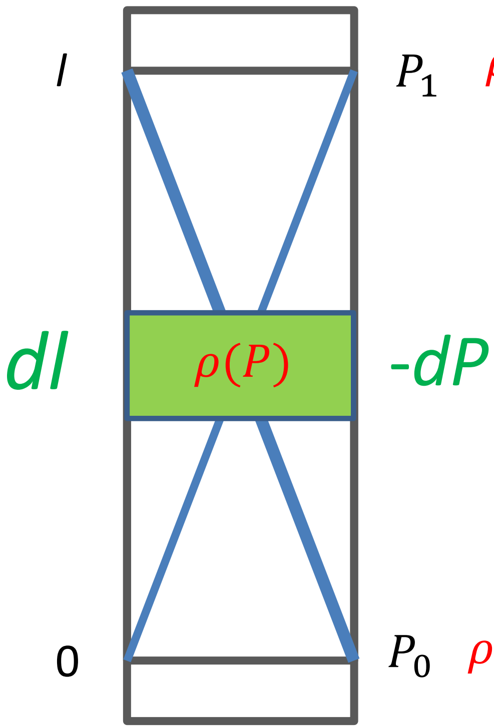
$$\rho = \frac{M_W P}{RT} = \mathrm{f(P, T)}$$

- The gas specific mass decreases along the column length because pressure decreases

- **Is this variation negligible?**

- Not in vacuum columns because $P_0 \ll 1\ atm$, thus $\Delta P \sim P_0 \Rightarrow P_1 \ll P_0 \quad \Rightarrow \quad \rho_1 \ll \rho_0$

Gas flow in packed vacuum columns



$$\rho_1 = \frac{M_W P_1}{RT}$$

$$\rho_0 = \frac{M_W P_0}{RT}$$

<u>Carman-Kozeny method</u> applied to a differential column volum with length *dl*

$$\frac{R_1}{\rho u_1^2} \frac{S_B \rho(P) u^2}{e^3} = \frac{(-dP)}{dl}$$

$$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{0{,}4}{Re_1^{0{,}1}}$$

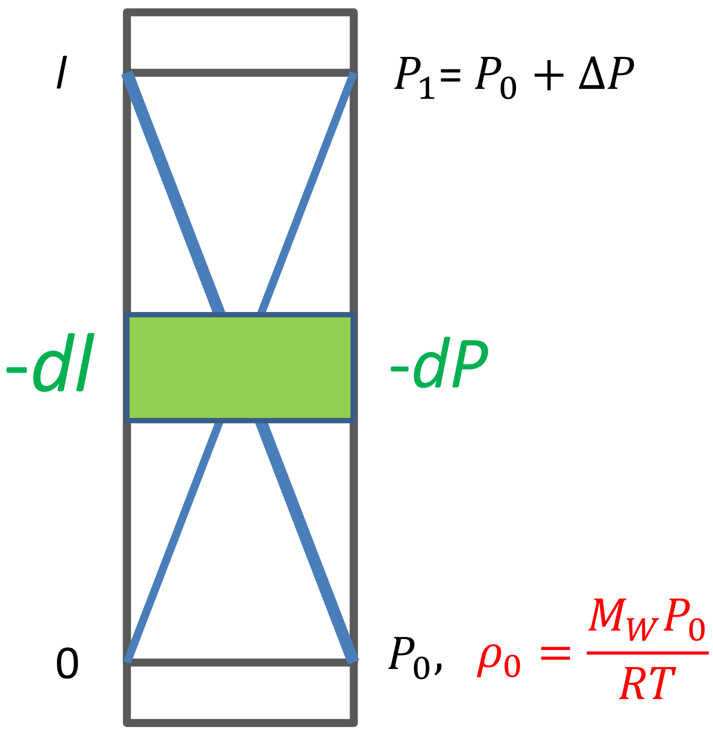
Constant along column length

$$Re_1 = \frac{\rho(P)u}{S_B \mu} = \frac{G}{S_B \mu}$$

Constant along column length

Note that $\rho(P)$ and u may change along column length, but their multiplication is constant, $\mathrm{G} = \rho(P)u = const$

Gas flow in packed vacuum columns



<u>Carman-Kozeny</u> method after integration:

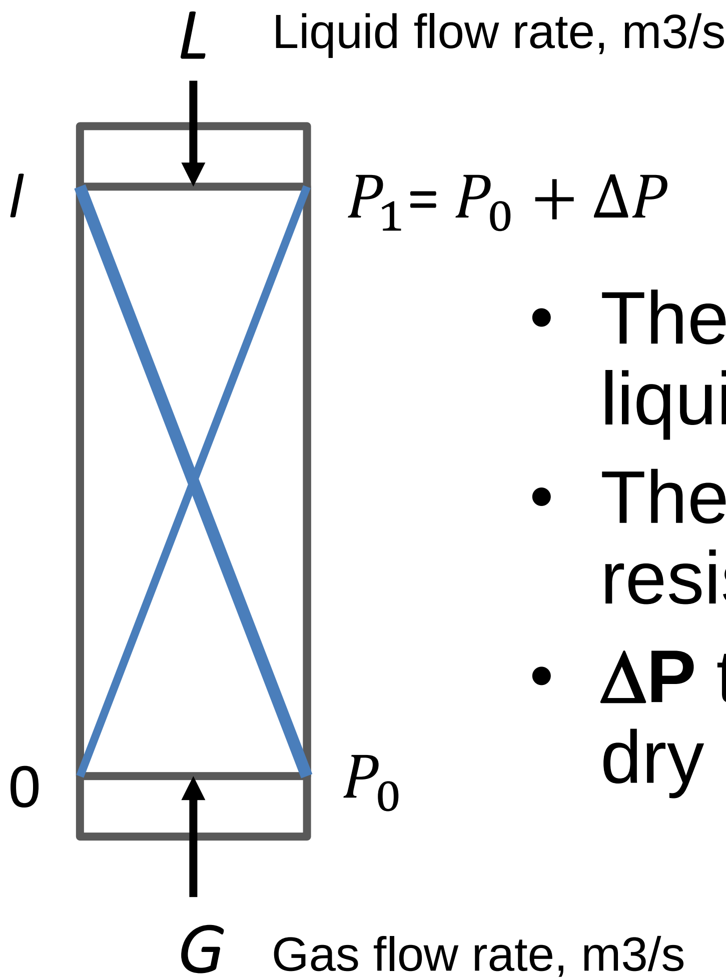
$$\boxed{\frac{R_1}{\rho u_1^2}\frac{S_B G^2}{e^3 \rho_0} = \frac{P_0^2 - P_1^2}{2 l P_0}}$$

For full particles, Carman, otherwise take Sawitowski or Ergun

$$\frac{R_1}{\rho u_1^2} = \frac{5}{Re_1} + \frac{0{,}4}{Re_1^{0{,}1}}$$

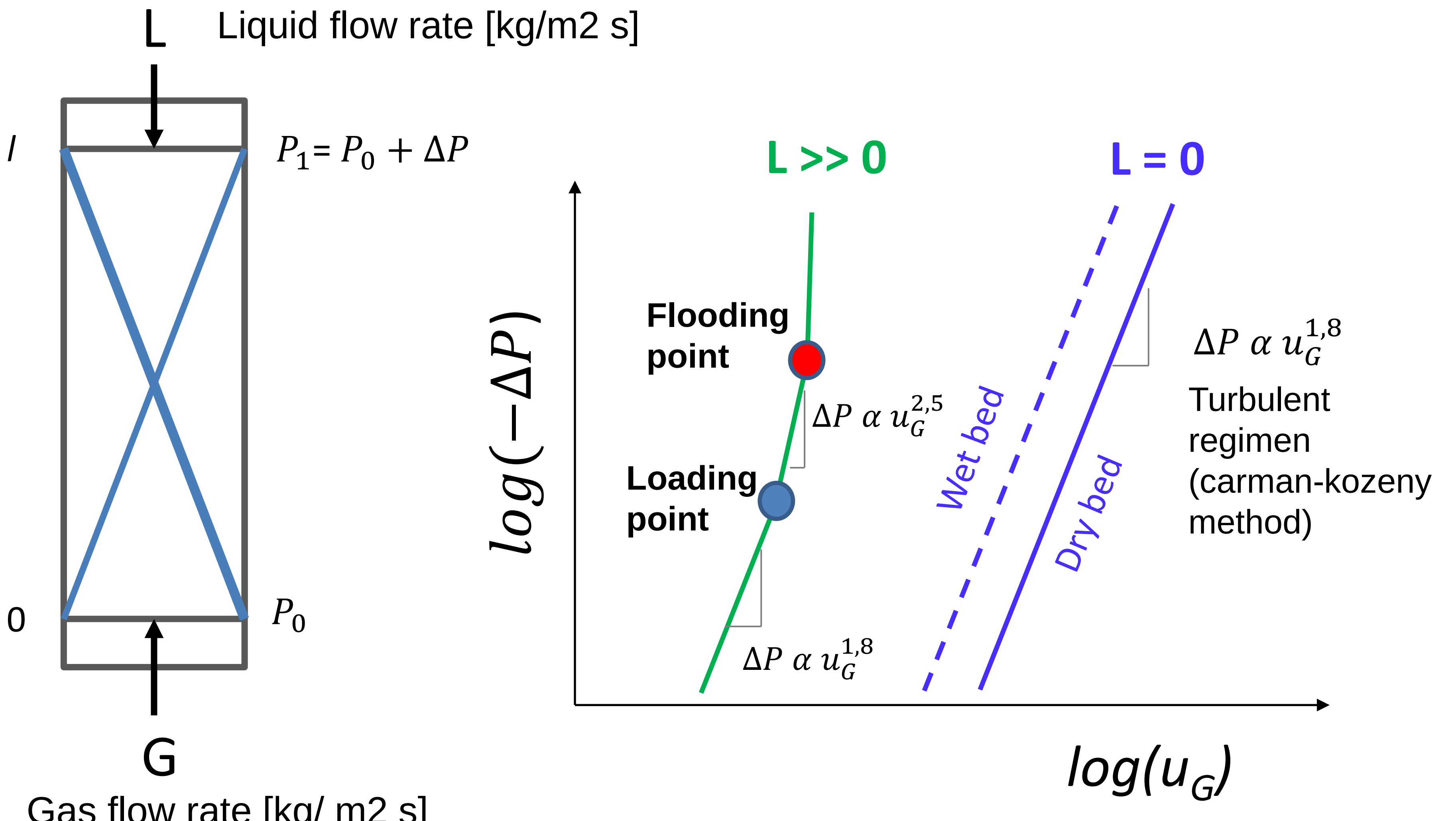
$$Re_1 = \frac{G}{S_B \mu}$$

Gas-liquid in counterflow in packed columns

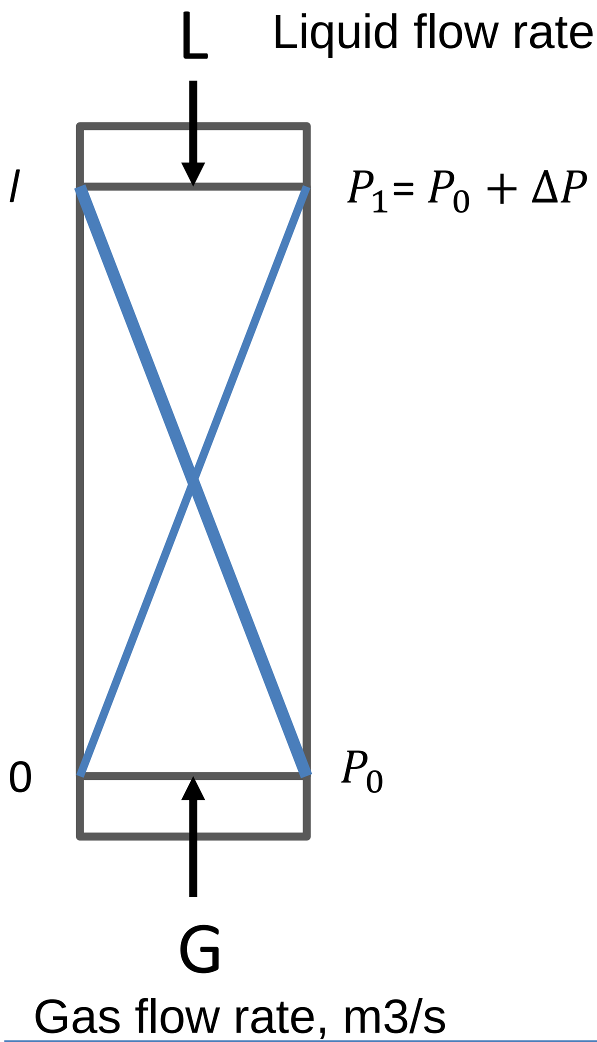


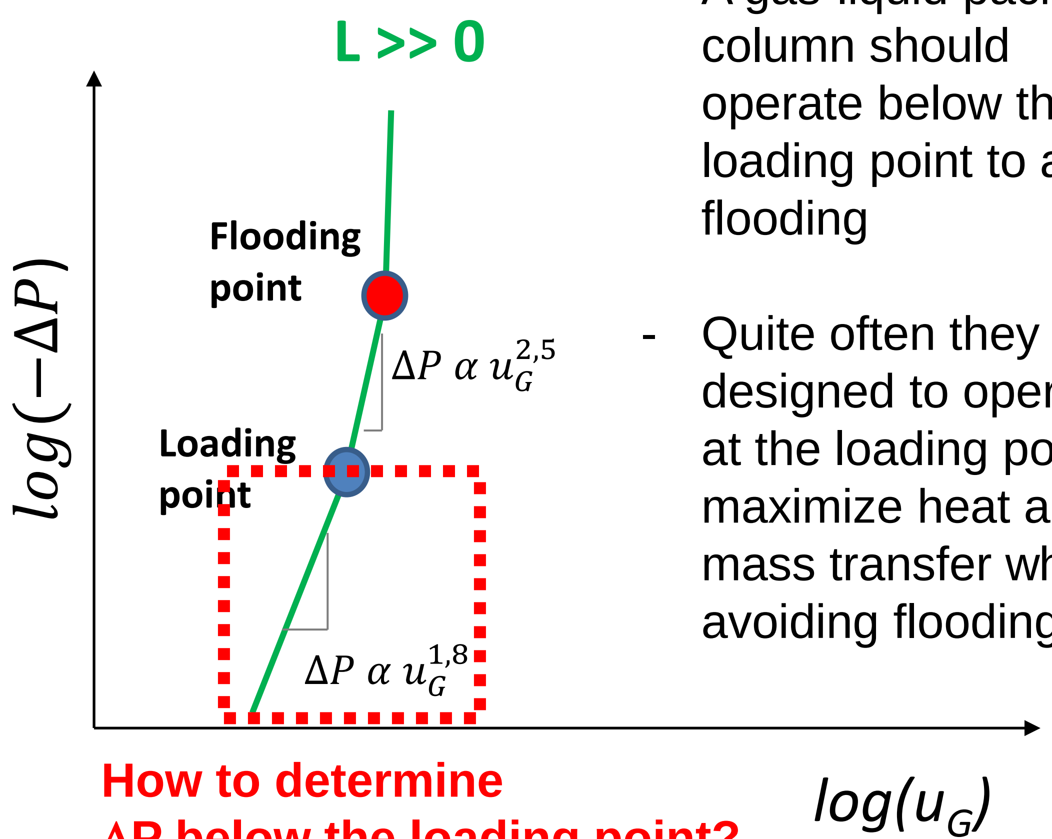
- The **ΔP** refers to the gas (not to the liquid)
- The liquid offers additional resistance to the flow of the gas
- **ΔP** tends to increase in relation to a dry column

Gas-liquid in counterflow in packed columns



Gas-liquid in counterflow in packed columns





How to determine ΔP below the loading point?

- A gas-liquid packed column should operate below the loading point to avoid flooding
- Quite often they are designed to operate at the loading point to maximize heat and/or mass transfer while avoiding flooding

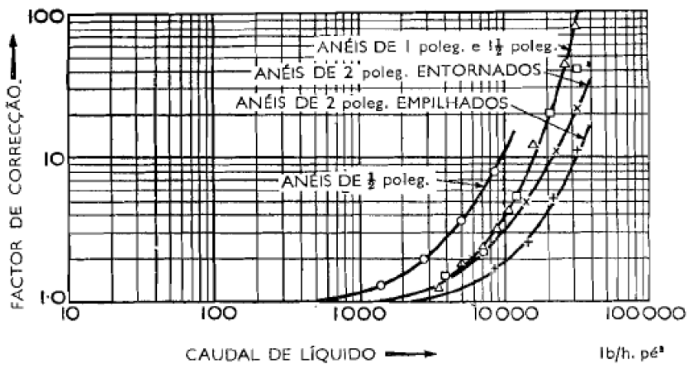
Carman-kozeny method

Carman-kozeny method to determine ΔP below the loading point

Step 1. Calculate ΔP for the dry column by the Carman-Kozeny method

$$\frac{R_1}{\rho u_1^2}\frac{S_B \rho u^2}{e^3} = \frac{(-\Delta P)}{l}$$

Step 2. Correction for the flow of liquid



$$\Delta P_{L>0} = \Delta P_{L=0} \times F_{corr}$$

Morris&Jackson method

Morris&Jackson method to determine ΔP below the loading point

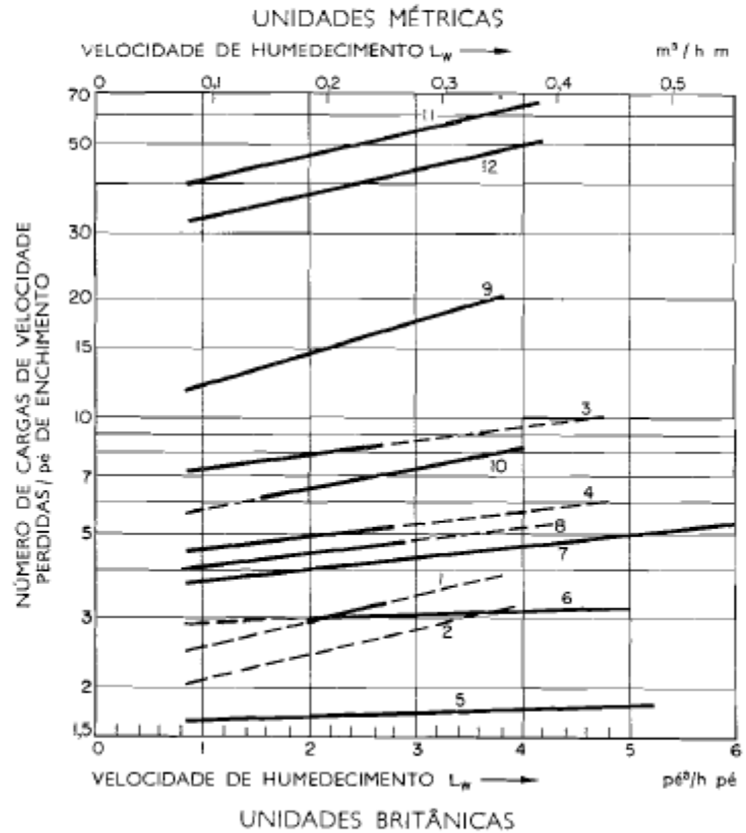
$$\boxed{\frac{-\Delta P}{l} = \frac{1}{2}\rho_G u_G^2 \boldsymbol{N}}$$

N – Number of velocity heads lost per unit of bed length, m^{-1}, it is a function of the wetting flow rate:

$$N = f(L_w)$$

L_w - Wetting flow rate, m^2/s

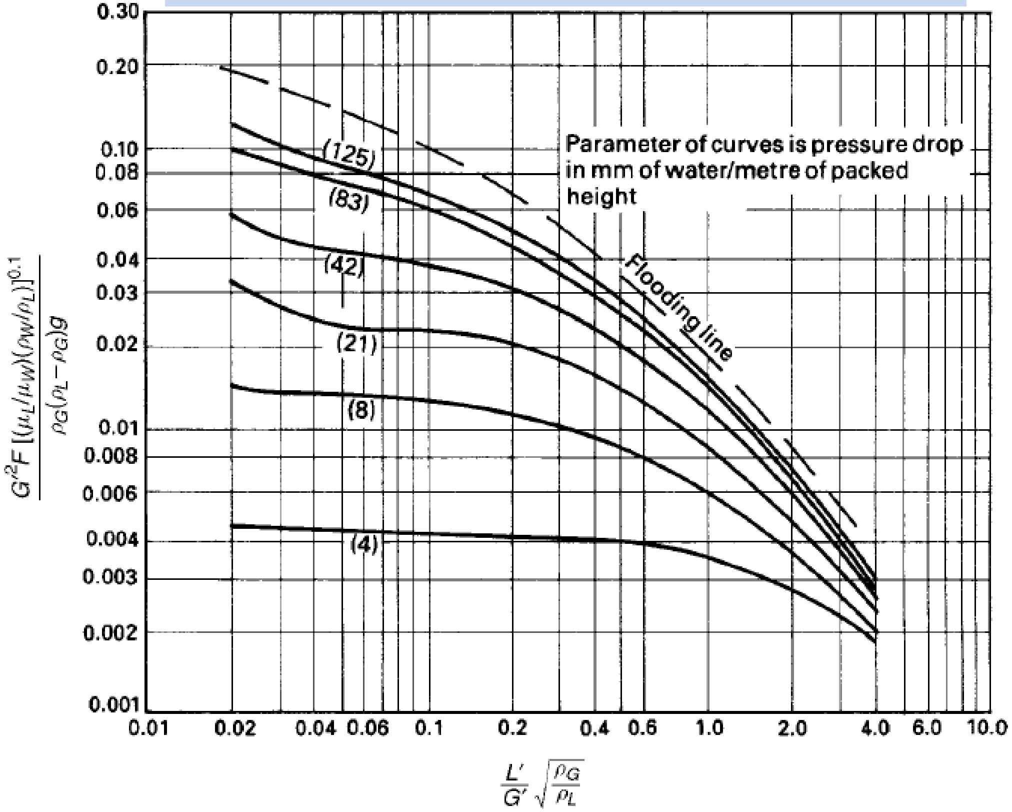
$$L_w = \frac{u_L}{S_B} = \frac{L}{S_B \rho_L}$$



Eckert generalized pressure drop method

Eckert generalized pressure drop method to determine ΔP below the flooding point

J.M. Coulson and J.F. Richardson pp 226

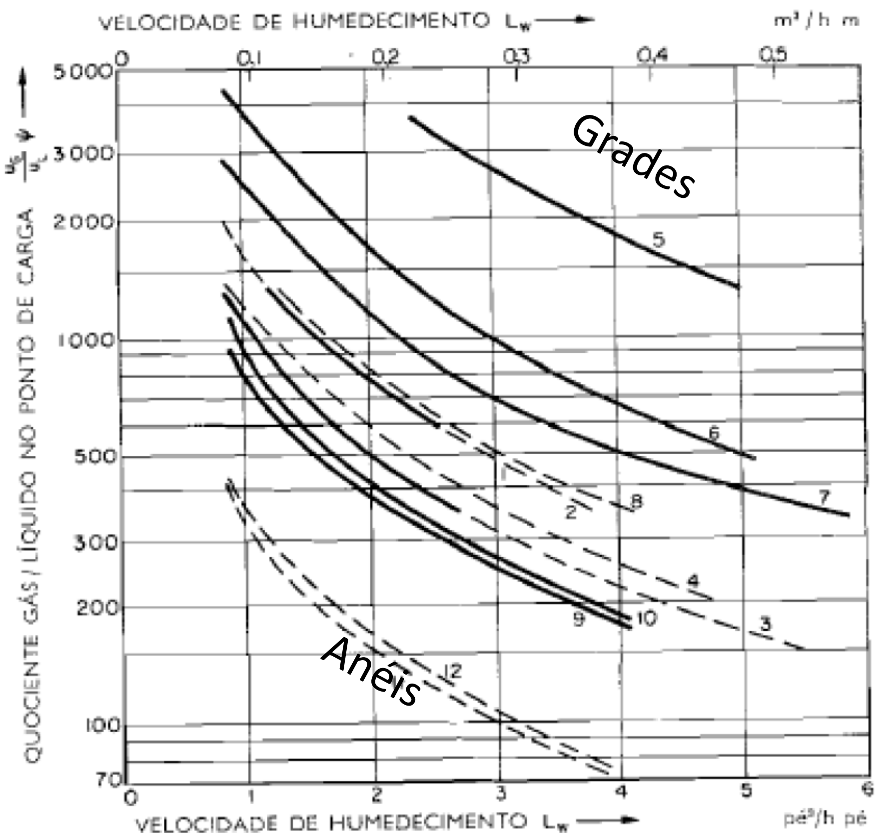


- This chart shows Pressure drop isolines (mm H20/m bed) as function of L/G ratios
- Properties of the packing are considered using the packing factor, *F*

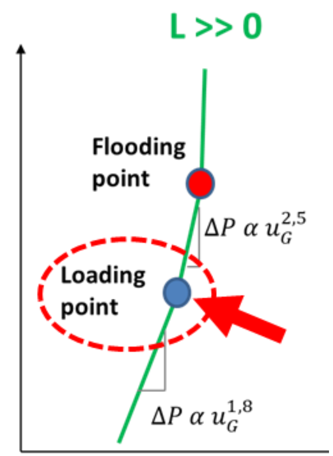
Graphical determination of the **loading point**

$$\frac{u_G}{u_L}\psi$$

$$\psi = \sqrt{\frac{\rho_G}{\rho_L}}$$

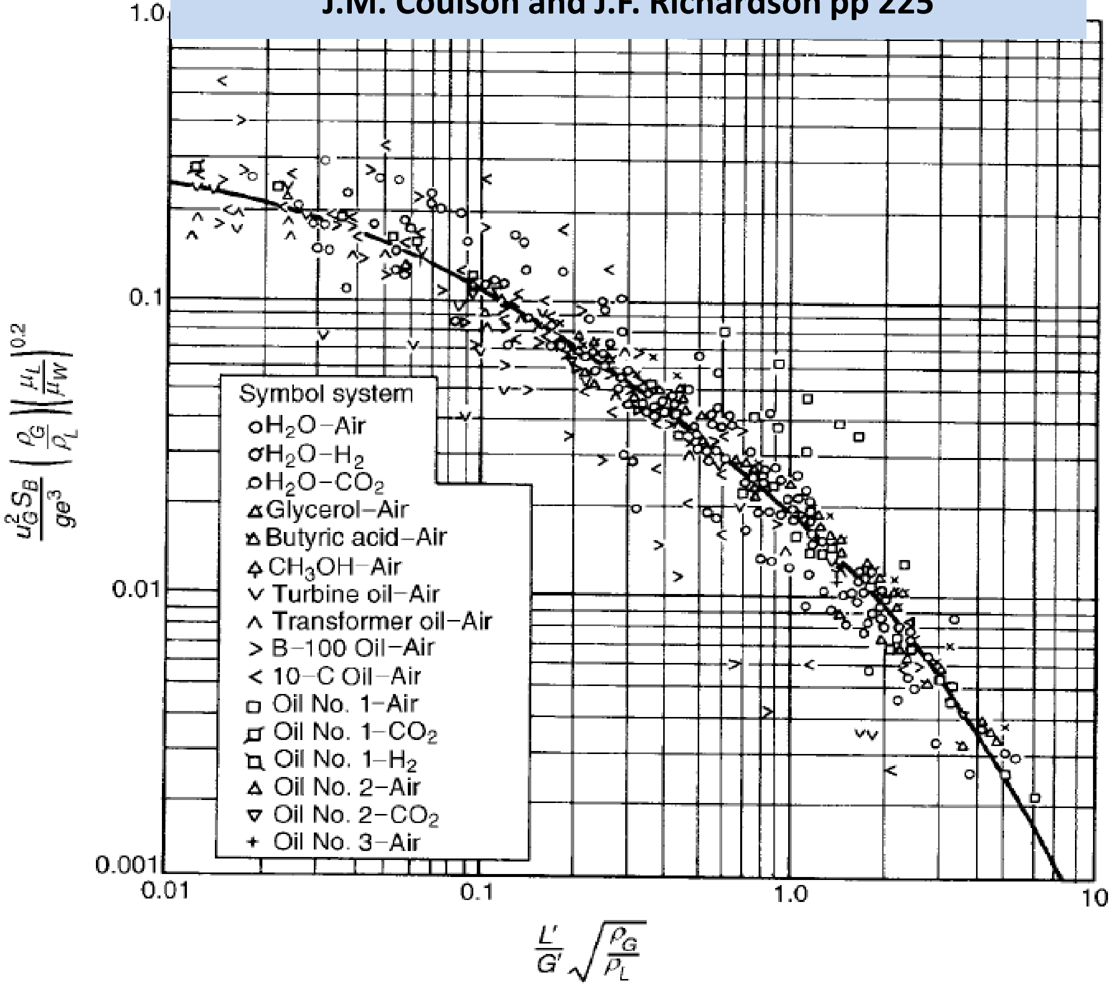


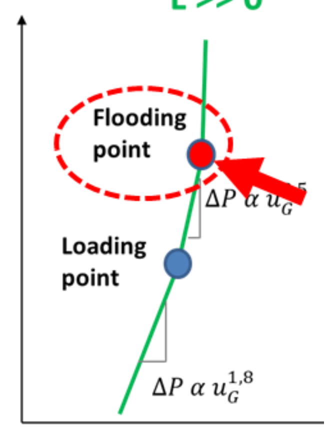
$$L_w = \frac{u_L}{S_B} = \frac{L}{S_B\rho_L}$$



Graphical determination of the flooding point

J.M. Coulson and J.F. Richardson pp 225





$$\ln\left(\frac{u_G^2 S_B}{g e^3}\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\mu}{\mu_W}\right)^{0,2}\right) = -4\left(\frac{L}{G}\sqrt{\frac{\rho_G}{\rho_L}}\right)$$

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Economic considerations

In the design of a packed bed column, the inlet mass flowrate, $\dot{\boldsymbol{m}} = \mathbf{GA}$ **(kg/s)**, where A is the cross-section area of the colums, is normally fixed as **design condition**

$$\boxed{GA = const}$$

Trade-off between G and A: By increasing G, A decreases, thus the capital costs decrease. However, $(-\Delta P)\ \alpha\ G^{1,8}$ increases, thus operating costs (energy) increase

<u>Consequences:</u>

- Gas-liquid counterflow columns are **designed to operate at the loading point** to maximize heat and/or mass transfer while avoiding flooding. Operating at the loading point minimizes the A, thus minimizes the capital costs, at the expense of higher operating costs.
- In high pressure counterflow columns, the column wall thickness is much higher thus capital costs are generally higher. They are allowed to operate above the loading point (~70% flooding point) such as to further decrease A and the cost of the column. The operating costs will however increase due to the much higher pressure drop .

Exercises VI.1-VI-6

VI - FLUXO DE FLUIDOS ATRAVÉS DE LEITOS GRANULARES E COLUNAS DE ENCHIMENTO

1. Numa fábrica de ácido sulfúrico pelo processo contacto o convertidor secundário é um convertidor do tipo de tabuleiros de 2.3 m de diâmetro, com o catalisador disposto em três camadas de 0.45 m de espessura cada. O catalisador está na forma de pastilhas cilíndricas com 9.5 mm de diâmetro e 9.5 mm de comprimento. A fracção de vazios (porosidade) é 0.35. O gás entra no convertidor a 400 °C e sai a 445 °C. A sua composição de entrada é:

SO_3 6.6; SO_2 1.7; O_2 10.0; N_2 81.7% molar,

e a sua composição de saída

SO_3 8.2; SO_2 0.2; O_2 9.3; N_2 82.3% molar .

O caudal de gás é 0.68 kg m^{-2} s^{-1}. Calcular a queda de pressão através do convertidor.
$\mu = 0.032$ mN s m^{-2}

2. Usa-se uma coluna de 0.6 m de diâmetro e 4 m de altura cheia de anéis Raschig cerâmicos de 25.4 mm, num processo de absorção de gases realizado à pressão atmosférica e 20 °C. Se puder considerar-se que o líquido e o gás têm as propriedades da água e do ar, e se os seus caudais forem 6.5 e 0.6 kg m^{-2} s^{-1} respectivamente, qual será a queda de pressão através da coluna?

(a) Usar o método de Carman,
(b) outro método e comparar os resultados obtidos.
(c) De quanto se pode aumentar o caudal de líquido até a coluna se inundar?

3. Mostre como se pode modificar uma equação para a perda de pressão numa coluna com enchimento para ser válida para casos em que a pressão total e a queda de pressão sejam da mesma ordem de grandeza. Exemplifique com a correlação de Ergun.

4. Usa-se uma coluna com enchimento, com 1.2 m de diâmetro e 9 m de altura cheia com anéis Raschig de 25.4 mm, para destilação sob vácuo de uma mistura de isómeros de peso molecular 155 g/mol. A temperatura média é 100 °C , a pressão no condensador é mantida a 0.13 kN/m^2 e a pressão no destilador varia entre 1.3 e 3.3 kN/m^2.

(a) Obtenha uma expressão para a queda de pressão supondo que não é apreciavelmente afectada pelo caudal de líquido e que pode ser calculada usando uma forma modificada da equação de Carman.

(b) Mostrar que na gama de pressões usadas, a queda de pressão é aproximadamente directamente proporcional ao caudal em massa de vapor e calcular a queda de pressão para um caudal de vapor de 0.125 kg m^{-2} s^{-1}.

Dados: Área específica do enchimento: $S_B = 656$ m^2/m^3
Porosidade média do leito: e=0.71
Viscosidade do vapor: μ=0.018 cP
Volume molecular = 22.4 m^3/kmol

5. Usa-se uma coluna com enchimento, com 1.22 m de diâmetro e 9 m de altura e cheia com anéis Raschig de 25.4 mm, para destilação sob vácuo de uma mistura de isómeros de peso molecular 155 g/mol. A temperatura média é 100 °C , a pressão no condensador é mantida a 0.13 kN/m^2 e a pressão no destilador é 33 kN/m^2. Obtenha uma expressão para a queda de pressão supondo que não é apreciavelmente afectada pelo caudal de líquido e que pode ser calculada usando uma forma modificada da equação de Ergun. Mostrar que na gama de pressões usadas, a queda de pressão é aproximadamente directamente proporcional ao caudal em massa de vapor e calcule aproximadamente a o caudal em massa de vapor para as condições de operação da coluna.

6. Dois líquidos orgânicos sensíveis ao calor (peso molecular médio=155 g/mol) vão ser separados por destilação sob vácuo numa coluna com 10 cm de diâmetro cheia com anéis Raschig cerâmicos de 6.35 mm (0.25 polegadas). O número de pratos teóricos necessários é 16 e verificou-se que o HEPT é 150 mm. Se o caudal de produto for 5 g/s com um quociente de refluxo de 8, calcular a pressão no condensador para que a temperatura no destilador não exceda 395 K (equivalente a uma pressão de 8 kN/m^2). Considere que S=800 m^2/m^3, μ=0.02 cP, e=0.72 e despreze as variações de temperatura e a correcção para o caudal de líquido.