

AM 1 - PO  
Resolução Ficha 3 e 4

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## Part I

### Ficha 3

7 - d)  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^2+k}{n^3+1}$  incompleto

$$\Rightarrow \lim_{n \rightarrow \infty} n \frac{n^2+1}{n^3+1} \leq \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^2+k}{n^3+1} \leq \lim_{n \rightarrow \infty} n \frac{n^2+n}{n^3+1} \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow \infty} n \frac{1/n + 1/n^2}{1 + 1/n^2} \leq \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^2+k}{n^3+1} \leq \lim_{n \rightarrow \infty} n \frac{1/n^2 + 1/n^3}{1 + 1/n^4}$$

### Questão 8

8 - a)  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{(2n+(-1)^n)}\right)^n$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{(2n+1)}\right)^n &\leq \lim_{n \rightarrow \infty} \left(1 + \frac{1}{(2n+(-1)^n)}\right)^n \leq \\ &\leq \lim_{n \rightarrow \infty} \left(1 + \frac{1}{(2n-1)}\right)^n \Rightarrow \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{(2n+1)}\right)^{2n+1}\right)^{\frac{n}{2n+1}} \leq \\ &\leq \lim_{n \rightarrow \infty} \left(1 + \frac{1}{(2n+(-1)^n)}\right)^n \leq \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{(2n-1)}\right)^{2n-1}\right)^{\frac{n}{2n-1}} \Rightarrow \\ \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{(2n+(-1)^n)}\right)^n &= \sqrt{e} \end{aligned}$$

$$8 - b) \quad \lim_{n \rightarrow \infty} \left(1 + \frac{2+(-1)^n}{n}\right)^{\sqrt{n}}$$

$$\begin{aligned} &\Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}} \leq \lim_{n \rightarrow \infty} \left(1 + \frac{2+(-1)^n}{n}\right)^{\sqrt{n}} \leq \lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^{\sqrt{n}} \Rightarrow \\ &\Rightarrow \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n\right)^{\sqrt{n}/n} \leq \lim_{n \rightarrow \infty} \left(1 + \frac{2+(-1)^n}{n}\right)^{\sqrt{n}} \leq \\ &\leq \lim_{n \rightarrow \infty} \left(\left(1 + \frac{3}{n}\right)^n\right)^{\sqrt{n}/n} \Rightarrow (e^3)^0 \leq \lim_{n \rightarrow \infty} \left(1 + \frac{2+(-1)^n}{n}\right)^{\sqrt{n}} \leq (e^1)^0 \Rightarrow \\ &\Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{2+(-1)^n}{n}\right)^{\sqrt{n}} = 1 \end{aligned}$$

## Part II

### Ficha 4

**Questão 1** Determine os sublimites das seguintes sucessões limitadas indicando os seus limites superior e inferior

$$1 - a) \quad \cos(n\pi/4)$$

$$\cos(n\pi/4) \in \{-1, -\sqrt{2}/2, 0, \sqrt{2}/2, 1\} \quad \forall n \in \mathbb{N};$$

$$\overline{\lim} a_n = 1; \quad \underline{\lim} a_n = -1$$

$$1 - b) \quad (1 + (-1)^n/n)^n$$

$$\text{sublimites} = \{e, e^{-1}\};$$

$$\overline{\lim} b_n = e; \quad \underline{\lim} b_n = e^{-1}$$

**1 - c)**  $n \sin \left( \frac{1+(-1)^n}{n} \right)$  **Incompleto**

$$n \sin \left( \frac{1+(-1)^n}{n} \right) = \begin{cases} n \sin (2/n) & \forall n \text{ par} \\ n \sin (0/n) & \forall n \text{ impar} \end{cases} ;$$

$$\lim_{n \rightarrow \infty} n \sin(0) = 0;$$

$$\lim_{n \rightarrow \infty} n \sin 2 = \lim_{n \rightarrow \infty} \frac{2/n}{2/n} 2 = 2 \implies \text{sublim} = \{0, 2\}$$

...

**1 - d)**  $\arctan((-1)^n n)$

$$\arctan((-1)^n n) = \begin{cases} \arctan(n) & \forall n \text{ par} \\ \arctan(-n) & \forall n \text{ impar} \end{cases} \implies$$

$$\implies \begin{cases} \lim_{n \rightarrow \infty} \arctan(n) = \pi/2 \\ \lim_{n \rightarrow \infty} \arctan(-n) = -\pi/2 \end{cases} \implies$$

$$\implies \text{sublim}(e_n) = \{-\pi/2, \pi/2\}; \overline{\lim} e_n = \pi/2; \underline{\lim} e_n = -\pi/2$$