Chapter 3.
Motion of particles in a fluid

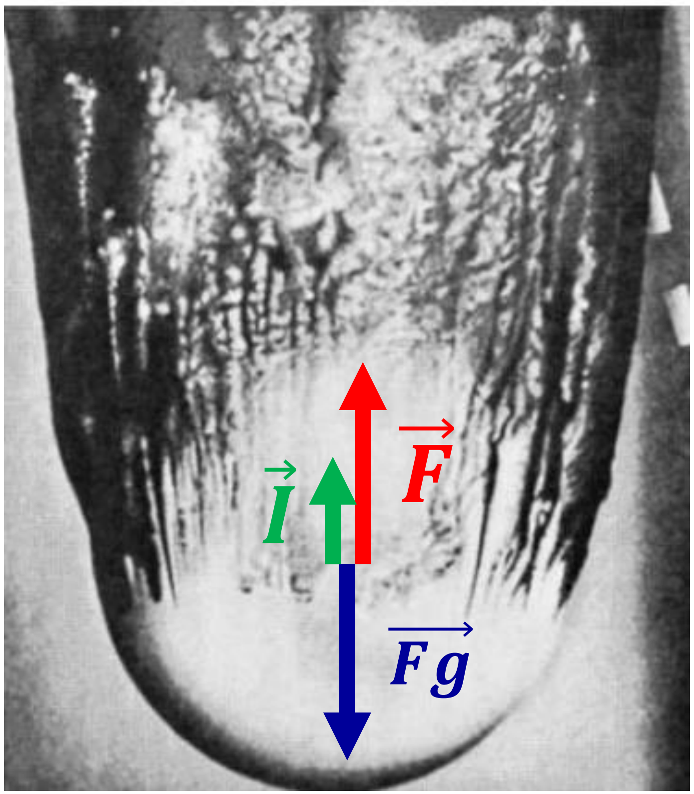
3.1 Free fall of a sphere in a fluid
3.2 Skin friction and form drag, Stoke's law
3.3 Friction factor over particle Re', Newton's law
3.4 Terminal fall velocity
3.5 Elutriation: single column and multistage
3.6 Extension to non-spherical particles, drops and bubbles
(***3.7*** Transient motion of particles) (later in the centrifugation chapter)

J.M. Coulson and J.F. Richardson pp 146 - 190

NOVA SCHOOL OF SCIENCE & TECHNOLOGY

Sphere free fall in a fluid

Consider a single spherical particle with diameter, *d* (m), and specific mass, ρ_S (kg/m^3) settling at a velocity, *u* (m/s), in a stationary fluid with specific mass, ρ (kg/m^3) and viscosity μ $(Pa.s)$. **Which are the forces acting on the shere**?



$\vec{F}$ – Drag force [PT: **força de atrito** ou força de arrasto]

$\vec{I}$ – Buoyancy force [PT: força de impulsão]

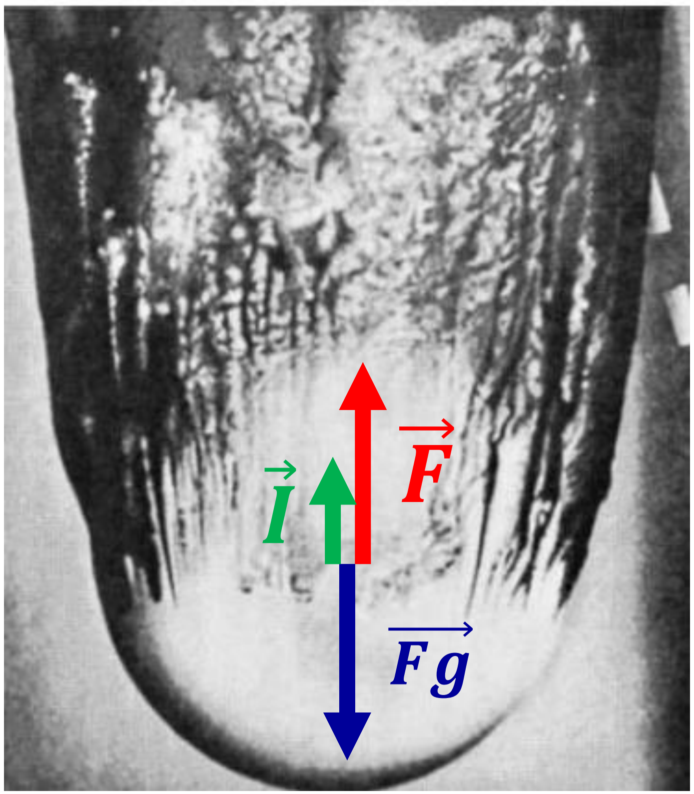
$\vec{Fg}$– Gravity force [PT: força gravítica]

$(\vec{Fg}-\vec{I}$= apparent weight [PT: peso aparente])

$$\vec{Fg}+\vec{I}+\vec{F}=0 : uniform\ movement\ (a=0)$$
$$\vec{Fg}+\vec{I}+\vec{F}\neq 0 : acelerated\ movement\ (a\neq 0)$$

Sphere free fall in a fluid

Consider a single spherical particle with diameter, *d* (m), and specific mass, ρ_S (kg/m^3) settling at a velocity u (m/s) in a stationary fluid with specific mass, ρ (kg/m^3) and viscosity μ $(Pa.s)$. **Which are the forces acting on the shere**?



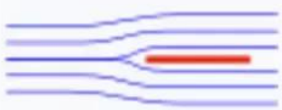
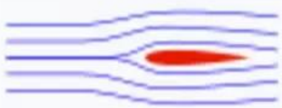
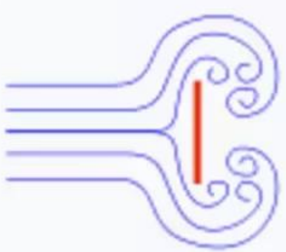
$\vec{F}$ – Drag force (in the following slides)

$\vec{Fg}$– Gravity force: $$F_g = \frac{\pi d^3}{6}\rho_s g$$

$\vec{I}$ – Buoyancy force: $$I = \frac{\pi d^3}{6}\rho g$$

$\vec{Fg}$– $\vec{I}$= apparent weight: $$F_{g,a} = \frac{\pi d^3}{6}(\rho_s - \rho)g$$

Drag force: skin *versus* form drag

Shape and flow	Form Drag	Skin friction
	0%	100%
	~10%	~90%
	~90%	~10%
	100%	0%

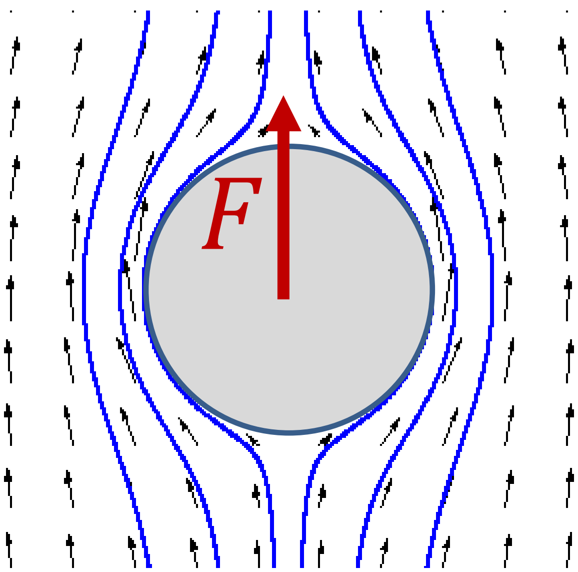
Let's consider the flow of a fluid around a solid body. The fluid will exert 2 types of drag forces on the body. These two forces always occur simultaneously although with different degrees:

Skin friction (atrito de superfície) is due to the shear stress of a viscous fluid on the body surface.

Drag form (atrito de forma) is caused by the shape and size of the body. Pressure variations between the head and back of the body appear, causing the form drag force.

Stoke's law

Stoke's law applies to the theoretical case of skin friction, which is predominant in streamline laminar flow. It was deduced from *ab initio* First principles by Stokes



$$F = 3\pi\mu u d \quad \text{[N]}$$

Particle Reynolds - Re'

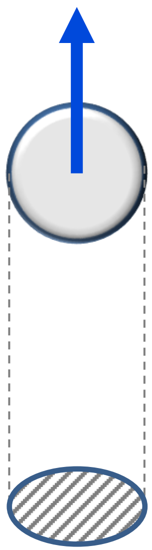
$$Re' = \frac{\rho u d}{\mu} < 0.2 \quad \text{(laminar flow)}$$

μ – Fluid viscosity (Pa.s)
u – relative velocity fluid/sphere (m/s)
d – sphere diameter (m)

Friction factor $\frac{R'}{\rho u^2}$ over particle Re'

Definition of Friction factor (dimensionless)

Sphere



Projected area in a plan

$$\frac{\pi d^2}{4}$$

$$F$$

Drag force (**N**)

$$\div\ \pi d^2/4$$

$$R' = \frac{F}{\pi d^2/4}$$

Drag force per unit of projected area (**N/m2**)

$$\div\ \rho u^2$$

$$\boxed{\frac{\boldsymbol{R'}}{\boldsymbol{\rho u^2}} = \frac{\boldsymbol{F}}{\boldsymbol{\rho u^2(\pi d^2/4)}}}$$

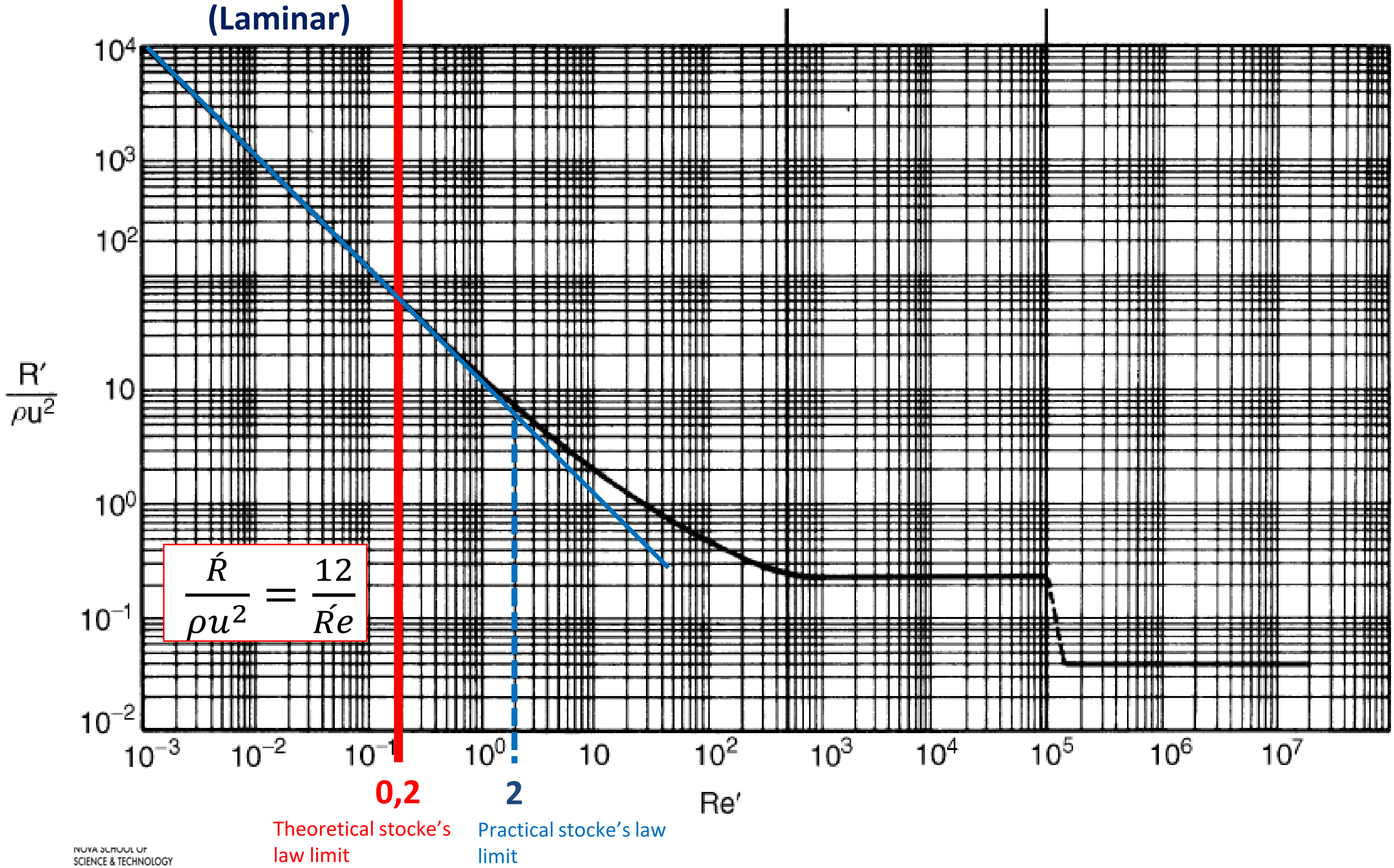
Friction factor (dimensionless)

$$\times 2$$

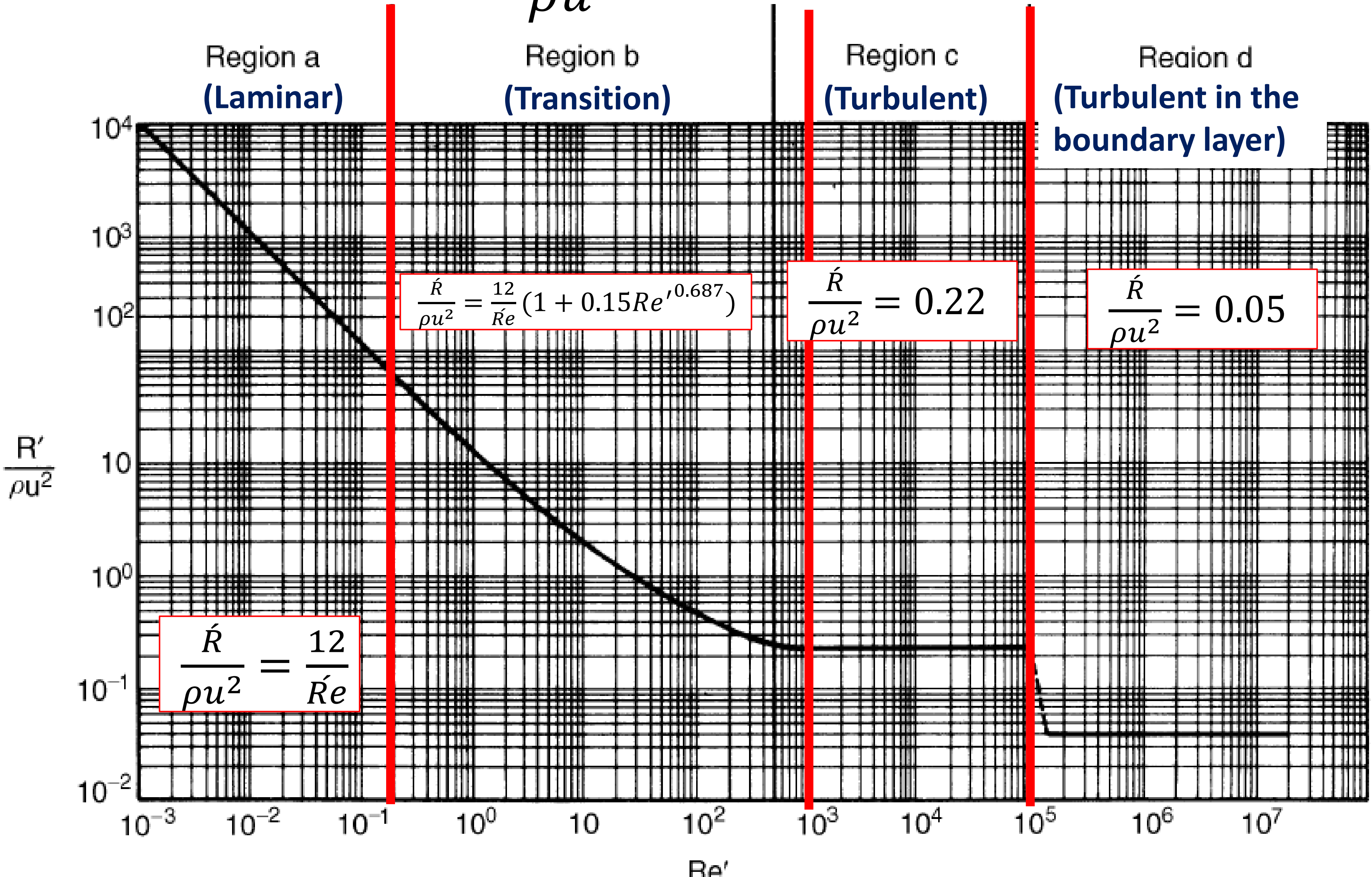
$$C_D = 2\frac{R'}{\rho u^2}$$

Drag coefficient (**dimensionless**)

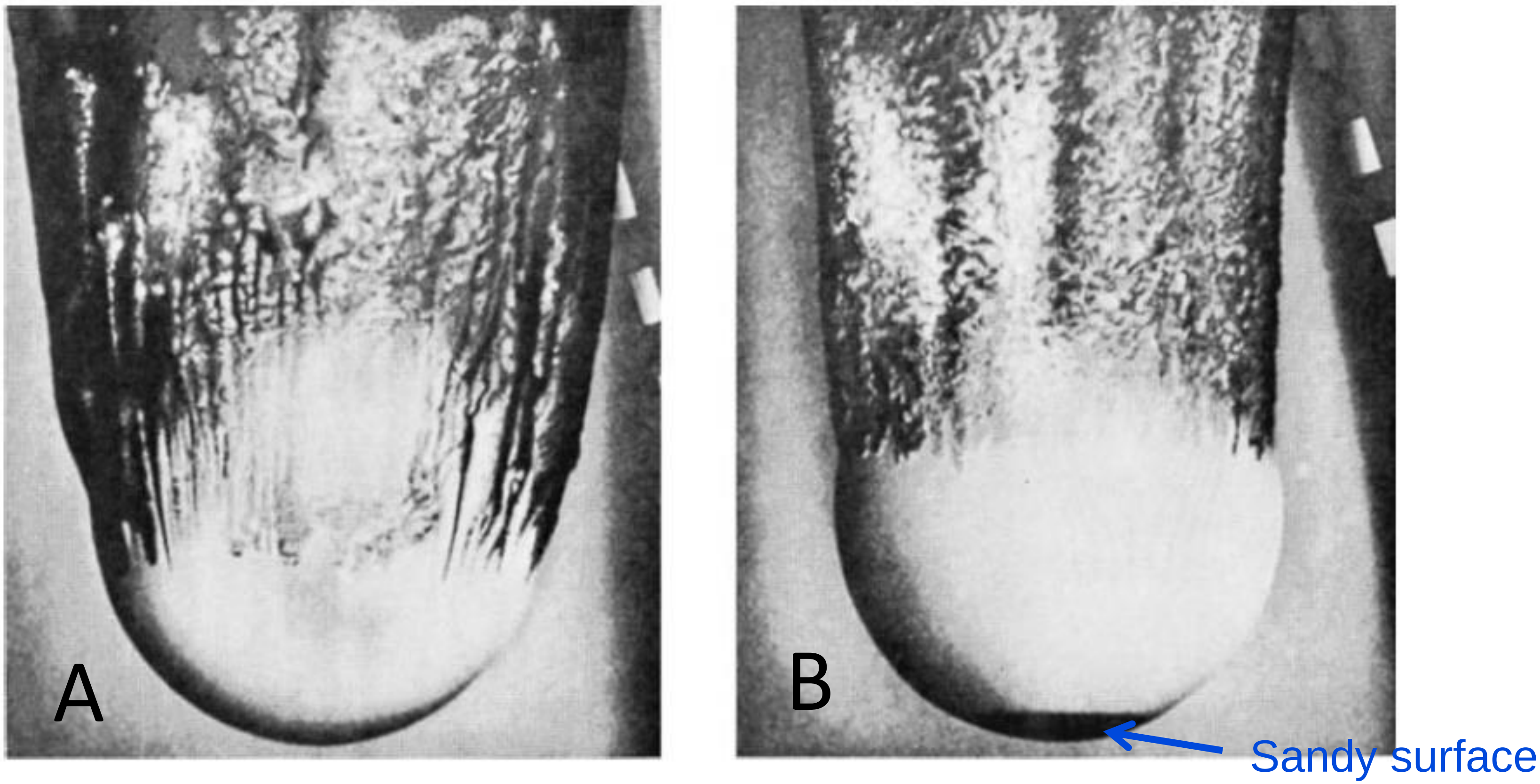
Friction factor $\frac{R'}{\rho u^2}$ over particle Re'



Friction factor $\frac{R'}{\rho u^2}$ over particle Re'



It has been shown that surface roughness causes a drag force drop for the same Reynolds numbers. **Why?**



Sandy surface at the head of the particle (experiment B) disrupts streamline flow resulting in higher turbulence than case A. Less eddies are formed in the back of the sphere in experiment B, causing a total drag force reduction in comparison to experiment A

Friction factor $\frac{R'}{\rho u^2}$ over particle Re'

Laminar:	$10^{-4} < R_e' \leq 0.2$	$\frac{R'}{\rho u^2} = \frac{12}{Re'}$	(skin friction)
Transition:	$0.2 < R_e' \leq 10^3$	$\frac{R'}{\rho u^2} = \frac{12}{Re'}(1 + 0.15Re'^{0.687})$	
Turbulent:	$10^3 < R_e' \leq 10^5$	$\frac{R'}{\rho u^2} = 0.22$	(form drag)
Turbulent inside the boundary layer:	$10^5 < R_e'$	$\frac{R'}{\rho u^2} = 0.05$	(form drag)

Drag force: stoke's law

The drag factor over Re' in laminar flow is mathematically equivalente to the stoke's law

$$\frac{F}{(\pi d^2/4)\,\rho u^2} = \frac{3\pi\mu u d}{(\pi d^2/4)\,\rho u^2}$$

$$\Leftrightarrow$$

$$\boxed{\frac{\acute{R}}{\rho u^2} = \frac{12}{\acute{R}e}}$$

Dimensionless

Drag force: Newton's law

Newton's law is valid in turbulent flow and was deduced from the experimentally determined curve of friction factor over Re'

Turbulent: $10^3 \leq \acute{R}_e \leq 10^5$ (form drag)

$$\frac{\acute{R}}{\rho u^2} = 0.22$$

$$\times \pi d^2/4 \times \rho u^2$$

$$\boxed{F = 0.055\pi d^2 \rho u^2}$$

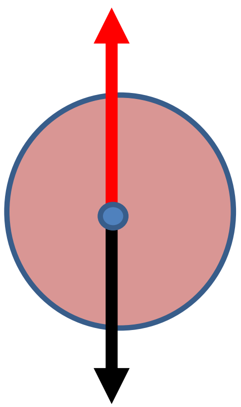
Note there is no viscosity in Newton's law!!!

Terminal fall velocity, u_0

If the drag force equals the apparent weight of the sphere then the accelaration is zero and the sphere settles at a constant velocity u_0:

$\vec{F}$ – Drag force

$$F = \frac{\pi d^3}{6}(\rho_s - \rho)g$$



Apparent weight

$$\frac{\pi d^3}{6}(\rho_s - \rho)g$$

For laminar flow ($\acute{R}e < 0{,}2$), then stoke's law holds:

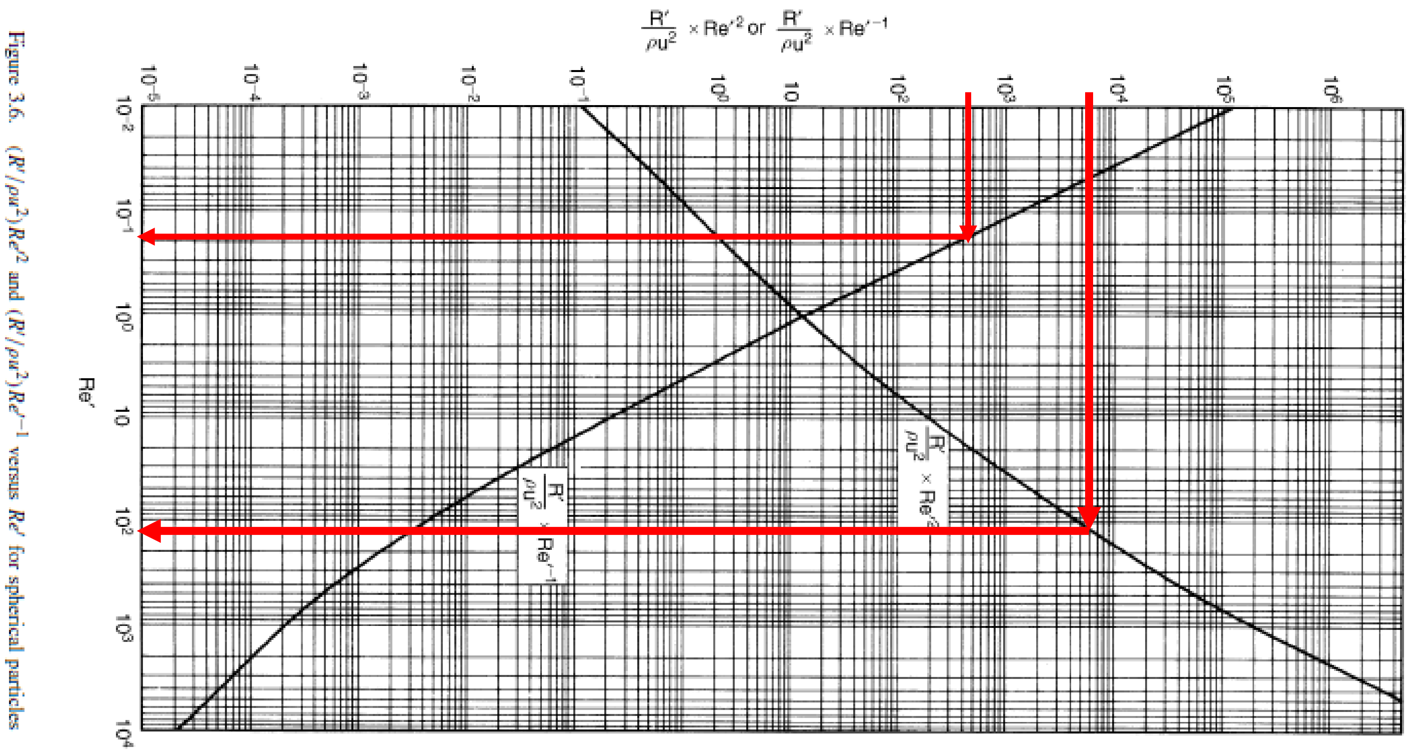
$$3\pi\mu u_0 d = \frac{\pi d^3}{6}(\rho_s - \rho)g \qquad \boxed{u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu}} \qquad \acute{R}e < 0{,}2$$

For turbulent flow ($10^3 \le \acute{R}e \le 10^5$), then Newton's law holds:

$$0.055\pi d^2 \rho u_0^2 = \frac{\pi d^3}{6}(\rho_s - \rho)g \qquad \boxed{u_0 = \sqrt{\frac{3d(\rho_s - \rho)g}{\rho}}}$$

$$10^3 \le \acute{R}e \le 10^5$$

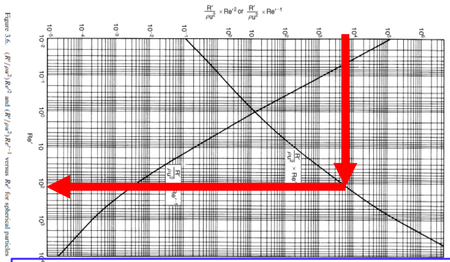
Terminal fall velocity, u_0: graphical method



Taken from **J.M. Coulson and J.F. Richardson (1965) pp. 158**

Terminal fall velocity, u_0: graphical method

Case 1. Condition for terminal fall velocity: $F = \frac{\pi d_i^3}{6}(\rho_s - \rho)g \Leftrightarrow$

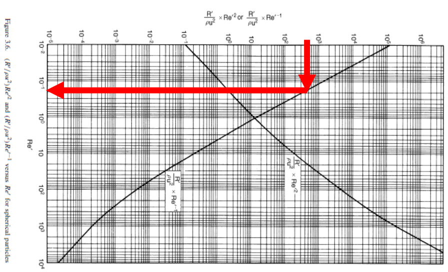


By manipulating and rearranging it may be shown:

$$\Leftrightarrow \frac{\acute{R}}{\rho u^2}\acute{R}e^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2} = \frac{2}{3}Ga$$

If d is known $\Rightarrow$ calculate $\frac{\acute{R}}{\rho u^2}\acute{R}e^2$ $\Rightarrow$Take from picture $\acute{R}e$ $\Rightarrow$take from $\acute{R}e$ the value of u_0

Case 2. Condition for terminal fall velocity: $F = \frac{\pi d_i^3}{6}(\rho_s - \rho)g \Leftrightarrow$



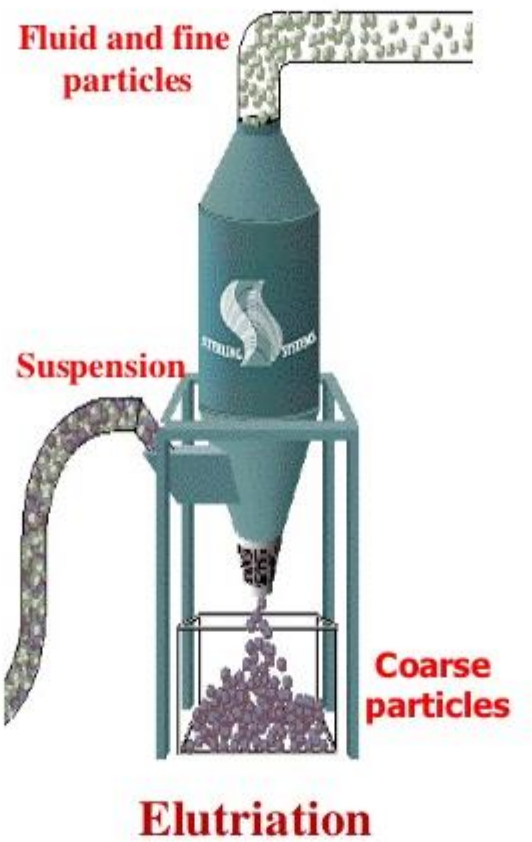
By manipulating and rearranging it may be shown:

$$\Leftrightarrow \frac{\acute{R}}{\rho u^2}\acute{R}e^{-1} = \frac{2(\rho_s - \rho)\mu g}{3\rho^2 u^3}$$

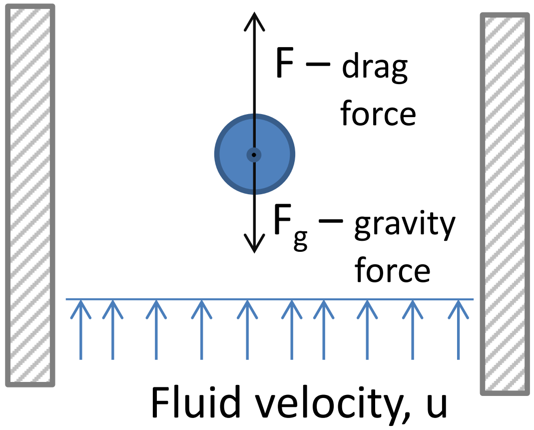
If u_0 is known $\Rightarrow$ calculate $\frac{\acute{R}}{\rho u^2}\acute{R}e^{-1}$ $\Rightarrow$Take from picture $\acute{R}e$ $\Rightarrow$take from $\acute{R}e$ the value of d

Ga – Galileo number (dimensionless)

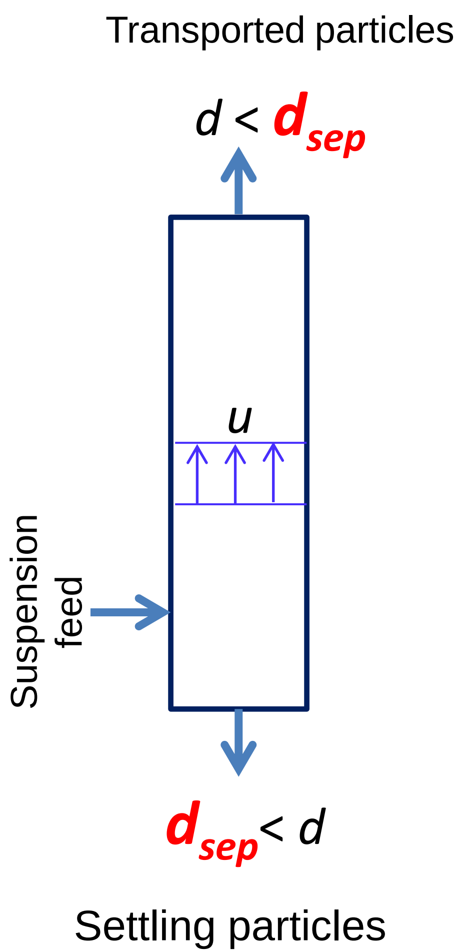
Elutriation



<u>Elutriation</u> is a particle classification and/or separation process based on the density and size of particles through the motion of a carrying fluid (gas or liquid). The smaller and less dense particles will be dragged out on the top of the column (**fine particles stream**). The bigger and more dense particles will settle in the bottom of the column (**coarse particles stream**).



Elutriation: single column



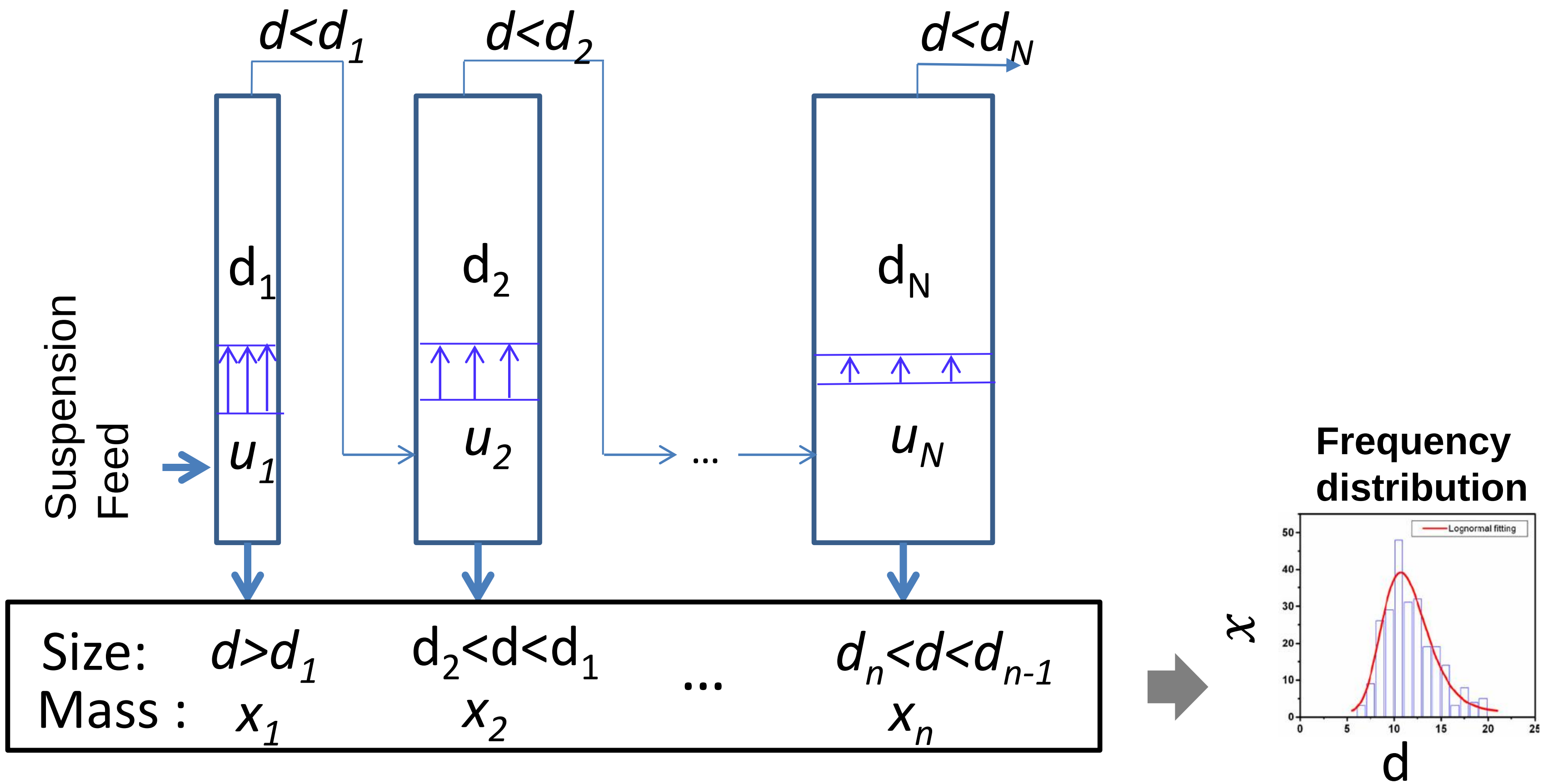
- Elutriation operates in the range 1 - 50 µm
- Thus operation is typically laminar, $\acute{R}e < 0{,}2$
- d_{sep} is the critical separation size of solids
- How to determine d_{sep} ?
- Particles (be it spheres) going up? $F > \dfrac{\pi d_{sep}^3}{6}(\rho_s - \rho)g$
- Particles going down? $F < \dfrac{\pi d_{sep}^3}{6}(\rho_s - \rho)g$
- Critical separation size (note that stoke's law holds):

$$F = 3\pi\mu u d_{sep} = \frac{\pi d_{sep}^3}{6}(\rho_s - \rho)g$$ (particles staying in the column)

$$\boxed{d_{sep} = \sqrt{\frac{18\mu u}{(\rho_s - \rho)g}}} \qquad \acute{R}e < 0{,}2$$

Elutriation: multi-stage with *N* columns

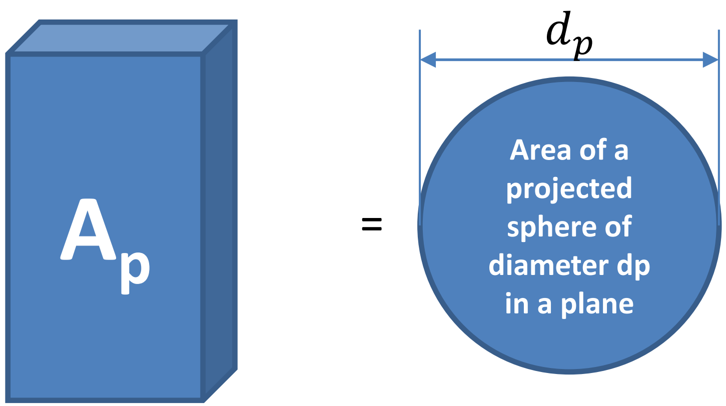
Cross section area increases from column 1 to *N*: $A_1 < A_2 < \cdots < A_N$
Fluid velocity decreases from column 1 to *N*: $u_1 > u_2 > \cdots > u_N$
Critical separation size decreases from column 1 to *N*: $d_1 > d_2 > \cdots > d_N$



Non-spherical geometry: Heywood method

Heywood method (Coulson pp. 166) is a 6-steps procedure:

Step 1. Determine the mean projected diameter of particle, d_p



$$A_p = \frac{\pi d_p^2}{4} \Leftrightarrow \boxed{d_p = \sqrt{\frac{4A_p}{\pi}}}$$

Which face to choose? The one with largest Ap !

Step 2. Determine the volume factor, k'

$$V_p = \acute{k} d_p^3 \Leftrightarrow \boxed{\acute{k} = \frac{V_p}{d_p^3}}$$

Non-spherical geometry: Heywood method

Step 3. Redo force balances for non-spherical geometry in its dimensionless form

$$\acute{R}A_p = V_p\,(\rho_s - \rho)g \Leftrightarrow \qquad \acute{R}\frac{\pi d_p^2}{4} = \acute{k}d_p^3(\rho_s - \rho)g$$

$$\Rightarrow \boxed{\frac{\acute{R}}{\rho u^2}\acute{R}e^2 = \frac{4\acute{k}\rho d_p^3(\rho_s - \rho)g}{\mu^2\pi}}$$

$$\Rightarrow \boxed{\frac{\acute{R}}{\rho u^2}\acute{R}e^{-1} = \frac{4\acute{k}\mu(\rho_s - \rho)g}{\rho^2\pi u^3}}$$

Step 4. Determine Reynolds, $log_{10}(\acute{Re})$, from Figure 3.6 or Table 3.4-3.5 as if a spherical particle (pp. 157, 158, 161)

Non-spherical geometry: Heywood method

Step 5. Additive corrections of $log_{10}(R\acute{e})$ obtained in **step 4** (spherical particals) using Tables 3.7-3.8 due to non-spherical geometry (pp. 166-167)

Table 3.7. Corrections to $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{2}$	−0.022	−0.002	+0.032	+0.131
$\bar{1}$	−0.023	−0.003	+0.030	+0.131
0	−0.025	−0.005	+0.026	+0.129
1	−0.027	−0.010	+0.021	+0.122
2	−0.031	−0.016	+0.012	+0.111
2.5	−0.033	−0.020	0.000	+0.080
3	−0.038	−0.032	−0.022	+0.025
3.5	−0.051	−0.052	−0.056	−0.040
4	−0.068	−0.074	−0.089	−0.098
4.5	−0.083	−0.093	−0.114	−0.146
5	−0.097	−0.110	−0.135	−0.186
5.5	−0.109	−0.125	−0.154	−0.224
6	−0.120	−0.134	−0.172	−0.255

Table 3.8. Corrections to $\log Re'$ as a function of $\{\log(R'/\rho u^2)Re'^{-1}\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^{-1}\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{4}$	+0.185	+0.217	+0.289	
$\bar{4}.5$	+0.149	+0.175	+0.231	
$\bar{3}$	+0.114	+0.133	+0.173	+0.282
$\bar{3}.5$	+0.082	+0.095	+0.119	+0.170
$\bar{2}$	+0.056	+0.061	+0.072	+0.062
$\bar{2}.5$	+0.038	+0.034	+0.033	−0.018
$\bar{1}$	+0.028	+0.018	+0.007	−0.053
$\bar{1}.5$	+0.024	+0.013	−0.003	−0.061
0	+0.022	+0.011	−0.007	−0.062
1	+0.019	+0.009	−0.008	−0.063
2	+0.017	+0.007	−0.010	−0.064
3	+0.015	+0.005	−0.012	−0.065
4	+0.013	+0.003	−0.013	−0.066
5	+0.012	+0.002	−0.014	−0.066

Step 6. Obtain $\boldsymbol{u_o}$ or ***d*** from $log_{10}(R\acute{e})$ obtained in **step 5**

Bubbles and Drops

Consider an Air Bubble with diameter, *d,* freely rising in water. When the forces are equal, the buble will rise at a constant velocity, u_0. In laminar flow stoke's law is applied with Hardmard correction to compensate for shape variations and internal recirculations.

$$\frac{\pi d^3}{6}(\rho_{air}-\rho)g<0$$

F

$$u_0 = \frac{d^2(\rho_s-\rho)g}{18\mu} \boldsymbol{Q}$$

$$1 < \boldsymbol{Q} = \frac{3\mu+3\mu_{Air}}{2\mu+3\mu_{Air}} < 1.5$$

Hardmard correction to stoke's law valid only in laminar conditions