AM3C –

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Conteúdo

Questão 6

Determine a deflexão $u(x,t)$ da corda vibrante de comprimento $L=\pi$, extremidades fixas, com $c^2 = 1$ supondo uma velocidade inicial igual a zero e com uma deflexão inicial dada por $f(x) = 0.01\,x\,(\pi{-}x)$.

Resposta

$$u(x,0) = f(x) = 0.01\,x\,(\pi - x);$$

$$\frac{\mathrm{d}^2 u}{\mathrm{d}t^2} = c^2\,\frac{\mathrm{d}^2 u}{\mathrm{d}x^2};$$

$$u(x,t) = \sum_{h=1}^{+\infty} A_h\,\sin(h\,x)\,\cos(h\,t);$$

$$A_h = \frac{2}{\pi}\int_0^{\pi} f(x)\,\sin(h\,x)\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi}\left(0.01\,x\,(\pi - x)\right)\sin(h\,x)\,\mathrm{d}x =$$
$$= \frac{0.02}{\pi}\left(\pi\int_0^{\pi} x\,\sin(h\,x)\,\mathrm{d}x - \int_0^{\pi} x^2\,\sin(h\,x)\,\mathrm{d}x\right)$$

$$\pi\int_0^{\pi} x\,\sin(h\,x)\,\mathrm{d}x = \pi\left(-\frac{x}{h}\cos(h\,x) + \frac{1}{h^2}\,\sin(h\,x)\right)\Bigg|_0^{\pi} = \cdots = (-1)^{h+1}\,\frac{\pi^2}{h} \tag{1}$$

$$-\int_0^{\pi} x^2\,\sin(h\,x)\,\mathrm{d}x = \cdots = -\left(-\frac{x^2}{h}\,\cos(h\,x) + \frac{2}{h^2}\,x\,\sin(h\,x) + \frac{2}{h^3}\,\cos(h\,x)\right)\Bigg|_0^{\pi} =$$
$$= -\begin{pmatrix} -\frac{x^2}{h}\,\cos(h\,\pi) + \frac{2}{h^2}\,\pi\,\sin(h\,\pi) + \frac{2}{h^3}\,\cos(h\,\pi) \\ -\frac{x^2}{h}\,\cos(h*0) + \frac{2}{h^2}*0*\sin(h*0) + \frac{2}{h^3}\,\cos(h*0) \end{pmatrix} =$$
$$= -\left((-1)^{n+1}\,\frac{\pi}{h} + \frac{2}{h^3}\,((-1)^n - 1)\right) = \begin{cases} +\pi^2/h & n \text{ par} \\ -\pi^2/h + 4/h^3 & n \text{ impar} \end{cases} \tag{2}$$