

# AM3C –

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## Conteúdo

Questão 6 . . . . . 2

## Questão 6

Determine a deflexão  $u(x, t)$  da corda vibrante de comprimento  $L = \pi$ , extremidades fixas, com  $c^2 = 1$  supondo uma velocidade inicial igual a zero e com uma deflexão inicial dada por  $f(x) = 0.01 x (\pi - x)$ .

### Resposta

$$u(x, 0) = f(x) = 0.01 x (\pi - x);$$

$$\frac{d^2 u}{dt^2} = c^2 \frac{d^2 u}{dx^2};$$

$$u(x, t) = \sum_{h=1}^{+\infty} A_h \sin(hx) \cos(ht);$$

$$\begin{aligned} A_h &= \frac{2}{\pi} \int_0^\pi f(x) \sin(hx) dx = \frac{2}{\pi} \int_0^\pi (0.01 x (\pi - x)) \sin(hx) dx = \\ &= \frac{0.02}{\pi} \left( \pi \int_0^\pi x \sin(hx) dx - \int_0^\pi x^2 \sin(hx) dx \right) \end{aligned}$$

$$\pi \int_0^\pi x \sin(hx) dx = \pi \left( -\frac{x}{h} \cos(hx) + \frac{1}{h^2} \sin(hx) \right) \Big|_0^\pi = \dots = (-1)^{h+1} \frac{\pi^2}{h} \quad (1)$$

$$\begin{aligned} - \int_0^\pi x^2 \sin(hx) dx &= \dots = - \left( -\frac{x^2}{h} \cos(hx) + \frac{2}{h^2} x \sin(hx) + \frac{2}{h^3} \cos(hx) \right) \Big|_0^\pi = \\ &= - \left( -\frac{x^2}{h} \cos(h\pi) + \frac{2}{h^2} \pi \sin(h\pi) + \frac{2}{h^3} \cos(h\pi) \right) = \\ &= - \left( -\frac{x^2}{h} \cos(h * 0) + \frac{2}{h^2} * 0 * \sin(h * 0) + \frac{2}{h^3} \cos(h * 0) \right) = \\ &= - \left( (-1)^{n+1} \frac{\pi}{h} + \frac{2}{h^3} ((-1)^n - 1) \right) = \begin{cases} +\pi^2/h & n \text{ par} \\ -\pi^2/h + 4/h^3 & n \text{ impar} \end{cases} \quad (2) \end{aligned}$$

Handwritten mathematical work on green chalkboard showing the derivation of the Fourier coefficients  $A_h$  for the deflection  $u(x, t)$  of a vibrating string.

**Left Column:**

- ① + ② =  $\begin{cases} -\frac{\pi^2}{h} + \frac{\pi^2}{h} = 0 & \text{se } h \text{ par} \\ \frac{\pi^2}{h} - \frac{\pi^2}{h} + \frac{4}{h^3} & \text{se } h \text{ impar} \end{cases}$
- $A_{h=0} = 0$  se  $h$  par
- $A_{2k-1} = \frac{0.02}{\pi} \cdot \frac{4}{(2k-1)^3} = \frac{0.08}{\pi} \cdot \frac{1}{(2k-1)^3}$
- $u(x, t) = \frac{0.08}{\pi} \sum_{k=1}^{+\infty} \frac{1}{(2k-1)^3} \sin[(2k-1)x] \cos[(2k-1)t]$

**Right Column:**

- ①  $= \pi \int_0^\pi x \sin(hx) dx = \pi \left[ -\frac{x}{h} \cos(hx) + \frac{1}{h^2} \sin(hx) \right]_0^\pi = \dots = (-1)^{h+1} \frac{\pi^2}{h}$
- ②  $= - \int_0^\pi x^2 \sin(hx) dx = - \left[ -\frac{x^2}{h} \cos(hx) + \frac{2}{h^2} x \sin(hx) + \frac{2}{h^3} \cos(hx) \right]_0^\pi = \dots = \begin{cases} +\pi^2/h & n \text{ par} \\ -\pi^2/h + 4/h^3 & n \text{ impar} \end{cases}$