Introduction

1 – Batch Reactor (BSTR)

2 – Continuous Reactor (CSTR)

3 – Plug Flow Reactor (PFR)

3.1 - Definition

3.2 - Material balances

3.3 - Kinetics

3.3 - Productivity

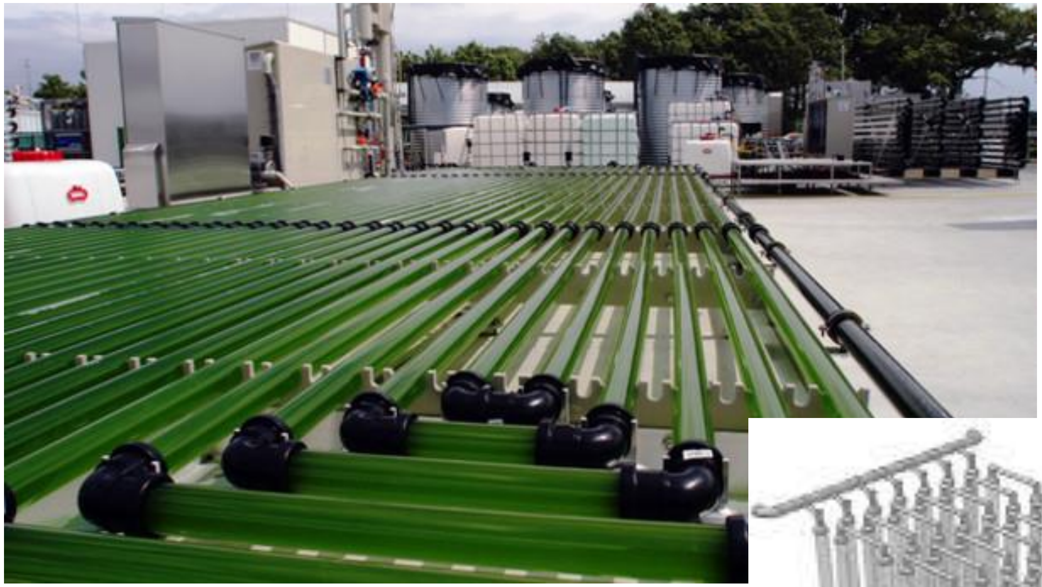
3.4 - PFR/CSTR comparison

3.5 – Discussion about PFR

3 – Plug Flow Reactor

3.1 – Definition

Plug flow photobioreactor



3 – Plug Flow Reactor

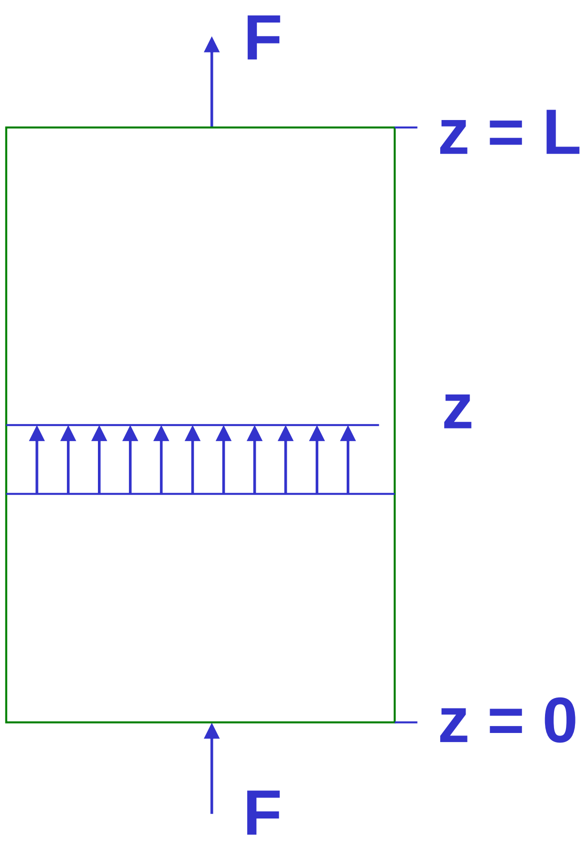
3.1 – Definition

Plug flow anaerobic digestor

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



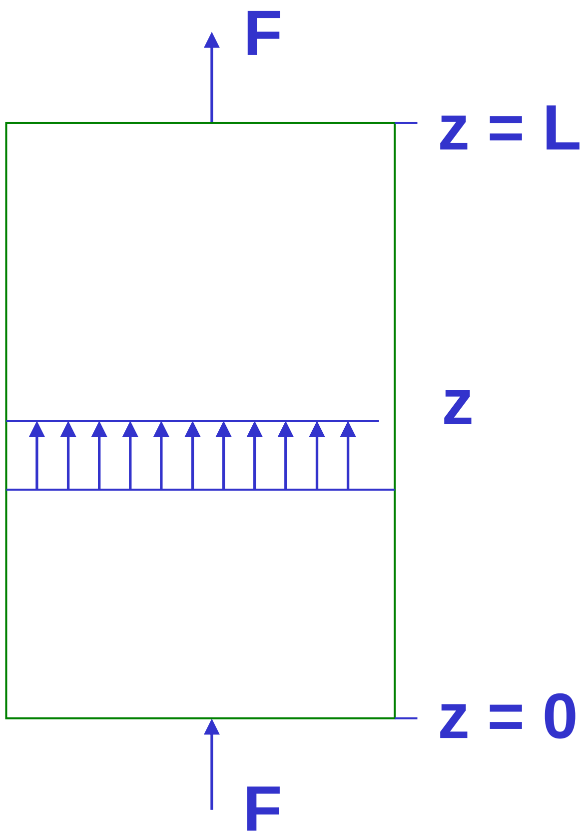
Recovery of the product at the top

Inoculation and introduction of nutrients into the base

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



z – position on a vertical axis (m)

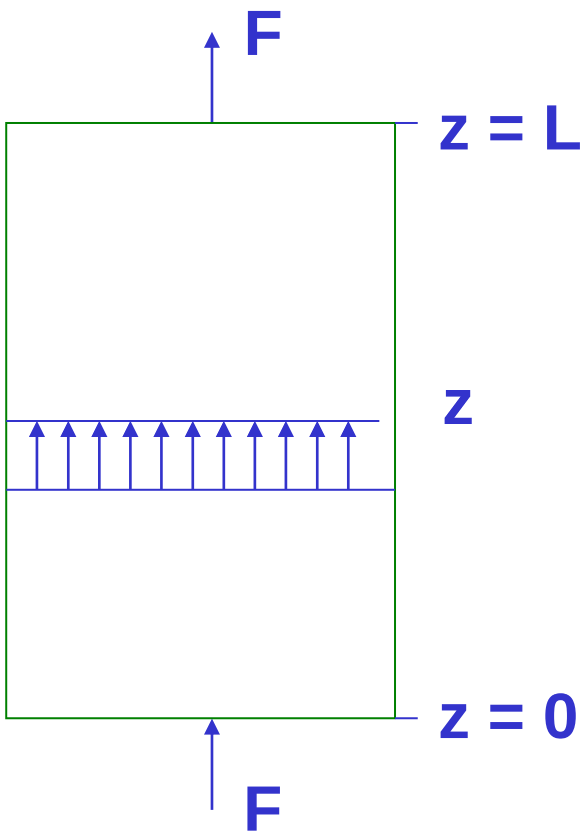
L - column height (m)

F - fluid flow rate in ascending flow (m^3/h)

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



z – position on a vertical axis (m)

L - column height (m)

F - fluid flow rate in ascending flow (m^3/h)

A - cross section area (m^2)

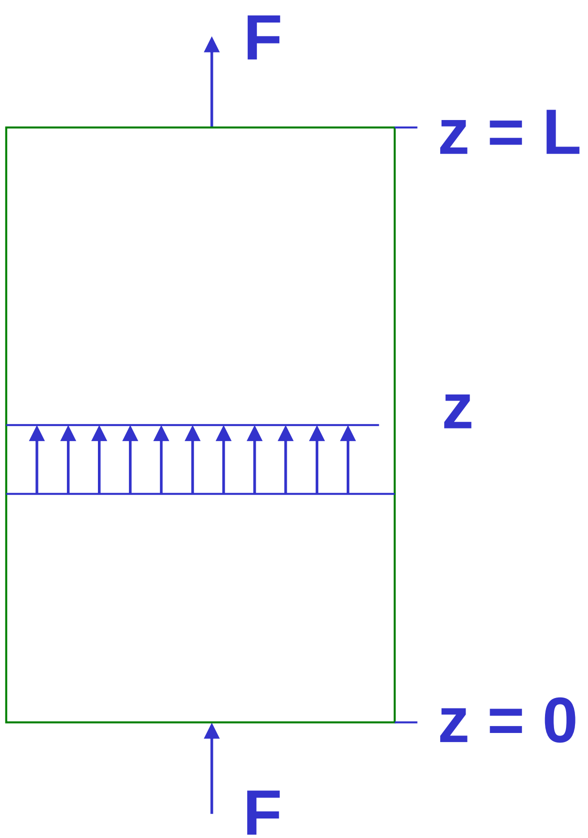
$$A = \frac{\pi\, d^2}{4}$$

d - diameter of the cylindrical column (m)

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



z – position on a vertical axis (m)

L - column height (m)

F - fluid flow rate in ascending flow (m^3/h)

A - cross section area (m^2)

$$A = \frac{\pi\, d^2}{4}$$

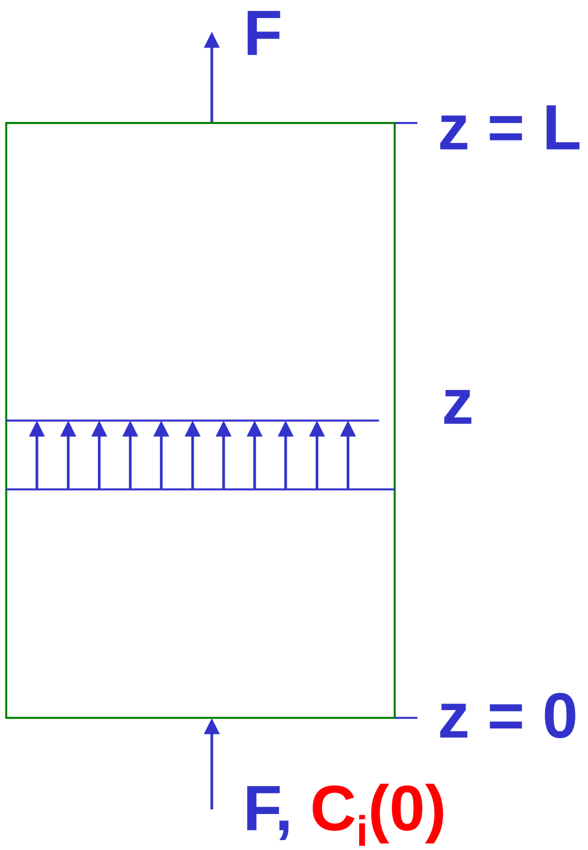
d - diameter of the cylindrical column (m)

V = AL - column volume (m^3)

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous

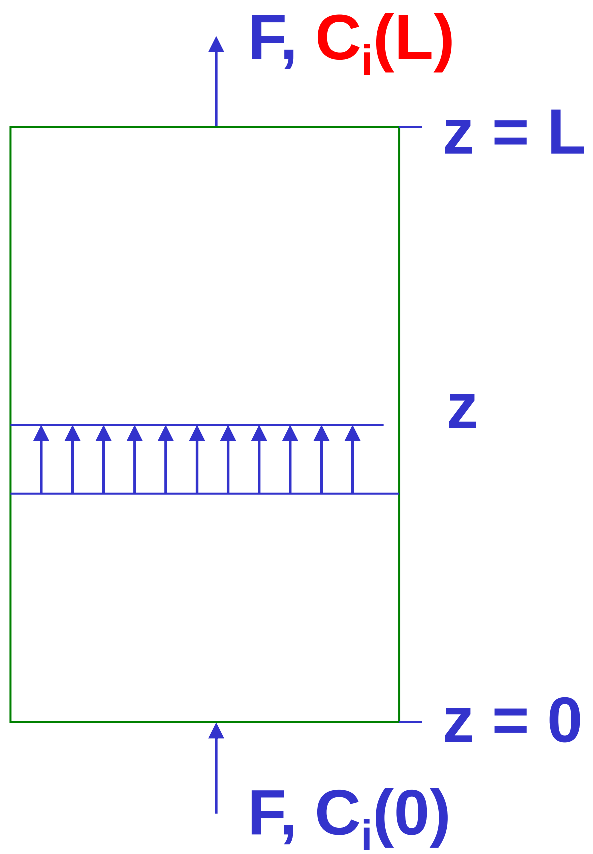


Ci(0) - concentration of a generic i component at the base of the column (kg/m^3)

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



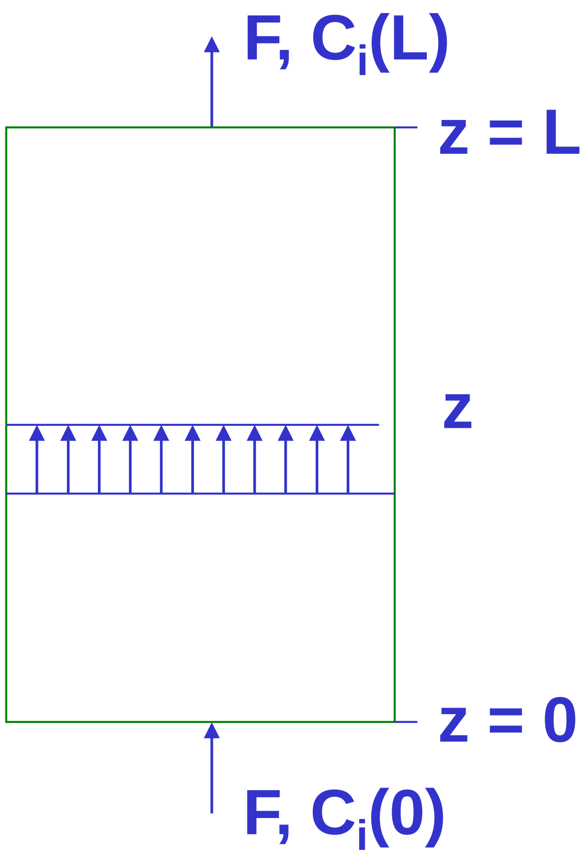
Ci(0) - concentration of a generic i component at the base of the column (kg/m^3)

Ci(L) - concentration of a generic i component at the top of the column (Kg/m^3)

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



Ci(0) - concentration of a generic i component at the base of the column (kg/m^3)

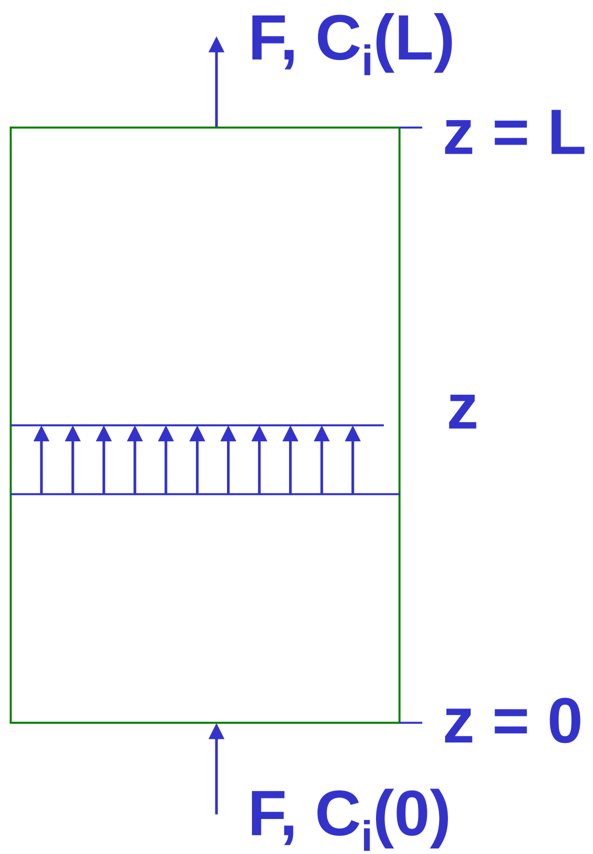
Ci(L) - concentration of a generic i component at the top of the column (Kg/m^3)

Ci(z) - concentration of a generic i component in a z position of the column (Kg/m^3)

3.1 – Definition

Geometry: cylindrical column

Operation: Continuous



Ci(0) - concentration of a generic i component at the base of the column (kg/m^3)

Ci(L) - concentration of a generic i component at the top of the column (Kg/m^3)

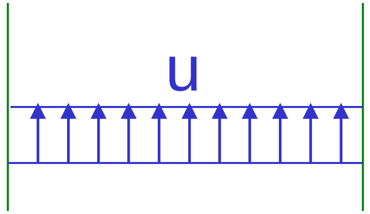
Ci(z) - concentration of a generic i component in a z position of the column (Kg/m^3)

u - axial velocity of the fluid inside the column (m/s)

$$\boxed{u = \frac{F}{A}}$$

3 – Plug Flow Reactor

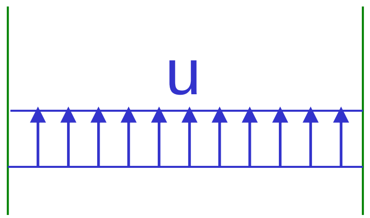
3.1 – Definition



Velocity profile in plug flow **= CONSTANT**

Therefore:
all fluid elements move at the same velocity u.

3.1 – Definition



Velocity profile in plug flow **= CONSTANT**

Therefore:
all fluid elements move at the same velocity u.

⇨ plug flow $$Re = \frac{\rho\, u\, d}{\mu} > 2000$$ (Nº Reynolds)

ρ – Specific mass of fluid (Kg/m^3)

u – axial velocity of the fluid (m/s)

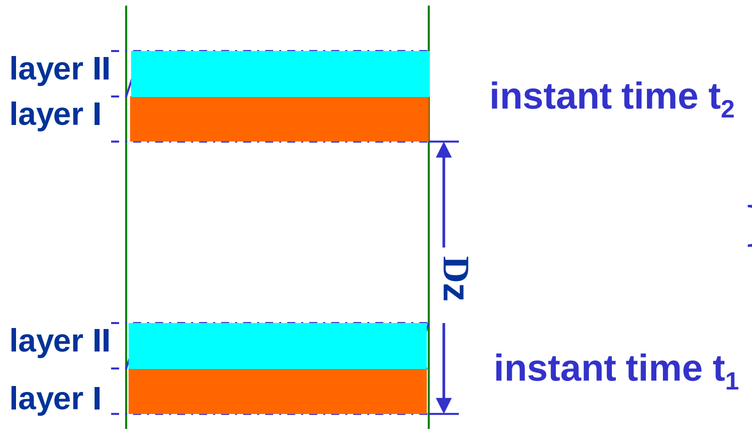
d – column diameter (m)

μ – Viscosity of the fluid (Pa.s)

3 – Plug Flow Reactor

3.1 – Definition

Displacement analysis of 2 adjacent fluid layers



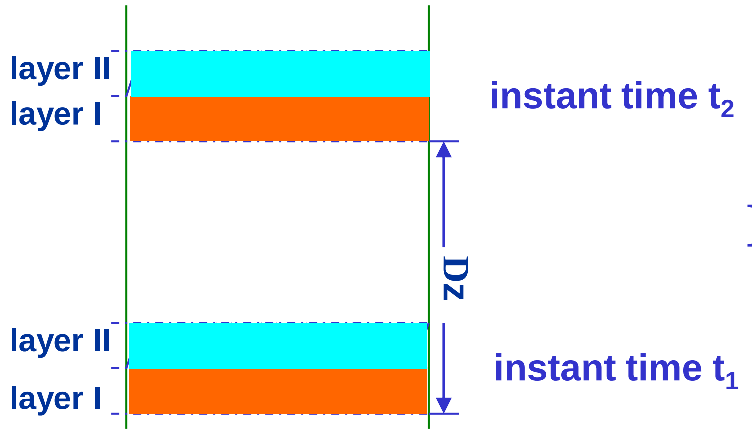
$$Dz = u\,(t_2 - t_1)$$

all fluid elements move at the same velocity u → Two adjacent fluid layer never mix

3 – Plug Flow Reactor

3.1 – Definition

Displacement analysis of 2 adjacent fluid layers

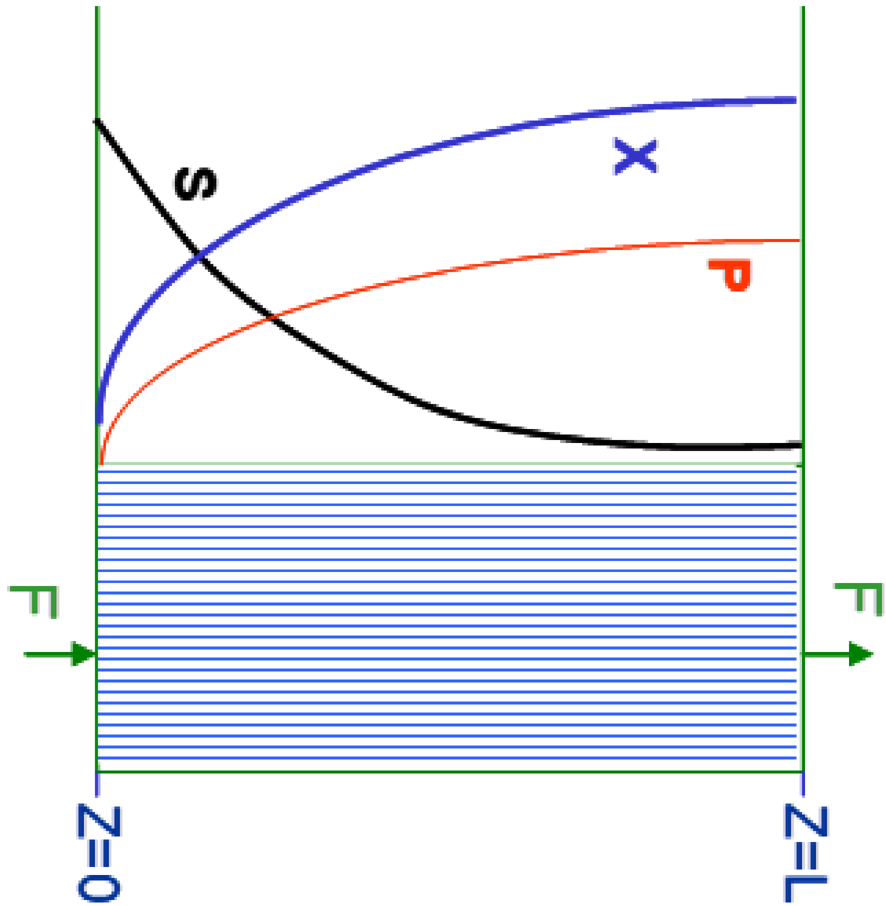


$$Dz = u\,(t_2 - t_1)$$

all fluid elements move at the same velocity u → Two adjacent fluid layer never mix

PFR: TOTAL SEGREGATION
CSTR: PERFECT MIX

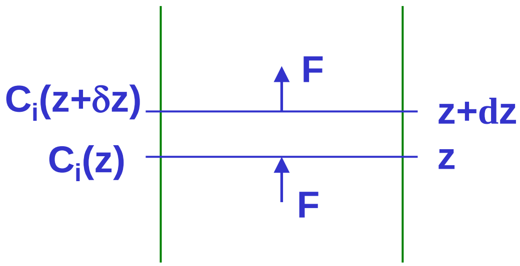
3.1 – Definition



3 – Plug Flow Reactor

3.2 – Material balances

Material balance to the infinitesimal section of the column with height dz a generic ‘i’ component

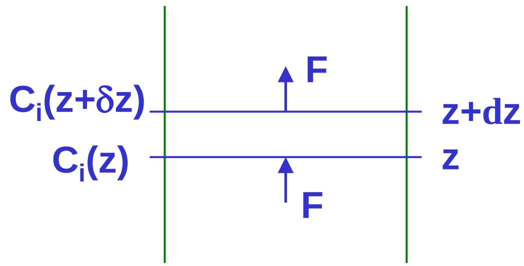


Mass of ‘i’ that enters z per unit of time + **Mass of ‘i’ produced by reaction in Adz volume per unit of time** = **Mass of “i” leaving z+dz per unit of time**

3 – Plug Flow Reactor

3.2 – Material balances

Material balance to the infinitesimal section of the column with height dz a generic ‘i’ component



Mass of ‘i’ that enters z per unit of time + **Mass of ‘i’ produced by reaction in Adz volume per unit of time** = **Mass of “i” leaving z+dz per unit of time**

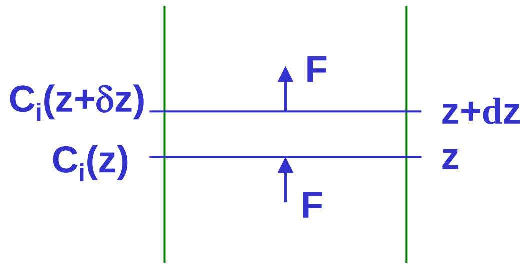
$$FC_i(z) \quad + \quad r_i\,(z)Adz \quad = \quad FC_i\,(z+dz)$$

Note: r_i – volumetric rate of i production

3 – Plug Flow Reactor

3.2 – Material balances

Material balance to the infinitesimal section of the column with height dz a generic ‘i’ component



Mass of ‘i’ that enters z per unit of time + **Mass of ‘i’ produced by reaction in Adz volume per unit of time** = **Mass of “i” leaving z+dz per unit of time**

$$FC_i(z) \quad + \quad r_i\,(z)Adz \quad = \quad FC_i\,(z+dz)$$

It's only true if dz is infinitely small

Note: r_i – volumetric rate of i production

3.2 – Material balances

$$F\, C_i\, (z) + \; r_i\, (z)\, A\, dz \; = F\, C_i\, (z + dz)$$

3.2 – Material balances

$$F\,C_i\,(z) + \; r_i\,(z)\,A\,dz \; = F\,C_i\,(z+dz)$$

$$\Leftrightarrow r_i\,(z)\,A\,dz \; = F\,C_i\,(z+dz) - F\,C_i\,(z)$$

3.2 – Material balances

$$F\,C_i\,(z) + r_i\,(z)\,A\,dz = F\,C_i\,(z+dz)$$

$$\Leftrightarrow r_i\,(z)\,A\,dz = F\,C_i\,(z+dz) - F\,C_i\,(z)$$

$$\Leftrightarrow r_i\,(z) = \frac{F}{A}\,\frac{C_i\,(z+dz) - C_i\,(z)}{dz}$$

3.2 – Material balances

$$F\,C_i\,(z) + r_i\,(z)\,A\,dz = F\,C_i\,(z+dz)$$

$$\Leftrightarrow r_i\,(z)\,A\,dz = F\,C_i\,(z+dz) - F\,C_i\,(z)$$

$$\Leftrightarrow r_i\,(z) = \frac{F}{A}\,\frac{C_i\,(z+dz) - C_i\,(z)}{dz}$$

$$\left(u = \frac{F}{A}\right)$$

$$\Leftrightarrow r_i\,(z) = u\,\frac{C_i\,(z+dz) - F\,C_i\,(z)}{dz}$$

3.2 – Material balances

$$F\,C_i\,(z) + r_i\,(z)\,A\,dz = F\,C_i\,(z + dz)$$

$$\Leftrightarrow r_i\,(z)\,A\,dz = F\,C_i\,(z + dz) - F\,C_i\,(z)$$

$$\Leftrightarrow r_i\,(z) = \frac{F}{A}\,\frac{C_i\,(z + dz) - C_i\,(z)}{dz}$$

$$\Leftrightarrow r_i\,(z) = u\,\frac{C_i\,(z + dz) - F\,C_i\,(z)}{dz}$$

$$\Leftrightarrow \lim_{dz \to 0} r_i\,(z) = u\,\lim_{dz \to 0} \frac{C_i\,(z + dz) - F\,C_i\,(z)}{dz}$$

3.2 – Material balances

$$F\,C_i\,(z) + \; r_i\,(z)\,A\,dz \; = F\,C_i\,(z + dz)$$

$$\Leftrightarrow r_i\,(z)\,A\,dz \; = F\,C_i\,(z + dz) - F\,C_i\,(z)$$

$$\Leftrightarrow r_i\,(z) \; = \frac{F}{A}\,\frac{C_i\,(z + dz) - \; C_i\,(z)}{dz}$$

$$\Leftrightarrow r_i\,(z) = u\,\frac{C_i\,(z + dz) - F\,C_i\,(z)}{dz}$$

$$\Leftrightarrow \lim_{dz \to 0} r_i\,(z) = u\,\lim_{dz \to 0}\frac{C_i\,(z + dz) - F\,C_i\,(z)}{dz}$$

$$\Leftrightarrow r_i = u\,\frac{dC_i}{dz}$$

3.2 – Material balances

$$F\,C_i\,(z) + r_i\,(z)\,A\,dz = F\,C_i\,(z + dz)$$

$$\Leftrightarrow r_i\,(z)\,A\,dz = F\,C_i\,(z + dz) - F\,C_i\,(z)$$

$$\Leftrightarrow r_i\,(z) = \frac{F}{A}\,\frac{C_i\,(z + dz) - C_i\,(z)}{dz}$$

$$\Leftrightarrow r_i\,(z) = u\,\frac{C_i\,(z + dz) - F\,C_i\,(z)}{dz}$$

$$\Leftrightarrow \lim_{dz \to 0} r_i\,(z) = u \lim_{dz \to 0} \frac{C_i\,(z + dz) - F\,C_i\,(z)}{dz}$$

$$\Leftrightarrow \boxed{r_i = u\,\frac{dC_i}{dz}}$$

Eq. of material to component 'i' em PFR

3 – Plug Flow Reactor

3.3 – Kinetics

Example: product formation associated with growth (Type I)

Reaction: $S \longrightarrow X + P$

Kinetics: $$\mu = \frac{\mu_{max} S}{k_m + S}$$ (assuming Monod kinetics)

3.3 – Kinetics

Example: product formation associated with growth (Type I)

Reaction: $S \rightarrow X + P$

Kinetics: $$\mu = \frac{\mu_{max} S}{k_m + S}$$ (assuming Monod kinetics)

Material balances: $$u \frac{dX}{dz} = \mu X - k_d X - k_e X$$

3.3 – Kinetics

Example: product formation associated with growth (Type I)

Reaction: $S \longrightarrow X + P$

Kinetics: $$\mu = \frac{\mu_{max} S}{k_m + S}$$ (assuming Monod kinetics)

Material balances:

$$u \frac{dX}{dz} = \mu X - k_d X - k_e X$$

$$u \frac{dS}{dz} = -\frac{\mu X}{y'_{XS}} - m_S X$$

3.3 – Kinetics

Example: product formation associated with growth (Type I)

Reaction: $S \longrightarrow X + P$

Kinetics: $$\mu = \frac{\mu_{max} S}{k_m + S}$$ (assuming Monod kinetics)

Material balances:

$$u\frac{dX}{dz} = \mu X - k_d X - k_e X$$

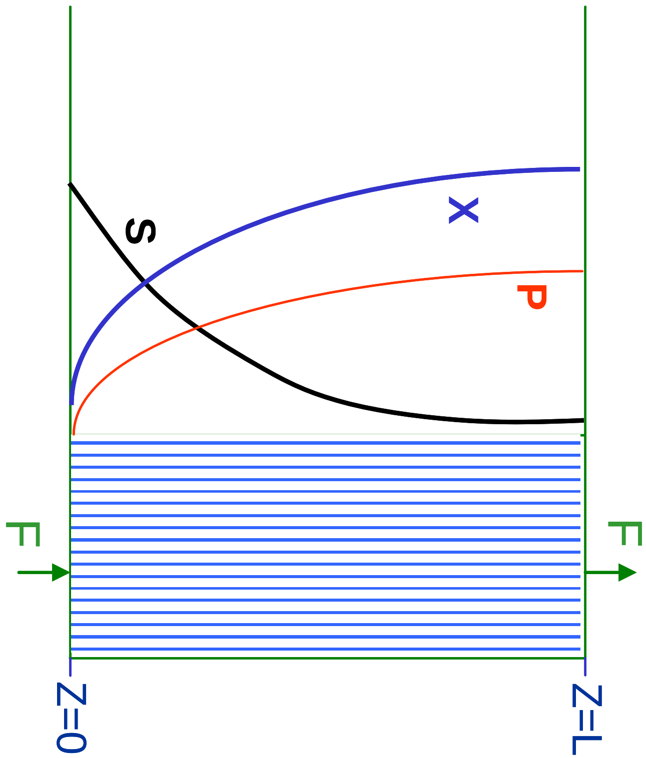
$$u\frac{dS}{dz} = -\frac{\mu X}{y'_{XS}} - m_S X$$

$$u\frac{dP}{dz} = \frac{\mu X}{y'_{XP}}$$

Analytical integration just for the case de μ≅μmax (high excess of S)

3.3 – Kinetics

Example: product formation associated with growth (Type I)



Typical Concentration Profile for Type I Product

- All concentrations depend on the ‘z’ position.
- Substrate concentration (S) decreases from bottom to top.
- Product (P) and biomass (X) concentrations increase from bottom to top.

3.4 – Productivity

Volumetric productivity of product

$$\boxed{\mathrm{Prod} = \frac{\mathrm{F\,P(z = L)}}{\mathrm{V}} = DP(z = L)}$$

g product l^{-1} h^{-1}

F	Flow rate (l/h)
P(z=L)	conc. of product at the top of the column (at the exit of the reactor) (g/l)
V	reactor volume (1)

Important note

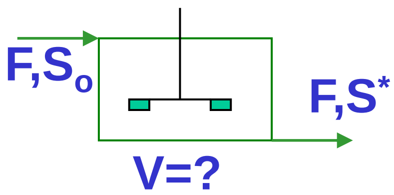
A design or optimization study should look for the maximum product concentration to occur at the top of the reactor.

3 – Plug Flow Reactor

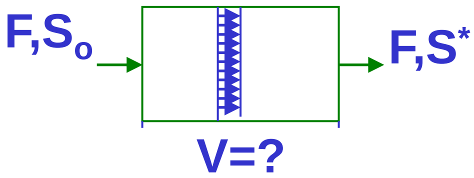
3.4 – Comparison of PFR with CSTR

Objective: convert So to S* → Volume needed?

CSTR



PFR

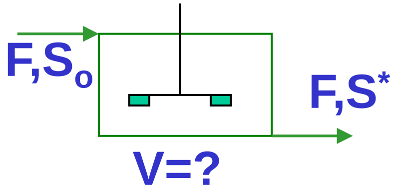


3 – Plug Flow Reactor

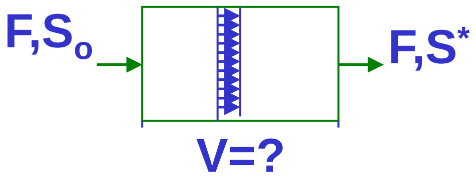
3.4 – Comparison of PFR with CSTR

Objective: convert So to S* → Volume needed?

CSTR



PFR



Material balance to the substrate:

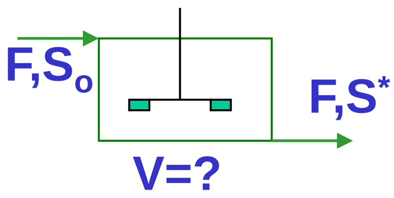
$$FS_0 - r_s V - FS^* = 0$$

$$\boxed{V = \frac{F(S_0 - S^*)}{r_s}}$$

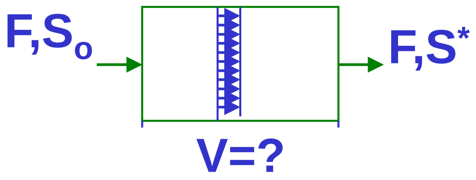
3.4 – Comparison of PFR with CSTR

Objective: convert So to S* → Volume needed?

CSTR



PFR



Material balance to the substrate:

$$FS_0 - r_sV - FS^* = 0$$

$$\boxed{V = \frac{F(S_0 - S^*)}{r_s}}$$

$$u\frac{dS}{dz} = -r_s \quad \Rightarrow$$

3.4 – Comparison of PFR with CSTR

$$u\frac{dS}{dz} = -r_s \Leftrightarrow u\,dS = -r_s\,dz$$

$$\Leftrightarrow -\frac{ds}{r_s} = \frac{dz}{u}$$

$$\Leftrightarrow -\int_{S_0}^{S^*}\frac{ds}{r_s} = \int_0^L \frac{dz}{u} = \frac{L-0}{u} = \frac{L\,A}{u\,A} = \frac{V}{F}$$

3.4 – Comparison of PFR with CSTR

$$u\frac{dS}{dz} = -r_s \Leftrightarrow u\,dS = -r_s\,dz$$

$$\Leftrightarrow -\frac{ds}{r_s} = \frac{dz}{u}$$

$$\Leftrightarrow -\int_{S_0}^{S^*} \frac{ds}{r_s} = \frac{V}{F}$$

$$\Leftrightarrow V = \frac{F(S_0 - S^*)}{\overline{r_s}}$$

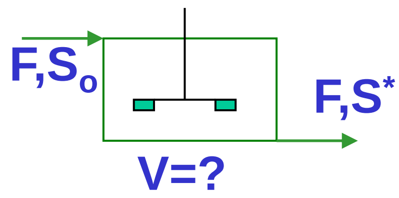
$$\bar{r} = \frac{\int_{S^*}^{S_0} r_S(S)dS}{S_0 - S^*}$$

3 – Plug Flow Reactor

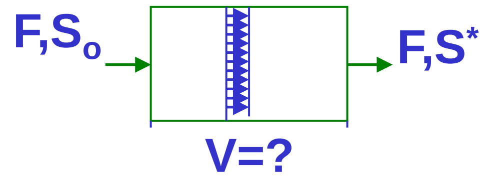
3.4 – Comparison of PFR with CSTR

Objective: convert So to S* → Volume needed?

CSTR



PFR



Material balance to the substrate:

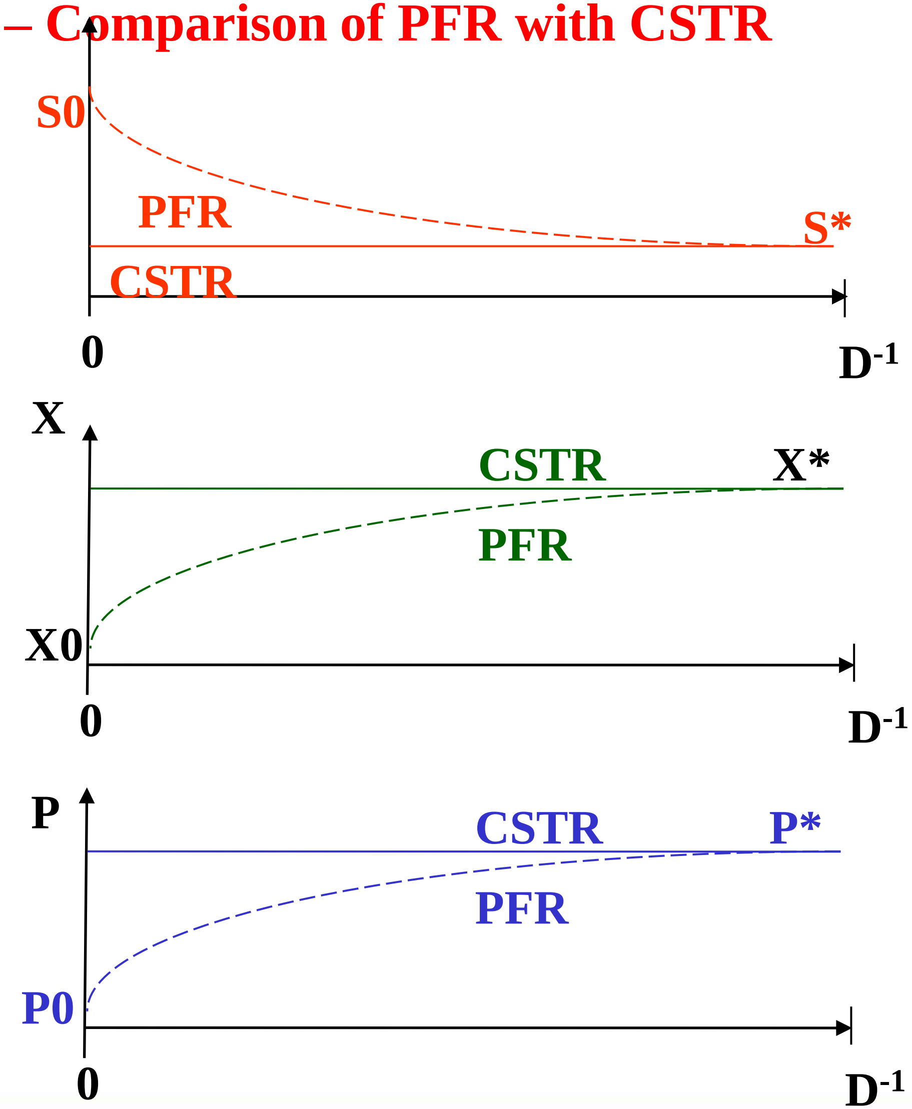
$$FS_0 - r_s V - FS^* = 0$$

$$u\frac{dS}{dz} = -r_s$$

$$\boxed{V = \frac{F(S_0 - S^*)}{r_s}}$$

$$\boxed{V = \frac{F(S_0 - S^*)}{\bar{r}_s}}$$

3.4 – Comparison of PFR with CSTR



$S_{PFR} \geq S_{CSTR}$

$X_{PFR} \leq X_{CSTR}$

$P_{PFR} \leq P_{CSTR}$

3.4 – Comparison of PFR with CSTR

Case 1: Negligible growth ($X_{PFR}= X_{CSTR}=$ constant over time)

	CSTR		PFR
	$V = \frac{F(S_0 - S^*)}{r\ (S^*)}$		$V = \frac{F(S_0 - S^*)}{\bar{r}_S}$
Kinetics	S_{CSTR}	$<$	S_{PFR}
	P_{CSTR}	$>$	P_{PFR}
Order 0 $r_s = k_0$ (indepedent of S)	$=$		$=$
Order 'n' $r_s = k\ S^n$	---		+++
Michaelis-Menten $r_s = \frac{r_{s\max} S}{K_m + S}$	---		+++
Inhibition by S (S ↗ r_s ↘)	+++		---
Inhibition by produt (P ↗ rs ↘)	---		+++

Note: the signs "+++" refer to the best reactor and "---" to the worst reactor

3.4 – Comparison of PFR with CSTR

Case 2: significant cell growth (i.e. autocatalytic kinetics)

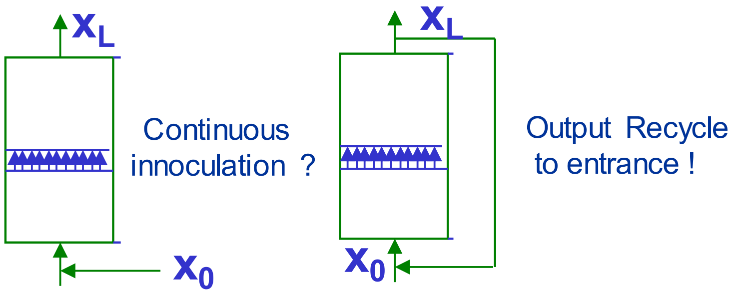
	CSTR		PFR
	$V = \dfrac{F(S_0 - S^*)}{r\ (S^*)}$		$V = \dfrac{F(S_0 - S^*)}{\bar{r}_S}$
	X_{CSTR}	>	X_{PFR}
Autocatalitic $r_s = v_s\ X$	+++		---

∴Biocatalysis with cells (bacteria, fungi, animal cell lines) CSTR tends to be more productive!!!!!!!!!

∴ Biocatalysis with enzymes PFR tends to be more productive!!!!!!!!!

3.5 – Discussion about PFR

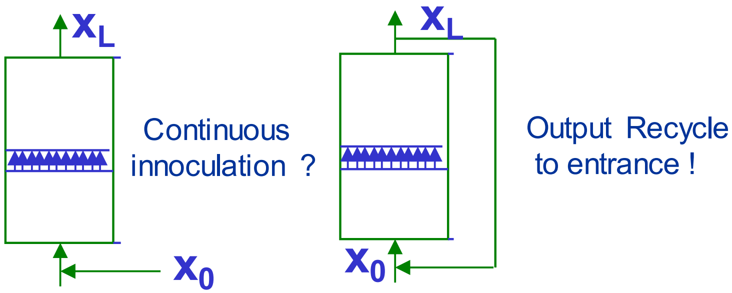
1. How to do Innoculation?



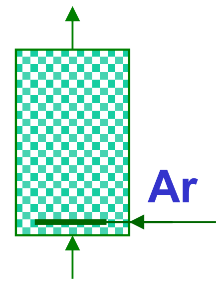
3 – Plug Flow Reactor

3.5 – Discussion about PFR

1. How to do Innoculation?



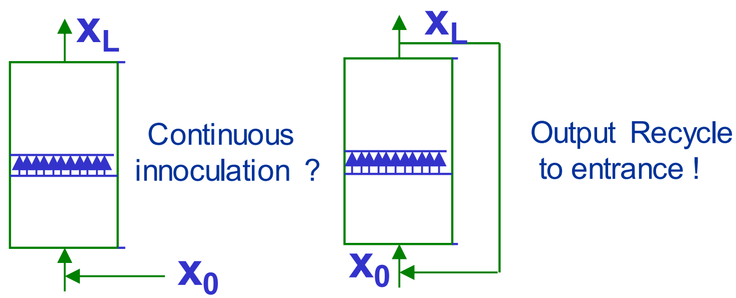
2. How to make areation?



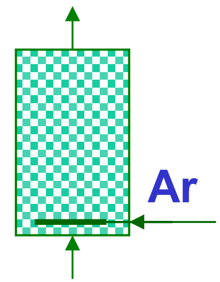
Bubbling at the bottom of the reactor promotes mixing.
PFR tends to CSTR

3.5 – Discussion about PFR

1. How to do Innoculation?



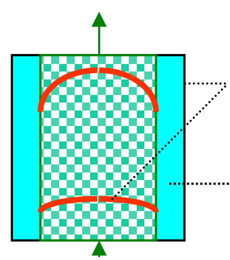
2. How to make areation?



Bubbling at the bottom of the reactor promotes mixing.
PFR tends to CSTR

3. How to control temperature?

heat transport less efficient than in CSTR



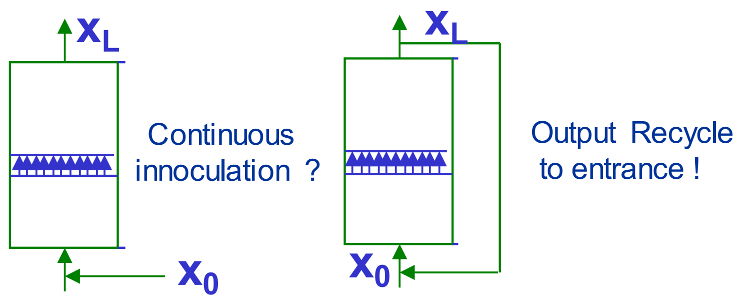
Evolution of temperature profile

Cooling jacket

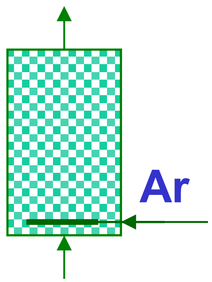
3 – Plug Flow Reactor

3.5 – Discussion about PFR

1. How to do Innoculation?



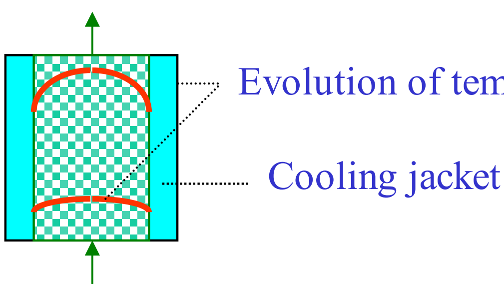
2. How to make areation?



Bubbling at the bottom of the reactor promotes mixing.
PFR tends to CSTR

3. How to control temperature?

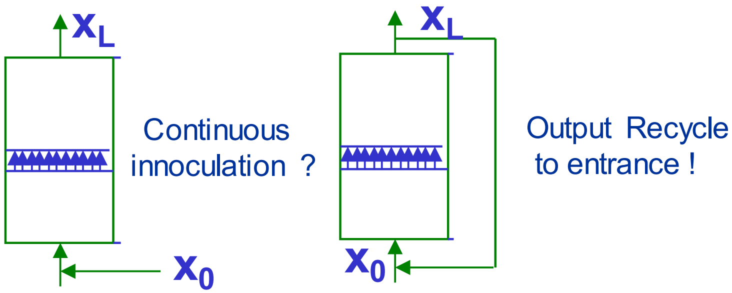
heat transport less efficient than in CSTR



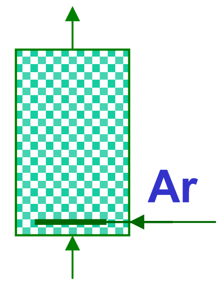
4. How to control pH?

3.5 – Discussion about PFR

1. How to do Innoculation?



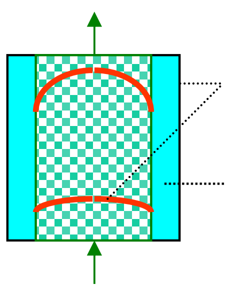
2. How to make areation?



Bubbling at the bottom of the reactor promotes mixing.
PFR tends to CSTR

3. How to control temperature?

heat transport less efficient than in CSTR



Evolution of temperature profile

Cooling jacket

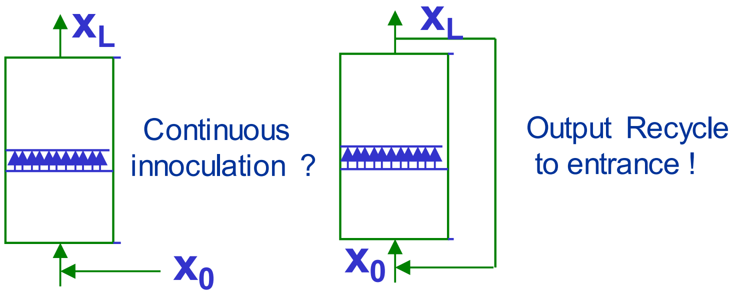
4. How to control pH?

5. How to control the concentration of any component?

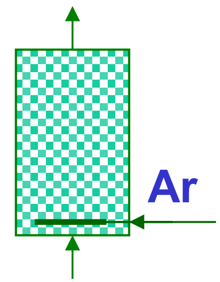
3 – Plug Flow Reactor

3.5 – Discussion about PFR

1. How to do Innoculation?



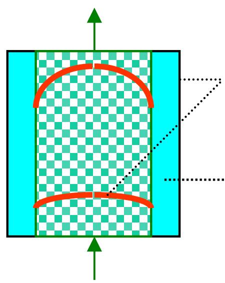
2. How to make areation?



Bubbling at the bottom of the reactor promotes mixing.
PFR tends to CSTR

3. How to control temperature?

heat transport less efficient than in CSTR



Evolution of temperature profile

Cooling jacket

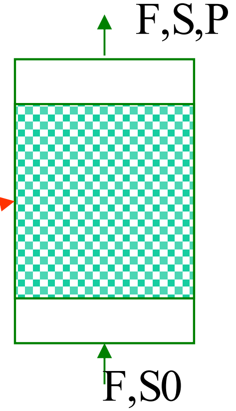
4. How to control pH?

5. How to control the concentration of any component?

> PFR with cells in suspension (and remarkable cell growth) is an impractical construct. It can, however, occur in practice in association with other bioreactors (example: plug flow associated with CSTRs)

3.5 – Discussion about PFR

Exception: tubular bioreactor with immobilized cells (or enzymes)



solid support
Cells grow adherent to solid support or 'incarcerated' in a polymer matrix

- Very stable cultures that remain viable for long periods of time (months, years)
- Cells at rest (low maintenance)
- High cell density (much higher than cells in suspension). Cells grow adherent and form biofilms
- After a growth phase, cell density remains constant over time (new cells simply replace the dying cells)
- Higher dilution rates because washout cannot occur

μ negligible
$\Rightarrow$kinetically favorable to the PFR regarding the CSTR