AM 1 - Resolução ficha 3

03/29

Contents

Exercício 1 3

Exercício 2 3

Exercício 3 3

3 - a) . . . 3
3 - b) . . . 3
3 - c) . . . 3
3 - d) . . . 3
3 - e) . . . 3
3 - f) . . . 3
3 - g) . . . 3
3 - h) . . . 4

Exercício 4 4

Exercício 5 4

5 - a) . . . 4
5 - b) . . . 4
5 - c) . . . 4
5 - d) . . . 4
5 - e) . . . 5
5 - f) . . . 5
5 - g) . . . 5
5 - h) . . . 5

5 - i) 5
5 - j) 5

Exercício 6 **5**

Exercício 1

Exercício 2

Exercício 3

3 - a)

3 - b)

3 - c)

3 - d)

$$\lim_{n\to\infty} \frac{n\,\cos(n^2)}{n^2+1} = \lim_{n\to\infty} \frac{n^{-1}\cos(n^2)}{1+1/n^2} = 0$$

3 - e)

$$\lim_{n\to\infty} \frac{2\,n\,e^{1/n}}{\sqrt{n^2+5}} = \lim_{n\to\infty} \frac{2\,e^{1/n}}{\sqrt{1+5/n^2}} = 2$$

3 - f)

$$\lim_{n\to\infty} \frac{5^n+4^n}{2\,5^{n+1}+1} = \lim_{n\to\infty} \frac{1/5+(4/5)^n\,1/5}{2+1/5^{n+1}} = \frac{1}{10}$$

3 - g)

$$\lim_{n\to\infty} \frac{3^n+e^n}{3^n+\pi^n} = \lim_{n\to\infty} \frac{\dfrac{3^n}{\pi^n}+\left(\dfrac{e}{\pi}\right)^n}{\dfrac{3^n}{\pi^n}+1} = 0$$

3 - h)

$$\lim_{n\to\infty} \frac{n^2\, 9^n + n^3}{(n+2)^2\, 3^{2n+1}} = \lim_{n\to\infty} \frac{n^2\, 9^n + n^3}{(n+2)^2\, 9^n\, 3} = \lim_{n\to\infty} \frac{1 + n/9^n}{(1+2/n)^2\; 3} = 1/3$$

Exercício 4

Exercício 5

5 - a)

$$\lim_{n\to\infty} \sqrt{n+5} - \sqrt{2n-1} = \lim_{n\to\infty} \frac{n+5-2n+1}{\sqrt{n+5}+\sqrt{2n-1}} =$$

$$= \lim_{n\to\infty} \frac{-1+6/n}{\sqrt{1/n+5/n^2}+\sqrt{2/n-1/n^2}} = -\infty$$

5 - b)

$$\lim_{n\to\infty} \sqrt{n^2+3\,n} - \sqrt{n^2-n+1} = \lim_{n\to\infty} \frac{n^2+3\,n-n^2+n-1}{\sqrt{n^2+3\,n}+\sqrt{n^2-n+1}} =$$

$$= \lim_{n\to\infty} \frac{4\,n-1}{\sqrt{n^2+3\,n}+\sqrt{n^2-n+1}} = \lim_{n\to\infty} \frac{4-1/n}{\sqrt{1+3/n}+\sqrt{1-1/n+1/n^2}} = 2$$

5 - c)

5 - d)

$$\lim_{n\to\infty} \sqrt{\ln(e^4\, n+1)} - \sqrt{\ln(n+2)} = \lim_{n\to\infty} \frac{\ln(e^4\, n+1) - \ln(n+1)}{\sqrt{\ln(e^4\, n+1)}+\sqrt{\ln(n+2)}} =$$

$$= \lim_{n\to\infty} \frac{\ln\left(\dfrac{e^4\, n+1}{n+1}\right)}{\sqrt{\ln(e^4\, n+1)}+\sqrt{\ln(n+2)}} = \lim_{n\to\infty} \frac{\ln\left(\dfrac{e^4+1/n}{1+1/n}\right)}{\sqrt{\ln(e^4\, n+1)}+\sqrt{\ln(n+2)}} = 0$$

5 - e)

$$\lim_{n\to\infty} n\ln(n+1) - n\ln(n) = \lim_{n\to\infty} n\ln\left(\frac{n+1}{n}\right) = \lim_{n\to\infty} \ln(1+1/n)^n = \ln(e^1) = 1$$

5 - f)

5 - g)

5 - h)

5 - i)

5 - j)

$$\lim_{n\to\infty} (1+1/n)^{n^2} (1-1/n)^{n^2} = \lim_{n\to\infty} \left(1-1/n^2\right)^{n^2} = e^{-1}$$

Exercício 6 Extra

$$\lim_{n\to\infty} \frac{n\,3^n + e^n}{(n+\sqrt{n})\,3^{n+1} + n^{100}} = \lim_{n\to\infty} \frac{1+\frac{e^n}{n\,3^n}}{3+\frac{3}{\sqrt{n}}+\frac{n^{99}}{3^n}} = \lim_{n\to\infty} \frac{1+n^{-1}\left(\frac{e}{3}\right)^n}{3+\frac{3}{\sqrt{n}}+\frac{n^{99}}{3^n}} = 1/3$$

Nota: funções exponenciais crescem sempre mais rápido que qualquer outra