FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE NOVA DE LISBOA
Departamento de Matemática

2º TESTE DE ANÁLISE MATEMÁTICA II-C- REPETIÇÃO 2020/2021
28 DE JANEIRO DE 2021

VERSÃO 1

PARA RESPONDER ÀS QUESTÕES DO GRUPO I ESCOLHA A LETRA CORRESPONDENTE À ÚNICA ALTERNATIVA CORRECTA (de A a F).

GRUPO I

[*1,5 valores*] 1. O integral repetido

$$\int_0^1 \left(\int_{\sqrt{1-x^2}}^{\sqrt{9-9x^2}} \cos(xy)\, dy \right) dx,$$

utilizando a ordem de integração inversa da apresentada, pode ser calculado a partir de:

A. $\int_0^1 \left(\int_{\sqrt{1-y^2}}^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx \right) dy + \int_1^3 \left(\int_0^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx \right) dy,$

B. $\int_0^1 \left(\int_{\sqrt{1-y^2}}^{\sqrt{9-9y^2}} \cos(xy)\, dx \right) dy + \int_1^3 \left(\int_0^{3\sqrt{1-y^2}} \cos(xy)\, dx \right) dy,$

C. $\int_0^1 \left(\int_0^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx \right) dy + \int_1^3 \left(\int_0^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx \right) dy,$

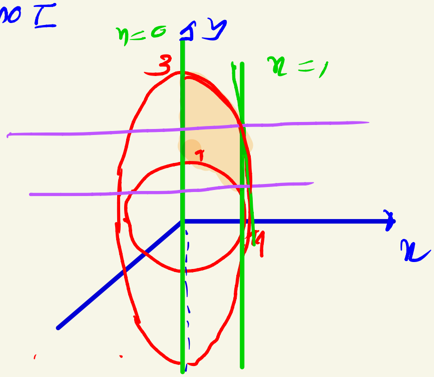
D. $\int_0^1 \left(\int_0^{3\sqrt{1-y^2}} \cos(xy)\, dx \right) dy + \int_1^3 \left(\int_0^{3\sqrt{1-y^2}} \cos(xy)\, dx \right) dy,$

E. $\int_0^1 \left(\int_0^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx \right) dy + \int_1^3 \left(\int_{\sqrt{1-y^2}}^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx \right) dy,$

F. $\int_0^1 \left(\int_0^{3\sqrt{1-y^2}} \cos(xy)\, dx \right) dy + \int_1^3 \left(\int_{\sqrt{1-y^2}}^{\sqrt{9-9y^2}} \cos(xy)\, dx \right) dy,$

Grupo I

1.



$x=1$

$$\int_0^1 \left(\int_{\sqrt{1-x^2}}^{\sqrt{9-9x^2}} \cos(xy)\, dy \right) dx$$

$x=0$

$y=\sqrt{9-9x^2} \Rightarrow x^2+\frac{y^2}{9}=1$

$y=\sqrt{1-x^2} \Rightarrow y^2+x^2=1$

$$\int_0^1 dy \int_{\sqrt{1-y^2}}^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx + \int_1^3 dy \int_0^{\sqrt{1-\frac{y^2}{9}}} \cos(xy)\, dx$$

[*1,5 valores*] 2. Seja $D \subset \mathbb{R}^3$ o domínio fechado, limitado superiormente pelo parabolóide $z = 1 - (x^2 + y^2)$ e inferiormente pelo parabolóide $z = x^2 + y^2$. O volume do domínio D pode ser calculado a partir do seguinte integral triplo:

A. $\int_0^{\frac{\sqrt{2}}{2}} (\int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} (\int_{1-(x^2+y^2)}^{x^2+y^2} dz)dy)dx$

B. $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (\int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} (\int_{x^2+y^2}^{1-(x^2+y^2)} dz)dy)dx$

C. $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (\int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} (\int_{x^2+y^2}^{1-(x^2+y^2)} (1-2(x^2+y^2))\, dz)dy)dx$

D. $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (\int_0^{\sqrt{\frac{1}{2}-x^2}} (\int_{x^2+y^2}^{1-(x^2+y^2)} dz)dy)dx$

E. $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (\int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} (\int_{x^2+y^2}^{1-(x^2+y^2)} (2(x^2+y^2)-1)\, dz)dy)dx$

F. $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (\int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} (\int_{1-(x^2+y^2)}^{x^2+y^2} dz)dy)dx$

[*1,5 valores*] 3. Seja L uma linha admitindo a representação paramétrica regular

$$\begin{cases} x = \cos t \\ y = \sin t \qquad\qquad 0 \le t \le \dfrac{\pi}{2} \\ z = log(1+t) \end{cases}$$

percorrida no sentido crescente do parâmetro t, $\varphi(x,y,z) = e^{xy+yz+zx}$ e

$$\vec{u} = u_1(x,y,z)\vec{i} + u_2(x,y,z)\vec{j} + u_3(x,y,z)\vec{k} = \nabla\varphi(x,y,z).$$

Seja

$$I = \int_L u_1(x,y,z)dx + u_2(x,y,z)dy + u_3(x,y,z)dz.$$

Então:

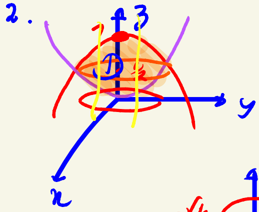
A. $I = -\dfrac{\pi}{2}$ **B.** $I = 0$ **C.** $I = \dfrac{\pi}{2}$ **D.** $I = \pi$ **E.** $I = \dfrac{3}{2}\pi$ **F.** $I = 2\pi$

[*1,5 valores*] 4. Seja $g(x,y,z)$ um campo escalar, definido e admitindo derivadas parciais contínuas até à segunda ordem num subconjunto aberto A de $\mathbb{R}^3$. Uma expressão de $\nabla \cdot \nabla g^2$ em função de g, $\|\nabla g\|$ e $\nabla^2 g$, é:

A. $2\|\nabla g\| + 2\nabla^2 g$ **B.** $2g\|\nabla g\| + 2\nabla^2 g$ **C.** $2g\|\nabla g\| + 2g\nabla^2 g$

D. $2\|\nabla g\|^2 + 2g\nabla^2 g$ **E.** $2g\|\nabla g\|^2 + 2g\nabla^2 g$ **F.** $2\|\nabla g\|^2 + 2\nabla^2 g$

2.

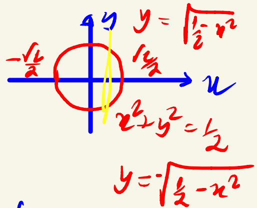


Intersecao

$$\begin{cases} z-1=-(x^2+y^2) \\ z=x^2+y^2 \end{cases} \quad \begin{cases} z-1=-z \\ — \end{cases} \quad \begin{cases} 2z=1 \\ — \end{cases} \quad \begin{cases} z=\frac{1}{2} \end{cases}$$

$$\downarrow$$

$$x^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$



$$Vol(D)=\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} dx \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} dy \int_{x^2+y^2}^{1-(x^2+y^2)} 1 \, dz \quad //$$

3. $\begin{cases} x=\cos t \\ y=\operatorname{sen} t \\ z=\log(1+t) \end{cases}$ $0 \le t \le \frac{\pi}{2}$ $\quad \varphi(t)=e^{xy+yz+zx}$ e

$$\vec{u}(x,y,z)=u_1(x,y,z)\vec{i}+u_2(x,y,z)\vec{j}+u_3(x,y,z)\vec{k}=\nabla\varphi(x,y,z)$$

Seja

$$I=\int_L u_1(x,y,z)\,dx+u_2(x,y,z)\,dy+u_3(x,y,z)\,dz=\int_L \vec{u}(P)\,dP$$

Sejam $A=(\cos 0, \operatorname{sen} 0, \log(1+0))=(1,0,0)$ e $B=(\cos\frac{\pi}{2}, \operatorname{sen}\frac{\pi}{2}, \log(1+\frac{\pi}{2}))$ $=(0,1,\log(1+\frac{\pi}{2}))$ o inicio e o fim da linha L e $\varphi(x,y,z)$ a função potencial de $\vec{u}$ então

$$\int_L \vec{u}(P)\,dP=\varphi(B)-\varphi(A)=\varphi(0,1,\log(1+\tfrac{\pi}{2}))-\varphi(1,0,0)$$
$$=e^{0+\log(1+\frac{\pi}{2})+0}-e^0=\cancel{1}+\frac{\pi}{2}-\cancel{1}=\frac{\pi}{2}$$

Num campo conservativo a calculo é independente do caminho, só depende dos extremos.

4. $g(x,y,z)$ campo escalar

$\nabla \cdot \nabla g^2$ em função de g, $\|\nabla g\|$ e $\nabla^2 g$

$$\nabla g^2 = \frac{\partial g^2}{\partial x}\vec{i} + \frac{\partial g^2}{\partial y}\vec{j} + \frac{\partial g^2}{\partial z}\vec{k}$$

$$\nabla \cdot \nabla g^2 = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right) \cdot \left(\frac{\partial g^2}{\partial x}\vec{i} + \frac{\partial g^2}{\partial y}\vec{j} + \frac{\partial g^2}{\partial z}\vec{k}\right)$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{i} + \frac{\partial}{\partial z}\vec{k}\right) \cdot \left(2g\frac{\partial g}{\partial x}\vec{i} + 2g\frac{\partial g}{\partial y}\vec{j} + 2g\frac{\partial g}{\partial z}\vec{k}\right.$$

$$= \frac{\partial}{\partial x}\left(2g\frac{\partial g}{\partial x}\right) + \frac{\partial}{\partial y}\left(2g\frac{\partial g}{\partial y}\right) + \frac{\partial}{\partial z}\left(2g\frac{\partial g}{\partial z}\right)$$

$$= 2\left(\underline{\frac{\partial g}{\partial x}\frac{\partial g}{\partial x}} + g\underline{\frac{\partial^2 g}{\partial x^2}}\right) + 2\left(\underline{\frac{\partial g}{\partial y}\frac{\partial g}{\partial y}} + g\underline{\frac{\partial^2 g}{\partial y^2}}\right) + 2\left(\underline{\frac{\partial g}{\partial z}\frac{\partial g}{\partial z}} + g\underline{\frac{\partial^2 g}{\partial z^2}}\right)$$

$$= 2\left(\underline{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2}\right) + 2g\left(\underline{\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}}\right)$$

$$= 2\|\nabla g\|^2 + 2g\,\nabla^2 g \quad //$$

$$\|\nabla g\| = \sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2} \Leftrightarrow \|\nabla g\|^2 = \left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2$$

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}$$

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VERSÃO 1

GRUPO II

[*3 valores*] 1. Considere o conjunto $A = \{(x,y) \in \mathbb{R}^2 : x^2 + \frac{y^2}{4} \geq 1 \wedge \frac{x^2}{4} + \frac{y^2}{16} \leq 1 \wedge y \geq 0\}$ e seja L a fronteira de A percorrida no sentido direto. Determine

$$\int_L e^{x^2} dx + x^2 y \, dy$$

a partir do cálculo de um integral duplo. (Sug: Utilize coordenadas elípticas.)

[*3 valores*] 2. Determine o volume do domínio $D \subset \mathbb{R}^3$ fechado, limitado lateralmente pela superfície cónica $z = 4 - \sqrt{x^2 + y^2}$ e compreendido entre os planos $z = 2$ e $z = 3$, utilizando as coordenadas cilíndricas

$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \\ z = z. \end{cases}$$

GRUPO III

[*3 valores*] 1. Determine a área da superfície de equações paramétricas

$$\begin{cases} x = \sin u \cos v, & \frac{\pi}{6} \leq u \leq \frac{\pi}{3} \\ y = \sin u \sin v, & 0 \leq v \leq 2\pi \\ z = \cos u. & \end{cases}$$

Identifique a superfície.

[*3 valores*] 2. Determine $\displaystyle\iint_S \nabla \times (y\vec{i} + z\vec{j}) \, . \, \vec{n} dS$,

na face "exterior"da porção da superfície esférica $x^2 + y^2 + z^2 = 1$ com $-1 \leq z \leq \frac{\sqrt{3}}{2}$, **através do cálculo de um integral curvilíneo**.

[*2 valores*] $\cancel{3.}$ Determine e caracterize os pontos de estacionaridade da função

$$f(x,y) = \log(1 + x^2 + y^2) - \int_0^x \frac{2t}{1+t^4} dt.$$

Grupo II

1. $A = \left\{(x,y)\in\mathbb{R}^2 : x^2+\frac{y^2}{4}\ge 1 \wedge \frac{x^2}{4}+\frac{y^2}{16}\le 1 \wedge y \ge 0\right\}$

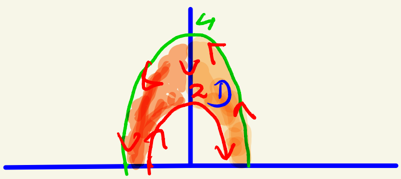
L fronteira de A no sentido direto,

$$\int_L e^{x^2}dx + x^2y\,dy$$

a partir do integral duplo.

Sug: use coordenadas elípticas

$$\int_L \underbrace{e^{x^2}}_{\varphi(x,y)}dx + \underbrace{x^2y}_{\psi(x,y)}dy = \iint_D \left(\frac{\partial\psi}{\partial x} - \frac{\partial\varphi}{\partial y}\right)dx\,dy = \iint_D (2xy-0)\,dx\,dy$$



$$\begin{cases} x = \rho\cos\theta \\ y = \frac{2}{1}\rho\,\text{sen}\,\theta \end{cases} \qquad \begin{array}{l} 0\le\theta\le\pi \\ 1\le\rho\le 2 \end{array} \qquad |J| = 2\rho$$

$$\iint_D 2xy\,dx\,dy = \int_0^\pi d\theta \int_1^2 2\,\rho\cos\theta\; 2\rho\,\text{sen}\,\theta\; 2\rho\, d\rho =$$

$$= 8\int_0^\pi d\theta \int_1^2 \rho^3\cos\theta\,\text{sen}\,\theta\, d\rho$$

$$= 8\int_0^\pi \cos\theta\,\text{sen}\,\theta \left[\frac{\rho^4}{4}\right]_1^2 d\theta = 8\times\frac{15}{4}\int_0^\pi \cos\theta\,\text{sen}\,\theta\, d\theta$$

$$= \frac{30}{2}\int_0^\pi \underbrace{2\cos\theta\,\text{sen}\,\theta}_{\text{sen}(2\theta)}\, d\theta = 15\left[-\frac{\cos(2\theta)}{2}\right]_0^\pi = 0$$

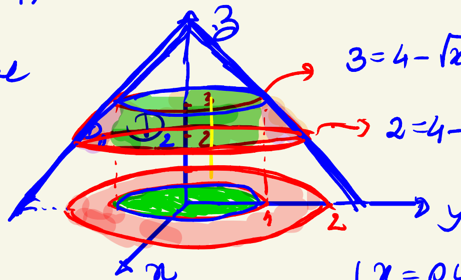
2.

$z = 4 - \sqrt{x^2+y^2}$ limita lateralmente

$\left.\begin{array}{l} z = 2 \\ z = 3 \end{array}\right\}$ O domínio está compreendido entre estes planos

$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\,\theta \\ z = z \end{cases}$ Utilizar as coordenadas cilíndricas para determinar o volume de D

Superfície cónica



$z = 4 - \sqrt{x^2+y^2} \Leftrightarrow x^2+y^2 = 1$

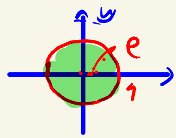
$2 = 4 - \sqrt{x^2+y^2} \Leftrightarrow x^2+y^2 = 2^2$

$\begin{array}{l} 0 \le \theta \le 2\pi \\ 2 \le z \le 3 \\ 0 \le \rho \le 4 - z \end{array}$

$D_1 \rightarrow$ domínio a verde

$D_2 \rightarrow$ " a vermelho

$D_1 \begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\,\theta \\ z = z \end{cases}$

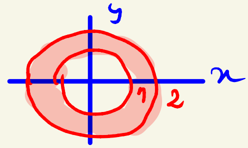


$0 \le \theta \le 2\pi$

$0 \le \rho \le 1 \quad |J| = \rho$

$2 \le z \le 3$

$D_2 \begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\,\theta \\ z = z \end{cases}$ $\begin{array}{l} 0 \le \theta \le 2\pi \\ 1 \le \rho \le 2 \quad |J| = \rho \\ 2 \le z \le 4 - \rho \end{array}$



$$\iiint_{D_1} dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^1 d\rho \int_2^3 \rho\, dz = 2\pi \int_0^1 \left[\frac{\rho^2}{2}\right] d\rho = \pi$$

$$\iiint_{D_2} dx\,dy\,dz = \int_0^{2\pi} d\theta \int_1^2 d\rho \int_2^{4-\rho} \rho\, dz = 2\pi \int_1^2 (4-\rho-2)\rho\, d\rho$$

$$= 2\pi \int_1^2 (2-\rho)\rho\, d\rho = 2\pi \left[\rho^2 - \frac{\rho^3}{3}\right]_1^2 = 2\pi\left(2^2 - \frac{2^3}{3} - 1 + \frac{1}{3}\right)$$

$$= 2\pi\left(4 - \frac{8}{3} - \frac{2}{3}\right) = 2\pi\left(\frac{12-8-2}{3}\right) = 2\pi \times \frac{2}{3} = \frac{4\pi}{3}$$

$$\text{Volume}(D) = \pi + \frac{4\pi}{3} = \frac{7\pi}{3}$$

ou alternativamente →

(+ simples)

$\longrightarrow$

$$D \begin{cases} x = \rho\cos\theta & \cancel{0\le\theta} \le 2\pi \\ y = \rho\sin\theta & 2 \le z \le 3 \\ z = z & 0 \le \rho \le 4-z \end{cases} \qquad |J| = \rho$$

$$\iiint_D dx\,dy\,dz = \int_0^{2\pi} d\theta \int_2^3 dz \int_0^{4-z} \rho\, d\rho$$

$$= 2\pi \int_2^3 \left[\frac{\rho^2}{2} \right]_0^{4-z} dz$$

$$= 2\pi \int_2^3 \left(\frac{4-z}{2} \right)^2 dz = \frac{2\pi}{2} \int_2^3 16 - 8z + z^2\, dz$$

$$= \pi \left[16z - 4z^2 + \frac{z^3}{3} \right]_2^3 = \dots = \frac{7\pi}{3} !!$$

Grupo $\boxed{II}$

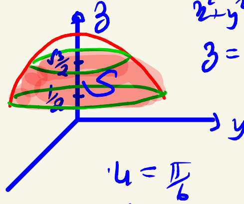
1. Área da superfície e identifique a superfície

$$\begin{cases} x = \operatorname{sen} u \cos v \\ y = \operatorname{sen} u \operatorname{sen} v \\ z = \cos u \end{cases} \qquad \frac{\pi}{6} \le u \le \frac{\pi}{3}, \quad 0 \le v \le 2\pi$$

$u = \frac{\pi}{3}$

$x^2+y^2 = \operatorname{sen}^2 \frac{\pi}{3}$

$z = \cos\frac{\pi}{3} = \frac{1}{2}$



$u = \frac{\pi}{6}$

$x^2+y^2 = \operatorname{sen}^2 \frac{\pi}{6}$

$z = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

—— // —— —— // ——

$$\text{Área}(S) = \iint_A \sqrt{EG - F^2}\, du\, dv \quad \boxed{\text{ou}}$$

$$\text{Área}(S) = \iint_A \left\| \frac{\partial P}{\partial u} \times \frac{\partial P}{\partial v} \right\| du\, dv$$

$$E = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$$

$$G = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$$

$$F = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}$$

$$E = (\cos u \cos v)^2 + (\cos u \operatorname{sen} v)^2 + (-\operatorname{sen} u)^2$$
$$= \cos^2 u + \operatorname{sen}^2 u = 1$$

$$G = (-\operatorname{sen} u \operatorname{sen} v)^2 + (\operatorname{sen} u \cos v)^2 + 0 = \operatorname{sen}^2 u$$

$$F = \cos u \cos v \operatorname{sen} u(-\operatorname{sen} v) + \cos u \operatorname{sen} v(\operatorname{sen} u \cos v) + 0$$
$$= -\cos u \cos v \operatorname{sen} u \operatorname{sen} v + \cos u . \cos v \operatorname{sen} u \operatorname{sen} v = 0$$

$$\text{área}(S) = \iint_A \sqrt{\operatorname{sen}^2 u - 0}\, du\, dv = \int_0^{2\pi} dv \int_{\pi/6}^{\pi/3} \operatorname{sen} u\, du$$

$$= 2\pi \Big[-\cos u\Big]_{\pi/6}^{\pi/3} = 2\pi\left(-\cos\frac{\pi}{3} + \cos\frac{\pi}{6}\right)$$

$$= 2\pi\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \pi(-1+\sqrt{3}) \, //$$

2.

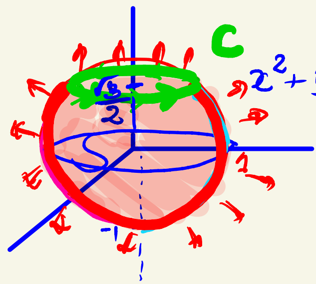
$$\iint_S \overbrace{\nabla\times(y\vec{i}+z\vec{j})}^{\text{rot}\,(y\vec{i}+z\vec{j})}\cdot\vec{n}\,dS$$

face exterior de S $\begin{cases} x^2+y^2+z^2=1 \\ -1\leq z\leq \frac{\sqrt{3}}{2}\end{cases}$

Usando um integral curvilíneo.

T. Stokes

$$\iint_S \text{rot}(y\vec{i}+z\vec{j})\cdot\vec{n}\,dS = \int_C (\underbrace{y}_{u_1}\vec{i}+\underbrace{z}_{u_2}\vec{j})\,dP$$



$x^2+y^2+\frac{3}{4}=1 \Rightarrow x^2+y^2=\frac{1}{4}$

C $x^2+y^2=\frac{1}{4}$ $\begin{cases} x=\frac{1}{2}\cos t \\ y=\frac{1}{2}\,\text{sen}\, t \\ z=\frac{\sqrt{3}}{2}\end{cases}$ $0\leq t\leq 2\pi$ sentido positivo

↳ Bordo de S

$$\int_C (y\vec{i}+z\vec{j})\,dP = \int_0^{2\pi}\left(\frac{1}{2}\,\text{sen}\,t\cdot\left(-\frac{1}{2}\,\text{sen}\,t\right)+\frac{\sqrt{3}}{2}\cdot\frac{1}{2}\cos t\right)dt$$

$$= -\frac{1}{4}\int_0^{2\pi}\overbrace{\text{sen}^2 t}^{\frac{1}{2}-\frac{1}{2}\cos 2t}\,dt+\frac{\sqrt{3}}{4}\int_0^{2\pi}\cos t\,dt$$

$$= -\frac{1}{4}\left[\frac{1}{2}t-\frac{\text{sen}(2t)}{4}\right]_0^{2\pi}+\frac{\sqrt{3}}{4}\left[\text{sen}\,t\right]_0^{2\pi} = -\frac{\pi}{4}$$