AM 1 - Resolução Ficha 5
Limites e Continuidade de funções

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Conteúdo

Questão 1

1 - f) $\lim_{x\to\infty} e^{\cos(x)}/x$

$= \lim\limits_{x\to\infty} e^{\cos(x)} * \lim\limits_{x\to\infty} 1/x = \lim\limits_{x\to\infty} e^{\cos(x)} * 0;\ e^{\cos(x)}$ é uma função limitada $\implies$

$\implies \lim\limits_{x\to\infty} e^{\cos(x)}/x = 0$

1 - g) $\lim_{x\to 0^+} e^{1/x}$

$= e^{\lim_{x\to 0^+} 1/x} = e^{+\infty} = +\infty$

1 - h) $\lim_{x\to 0^-} e^{1/x}$

$= e^{\lim_{x\to 0^-} 1/x} = e^{-\infty} = 0$

Questão 2 Prove $\nexists\, f(x)$

2 - a) $\nexists\, a(x) = x/|x| : x = 0$

$\iff \lim\limits_{x\to 0^+} a(x) = 1 \neq \lim\limits_{x\to 0^-} a(x) = -1$

2 - b) $\nexists\, b(x) = \frac{x^2-1}{|x-1|} : x = 1$

$\iff \lim\limits_{x\to 1^+} b(x) = \lim\limits_{x\to 1^+} \dfrac{(x+1)(x-1)}{x-1} = 2 \neq \lim\limits_{x\to 1^-} b(x) =$

$= \lim\limits_{x\to 1^-} \dfrac{(x+1)(x-1)}{1-x} = \lim\limits_{x\to 1^-} \dfrac{-(x+1)}{1} = -2$

2 - c) $\nexists\, c(x) = \arctan(e^{1/x}) : x = 0$

$\iff \lim\limits_{x\to 0^+} c(x) = \arctan(e^{\lim_{x\to 0^+} 1/x}) = \arctan(e^{\infty}) = \arctan(\infty) = \pi/2 \neq$

$\neq \lim\limits_{x\to 0^-} c(x) = \arctan(e^{\lim_{x\to 0^-} 1/x}) = \arctan(e^{-\infty}) = \arctan(0) = 0$

2 - d) $\nexists\, d(x) = e^{\cos(x)} : x = +\infty$ **Incompleta**

$\iff \lim_{x\to\infty} d(x);\ x_n = 2\,n\,\pi \quad \forall\, n \in \mathbb{N}$

$\cdots$

Questão 3

3 - a) $\lim_{x\to 0} \tan(x)/x$ **Incompleto**

$$= \lim_{x\to 0} \cos^{-}1(x)\frac{\sin(x)}{x} = \lim_{x\to 0} \cdots$$

3 - b) $\lim_{x\to 0}(1 - e^{2\,x})/x$ **Incompleto**

$$= \cdots$$

3 - c) $\lim_{x\to 0}(1 - e^{2x})/\sin(3\,x)$ **Incompleto**

$$= \lim_{x\to 0} \frac{1 - e^{2x}}{2\,x}\frac{3\,x}{\sin(3\,x)}\frac{2\,x}{3\,x} = \cdots$$

3 - d) $\lim_{x\to 0} \ln(x+1)/x$ **Incompleto**

$$x + 1 = e^{y} \implies \lim_{x\to 0} \ln(x+1)/x = \lim_{x\to 0} \frac{y}{e^{y} - 1} = \cdots$$

3 - e) $\lim_{x\to 0}(1 - \cos(3\,x))/x^2$

$$= \lim_{x\to 0} \frac{1 - \cos^2(3\,x)}{x^2(1 + \cos(3\,x))} = \lim_{x\to 0} \frac{\sin^2(3\,x)}{x^2(1 + \cos(3\,x))} = \lim_{x\to 0} \frac{\sin^2(3\,x)}{(3\,x)^2}\frac{9\,x^2}{x^2(1 + \cos(3\,x))} =$$
$$= \lim_{x\to 0} \left(\frac{\sin(3\,x)}{3\,x}\right)^2 \lim_{x\to 0} \frac{9}{1 + \cos(3\,x)} = 9/2$$