SECTION 4

FLOW OF FLUIDS THROUGH GRANULAR BEDS AND PACKED COLUMNS

Problem 4.1

In a contact sulphuric acid plant the secondary converter is a tray type converter, 2·3 m in diameter with the catalyst arranged in three layers, each 0·45 m thick. The catalyst is in the form of cylindrical pellets 9·5 mm in diameter and 9·5 mm long. The void fraction is 0·35. The gas enters the converter at 675 K and leaves at 720 K. Its inlet composition is

$$SO_3\ 6{\cdot}6,\quad SO_2\ 1{\cdot}7,\quad O_2\ 10{\cdot}0,\quad N_2\ 81{\cdot}7\ \text{mol}\%$$

and its exit composition

$$SO_3\ 8{\cdot}2,\quad SO_2\ 0{\cdot}2,\quad O_2\ 9{\cdot}3,\quad N_2\ 82{\cdot}3\ \text{mol}\%$$

The gas flow rate is 0·68 kg/m²s. Calculate the pressure drop through the converter.

$$\mu = 0{\cdot}032\ \text{mN s/m}^2$$

Solution

This problem will be solved by three different methods.

(a) *Chilton and Colburn*

$$Re' = \rho u d/\mu \quad \text{and} \quad \frac{(-\Delta P)}{2\rho u^2}\frac{d}{l} = \phi'_1$$

For $Re' < 40$: $\qquad \phi'_1 = 850/Re'$

For $Re' > 40$: $\qquad \phi'_1 = 38/Re'^{0{\cdot}15}$

The mean molecular weight

$$= (0{\cdot}066 \times 80) + (0{\cdot}017 \times 64) + (0{\cdot}1 \times 32) + (0{\cdot}817 \times 28)$$

$$= 32{\cdot}44\ \text{kg/kmol} \quad \text{at the inlet,}$$

and at the outlet:

$$(0{\cdot}082 \times 80) + (0{\cdot}002 \times 64) + (0{\cdot}093 \times 32) + (0{\cdot}823 \times 28) = 32{\cdot}71\ \text{kg/kmol}$$

Inlet temperature = 675 K.
Outlet temperature = 720 K.

$\therefore$ average temperature $= 697{\cdot}5$ K.

Average molecular weight $= 32{\cdot}58$ kg/kmol. $\rho = ?$

$PV = nRT$

$\rho = \frac{MP}{RT}$

$$\text{Average gas density} = \frac{32{\cdot}58}{22{\cdot}4} \times \frac{273}{697{\cdot}5} = 0{\cdot}569\ \text{kg/m}^3. \; ?$$

$$\text{Reynolds number} = \rho u d/\mu = Gd/\mu$$

$$= 0{\cdot}68 \times 9{\cdot}5 \times 10^{-3}/0{\cdot}032 \times 10^{-3}$$

$$= 202$$

$$-\Delta P = \frac{2\rho u^2 l}{d} \times \frac{38}{(202)^{0{\cdot}15}}$$

Average gas velocity $u = 0{\cdot}68/0{\cdot}569 = 1{\cdot}20$ m/s

$$\therefore \quad -\Delta P = \frac{2 \times 0{\cdot}569 \times (1{\cdot}20)^2 \times (3 \times 0{\cdot}45)}{9{\cdot}5 \times 10^{-3}} \times \frac{38}{(202)^{0{\cdot}15}}$$

$$= 3{\cdot}99 \times 10^3\ \text{N/m}^2$$

$$= \underline{\underline{4{\cdot}0\ \text{kN/m}^2}}$$

(b) *Rose*

Rose defined Re' in the same way as Chilton and Colburn but instead of ϕ'_1 used ϕ_1, i.e. $-\Delta P d/\rho u^2 l$. Defining d as the diameter of a sphere with the same specific surface as the material forming the bed, his correlation was presented as:

$$\phi_1 = 1000/Re' + 125/Re'^{0{\cdot}5} + 14$$

For cylindrical pellets of length = diameter = d:

$$S = \left(2\frac{\pi}{4}d^2 + \pi d^2\right)\Big/\frac{\pi}{4}d^3 = 6/d$$

which is the same as for spheres.

$$\therefore \quad d = 9{\cdot}5\ \text{mm} \quad \text{and} \quad Re' = 202 \quad \text{as before.}$$

Then $\qquad \phi_1 = 1000/202 + 125/202 + 14$

$$= 27{\cdot}7$$

$$\therefore \quad \Delta P = 27{\cdot}7 \times 0{\cdot}569 \times (1{\cdot}20)^2 \times (3 \times 0{\cdot}45)/9{\cdot}5 \times 10^{-3}$$

$$= 3{\cdot}23 \times 10^3\ \text{N/m}^2$$

$$= \underline{\underline{3{\cdot}2\ \text{kN/m}^2}}$$

(c) *Carman*

$$\frac{R}{\rho u^2} = \frac{e^3}{5(1-e)} \frac{\Delta P}{l} \frac{1}{\rho u^2} \qquad \text{(equation 4.13)}$$

$$\frac{R}{\rho u^2} = 5/Re_1 + 0{\cdot}4/Re_1^{0\cdot1} \qquad \text{(equation 4.14)}$$

$$Re_1 = \frac{G}{S(1-e)\mu} \qquad \text{(equation 4.11)}$$

$$S = 6/d = 6/9{\cdot}5 \times 10^{-3} = 631\ \mathrm{m^2/m^3}$$

$$\therefore \quad Re_1 = 0{\cdot}68/631 \times 0{\cdot}65 \times 0{\cdot}032 \times 10^{-3} = 51{\cdot}8$$

$$\therefore \quad \frac{R}{\rho u^2} = \frac{5}{51{\cdot}8} + \frac{0{\cdot}4}{(51{\cdot}8)^{0\cdot1}} = 0{\cdot}366$$

From equation 4.13:

$$\begin{aligned}\Delta P &= 0{\cdot}366 \times 631 \times 0{\cdot}65 \times (3 \times 0{\cdot}45) \times 0{\cdot}569 \times (1{\cdot}20)^2/(0{\cdot}35)^3 \\ &= 3{\cdot}87 \times 10^3\ \mathrm{N/m^2} \\ &= \underline{\underline{3{\cdot}9\ \mathrm{kN/m^2}}}\end{aligned}$$

Problem 4.2

Show how an equation for the pressure drop in a packed column can be modified to apply to cases where the total pressure and the pressure drop are of the same order of magnitude.

Two heat-sensitive organic liquids (average molecular weight = 155 kg/kmol) are to be separated by vacuum distillation in a 100 mm diameter column packed with 6 mm stoneware Raschig rings. The number of theoretical plates required is 16 and it has been found that the HETP is 150 mm. If the product rate is 5 g/s at a reflux ratio of 8, calculate the pressure in the condenser so that the temperature in the still does not exceed 395 K (equivalent to a pressure of 8 kN/m²). Assume $a = 800\ \mathrm{m^2/m^3}$, $\mu = 0{\cdot}02\ \mathrm{mN\ s/m^2}$, $e = 0{\cdot}72$, and neglect the temperature changes and the correction for liquid flow.

Solution

The modified Reynolds number Re_1 is defined by:

$$Re_1 = \frac{u\rho}{S(1-e)\mu} = \frac{G}{S(1-e)\mu} \qquad \text{(equation 4.11)}$$

Ergun's equation (4.18) may be rewritten as:

$$\frac{R}{\rho u^2} = 4{\cdot}17/Re_1 + 0{\cdot}29 \qquad \text{(equation 4.19)}$$

Hence

$$\frac{R}{\rho u^2} = \frac{4{\cdot}17S(1-e)\mu}{G} + 0{\cdot}29$$

Equation 4.13 states:

$$\frac{R}{\rho u^2} = \frac{e^3}{S(1-e)} \frac{(-\mathrm{d}P)}{\mathrm{d}l} \frac{1}{\rho u^2}$$

$$= \frac{e^3}{S(1-e)} \frac{(-\mathrm{d}P)}{\mathrm{d}l} \frac{\rho}{G^2}$$

$$-\rho \frac{\mathrm{d}P}{\mathrm{d}l} = \frac{R}{\rho u^2} \frac{S(1-e)}{e^3} G^2$$

$$-\int \rho \,\mathrm{d}P = \frac{R}{\rho u^2} \frac{S(1-e)}{e^3} G^2 \int \mathrm{d}l$$

$$= \frac{R}{\rho u^2} \frac{S(1-e)}{e^3} G^2 l$$

In this problem $a = 800\ \mathrm{m^2/m^3} = S(1-e)$

Product rate $= 0{\cdot}5$ g/s and if the reflux ratio $= 8$, then:

Vapour rate $= 4{\cdot}5$ g/s

and $G = 4{\cdot}5 \times 10^{-3}/(\pi/4)(0{\cdot}1)^2 = 0{\cdot}573\ \mathrm{kg/m^2\,s}$

$\mu = 0{\cdot}02 \times 10^{-3}\ \mathrm{N\,s/m^2}$

$e = 0{\cdot}72$

$S \quad (1-e) \quad \mu$

$\therefore \quad Re_1 = 0{\cdot}573/(800 \times 0{\cdot}28 \times 0{\cdot}02 \times 10^{-3}) = 128$

$$\frac{R}{\rho u^2} = 4{\cdot}17/128 + 0{\cdot}29 = 0{\cdot}32$$

Hence $-\int \rho \,\mathrm{d}P = 0{\cdot}32 \times 800 \times 0{\cdot}28 \times (0{\cdot}573)^2 \times 2{\cdot}4/(0{\cdot}72)^3$

since $l = 16 \times 0{\cdot}15 = 2{\cdot}4$ m

$$-\int \rho \,\mathrm{d}P = 151{\cdot}3$$

Now $\rho = \rho_s \times P/P_s$ $\quad \rho$ si se considera T ~~…~~ constante

where s refers to the still.

The vapour density in the still

$$\rho_s = \frac{155}{22{\cdot}4} \times \frac{273}{395} \times \frac{P_s}{101{\cdot}3 \times 10^3} = 4{\cdot}73 \times 10^{-5} P_s$$

$$\rho = 4{\cdot}73 \times 10^{-5} P$$

$$-\int_{P_c}^{P_s} \rho \,\mathrm{d}P = -\int_{P_c}^{P_s} 4{\cdot}73 \times 10^{-5} P \,\mathrm{d}P = 4{\cdot}73 \times 10^{-5}(P_s^2 - P_c^2)$$

where P_c = condenser pressure.

Now $(P_s - P_c) = -\Delta P$ and if $P_s \simeq -\Delta P$:

$$(P_s^2 - P_c^2) \simeq \Delta P^2$$

$$\Delta P^2 = 151{\cdot}3/(4{\cdot}73 \times 10^{-5})$$

$$\Delta P = 1790\ \mathrm{N/m^2}$$

Now $$P_s = 8000\ \mathrm{N/m^2}$$

$\therefore$ $$P_c = 6210\ \mathrm{N/m^2} = \underline{\underline{6{\cdot}2\ \mathrm{kN/m^2}}}$$

Problem 4.3

A column 0·6 m diameter and 4 m tall, packed with 25 mm ceramic Raschig rings, is used in a gas absorption process carried out at atmospheric pressure and 293 K. If the liquid and gas can be considered to have the properties of water and air, and their flow rates are 6·5 and 0·6 kg/m²s respectively, what will be the pressure drop across the column?

Use (a) Carman's method, (b) one other method, and compare the results obtained. How much can the liquid rate be increased before the column will flood?

Solution

(a) *Carman's method*

Equation 4.17 presents Carman's correlation for flow through randomly packed beds as:

$$R/\rho u^2 = 5/Re_1 + 1{\cdot}0/Re_1^{0{\cdot}1} \qquad \text{(equation 4.17)}$$

where $$R/\rho u^2 = \frac{e^3}{S(1-e)}\frac{\Delta P}{l}\frac{1}{\rho u^2} \qquad \text{(equation 4.13)}$$

and $$Re_1 = \frac{G}{S(1-e)\mu} \qquad \text{(equation 4.11)}$$

Using the data given in the problem:

$$\rho_{\mathrm{air}} = \frac{29}{22{\cdot}4} \times \frac{273}{293} = 1{\cdot}21\ \mathrm{kg/m^3}$$

$$G = 0{\cdot}6\ \mathrm{kg/m^2 s}$$

$\therefore$ $$u = 0{\cdot}6/1{\cdot}21 = 0{\cdot}496\ \mathrm{m/s}$$

From Problem 4.5 for 25 mm Raschig rings,

$$S = 656\ \mathrm{m^2/m^3} \quad \text{and} \quad e = 0{\cdot}71$$

$\therefore$ $$Re_1 = 0{\cdot}6/656 \times 0{\cdot}29 \times 0{\cdot}018 \times 10^{-3} = 175$$

$$\frac{R}{\rho u^2} = \frac{(0{\cdot}71)^3}{656 \times 0{\cdot}29} \times \frac{(-\Delta P)}{4} \times \frac{1}{1{\cdot}21 \times (0{\cdot}496)^2} = 1{\cdot}71 \times 10^{-3} P$$

$$1{\cdot}71 \times 10^{-3}(-\Delta P) = 5/175 + 1{\cdot}0/(175)^{0{\cdot}1} = 0{\cdot}625$$

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and $$\Delta P = 365\ \text{N/m}^2$$

Figure 4.23 may be used to allow for the effect of liquid flow. The correction factor is found from Fig. 4.23 to be 1·8 so that:

$$P = 365 \times 1{\cdot}8 = \underline{\underline{670\ \text{N/m}^2}}$$

(b) *Morris and Jackson's method*

The wetting rate $L_w = u_L/S_B = 6{\cdot}5/1000 \times 190 = 3{\cdot}42 \times 10^{-5}\ \text{m}^3/\text{s m}^2$

since $$S_B = S(1 - e) = 190\ \text{m}^2/\text{m}^3$$

From Fig. 4.25 the number of velocity heads lost $= N = 1050$

Then
$$\Delta P = 0{\cdot}5 N \rho_G u_G^2 l \qquad \text{(equation 4.36)}$$
$$= 0{\cdot}5 \times 1050 \times 1{\cdot}21 \times 0{\cdot}496^2 \times 4$$
$$= \underline{\underline{625\ \text{N/m}^2}}$$

(c) Figure 4.28 may be used to find at what liquid flow rate the column will flood.

$$\frac{u_G^2 S_B}{ge^3}\frac{\rho_G}{\rho_L}\left(\frac{\mu_L}{\mu_w}\right)^{0{\cdot}2} = \frac{0{\cdot}496^2 \times 190 \times 1{\cdot}21}{9{\cdot}81 \times (0{\cdot}71)^3 \times 1000} = 0{\cdot}0161$$

From the flooding curve, $$\frac{L}{G}\sqrt{\frac{\rho_G}{\rho_L}} = 1{\cdot}2$$

and $$L = 1{\cdot}2 \times 0{\cdot}6\sqrt{(1000/1{\cdot}21)}$$
$$= \underline{\underline{20{\cdot}7\ \text{kg/m}^2\ \text{s}}}$$

Problem 4.4

A packed column, 1·2 m in diameter and 9 m tall, and packed with 25 mm Raschig rings, is used for the vacuum distillation of a mixture of isomers of molecular weight 155 kg/kmol. The mean temperature is 373 K, the pressure at the top of the column is maintained at 0·13 kN/m^2, and the still pressure ranges between 1·3 and 3·3 kN/m^2. Obtain an expression for the pressure drop on the assumption that it is not appreciably affected by the liquid flow and can be calculated using a modified form of Carman's equation (4.17). Show that, over the range of operating pressures used, the pressure drop is approximately directly proportional to the mass rate of flow of vapour, and calculate the pressure drop at a vapour rate of 0·125 kg/m^2 s.

Data: Specific surface of packing $S = 650\ \text{m}^2/\text{m}^3$.

Mean voidage of bed $e = 0{\cdot}71$.

Viscosity of vapour $= 0{\cdot}018\ \text{mN s/m}^2$.

Molecular volume $= 22{\cdot}4\ \text{m}^3/\text{kmol}$.

Solution

The proof that the pressure drop is approximately proportional to the mass flow rate of vapour is illustrated in Problem 4.5. Using the data specified in this problem:

$$Re_1 = G/S(1-e)\mu$$
$$= 0{\cdot}125/660 \times 0{\cdot}29 \times 0{\cdot}018 \times 10^{-3} = 36{\cdot}3$$

The modified Carman's equation states:

$$R/\rho u^2 = 5/Re_1 + 1/Re_1^{0 \cdot 1}$$
$$= 5/36{\cdot}3 + 1/(36{\cdot}3)^{0 \cdot 1} = 0{\cdot}836$$

As in Problem 4.2:

$$\frac{R}{\rho u^2} = \frac{e^3}{S(1-e)} \frac{(-\mathrm{d}P)}{\mathrm{d}l} \frac{1}{\rho u^2} \qquad \text{(equation 4.13)}$$

$$= \frac{e^3}{S(1-e)} \frac{(-\mathrm{d}P)}{\mathrm{d}l} \frac{\rho}{G^2}$$

$$\therefore \quad -\int \rho \, \mathrm{d}P = \frac{R}{\rho u^2} \frac{S(1-e)}{e^3} G^2 l$$

$$= 0{\cdot}836 \times 650 \times 0{\cdot}29 \times 9G^2/(0{\cdot}71)^3$$

$$= 3690G^2$$

Now $\rho/P = \rho_s/P_s$ where subscript s refers to the still,

$$\rho_s = \frac{155}{22{\cdot}4} \times \frac{273}{373} \times \frac{P_s}{101{\cdot}3 \times 10^3} = 5 \times 10^{-5} P_s$$

$$\rho_s/P_s = 5 \times 10^{-5}$$

$$\therefore \quad \rho = 5 \times 10^{-5} P$$

$$-\int_{P_c}^{P_s} \rho \, \mathrm{d}P = 2{\cdot}5 \times 10^{-5}(P_s^2 - P_c^2)$$

Now $P_s - P_c = -\Delta P$, and if $\Delta P \simeq -P_s$, $\quad (P_s^2 - P_c^2) \simeq \Delta P^2$

$$\therefore \quad -\int_{P_c}^{P_s} \rho \, \mathrm{d}P = 2{\cdot}5 \times 10^{-5} \Delta P^2 = 3690G^2$$

i.e. $\underline{\underline{\Delta P \simeq G}}$

If $G = 0{\cdot}125$, $-\Delta P = [3690 \times (0{\cdot}125)^2/2{\cdot}5 \times 10^{-5}]^{0 \cdot 5}$
$= 1520 \text{ N/m}^2$
$= \underline{\underline{1{\cdot}52 \text{ kN/m}^2}}$

Problem 4.5

A packed column, 1·22 m in diameter and 9 m tall, and packed with 25 mm Raschig rings, is used for the vacuum distillation of a mixture of isomers of molecular weight 155 kg/kmol The mean temperature is 373 K, the pressure at the top of the column is maintained at 0·13 kN/m^2, and the still pressure is 1·3 kN/m^2. Obtain an expression for the pressure drop on the assumption that it is not appreciably affected by the liquid flow and can be calculated using the modified form of Carman's equation.

Show that, over the range of operating pressures used, the pressure drop is approximately directly proportional to the mass rate of flow of vapour, and calculate approximately the flow rate of vapour.

Data: Specific surface of packing $S = 656\ \mathrm{m^2/m^3}$.
Mean voidage of bed $e = 0{\cdot}71$.
Viscosity of vapour $\mu = 0{\cdot}018\ \mathrm{mN\,s/m^2}$.
Kilogram molecular volume $= 22{\cdot}4\ \mathrm{m^3/kmol}$.

Solution

The modified form of Carman's equation states:

$$R/\rho u^2 = 5/Re_1 + (1/Re_1)^{0\cdot 1} \qquad \text{(equation 4.17)}$$

where

$$Re_1 = G/S(1-e)\mu$$

$$Re_1 = \frac{G}{656(1-0{\cdot}71)\times 0{\cdot}018\times 10^{-3}} = 292G$$

$$\therefore \quad \frac{R}{\rho u^2} = \frac{5}{292G} + \left(\frac{1}{292G}\right)^{0\cdot 1} = \frac{0{\cdot}017}{G} + \frac{0{\cdot}57}{G^{0\cdot 1}}$$

Now

$$\frac{R}{\rho u^2} = \frac{e^3}{S(1-e)}\frac{(-\mathrm{d}P)}{\mathrm{d}l}\frac{\rho}{G^2}$$

and as in the previous problem, by substitution:

$$-\int \rho\, \mathrm{d}P = 81{\cdot}4G + 2730G^{1\cdot 9}$$

As before,

$$-\int \rho\, \mathrm{d}P = 2{\cdot}5\times 10^{-5}\Delta P^2$$

$$\therefore \quad \Delta P^2 = 3{\cdot}26\times 10^6 G + 1{\cdot}09\times 10^8 G^{1\cdot 9}$$

If the first term is neglected:

$$\underline{-\Delta P = 1{\cdot}05\times 10^4 G^{0\cdot 95}}$$

If $-\Delta P = 1300 - 130 = 1170\ \mathrm{N/m^2}$:

$$\underline{G = 0{\cdot}099\ \mathrm{kg/m^2\,s}}$$