Problem 4.1.

Consider a cell culture with negligible growth ($\mu \approx 0$) characterized by the following biological reaction:

Reaction: $S \rightarrow P\ (+\ X)$

Kinetic: $$v_s = \frac{v_{s,max} S}{K_S + S}$$

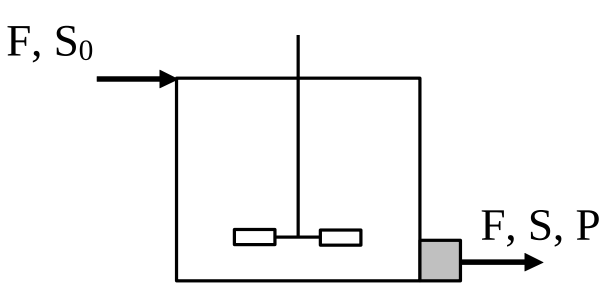
$v_{s,max} = 0.6$ g-subs g-cel^{-1} h^{-1}

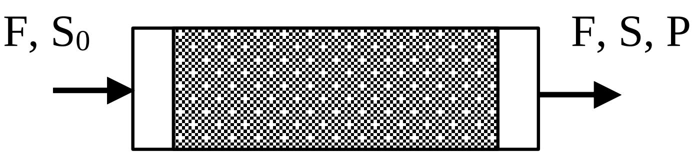
$K_S = 0.01$ g/L

$Y'_{p/s} = 0.2$ g-prod/g-subs

(negligible maintenance)

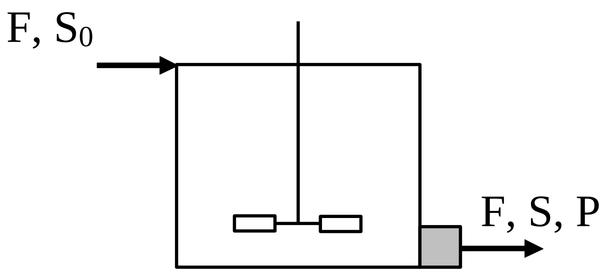
Consider the Bioreactors indicated in the figure. In the case of the CSTR, a filter in the output current prevents the Bioreactor from being washed. In the case of PFR, cells are immobilized on a solid support without diffusional limitations for the transport of 'S' and 'P'. In either case the cells are evenly distributed with concentration X=12.1 g/l.





a) Size the Bioreactors indicated in the figure for a productivity of 60 g product/h knowing that the input current has a substrate concentration So=13.2 g/l and that its conversion is 95%. Consider that the substrate is in large excess in each of the reactors and that consequently the kinetics is of order 0.

CSTR



Balance to the substrate

$$\frac{F}{V} S_0 - \frac{F}{V} S - r_s = 0$$

r_s = volumetric substrate consumption rate

$$F\,S_0 - F\,S - r_s\,V = 0$$

$$F\,(S_0 - S) - r_s\,V = 0$$

$$r_s = V_s\,X$$

V_s = specific substrate consumption rate

$$F\,(S_0 - S) - V_s\,X\,V = 0$$

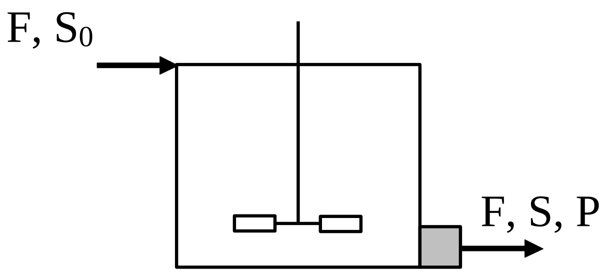
Calcular S

$$S_0 = 13{,}2\ \mathrm{g/l}$$

$$\Big\downarrow \textit{95\% conversion}$$

$$S = 0{,}66\ \mathrm{g/l}$$

CSTR



$$S \gg K_s$$

$$0.66 \text{ g/L} \gg 0.01 \text{ g/L}$$

$$V_s = V_{s,max} \frac{S}{K_s + S}$$

$$V_s \approx V_{s,max}$$

$$F\,(S_0 - S) - V_{s,max}\,X\,V = 0$$

$$\mathrm{S_0} = 13{,}2\ \mathrm{g/l}$$

$$\Big\downarrow \textit{95\% conversion}$$

$$\mathrm{S} = 0{,}66\ \mathrm{g/l}$$

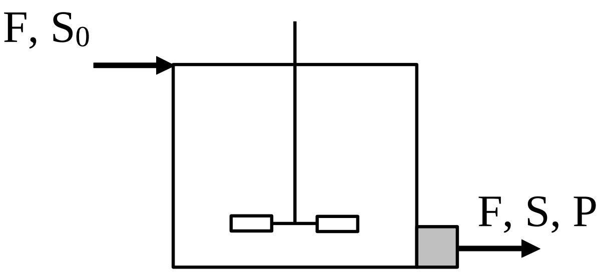
$$\mathrm{Y_{p/s}} = \frac{\Delta \mathrm{P}}{\Delta \mathrm{S}} = \frac{P - P_0}{S_0 - S}$$

$$\mathrm{P} = 2{,}51\ \mathrm{g/l}$$

$$\mathrm{F\,P} = 60\ \mathrm{g/h}$$

$$\mathrm{F} = 23{,}9\ \mathrm{l/h}$$

CSTR



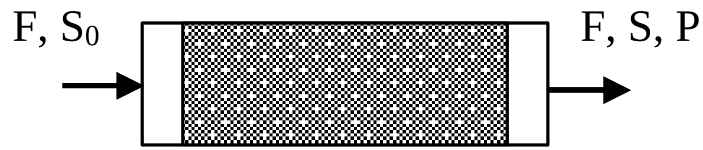
$$S \gg K_s \qquad V_s = V_{s,max}\frac{S}{K_s + S}$$

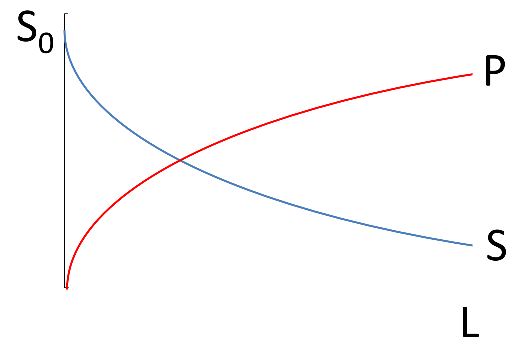
$$V_s \approx V_{s,max}$$

$$F\,(S_0 - S) - V_{s,max}\,X\,V = 0$$

$$V = 41{,}3\ \mathrm{l}$$

PFR





Balance to the substrate

$$u \frac{dS}{dz} = -r_s$$

$$u \frac{dS}{dz} = -V_s\, X$$

$$u\, dS = -V_{s,max}\, X\, dz$$

$$u \int_{S_0}^{S} dS = -V_{s,max}\, X \int_{0}^{L} dz$$

$$u\,(S - S_o) = -V_{s,max}\, X\,(L - 0)$$

$$u\,(S - S_o) = -V_{s,max}\, X\; L$$

$$\frac{F}{A}\,(S - S_o) = -V_{s,max}\, X\; L$$

$$F\,(S - S_o) = -V_{s,max}\, X\; L\; A$$

$$F\,(S - S_o) = -V_{s,max}\, X\; V$$

$$V = 41{,}3\ l$$

b) Size the reactors for the same conditions as in point a) considering that the saturation constant is Ks= 25 g/l, S_0=2.5 g/l and X=1.21 g/l.

$$S_0 = 2{,}5 \text{ g/l}$$

$$\Big\downarrow \text{ 95\% conversion}$$

$$S = 0{,}125 \text{ g/l}$$

$$Y_{p/s} = \frac{\Delta P}{\Delta S} = \frac{P - P_0}{S_0 - S}$$

$$P = 0{,}475 \text{ g/l}$$

60 g/h

$$F\,P = 60 \text{ g/h}$$

$$F = 126{,}3 \text{ l/h}$$

CSTR

$$S \ll K_s$$

$$0.125 \text{ g/L} \ll 25 \text{ g/L}$$

$$V_s = V_{s,max} \frac{S}{K_s + S}$$

$$V_s \approx \frac{V_{s,max}}{K_s} S$$

$$V_s \approx 0{,}003 \text{ gS/gX.h}$$

$$F\,(S_0 - S) - V_s\,X\,V = 0$$

$$V = 83322 \text{ l} = 83{,}3 \text{ m}^3$$

PFR

Balance to the substrate

$$u\,\frac{dS}{dz} = -\,r_s$$

$$u\,\frac{dS}{dz} = -\,V_s\,X$$

$$u\,\frac{dS}{S} = -\,\frac{V_{s,max}}{K_s}\,X\,dz$$

$$u\int_{S_0}^{S}\frac{dS}{S} = -\,\frac{V_{s,max}}{K_s}\,X\int_{o}^{L}dz$$

$$u\ln\frac{S}{S_0} = -\,\frac{V_{s,max}}{K_s}\,X\,(L - 0)$$

$$\frac{F}{A}\ln\frac{S}{S_0} = -\,\frac{V_{s,max}}{K_s}\,X\,L$$

$$F\ln\frac{S}{S_0} = -\,\frac{V_{s,max}}{K_s}\,X\,L\,A$$

$$F\ln\frac{S}{S_0} = -\,\frac{V_{s,max}}{K_s}\,X\,V$$

$$V = 13029\ \mathrm{l} = 13{,}0\ \mathrm{m}^3$$

c) Comment on the results obtained in a) and b)

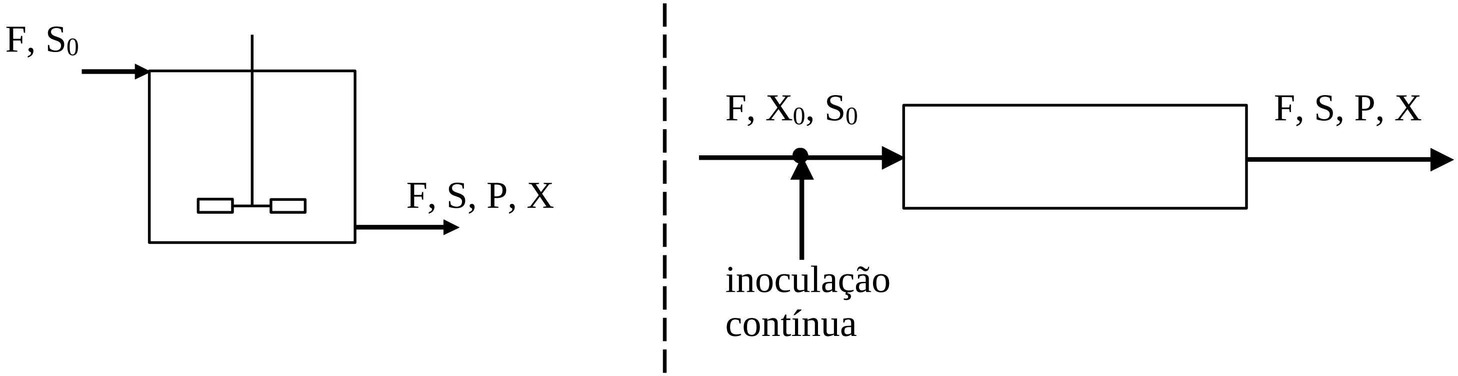
When cell growth is nigligible

and $S \gg K_s$ $V_{CSTR} \approx V_{PFR}$

and $S \leq K_s$ $V_{CSTR} > V_{PFR}$

Problema 4.2.

Consider the following Biorreactors



In which the biological reaction occurs

Reaction: $S \rightarrow P + X$

kinetic: $$\mu = \frac{\mu_{max} S}{K_S + S}$$

$\mu_{max}=0.3\ h^{-1}$

$K_S=0.01$ g/L

$Y'_{xs}=0.5$ g cell/g subs

$Y'_{xp}=4.3$ g cell/g prod

(negligible maintenance)

It is intended to dimension each one of the Bioreactors to treat a stream with $S_0=150$ g/l and to convert 90% of substrate into product and cells. The size of the reactor must be such that the absolute productivity is 1 kg of product per hour.

a) Size a CSTR for the specified conditions knowing that the reactor is initially inoculated with 1 g/l of cells after which the process converges to steady state.

steady-state

$$\mu = D = \frac{F}{V}$$

Calcular μ

$$\mu = \frac{\mu_{max} S}{K_s + S} = 0{,}2998\ h^{-1}$$

Calcular S

$$S_0 = 150\ \mathrm{g/l}$$

$$\Big\downarrow \textit{90\% conversion}$$

$$S = 15\ \mathrm{g/l}$$

Calcular F

$$1\ \mathrm{kg/h}$$

$$F\,P = 1000\ \mathrm{g/h}$$

$$F = 63{,}7\ \mathrm{l/h}$$

Calcular P

$$Y_{x/p} = \frac{\Delta X}{\Delta P} = \frac{X - X_0}{P - P_0}$$

$$P = 15{,}7\ \mathrm{g/l}$$

Calcular X

$$Y_{x/s} = \frac{\Delta X}{\Delta S} = \frac{X - X_0}{S_0 - S}$$

$$X = 67{,}5\ \mathrm{g/l}$$

$$D = \frac{F}{V}$$

$$V = 213{,}8\ l$$

b) Size a PFR for the same conditions but with continuous inoculation with X_0= 1 g/l. Compare the result with the one obtained in point a) and comment.

Balance to the biomass

$$u\,\frac{dX}{dz} = r_x = \mu\,X$$

$$\mu = \frac{\mu_{max}\,S}{K_s + S} = \frac{\mu_{max}\,S}{K_s + S} \approx \mu_{max}$$

$$S \gg K_s$$

$$u\,\frac{dX}{X} = \mu_{max}\,dz$$

$$u\int_{X_0}^{X}\frac{dX}{X} = \mu_{max}\int_0^L dz$$

$$F\ln\frac{X}{X_0} = \mu_{max}\,L\,A$$

$$u\ln\frac{X}{X_0} = \mu_{max}\,L$$

$$F\ln\frac{X}{X_0} = \mu_{max}\,V$$

$$\frac{F}{A}\ln\frac{X}{X_0} = \mu_{max}\,L$$

$$V = 897{,}6\ \mathrm{l}$$

c) Determine the length and diameter of the PFR from point b). Consider that the physical properties of the culture medium are similar to those of water.

$$\mathrm{Re} = \frac{\rho\, \mathrm{u}\, \mathrm{d}}{\mu}$$

Re = Reynolds number $\qquad \mathrm{Re} < 2000$ *plug flow*

ρ = density = $1000\ \mathrm{kg/m^3}$ *water*

μ = viscosity = $10^{-3}\ \mathrm{Pa.s}$ *water*

$1\ \mathrm{Pa.s} = 1\mathrm{kg/m.s}$

$$\mathrm{u} = \frac{\mathrm{F}}{\mathrm{A}} = \frac{F}{\frac{\pi\, d^2}{4}} = \frac{4\, F}{\pi\, d^2}$$

$$\mathrm{F} = 63{,}7\ \mathrm{l/h} = 1{,}769 \times 10^{-5}\ \mathrm{m^3/s}$$

$$\mathrm{Re} = \frac{\rho\ \mathrm{u}\ \mathrm{d}}{\mu} = \frac{1000\ kg/m^3 \times \dfrac{4\ F}{\pi d^2}\ m^3/s \times d}{10^{-3}\ kg/m.s} = \frac{0{,}071}{\pi \times 10^{-3} \times d}$$

$$\mathrm{Re} < 2000 \qquad \frac{0{,}071}{\pi \times 10^{-3} \times d} < 2000$$

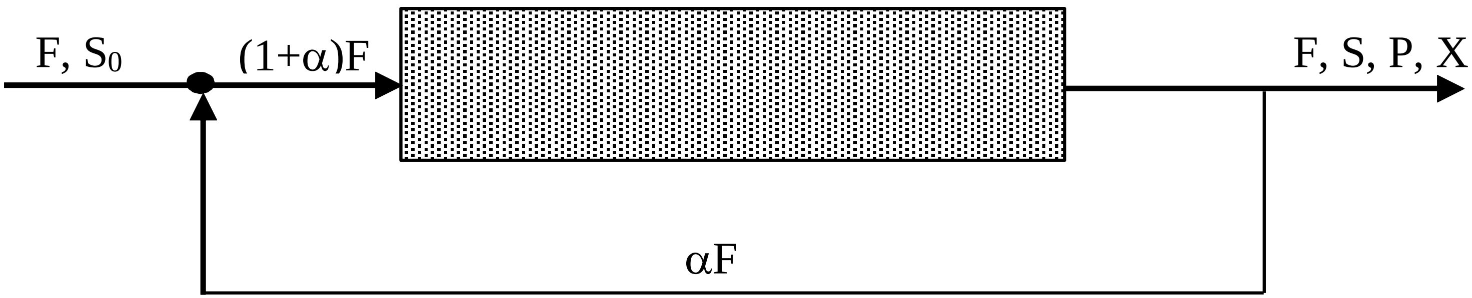
$$\mathrm{d} > 1{,}1\ cm$$

$$\mathrm{V} = \mathrm{A\ L}$$

$$\mathrm{L} = 9444\ \mathrm{m}$$

Problema 4.3.

Consider a piston Bioreactor with backflow from the outlet to the inlet as indicated in the figure



with α the ratio (reflow flow)/(inflow flow). The input current has S0=100g/l and the flow through the system is F=10 l/h. The intended conversion is 90% of the input substrate. Consider that the Bioreactor is always operated with a large excess of substrate so $\mu=\mu max=0.3$ h-1. The yields are Y’x/s=0.5 and Y’x/p=7.1 (negligible maintenance).

a) Calculate the concentrations of S, P and X at the exit of the system.

Calcular S

$$S_0 = 100 \text{ g/l}$$

$$\Big\downarrow \textit{90\% conversion}$$

$$S = 10 \text{ g/l}$$

Calcular X

$$Y_{x/s} = \frac{\Delta X}{\Delta S} = \frac{X - X_0}{S_0 - S}$$

$$X = 45 \text{ g/l}$$

Calcular P

$$Y_{x/p} = \frac{\Delta X}{\Delta P} = \frac{X - X_0}{P - P_0}$$

$$P = 6{,}34 \text{ g/l}$$

b) If you choose α =0.1, what volume is needed to achieve the desired conversion?

$$\alpha = 0{,}1 \qquad V = \frac{F\ ln\frac{X}{X_0}}{\mu_{max}} \qquad F \rightarrow (1+\alpha)\,F$$

Balance to the biomass

inlet $\alpha\,X$

outlet $X + \alpha\,X$

$$\frac{X_{out}}{X_{in}} = \frac{X + \alpha\,X}{\alpha\,X} = \frac{1+\alpha}{\alpha}$$

$$V = \frac{(1+\alpha)\,F\,\ln\frac{1+\alpha}{\alpha}}{\mu_{max}} = 87{,}9\ l$$

c) If you choose α =2, what volume is needed to achieve the desired conversion?

$$V = \frac{(1+\alpha)\, F \ln \frac{1+\alpha}{\alpha}}{\mu_{max}} = 40{,}5 \text{ l}$$

d) Compare and comment on the results of b) and c)

$\alpha = 0{,}1$ $\qquad V = 87{,}9 \text{ l}$

$\alpha = 2$ $\qquad V = 40{,}5 \text{ l}$

=> increasing α → reduction of the PFR volume