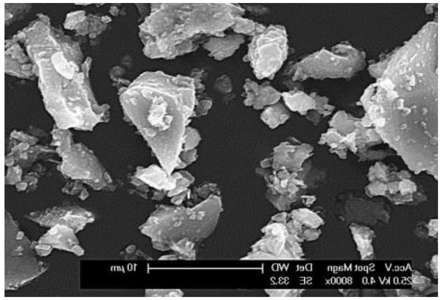
Chapter 1. Introduction

1.1 Properties of single particles: size, shape
1.2 Particle size distribution: mean diameter
1.3 Methods to measure particle size: sieving, elutriation

J.M. Coulson and J.F. Richardson pp 1 - 17

Physichochemical properties of solid particles

Single particles



Cement particles

Escherichia coli (single cell) TEM image

Size, Shape, hardness, compressive resistence, electrical charge, (intraparticle) porosity, …

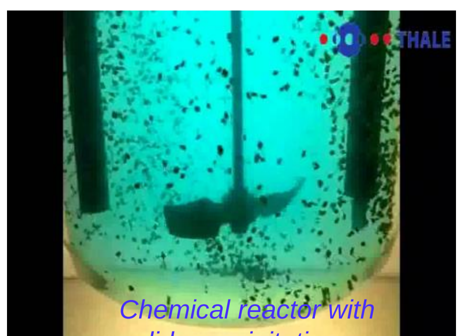
Bulk solids

Bulk cement

Particle size distribution, (interparticle) porosity, humidity, agglomeration, flowability, …

Solids suspensions

Heterogeneuous mixture of a fluid (gas or liquid) with a solid

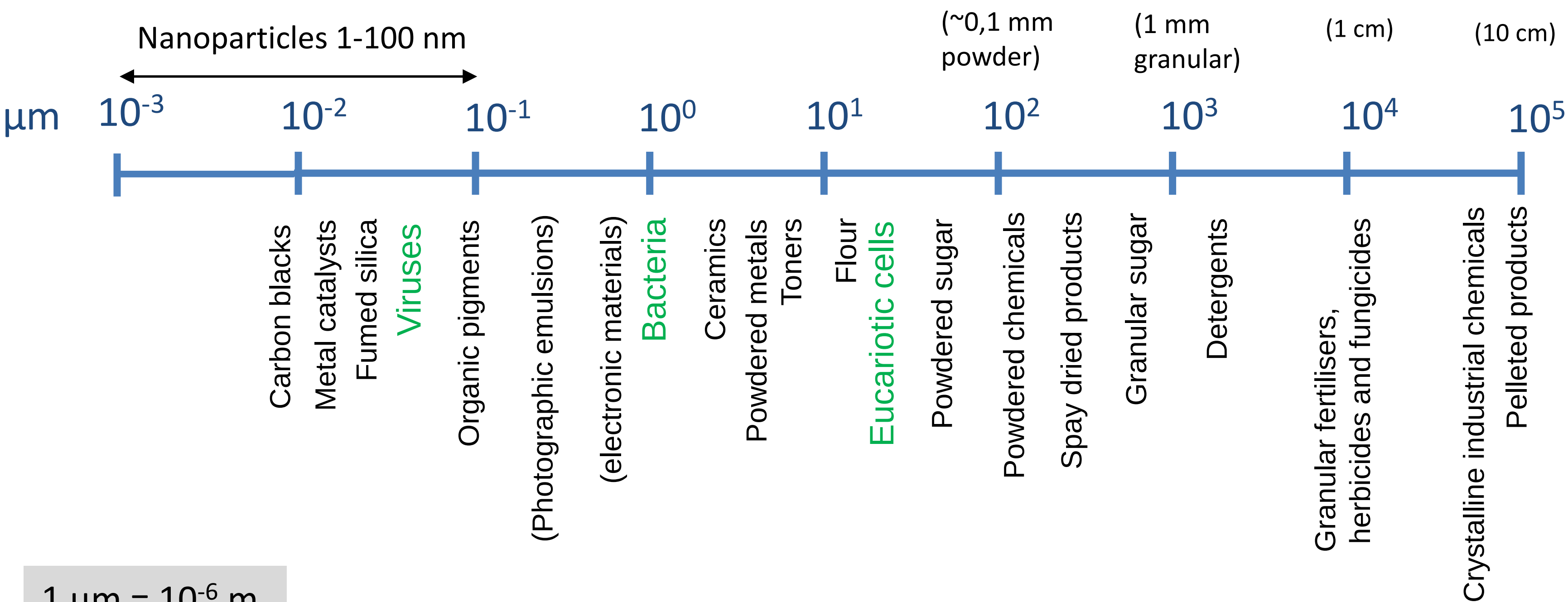


Chemical reactor with solids precipitation

Particle size distribution, Concentration of solids, viscosity of suspension, flocculation, settleability, …

Properties of single particles: size

Examples of particles in Chemical and Biological Engineering and respective sizes (adapted from Fig 1.2 in Coulson and Richardon)



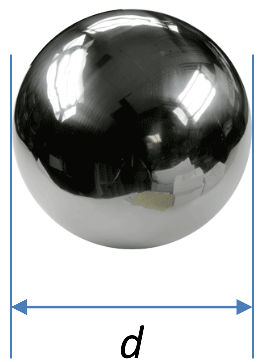
1 µm = 10^{-6} m

Properties of single particles: shape

Regular shape.

Regular shapes have well-defined geometry by a mathematical equation, thus their volume, surface area, etc… may be calculated by well defined mathematical formulas. For example, a sphere:

Sphere



Volume: $$\frac{\pi d^3}{6} = \frac{4\pi r^3}{3}$$

Surface area: $$\pi d^2 = 4\pi r^2$$

Projected area in a plane: $$\frac{\pi d^2}{4} = \pi r^2$$

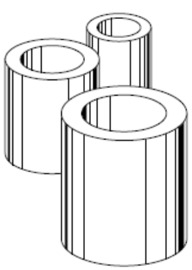
- A sphere is a special regular shape because it is the only shape that is completely symmetrical irrespective of orientation.
- A cube or a cylinder are also regular shapes but their symmetry depends on orientation

Properties of single particles: shape

Complex regular shapes

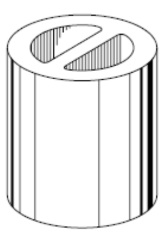
- Complex regular shapes are many times used in packed columns in chemical and biological engineering (distillation columns, absorption columns, chromatographic columns, packed bed reactors, etc..)
- **They are typically designed to maximize surface area per unit volume!!!!**
- They enable improved packing properties, e.g. regular packing with more particles per unit volume
- More expensive but with higher performance
- (More to this in chapter 5. Flow of fluids in packed columns)

Raschig Rings



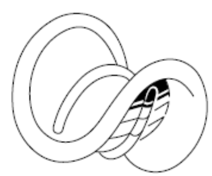
(a)

Lessing Rings



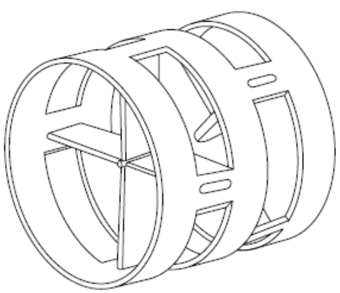
(b)

Berl Saddle



(c)

Pal Ring



(d)

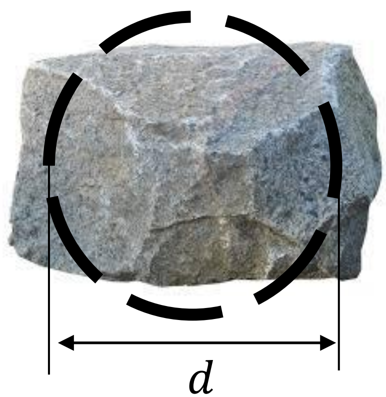
Nutter Ring

(f)

Properties of single particles: shape

Irregular shape: Cannot be defined by a mathematical equation. Undefined size that varies with orientation. Several possibilities to define **a characteristic dimension, d:**

- Diameter of sphere with same volume as particle (see below)
- Diameter of sphere with same surface as particle
- Diameter of sphere with same surface per unit volume as particle

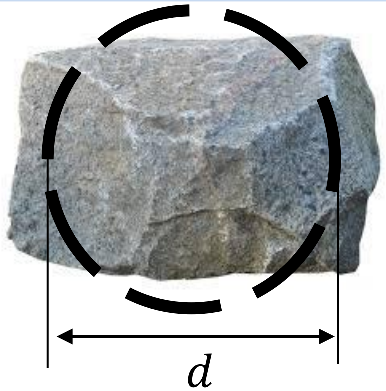


Defining the characteristic dimension of particle, d, as the diameter of an hypothetical sphere with the same volume of particle

$$V_{particle} = V_{shpere} = \frac{\pi d^3}{6}$$

$$\boxed{d = \sqrt[3]{\frac{6V_{particle}}{\pi}}}$$

Properties of single particles: shape



For any **irregular particle** with characteristic dimension, d, shape and size related characteristic properties may be quantified that assume particle symmetry.

-Particle length [L, m]:

$$L = d$$

-Particle surface area [S, m^2]:

$$S = k'd^2$$

surface factor - k' ($k' = \pi$ for spheres)

-Particle volume [V, m^3] :

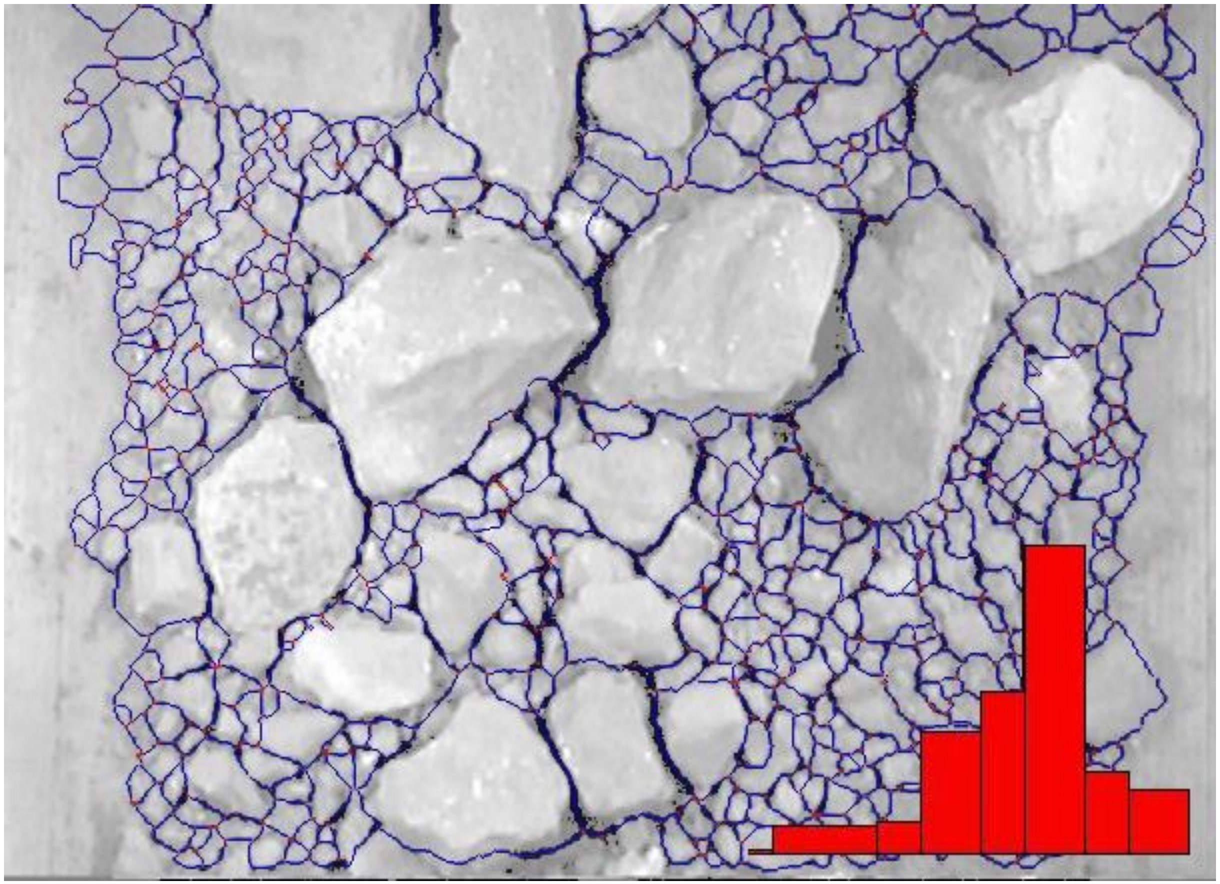
$$V = k''d^3$$

volume factor - k'' ($k'' = \pi/6$ for spheres)

-Particle mass [m, kg] :

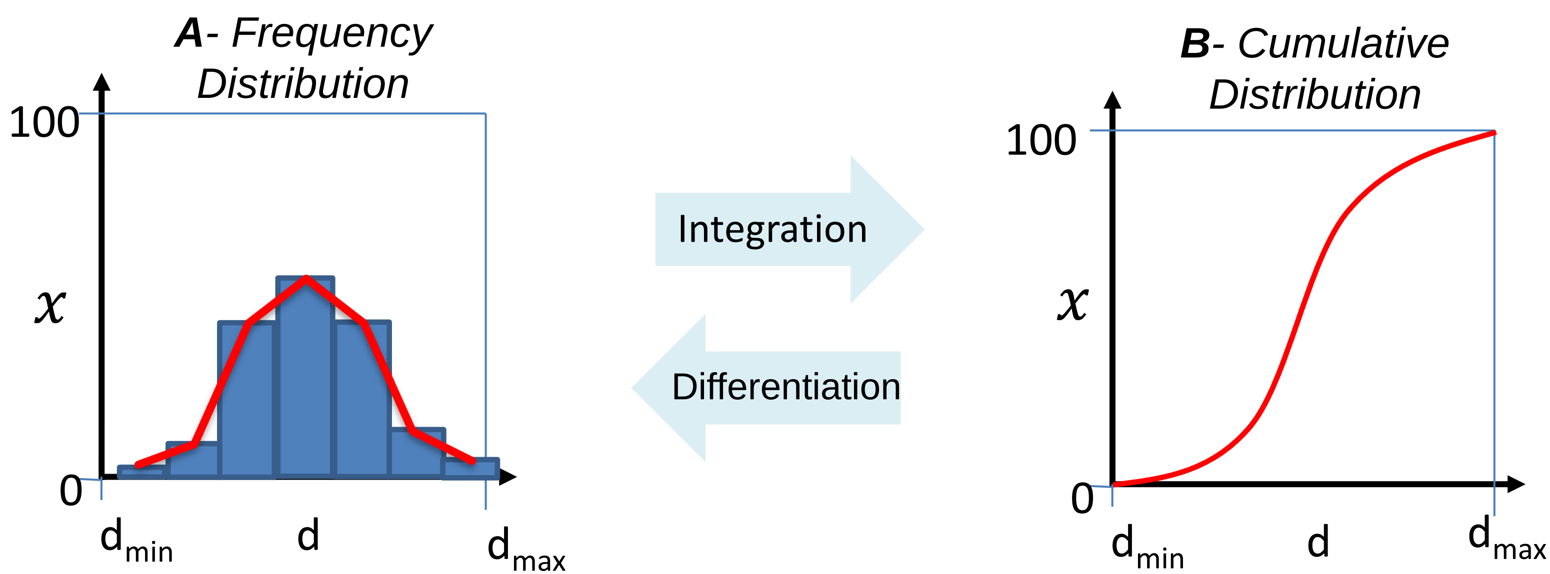
$$m = \rho_s V = \rho_s k'' d^3$$

specific mass of solid [kg/m3]- ρ_s



Particle size distribution

Bulk solids or solids suspensions typically contain particles with many different sizes. **Particle size distributions** characterize the mixture in terms of quantity of particles (e.g. mass fracion, x) as function of respective size (e.g. diameter, d). There are two ways to represent particle size distribution: **A** – Frequency distribution, **B** – Cumulative distribution



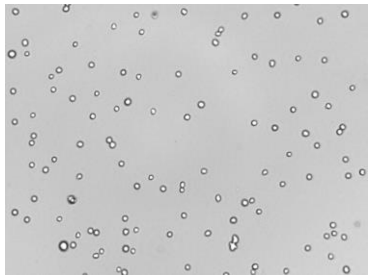
{Any point in crurve reads mass fraction, x, with size = d}

{Any point in crurve reads mass fraction, x, with size $\leq d$}

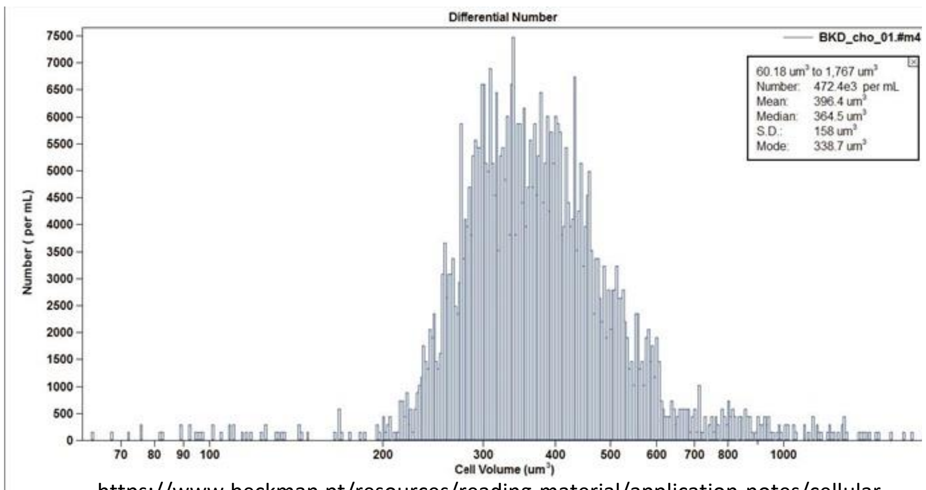
Particle size distribution

Examples: (1) Chinese Hamster Ovary (CHO) cells, (2) Portland cement

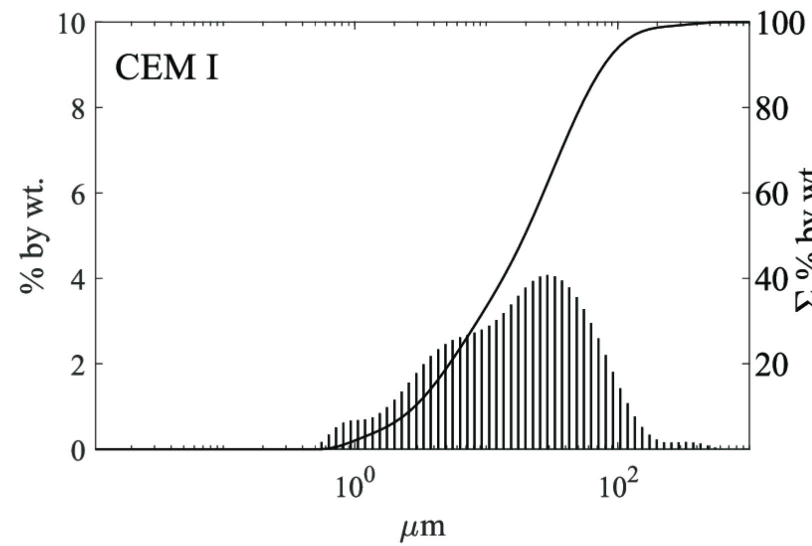
(1) CHO cells



(2) Portland cement



https://www.beckman.pt/resources/reading-material/application-notes/cellular-analysis-using-the-coulter-principle



The Influence of Ambient Temperature on High Performance Concrete Properties, DOI:
•10.3390/ma13204646

Particle size distribution

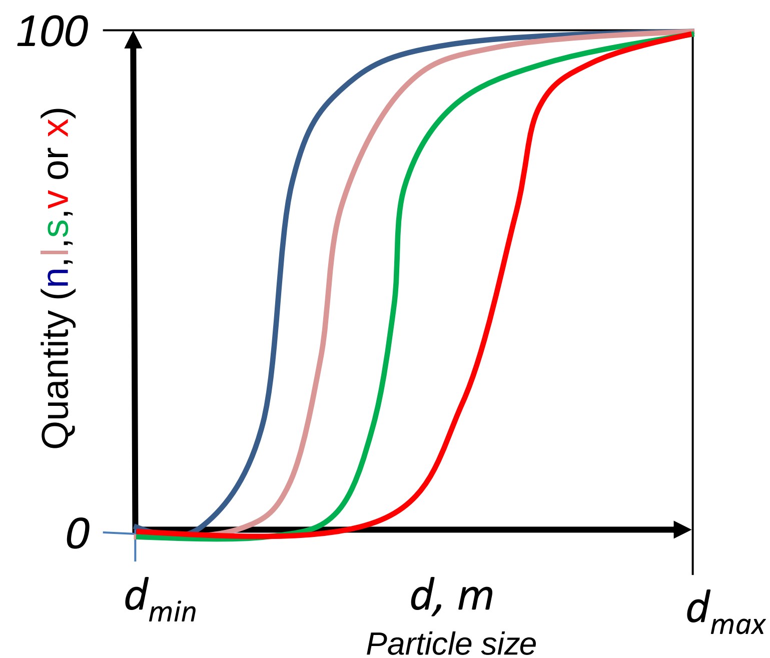
In particle size distribution curves, the "quantity" of particles may be defined in different ways. This results in different distribution curves for the same solids sample.

The same solids sample

Different ways to measure quantity

- ***n*** – number fraction
- ***l*** – length fraction
- ***s*** – surface fraction
- ***v*** – volume fraction =
- ***x*** – weight fraction

Different distribution curves



Particle size distribution

Any quantitities *n*, *x*, *v*, *s* or *l* may be converted into any of the other based on particle size and geometric considerations

Any fraction i of ni particles with size di

Number		Length, m		Surface, m^2		Volume, m^3		Mass, kg
$n_i, \ d_i$	⇨	$= n_i d_i$	⇨	$= n_i(\dot{k} d_i^2)$	⇨	$= n_i(\ddot{k} d_i^3)$	⇨	$= n_i(\rho \ddot{k} d_i^3)$

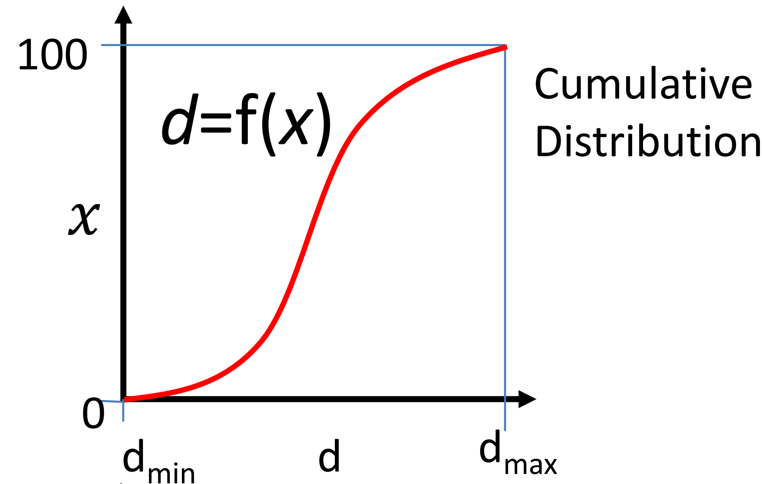
number fraction	Length fraction	Surface fraction	Volume fraction = Mass fraction
$n_i,$	$l_i = \frac{n_i d_i}{\sum n_k d_k}$	$s_i = \frac{n_i d_i^2}{\sum n_k d_k^2}$	$v_i = \frac{n_i d_i^3}{\sum n_k d_k^3} = x_i = \frac{n_i d_i^3}{\sum n_k d_k^3}$

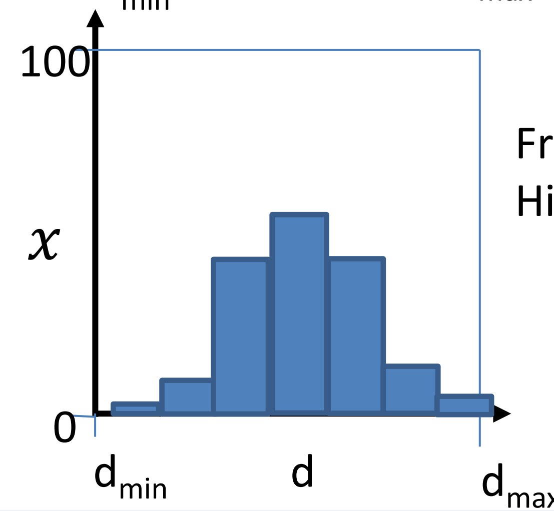
Particle size distribution: mean diameter

Mean diameter based on weight, $\bar{d}_x$

$$\bar{d}_x = \frac{\int d\, dx}{\int dx}$$

$$\bar{d}_x = \frac{\sum d_i x_i}{\sum x_i}$$





Particle size distribution: mean diameter

$\bar{d}_x$ - Mean diameter based on weight:

$$\bar{d}_x = \frac{\int d\ dx}{\int dx} \qquad \bar{d}_x = \frac{\sum d_i x_i}{\sum x_i}$$

$\bar{d}_n$ - Mean diameter based on number:

$$\bar{d}_n = \frac{\int d\ dn}{\int dn} \qquad \bar{d}_n = \frac{\sum d_i n_i}{\sum n_i}$$

$\bar{d}_V$ - Mean diameter based on volume:

$$\bar{d}_V = \frac{\int d\ dv}{\int dv} \qquad \bar{d}_V = \frac{\sum d_i v_i}{\sum v_i}$$

$\bar{d}_S$ - Mean diameter based on surface:

$$\bar{d}_S = \frac{\int d\ ds}{\int ds} \qquad \bar{d}_S = \frac{\sum d_i s_i}{\sum s_i}$$

$\bar{d}_L$ - Mean diameter based on length:

$$\bar{d}_L = \frac{\int d\ dl}{\int dl} \qquad \bar{d}_L = \frac{\sum d_i l_i}{\sum l_i}$$

Particle size distribution: mean diameter

	Measurement in weight, x	Measurement in number, n
Mean diameter based on volume (weight) $\bar{d}_x = \bar{d}_v$	$= \dfrac{\sum x_i d_i}{\sum x_i}$	$= \dfrac{\sum n_i d_i^4}{\sum n_i d_i^3}$
Mean diameter based on surface $\bar{d}_S$	$= \dfrac{\sum x_i}{\sum \frac{x_i}{d_i}}$	$= \dfrac{\sum n_i d_i^3}{\sum n_i d_i^2}$
Mean diameter based on length $\bar{d}_L$	$= \dfrac{\sum \frac{x_i}{d_i}}{\sum \frac{x_i}{d_i^2}}$	$= \dfrac{\sum n_i d_i^2}{\sum n_i d_i}$

Particle size distribution: mean diameter

Mean volume diameter (diâmetro do volume médio), $\acute{d}_{volume}$

It's a different way to calculate a representive sample size where all particles are assumed to have the same size

$$(\ddot{k}\acute{d}_{volume}^{3})\sum n_i = \sum n_i(\ddot{k}d_i^3)$$

Mean volume

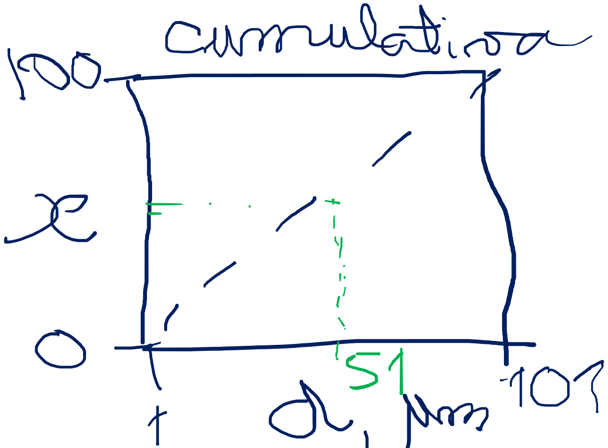
Real sample volume

$$\boxed{\acute{d}_{volume} = \sqrt[3]{\frac{\sum n_i d_i^3}{\sum n_i}}}$$

Particle size distribution: mean diameter

	Measurements in weight fraction, x	Measurements in number fraction, n
Mean volume (weight) diameter $\acute{d}_x = \acute{d}_v$	$= \sqrt[3]{\dfrac{\sum x_i}{\sum \frac{x_i}{d_i^3}}}$	$= \sqrt[3]{\dfrac{\sum n_i d_i^3}{\sum n_i}}$
Mean surface diameter $\acute{d}_S$	$= \sqrt{\dfrac{\sum \frac{x_i}{d_i}}{\sum \frac{x_i}{d_i^3}}}$	$= \sqrt{\dfrac{\sum n_i d_i^2}{\sum n_i}}$
Mean length diameter $\acute{d}_L$	$= \dfrac{\sum \frac{x_i}{d_i^2}}{\sum \frac{x_i}{d_i^3}}$	$= \dfrac{\sum n_i d_i}{\sum n_i}$

Problema I.3



$$\boxed{x = d - 1}$$

$$d = x + 1$$

b) $$\bar{d}_S = \frac{\int_0^{100} \cancel{d}\, \cancel{A} \frac{dx}{\cancel{d}}}{\int_0^{100} \cancel{A} \frac{dx}{d}} = 21,7\ \mu m$$

$$\begin{cases} dS = dn\, K' d^2 \\ dx = dn\, \rho K'' d^3 \end{cases} \Rightarrow dS = A \times \frac{dx}{d}, \quad A = \frac{K'}{\rho K''}$$

$n_i \quad d_i$

$dn \quad d$

$l_i = n_i d_i$

$dl = dn\, d$

$s_i = n_i K' d_i^2$

$$\boxed{dS = dn\, K' d^2}$$

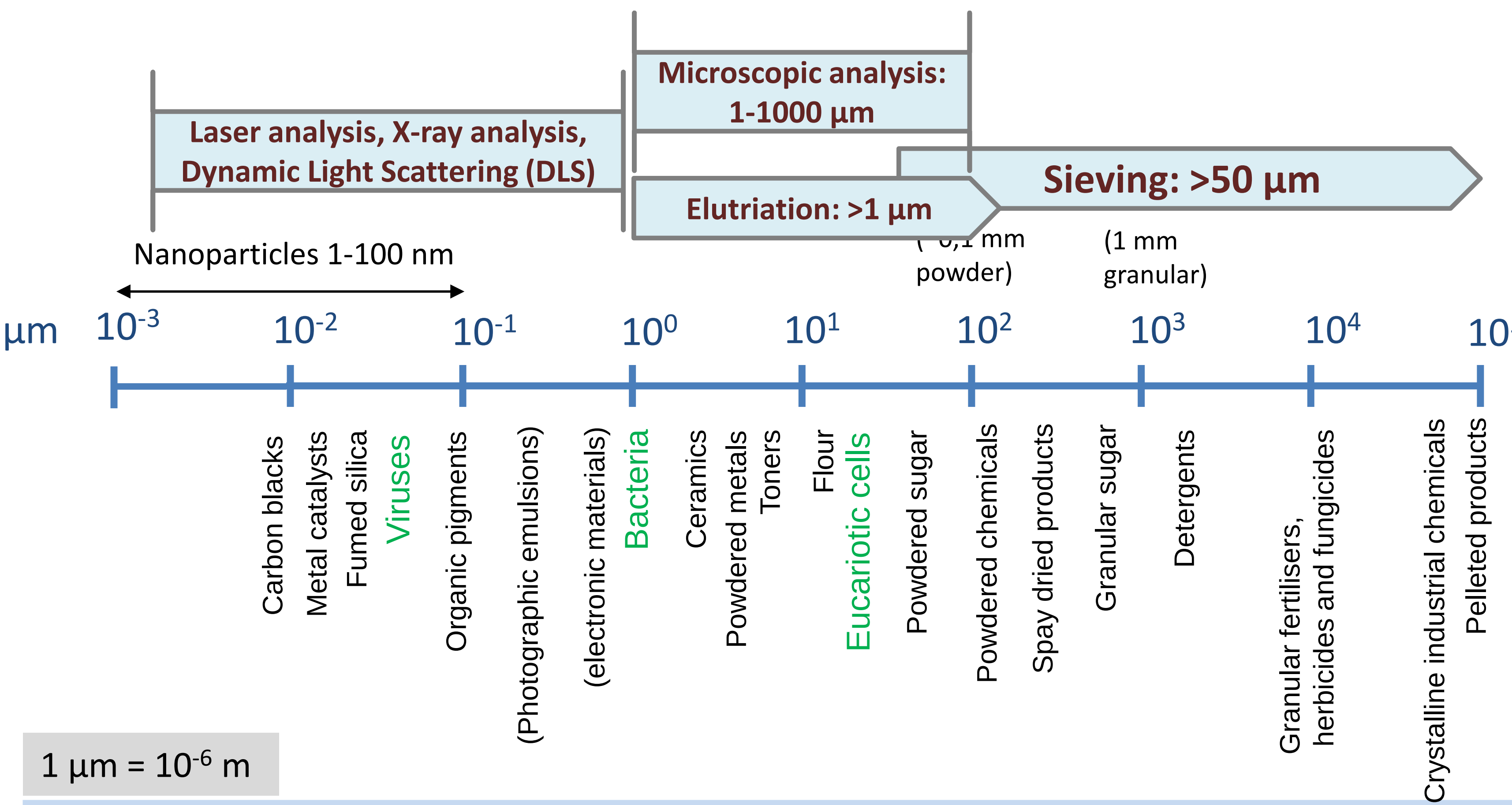
$V_i = n_i K'' d_i^3$

$x_i = n_i \rho K'' d_i^3$

$dV = dn\, K'' d^3$

$$\boxed{dx = dn\, \rho K'' d^3}$$

Methods to measure particle size



Methods to measure particle size: sieving

- Sievers consist of a sequence of large-to-low mesh sizes to separate particles based on their size
- Mechanical vibration is applied to avoid agglomeration of particles
- Laboratory sievers are used as a certified Analytical technique to determine particle size distribution in the range > 50 µm
- Industrial sievers are used to separate solids of different sizes in discontinuous or continuous mode

Lab Siever

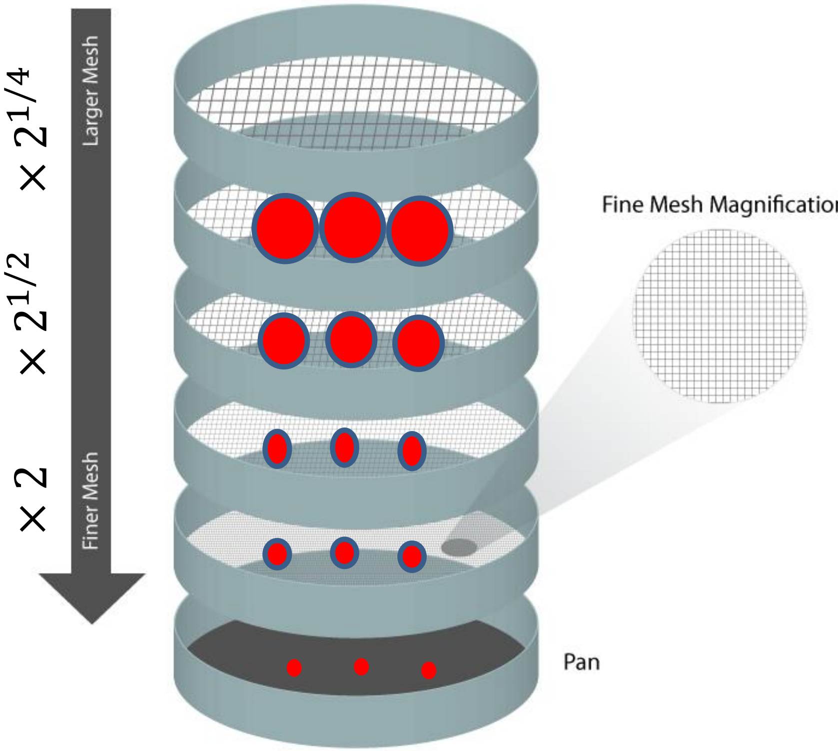
Industrial Siever

Methods to measure particle size: sieving

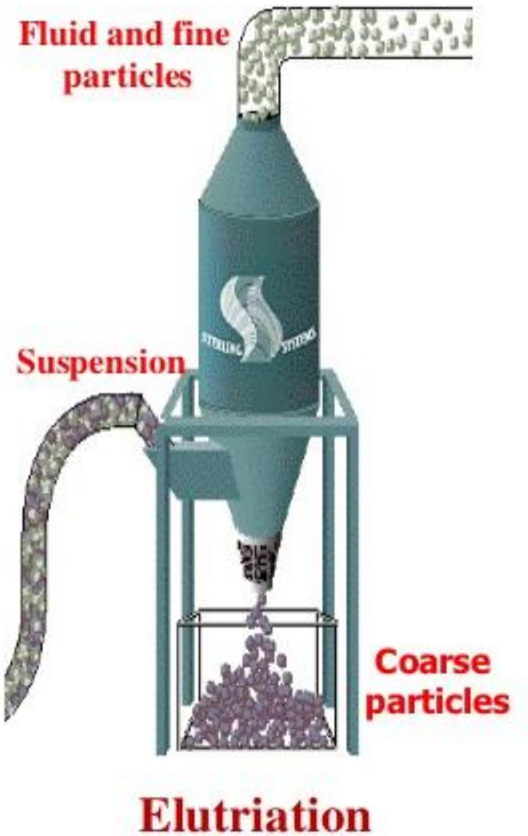
Standardized sequence of mesh sizes

Table 1.1. Standard sieve sizes

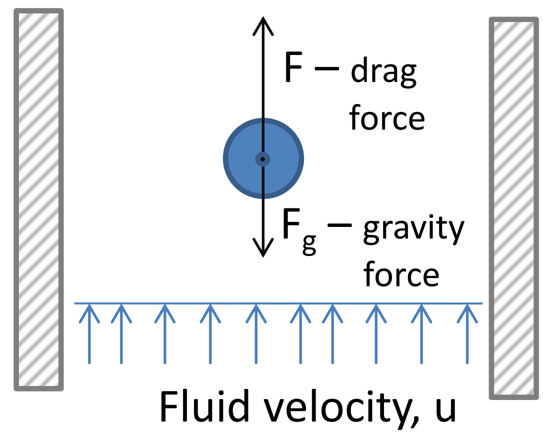
British fine mesh (B.S.S. 410)[3]			I.M.M.[4]			U.S. Tyler[5]			U.S. A.S.T.M.[5]		
Sieve no.	Nominal aperture		Sieve no.	Nominal aperture		Sieve no.	Nominal aperture		Sieve no.	Nominal aperture	
	in.	µm		in.	µm		in.	µm		in.	µm
						325	0.0017	43	325	0.0017	44
						270	0.0021	53	270	0.0021	53
300	0.0021	53				250	0.0024	61	230	0.0024	61
240	0.0026	66	200	0.0025	63	200	0.0029	74	200	0.0029	74
200	0.0030	76							170	0.0034	88
170	0.0035	89	150	0.0033	84	170	0.0035	89			
150	0.0041	104				150	0.0041	104	140	0.0041	104
120	0.0049	124	120	0.0042	107	115	0.0049	125	120	0.0049	125
100	0.0060	152	100	0.0050	127	100	0.0058	147	100	0.0059	150
			90	0.0055	139	80	0.0069	175	80	0.0070	177
85	0.0070	178	80	0.0062	157	65	0.0082	208	70	0.0083	210
			70	0.0071	180				60	0.0098	250
72	0.0083	211	60	0.0083	211	60	0.0097	246	50	0.0117	297
60	0.0099	251							45	0.0138	350
52	0.0116	295	50	0.0100	254	48	0.0116	295	40	0.0165	420
			40	0.0125	347	42	0.0133	351	35	0.0197	500
44	0.0139	353				35	0.0164	417	30	0.0232	590
36	0.0166	422	30	0.0166	422	32	0.0195	495			
30	0.0197	500				28	0.0232	589			
25	0.0236	600									
22	0.0275	699	20	0.0250	635	24	0.0276	701	25	0.0280	710
18	0.0336	853	16	0.0312	792	20	0.0328	833	20	0.0331	840
16	0.0395	1003				16	0.0390	991	18	0.0394	1000
14	0.0474	1204	12	0.0416	1056	14	0.0460	1168	16	0.0469	1190
12	0.0553	1405	10	0.0500	1270	12	0.0550	1397			
10	0.0660	1676	8	0.0620	1574	10	0.0650	1651	14	0.0555	1410
8	0.0810	2057				9	0.0780	1981	12	0.0661	1680
7	0.0949	2411				8	0.0930	2362	10	0.0787	2000
6	0.1107	2812	5	0.1000	2540	7	0.1100	2794	8	0.0937	2380
5	0.1320	3353				6	0.1310	3327			
						5	0.1560	3962	7	0.1110	2839
						4	0.1850	4699			
									6	0.1320	3360
									5	0.1570	4000
									4	0.1870	4760



Methods to measure particle size: elutriation



<u>Elutriation</u> is a particle classification and/or separation process based on the density and size of particles through the motion of a carrying fluid (gas or liquid). The smaller and less dense particles will be dragged out on the top of the column (**fine particles stream**). The bigger and more dense particles will settle in the bottom of the column (**coarse particles stream**).



Exercises I.1-4

I - CLASSIFICAÇÃO DE PARTÍCULAS SÓLIDAS

1. A análise por peneiração duma amostra de um sólido finamente moído produziu o seguinte resultado:

Dimensão da abertura (mm)		Percentagem do produto (% em número)
Passando por	6.00	100
Retido em	4.00	6
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	28
Retido em	0.25	17
Retido em	0.125	5
Passando por	0.125	3

Determine e represente graficamente:

a) Curva de distribuição de frequência em número
b) Curva de distribuição cumulativa em número
c) Complemente os gráficos anteriores com as distribuições em comprimento, superfície, volume e peso.

Calcule o tamanho médio da amostra de sólido:

d) Determine os diâmetros médios baseados em comprimento, superfície, volume e peso
e) Determine o diâmetro do comprimento médio, superfície média, volume médio e peso médio

2. A análise por peneiração duma amostra de um sólido finamente moído produziu o seguinte resultado:

Dimensão da abertura (mm)		Percentagem do produto (% em peso)
Passando por	12.00	100
Retido em	8.00	35
Retido em	4.00	32
Retido em	2.00	17
Retido em	1.00	8
Passando por	1.00	8

a) Determine os diâmetros médios baseados em comprimento, superfície, volume e peso
b) Determine o diâmetro do comprimento médio, superfície média, volume médio e peso médio

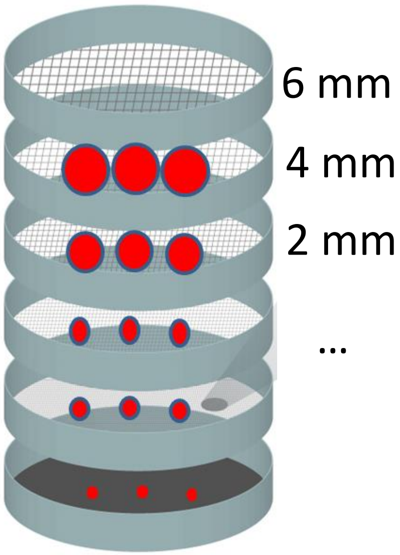
3. A análise granulométrica de um material em pó numa base de peso é representada por uma linha recta que vai de 0% em peso na dimensão de partícula de 1 mícron até 100% em peso na dimensão de partícula de 101 mícrons.

a) Calcular o diâmetro médio em volume das partículas que constituem o sistema.
b) Calcular o diâmetro médio superficial das partículas que constituem o sistema.

4. As equações que dão a curva de distribuição de números para um material em pó são $dn/dd = d$ para a gama de tamanhos de 0-10 mícrons, e $dn/dd = 100\,000/d^4$ para a gama de tamanhos de 10-100 mícrons. Esboçar as curvas de distribuição de número, superfície e peso. Calcular o diâmetro médio superficial para o pó.

Explicar sucintamente o modo como se obteriam experimentalmente os dados para a construção destas curvas.

Exercise I.1 Sieving analysis



Mesh size (mm)		Particle count (%)
Passando por	6.00	100
Retido em	4.00	6
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	28
Retido em	0.25	17
Retido em	0.125	5
Passando por	0.125	3

Exercise I.1 Sieving analysis

Raw data → Particle size distribution

Siever size (mm)		Particle count (%)
Passando por	6.00	100
Retido em	4.00	6
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	28
Retido em	0.25	17
Retido em	0.125	5
Passando por	0.125	3

Particle size (mm)		Particle count (%)
	-	-
	(6.00+4.00)/2=5.00	6
	(4.00+2.00)/2=3.00	18
	(2.00+0.75)/2=1.375	23
	(0.75+0.50)/2=0.625	28
	(0.50+0.25)/2=0.375	17
	(0.25+0.125)/2=0.1875	5
	(0.125+0)/2=0.0625	3

Size distribution