EB – Summary

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Conteúdo

Batch Reactor

1 Cell growth

Volumetric growth rate

$$\boldsymbol{r_x} = \boldsymbol{\mu}\,\boldsymbol{x} \qquad \begin{cases} (\boldsymbol{r_x}) = \mathrm{g\,(X)/L\,h} \\ (\boldsymbol{\mu}) = \mathrm{h}^{-1} \end{cases}$$

Specific cell growth rate

$$\boldsymbol{\mu} = \frac{\boldsymbol{\mu}_{\max}\,\boldsymbol{S}}{\boldsymbol{K_s} + \boldsymbol{S}}$$

2 Substrate consumption

Volumetric rate of consumption

$$\boldsymbol{r_s = Y_{x/s}^{-1}\,\mu\,x = \frac{V_{\max}\,S}{K_m + S} = Y_{x/s}'^{-1}\,\mu\,x + m\,x}$$

$$\begin{cases} (\boldsymbol{r_s}) = \mathrm{g\,(S)/L\,h} \\ (\boldsymbol{V_{s\ \max}}) = \mathrm{g\,(S)/g\,(X)\,h} \\ (\boldsymbol{m}) = \mathrm{g\,(S)/g\,(X)\,h}\ \text{(maintenance coefficient)} \end{cases}$$

Specific rate of substrate consumption

$$\boldsymbol{V_s = \frac{r_s}{x} = Y_{x/s}\,\mu = Y_{x/s}'\,\mu + m}$$

$$\begin{cases} (\boldsymbol{V_s}) = \mathrm{g\,(S)/g\,(X)\,h} \end{cases}$$

3 Product formation

Product associated with growth

$$r_P = \frac{\mathrm{d}P}{\mathrm{d}t} = Y_{p/x}\,\mu\,x; \quad V_P = x^{-1}\frac{\mathrm{d}P}{\mathrm{d}t} = Y_{p/x}\,\mu$$

$$\begin{cases} (r_P) = \mathrm{g\,(P)/L\,h} \text{ (volumetric product production rate)} \\ (v_P) = \mathrm{g\,(P)/g\,(X)\,h} \text{ (specific product production rate)} \end{cases}$$

Product partially associated with growth

$$r_p = \frac{\mathrm{d}P}{\mathrm{d}t} = \alpha\,\mu\,x + \beta\,x; \quad V_P = x^{-1}\,\frac{\mathrm{d}P}{\mathrm{d}t} = \alpha\,\mu + \beta;$$

$$\alpha = Y'_{P/s}$$

$$\begin{cases} (\alpha) = \mathrm{g\,(P)/g\,(X)} \text{ (true yeld coefficient of product formation)} \\ (\beta) = \mathrm{g\,(P)/g\,(X)\,h} \text{ (specific product formation rate due to maintena} \end{cases}$$

Non-growth Associated Product

$$r_P = A\,x$$

Continuous Reactor

4 Mass balance to cell concentration

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{F_0}{V}\,x_0 - \frac{F}{V}\,x + \mu\,x - k_d\,x$$

$$\lim_{x=0,\,\mu\gg k_d} \frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{F}{V}\,x + \mu\,x = -D\,x + \mu\,x$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0 \wedge D = \mu \quad \text{(Steady state)}$$

$$\begin{cases} \dfrac{F_0}{V}\,x_0 : \text{Cells entering the reactor} \\ \dfrac{F}{V}\,x : \text{Cells exiting the reactor} \\ \mu\,x : \text{Cell growth} \\ k_d\,x : \text{Cell death} \\ (D) = \mathrm{h}^{-1}\ \text{(Dilution rate)} \end{cases}$$

5 Mass balance to the substrate

$$
\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= \frac{F\,S_0}{V} - \frac{F\,S}{V} - \frac{\mu\,x}{Y_{x/s}} - m\,x = \\
&= D\,S_0 - D\,S - \frac{\mu\,x}{Y_{x/s}} - m\,x; \quad \left(\frac{F}{V} = D\right) \\
\lim_{m \ll \mu} \frac{\mathrm{d}S}{\mathrm{d}t} &= D\,(S_0 - S) - \frac{\mu\,x}{Y_{x/s}} \\
\frac{\mathrm{d}S}{\mathrm{d}t} &= 0 \wedge Y_{x/s}\,(S_0 - S) = x \quad \text{(Steady state)} \\
&\begin{cases}
\frac{F\,S_0}{V} : \text{Substrate entering the reactor} \\
\frac{F\,S}{V} : \text{Substrate exitting the reactor} \\
\frac{\mu\,x}{Y_{x/s}} : \text{Substrate used for cell growth} \\
m\,x : \text{Substrate used for maintenance}
\end{cases}
\end{aligned}
$$

6 relationship between substrate concentration and cell concentration with dilution rate

$$\mu = \frac{\mu_{\max} S}{K_S + S}$$

Continuous reactor on steady state

$$\mu = D = \frac{\mu_{\max} S}{K_S + S} \implies S = \frac{K_s D}{\mu_{\max} - D}$$

$$x = Y_{x/s}\left(S_0 - \frac{K_s D}{\mu_{\max} - D}\right)$$

$$D_c = \frac{\mu_{\max} S_0}{K_s + S_0} \quad \text{(Critical washout rate)}$$

D_c Critical washout rate, when $x = 0 \land D \sim \mu_{\max}$

7 Cell Productivity

$$D_X$$

$$D_{x\ \max} = \mu_{\max}\left(1 - \sqrt{\frac{\boldsymbol{k_s}}{\boldsymbol{k_s} + \boldsymbol{S_0}}}\right)$$

8 Effect of the maintenance coefficient

Negligible cell maintenance

$$\boldsymbol{x = Y_{x/s} \left(S_0 - \frac{k_s\, D}{\mu_{\max} - D} \right)}$$

Considering cell maintenance

$$\boldsymbol{x = \frac{D\,(S_0 - S)}{Y'_{x/s}\, D + m}}$$

Graph Plotting $X \times D$ whe can see the black curve of negligible cell maintenance in full, at the start of the curve ($D \to 0$) whe can see above the black line, a gree line of production of intracelular reserves and below a blue curve which considers cell maintenance, both merge at the same point with the black curve.

9 Product Production

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -D\,P + Y_{p/x}\,\mu\,x$$

Product associated with growth

$$D\,P = Y_{p/x}\,\mu\,x \quad \text{(At steady-state)}$$
$$V_P = Y_{P/x}\,\mu$$
$$\begin{cases} (D\,P) = \mathrm{g\,(P)/L\,h} \text{ (Volumetric Productivity)} \\ (V_P) = \mathrm{g\,(P)/g\,(X)\,h} \text{ (Specific productivity)} \end{cases}$$

Production partially associated to growth   Mass balance to the substrate:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = D\,S_0 - D\,S - \frac{\mu\,x}{Y'_{x/s}} - \frac{r_p}{Y'_{p/x}} - m\,x$$
$$D\,P = (Y'_{p/x}\,\mu + m_p)\,X$$

r_p volumetric rate of production formation (g (P)/L h)

10 Cell recirculation reactors

With a decanter Reactor with spinning rotor, output connects to the decantor that outputs to the reactor.

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{F_r}{V}\,x_r - \frac{F_0 + F_r}{V}\,x_1 + \mu\,x_1$$

$$\frac{\mathrm{d}X}{\mathrm{d}t} = 0 = F_r\,x_r - (F_0 + F_r)\,x_1 + \mu\,x_1\,V \quad \text{(At steady state)}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = D\,(S_0 - S) - \frac{\mu\,x_1}{Y_{x/s}} = 0 \quad \text{(Balance to the substrate)}$$

$\frac{F_r}{V}\,x_r$ Biomass entrering the reactor

$\frac{F_0+F_r}{V}\,x_1$ Biomass exitting the reactor

$\mu\,x_1$ Cell growth

Membrane bioreactors

Submerged Feed input to reactor with the membrane submerged, the output comes from the membrane

With cell recirculation Feed inputs to reactor which outputs to membrane that has two outputs, one back to reactor other to permeate

Plug flow reactor

11 Definition

Geometry Cylindrical

Operation Continuous

Image Continuous reaction column, F enters at the bottom and leaves at the top, z goes from zero to the top where $z = L$, on the cross section we see a derivative with many arrows pointing to the same direction as F

- F enters at the bottom and leaves at the top
- z at bottom $z = 0$ at top $z = L$
- $(C_i(z)) = \mathrm{kg/m^3}$ Concentration of a generic i component at heigh z of the column
- $(u) = \mathrm{m/s}$ axial velocity of the fluid inside the column

$(z) = \mathrm{m}$	Position on a vertical axis
$(L) = \mathrm{m}$	Column height
$(F) = \mathrm{m^3/h}$	fluid flow rate in ascending flow
$(A) = \mathrm{m^2}$	Cross section area
$(d) = \mathrm{m}$	Diameter of the cylindrical column
$V = A\,L$	Column volume

Velocity of plug flow

$$\boldsymbol{u = F/A = \text{constant}}$$

All fluid move at the same velocity u

Reynald at plug flow

$$\boldsymbol{Re = \frac{\rho\, u\, d}{\mu} > 2000}$$

ρ Specific mass of fluid $(\mathrm{kg/m^3})$

u axial velocity of the fluid $(\mathrm{m/s})$

d column diameter (m)

μ Viscosity of the fluid $(\mathrm{Pa\,s})$

Note

PFR Total segregation

CSTR Perfect mix

12 Material Balances

Material balance to the infinitesimal section of the column with height $\mathrm{d}z$ a generic 'i' component

$$\boldsymbol{F\,C_i(z) + r_i(z)\,A\ \mathrm{d}z = F\,C_i(z + \mathrm{d}z)}$$

$$\boldsymbol{r_i = u\,\frac{\mathrm{d}C_i}{\mathrm{d}z}}$$

$F\,C_i(z)$ Mass of 'i' that enters z per unit of time

$r_i(z)\,A\ \mathrm{d}z$ Mass of 'i' produced by reaction in $A\ \mathrm{d}z$ volume per unit of time, it's only tryue if $\mathrm{d}z$ is infinitely small

$F\,C_i(z + \mathrm{d}z)$ Mass of unit 'i' leaving $z + \mathrm{d}z$ per unit of time

13 Kinetics

title Example: Product formation associated with growth (type I)

Reaction

S -> X + P

Kinetics (assuming monod kinetics)

$$\mu = \frac{\mu_{\max} S}{k_m + S}$$

Material balances

$$u \frac{\mathrm{d}x}{\mathrm{d}z} = \mu x - k_d x - k_e x$$
$$u \frac{\mathrm{d}S}{\mathrm{d}z} = -\frac{\mu x}{Y'_{x/s}} - m_s x$$
$$u \frac{\mathrm{d}P}{\mathrm{d}z} = \frac{\mu x}{Y'_{x/p}}$$

Note analytical integration just for the case $\mu \cong \mu_{\max}$ (or high excess of S)

14 Productivity

$$\text{Prod} = \frac{\boldsymbol{F}\,\boldsymbol{P}(\boldsymbol{z} = \boldsymbol{L})}{\boldsymbol{V}} = \boldsymbol{D}\,\boldsymbol{P}(\boldsymbol{z} = \boldsymbol{L})$$

$$(\text{Prod}) = \mathrm{g}\,(\mathrm{P})/\mathrm{L}\,\mathrm{h}$$

15 Comparisson of PFR with CSTR

graph 1: $S \times D^{-1}$ CSTR is constant while PFR starts at S0 and decrases to CSTR, $S_{PFR} \geq S_{CSTR}$
graph 2: $X \times D^{-1}$ CSTR is a constant, PFR starts below at X0 and grows to meet CSTR at X*, $X_{PFR} \leq X_{CSTR}$
Graph 3: $P \times D^{-1}$ CSTR is a constant, PFR starts below at P0 and grows to meet CSTR at P* $P_{PFR} \leq P_{CSTR}$

16 Comparisson of PFR with CSTR

Case 1: negligible gorwth X is equal

- Order 0 $r_s = k_0$ independent of S
- Order n $r_s = k\, S^n$ CSTR<PFR
- michaelis-menten $r_s = \frac{r_{s\ \max} S}{k_m\, S}$ CSTR<PFR
- Inihibition by S, S greater, Rs lower, CSTR>PFR
- Inihibition by product P greater rs lower, CSTR<PFR

Autocalytic

$$\boldsymbol{r_S = V_S\, X}$$
$$\boldsymbol{X_{CSTR} > X_{PFR}}$$
$$\boldsymbol{V_{CSTR} = \frac{F(S_0 - S^*)}{r(S^*)}}$$
$$\boldsymbol{V_{PFR} = \frac{F(S_0 - S^*)}{\bar{r}_S}}$$

17 Discussion about PFR

Exception: Tubular bioreactor with immobilized cells (or enzymes)

- Very stable cultures that remain viable for long periods of time (months, years)
- Cells ate rest (low maintenance)
- High cell density (much higher than cells in suspension). Cells grow adherent and form biofilms
- After a growth phase, cell density remains constant over time (new cells simply replace the dying cells)
- Higher dilution rates because washout cannot occour
- μ negligible $\implies$ Kinectically favorable to the PFR regarding the CSTR state
- Solid support cells grow adherent to solid support or incarcerated in a polymer matrix