NOVA SCHOOL OF SCIENCE & TECHNOLOGY

Course: OSF

Operações Sólido Fluido

Solid Fluid Operations

LEQB/MIEQB, 2023/24

Chemical and Biological Engineering Section, Department of Chemistry, FCTNOVA

Instructors

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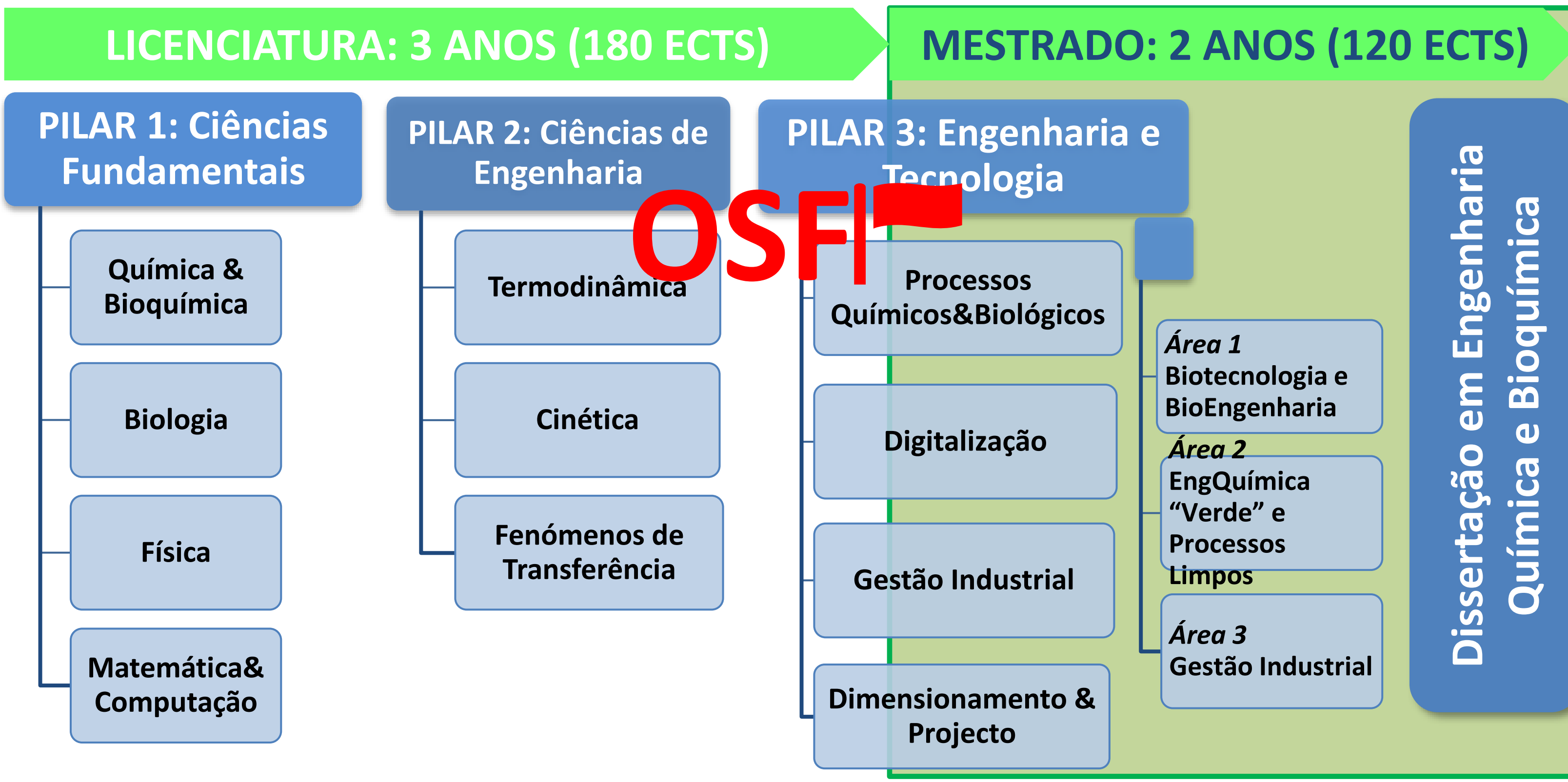
OSF topics of study

Mechanisms and processes involving the handling and/or processing of bulk solids (e.g. powders, granules, pellets) and fluids (gas, liquid, flow deforming material) in the chemical & biological industry.



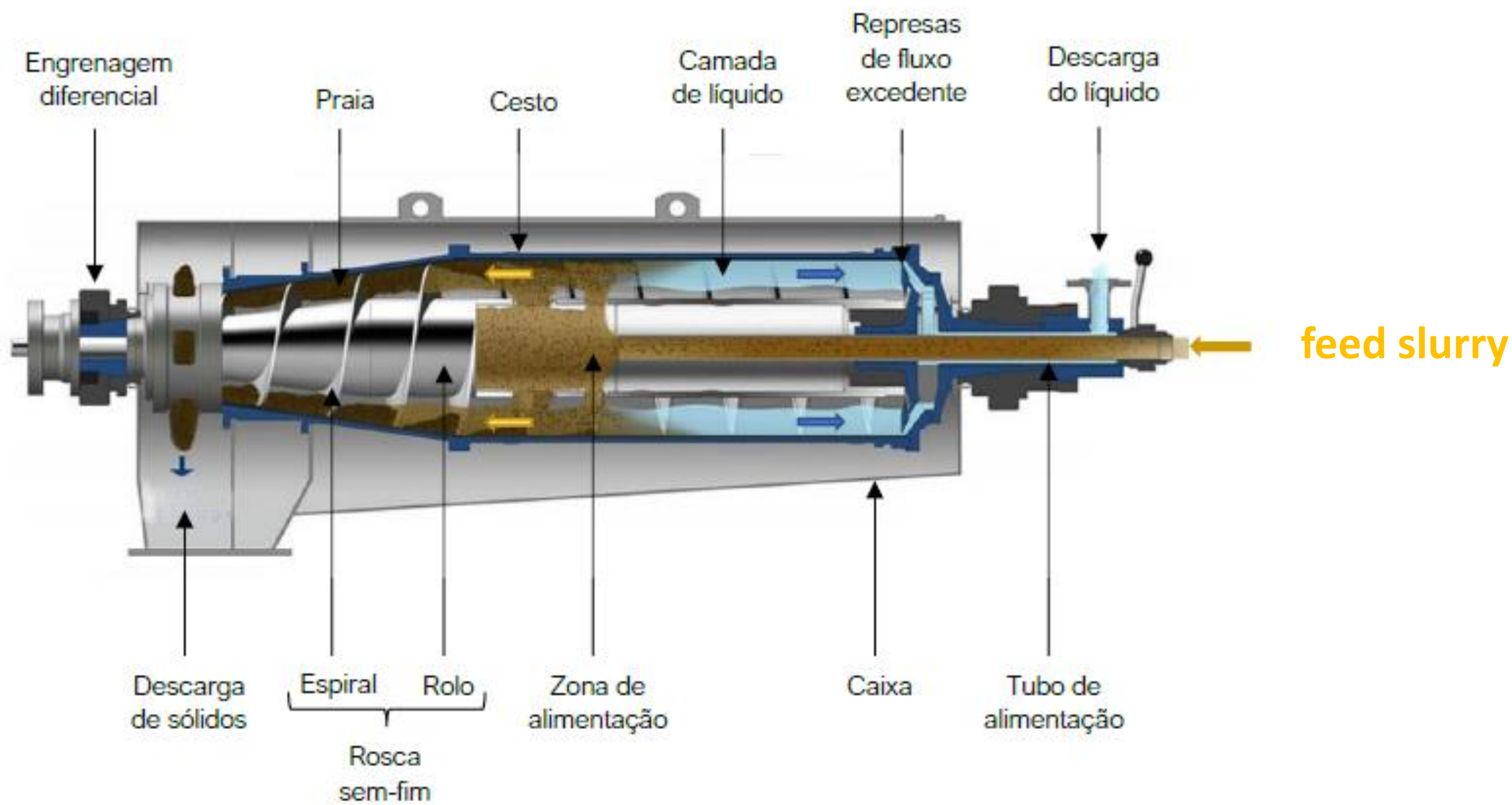
https://www.thechemicalengineer.com/features/bulk-solids-handling-perspective-on-a-professional-blind-spot/

“It is estimated that >70% of everything we use or consume involves **bulk solids** handling somewhere in its lifecycle”

Example 1: Design of a settling tank

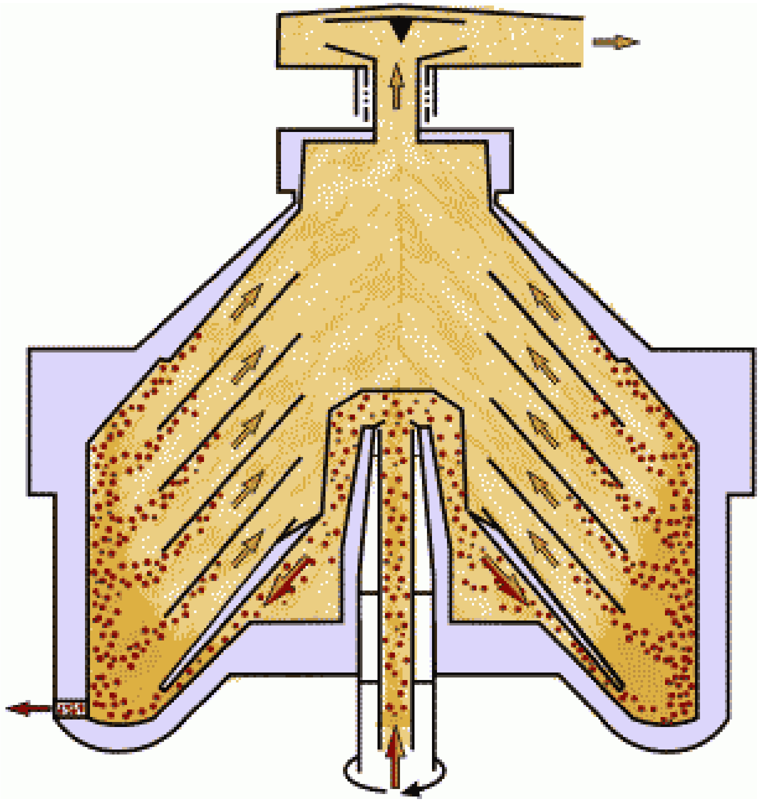
Example 2: Decanter centrifuge

Continuous separation of the solids and liquids in a feed slurry by centrifugation



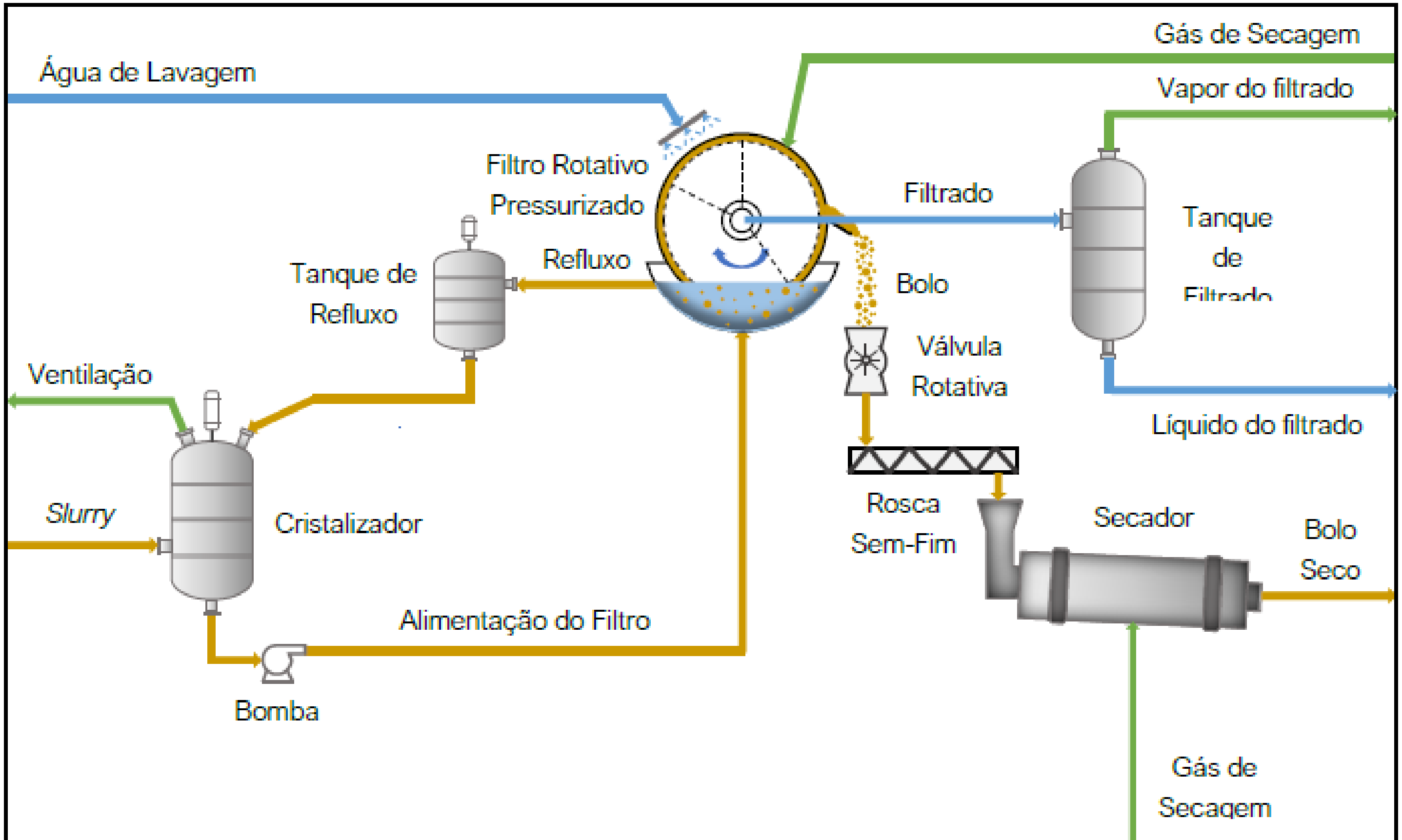
Adapted from: https://www.flottweg.com/pt/linha-de-produtos/decanter/

Example 3: Disk stack centrifuge



https://www.youtube.com/watch?v=GhT_N_-TlBY

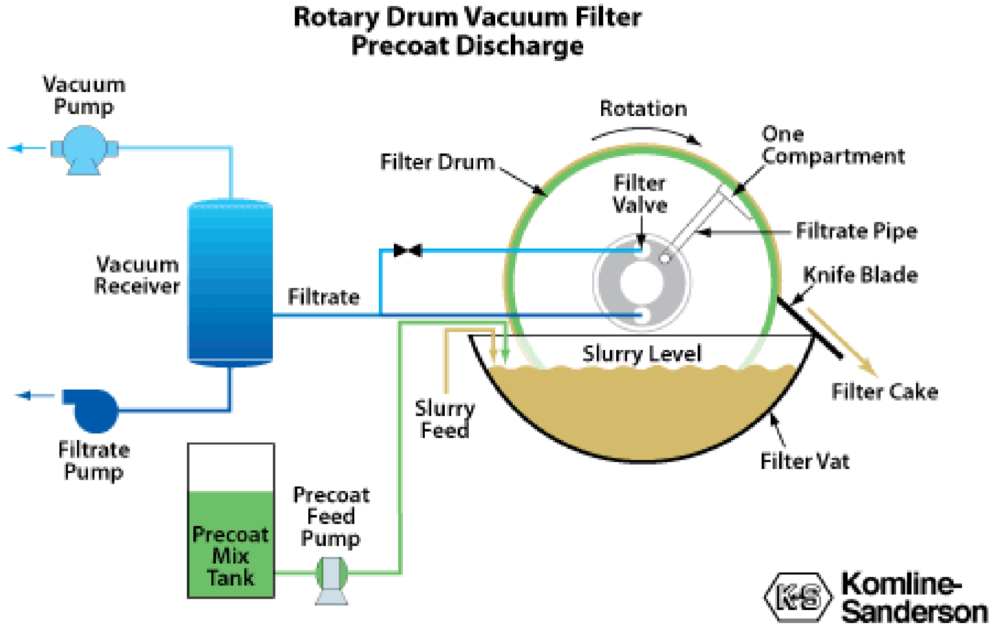
Example 4: Pressurized Rotary Filter

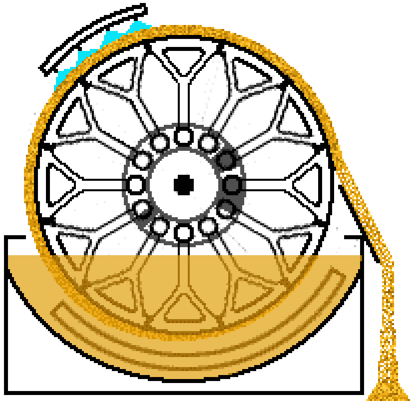


Adapted from a Purified Terephthalic Acid (PTA) process flowsheet

Example 5: Rotary vacuum filter

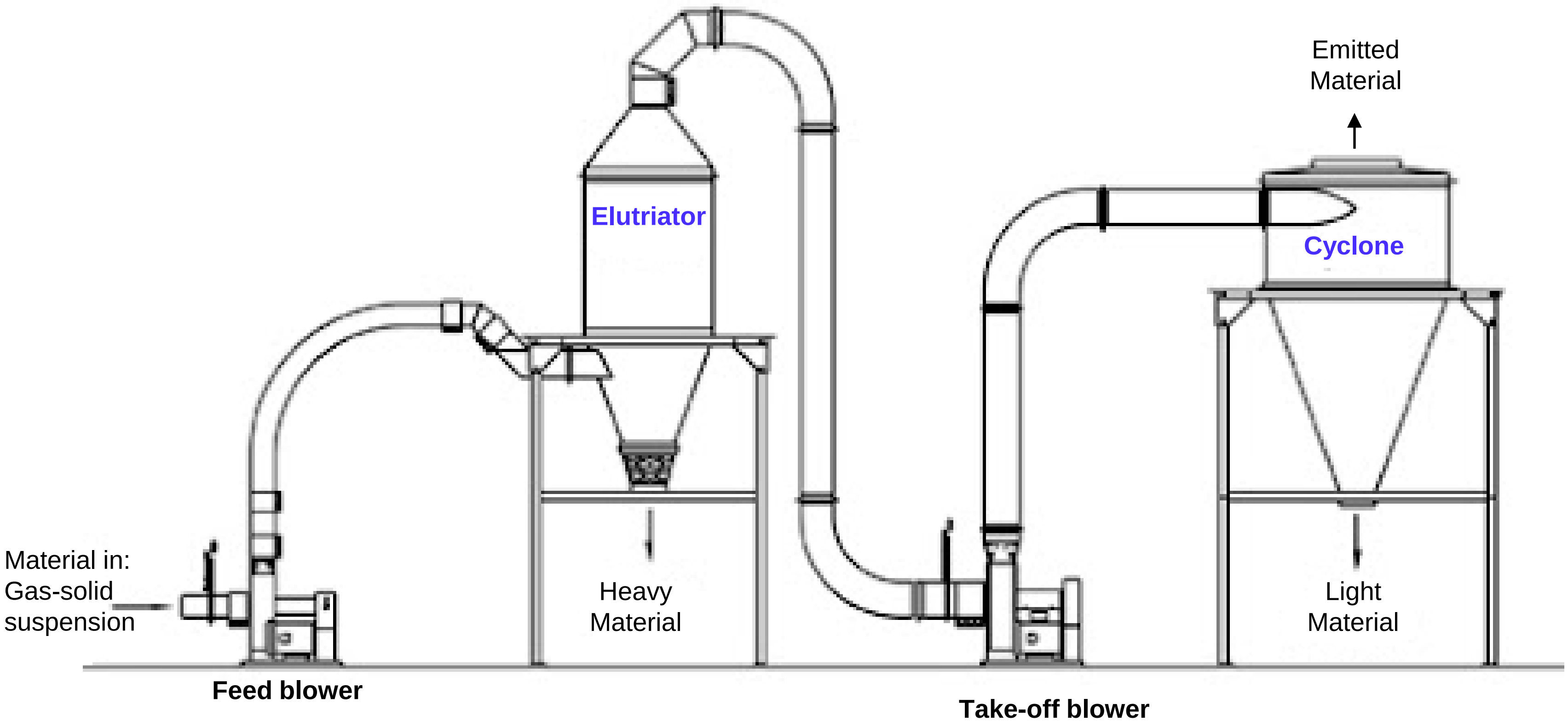
Continuous separation of a slurry feed in its solid and liquid components by filtration. Integrated filtration + washing + drying of the solids cake.





Example 6: Gas elutriator + Cyclone

This plant separates heavy solids from light solids in a two-stages separation process. The first stage is an elutriator that uses the gravity force to separate the heavy/light solids streams. The second state is a cyclone separation unit that uses the centrifugal force to separate the heavy/light solids streams (more to Cyclones in Chapter 7)



Syllabus

Chapter 1. Characterization of solid particles

General concepts. Single particles characterization. Particle size distributions based on weight, number and surface area. Mean size based on number or weight data. Particles shape. Methods for particle size measurement.

Chapter 2. Size reduction of solids

Mechanisms for size reduction. Dynamics of size distribution. Energy for size reduction. Equipment for size reduction.

Laboratory experimente + report; 2 weeks (see CLIP for exact dates); frequency mandatory

Chapter 3. Motion of particles in a fluid

Characterization of flow around a sphere (laminar and turbulent flow). Skin and form drag friction. Stokes law. Newton law. Terminal settling velocities. Extension to non-spherical particles. Transient motion of particles: vertical acceleration under gravity.

Syllabus

Chapter 4. Sedimentation

Free and hindered settling. Fine and coarse settling. The thickening process. Kynch method. Design of settling processes.

Chapter 5. Centrifugal separation

Types of centrifuges. Mechanical design. Fluid pressure and liquid surface form. Separation of two liquids. Separation between suspension solids and liquids. Filtration using centrifuges. Prediction of separation efficiency. Gas cleaning. Cyclone design. The theoretical cut-off model. Recovery efficiency. Pressure drop in cyclones. Electrostatic separators.

Teste-1: Chapters 1-5

Syllabus

Chapter 6. Flow of fluids in packed columns.

Characterization of flow in packed columns. Characterization of packings. Calculation of friction factors and pressure drop. Extension to vacuum columns. Economical design of packed bed columns. Heat and mass transfer.

Chapter 7. Fluidization

Description of fluidization phenomena. Gas and liquid fluidization. Bubbling behavior. Calculation of minimum fluidizing velocity. Calculation of bed expansion.

Chapter 8. Filtration

Filtration theory. The general filtration equation. Cake and filter resistance. Compressible and incompressible cakes. Filtration equipment. Design of plate and frame filters and design of rotary vacuum filters

Teste-2: Chapters 6-8

Frequency & grading

Frequency

Mandatory frequency to P (laboratory) and 2/3 of TP.

Continuous grading

A = Laboratory experiment + report; Mandatory

B = (Test-1 + Test-2)/2; Minimum grade = 9.5

Final grade = 20% **A** + 80% **B**

Final exame

C - All topics in a single exame at the end of the semester;

Final grade = 20% **A** + 80% **C**

Bibliography

1. **J.M. Coulson and J.F. Richardson**, Chemical Engineering, II Vol., 2ª Ed., 1965, Pergamon Press, London
2. **J. P. K. Seville, U. Tüzün and R. Clift**, Processing particulate solids, 1ª Ed., 1997, Blackie Academic & Professional, London, UK, ISBN: 0751403768
3. **Philip A Schweitzer**, Handbook of Separation Techniques for Chemical Engineers, 3ª Ed, 1996, McGraw-Hill, New York, NY, ISBN: 0070570612
4. **Albert Rushton, Anthony S. Ward, Richard G. Holdich**, Solid-Liquid Filtration and Separation Technology (Hardcover), 2ª Ed, 2000, Wiley-VCH, Germany

Teaching material

THEORY

- [THE BEST] **J.M. Coulson and J.F. Richardson**, Chemical Engineering, II Vol., 2ª Ed., 1965, Pergamon Press, London
- Slides (always work in progress)

PROBLEM-SOLVING (TP)

- **List of exercises @ CLASSROOM (**taken from **J.M. Coulson and J.F. Richardson**, Chemical Engineering, II Vol., 2ª Ed., 1965, Pergamon Press, London)
- **List of exercises @ HOME (**taken from **J.M. Coulson and J.F. Richardson**, Chemical Engineering, II Vol., 2ª Ed., 1965, Pergamon Press, London)
- Calculating machine or LAPTOP

NOTE: Solution of exercises is also available from J.M. Coulson and J.F. Richardson, Chemical Engineering, II Vol., 2ª Ed., 1965, Pergamon Press, London

Chapter 1. Characterization of solid particles

1.1 Properties of single particles: size, shape

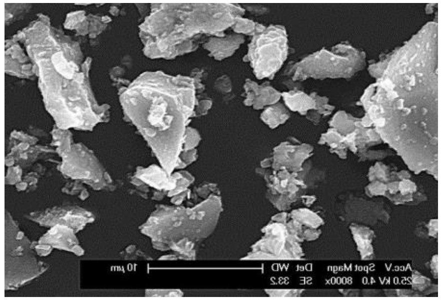
1.2 Particle size distribution: mean diameter

1.3 Methods to measure particle size: sieving, elutriation

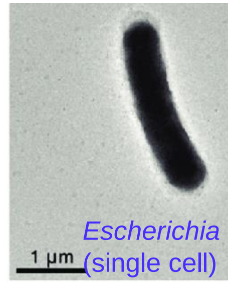
J.M. Coulson and J.F. Richardson pp 1 - 17

Physichochemical properties of solid particles

Single particles



Cement particles



Escherichia coli (single cell) TEM image

Size, Shape, hardness, compressive resistance, electrical charge, (intraparticle) porosity, …

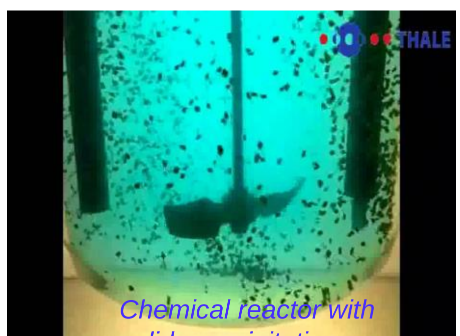
Bulk solids

Bulk cement

Particle size distribution, (interparticle) porosity, humidity, agglomeration, flowability, …

Solids suspensions

Heterogeneous mixture of a fluid (gas or liquid) with a solid

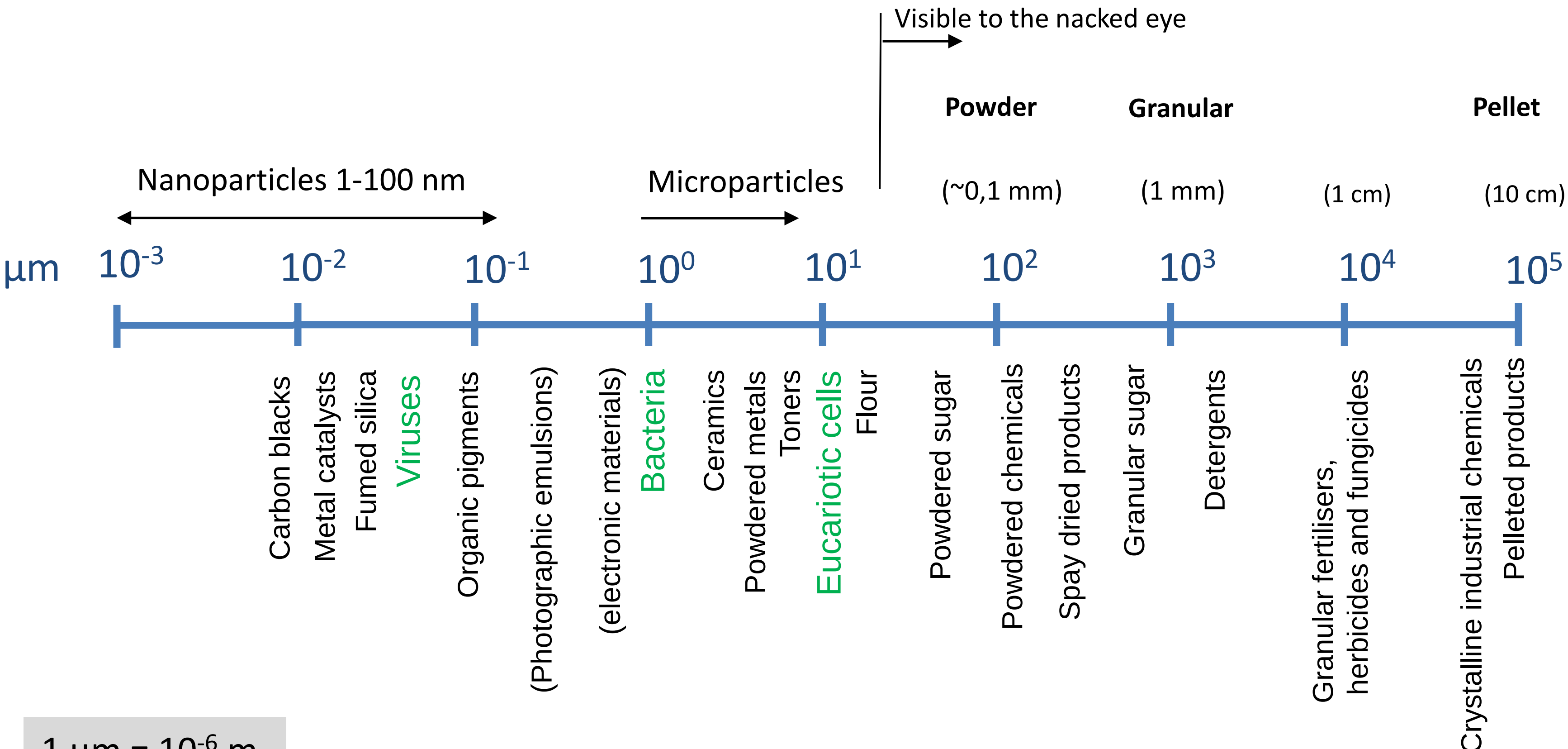


Chemical reactor with solids precipitation

Particle size distribution, Concentration of solids, viscosity of suspension, flocculation, settleability, …

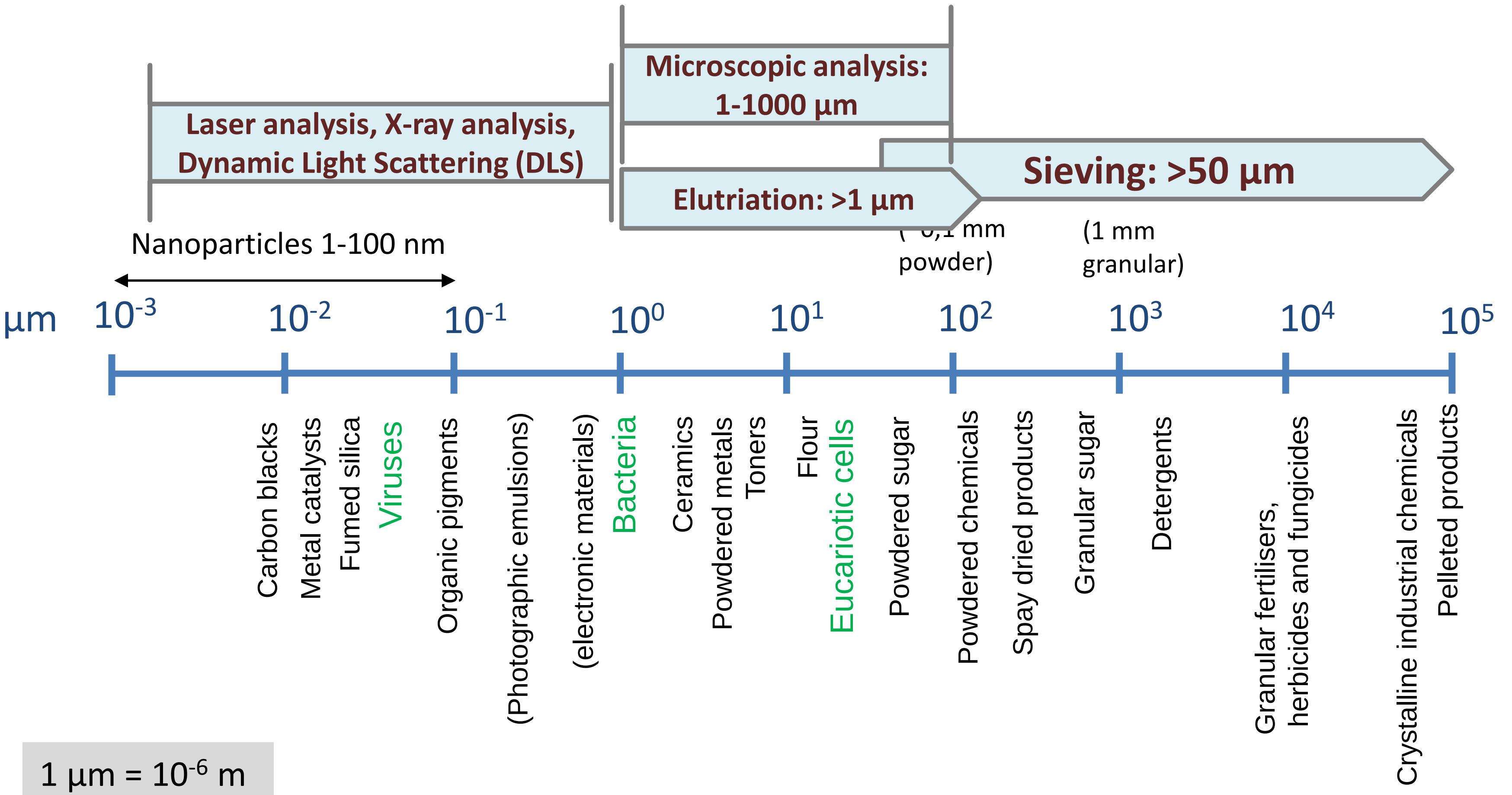
Properties of single particles: size

Examples of particles in Chemical and Biological Engineering and respective sizes (adapted from Fig 1.2 in Coulson and Richardon)



1 µm = 10^{-6} m

Methods to measure particle size



Methods to measure particle size: sieving

- Sievers consist of a sequence of large-to-low mesh sizes to separate particles based on their size
- Mechanical vibration is applied to avoid agglomeration of particles
- Laboratory sievers are used as a certified Analytical technique to determine particle size distribution in the range > 50 µm
- Industrial sievers are used to separate solids of different sizes in discontinuous or continuous mode

Lab Siever

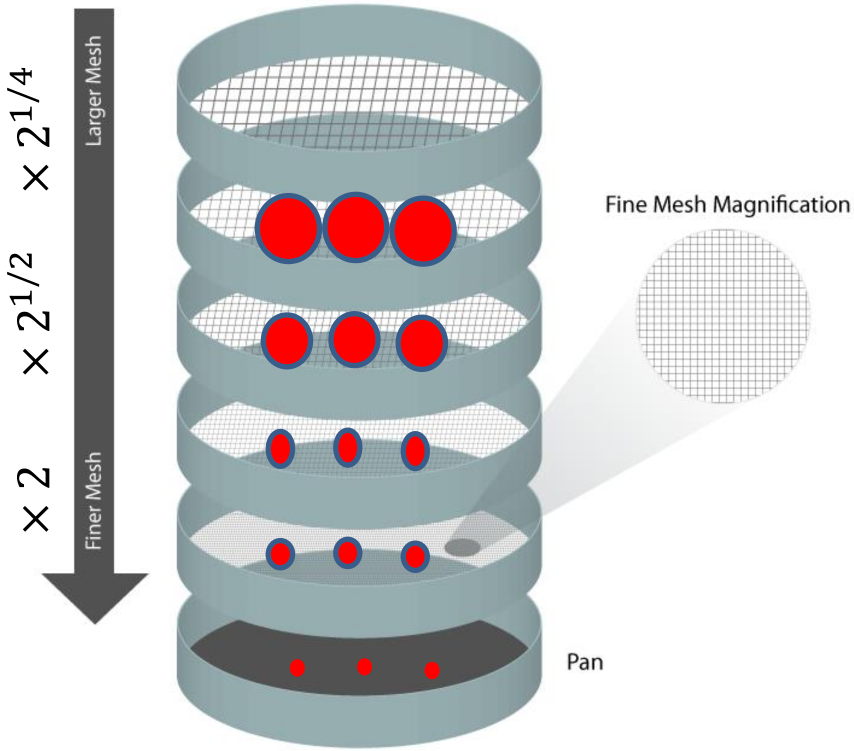
Industrial Siever

Methods to measure particle size: sieving

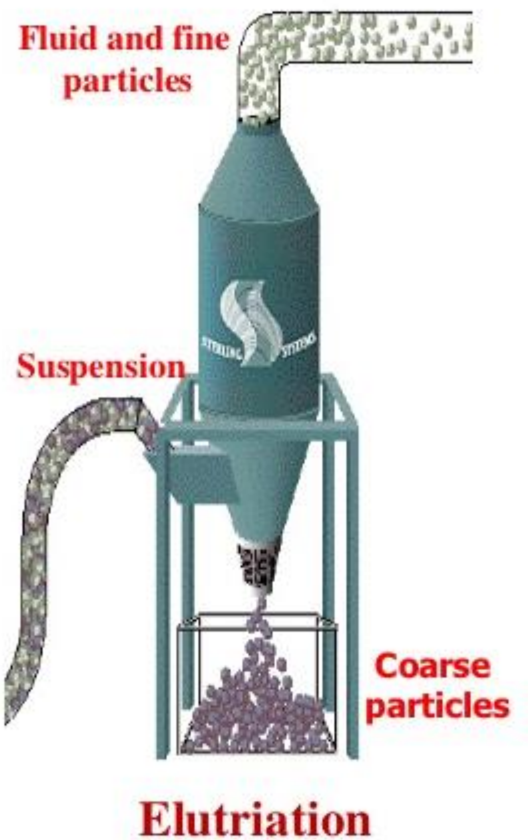
Standardized sequence of mesh sizes

Table 1.1. Standard sieve sizes

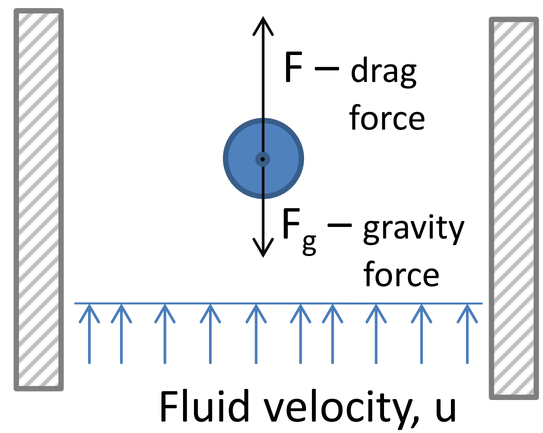
British fine mesh (B.S.S. 410)[3]			I.M.M.[4]			U.S. Tyler[5]			U.S. A.S.T.M.[5]		
Sieve no.	Nominal aperture in.	Nominal aperture µm	Sieve no.	Nominal aperture in.	Nominal aperture µm	Sieve no.	Nominal aperture in.	Nominal aperture µm	Sieve no.	Nominal aperture in.	Nominal aperture µm
						325	0.0017	43	325	0.0017	44
						270	0.0021	53	270	0.0021	53
300	0.0021	53				250	0.0024	61	230	0.0024	61
240	0.0026	66	200	0.0025	63	200	0.0029	74	200	0.0029	74
200	0.0030	76							170	0.0034	88
170	0.0035	89	150	0.0033	84	170	0.0035	89			
150	0.0041	104				150	0.0041	104	140	0.0041	104
120	0.0049	124	120	0.0042	107	115	0.0049	125	120	0.0049	125
100	0.0060	152	100	0.0050	127	100	0.0058	147	100	0.0059	150
			90	0.0055	139	80	0.0069	175	80	0.0070	177
85	0.0070	178	80	0.0062	157	65	0.0082	208	70	0.0083	210
			70	0.0071	180				60	0.0098	250
72	0.0083	211	60	0.0083	211	60	0.0097	246	50	0.0117	297
60	0.0099	251							45	0.0138	350
52	0.0116	295	50	0.0100	254	48	0.0116	295	40	0.0165	420
			40	0.0125	347	42	0.0133	351	35	0.0197	500
44	0.0139	353				35	0.0164	417	30	0.0232	590
36	0.0166	422	30	0.0166	422	32	0.0195	495			
30	0.0197	500				28	0.0232	589			
25	0.0236	600									
22	0.0275	699	20	0.0250	635	24	0.0276	701	25	0.0280	710
18	0.0336	853	16	0.0312	792	20	0.0328	833	20	0.0331	840
16	0.0395	1003				16	0.0390	991	18	0.0394	1000
14	0.0474	1204	12	0.0416	1056	14	0.0460	1168	16	0.0469	1190
12	0.0553	1405	10	0.0500	1270	12	0.0550	1397			
10	0.0660	1676	8	0.0620	1574	10	0.0650	1651	14	0.0555	1410
8	0.0810	2057				9	0.0780	1981	12	0.0661	1680
7	0.0949	2411				8	0.0930	2362	10	0.0787	2000
6	0.1107	2812	5	0.1000	2540	7	0.1100	2794	8	0.0937	2380
5	0.1320	3353				6	0.1310	3327			
						5	0.1560	3962	7	0.1110	2839
						4	0.1850	4699			
									6	0.1320	3360
									5	0.1570	4000
									4	0.1870	4760



Methods to measure particle size: elutriation



<u>Elutriation</u> is a particle classification and/or separation process based on the density and size of particles through the motion of a carrying fluid (gas or liquid). The smaller and less dense particles will be dragged out on the top of the column (**fine particles stream**). The bigger and more dense particles will settle in the bottom of the column (**coarse particles stream**).



Properties of single particles: shape

Regular shape.

Regular shapes have well-defined geometry by a mathematical equation, thus their volume, surface area, etc… may be calculated by well defined mathematical formulas. For example, a sphere:

Sphere

Volume: $$\frac{\pi d^3}{6} = \frac{4\pi r^3}{3}$$

Surface area: $$\pi d^2 = 4\pi r^2$$

Projected area in a plane: $$\frac{\pi d^2}{4} = \pi r^2$$

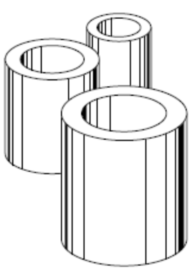
- A sphere is a special regular shape because it is the only shape that is completely symmetrical irrespective of orientation.
- A cube or a cylinder are also regular shapes but their symmetry depends on orientation

Properties of single particles: shape

<u>Complex regular shapes</u>

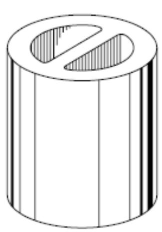
- Complex regular shapes are many times used in packed columns in chemical and biological engineering (distillation columns, absorption columns, chromatographic columns, packed bed reactors, etc..)
- **They are typically designed to maximize surface area per unit volume!!!!**
- They enable improved packing properties, e.g. regular packing with more particles per unit volume
- More expensive but with higher performance
- (More to this in <u>chapter 5. Flow of fluids in packed columns</u>)

Raschig Rings



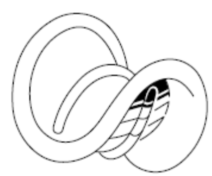
(a)

Lessing Rings



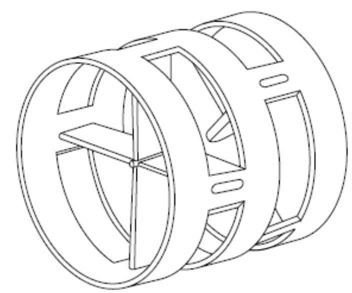
(b)

Berl Saddle



(c)

Pal Ring



(d)

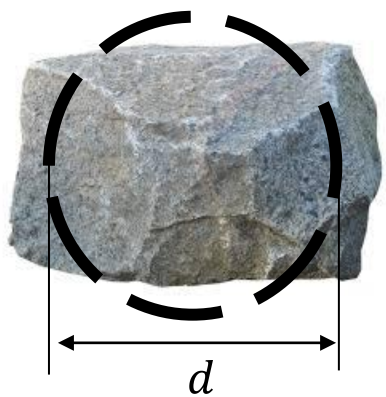
Nutter Ring

(f)

Properties of single particles: shape

<u>Irregular shape</u>: Cannot be defined by a mathematical equation. Undefined size that varies with orientation. Several possibilities to define **a characteristic dimension, $\boldsymbol{d}$:**

- Diameter of sphere with same volume as particle (see below)
- Diameter of sphere with same surface as particle
- Diameter of sphere with same surface per unit volume as particle

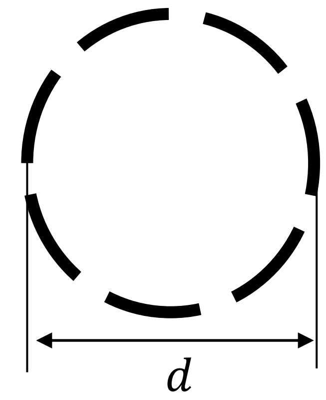


Defining the characteristic dimension of particle, d, as the diameter of an hypothetical sphere with the same volume of particle

$$V_{particle} = V_{shpere} = \frac{\pi d^3}{6} \qquad \boxed{d = \sqrt[3]{\frac{6V_{particle}}{\pi}}}$$

Properties of single particles

For any single particle with characteristic dimension (d), irrespective of its **regular** or **irregular** shape**,** several properties may be quantified as follows:



-**Particle length [*L*, m]:** $$L = d$$

-**Particle surface area [*S*, m²]:** $$S = k'd^2$$

surface factor - k' ($k' = \pi$ for spheres)

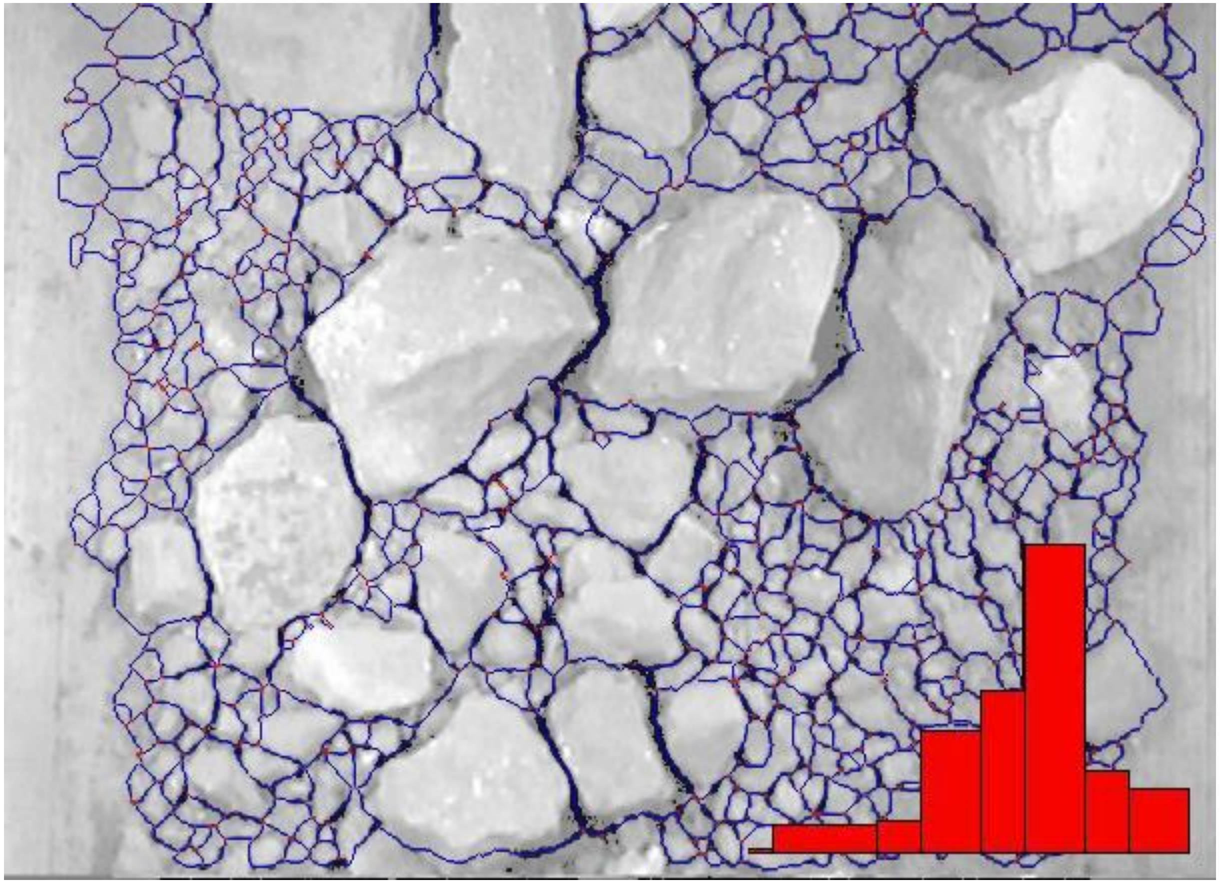
-**Particle volume [*V*, m³] :** $$V = k''d^3$$

volume factor - k'' ($k'' = \pi/6$ for spheres)

-**Particle mass [*m*, kg] :** $$m = \rho_s V = \rho_s k''d^3$$

specific mass of solid [kg/m3]- ρ_s

Particle size distribution



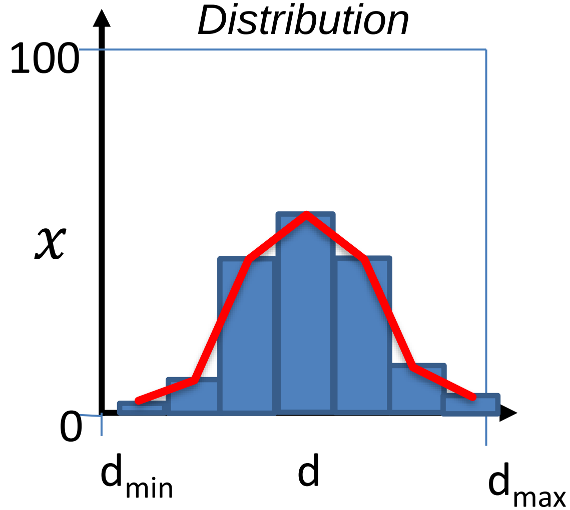
Bulk solids or solid suspensions typically comprehend particles of many different sizes

Particle size distributions provide detailed information of particle sizes and respective quantities in a given sample

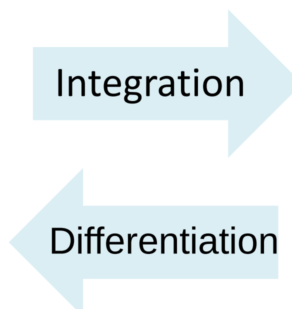
Particle size distribution

Bulk solids or solids suspensions typically contain particles with many different sizes. **Particle size distributions** characterize the mixture in terms of quantity of particles (e.g. mass fraction, x) as function of respective size (e.g. diameter, d). There are two ways to represent particle size distribution: **A** – Frequency distribution, **B** – Cumulative distribution

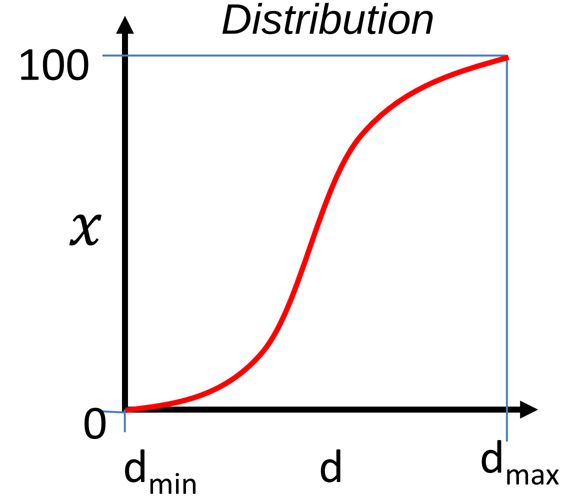
***A**- Frequency Distribution*



{Any point in curve reads mass fraction, x, with size = d}



***B**- Cumulative Distribution*

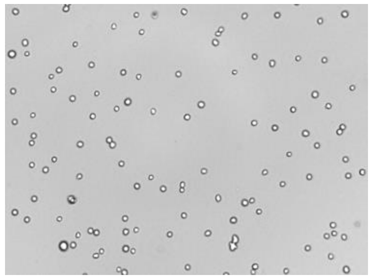


{Any point in curve reads mass fraction, x, with size $\leq d$}

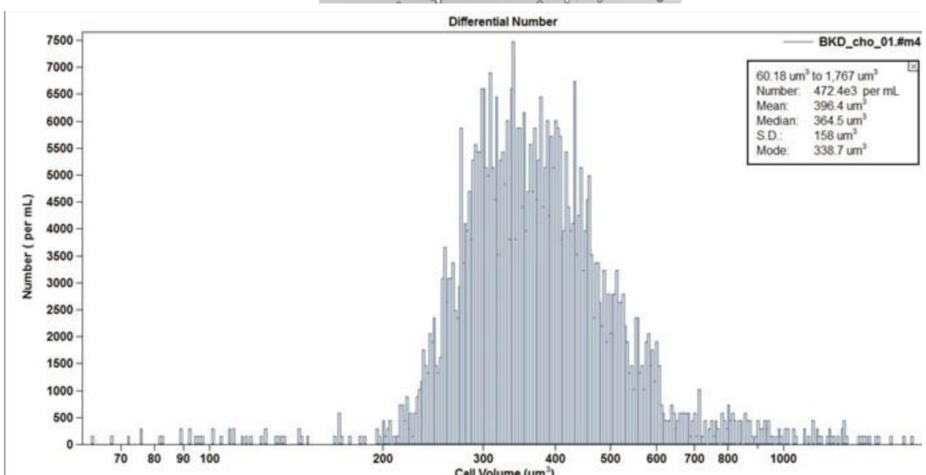
Particle size distribution

Examples: (1) Chinese Hamster Ovary (CHO) cells, (2) Portland cement

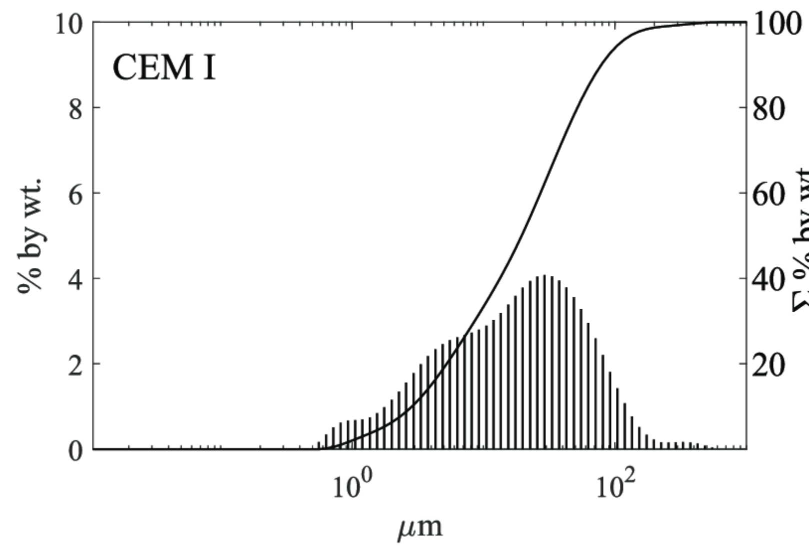
(1) CHO cells



(2) Portland cement



https://www.beckman.pt/resources/reading-material/application-notes/cellular-analysis-using-the-coulter-principle



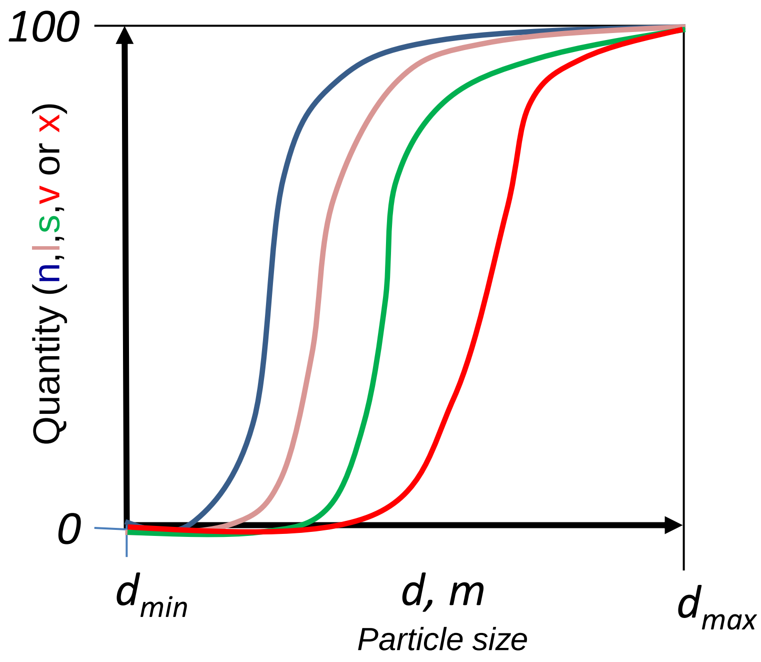
The Influence of Ambient Temperature on High Performance Concrete Properties, DOI:
•10.3390/ma13204646

Particle size distribution

In particle size distribution curves, the distribution base (e.g. “quantity” of particles) may be defined in different ways. This results in different distribution curves for the same solids sample.

The same solids sample

Different distribution curves



Different ways to measure quantity

- ***n*** – number fraction
- ***l*** – length fraction
- ***s*** – surface fraction
- ***v*** – volume fraction =
- ***x*** – weight fraction

Particle size distribution

Any quantities *n*, *x*, *v*, *s* or *l* may be converted into any of the other based on particle size and geometric considerations. Suppose the solid sample is fractionated in fractions of equal sized particles:

Any fraction i of ni particles with size di

Number $n_i, \ d_i$ ⇨ Length, m $= n_i d_i$ ⇨ Surface, m^2 $= n_i(\dot{k} d_i^2)$ ⇨ Volume, m^3 $= n_i(\ddot{k} d_i^3)$ ⇨ Mass, kg $= n_i(\rho \ddot{k} d_i^3)$

number fraction $n_i,$

Length fraction $l_i = \dfrac{n_i d_i}{\sum n_k d_k}$

Surface fraction $s_i = \dfrac{n_i d_i^2}{\sum n_k d_k^2}$

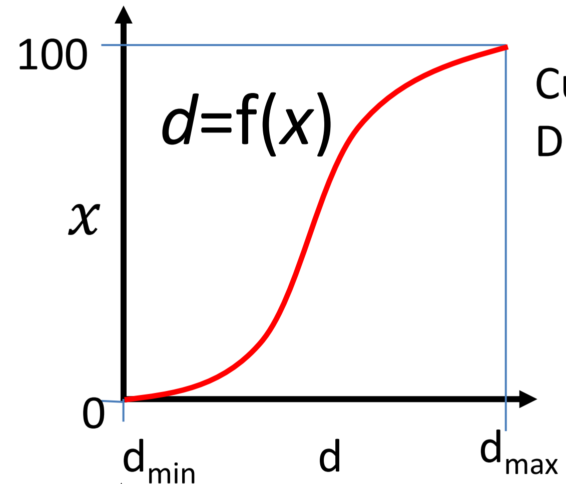
Volume fraction = Mass fraction $v_i = \dfrac{n_i d_i^3}{\sum n_k d_k^3} = x_i = \dfrac{n_i d_i^3}{\sum n_k d_k^3}$

Particle size distribution: mean diameter

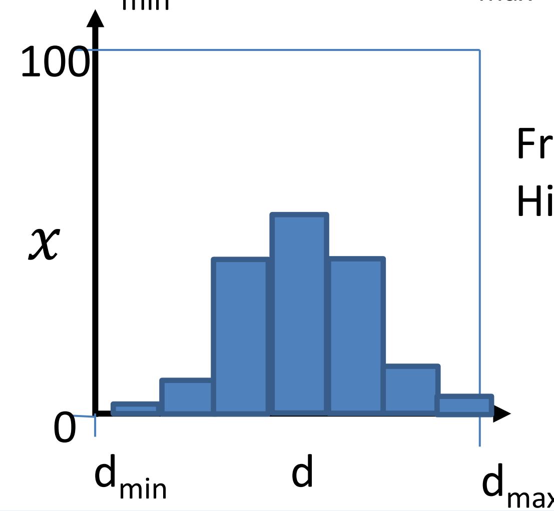
<u>Mean diameter based on weight, $\bar{d}_x$</u>

$$\bar{d}_x = \frac{\int d\,dx}{\int dx}$$

$$\bar{d}_x = \frac{\sum d_i x_i}{\sum x_i}$$



Cumulative Distribution



Frequency Histogram

Particle size distribution: mean diameter

$\bar{\boldsymbol{d}}_{\boldsymbol{x}}$ - Mean diameter based on weight:

$$\bar{d}_x = \frac{\int d\ dx}{\int dx} \qquad \bar{d}_x = \frac{\sum d_i x_i}{\sum x_i}$$

$\bar{\boldsymbol{d}}_{\boldsymbol{n}}$ - Mean diameter based on number:

$$\bar{d}_n = \frac{\int d\ dn}{\int dn} \qquad \bar{d}_n = \frac{\sum d_i n_i}{\sum n_i}$$

$\bar{\boldsymbol{d}}_{\boldsymbol{V}}$ - Mean diameter based on volume:

$$\bar{d}_V = \frac{\int d\ dv}{\int dv} \qquad \bar{d}_V = \frac{\sum d_i v_i}{\sum v_i}$$

$\bar{\boldsymbol{d}}_{\boldsymbol{S}}$ - Mean diameter based on surface:

$$\bar{d}_S = \frac{\int d\ ds}{\int ds} \qquad \bar{d}_S = \frac{\sum d_i s_i}{\sum s_i}$$

$\bar{\boldsymbol{d}}_{\boldsymbol{L}}$ - Mean diameter based on length:

$$\bar{d}_L = \frac{\int d\ dl}{\int dl} \qquad \bar{d}_L = \frac{\sum d_i l_i}{\sum l_i}$$

Particle size distribution: mean diameter

	Measurement in weight, x	Measurement in number, n
Mean diameter based on volume (weight) $\bar{d}_x = \bar{d}_v$	$= \dfrac{\sum x_i d_i}{\sum x_i}$	$= \dfrac{\sum n_i d_i^4}{\sum n_i d_i^3}$
Mean diameter based on surface $\bar{d}_S$	$= \dfrac{\sum x_i}{\sum \frac{x_i}{d_i}}$	$= \dfrac{\sum n_i d_i^3}{\sum n_i d_i^2}$
Mean diameter based on length $\bar{d}_L$	$= \dfrac{\sum \frac{x_i}{d_i}}{\sum \frac{x_i}{d_i^2}}$	$= \dfrac{\sum n_i d_i^2}{\sum n_i d_i}$

Particle size distribution: mean diameter

Mean volume diameter ("diâmetro do volume médio"), d'_v

It's a different way to calculate a representative sample size. First the particle mean volume is computed. Then the diameter of the mean volume is taken as the representative size of the sample. For a sample with i=1,…,N fractions of n_i particles with equal size d_i :

$$\text{Mean volume} = \left(k'' {d'}_v^3\right) = \sum n_i \left(k'' d_i^3\right) / \sum n_i$$

Real sample volume

Total number of particles

Mean volume diameter = $$\boxed{d'_v = \sqrt[3]{\frac{\sum n_i d_i^3}{\sum n_i}}}$$

Particle size distribution: mean diameter

	Measurements in weight fraction, *x*	Measurements in number fraction, *n*
Mean volume (weight) diameter $d'_x = d'_v$	$= \sqrt[3]{\dfrac{\sum x_i}{\sum \dfrac{x_i}{d_i^3}}}$	$= \sqrt[3]{\dfrac{\sum n_i d_i^3}{\sum n_i}}$
Mean surface diameter d'_S	$= \sqrt{\dfrac{\sum \dfrac{x_i}{d_i}}{\sum \dfrac{x_i}{d_i^3}}}$	$= \sqrt{\dfrac{\sum n_i d_i^2}{\sum n_i}}$
Mean length diameter d'_L	$= \dfrac{\sum \dfrac{x_i}{d_i^2}}{\sum \dfrac{x_i}{d_i^3}}$	$= \dfrac{\sum n_i d_i}{\sum n_i}$

Exercises I.1-4

I - CLASSIFICAÇÃO DE PARTÍCULAS SÓLIDAS

1. A análise por peneiração duma amostra de um sólido finamente moído produziu o seguinte resultado:

Dimensão da abertura (mm)		Percentagem do produto (% em número)
Passando por	6.00	100
Retido em	4.00	6
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	28
Retido em	0.25	17
Retido em	0.125	5
Passando por	0.125	3

Determine e represente graficamente:

a) Curva de distribuição de frequência em número
b) Curva de distribuição cumulativa em número
c) Complemente os gráficos anteriores com as distribuições em comprimento, superfície, volume e peso.

Calcule o tamanho médio da amostra de sólido:

d) Determine os diâmetros médios baseados em comprimento, superfície, volume e peso
e) Determine o diâmetro do comprimento médio, superfície média, volume médio e peso médio

2. A análise por peneiração duma amostra de um sólido finamente moído produziu o seguinte resultado:

Dimensão da abertura (mm)		Percentagem do produto (% em peso)
Passando por	12.00	100
Retido em	8.00	35
Retido em	4.00	32
Retido em	2.00	17
Retido em	1.00	8
Passando por	1.00	8

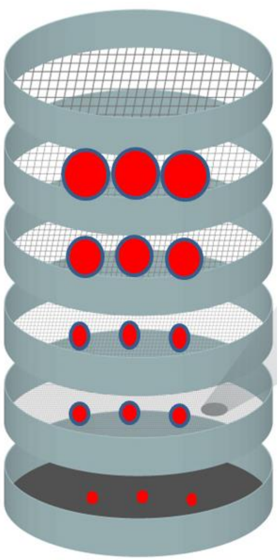
a) Determine os diâmetros médios baseados em comprimento, superfície, volume e peso
b) Determine o diâmetro do comprimento médio, superfície média, volume médio e peso médio

3. A análise granulométrica de um material em pó numa base de peso é representada por uma linha recta que vai de 0% em peso na dimensão de partícula de 1 micron até 100% em peso na dimensão de partícula de 101 microns.
a) Calcular o diâmetro médio em volume das partículas que constituem o sistema.
b) Calcular o diâmetro médio superficial das partículas que constituem o sistema.

4. As equações que dão a curva de distribuição de números para um material em pó são $dn/dd = d$ para a gama de tamanhos de 0-10 microns, e $dn/dd = 100\,000/d^4$ para a gama de tamanhos de 10-100 microns. Esboçar as curvas de distribuição de número, superfície e peso. Calcular o diâmetro médio superficial para o pó.

Explicar sucintamente o modo como se obteriam experimentalmente os dados para a construção destas curvas.

Exercise I.1 Sieving analysis



Mesh size (mm)		Particle count (%)
Passando por	6.00	100
Retido em	4.00	6
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	28
Retido em	0.25	17
Retido em	0.125	5
Passando por	0.125	3

Exercise I.1 Sieving analysis

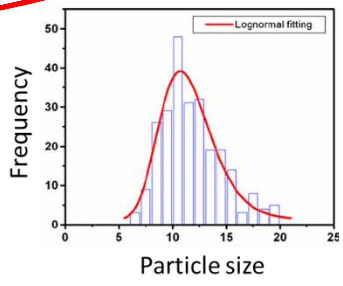
Raw data

Particle size distribution

Siever size (mm)		Particle count (%)
Passando por	6.00	100
Retido em	4.00	6
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	28
Retido em	0.25	17
Retido em	0.125	5
Passando por	0.125	3

Particle size (mm)	Particle count (%)
-	-
(6.00+4.00)/2=5.00	6
(4.00+2.00)/2=3.00	18
(2.00+0.75)/2=1.375	23
(0.75+0.50)/2=0.625	28
(0.50+0.25)/2=0.375	17
(0.25+0.125)/2=0.1875	5
(0.125+0)/2=0.0625	3

Size distribution



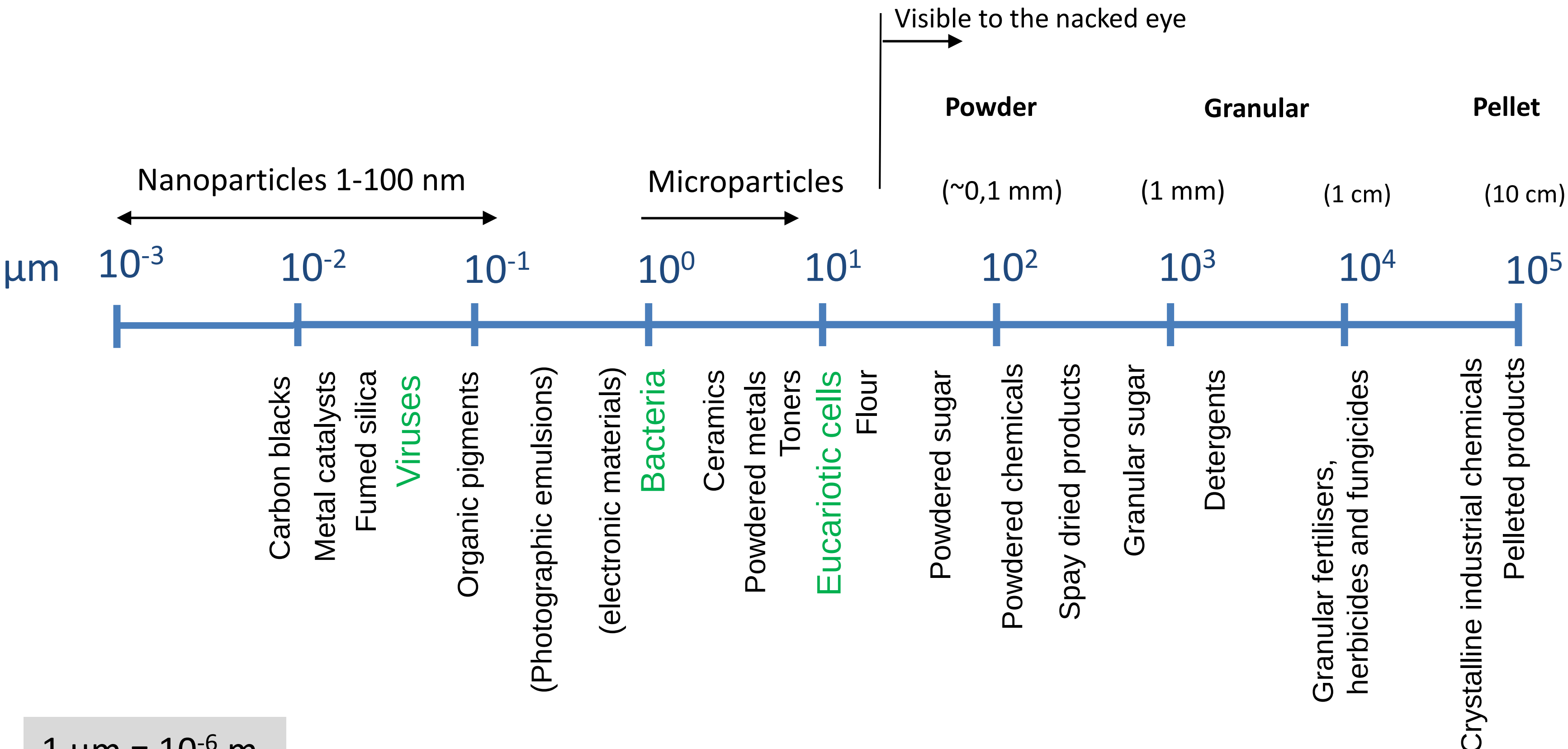
Chapter 2. Size reduction of solids

2.1 Introductory concepts
2.2 Mechanisms of size reduction
2.3 Energy for size reduction
2.4 Equipment for size reduction
2.5 Exercises.

J.M. Coulson and J.F. Richardson pp 95 - 144

Introduction

Examples of particles in Chemical and Biological Engineering and respective sizes (adapted from Fig 1.2 in Coulson and Richardon)



1 µm = 10^{-6} m

Introduction

In chemical and biological engineering, it is often necessary either to decrease or to increase the particle size. For example, the starting material is too coarse, and possibly in the form of large rocks, and the final product needs to be a fine powder. The particle size will have to be progressively reduced in stages.

Feed: Large rocks

Sequence of size reduction stages

Stage 1
- Coarse reduction

Stage 2
- Intermediate reduction

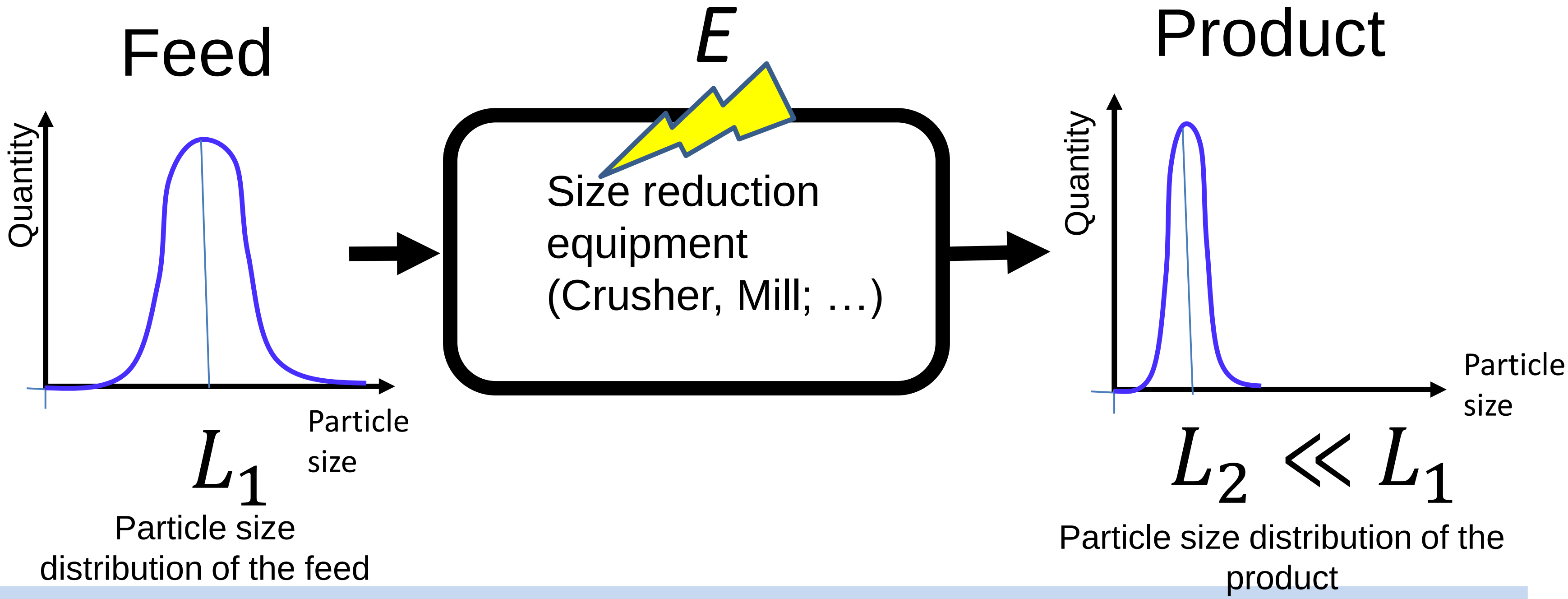
Stage 3
- Fine reduction

Machines of very different characters are required in each stage.

Product: Fine powder

Size reduction unit operation

A size reduction unit operation (a crusher or a mill) spends energy (**E**) to reduce the size of solids from the average size $\boldsymbol{L_1}$ to the final average size $\boldsymbol{L_2 \ll L_1}$



Solids properties that affect size reduction

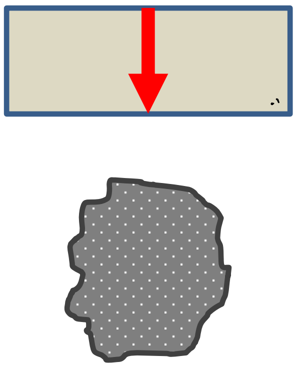
Solid properties that highly impact the performance of a size reduction process and that influences the choice of the equipment:

- **Size of the feed-** depending on the size of the feed and product, reduction is classified as coarse, intermediate and fine reduction; the energy needed is very different
- **Compressive strength -** minimum compressive (slow) strength that causes solid fracture
- **Brittleness ("fragilidade"):** Brittle materials are characterized by little deformation, poor capacity to resist impact and vibration of load, high compressive strength, and low tensile strength. Most of inorganic non-metallic materials are brittle materials, e.g. glass
- **Stickiness ("pegajosidade"):** Stickiness is a property that causes considerable difficulty in reducing the size because the material gets to adhere to the equipment
- **Soapiness (propriedade de sabão)**: a measure of the friction coefficient of the material surface
- **Humidity**: humidity content between 5-50% should be avoided as solids tend to cake, do not flow well; energy increases
- **Friability ("Friabilidade"):** it is the tendency of the material to fracture during normal handling; a crystalline material will break along well defined planes; the energy to break the material will increase as the material gets smaller.

Types of forces in size reduction equipment

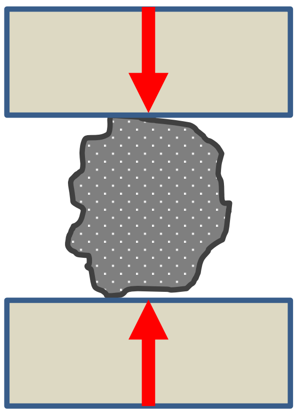
Four main types of forces develop simultaneously in any size reduction equipment. Different machines will however develop a predominant type of force.

1- Impact



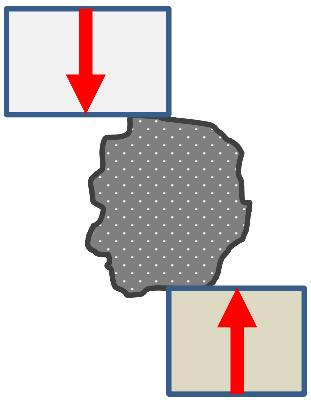
particle concussion by a single rigid force

2 - Compression



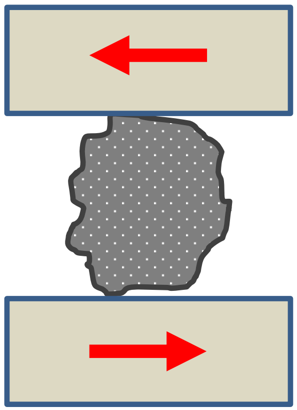
particle disintegration by two rigid forces

3 - Shear



produced by a fluid or by particle–particle interaction

4 - Attrition



arising from particles scraping against one another or against a rigid surface

Compressive strength, f_c [N/m2]

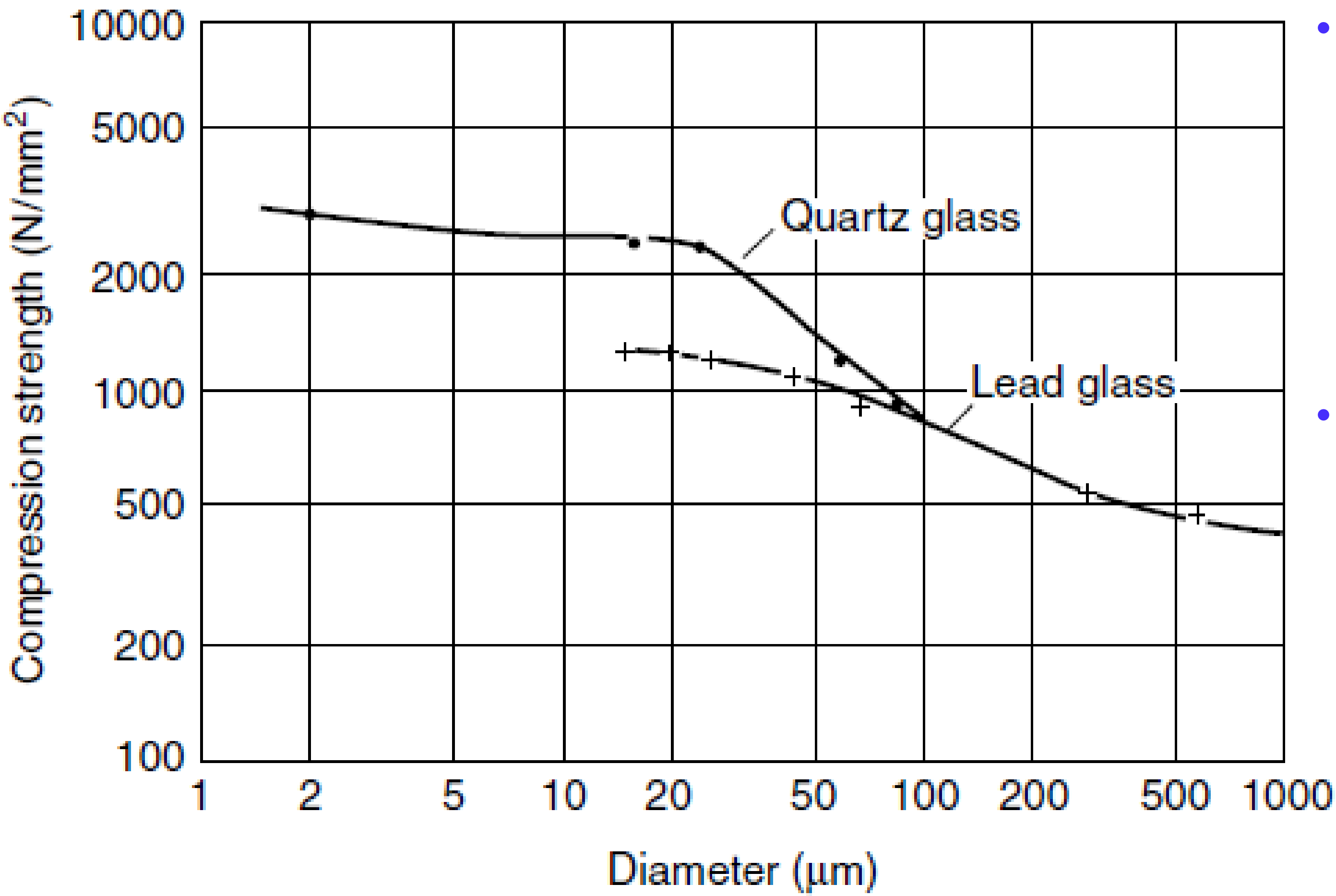
Compressive strength (resistência ao esmagamento) , f_c [N/m2] is the compressive force required to cause rupture of the particle

Rock	Tensile Strength (MPa)	Compressive Strength (MPa)
Limestone	18.00 ± 0.62 (20)	41.45 ± 3.52 (4)
Sandstone	19.17 ± 0.21 (23)	77.59 ± 1.59 (5)
Sandstone	23.10 ± 0.48 (19)	80.83 ± 2.21 (10)
Sandstone	24.21 ± 0.83 (8)	90.48 ± 3.86 (4)
Mudstone	35.17 ± 3.17 (4)	50.07 ± 3.79 (4)
Limestone	36.28 ± 1.24 (24)	142.55 ± 6.14 (5)
Limestone	38.76 ± 2.69 (23)	142.97 ± 19.10 (8)
Ironstone	44.28 ± 4.48 (5)	190.69 ± 17.93 (4)
Sandstone	65.66 ± 0.83 (11)	167.66 ± 9.86 (5)

Table 2. Mean tensile strength and compressive strength for selected sedimentary rock types (after Johnson and Degraff, 1988).

Compressive strength, f_c [N/m2]

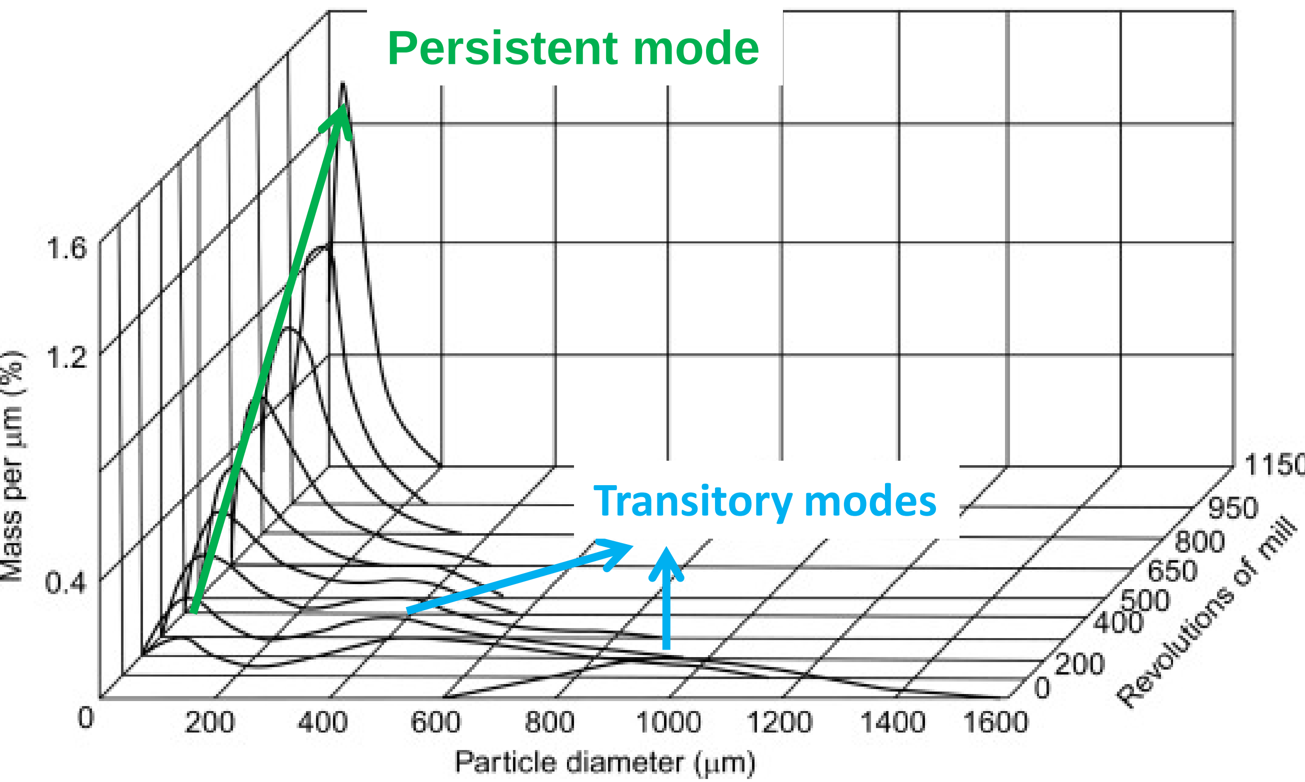
(taken from Coulson & Richardon)



- The compressive strength, f_c [N/m2], of a solid material tends to increase as the size of the solid decreases
- As consequence, the energy per unit mass [J/Kg] spent to reduce the size of a given material is higher in the small size range

Transition and persistent modes

Effect of progressive gridding on the solid size distribution. The size distribution dynamically changes with mill revolutions



- **Persistent mode** (stable final size) is determined by the internal structure of the material
- The relative quantities **persistent** and **transitory modes** is determined by the size reduction machine (predominant type of force and power)

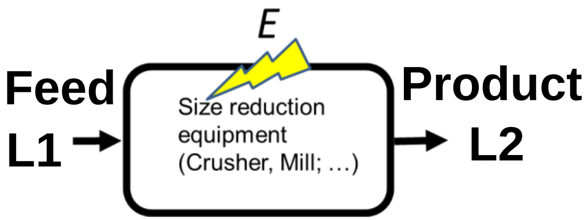
(taken from Coulson & Richardson)

Energy for size reduction

Very low energy utilization efficiency between 0,1 – 2,0%

- **In producing inelastic deformation which results in effective size reduction L1→L2 and creation of new surface.**
- In producing elastic deformation of the particles before fracture occurs
- In causing elastic deformation of the equipment.
- In friction between particles, and between particles and the machine.
- In noise, heat and vibration in the plant, and
- In friction losses in the plant itself.

Energy for size reduction



Depending on the size of the feed and of the desired product, reduction equipment is classified as: (1)Fine, (2)Intermediate, (3)Coarse; different laws of energy are applied in each case

	FINE	INTERMEDIATE	COARSE
Feed size (L1)	5-2 mm	50-5 mm	1500-40 mm
Product size (L2)	<0,1 mm *(powder)*	5-0,1 *(granular/powder)*	50-5 mm *(large/granular)*
Examples of equipment	**Ball mill** Buhrstone mill Roller mill NEI pendulum mill Griffin mill Ring roller mill Tube mill …	**Crushing rolls** Disc crusher Edge runner mill Hammer mill Single roll crusher Pin mill Symons disc crusher …	**Blake jaw crusher** Stag jaw crusher Dodge jaw crusher Gyratory crusher …

Energy laws

Empirical law of energy for size reduction:

$$\frac{dE}{dL} = -C\ L^{p}$$

E – energy spent for size reduction [kJ/kg]
L – size of solids [m]
C – empirical constant related to the solid properties and equipment properties
p - empirical constant related to the size of solids

$p = -1$: coarse reduction, $p = -3/2$: intermediate reduction, $p = -2$: fine reduction

Energy laws

Rittinger's law (*p*=-2)

$$E = C\left(\frac{1}{L_2} - \frac{1}{L_1}\right)$$

$$E = K_R f_c\left(\frac{1}{L_2} - \frac{1}{L_1}\right)$$

Fine reduction

- Energy is utilized more efficiently
- Energy increases as the feed size decreases

Bond's law (*p* = -3/2)

$$E = 2C\left(\frac{1}{\sqrt{L_2}} - \frac{1}{\sqrt{L_1}}\right)$$

$$E = E_i\sqrt{\left(\frac{100}{L_2}\right)}\left(1 - \frac{1}{\sqrt{q}}\right)$$

Intermediate reduction

- Intermediate efficiency
- Energy increases as the feed size decreases

Kick's law (*p* = -1)

$$E = C\,ln\left(\frac{L_1}{L_2}\right)$$

$$E = K_K f_c\,ln\left(\frac{L_1}{L_2}\right)$$

Coarse reduction

- Energy is utilized less efficiently
- Energy increases with the ratio of feed/product sizes

Energy laws

Rittinger's law	Bond's law	Kick's law
$E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1}\right)$	$E = E_i \sqrt{\left(\frac{100}{L_2}\right)}\left(1 - \frac{1}{\sqrt{q}}\right)$	$E = K_K f_c \, ln\left(\frac{L_1}{L_2}\right)$
Fine reduction	Intermediate reduction	Coarse reduction

E – energy spent for size reduction, [KJ/kg]

$\boldsymbol{K_R, K_K}$– Rittinger, Kick constant respectively; empririal constant related to the equipment; without physical meaning

$\boldsymbol{f_C}$– Compressive strength [MPa]; caracterizes the solid material that is being reduced

For bond's law only:

$\boldsymbol{E_i}$ - the work index: amount of energy required to reduce unit mass of material from **L1=∞** to a size **L2=100 μm**

$\boldsymbol{q = L_1/L_2}$

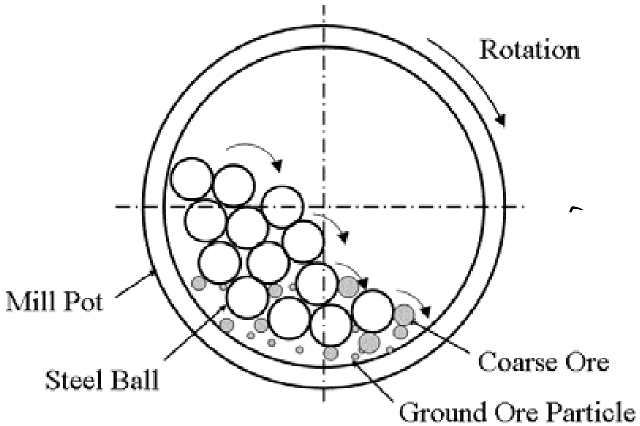
Energy laws

Determination of average size (L1, L2)

- **L1** or **L2** are calculated as the **mean diameter based on volume** (d_V - see chapter 1) from the feed and product size distributions

- Alternatively, **Bond's diameter** (d_{Bond}) provides a good estimate. **Bond's diameter** (d_{Bond}) is defined as the mesh size through which 80% of material passes, in a sieving characterization experiment

Ball mill (Moinho de bolas)



- Hollow rotating cylindrical chamber
- Griding balls inside [0,3-0,4 v/v]
- The balls are typically made of steel
- Prevalent forces: **Impact/Attrition**
- Classified as **fine size reduction**
- Rittinger's energy law

$$E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1} \right)$$

Ball mill (Moinho de bolas)

- A **ball mill** has a critical rotation speed $(w_c, rad/s)$ that must be avoided. At the critical point, the ball (with mass m) is subject to a centrifugal force (mu^2/r) equal to the gravitational force (mg)

$$m\frac{u^2}{r} = mg \Leftrightarrow m\, rw_c^2 = mg \Leftrightarrow$$

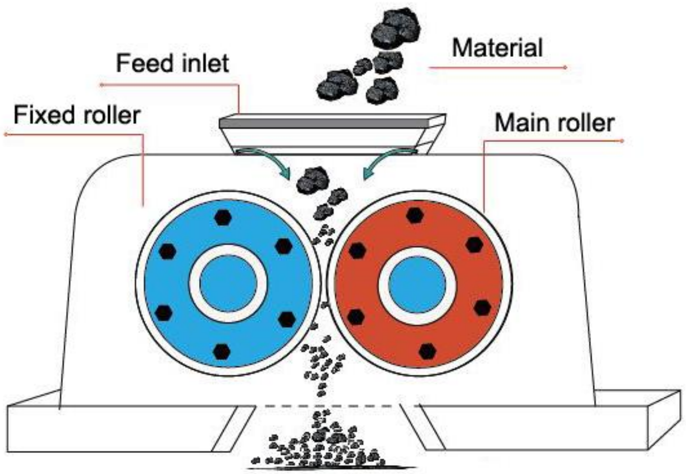
$$\boxed{w_c = \sqrt{\frac{g}{r}}}$$

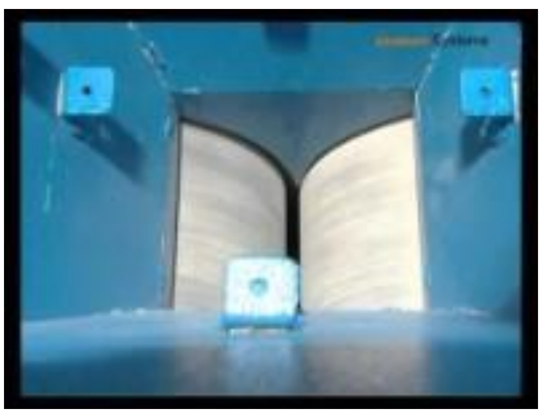
- The optimal rotation speed $(w_o, rad/s)$ should be chosen below the critical value $(w_c, rad/s)$ in order to maximize milling efficiency:

$$\boxed{w_o \sim [1/2, 3/4] \times w_c}$$

w_c - critical rotation speed [rad/s], *r* – mill internal radius [m], *g* = 9,81 [m/s^2]

Crushing rolls (triturador de rolos)

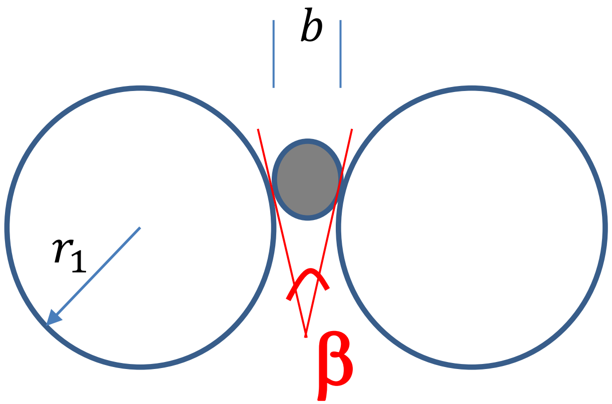




- Two parallel rollers (compact cylinders) rotating in opposite direction (inwards)
- In its simple version, only one of the rollers is mechanically driven
- The distance between the rollers may be adjusted to the feed size and desired product size
- Main forces: **compressive/attrition**
- Classified as **intermediate size reduction**
- Bond's energy law:

$$E = E_i \sqrt{\left(\frac{100}{L_2}\right)}\left(1 - \frac{1}{\sqrt{q}}\right)$$

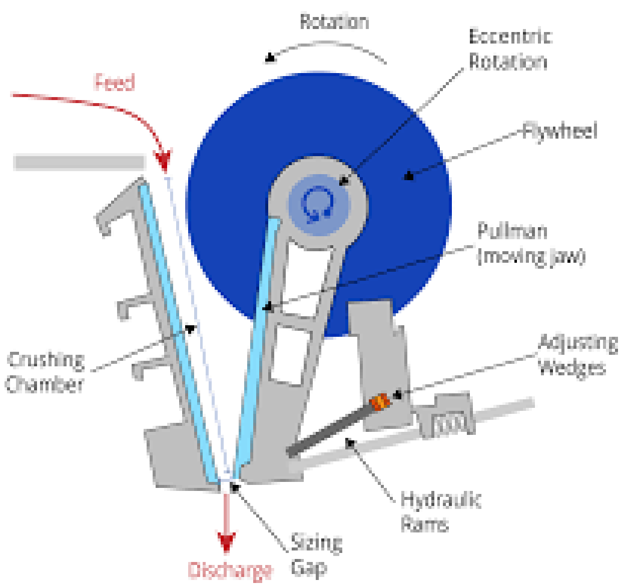
Crushing rolls (triturador de rolos)



$$\cos\left(\frac{\beta}{2}\right) = \frac{r_1 + b/2}{r_1 + r_2}$$

- Roll radius, $\boldsymbol{r_1}$
- The distance between the rolls, $\boldsymbol{b}$, must be adjusted to the feed particle size, $\boldsymbol{r_2}$
- **Nip angle,** β, should be at most 31º degrees

Jaw crusher (Triturador de maxilas)



- Two jaws with compatible toothed surface
- Fixed jaw + swing jaw
- Angle of nip ~30º degree
- Main force applied: <u>**compressive**</u>
- Classified as <u>**coarse reduction**</u>
- Kick's energy law:

$$E = K_K f_c \, ln\left(\frac{L_1}{L_2}\right)$$

Exercises II.1–7

II - REDUÇÃO DA GRANULOMETRIA DE SÓLIDOS

1. Tritura-se um material num triturador de maxilas Blake e reduz-se o tamanho médio das partículas de 50 mm para 10 mm, com um consumo de energia de 13.0 kw s kg^{-1}. Qual será o consumo de energia necessário para triturar o mesmo material do tamanho médio 75 mm até à dimensão média de 25 mm,

(a) supondo aplicável a lei de Rittinger, e
(b) supondo aplicável a lei de Kick?

Qual destes resultados considera de maior confiança e porquê?

2. Usou-se um triturador para triturar um material cuja resistência à compressão era de 22.5 MN/m^2. O tamanho da alimentação era menor que 50 mm, maior que 40 mm e a potência necessária era 13.0 kw s kg^{-1}. A análise por peneiração do produto produziu o seguinte resultado:

Dimensão da abertura (mm)		Percentagem do produto (% em número)
Passando por	6.00	100
Retido em	4.00	26
Retido em	2.00	18
Retido em	0.75	23
Retido em	0.50	8
Retido em	0.25	17
Retido em	0.125	3
Passando por	0.125	5

Qual seria a potência necessária para triturar um kg por segundo de um material com resistência à compressão de 45 MN/m^2 a partir de uma alimentação de menor que 45 mm, maior que 40 mm para dar um produto de tamanho médio de 0.50 mm?

3. Um triturador para moer cal de 70 MN/m^2 de resistência à compressão desde o tamanho médio de 6 mm de diâmetro até ao tamanho médio de 0.1 mm de diâmetro, precisa de ter 9 kw. A mesma máquina usa-se para triturar domolite ao mesmo ritmo de produção desde o tamanho médio de 6 mm de diâmetro até um produto que contém 20% com um diâmetro médio de 0.25 mm, 60% com um diâmetro médio de 0.125 mm, tendo o restante um diâmetro médio 0.085 mm. Fazer a estimativa da potência em kw necessária para accionar o triturador, supondo que a resistência ao esmagamento da domolite é 100 MN/m^2 e que a trituração obedece à lei de Rittinger.

4. Se se regularem uns rolos de moagem de 1 m de diâmetro de tal modo que as superfícies de moagem fiquem à distância de 12.5 mm e o ângulo de presa for 31°, qual é o tamanho máximo de partículas que se deveria introduzir nos rolos?

Se a capacidade real da máquina é 12% da teórica, calcular o ritmo de produção em kg por segundo, quando a funcionar a 2.0 Hz, se a superfície de trabalho dos rolos tiver 0.4 m de comprimento e se a alimentação pesar 2500 kg/m^3.

5. Um triturador mói sal desde um tamanho médio de partícula de 45 mm até um produto assim

Dimensão (mm)	% do produto em número
12.5	0.5
7.5	7.5
5.0	45.0
2.5	19.0
1.5	16.0
0.75	8.0
0.40	3.0
0.20	1.0

e ao fazer isto consome 21 kJ/kg de material triturado.

Calcular a potência necessária para triturar o mesmo material ao mesmo caudal, a partir de uma alimentação com um tamanho médio de 25 mm até um produto com com um tamanho médio de 1mm.

6. Um moinho de bolas com 1.2 m de diâmetro está a trabalhar a 0.80 Hz verificando-se que o moinho não está a trabalhar satisfatoriamente. Sugere alguma modificação nas condições de funcionamento?

7. É preciso fornecer 3 kw a uma máquina para esta triturar material ao caudal de 0.3 kg/s desde cubos de 12.5 mm até um produto com os seguintes tamanhos (% em número):

80%	3.175 mm
10%	2.5 mm
10%	2.25 mm

Que potência em kw teria de fornecer-se a esta máquina para triturar 0.3 kg/s do mesmo material de cubos de 7.5 mm até cubos de 2.0 mm?

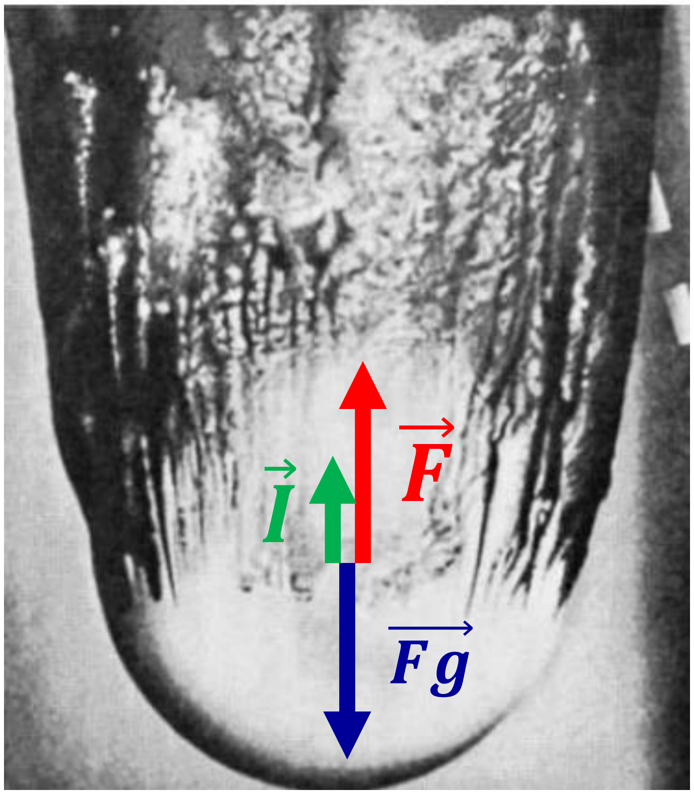
Chapter 3.
Motion of particles in a fluid

3.1 Free fall of a sphere in a fluid
3.2 Skin friction and form drag, Stoke's law
3.3 Friction factor over particle Re', Newton's law
3.4 Terminal fall velocity, u_0
3.5 Elutriation: single column and multistage
3.6 Extension to non-spherical particles, drops and bubbles
(3.7 Transient motion of particles) (later in the centrifugation chapter)

J.M. Coulson and J.F. Richardson pp 146 - 190

Sphere free fall in a fluid

Consider a single spherical particle with diameter, *d* (m), and specific mass, ρ_S (kg/m^3) settling at a velocity, *u* (m/s), in a stationary fluid with specific mass, ρ (kg/m^3) and viscosity μ $(Pa.s)$. **Which are the forces acting on the shere**?



$\vec{F}$ – Drag force [PT: **força de atrito** ou força de arrasto]

$\vec{I}$ – Buoyancy force [PT: força de impulsão]

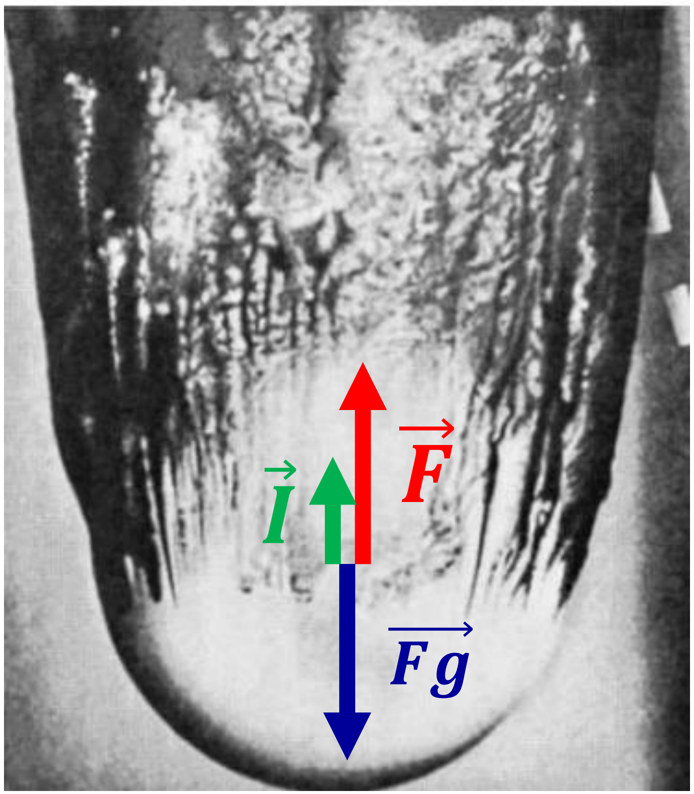
$\overrightarrow{Fg}$ – Gravity force [PT: força gravítica]

($\overrightarrow{Fg} - \vec{I}$ = apparent weight [PT: peso aparente])

$$\overrightarrow{Fg} + \vec{I} + \vec{F} = 0 : uniform\ movement\ (a = 0)$$
$$\overrightarrow{Fg} + \vec{I} + \vec{F} \neq 0 : acelerated\ movement\ (a \neq 0)$$

Sphere free fall in a fluid

Consider a single spherical particle with diameter, *d* (m), and specific mass, ρ_S (kg/m^3) settling at a velocity u (m/s) in a stationary fluid with specific mass, ρ (kg/m^3) and viscosity μ $(Pa.s)$. **Which are the forces acting on the shere**?



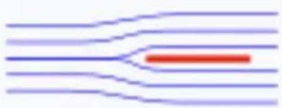
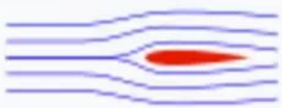
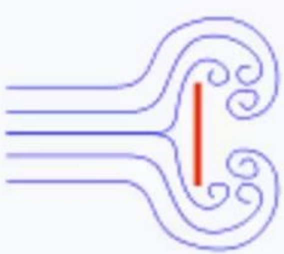
$\vec{F}$ – Drag force (in the following slides)

$\overrightarrow{Fg}$– Gravity force: $$F_g = \frac{\pi d^3}{6}\rho_s g$$

$\vec{I}$ – Buoyancy force: $$I = \frac{\pi d^3}{6}\rho g$$

$\overrightarrow{Fg}$– $\vec{I}$= apparent weight: $$F_{g,a} = \frac{\pi d^3}{6}(\rho_s - \rho)g$$

Drag force: skin *versus* form drag

Shape and flow	Form Drag	Skin friction
	0%	100%
	~10%	~90%
	~90%	~10%
	100%	0%

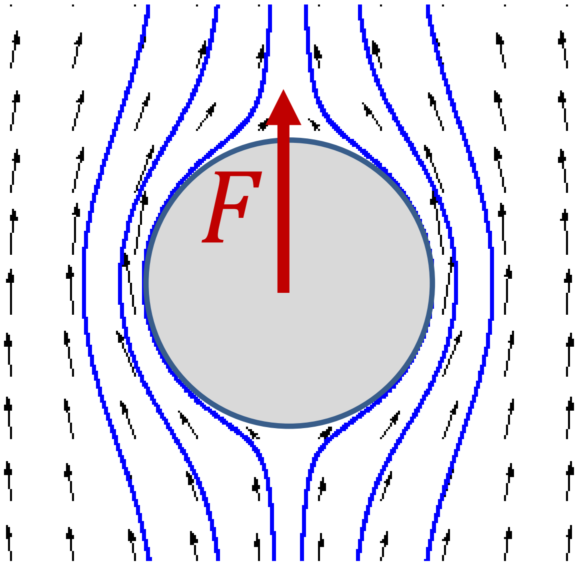
Let's consider the flow of a fluid around a solid body. The fluid will exert 2 types of drag forces on the body. These two forces always occur simultaneously although at different degrees:

Skin friction (*atrito de superfície*) is due to the shear stress of a viscous fluid on the body surface.

Drag form (*atrito de forma*) is caused by the shape and size of the body. Pressure variations between the head and back of the body appear, causing the form drag force.

Stoke's law

Stoke's law applies to the theoretical case of skin friction, which is predominant in streamline laminar flow. It was deduced from *ab initio* First principles by Stokes



$$F = 3\pi\mu u d \quad [N]$$

Particle Reynolds - Re'

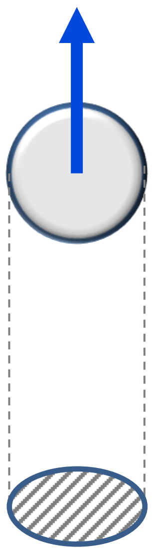
$$Re' = \frac{\rho u d}{\mu} < 0.2 \quad \text{(laminar flow)}$$

μ – Fluid viscosity (Pa.s)
u – relative velocity fluid/sphere (m/s)
d – sphere diameter (m)

Friction factor $\frac{R'}{\rho u^2}$ over particle Re’

Definition of Friction factor (dimensionless)

Sphere



Projected area
in a plan

$$\frac{\pi d^2}{4}$$

$$F$$

Drag force (**N**)

$\div \pi d^2/4$

$$R' = \frac{F}{\pi d^2/4}$$

Drag force per unit of projected area (**N/m2**)

$\div \rho u^2$

$$\boxed{\frac{\boldsymbol{R'}}{\boldsymbol{\rho u^2}} = \frac{\boldsymbol{F}}{\boldsymbol{\rho u^2(\pi d^2/4)}}}$$

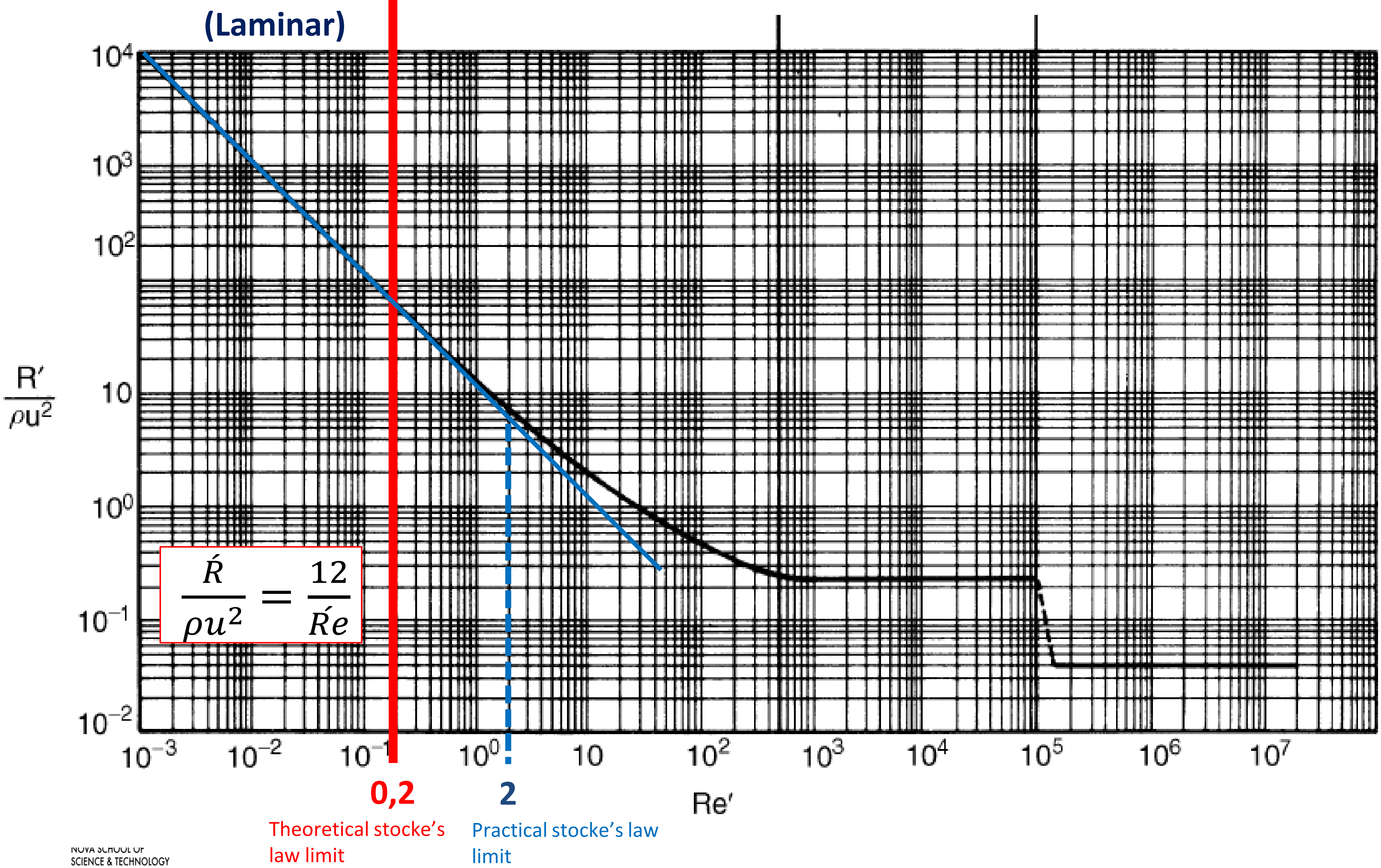
Friction factor (dimensionless)

$\times 2$

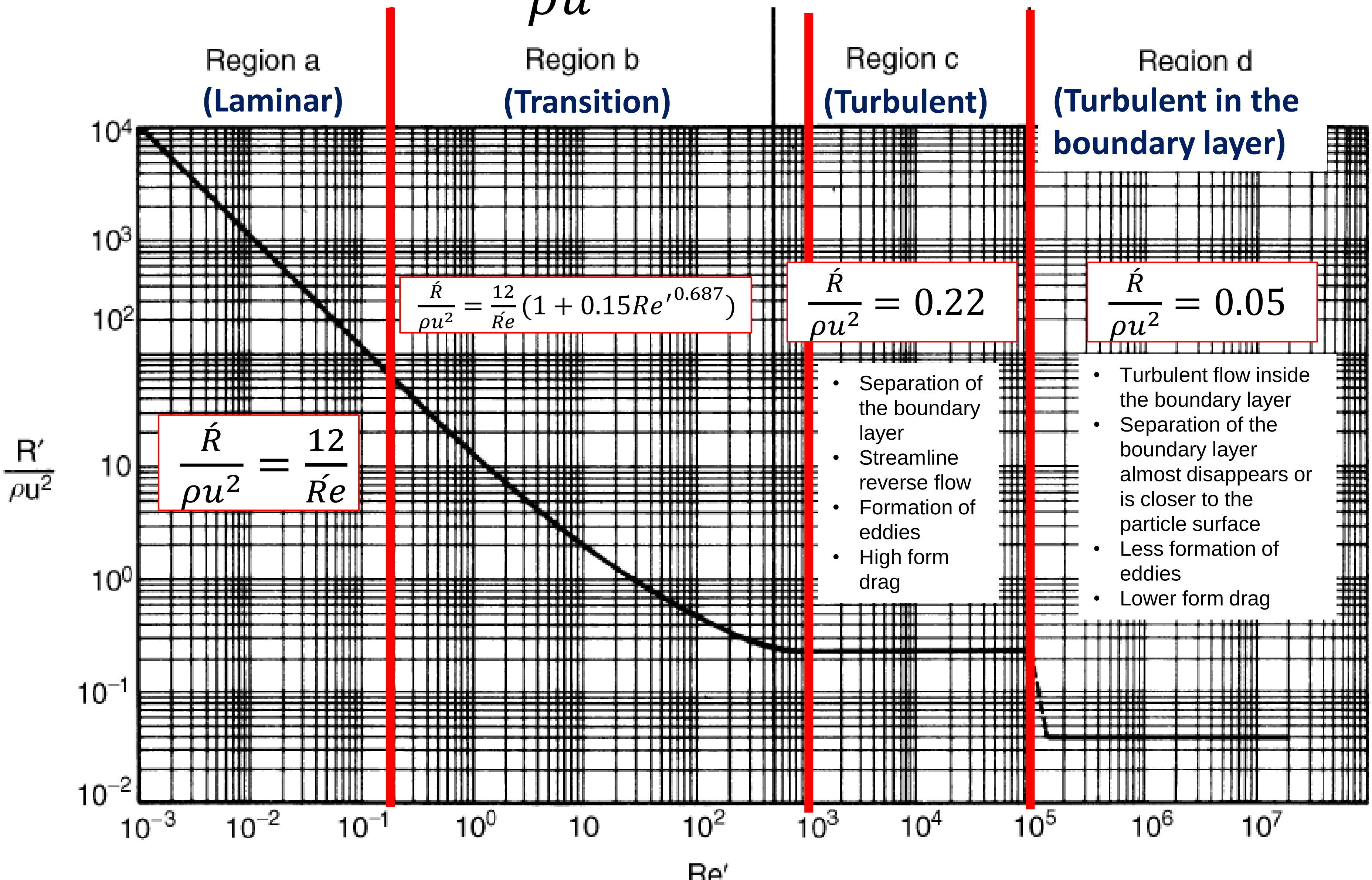
$$C_D = 2\frac{R'}{\rho u^2}$$

Drag coefficient (**dimensionless**)

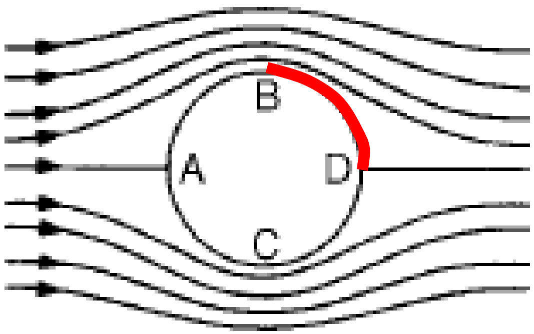
Friction factor $\frac{R'}{\rho u^2}$ over particle Re'



Friction factor $\frac{R'}{\rho u^2}$ over particle Re'

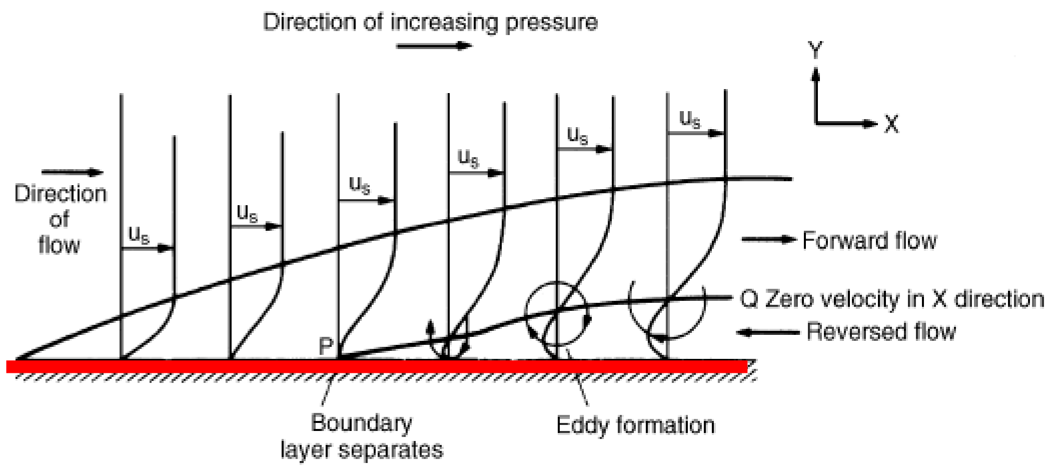


Flow of fluid around a rigid body



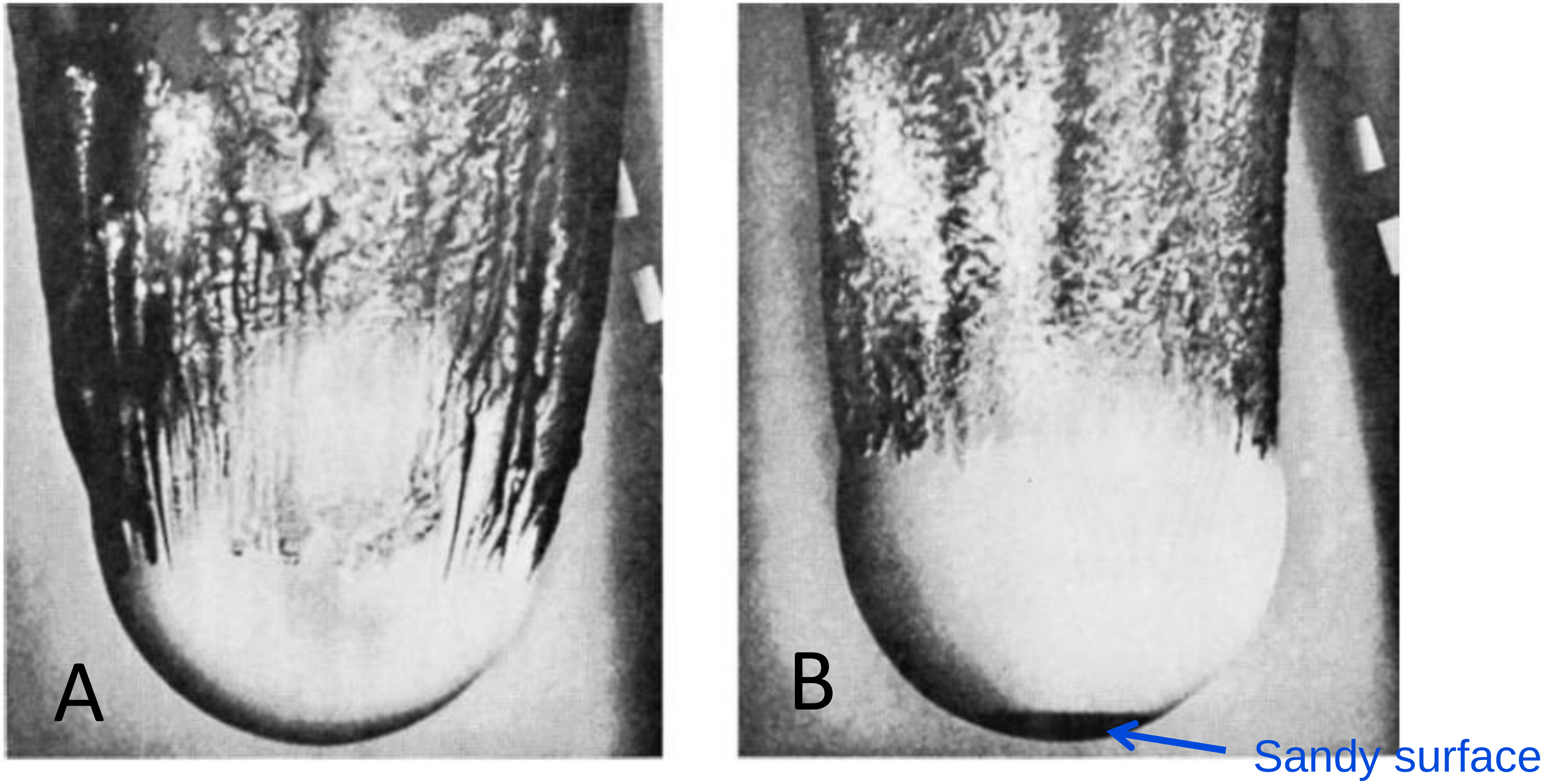
In a given flow streamline from B to D (or from C to D):

- Fluid velocity decreases
- Fluid pressure increases



In region c) the flow is turbulent in the bulk but is laminar in the boundary layer. Due to the increase of pressure in the particle tail, the boundary layer separates, resulting in reverse flow. This originates the formation of eddies and to high energy dissipation. This greatly increases the pressure difference between head and tail thereby greatly increasing the form drag

It has been shown that surface roughness causes a drag force drop for the same Reynolds numbers. **Why?**



Sandy surface at the head of the particle (experiment B) disrupts streamline flow resulting in higher turbulence than case A. Less eddies are formed in the back of the sphere in experiment B, causing a total drag force reduction in comparison to experiment A

Friction factor $\frac{R'}{\rho u^2}$ over particle Re'

Laminar: $10^{-4} < R_e \leq 0.2$ $\frac{\acute{R}}{\rho u^2} = \frac{12}{\acute{R}e}$ (skin friction)

Transition: $0.2 < R_e \leq 10^3$ $\frac{\acute{R}}{\rho u^2} = \frac{12}{\acute{R}e}(1 + 0.15Re'^{0.687})$

Turbulent: $10^3 < R_e \leq 10^5$ $\frac{\acute{R}}{\rho u^2} = 0.22$ (form drag)

Turbulent inside the boundary layer: $10^5 < \acute{R}_e$ $\frac{\acute{R}}{\rho u^2} = 0.05$ (form drag)

Drag force: stoke’s law

The drag factor over Re’ in laminar flow is mathematically equivalente to the stoke’s law

$$\frac{F}{(\pi d^2/4)\,\rho u^2} = \frac{3\pi\mu u d}{(\pi d^2/4)\,\rho u^2}$$

$$\Leftrightarrow$$

$$\boxed{\frac{\acute{R}}{\rho u^2} = \frac{12}{\acute{R}e}}$$

Dimensionless

Drag force: Newton's law

Newton's law is valid in turbulent flow and was deduced from the experimentally determined curve of friction factor over Re'

Turbulent: $10^3 \leq \acute{R}_e \leq 10^5$ (form drag)

$$\frac{\acute{R}}{\rho u^2} = 0.22$$

$$\times \pi d^2/4 \times \rho u^2$$

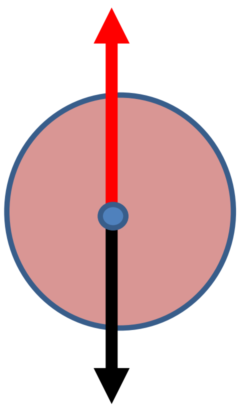
$$\boxed{F = 0.055 \pi d^2 \rho u^2}$$

Note there is no viscosity in Newton's law!!!

Terminal fall velocity, u_0

If the drag force equals the apparent weight of the sphere then the accelaration is zero and the sphere settles at a constant velocity u_0:

$\vec{F}$ – Drag force



Apparent weight

$$\frac{\pi d^3}{6}(\rho_s - \rho)g$$

$$F = \frac{\pi d^3}{6}(\rho_s - \rho)g$$

For laminar flow ($\acute{R}e < 0{,}2$), then stoke's law holds:

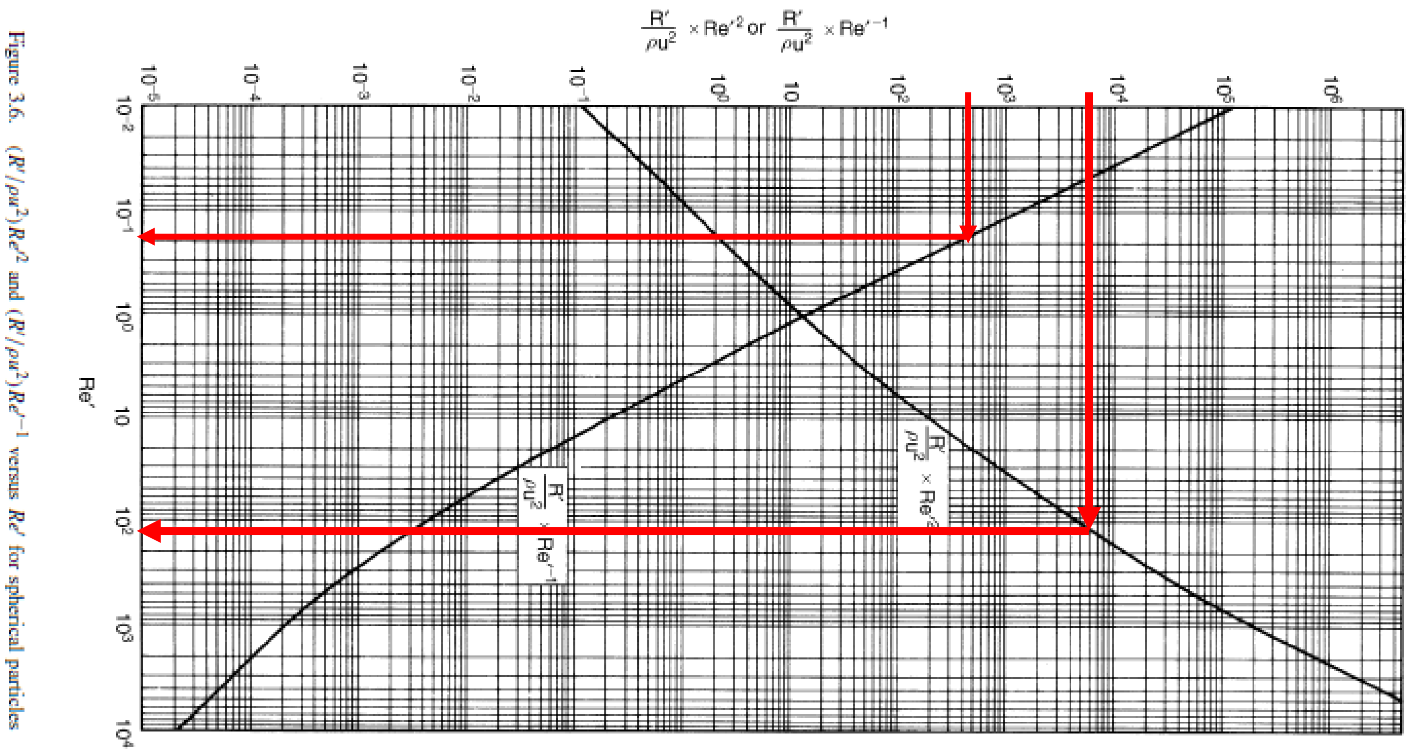
$$3\pi\mu u_0 d = \frac{\pi d^3}{6}(\rho_s - \rho)g \qquad \boxed{u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu}} \quad \acute{R}e < 0{,}2$$

For turbulent flow ($10^3 \leq \acute{R}e \leq 10^5$), then Newton's law holds:

$$0.055\pi d^2 \rho u_0^2 = \frac{\pi d^3}{6}(\rho_s - \rho)g \qquad \boxed{u_0 = \sqrt{\frac{3d(\rho_s - \rho)g}{\rho}}}$$

$$10^3 \leq \acute{R}e \leq 10^5$$

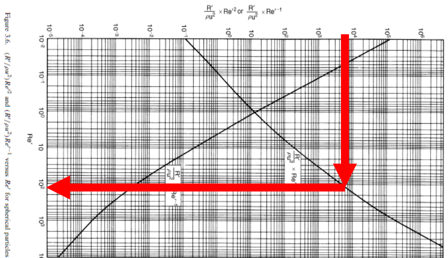
Terminal fall velocity, u_0: graphical method



Taken from **J.M. Coulson and J.F. Richardson (1965) pp. 158**

Terminal fall velocity, u_0: graphical method

Case 1. Condition for terminal fall velocity: $F = \frac{\pi d_i^3}{6}(\rho_s - \rho)g \Leftrightarrow$

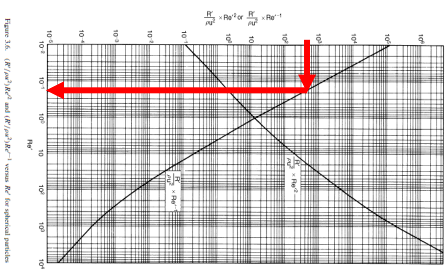


By manipulating and rearranging it may be shown:

$$\Leftrightarrow \frac{\acute{R}}{\rho u^2}\acute{R}e^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2} = \frac{2}{3}Ga$$

If $\mathbf{d}$ is known $\Rightarrow$ calculate $\frac{\acute{R}}{\rho u^2}\acute{R}e^2$ $\Rightarrow$Take from picture $\acute{R}e$ $\Rightarrow$take from $\acute{R}e$ the value of $\boldsymbol{u_0}$

Case 2. Condition for terminal fall velocity: $F = \frac{\pi d_i^3}{6}(\rho_s - \rho)g \Leftrightarrow$



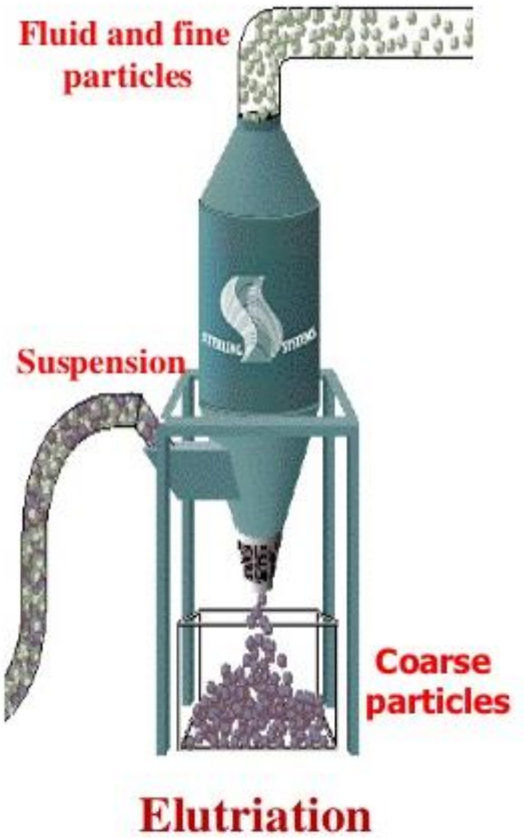
By manipulating and rearranging it may be shown:

$$\Leftrightarrow \frac{\acute{R}}{\rho u^2}\acute{R}e^{-1} = \frac{2(\rho_s - \rho)\mu g}{3\rho^2 u^3}$$

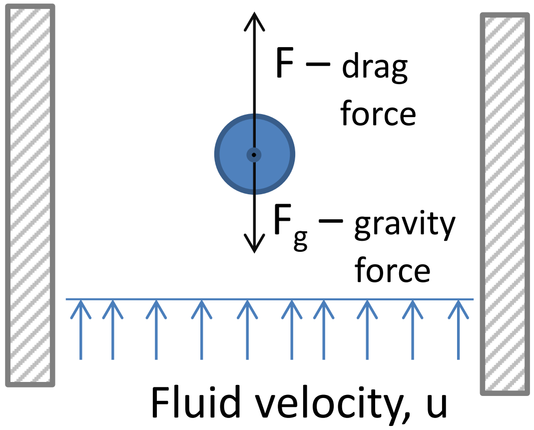
If $\boldsymbol{u_0}$ is known $\Rightarrow$ calculate $\frac{\acute{R}}{\rho u^2}\acute{R}e^{-1}$ $\Rightarrow$Take from picture $\acute{R}e$ $\Rightarrow$take from $\acute{R}e$ the value of $\mathbf{d}$

Ga – Galileo number (dimensionless)

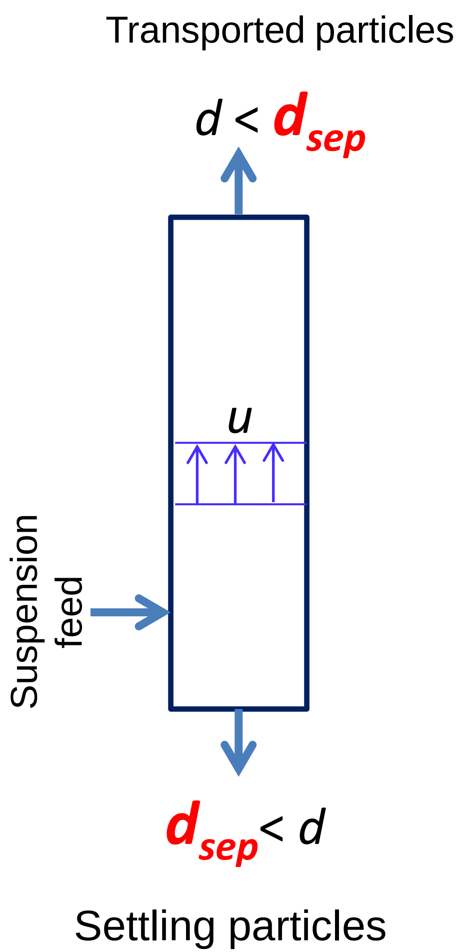
Elutriation



Elutriation is a particle classification and/or separation process based on the density and size of particles through the motion of a carrying fluid (gas or liquid). The smaller and less dense particles will be dragged out on the top of the column (**fine particles stream**). The bigger and more dense particles will settle in the bottom of the column (**coarse particles stream**).



Elutriation: single column



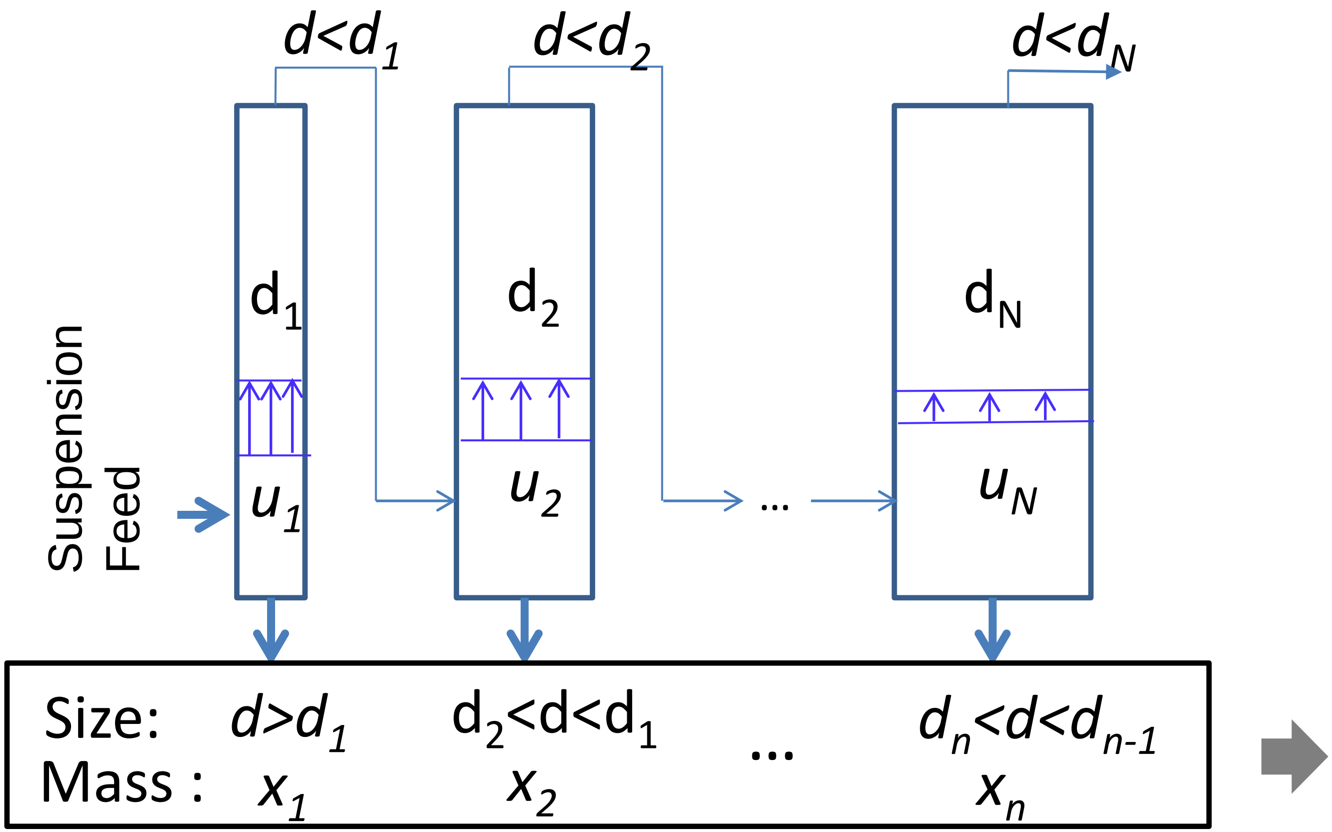
- Elutriation operates in the range 1 - 50 µm
- Thus operation is typically laminar, $\acute{R}e < 0{,}2$
- d_{sep} is the critical separation size of solids
- How to determine d_{sep} ?
- Particles (be it spheres) going up? $F > \frac{\pi d^3}{6}(\rho_s - \rho)g$
- Particles going down? $F < \frac{\pi d^3}{6}(\rho_s - \rho)g$
- Critical separation size, d_{sep}, (note that stoke's law holds):

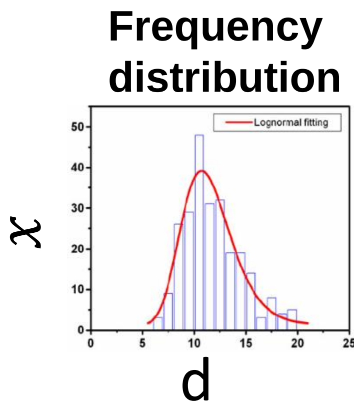
$$F = 3\pi\mu u d_{sep} = \frac{\pi d_{sep}^3}{6}(\rho_s - \rho)g$$ (particles staying in the column)

$$\boxed{d_{sep} = \sqrt{\frac{18\mu u}{(\rho_s - \rho)g}}} \qquad \acute{R}e < 0{,}2$$

Elutriation: multi-stage with *N* columns

Cross section area increases from column 1 to *N*: $A_1 < A_2 < \cdots < A_N$
Fluid velocity decreases from column 1 to *N*: $u_1 > u_2 > \cdots > u_N$
Critical separation size decreases from column 1 to *N*: $d_1 > d_2 > \cdots > d_N$

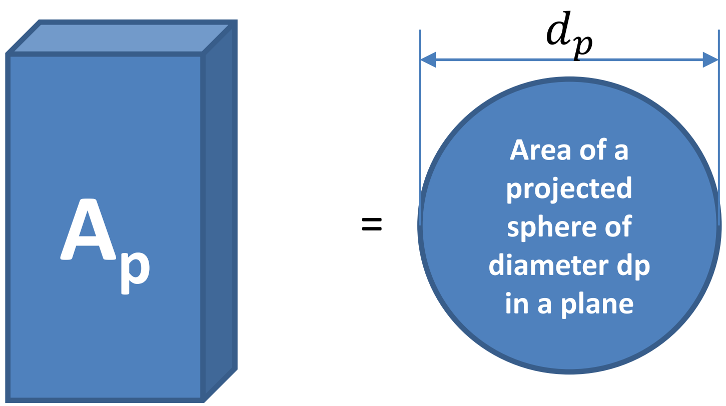




Non-spherical geometry: Heywood method

Heywood method (Coulson pp. 166) is a 6-steps procedure:

Step 1. Determine the mean projected diameter of particle, d_p



$$A_p = \frac{\pi d_p^2}{4} \Leftrightarrow \boxed{d_p = \sqrt{\frac{4A_p}{\pi}}}$$

Which face to choose? The one with largest Ap !

Step 2. Determine the volume factor, k'

$$V_p = k\acute{d}_p^3 \Leftrightarrow \boxed{\acute{k} = \frac{V_p}{d_p^3}}$$

Non-spherical geometry: Heywood method

Step 3. Redo force balances for non-spherical geometry in its dimensionless form

$$\acute{R} A_p = V_p\,(\rho_s - \rho)g \Leftrightarrow \qquad \acute{R}\frac{\pi d_p^2}{4} = \acute{k} d_p^3(\rho_s - \rho)g$$

$$\Rightarrow \boxed{\frac{\acute{R}}{\rho u^2}\acute{R}e^2 = \frac{4\acute{k}\rho d_p^3(\rho_s-\rho)g}{\mu^2\pi}}$$

$$\Rightarrow \boxed{\frac{\acute{R}}{\rho u^2}\acute{R}e^{-1} = \frac{4\acute{k}\mu(\rho_s-\rho)g}{\rho^2\pi u^3}}$$

Step 4. Determine Reynolds, $log_{10}(\acute{Re})$, from Figure 3.6 or Table 3.4-3.5 as if a spherical particle (pp. 157, 158, 161)

Non-spherical geometry: Heywood method

Step 5. Additive corrections of $log_{10}(R\acute{e})$ obtained in **step 4** (spherical particals) using Tables 3.7-3.8 due to non-spherical geometry (pp. 166-167)

Table 3.7. Corrections to $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{2}$	−0.022	−0.002	+0.032	+0.131
$\bar{1}$	−0.023	−0.003	+0.030	+0.131
0	−0.025	−0.005	+0.026	+0.129
1	−0.027	−0.010	+0.021	+0.122
2	−0.031	−0.016	+0.012	+0.111
2.5	−0.033	−0.020	0.000	+0.080
3	−0.038	−0.032	−0.022	+0.025
3.5	−0.051	−0.052	−0.056	−0.040
4	−0.068	−0.074	−0.089	−0.098
4.5	−0.083	−0.093	−0.114	−0.146
5	−0.097	−0.110	−0.135	−0.186
5.5	−0.109	−0.125	−0.154	−0.224
6	−0.120	−0.134	−0.172	−0.255

Table 3.8. Corrections to $\log Re'$ as a function of $\{\log(R'/\rho u^2)Re'^{-1}\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^{-1}\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{4}$	+0.185	+0.217	+0.289	
$\bar{4}.5$	+0.149	+0.175	+0.231	
$\bar{3}$	+0.114	+0.133	+0.173	+0.282
$\bar{3}.5$	+0.082	+0.095	+0.119	+0.170
$\bar{2}$	+0.056	+0.061	+0.072	+0.062
$\bar{2}.5$	+0.038	+0.034	+0.033	−0.018
$\bar{1}$	+0.028	+0.018	+0.007	−0.053
$\bar{1}.5$	+0.024	+0.013	−0.003	−0.061
0	+0.022	+0.011	−0.007	−0.062
1	+0.019	+0.009	−0.008	−0.063
2	+0.017	+0.007	−0.010	−0.064
3	+0.015	+0.005	−0.012	−0.065
4	+0.013	+0.003	−0.013	−0.066
5	+0.012	+0.002	−0.014	−0.066

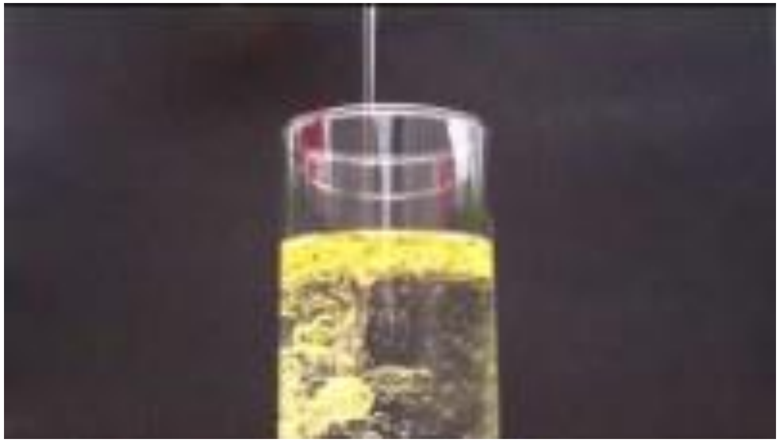
Step 6. Obtain $\boldsymbol{u_o}$ or ***d*** from $log_{10}(R\acute{e})$ obtained in **step 5**

Bubbles and Drops

Consider an **gas bubble** or an **oil drop** with diameter, *d,* freely rising in water. When the forces are equal, the bubble or drop will rise at a constant velocity, u_0. The gas bubble or oil drop do not behave as a rigid body. Their shape will adjust to the movement

Air in water

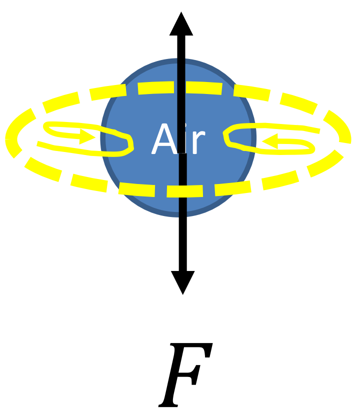
Oil in water



Bubbles and Drops

When the forces are balanced, the bubble or drop will rise at a constant velocity, u_0. In laminar flow, Stoke's law applies with Hardmard correction to compensate for shape variations and internal recirculation.

$$\frac{\pi d^3}{6}(\rho_{bubble}-\rho)g<0$$



Hardmard correction to Stoke's law valid only in laminar flow

$$F = 3\pi\mu u d/Q$$

$$1 < \boldsymbol{Q} = \frac{\boldsymbol{3\mu+3\mu_{bubble}}}{\boldsymbol{2\mu+3\mu_{bubble}}} < 1.5$$

$$\boxed{u_0 = \frac{d^2(\rho_s-\rho)g}{18\mu}\boldsymbol{Q}}$$

Exercises

III - MOVIMENTO DE PARTÍCULAS NUM FLUIDO

1. Sujeita-se a elutriação uma mistura finamente moída de galena e calcário na proporção de 1 para 4 em peso, mediante uma corrente ascendente de água, que flui a 0.5 cm/s. Supondo que a distribuição de tamanhos é a mesma para ambos os materiais e corresponde à que se indica no quadro seguinte, faça a estimativa da percentagem de galena no material arrastado e no material que fica para trás. Considere a viscosidade absoluta da água igual a 1 mN s m^{-2} e use a equação de Stokes.

Diâmetro (mícrons)	20	30	40	50	60	70	80	100
% em peso de finos	15	28	48	54	64	72	78	88

Dados: densidade da galena =7.5; densidade do calcáreo=2.7

2. Calcular a velocidade limite de uma bola de aço com 2 mm de diâmetro (massa específica = 7.87 g/cm^3) em óleo (massa específica 0.9 g/cm^3 , viscosidade 50 mN s m^{-2}).

3. Qual será a velocidade de sedimentação de uma partícula de aço esférica, com 0.40 mm de diâmetro, num óleo de densidade 0.82 e viscosidade 10 mN s m^{-2} ? A densidade do aço é 7.87.

4. Quais são as velocidades de sedimentação de placas de mica com 1 mm de espessura e áreas na gama de 6 a 600 mm^2 num óleo de densidade 0.82 e viscosidade 10 mN s m^{-2}. A densidade da mica é 3.0.