CN A – Teste 2023.1 Resolução

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Conteúdo

Questão 1

$$\boldsymbol{x = 1/13 \qquad \bar{x} = 0.0769}$$

$$\boldsymbol{g(x) = \frac{1}{2/25 - x}; \qquad r_{g(x)} \approx \left|\frac{x\, g'(x)}{g(x)}\right| r_x, \quad g(x) \neq 0}$$

Resposta c

$$\frac{r_{g(x)}}{r_x} \approx \left|\frac{x\, g'(x)}{g(x)}\right| = \left|\frac{(1/13)(\,(-1)\,(2/25 - 1/13)^{-2}(-1))}{(2/25 - 1/13)}\right| = 2640625$$

$\therefore$ é mal condicionada

Questão 2

$$[a, b] \quad a < x_0 < x_1 < \cdots < x_n = b$$

$$f_{(x_i)}, i = 1, \ldots, n \quad n \geq 3$$

$$\begin{cases} p \text{ polinomio lagrange grau n} \\ q \text{ polinomio 2 grau minimos quadrados} \\ S \text{ spline cubico para } x_i \end{cases}$$

Resposta c

Questão 3

Considere a tabela para f

x_i	0	1	3
$f(x_i)$	c	-1	2

$$\boldsymbol{c \in \mathbb{R} \quad p_1(x)\ \text{approx f por min quadr}: p_1(x_i) = f(x_i), i = 0, 1, 2}$$

Resposta a

$$p_{1\,(x_i)} = \alpha_0 + \alpha_1\, x_i$$

$$\begin{cases} \alpha_0 + \alpha_1\,(x_0 + x_1 + x_2) = y_0 + y_1 + y_2 \\ \alpha_0\,(x_0 + x_1 + x_2) + \alpha_1\,(x_0^2 + x_1^2 + x_2^2) = x_0\,y_0 + x_1\,y_1 + x_2\,y_2 \\ \alpha_0\,(x_0^2 + x_1^2 + x_2^2) + \alpha_1\,(x_0^3 + x_1^3 + x_2^3) = x_0^2\,y_0 + x_1^2\,y_1 + x_2^2\,y_2 \end{cases} \quad =$$

$$= \begin{cases} \alpha_0 + \alpha_1\,(1+3) = c - 1 + 2 \\ \alpha_0\,(1+3) + \alpha_1\,(1+9) = -1 + 3*2 \\ \alpha_0\,(1+9) + \alpha_1\,(1+27) = 1*-1+9*2 \end{cases} \quad \implies$$

$$\implies \begin{cases} 5/2 = c \\ \alpha_1 = 18/12 = 3/2 \\ \alpha_0 = 1.7 - \alpha_1\,28/10 = -5/2 \end{cases}$$

Questão 4

$$\boldsymbol{I = \int_{-1}^{1} f(x)\ \mathrm{d}x}$$

$$\boldsymbol{\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2} = C,\ \forall\, x \in \mathbb{R}, C \neq 0}$$

Resposta b

$$I_T \approx \frac{h}{2}\left(f_0 + f_1\right) = \frac{1}{2}\left(f_0 + f_1\right)$$

$$I_{PM} \approx h\, f_{\left(\frac{x_0 + x_1}{2}\right)} = f_{\left(\frac{-1+1}{2}\right)} = f_0$$

Questão 5

$$\boldsymbol{I = \int_a^b f(x)\ \mathrm{d}x; \qquad J = \int_a^b p_2(x)\ \mathrm{d}x}$$
$$\boldsymbol{x = \{a, (a+b)/2, b\}}$$

Resposta b

$$I_S \approx \frac{(b-a)/2}{3}\,(f_0 + 4\,f_1 + f_2)$$

$$I_T \approx \frac{(b-a)/2}{2}\,(f_0 + f_1)$$

Questão 6

Considere a seguinte tabela de val da fun g

x_i	-2	-1	4	5
$g(x_i)$	$-1/3$	$-1/24$	$8/3$	$1/8$

Q6 a.

Pol de N com diff div interp da tabela

Resposta

$$p_{n\,(x)} = g_0 + \sum_{i=0}^{n-1} \left(\prod_{j=0}^{i} x - x_j \right) g_{[x_0,\ldots,x_{i+1}]}$$

Questão 7

Considere o integral

$$\boldsymbol{I = \int_0^3 \ln(x^2)\,\mathrm{d}x}$$

Q7 a.

Determine $\hat{I}$ por ponto med comp h=1

Q7 b.

$$\hat{I}_S, n = 2$$

$$\hat{I}_S \approx \frac{h}{3}\,(f_0 + 4\,f_1 + f_2) = \frac{(3-1)/2}{3}\,(\ln(0^2) + 4\,\ln(1^2) + \ln(2^2))$$

Questão 8

Seja S a função definida por

$$\boldsymbol{S}\begin{cases}\boldsymbol{-x^3-6\,x^2-8\,x+2,} & \boldsymbol{-2\le x<-1}\\ \boldsymbol{\alpha\,x^3+\beta\,x+4,} & \boldsymbol{-1\le x<0}\\ \boldsymbol{-2\,x+4,\quad 0\le x<1}\end{cases}$$

Det α e β de forma que seja spline cubico interp, s é spline nat?

Resposta

$$\lim_{x\to -1^-} S(x) = 1-6+8 = 3 = \lim_{x\to -1^+} S(x) = -\alpha-\beta+4$$

$$\lim_{x\to 0^-} S(x) = 4 = \lim_{x\to 0^+} S(x) = 4$$

$$\frac{\mathrm{d}S(x)}{\mathrm{d}x} = \begin{cases}-3\,x^2-12\,x-8, & -2\le x<-1\\ 3\,\alpha\,x^2+\beta, & -1\le x<0\\ -2,\quad 0\le x<1\end{cases}$$

$$\lim_{x\to -1^-}\frac{\mathrm{d}S(x)}{\mathrm{d}x} = -3+12-8 = 1 = \lim_{x\to -1^+}\frac{\mathrm{d}S(x)}{\mathrm{d}x} = 3\,\alpha+\beta$$

$$\lim_{x\to 0^-}\frac{\mathrm{d}S(x)}{\mathrm{d}x} = \beta = \lim_{x\to 0^+}\frac{\mathrm{d}S(x)}{\mathrm{d}x} = -2$$

$$\frac{\mathrm{d}^2S(x)}{\mathrm{d}x^2} = \begin{cases}-6\,x-12, & -2\le x<-1\\ 6\,\alpha\,x, & -1\le x<0\\ 0,\quad 0\le x<1\end{cases}$$

$$\lim_{x\to -1^-}\frac{\mathrm{d}^2S(x)}{\mathrm{d}x^2} = 6-12 = -6 = \lim_{x\to -1^+}\frac{\mathrm{d}^2S(x)}{\mathrm{d}x^2} = -6\,\alpha$$

$$\lim_{x\to 0^-}\frac{\mathrm{d}^2S(x)}{\mathrm{d}x^2} = 0 = 0$$

$$\implies \begin{cases}\beta = -2\\ -\alpha-\beta+4 = -\alpha+2+4 = 3 \implies \alpha = 3\\ 1 \neq 3\,\alpha+\beta = 3*3-2 = 7\end{cases}$$

$\therefore$ não interpola nem é spline nat