SECTION 1

PARTICULATE SOLIDS

Problem 1.1

The size analysis of a powdered material on a weight basis is represented by a straight line from 0% weight at 1 μm particle size to 100% weight at 101 μm particle size. Calculate the mean surface diameter of the particles constituting the system.

Solution

From equation 1.15 the surface mean diameter is given by:

$$d_s = 1/\Sigma(x_1/d_1)$$

Since the size analysis is represented by the continuous line:

$$d = 100x + 1 \qquad \text{(Fig. 1.7)}$$

$$d_s = 1\Big/\int_0^1 \mathrm{d}x/d$$

$$= 1\Big/\int_0^1 \mathrm{d}x/(100x+1)$$

$$= 100/\ln 101$$

$$= \underline{\underline{21{\cdot}7\ \mu\mathrm{m}}}$$

Problem 1.2

The equations giving the number distribution curve for a powdered material are $\mathrm{d}n/\mathrm{d}d = d$ for the size range 0 to 10 μm and $\mathrm{d}n/\mathrm{d}d = 100{,}000/d^4$ for the size range 10 to 100 μm. Sketch the number, surface, and weight distribution curves. Calculate the surface mean diameter for the powder.

Explain briefly how the data for the construction of these curves would be obtained experimentally.

Solution

For the range, $d = 0 - 10\,\mu m$,

$$\frac{dn}{dd} = d$$

On integration, $$n = 0.5d^2 + c_1 \qquad \text{(i)}$$

where c_1 is the constant of integration.

For the range, $d = 10 - 100\,\mu m$,

$$\frac{dn}{dd} = 10^5 d^{-4}$$

On integration, $$n = c_2 - 0.33 \times 10^5 d^{-3} \qquad \text{(ii)}$$

when $d = 0$, $n = 0$, and from (i) $\quad c_1 = 0$

when $d = 10\,\mu m$, in (i) $\quad n = 0.5 \times 10^2 = 50$

in (ii) $\quad 50 = c_2 - 0.33 \times 10^5 \times 10^{-3}$

and $\quad c_2 = 83.0$

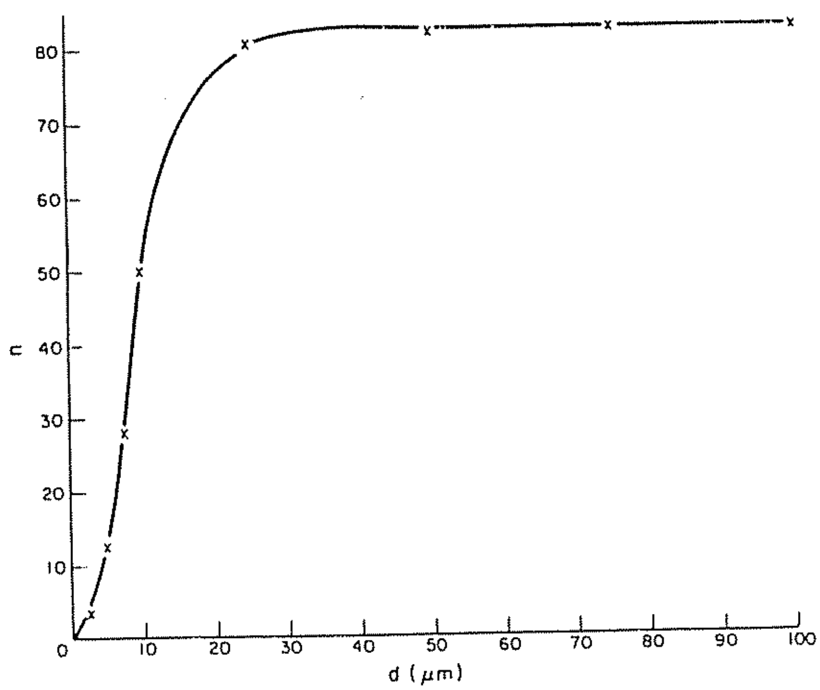

FIG. 1a

Thus for $d = 0 - 10\,\mu m$, $\quad n = 0.5d^2$

and for $d = 10 - 100\,\mu m$, $\quad n = 83.0 - 0.33 \times 10^5 d^{-3}$

Using these equations, values of n are obtained as follows:

d (μm)	n	d (μm)	n
0	0	10	50·0
2·5	3·1	25	80·9
5·0	12·5	50	82·7
7·5	28·1	75	82·9
10·0	50·0	100	83·0

and these data are plotted in Fig. 1a.

From this plot, values of d are obtained for chosen values of n and hence n_1 and d_1 obtained for each increment of n. Values of $n_1 d_1^2$ and $n_1 d_1^3$ are calculated and the totals obtained. The surface area of the particles in the increment is then given by:

$$s_1 = n_1 d_1^2 / \Sigma n_1 d_1^2$$

and s is then found as Σs_1. Similarly the weight of the particles $x = \Sigma x_1$ where:

$$x_1 = n_1 d_1^3 / \Sigma n_1 d_1^3$$

The results are given as follows:

n	d	n_1	d_1	$n_1 d_1^2$	$n_1 d_1^3$	s_1	s	x_1	x
0	0								
		20	3·1	192	596	0·014	0·014	0·001	0·001
20	6·2								
		20	7·6	1155	8780	0·085	0·099	0·021	0·022
40	9·0								
		10	9·5	903	8573	0·066	0·165	0·020	0·042
50	10·0								
		10	10·7	1145	12250	0·084	0·249	0·029	0·071
60	11·4								
		5	11·75	690	8111	0·057	0·300	0·019	0·090
65	12·1								
		5	12·85	826	10609	0·061	0·361	0·025	0·115
70	13·6								
		2	14·15	400	5666	0·029	0·390	0·013	0·128
72	14·7								
		2	15·35	471	7234	0·035	0·425	0·017	0·145
74	16·0								
		2	16·75	561	9399	0·041	0·466	0·022	0·167
76	17·5								
		2	18·6	692	12870	0·057	0·517	0·030	0·197
78	19·7								
		2	21·2	890	18877	0·065	0·582	0·044	0·241
80	22·7								
		1	24·1	581	14000	0·043	0·625	0·033	0·274
81	25·5								
		1	28·5	812	23150	0·060	0·685	0·055	0·329
82	31·5								
		1	65·75	4323	284240	0·316	1·000	0·670	1·000
83	100								
				13641	424355				

Values of s and x are plotted as functions of d in Fig. 1b.
Surface mean diameter,

$$d_s = \Sigma(n_1 d_1^3)/\Sigma(n_1 d_1^2) = 1/\Sigma(x_1/d_1)$$

$$= \int d^3\,\mathrm{d}n \Big/ \int d^2\,\mathrm{d}n$$

For $0 < d < 10\,\mu\mathrm{m}$, $\quad \mathrm{d}n = d\,\mathrm{d}d$

For $10 < d < 100\,\mu\mathrm{m}$, $\quad \mathrm{d}n = 10^5 d^{-4}\,\mathrm{d}d$

$$d_s = \left(\int_0^{10} d^4\,\mathrm{d}d + \int_{10}^{100} 10^5 d^{-1}\,\mathrm{d}d\right) \Big/ \left(\int_0^{10} d^3\,\mathrm{d}d + \int_{10}^{100} 10^5 d^{-2}\,\mathrm{d}d\right)$$

$$= ([d^5/5]_0^{10} + 10^5[\ln d]_{10}^{100})/([d^4/4]_0^{10} + 10^5[-d^{-1}]_{10}^{100})$$

$$= (2 \times 10^4 + 2{\cdot}303 \times 10^5)/(2{\cdot}5 \times 10^3 + 9 \times 10^3)$$

$$= \underline{\underline{21{\cdot}8\,\mu\mathrm{m}}}$$

The size range of a material is determined by sieving for relatively large particles and by sedimentation methods for particles which are too fine for sieving. The use of such a method is described in Chapter 1, Volume 2.

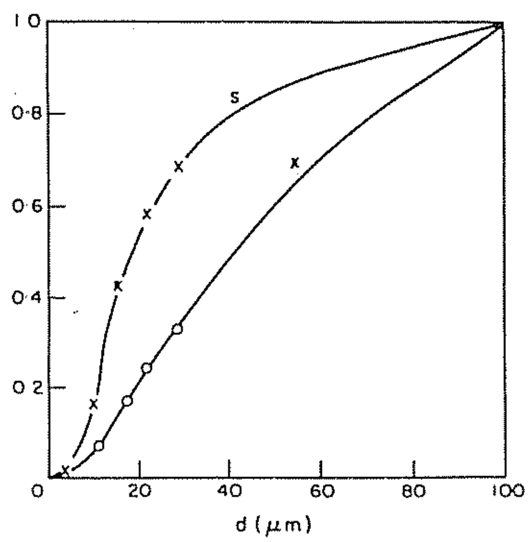

FIG. 1b

Problem 1.3

The fineness characteristic of a powder on a cumulative basis is represented by a straight line from the origin to 100% undersize at particle size 50 μm. If the powder is initially dispersed uniformly in a column of liquid, calculate the proportion by weight which remains in suspension at a time interval from commencement of settling such that a 40 μm particle would fall the total height of the column.

Solution

For flow under streamline conditions, the velocity is proportional to the diameter squared. Hence the time taken for a 40 μm particle to fall a height h m is:

$$h/40^2 k \text{ s}$$

During this time, a particle of diameter d μm has fallen a distance:

$$kd^2 h/40^2 k = hd^2/40^2 \text{ m}$$

The proportion of particles of size d which are still in suspension $= (1 - d^2/40^2)$ and the fraction by mass of particles which are in suspension

$$= \int_0^{40} (1 - d^2/40^2) \, dw$$

Since $dw/dd = 1/50$, the mass fraction:

$$= (1/50) \int_0^{40} (1 - d^2/40^2) \, dd$$

$$= (1/50)[d - d^3/4800]_0^{40} = 0{\cdot}533$$

or 53·3% of the particles remain in suspension

Problem 1.4

In a mixture of quartz, specific gravity 2·65, and galena, specific gravity 7·5, the sizes of the particles range from 0·0052 to 0·025 mm.

On separation in a hydraulic classifier under free settling conditions, three fractions are obtained, one consisting of quartz only, one a mixture of quartz and galena, and one of galena only. What are the ranges of sizes of particles of the two substances in the mixed portion?

Solution

Use is made of equation 3.17, which may be written as:

$$u = kd^2(\rho_s - \rho)$$

For large galena, $u = k \times 0{\cdot}025^2 (7{\cdot}5 - 1{\cdot}0) = 0{\cdot}004\,06k$ mm/s

For small galena, $u = k \times 0{\cdot}0052^2 (7{\cdot}5 - 1{\cdot}0) = 0{\cdot}000\,175k$ mm/s

For large quartz, $u = k \times 0{\cdot}025^2 (2{\cdot}65 - 1{\cdot}0) = 0{\cdot}001\,03k$ mm/s

For small quartz, $u = k \times 0{\cdot}0052^2 (2{\cdot}65 - 1{\cdot}0) = 0{\cdot}000\,045k$ mm/s

If the time of settling was such that particles with a velocity equal to $0{\cdot}001\,03k$ mm/s settled, then the bottom product would contain quartz. This is not so and hence the maximum size of galena particles still in suspension is given by:

$$0.001\,03k = kd^2(7.5 - 1.0)$$

$$d = 0.0126\text{ mm}$$

Similarly if the time of settling was such that particles with a velocity equal to $0.000\,175k$ mm/s did not start to settle, then the top product would contain galena. This is not the case and hence the minimum size of quartz in suspension is given by:

$$0.000\,175k = kd^2(2.65 - 1.0)$$

or

$$d = 0.0103\text{ mm}$$

It may therefore be concluded that, assuming streamline conditions, the fraction of material in suspension, that is containing quartz *and* galena, is made up of particles of sizes in the range:

$$\underline{\underline{0.0103\text{–}0.0126\text{ mm}}}$$

Problem 1.5

It is desired to separate into two pure fractions a mixture of quartz and galena of a size range from 0.015 mm to 0.065 mm by the use of a hindered settling process. What is the minimum apparent density of the fluid that will give this separation? How will the viscosity of the bed affect the minimum required density?

(Density of galena = 7500 kg/m^3, density of quartz = 2650 kg/m^3.)

Solution

Assuming that the shapes of the galena and quartz particles are similar, then from equation 1.31, the required density of fluid for viscous conditions is given by:

$$(0.065/0.015) = [(7500 - \rho)/(2650 - \rho)]^{0.5}$$

or

$$\rho = 2377\text{ kg/m}^3$$

From equation 1.33, the required density for fully turbulent conditions is given by:

$$(0.065/0.015) = (7500 - \rho)/(2650 - \rho)$$

and

$$\rho = 1196\text{ kg/m}^3$$

Thus the minimum density of the fluid to effect the separation is $\underline{\underline{1196\text{ kg/m}^3}}$. This assumes that fully turbulent conditions prevail. As the viscosity is increased, the value of the Reynolds group will decrease and the required density will rise to the value of $\underline{\underline{2377\text{ kg/m}^3}}$ necessary for viscous conditions.

Problem 1.6

Write a short essay explaining the circumstances in which a particle size distribution would be determined by microscopical measurement or by sedimentation in a liquid. State the characteristics of these two methods of measurement.

The following table gives the size distribution of a dust as measured by the microscope. Convert these figures to obtain the distribution on a weight basis, and calculate the specific surface, assuming spherical particles of specific gravity 2·65.

Size range in μm	Number of particles in range
0–2	2000
2–4	600
4–8	140
8–12	40
12–16	15
16–20	5
20–24	2

Solution

The determination of a particle size distribution by microscopic and sedimentation techniques is discussed in section 1.2.2 to which reference should be made. In simple terms, sedimentation would be adopted for very small particles which are too fine for classification by either sieving or visual examination.

From equation 1.4, the mass fraction of particles of size d_1 is given by:

$$x_1 = n_1 k_1 d_1^3 \rho_s$$

where k_1 is a constant, n_1 the number of particles of size d_1, and ρ_s the density of the particles = 2650 kg/m^3.

$\Sigma x_1 = 1$ and hence the weight fraction is

$$x_1 = n_1 k_1 d_1^3 \rho_s / \Sigma n k d^3 \rho_s$$

In this case:

d	n	$kd^3 n\rho_s$	x
1	2000	5,300,000k	0·011
3	600	42,930,000k	0·090
6	140	80,136,000k	0·168
10	40	106,000,000k	0·222
14	15	109,074,000k	0·229
18	5	77,274,000k	0·162
22	2	56,434,400k	0·118
		Σ = 477,148,400k	Σ = 1·0

The surface mean diameter is given by equation 1.14:

$$d_s = \Sigma(n_1 d_1^3)/\Sigma(n_1 d_1^2)$$

The working is as follows:

d	n	nd^2	nd^3
1	2000	2000	2,000
3	600	5400	16,200
6	140	5040	30,240
10	40	4000	40,000
14	15	2940	41,160
18	5	1620	29,160
22	2	968	21,296
		$\Sigma = 21{,}968$	$\Sigma = 180{,}056$

$$d_s = 180{,}056/21{,}968 = 8{\cdot}20\,\mu\text{m}$$

This is the size of particle with the same specific surface as the mixture.

Volume of a particle $8{\cdot}20\,\mu$m diameter $= (\pi/6) \times 8{\cdot}20^3 = 288{\cdot}7\,\mu\text{m}^3$.

Surface area of a particle of $8{\cdot}20\,\mu$m diameter $= \pi \times 8{\cdot}20^2 = 211{\cdot}2\,\mu\text{m}^2$ and hence specific surface $= 211{\cdot}2/288{\cdot}7 = 0{\cdot}731\ \mu\text{m}^2/\mu\text{m}^3$

or $$\underline{\underline{0{\cdot}731 \times 10^6\ \text{m}^2/\text{m}^3}}$$

Problem 1.7

The performance of a solids mixer has been assessed by calculating the variance occurring in the weight fraction of a component amongst a selection of samples withdrawn from the mixture. The quality was tested at intervals of 30 s and the results are:

Sample variance	0·025	0·006	0·015	0·018	0·019
Mixing time (s)	30	60	90	120	150

If the component analysed is estimated to represent 20% of the mixture by weight and each of the samples removed contained approximately 100 particles, comment on the quality of the mixture produced and present the data in graphical form showing the variation of mixing index with time.

Solution

For a completely unmixed system, $s_0^2 = p(1-p)$ (equation 1.23)

$$= 0{\cdot}20(1 - 0{\cdot}20) = 0{\cdot}16$$

For a completely random mixture, $s_r^2 = p(1-p)/n$ (equation 1.22)

$$= 0{\cdot}20(1 - 0{\cdot}20)/100 = 0{\cdot}0016$$

The degree of mixing M is given by equation 1.24 as:

$$M = (s_0^2 - s^2)/(s_0^2 - s_r^2)$$

In this case, $M = (0{\cdot}16 - s^2)/(0{\cdot}16 - 0{\cdot}0016)$

$= 1{\cdot}01 - 6{\cdot}313s^2$

The calculated data are therefore:

t (s)	30	60	90	120	150
s^2	0·025	0·006	0·015	0·018	0·019
M	0·852	0·972	0·915	0·896	0·890

These data are plotted in Fig. 1c and it is clear that the degree of mixing is a maximum at $t = 60$ s.

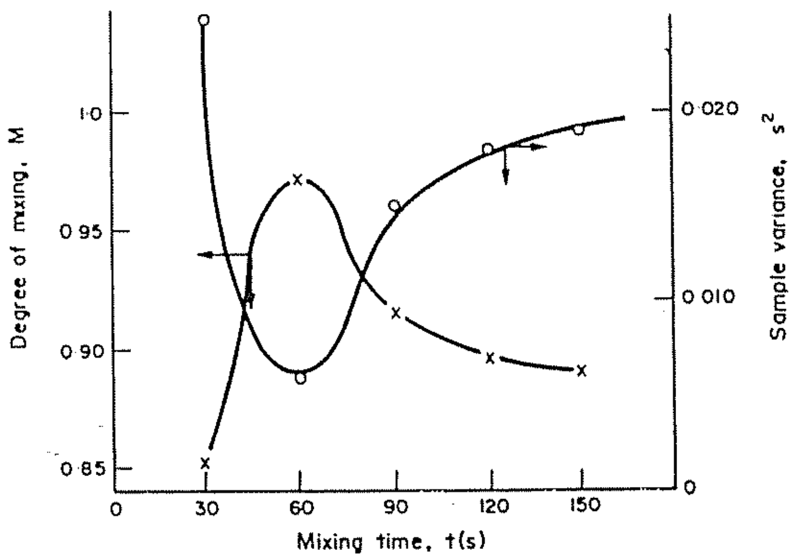

FIG. 1c