ALGA - Fichas Formativas: Resolução

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Conteúdo

Ficha Formativa 3

Questão 1

Q1.1)

(i) A

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ -1 & 1 & 3 & -1 \\ 1 & 0 & 1 & 2 \end{bmatrix} \xrightarrow[\substack{l_3 \mathrel{+}= l_1 \\ l_4 \mathrel{+}= -l_1 \\ l_2 \mathrel{+}= l_4 \\ l_3 \mathrel{+}= 2\, l_4}]{} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 5 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$\therefore \exists\, A^{-1}$

(ii) M

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & -1 & 1 \\ -1 & 4 & 1 \end{bmatrix} \xrightarrow[\substack{l_3 \mathrel{+}= l_2 \\ l_1 \mathrel{+}= -2\, l_2}]{} \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 1 \\ 0 & 3 & 2 \end{bmatrix}$$

$\therefore \nexists\, M^{-1}$

(iii) N

$$N \in \mathcal{M}_{3\times 4} \therefore \nexists\, N^{-1}$$

(iv) P

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow[l_3 \mathrel{+}= -l_1 l_2 \mathrel{+}= -l_1]{} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\therefore \exists\, P^{-1}$

(v) Q

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \xrightarrow[l_1 \mathrel{+}= -2\, l_2 l_3 \mathrel{+}= -l_2 l_2 \mathrel{+}= -l_3]{} \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

$\therefore \exists\, Q^{-1}$

Q1.2)

Pelas mesmas rasões que a): A, P, Q.

Q1.3)

$$\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \\ \boldsymbol{x_4} \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ \mathbf{3} \\ \mathbf{3} \\ \mathbf{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & | & 1 \\ 0 & 1 & 2 & 0 & | & 3 \\ -1 & 1 & 3 & -1 & | & 3 \\ 1 & 0 & 1 & 2 & | & 2 \end{bmatrix} \xrightarrow[\substack{l_3 \mathrel{+}= l_1 \\ l_4 \mathrel{+}= -l_1 \\ l_2 \mathrel{+}= l_4 \\ l_3 \mathrel{+}= 2\,l_4 \\ l_3 \mathrel{+}= -2\,l_2 \\ l_2 \mathrel{+}= 3\,l_3 \\ l_4 \mathrel{+}= l_3 - l_2 \\ l_1 \mathrel{+}= -l_2 + l_4}]{} \begin{bmatrix} 1 & 0 & 0 & 0 & | & 6 \\ 0 & 0 & 0 & 1 & | & -2 \\ 0 & 0 & -1 & 0 & | & -2 \\ 0 & -1 & 0 & 0 & | & 3 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 6 \\ -3 \\ 2 \\ -2 \end{bmatrix}$$

Q1.4)

$$\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{1} & \mathbf{1} & \mathbf{3} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{bmatrix} = \begin{bmatrix} \mathbf{2} \\ \mathbf{-1} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & | & 2 \\ 0 & 2 & 1 & 1 & | & -1 \\ -1 & 1 & 1 & 3 & | & 0 \end{bmatrix} \xrightarrow[\substack{l_3 += l_1 \\ l_3 += -l_2}]{} \begin{bmatrix} 1 & 1 & 0 & 1 & | & 2 \\ 0 & 2 & 1 & 1 & | & -1 \\ 0 & 0 & 0 & 3 & | & 3 \end{bmatrix}$$

$$\implies r(N) = r(N|C) < 4$$

$\therefore$ Sistema possível indeterminado com grau de indeterminação 1

Questão 2

$$\boldsymbol{A_{(\alpha)} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & \alpha \\ 3 & 1 & 2\,\alpha \end{bmatrix} : \alpha \in \mathbb{R}}$$

Q2.1)

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & \alpha \\ 3 & 1 & 2\,\alpha \end{bmatrix} \xrightarrow[\substack{l_3 \mathrel{+}= -2\,l_2 \\ l_3 \mathrel{+}= l_1 \\ l_2 \mathrel{+}= -2\,l_1 + l_3}]{} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & \alpha - 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\therefore \alpha \neq 1$$

Q2.2)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\substack{l_2 \mathrel{+}= -2\,l_1 \\ l_3 \mathrel{+}= -3\,l_1 \\ l_3 \mathrel{+}= -l_2 \\ l_1 \mathrel{+}= -l_3}]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

Questão 3

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & | & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{0} & \boldsymbol{\alpha} & | & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{\beta} & \boldsymbol{\alpha - 2} & | & \boldsymbol{\beta} \\ \mathbf{4} & \boldsymbol{\beta} & \mathbf{0} & \mathbf{4} & | & \boldsymbol{\alpha - 4} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & | & 1 \\ 2 & 1 & 0 & \alpha & | & 2 \\ 0 & 1 & \beta & \alpha - 2 & | & \beta \\ 4 & \beta & 0 & 4 & | & \alpha - 4 \end{bmatrix} \xrightarrow[\substack{l_4 += -4\,l_1 \\ l_3 += -l_2 \\ l_2 += -2\,l_1 \\ l_3 += 2\,l_1 \\ l_3 \to l_3/\beta}]{} \begin{bmatrix} 1 & 0 & 0 & 1 & | & 1 \\ 0 & 1 & 0 & \alpha - 2 & | & 0 \\ 0 & 0 & 1 & 0 & | & 1 \\ 0 & \beta & 0 & 0 & | & \alpha \end{bmatrix}$$

Ficha Formativa 4

Questão 1

Q1.1)

(i) A

$$= -\det\begin{pmatrix}3 & 1 & -1\\ 2 & 5 & 1\\ 2 & 3 & 1\end{pmatrix} = \det\begin{pmatrix}2 & 5\\ 2 & 3\end{pmatrix} + 1\det\begin{pmatrix}3 & 1\\ 2 & 3\end{pmatrix} - 1\det\begin{pmatrix}3 & 1\\ 2 & 5\end{pmatrix} = 6 - 10 + (9$$

(ii) D

$$= \det\begin{pmatrix}3 & 3 & 5\\ 6 & 1 & 2\\ 4 & 1 & 6\end{pmatrix} - \det\begin{pmatrix}3 & 1 & 5\\ 6 & 1 & 2\\ 4 & 1 & 6\end{pmatrix} - 2\det\begin{pmatrix}3 & 1 & 5\\ 3 & 3 & 5\\ 4 & 1 & 6\end{pmatrix} - \det\begin{pmatrix}3 & 1 & 5\\ 3 & 3 & 5\\ 6 & 1 & 2\end{pmatrix} = (3 * (6$$

(iii) E

$$= 7 * 2 * -1 * -2 = 28$$

(iv) C

$$= -\det \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} = -2 \det \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = -2 * 1 = -2$$

Q1.6)

$$= -\det \begin{pmatrix} 4 & 1 & 0 \\ 2 & 5 & 2 \\ 2 & 5 & 2 \end{pmatrix} = 0$$

Questão 2

Q2.1)

$$= 15 * (-1) * (1/5) = -3$$

Q2.2)

$$= -\det\begin{pmatrix}1 & -1 & 0\\ 3 & 1 & 0\\ 0 & 1 & 1\end{pmatrix} = -\det\begin{pmatrix}1 & -1\\ 3 & 1\end{pmatrix} = -(1-(-3)) = -4$$

Q2.3)

$$= \det\begin{pmatrix}3 & -1 & 0\\ 2 & 1 & 0\\ 2 & 1 & 0\end{pmatrix} = 0$$

Q2.4)

$$\alpha = \frac{\det B}{\det A} = \frac{4\ \det B(1|1)}{3\ \det A(1|1)} = 4/3$$

Ficha Formativa 6

Questão 1

$$\left.\begin{array}{l} \boldsymbol{u_1} = (1,1,0,1) \\ \boldsymbol{u_2} = (0,0,1,0) \\ \boldsymbol{u_3} = (1,0,1,0) \end{array}\right\} \in \mathbb{R}^4$$

$$\boldsymbol{\mathcal{B}} = ((0,1,0,0),(0,0,-2,0),(1,1,0,0),(0,0,0,2))$$

$$\boldsymbol{\mathcal{B}'} = ((2,2,-4,1),(0,0,1,0),(0,-1,2,1),(0,0,2,-3))$$

Q1.1)

$$\boldsymbol{y(w) : (x,y,z,w) \in \langle u_1, u_2, u_3 \rangle}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \implies \alpha_3 = y \wedge \alpha_3 = w \implies y = w$$

Q1.2)

$$\boldsymbol{t : (u_1, u_2, u_3, (1, 0, 3, t)) \text{ é base de } \mathbb{R}^4}$$

$$t : r\left(\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 1 & 0 & 0 & t \end{bmatrix}\right) = 4; \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 1 & 0 & 0 & t \end{bmatrix} \xrightarrow[\substack{l_4 += -l_2 \\ l_1 += -l_4/t - l_2 \\ l_3 += -l_1 - l_4\, 3/t}]{} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & t \end{bmatrix} \implies$$
$$\implies t = \mathbb{R} \backslash \{1\}$$

Q1.3)

$$\boldsymbol{v \in \mathbb{R}^4 : \begin{array}{l} (u_1, u_2, u_3, v) \text{ é base de } \mathbb{R}^4 \wedge \\ \wedge \langle u_1, u_2, u_3, v \rangle (3, 0, -1, -1) = (2, -1, 1, -1) \end{array}}$$

$$v = (v_1, v_2, v_3, v_4) \wedge \begin{bmatrix} 0 & 0 & 1 & v_1 \\ 1 & 0 & 0 & v_2 \\ 0 & 1 & 0 & v_3 \\ 0 & 0 & 0 & v_4 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} \implies$$
$$\implies v = \begin{bmatrix} -1 - 2 \\ 3 + 1 \\ -1 \\ 1 \end{bmatrix} = (-3, 4, -1, 1)$$

Q1.4)

$$\boldsymbol{x : \mathcal{B}' v = x \land \mathcal{B} v = (0, 4, 4, 1)}$$

$$x = \mathcal{B}'\mathcal{B}^{-1}(0,4,4,1) = \begin{bmatrix} 2 & 2 & -4 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 2 & -3 \end{bmatrix} \frac{\operatorname{adj}\mathcal{B}}{\det\mathcal{B}} \begin{bmatrix} 0 \\ 4 \\ 4 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 2 & -4 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 2 & -3 \end{bmatrix} \left(\left(1 * (-1)^{1+2} \begin{vmatrix} 0 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{vmatrix} \right)^{-1} \begin{bmatrix} 4 & 0 & -4 & 0 \\ -4 & 0 & 8 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 0 \\ 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} -4/4 & 0 & 4/4 & 0 \\ 4/4 & 0 & -8/4 & 0 \\ 0 & 2/4 & 0 & 0 \\ 0 & 0 & 0 & -2/4 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 4 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 2 & -4 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 2 \\ -2/4 \end{bmatrix} = \begin{bmatrix} -33/2 \\ 2 \\ 23/2 \\ 11/2 \end{bmatrix}$$

Ficha Formativa 7

Questão 1

$$\{F, G\} \text{ subespaços } \mathbb{R}^4 :$$
$$F = \left\{(a, b, c, d) \in \mathbb{R}^4 : a = b + c \wedge d - 2a = 0\right\} \wedge$$
$$G = \langle(1,1,1,1),(1,0,2,3),(0,0,0,1)\rangle$$

$$\begin{aligned}
S_1 &= ((1,1,1,1)) \\
S_2 &= ((0,1,-1,0)) \\
S_3 &= ((1,1,0,2),(1,0,1,2)) \\
S_4 &= ((0,-1,1,0),(1,2,-1,2)), \\
S_5 &= ((1,1,0,0),(1,0,1,0),(0,0,0,2)) \\
S_6 &= ((1,1,0,2),(1,0,1,2),(1,2,-1,2)) \\
S_7 &= ((1,0,2,3),(1,1,1,1),(0,0,0,1)) \\
S_8 &= ((1,1,0,2),(1,0,1,2),(1,0,2,3),(0,0,0,1)) \\
S_9 &= ((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)) \\
S_{10} &= ((1,1,0,2),(1,0,1,2),(1,0,2,3),(1,1,1,1),(0,0,0,1))
\end{aligned}$$

Q1.1)

$$i : S_i \text{ gera } F \wedge F = \left\{(a,b,c,d) \in \mathbb{R}^4 : a = b + c \wedge d - 2a = 0\right\} =$$
$$= \{(b+c, b, c, 2\,a) : \{a,b,c\} \in \mathbb{R}\} =$$
$$= \langle(0,0,0,2),(1,1,0,0),(1,0,1,0),(0,0,0,0)\rangle =$$
$$= \langle(1,1,0,0),(1,0,1,0),(0,0,0,2)\rangle = \langle S_5 \rangle$$
$$\therefore i = \{5\}$$

Questão 2

$$i : S_i \text{ base de } F \wedge F \leq \langle(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\rangle \wedge$$
$$\wedge\, F \leq \langle(1,1,0,0),(1,0,1,0),(0,0,0,2)\rangle \wedge$$
$$\wedge\, F \leq \langle(1,1,0,0),(1,0,1,0),(0,0,0,2)\rangle$$