AM 2C – Exame 3: Resolução

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Conteúdo

Grupo I

Questão 1

Parabola→Vertice, foco e diretriz

$$\boldsymbol{x = 2 - y - y^2/4}$$

$$x = \frac{(y - y')^2}{4\,a} + x' \begin{cases} a = (4 * (-1/4))^{-1} = -1 \\ y' = -(-1) * 2 * (-1) = -2 \\ x' = 2 - \frac{(-2)^2}{4*(-1)} = 3 \end{cases}$$

$$\therefore x = \frac{(y + 2)^2}{-4} + 3 \begin{cases} X' = (3, -2) \\ F = (x' + a, y') = (3 - 1, -2) = (2, -2) \\ L \subset \mathbb{R}^2 : x = x' - a = 3 + 1 = 4 \end{cases}$$

Questão 2

Seja $f : \mathbb{R}^2 \to \mathbb{R}$ uma norma.

b) $f(\lambda\, x) = \lambda\, f(x)$

Questão 3

Considere o sistema de equações

$$\begin{cases} \log(x\,u^2) - y + 2\,v = 0 \\ x\,e^v - y\,v^2 + u = 0 \end{cases}$$

Define u e v como funções de x e y na viz de $P_0 = (x_0, y_0, u_0, v_0) = (1, 0, -1, 0)$

$$\frac{\frac{\partial f_1}{\partial x}}{\frac{\partial f_1}{\partial u}} = \frac{\frac{(u^2)}{x\,u^2}\log(e)}{\frac{x\,2\,u}{x\,u^2}\log(e)} = \frac{\log(e)}{-2\log(e)} = -1/2$$
$$\frac{\frac{\partial f_1}{\partial x}}{\frac{\partial f_1}{\partial v}} = \frac{\frac{(u^2)}{x\,u^2}\log(e)}{2} = \frac{\log(e)}{2}$$

$$\frac{\partial f_1}{\partial x}(P_0) = \frac{\left(u^2 + x\,2\,u\,\frac{\partial u}{\partial x}\right)}{x\,u^2}\log(e) + 2\,\frac{\partial v}{\partial x} = \frac{u_0^2\,\log(e)}{x_0\,u_0^2} + \frac{x_0\,2\,u_0\,\frac{\partial u}{\partial x}\,\log(e)}{x_0\,u_0^2} + 2\,\frac{\partial v}{\partial x} =$$
$$= \frac{\log(e)}{1} + \frac{2\,\frac{\partial u}{\partial x}\,\log(e)}{(-1)} + 2\,\frac{\partial v}{\partial x} = \log(e) - 2\,\frac{\partial u}{\partial x}\,\log(e) + 2\,\frac{\partial v}{\partial x} = 0$$

$$\frac{\partial f_2}{\partial x}(P_0) = e^{v_0} + x_0\,e^{v_0}\,\frac{\partial v}{\partial x} - y_0\,2\,v_0\,\frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} = 1 + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} = 0 \implies$$

$$\implies \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}\log(e)^{-1} + 1/2 = \left(-\frac{\partial u}{\partial x} - 1\right)\log(e)^{-1} + 1/2 =$$
$$= -\frac{\partial u}{\partial x}\log(e)^{-1} - \log(e)^{-1}1/2 \implies \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} = \frac{-\log(e)^{-1}1/2}{\log(e)^{-1} + 1}$$

Questão 4

Seja $g(s,t) = f(u,v)$ em q f é dif e $u = s^2 - t^2, v = t^2 - s^2$. Sabendo q g satisfaz a eq

$$\boldsymbol{(t+2)\frac{\partial g}{\partial s} + (s+2)\frac{\partial g}{\partial t} = h(s,t)\left(\frac{\partial f}{\partial u} - \frac{\partial f}{\partial v}\right)}$$

Questão 5

Considere o conjunto

$$\boldsymbol{A = \{(x,y) \in \mathbb{R}^2 : y \geq x \land x \geq y^2\}}$$

Seja L a fronteira de A percorrida no sentido +. O integral de linha

$$\boldsymbol{\int_L (x^3 - 2\,y)\,\mathrm{d}x + (2\,x - y^3)\,\mathrm{d}y}$$

Pode ser calc usando coord polares

$$\begin{cases} P_1 = (0,0) \\ P_2 : y = x = y^2 \implies P_2 = (1,1) \end{cases}$$

$$\begin{cases} P_1 = (0,0) \\ P_2 = (\sqrt{1+1}, \arccos(1/\sqrt{2})) = (\sqrt{2}, \pi/4) \end{cases}$$

$$\det \mathrm{J} = \rho$$

$$x = y^2 \implies \rho\cos\theta = \rho^2\sin^2\theta \implies \rho = \frac{\cos\theta}{\sin^2\theta}$$

$$\therefore \int_L (x^3 - 2\,y)\,\mathrm{d}x + (2\,x - y^3)\,\mathrm{d}y = \iint_A 4\,\mathrm{d}x\,\mathrm{d}y = \int_{\pi/4}^{\pi/2}\int_0^{\frac{\cot(\theta)}{\sin\theta}} 4\rho\,\mathrm{d}\rho\,\mathrm{d}\theta$$

Questão 6

plano tg ao cone elip $x^2 + 4\,y^2 = z^2$ no p (3,2,5)

$$
\begin{aligned}
&(x)(2\,x') + (y)(8\,y') - (z)(2\,z') = \\
&= 2\,x'\,x + 8\,y'\,y - 2\,z'\,z - 2\,x'^2 - 8\,y'^2 + 2\,z'^2 = \\
&= 6\,x + 16\,y - 10\,z = \\
&= 6\,x + 16\,y - 10\,z = 0 \implies \\
&\implies 3\,x + 8\,y - 5\,z = 0
\end{aligned}
$$

Questão 7

O integral repetido

$$\int_{-2}^{0}\int_{x}^{0} x^2 \ \mathrm{d}x + \int_{0}^{2}\int_{0}^{x} x^2 \ \mathrm{d}x$$

$\displaystyle\int_{-2}^{0}\int_{-2}^{y} x^2 \,\mathrm{d}x\,\mathrm{d}y + \int_{0}^{2}\int_{y}^{2} x^2 \,\mathrm{d}x\,\mathrm{d}y$

Grupo II

Questão 1

Considere a função real g, continua de 2 var

$$g(x,y) = \begin{cases} \frac{x^4+y^4}{x^2+y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}$$

Q1 a.

Determine $\frac{\partial g}{\partial x}(x, y), \forall\,(x, y) \in \mathbb{R}^2$

$$\frac{\partial g}{\partial x}(x, y) = \frac{4\,x^3(x^2 + y^2) - 2\,x(x^4 + y^4)}{(x^2 + y^2)^2}$$

Q1 b.

Estude a continuidade de $\frac{\partial g}{\partial x}(x, y)$ em (0,0)

$$\forall\, \delta > 0\, \exists\, \varepsilon > 0 : \left(\left(\forall (x, y) \neq (0, 0) \land \left\| \sqrt{x^2 + y^2} \right\| < \varepsilon \right) \implies |g(x, y) - 0| < \delta \right) \implies$$

$$\implies \left| \frac{x^4 + y^4}{x^2 + y^2} \right| \leq \frac{x^4 + 2\, x^2\, y^2 + y^4}{x^2 + y^2} = \frac{(x^2 + y^2)^2}{x^2 + y^2} = x^2 + y^2 \leq \varepsilon^2 = \delta$$

$$\therefore \varepsilon = \sqrt{\delta}$$

Q1 c.

Estude a dif de g em (0,0)

$$\frac{\partial g}{\partial x}(0,0) = \lim_{h\to 0} \frac{g(h,0)-g(0,0)}{h} = \lim_{h\to 0} \frac{g(h,0)-g(0,0)}{h} =$$
$$= \lim_{h\to 0} \frac{h^2}{h} = \lim_{h\to 0} h = 0 = \frac{\partial g}{\partial y}(0,0)$$

$$g(a,b)-g(0,0) = \frac{a^4+b^4}{a^2+b^2} - 0 =$$
$$= \frac{\partial g}{\partial x}(0,0)\,a + \frac{\partial g}{\partial y}(0,0)\,b + \varepsilon(a,b)\sqrt{a^2+b^2} = \varepsilon(a,b)\sqrt{a^2+b^2} \implies$$
$$\implies \varepsilon(a,b) = \frac{a^4+b^4}{(a^2+b^2)^2} = \frac{a^4+b^4}{a^4+2\,a^2\,b^2+b^4} \implies$$
$$\implies \lim_{a\to 0^+} \varepsilon(a,a) = \lim_{a\to 0^+} \frac{a^4+a^4}{(a^2+a^2)^2} = \lim_{a\to 0^+} \frac{2\,a^4}{4\,a^4} = 1/2 \neq 0$$

Questão 2

Considere a função $f : \mathbb{R}^2 \to \mathbb{R}$ definida por

$$f(x, y) = 2\,x^3 + x\,y^2 + 2\,x\,y$$

Q2 a.

Determine os extremos locais de f

$$\left\{(x,y)\in\mathbb{R}^2 : \begin{pmatrix} \dfrac{\partial f}{\partial x} = 6\,x + y^2 + 2\,y = 0 \\ \dfrac{\partial f}{\partial y} = 2\,x\,y + 2\,x = 0 \end{pmatrix}\right\} =$$

$$= \left\{(x,y)\in\mathbb{R}^2 : \begin{pmatrix} x = -\dfrac{y(y+2)}{6} \\ 2\,x\,(y+1) = 0 \\ y(y+2)(y+1) = 0 \end{pmatrix}\right\} =$$

$$= \{(0,0),(1/6,-1),(0,-2)\}$$

$$\det \mathrm{H}(f(x,y)) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x\,\partial y} \\ \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y\,\partial x} \end{vmatrix} = \begin{vmatrix} 6 & 2\,y+2 \\ 2\,x & 2\,y+2 \end{vmatrix} = (12-4\,x)(y+1)$$

$$\begin{cases} \det \mathrm{H}(f(0,0)) = 12 \wedge \frac{\partial^2 f}{\partial x^2} = 6 & \therefore \text{ minimo local} \\ \det \mathrm{H}(f(1/6,-1)) = 0 & \therefore \text{ indeterminado} \\ \det \mathrm{H}(f(0,-2)) = -12 & \therefore \text{ ponto de cela} \end{cases}$$

Q2 b.

Extremos locais restrita a

$$\left\{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{\mathbf{2}} : \boldsymbol{x} - \boldsymbol{y} = \mathbf{1}\right\}$$

e com x,y verificando $|x|\,, |y| < 4$

Grupo III

Questão 1

Calcule o integral de linha

$$\int_L 2\, y\, e^{z^2}\, \mathrm{d}x + (x^2 + y - z)\, \mathrm{d}y + (y + z)\, \mathrm{d}z$$