AM 1 - PO
Resolução Ficha 3 e 4

Felipe Pinto - 61387

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Contents

Part I

Ficha 3

7 - d) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^2+k}{n^3+1}$ incompleto

$$\implies \lim_{n\to\infty} n\frac{n^2+1}{n^3+1} \leq \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^2+k}{n^3+1} \leq \lim_{n\to\infty} n\frac{n^2+n}{n^3+1} \implies$$

$$\implies \lim_{n\to\infty} n\frac{1/n+1/n^2}{1+1/n^2} \leq \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^2+k}{n^3+1} \leq \lim_{n\to\infty} n\frac{1/n^2+1/n^3}{1+1/n^4}$$

Questão 8

8 - a) $\lim_{n\to\infty} \left(1+\frac{1}{(2\,n+(-1)^n)}\right)^n$

$$\implies \lim_{n\to\infty} \left(1+\frac{1}{(2\,n+1)}\right)^n \leq \lim_{n\to\infty} \left(1+\frac{1}{(2\,n+(-1)^n)}\right)^n \leq$$

$$\leq \lim_{n\to\infty} \left(1+\frac{1}{(2\,n-1)}\right)^n \implies \lim_{n\to\infty} \left(\left(1+\frac{1}{(2\,n+1)}\right)^{2\,n+1}\right)^{\frac{n}{2\,n+1}} \leq$$

$$\leq \lim_{n\to\infty} \left(1+\frac{1}{(2\,n+(-1)^n)}\right)^n \leq \lim_{n\to\infty} \left(\left(1+\frac{1}{(2\,n-1)}\right)^{2\,n-1}\right)^{\frac{n}{2\,n-1}} \implies$$

$$\implies \lim_{n\to\infty} \left(1+\frac{1}{(2\,n+(-1)^n)}\right)^n = \sqrt{e}$$

8 - b) $\lim_{n\to\infty}\left(1+\frac{2+(-1)^n}{n}\right)^{\sqrt{n}}$

$$\implies \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}} \leq \lim_{n\to\infty}\left(1+\frac{2+(-1)^n}{n}\right)^{\sqrt{n}} \leq \lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{\sqrt{n}} \implies$$
$$\implies \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^{\sqrt{n}/n} \leq \lim_{n\to\infty}\left(1+\frac{2+(-1)^n}{n}\right)^{\sqrt{n}} \leq$$
$$\leq \lim_{n\to\infty}\left(\left(1+\frac{3}{n}\right)^n\right)^{\sqrt{n}/n} \implies (e^3)^0 \leq \lim_{n\to\infty}\left(1+\frac{2+(-1)^n}{n}\right)^{\sqrt{n}} \leq (e^1)^0 \implies$$
$$\implies \lim_{n\to\infty}\left(1+\frac{2+(-1)^n}{n}\right)^{\sqrt{n}} = 1$$

Part II

Ficha 4

Questão 1 Determine os sublimites das seguintes sucessões limitadas indicando os seus limites superior e inferior

1 - a) $\cos(n\,\pi/4)$

$\cos(n\,\pi/4) \in \{-1, -\sqrt{2}/2, 0, \sqrt{2}/2, 1\}\ \forall\, n \in \mathbb{N};$
$\overline{\lim}\, a_n = 1;\ \underline{\lim}\, a_n = -1$

1 - b) $(1+(-1)^n/n)^n$

$\text{sublimites} = \{e, e^{-1}\};$
$\overline{\lim}\, b_n = e;\ \underline{\lim}\, b_n = e^{-1}$

1 - c) $n\,\sin\left(\frac{1+(-1)^n}{n}\right)$ Incompleto

$$n\,\sin\left(\frac{1+(-1)^n}{n}\right) = \begin{cases} n\,\sin(2/n) & \forall\, n \text{ par} \\ n\,\sin(0/n) & \forall\, n \text{ impar} \end{cases};$$

$$\lim_{n\to\infty} n\,\sin(0) = 0;$$

$$\lim_{n\to\infty} n\,\sin 2 = \lim_{n\to\infty} \frac{2/n}{2/n} 2 = 2 \implies \mathit{sublim} = \{0, 2\}$$

...

1 - d) $\arctan((-1)^n\, n)$

$$\arctan((-1)^n\, n) = \begin{cases} \arctan(n) & \forall\, n \text{ par} \\ \arctan(-n) & \forall\, n \text{ impar} \end{cases} \implies$$

$$\implies \begin{cases} \lim_{n\to\infty} \arctan(n) = \pi/2 \\ \lim_{n\to\infty} \arctan(-n) = -\pi/2 \end{cases} \implies$$

$$\implies \text{sublim}(e_n) = \{-\pi/2, \pi/2\};\; \overline{\lim}\, e_n = \pi/2;\; \underline{\lim}\, e_n = -\pi/2$$