

## Ficha 5 – Soluções

2.

- a.  $-3$ .
- b. Não existe.
- c. Não existe.
- d.  $0$ .
- e. Não existe.
- f.  $0$ .
- g.  $0$ .
- h.  $0$ .

3.

- a. Contínua em  $\mathbb{R}^2 \setminus \{(0,0)\}$ .
- b. Contínua em  $\mathbb{R}^2$ .
- c. Contínua em  $\mathbb{R}^2$ .
- d. Contínua em  $\mathbb{R}^2 \setminus \{(0,0)\}$ .

4.

- a.  $\frac{\partial f}{\partial x}(x, y) = e^x \sin(y) - \sin(x - 3y); \frac{\partial f}{\partial y}(x, y) = e^x \cos(y) + 3 \sin(x - 3y)$ .
- b.  $\sin(1)x - (e + 3 \sin(1))y + z = \cos(1) + \sin(1)$ .
- c.  $(e^x \sin(y) - \sin(x - 3y), e^x \cos(y) + 3 \sin(x - 3y))$ .
- d.  $\frac{e+2 \sin(1)}{\sqrt{2}}$

5.  $u \cos(x) \cos(y) - v \sin(x) \sin(y)$ .

6.

- a. Ambas as funções são contínuas em  $(0,0)$ .
- b.  $\frac{\partial f}{\partial x}(0,0) = 1, \frac{\partial f}{\partial y}(0,0) = 0, \frac{\partial g}{\partial x}(0,0) = 0$ . Não existe derivada parcial de  $g$  em  $(0,0)$ .

7.

- a. Contínua em  $(1,0)$ .
- b. Não existe  $D_{\vec{u}}f(1,0)$ .

8.

- a. Contínua em  $(0,0)$ .
- b.  $\frac{\partial f}{\partial x}(0,0) = 0, \frac{\partial f}{\partial y}(0,0) = 0$ .

c.  $D_{\vec{u}}f(0,0) = \frac{6}{5\sqrt{5}}$ . A função  $f$  não é diferenciável em  $(0,0)$ .

9.

a.  $\frac{\partial f}{\partial x}(0,0) = 0, \frac{\partial f}{\partial y}(0,0) = 0$ .

b.  $D_{\vec{u}}f(0,0) = \sqrt{2}$ . A função  $f$  não é diferenciável em  $(0,0)$ .

11.

a. 
$$\begin{bmatrix} \frac{1}{2\sqrt{2}} & -\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{\frac{3}{2}}}{2} & \frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\sqrt{2} \end{bmatrix}.$$

b.  $-\rho^2 \sin(\varphi)$ .

12.

a.  $r$ .

b.  $\frac{\partial u}{\partial r}\left(1, \frac{\pi}{4}, 0\right) = \frac{1}{\sqrt{2}} + \sqrt{2}, \frac{\partial u}{\partial \theta}\left(1, \frac{\pi}{4}, 0\right) = -\frac{1}{\sqrt{2}} + \sqrt{2}.$

c. 
$$\frac{\partial^2 u}{\partial r \partial \theta}(r, \theta, z) = \cos(\theta) \frac{\partial f}{\partial y}(r \cos(\theta), r \sin(\theta), z) - \sin(\theta) \frac{\partial f}{\partial x}(r \cos(\theta), r \sin(\theta), z) +$$
  

$$r \cos(2\theta) \frac{\partial^2 f}{\partial y \partial x}(r \cos(\theta), r \sin(\theta), z) + \frac{r}{2} \sin(2\theta) \left( \frac{\partial^2 f}{\partial y^2}(r \cos(\theta), r \sin(\theta), z) - \right.$$
  

$$\left. \frac{\partial^2 f}{\partial x^2}(r \cos(\theta), r \sin(\theta), z) \right),$$

$$\frac{\partial^2 u}{\partial r^2}(r, \theta, z) = \cos^2(\theta) \frac{\partial^2 f}{\partial x^2}(r \cos(\theta), r \sin(\theta), z) + \sin^2(\theta) \frac{\partial^2 f}{\partial y^2}(r \cos(\theta), r \sin(\theta), z) +$$
  

$$\sin(2\theta) \frac{\partial^2 f}{\partial y \partial x}(r \cos(\theta), r \sin(\theta), z).$$

15.  $\frac{\partial w}{\partial v}(u, v) = \frac{\partial f}{\partial x}(u + v, u \sin(v), v) + u \cos(v) \frac{\partial f}{\partial y}(u + v, u \sin(v), v) + \frac{\partial f}{\partial z}(u +$   
 $v, u \sin(v), v),$

$$\frac{\partial^2 w}{\partial u^2}(u, v) = \frac{\partial^2 f}{\partial x^2}(u + v, u \sin(v), v) + 2 \sin(v) \frac{\partial^2 f}{\partial y \partial x}(u + v, u \sin(v), v) +$$
  

$$\sin^2(v) \frac{\partial^2 f}{\partial y^2}(u + v, u \sin(v), v).$$

16.

a.  $\frac{\partial \varphi}{\partial \theta}(r, \theta) = r \left( -e^{r^3 \cos^3(\theta)} \sin(\theta) + 3r \cos(\theta)(1 + \sin(\theta)) \right).$

b.  $\frac{\partial \varphi}{\partial \theta}\left(2, \frac{\pi}{3}\right) = 6 + (3 - e)\sqrt{3}.$

17. 5,04.

18.

a. 0,93.

b. 2.

c. 1,1.

19.

a.  $z = 1$ .

b.  $x + y + z = \sqrt{3}$ .

20.  $\frac{dy}{dx}(1) = 1$ .

21.

a.  $\frac{dy}{dx}(1) = -\frac{7}{5}$ .

b.  $\frac{dy}{dx}(1) = -\frac{\pi}{2}$ .

22.

b.  $\frac{24+8\pi+\pi^2}{2\sqrt{2}(4+\pi)}$ .

23.

b.  $\frac{\partial \psi}{\partial y}(1,1) = \frac{1}{2}, \frac{\partial \psi}{\partial z}(1,1) = 0$ .

24.

b.  $\frac{\partial \psi}{\partial y}(1,1) = -1, \frac{\partial \psi}{\partial z}(1,1) = -1$ .

25.

a.  $\frac{\partial u}{\partial x}(x,y) = y \cos(xy) - \frac{1}{2}y \sin\left(\frac{x}{2}\right) \frac{\partial \varphi}{\partial v}\left(y \cos\left(\frac{x}{2}\right), \cos(xy)\right) - y \sin(xy) \frac{\partial \varphi}{\partial w}\left(y \cos\left(\frac{x}{2}\right), \cos(xy)\right)$ .

b.  $\frac{\partial u}{\partial x}\left(\frac{\pi}{3}, 2\right) = -2 + \frac{1}{\sqrt{3}}$ .

26.

a. A função não é diferenciável em  $(0,0)$ .

b. A função não é diferenciável em  $(0,0)$ .

c. A função é diferenciável em  $(0,0)$ .

27.

a.  $\frac{\partial f}{\partial x}(0,0) = 0, \frac{\partial f}{\partial y}(0,0) = 0$ .

b. A função é diferenciável em  $(0,0)$ .

28.  $(\pi + 1)x - z = -3$ .

30.

a.  $\frac{\partial H}{\partial x}(x, y, z) = \frac{x \cos(\sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}} \frac{\partial f}{\partial u} \left( \sin(\sqrt{x^2 + y^2}), \frac{1}{2}z \right),$

$$\frac{\partial H}{\partial y}(x, y, z) = \frac{y \cos(\sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}} \frac{\partial f}{\partial u} \left( \sin(\sqrt{x^2 + y^2}), \frac{1}{2}z \right),$$

$$\frac{\partial H}{\partial z}(x, y, z) = \frac{1}{2} \frac{\partial f}{\partial v} \left( \sin(\sqrt{x^2 + y^2}), \frac{1}{2}z \right).$$

b.  $\nabla H \left( \frac{\pi}{2}, 0, 2 \right) = \left( 0, 0, -\frac{1}{2} \right).$

34.  $3x + 8y - 5z = 0$ .

35.  $\{(0, 2, 0), (0, -2, 0)\}.$