AM 1 - Ficha 7
Diferenciabilidade

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Conteúdo

Questão 1

1 - a)

$$\boldsymbol{f_{1\,(x)} = \begin{cases} 0 & \text{se } x < 0 \\ x^2 & \text{se } x \geq 0 \end{cases}}$$

$f'_{1\,(x)} = \lim_{x \to 0^-} \dfrac{f'_{1\,(x)} - f_{1\,(0)}}{x - 0} = \lim_{x \to 0^-} \dfrac{0 - 0}{x} = 0 = \lim_{x \to 0^+} \dfrac{f'_{1\,(x)} - f_{1\,(0)}}{x - 0} = \lim_{x \to 0^+} \dfrac{x^2 - 0}{x} = 0$
$\therefore f'_{1\,(x)}$ é diferenciavel em 0

1 - b)

$$\boldsymbol{f_{2\,(x)} = \begin{cases} \sin(2\,x) & \text{se } x < 0 \\ e^{2\,x} - 1 & \text{se } x \geq 0 \end{cases}}$$

$f'_{2\,(x)} = \lim_{x \to 0^-} \dfrac{f'_{2\,(x)} - f_{2\,(0)}}{x - 0} = \lim_{x \to 0^-} \dfrac{\sin(2\,x) - 0}{x} = 2 = \lim_{x \to 0^+} \dfrac{f'_{2\,(x)} - f_{2\,(0)}}{x - 0} = \lim_{x \to 0^+} \dfrac{e^{2\,x} - 1 - 0}{x} = 2$
$\therefore f'_{2\,(x)}$ é diferenciavel em 0

1 - c)

$$\boldsymbol{f_{3\,(x)} = \begin{cases} |x|^{1.5} \sin(1/x) & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}}$$

$f'_{3\,(x)} = \lim_{x \to 0^-} \dfrac{f'_{3\,(x)} - f_{3\,(0)}}{x - 0} = \lim_{x \to 0^-} \dfrac{|x|^{1.5} \sin(1/x) - 0}{x} = \lim_{x \to 0^-} -|x|^{0.5} \sin(1/x) = 0 =$
$= \lim_{x \to 0^+} \dfrac{f'_{3\,(x)} - f_{3\,(0)}}{x - 0} = \lim_{x \to 0^+} \dfrac{|x|^{1.5} \sin(1/x) - 0}{x} = \lim_{x \to 0^+} |x|^{0.5} \sin(1/x) = 0$
$\therefore f'_{3\,(x)}$ é diferenciavel em 0

1 - d)

$$\boldsymbol{f_{4\,(x)} = \begin{cases} e^{-1/x^2} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}}$$

$f'_{4\,(x)} = \lim_{x \to 0^-} \dfrac{f'_{4\,(x)} - f_{4\,(0)}}{x - 0} = \lim_{x \to 0^-} \dfrac{e^{-1/x^2} - 0}{x} = \lim_{x \to 0^-} \dfrac{1/x}{e^{-(1/x)^2}} = 0 =$
$= \lim_{x \to 0^+} \dfrac{f'_{4\,(x)} - f_{4\,(0)}}{x - 0} = \lim_{x \to 0^+} \dfrac{e^{-1/x^2} - 0}{x} = \lim_{x \to 0^-} \dfrac{1/x}{e^{-(1/x)^2}} = 0$
$\therefore f'_{4\,(x)}$ é diferenciavel em 0

Questão 2

2 - a)

$$\boldsymbol{g_{1\,(x)} = \begin{cases} 0 & \text{se } x < 0 \\ x & \text{se } x \geq 0 \end{cases}}$$

$$g'_{1\,(x)} = \lim_{x\to 0^-} \frac{g'_{1\,(x)} - g_{1\,(0)}}{x-0} = \lim_{x\to 0^-} \frac{0-0}{x} = 0 = \lim_{x\to 0^+} \frac{g'_{1\,(x)} - g_{1\,(0)}}{x-0} = \lim_{x\to 0^+} \frac{x-0}{x} = 1$$

$\therefore g'_{1\,(x)}$ não é diferenciavel em 0

2 - b)

$$\boldsymbol{g_{2\,(x)} = \begin{cases} \sin(x) & \text{se } x < 0 \\ 1 - \cos(x) & \text{se } x \geq 0 \end{cases}}$$

$$g'_{2\,(x)} = \lim_{x\to 0^-} \frac{g'_{2\,(x)} - g_{2\,(0)}}{x-0} = \lim_{x\to 0^-} \frac{\sin(x) - 1 - \cos(0)}{x} = \lim_{x\to 0^-} \frac{\sin(x)}{x} = 1 =$$

$$= \lim_{x\to 0^+} \frac{g'_{2\,(x)} - g_{2\,(0)}}{x-0} = \lim_{x\to 0^+} \frac{1 - \cos(x) - 1 - \cos(0)}{x} = \lim_{x\to 0^+} \frac{1 - \cos^2(x)}{x(1+\cos(x))} =$$

$$= \lim_{x\to 0^+} \frac{\sin^2(x)}{x^2} * \lim_{x\to 0^+} \frac{x}{1+\cos(x)} = 0$$

$\therefore g'_{2\,(x)}$ não é diferenciavel em 0

2 - c)

$$\boldsymbol{g_{3\,(x)} = \sqrt{\sin(x)\,x} \quad x \in [-\pi, \pi]}$$

$$g'_{3\,(x)} = \lim_{x\to 0^-} \frac{g'_{3\,(x)} - g_{3\,(0)}}{x-0} = \lim_{x\to 0^-} \frac{\sqrt{\sin(x)\,x} - \sqrt{\sin(0)\,0}}{x} = \lim_{x\to 0^-} -\sqrt{\sin(x)\,x/x^2} = -1 =$$

$$= \lim_{x\to 0^+} \frac{g'_{3\,(x)} - g_{3\,(0)}}{x-0} = \lim_{x\to 0^+} \frac{\sqrt{\sin(x)\,x} - \sqrt{\sin(0)\,0}}{x} = \lim_{x\to 0^+} \sqrt{\sin(x)\,x/x^2} = 1$$

$\therefore g'_{3\,(x)}$ não é diferenciavel em 0

Questão 9

9 - a)

$$\boldsymbol{f_{(a)} = g_{(a)} = 0 \quad e \quad g'_{(a)} \neq 0}$$

$$\lim_{x \to a} \frac{f_{(x)}}{g_{(x)}} = \lim_{x \to a} \frac{\dfrac{f_{(x)} - f_{(a)}}{x - a}}{\dfrac{g_{(x)} - g_{(a)}}{x - a}} = \frac{f'_{(a)}}{g'_{(a)}}$$

9 - b)

$$\lim_{x \to 0} \frac{x^2 + x}{x\, e^x + \sin(x)};$$

$$f_{(0)} = 0^2 + 0 = 0;$$

$$g_{(0)} = 0 * e^0 + \sin(0) = 0;$$

$$g'_{(0)} = 0 * e^0 + \cos(0) = 1 \neq 0$$

$$\therefore \lim_{x \to 0} \frac{x^2 + x}{x\, e^x + \sin(x)} =$$

$$= \lim_{x \to 0} \frac{2 * x + 1}{e^x + x^2\, e^x + \cos(x)} = 1/2$$