CN A – Resolução Teste 2024.1

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17 de dezembro de 2024

Conteúdo

Questão 1

Erro relativo para approx $\hat{I}$

Resposta

$$r_{\hat{I}} = \frac{\left|\hat{I} - I\right|}{|I|} \leq \frac{|M|}{|I|} \therefore \text{ Alternativa d)}$$

Questão 2

- f def no intervalo $[0, 3]$
- $p_2(x) = 2\,x^2\,3\,x$ Pol de grau ≤ 2 interp de f em $\{0, 1, 2\}$
- $f[x_0, \ldots, x_4] = 4; x_3 = 3$
- $p_3(x) =?$ interp de f em $(x_i, f(x_i)), i = \{0, 1, 2, 3\}$

Resposta

$$p_{3\,(x)} = f(x_0) + \sum_{i=0}^{3-1} \left(\prod_{j=0}^{i} x - x_j \right) f[x_0, \ldots, x_{i+1}] =$$
$$= p_{2\,(x)} + f[x_0, x_1, x_2, x_3]\,(x - x_0)\,(x - x_1)\,(x - x_2) =$$
$$= 2\,x^2 + 3\,x + 4\,(x - 0)\,(x - 1)\,(x - 2) =$$
$$= 2\,x^2 + 3\,x + 4(x^3 - 2\,x^2 - x^2 + 2\,x) =$$
$$= x^3 * 4 - x^2 * 10 + x^1 * 11$$

Resposta: aliena c)

Questão 3

- $I = int_0^1 f(x)\,\mathrm{d}x$
- $I_T = 6.5$
- $I_{PM} = 4.25$
- $I_S = ?$

Resposta

Resposta b)

$$\hat{i} \approx \int_{a=0}^{b=1} f_{(x)}\,\mathrm{d}x = \frac{h}{3}\,(f_0 + 4\,f_1 + f_2) = \frac{1}{3}\,(2\left(\frac{h}{2}(f(0) + f(1))\right) + 4\,(h\,f(0.5))) =$$

$$= \frac{1}{3}\,(2\,I_T + 4\,I_{PM}) =$$

$$= \frac{1}{3}\,(2 * 6.5 + 4 * 4.25) = 10;$$

$$I_{PM} \approx h\,f_{(\frac{0+1}{2})} = h\,f(0.5) = 4.25;$$

$$I_T \approx \frac{h}{2}\,(f(0) + f(1)) = 6.5$$

Questão 4

- $p_4(x)$
- $x = \{0, 1, \ldots, 4\}$
- S spline cubic de f em 0, 2 e 4
- q é pol grau 4 por min quadrados

Resposta

Resposta alinea b)

Questão 5

- $f'(0) = 5$
- $f"(2) = -1$

x_i	0	1	2
$f(x_i)$	6	9	6

$$\boldsymbol{S(x) = \begin{cases} -2\,x^2 + 5\,x + 6, & 0 \leq x < 1 \\ 4\,x^3 - 18\,x^2 + 23\,x, & 1 \leq x \leq 2 \end{cases}}$$

Resposta

Resposta b)

$S(0) = -2\,(0)^3 + 5 * 0 + 6 = 6;$

$S(1) = -2\,(1)^3 + 5\,(1)6 = 9 =$
$= 4\,(1)^3 - 18 * (1)^2 + 23 * (1) = 9;$

$S(2) = 4\,(2)^3 - 18 * (2)^2 + 23 * (2) = 6$

$\therefore S$ interpola f;

$$\frac{\mathrm{d}S(x)}{\mathrm{d}x} \begin{cases} -4\,x + 5, & 0 \leq x < 1 \\ 12\,x^2 - 36\,x + 23, & 1 \leq x \leq 2 \end{cases};$$

$$\lim_{x \to 1^-} \frac{\mathrm{d}S(x)}{\mathrm{d}x} = -4 * 1 + 5 = 1 =$$
$$\neq \lim_{x \to 1^+} \frac{\mathrm{d}S(x)}{\mathrm{d}x} = 12 - 36 + 23 = -1;$$

S Não é spline

Questão 6

Considere a tab com val f

x_i	-2	0	1
$f(x_i)$	-42	-2	3

Q6 a.

Dete o pol de lagrange inter da tab

Resposta

$$p_3(x) = \sum_{i=0}^{3} y_i \, L_i(x) = -42 * L_0 - 2\, L_1 + 3\, L_2 =$$

$$= -42 * \frac{1}{2}(x^2 - 3\,x + 2) - 2\,(-(x^2 - 2\,x)) + 3\left(\frac{1}{2}(x^2 - x)\right);$$

$$L_i(x) = \prod_{j=0}^{i-1} \frac{x - x_j}{x_i - x_j} \prod_{j=i+1}^{3} \frac{x - x_j}{x_i - x_j};$$

$$L_0(x) = \prod_{j=0}^{0-1} \frac{x - x_j}{0 - x_j} \prod_{j=0+1}^{3} \frac{x - x_j}{0 - x_j} = \frac{x-1}{0-1}\,\frac{x-2}{0-2} = \frac{1}{2}(x^2 - 3\,x + 2);$$

$$L_1(x) = \prod_{j=0}^{1-1} \frac{x - x_j}{1 - x_j} \prod_{j=1+1}^{3} \frac{x - x_j}{1 - x_j} = \frac{x-0}{1-0}\,\frac{x-2}{1-2} = -(x^2 - 2\,x);$$

$$L_2(x) = \prod_{j=0}^{2-1} \frac{x - x_j}{2 - x_j} \prod_{j=2+1}^{3} \frac{x - x_j}{2 - x_j} = \frac{x-0}{2-0}\,\frac{x-1}{2-1} = \frac{1}{2}(x^2 - x)$$

Q6 b.

Approx p $f(-1)$ e majorante absoluto para f(-1)

- $|f^3(x)| \leq 1, \forall\, x \in [-2, 1]$

Resposta

$$f(-1) \approx p_3(-1) =$$

$$= -42 * \frac{1}{2}((-1)^2 - 3\,(-1) + 2) - 2\,(-((-1)^2 - 2\,(-1))) + 3\left(\frac{1}{2}((-1)^2 - (-1))\right) =$$

$$= 57$$

$$\eta_{f(-1)} \leq 1$$

$$r_{f(-1)} = \frac{|f(-1) - p_3(-1)|}{|f(-1)|}$$

Q6 c.

Pol de grau 1 por min quadrad

Resposta

$$p_{1,(x)} = \sum_{i=0}^{1} \alpha_i \, x^i;$$

$$\begin{bmatrix} \sum_{i=0}^{1} \left(\alpha_i \, \sum_{j=0}^{n} x_j^{i+k} \right) = \sum_{i=0}^{n} x_i^k \, y_i \\ k \in [0,1] \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\alpha_0 \, \sum_{j=0}^{3} x_j^{0+0} + \alpha_1 \, \sum_{j=0}^{3} x_j^{1+0} \right) = x_0^0 \, y_0 + x_1^0 \, y_1 + x_2^0 \, y_2 \\ \left(\alpha_0 \, \sum_{j=0}^{3} x_j^{0+1} + \alpha_1 \, \sum_{j=0}^{3} x_j^{1+1} \right) = x_0^1 \, y_0 + x_1^1 \, y_1 + x_2^1 \, y_2 \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\alpha_0 \, \sum_{j=0}^{3} x_j^{0+0} + \alpha_1 \, \sum_{j=0}^{3} x_j^{1+0} \right) = x_0^0 \, y_0 + x_1^0 \, y_1 + x_2^0 \, y_2 \\ \left(\alpha_0 \, \sum_{j=0}^{3} x_j^{0+1} + \alpha_1 \, \sum_{j=0}^{3} x_j^{1+1} \right) = x_0^1 \, y_0 + x_1^1 \, y_1 + x_2^1 \, y_2 \end{bmatrix}$$