Chapter 4. Sedimentation

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4.2 *Viscosity of suspensions*
4.2 *Settling kinetics*

- Selective,
- sludge line settling,
- coarse sedimentation

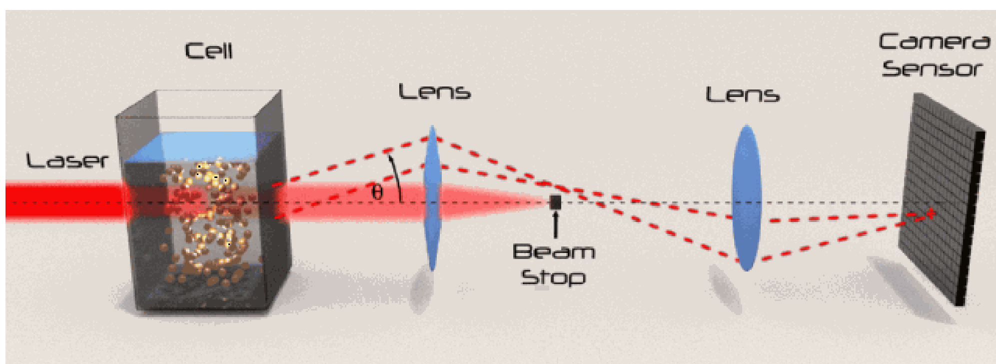
4.3 *Design of a settler*

- Settling operation,
- Material balances
- Kynch method,
- Clarification method

J.M. Coulson and J.F. Richardson pp 237 - 290

Introduction

- A **solids suspension** is a heterogeneous mixture of materials consisting of two phases:
  - **(1)** <u>**Dispersed phase (“fase dispersa”)**</u>: an insoluble solid phase composed by a large number of small particles
  - **(2)** <u>**Dispersion phase (“fase dispersante”)**</u>: A continuous dispersion phase**,** gas or liquid, in which the solid is dispersed
- Opposed to **solutions**, which are transparent homogeneous mixtures, solid suspensions present **turbidity**, i.e. a light beam is scattered by a given angle, θ, due to the presence of undissolved solid particles



Introduction

Solid suspensions may be classified according to the particle size and physicochemical stability as **colloidal**, **fine** and **coarse** suspension

Coarse: d > 100 μm (macroscopic solids e.g. sand)

Fine: 1 μm < d < 100μm (e.g. bacterial or eucariotic cells)

Less stable; **Settle freely**; Are filterable

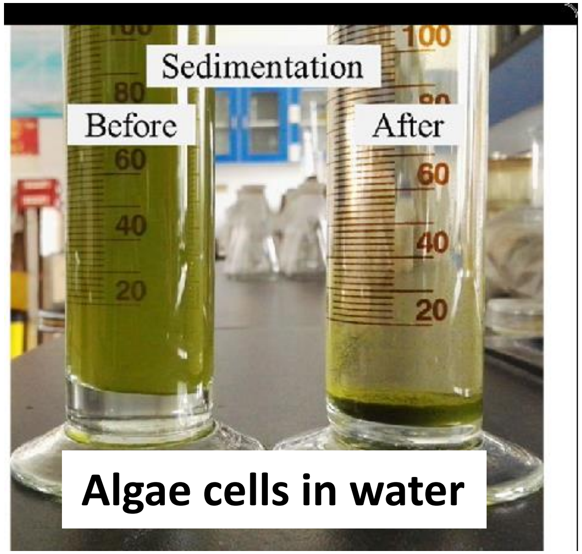
$\approx 1\ \mu m$

Colloid: 1 nm d < 1 μm (e.g. macromolecules such as polysaccharides, lipids and proteins)

Very stable; **don't settle freely**; are not filterable

Examples of solids suspensions

Colloid : 87% water + 13% solids (Lipids: 3.5%; Proteins: 3.4%; Carbohydrates: 4.8%; minerals). Negatively charged globules are formed that repel each other. It is a very stable heterogenous mixture and does not settle freely



Fine suspension: typically contain micron scale particles (very large macromolecules, cell debris and whole cells). **E.g. Dinoflagellate** (*Peridinium sp.*) microalgea has 20-20 μm length. They settle freely although very slowly.

Colloid (but…): Very small airborn solid particles with complex composition (including carbon particles and ignition source particles). Some larger particles settle freely. Other smaller particles are very stable and do not settle freely

Examples of solids suspensions

Mud in water

Solids suspension (coarse and/or fine) : Mud is soil in water. It is a very complex heterogeneous mixture of NOM (Natural Organic Matter) and minerals. Large particles do settle freely (some NOM does not settle freely)

Sugar cane crystalizer

Coarse suspension: sucrose crystals in the mm range in water. A crystallizer transforms a solution into a coarse solids suspension by imposing supersaturation conditions. The sugar crystals do settle freely and rapidly

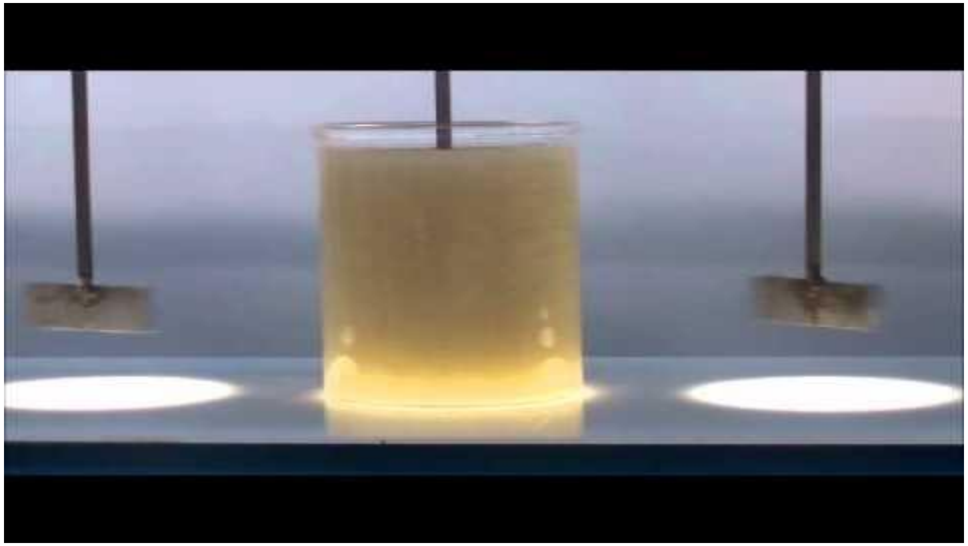
Airborn sawdust

Coarse suspension: large wood particles in the mm range in air. The wood particles do settle freely in air

Flocculation of coloidal suspensions

Flocculation agents are chemicals used to aid sedimentation. They are typically charged molecules that interact with the charged dispersed particles, neutralising them. A well known flocculant in water treatment plants is **Aluminium Sulfate: $Al_2 (SO_4)_3$**

- Colloidal suspensions are very stable due to repulsive forces between dispersed particles; they don't settle;
- The addition of flocculants destabilizes the colloidal suspension leading to the formation of flocs (this process is also known as aggregation or coagulation).
- Flocs are aggregated particles with occluded liquid, thus they have an intermediate specific mass between liquid and solid
- A flocculated suspension settles more or less easily while colloidal suspensions don't settle at all



Concentration of solids

Different ways to define concentration of solids in a suspension (***C***, ***X***, ***e***, ***1-e***, ***V***). Are these variables related?

C : mass concentration, kg-solid/m^3-suspension

X: mass fraction, kg-solid/kg-suspension

e: void fraction (dimensionless)

$$\boldsymbol{e} = \frac{void\ volume}{void\ volume + solids\ volume} = \frac{void\ volume}{total\ volume}$$

1-*e:* volumetric concentration of solids (v/v)

$$(\mathbf{1} - \boldsymbol{e}) = \frac{solids\ volume}{total\ volume}$$

V: mass ratio, e.g. (kg water/kg solids)

TSS – Total Suspended Solids

Step 1. Take a sample of suspension of volume (**V**)
Step 2. Dry and weight a clean filter (**W1**)
Step 2. Filter sample. All particles with size greater than 0.45 microns are captured by the filter
Step 3. Dry and weight filter with solids in a drying oven (**W2**)

$$TSS[\frac{kg}{m^3}] = \frac{W2 - W1}{V}$$

Suspension viscosity, μ_c

- The presence of solids typically increases the viscosity of the suspension, $\mu_c > \mu$
- Gentle stirring typically decreases the viscosity of the suspension

Diluted suspensions

Einstein Eq: $1 - e < 0.02$

$$\boxed{\frac{\mu_c}{\mu} = [1 + k''(1 - e)]}$$

Concentrated suspensions

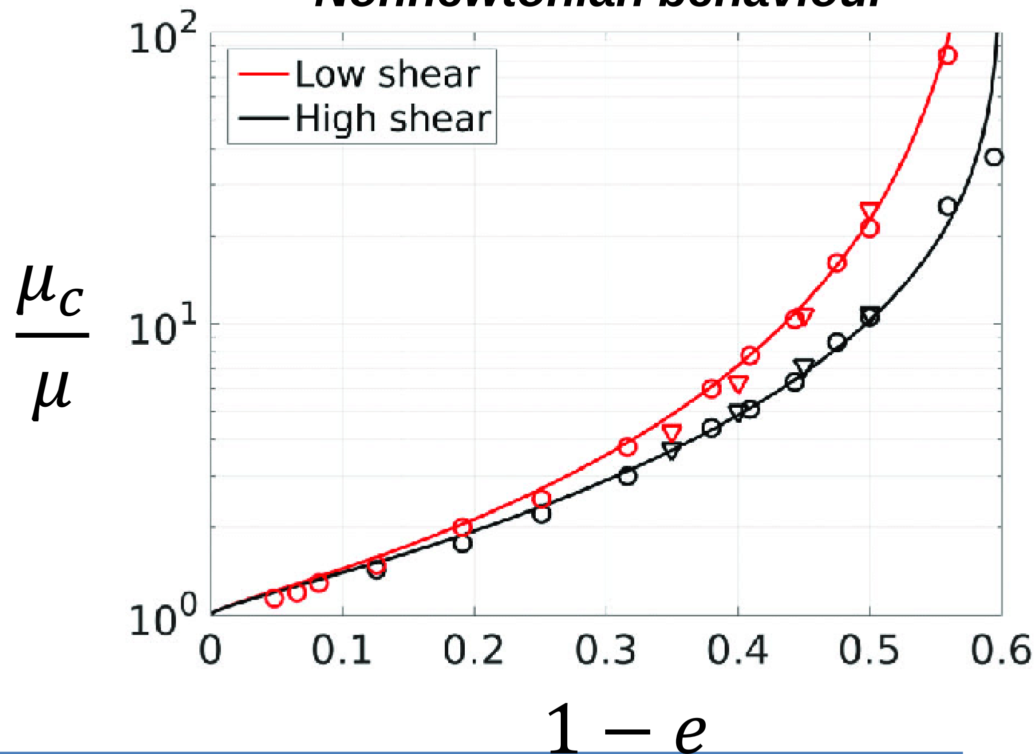
Vand Eq: $1 - e > 0.02$

$$\boxed{\frac{\mu_c}{\mu} = \cdot\, e^{\frac{k''(1-e)}{1-a'(1-e)}}}$$

$a' = 0.609\ (spheres)$

$k'' = 2.5\ (spheres)$

Nonnewtonian behaviour



Settling kinetics

<u>Case 1 – Fine suspensions:</u> 1 μm < d < 100μm (e.g. bacterial or eucariotic cells)

<u>Case 2 - coarse suspensions:</u> d > 0,1 mm (macroscopic solids e.g. sand)

Types of settling in fine suspensions

A - Selective settling: In diluted suspensions ($C \rightarrow 0$) , particles settle independently of each other according to their size, geometry and specific mass. Different particle sizes settle at *different velocities*.

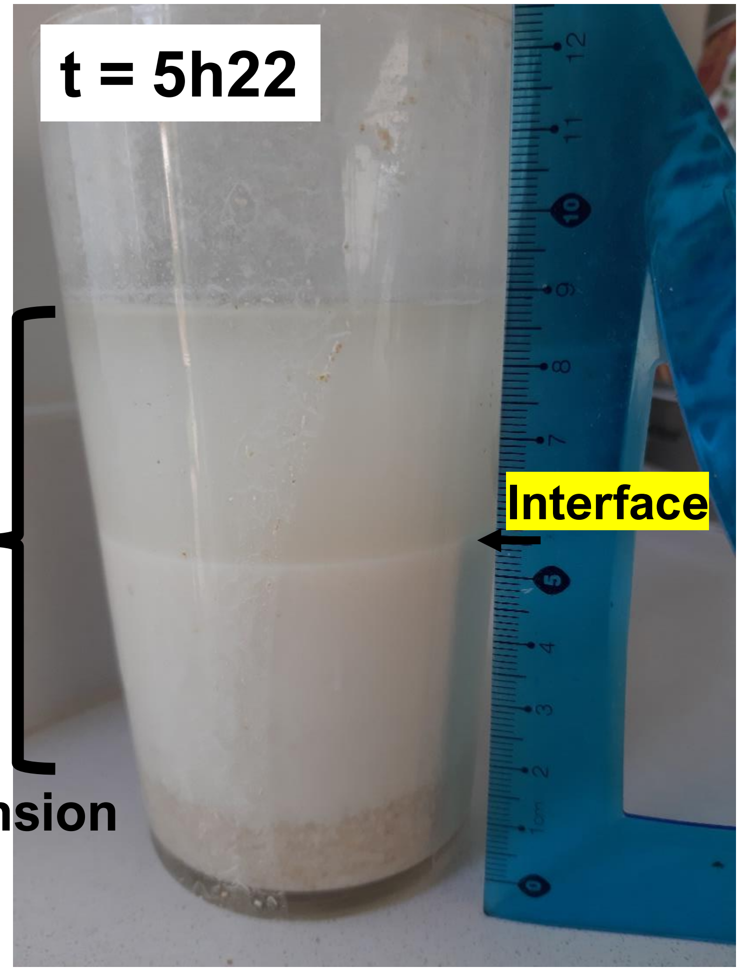
$$\boxed{u_c \cong u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu}}$$

>>Exercise III.4

B - Sludge line settling: In concentrated solids suspension ($C>>0$) with particle size differences less than 6:1, all particles settle at the *same velocity* and a very well-defined interface between clear liquid and solids region is formed.

$\therefore$ Larger particles are retarded while small particles are accelerated

Setlling kinetics experiment@home

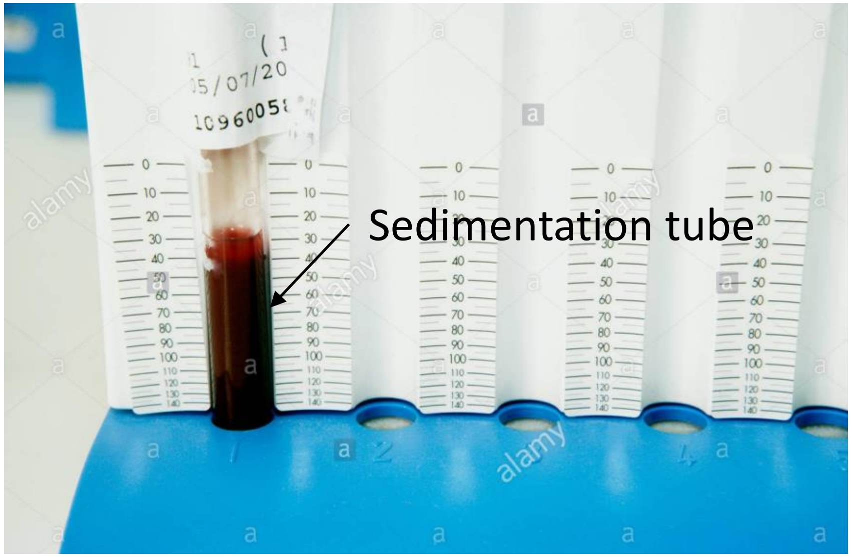


Sludge Line Settling

coarse suspension settling

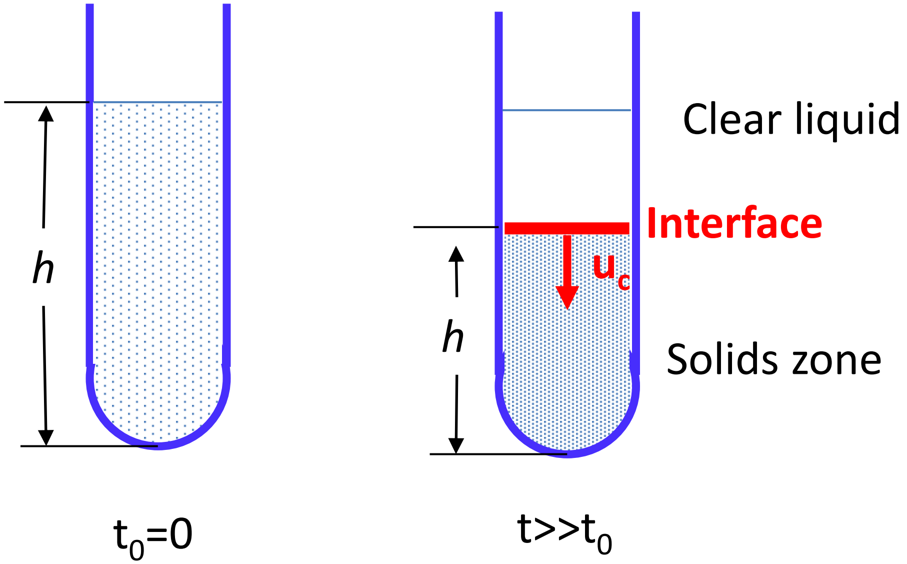
Sludge line settling (SLS)

How to experimentally determine the settling velocity when sludge line settling, u_c ?



This picture shows the experimental determination of erythrocytes settling rate in blood using a sedimentation tube

Experiment with a sedimentation tube



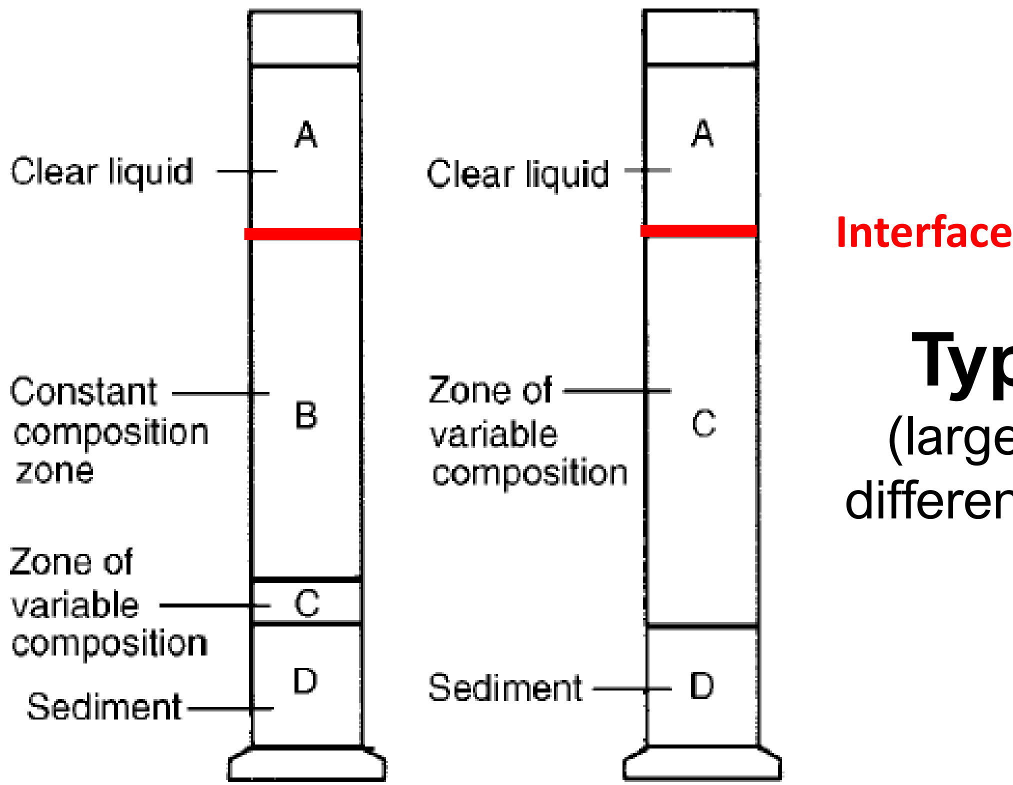
$$u_c = \frac{dh}{dt} \approx \frac{\Delta h}{\Delta t}$$

Sludge Line Settling (SLS)

There are 2 types of SLS. In **type 1 SLS** a pronounced constant composition zone is formed (zone B with constant solids concentration) whereas in **type 2 SLS** a pronounced variable composition zone (zone C) is formed and no zone B is formed.

Type 1 SLS

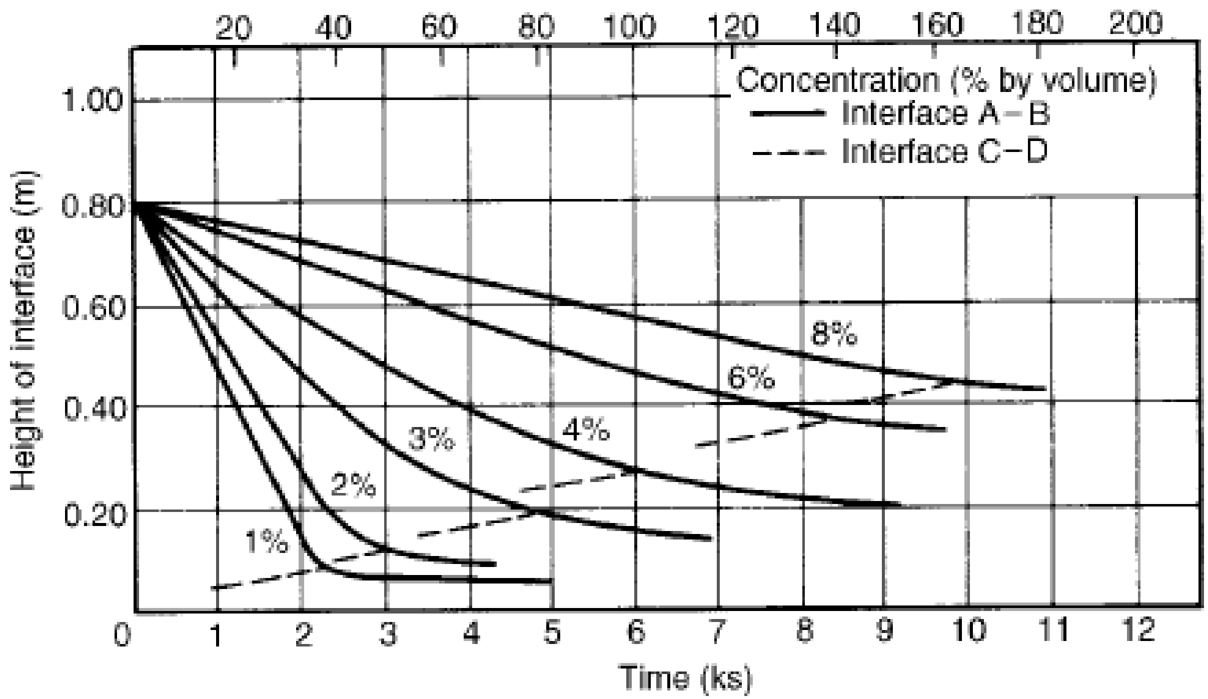
(much more frequent)



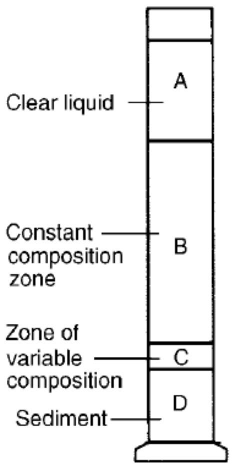
Type 2 SLS

(large particle size differences in zone C)

Effect of solids concentration in SLS



Effect of volumetric concentration of solids, (1-e)x100 (%), on interface high, H (m) in calcium carbonate suspensions (**M. Coulson and J.F. Richardson pp 242**)



(1) linear decay of interface A-C height, *H* (constant $\boldsymbol{u_c}$)

(2) the higher the solids concentration the lower is the slope (**u_c** decreases with *C*)

(3) after some time the interface height, *H*, slows down and stabilizes at the final value H_∞ (**u_c** ~ 0)

SLS kinetic models, u_c

Robinson Eq.:

$$u_c = \frac{K'' d^2 (\rho_s - \rho_c) g}{\mu_c}$$

Steinour Eq.: small uniform particles;

$$u_c = \frac{e^2 d^2 (\rho_s - \rho_c) g}{18\mu} f(e)$$

Case dependente tapioca in oil $f(e) = 10^{-1.82(1-e)}$

Hawsksley Eq.:

$$u_c = \frac{e d^2 (\rho_s - \rho_c) g}{18\mu_c}$$

***Note**:* ρ_c and μ_c are properties of the suspension. Also note that these equations show a dependency of u_c on solids concentration

***Question**: I*s the settling velocity, u_c, in SLS smaller or greater than the terminal fall velocity, u_o?

Settling kinetics

Case 1 – Fine suspensions: 1 μm < d < 100μm (e.g. bacterial or eucariotic cells)

Case 2 - coarse suspensions: d > 0,1 mm (macroscopic solids e.g. sand)

Coarse suspensions settling kinetics, u_c

- Settling velocity for coarse suspensions with d>0,1 mm

$$\boxed{\frac{u_c}{u_i} = e^n}$$

$$\log(u_0) = \log(u_i) + \frac{d}{d_t}$$

u_c – settling velocity, m/s
u_i – settling velocity for infinit dilution, m/s
e – void faction
n = coeficiente = $f(Re'_0, d/d_t)$
d – particle diameter
d_t – tube diameter
$Re'_0 = \frac{\rho u_0 d}{\mu}$ - particle Reynolds at u_0

Table 5.1. n as a function of Ga or Re'_0 and d/d_t

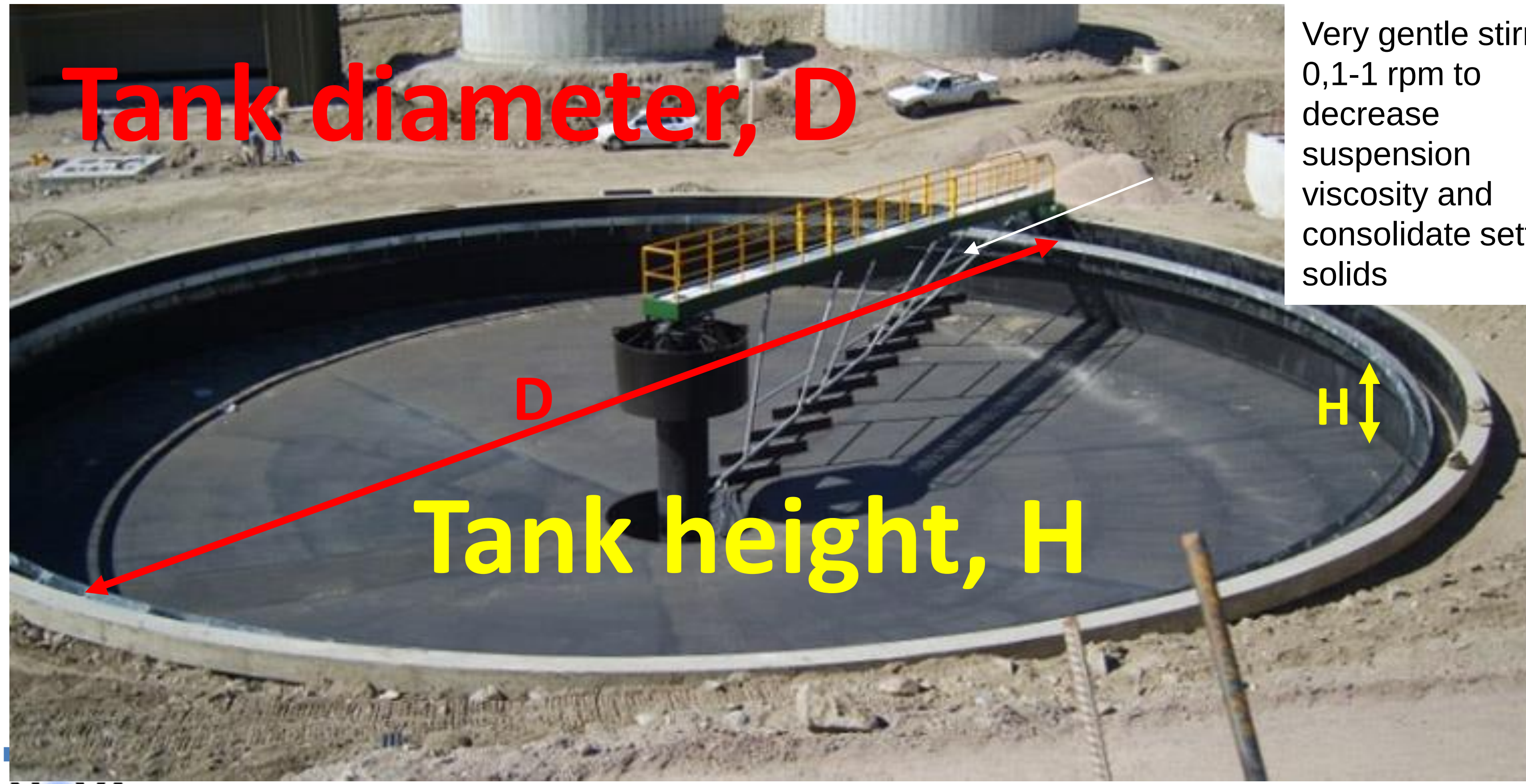
Range of Ga	Range of Re'_0	n as function of $Ga, d/d_t$	n as function of $Re'_0, d/d_t$
0–3.6	0–0.2	$4.6 + 20\, d/d_t$	$4.6 + 20\, d/d_t$
3.6–21	0.2–1	$(4.8 + 20\, d/d_t)\, Ga^{-0.03}$	$(4.4 + 18\, d/d_t)\, Re_0'^{-0.03}$
21–2.4 $\times 10^4$	1–200	$(5.5 + 23\, d/d_t)\, Ga^{-0.075}$	$(4.4 + 18\, d/d_t)\, Re_0'^{-0.1}$
2.4×10^4–8.3×10^4	200–500	$5.5 Ga^{-0.075}$	$4.4 Re_0'^{-0.1}$
$>8.3 \times 10^4$	>500	2.4	2.4

J.M. Coulson and J.F. Richardson, pp 272

Design of a settler

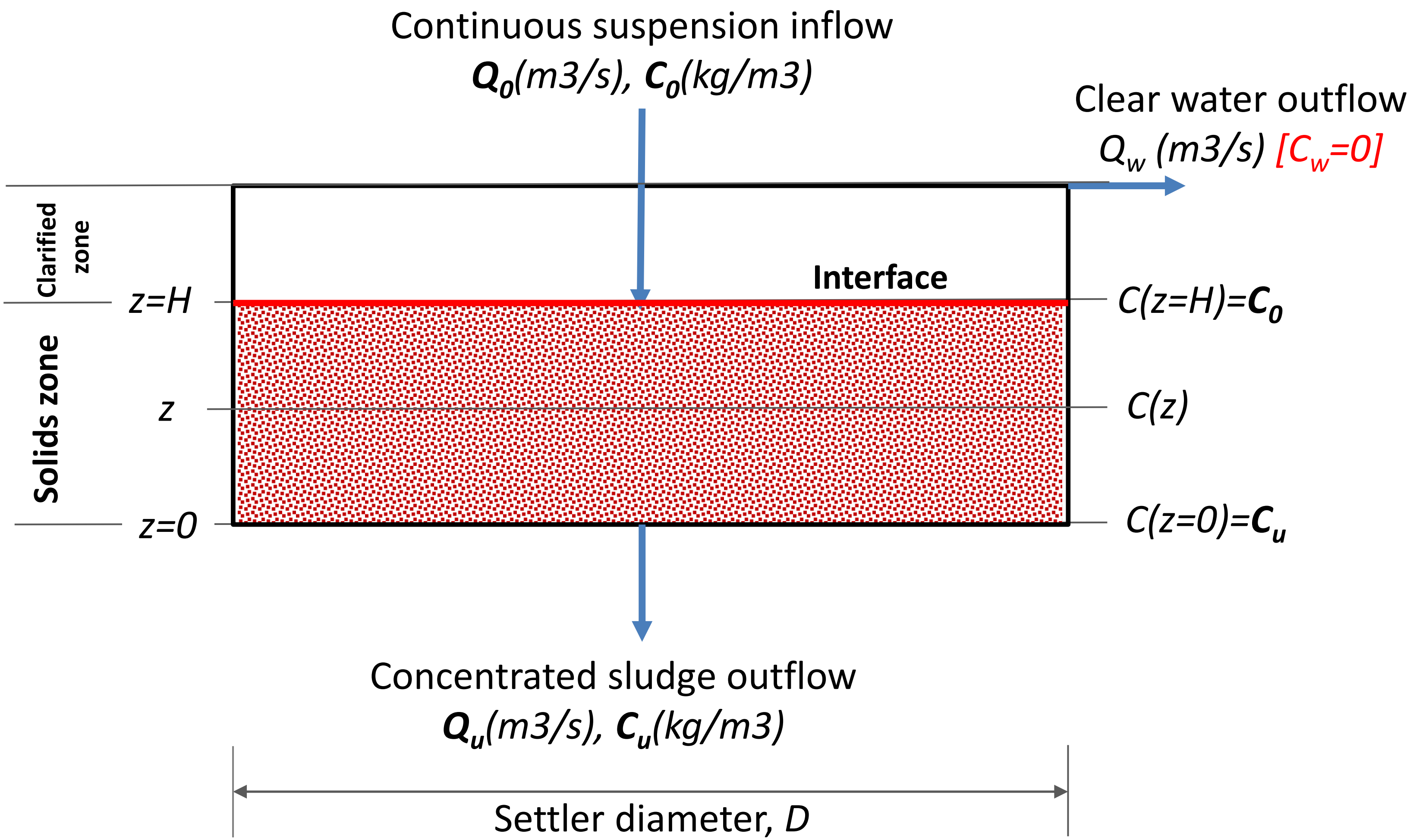
Design problem

Given a liquid-solid suspension inflow stream $\mathbf{Q_0}$, $\mathbf{C_0}$, what's the size of the settler (***H***, ***D***=?) such as to ensure 1) a solids free (clarified) outflow stream and 2) solids outflow stream with desired concentration $\mathbf{C_u}$



Very gentle stirring 0,1-1 rpm to decrease suspension viscosity and consolidate settled solids

Continuous settler



Material balances in steady-state

Hypotheses: no solids in the clear water outflow stream

Solids:

$$Q_0 C_0 = Q_u C_u$$

Solids in in the inflow stream, kg/s

Solids in in the underflow stream, kg/s

$$\boxed{Q_u = \frac{Q_0 C_0}{C_u} \ [\mathrm{m^3/s}]}$$

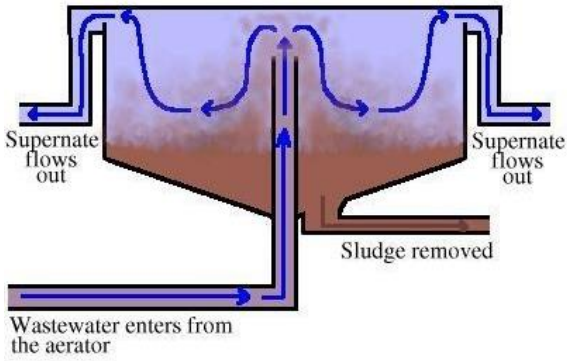
Liquid:

$$Q_w = Q_0 - Q_u$$

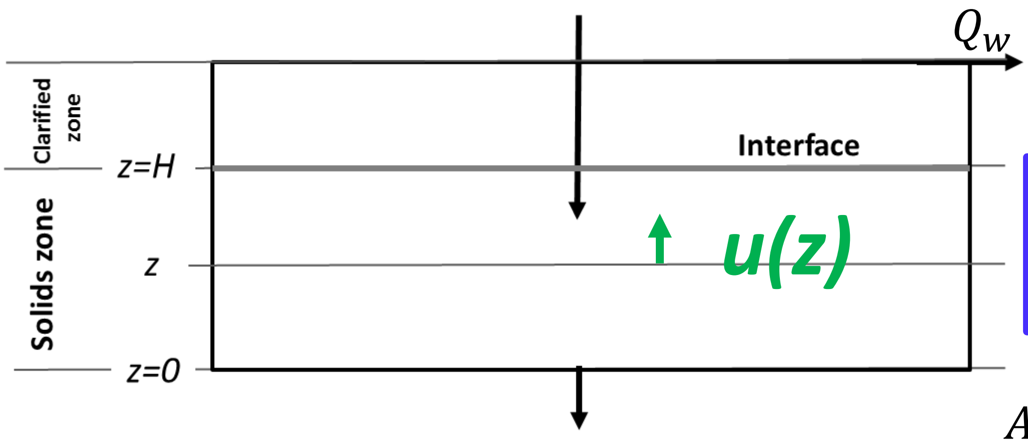
$$\boxed{Q_w = Q_u C_u \left(\frac{1}{C_0} - \frac{1}{C_u}\right) [\mathrm{m^3/s}]}$$

Flow of liquid

The solids are moving down due to gravity, taking the space of the liquid, which is impelled to move up.



$\therefore$ average upflow velocity of liquid, ***u(z) [m/s]***, at any plane z



$$\boxed{u(z) = \frac{Q_u C_u}{A}\left(\frac{1}{C(z)} - \frac{1}{C_u}\right)}$$

A – cross section area of the settling tank

Condition for a solids-free overflow stream

How to ensure a clear liquid overflow stream without any solids?

$$u(z) \leq u_c\big(C(z)\big) \quad \forall_z$$

upflow velocity of liquid, **u(z)**, must be smaller than the settling velocity at any plane z, otherwise the drag force will drag solids upwards

From the material balances:

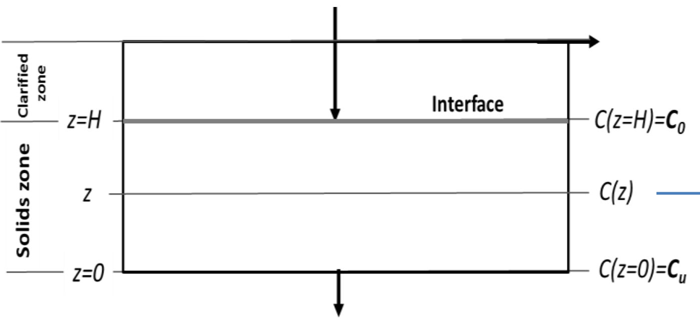
$$u(z) = \frac{Q_u C_u}{A}\left(\frac{1}{C(z)} - \frac{1}{C_u}\right)$$

$$\boxed{A \geq \frac{Q_o C_o}{u_c(C(z))}\left(\frac{1}{C(z)} - \frac{1}{C_u}\right) \quad \forall_z}$$

Large enough cross-section tank area to ensure a solids free overflow stream

Flux of solids, ψ, in a continuous settler

The flux of solids, ψ, applies only to the solids zone and corresponds to the mass flow of solids normalized by the cross-section area of the tank. It has units of kg/[s m2]. It has two terms: (1) the flux due to settling, and (2) the flux due to advective transport.



(1) Flux of solids due to settling

(2) Flux of solids due to advective transport

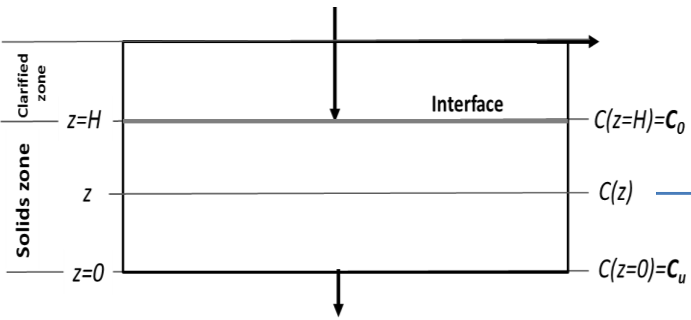
$$\psi(z) = u_c(C(z))C(z) + u_u C(z)$$

$$u_u = \frac{Q_u}{A}$$

∴ Average linear velocity [m/s] of suspension (downward direction) in the solids zone(it is a constant)

Flux of solids, ψ, in a continuous settler

The flux of solids, ψ, applies only to the solids zone and corresponds to the mass flow of solids normalized by the cross-section area of the tank. It has units of kg/[s m2]. It has two terms: (1) the flux due to settling, and (2) the flux due to transport.



$$\psi_0 = u_c(C_0)C_0 + u_u C_0$$

$$\psi(z) = u_c\big(C(z)\big)C(z) + u_u C(z)$$

$$\psi_u = 0 + u_u C_u$$

$$u_u = \frac{Q_u}{A}$$

At z=0 there is no sedimentation (tank wall physical barrier), thus flux of solids is caused solely by transport (i.e. underflow discharge stream)

Design by the Kynch method

Kynch condition: The flux of solids kg/(s m^2) in the underflow, ψ_u, must not exceed the flux of solids, $\psi(z)$, at any plane z of the continuous settler

$$\psi_u \leq \psi(z) = \underbrace{u_c(z)C(z)}_{\text{Flux of solids due to settling}} + \underbrace{u_u C(z)}_{\text{Flux of solids due to advective transport}} \quad \forall_z$$

Interpretation: The flux of solids in the underflow, ψ_u, must be chosen low enough by the designer to enable accumulation of solids inside the tank

Design by the Kynch method

$$\boxed{A^* = \frac{Q_u C_u}{\psi_u^*} = \frac{Q_0 C_0}{\psi_u^*}}$$

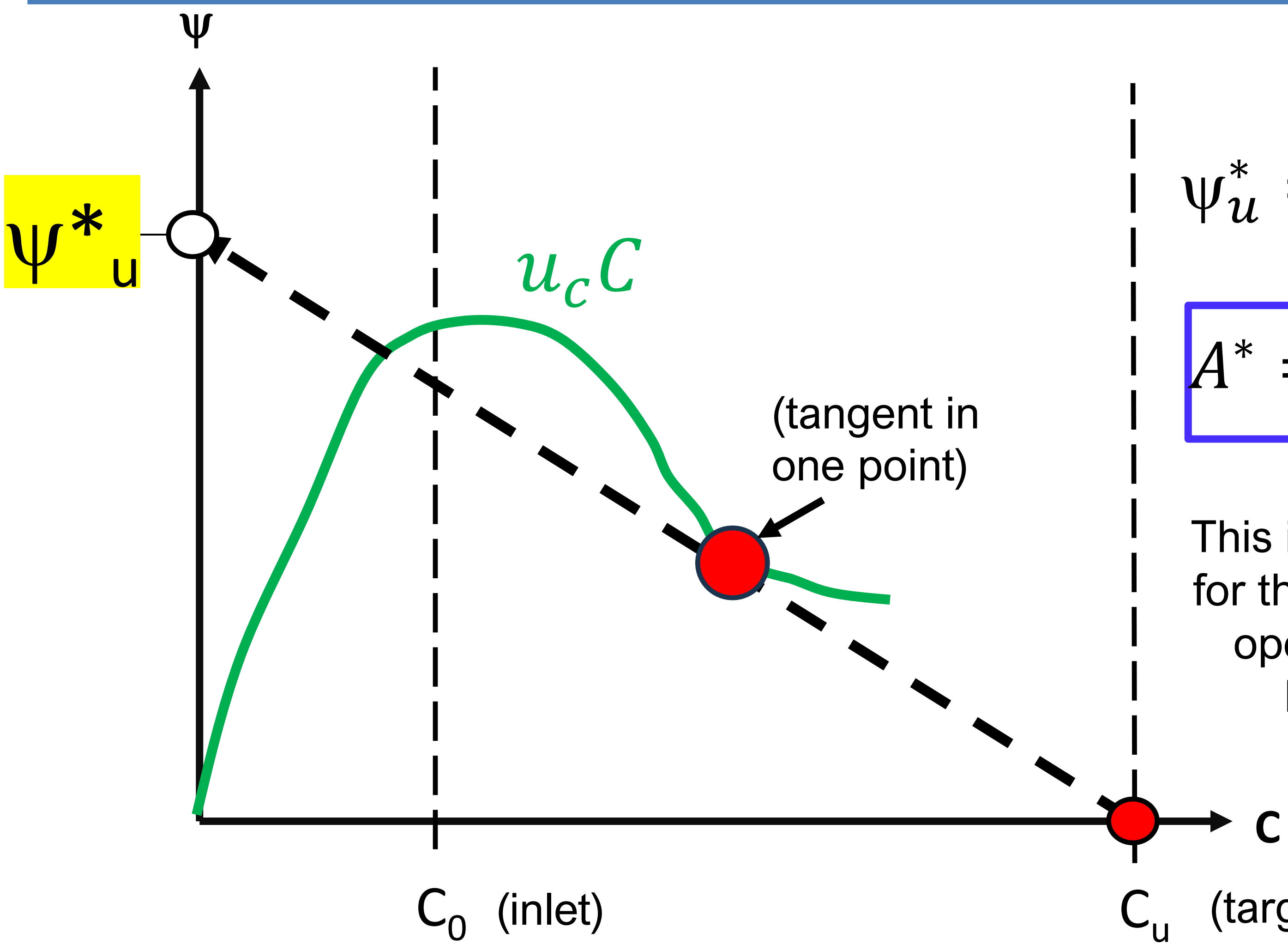
∴ find the maximum flux of solids, ψ_u^*, that obeys to Kynch condition and then calculate the minimum tank area, **A***

Kynch condition:

$$\psi_u \leq \psi(z) = u_c(z)C(z) + u_u C(z) \quad \forall_z$$

$$\psi_u = \frac{Q_u C_u}{A}$$ (flux of solids in the underflow)

Design by the Kynch method



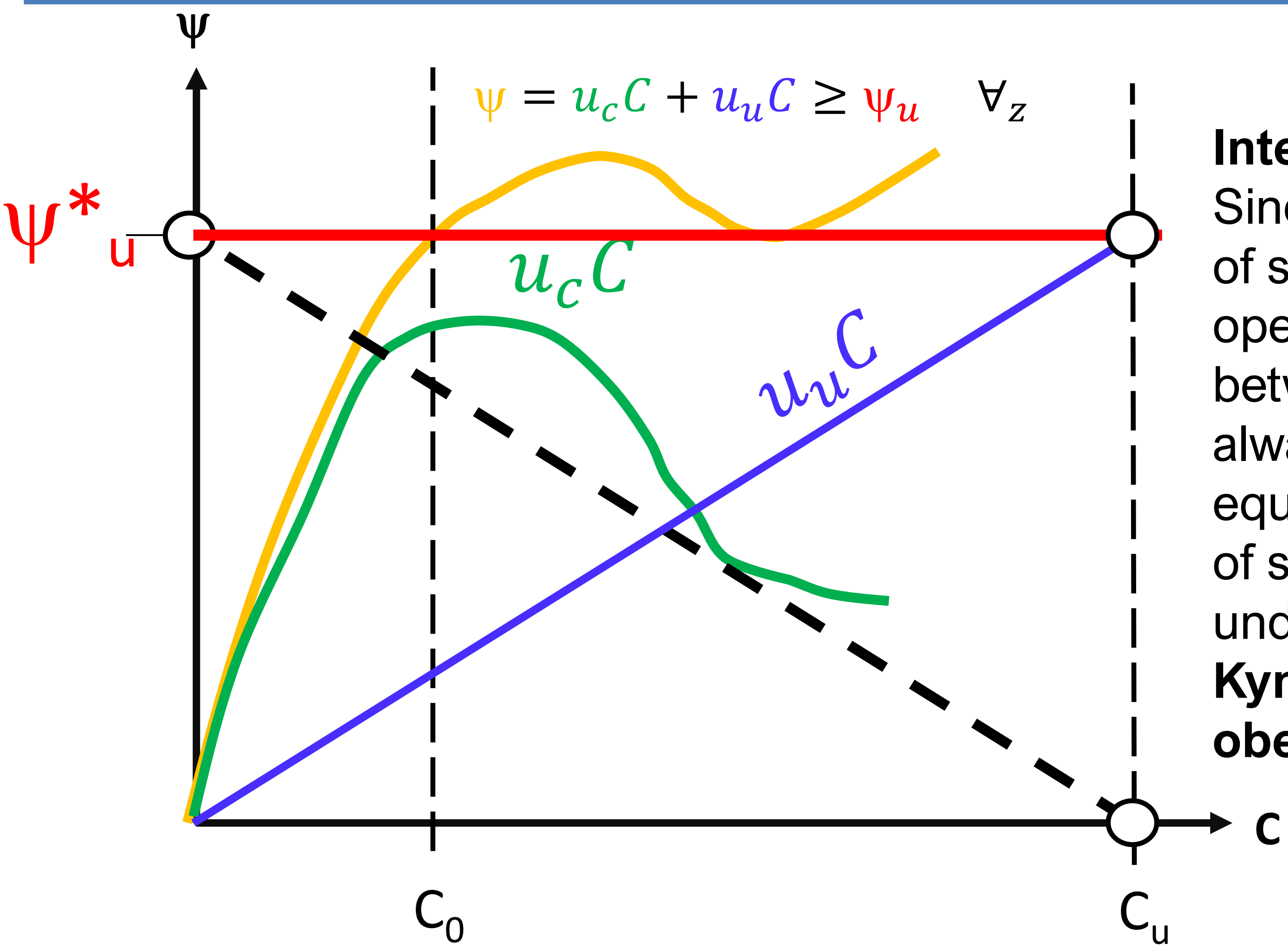
$$\psi_u^* = \frac{Q_u C_u}{A^*} = \frac{Q_0 C_0}{A^*}$$

$$\boxed{A^* = \frac{Q_u C_u}{\psi_u^*} = \frac{Q_0 C_0}{\psi_u^*}}$$

This is the minimum area for the settler to properly operate according to Kynch criterion.

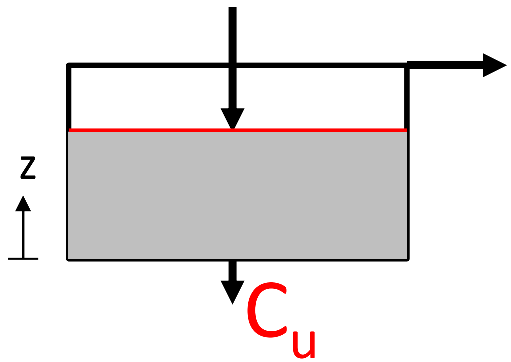
Design by the Kynch method

PROOF



Interpretation: Since the total flux of solids, ψ, at any operation point between [C0,Cu], is always greater or equal than the flux of solids in the underflow, ψ_u, **the Kynch criterion is obeyed!**

Height (H) of the settling tank



How to ensure the desired solids concentration , C_u, in the underflow stream?

Sludge residence time in the continuous settler [h] = Settling time needed to reach Cu in the batch sedimentation tube [h]

$$\frac{AH}{Q_0} = t_{batch,C_u}$$

$$\boxed{H = \frac{Q_0 \times t_{batch,C_u}}{A}}$$

∴ Height of the settling tank [m] that sets the sludge residence time sufficiently long for the solids to reach the desired concentration, Cu

Exercises IV.1 – IV-7

IV – SEDIMENTAÇÃO E ESPESSAMENTO

1. Um ensaio de decantação em tubo de ensaio foi realizado com uma suspensão de carbonato de cálcio (massa específica = 2710 kg/m^3) em água, cuja concentração inicial é de 236 g/l. Os resultados do ensaio vêm expressos na seguinte tabela.

Tempo (h)	Altura de interface (cm)
0	36.0
0.25	32.4
0.5	28.6
1.0	21.0
1.75	14.7
3.0	12.3
4.75	11.6
12.0	9.8
20.0	8.0

a) Determine a concentração de sólidos na zona de espessado em função do tempo
b) Determine a porosidade na zona de espessado em função do tempo.
c) Determine a velocidade de sedimentação em função da concentração de sólidos (represente graficamente).
d) Determine o fluxo (médio) de sólidos em função da concentração de sólidos.
e) Estime a velocidade de sedimentação usando uma lei apropriada assumindo que o diâmetro médio das partículas é 5 μm.

2. Pretende-se projectar um sedimentador a operar em contínuo para concentrar uma polpa com 5 kg de água por kg de sólidos numa lama que contenha 1.5 kg de água por kg de sólidos. O caudal de entrada ao sedimentador é 0.6 kg-sólido/s e a massa específica do sólido é 2500 kg/m3. Ensaios laboratoriais de decantação determinaram que o tempo necessário para concentrar de 5 kg-água/kg-sólido a 1,5 kg-água/kg-sólido é de 3 horas de acordo com o seguinte perfil de velocidade de sedimentação:

Concentração (kg de água/kg de sólidos)	5.0	4.2	3.7	3.1	2.5
Velocidade de sedimentação (mm/s)	0.17	0.10	0.08	0.06	0.042

a) Calcule o caudal de água em kg/s na corrente do clarificado.
b) Calcule a área mínima por forma a assegurar que não passa sólido para a corrente de clarificado.
c) Calcule o fluxo de sedimentação (em kg s^{-1} m^{-2})
d) Calcule a área mínima por forma a assegurar o fluxo de sólidos pretendido na corrente de espessado pelo método de Kynch.
e) Determine a altura e o diâmetro do sedimentador

3. Pretende-se dimensionar um sedimentador para tratar 0.1 m^3/s duma suspensão de sólidos com uma concentração de 150 kg/m^3. A concentração pretendida no espessado é de 1290 kg/m^3. Um ensaio laboratorial de decantação com a mesma suspensão forneceu os resultados indicados na tabela. O ensaio de decantação demonstrou que são necessárias 19 horas para a suspensão decantar duma concentração inicial de 150 kg/m3 a uma concentração final de 1290 kg/m3.

Concentração de sólidos (kg/m^3)	Velocidade de sedimentação (μm/s)
100	148
200	91
300	55.33
400	33.25
500	21.40
600	14.50
700	10.29
800	7.38
900	5.56
1000	4.20
1100	3.27

4. Um ensaio de decantação foi realizado com uma suspensão de carbonato de cálcio, cuja concentração inicial é de 236 g/l. Deseja-se calcular o diâmetro de um decantador com capacidade para processar 8 t/h de suspensão, alimentada ao decantador contendo 236 kg/m^3 de $CaCO_3$. A lama espessada deverá conter 550 kg/m^3 de $CaCO_3$. Os resultados do ensaio de decantação encontram-se na tabela abaixo.

Tempo (h)	Altura da interface (cm)
0	36.0
0.25	32.4
0.5	28.6
1.0	21.0
1.75	14.7
3.0	12.3
4.75	11.6
12.0	9.8
20.0	8.0