ALGA - Slides Anotações

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Conteúdo

Slide 1 – Matrizes

Slide 2 – Sistema de Equações Lineares

Slide 3 – Determinantes

Slide 4 – Espaços Vetoriais

1 Sequencia Geradora

$$\boldsymbol{U} \text{ gera } \boldsymbol{F} \implies \boldsymbol{F} = \langle \boldsymbol{U} \rangle$$

2 Base

$$\boldsymbol{U} \text{ base de } \boldsymbol{F} \implies \nexists \boldsymbol{u}_i : \boldsymbol{u}_i = \boldsymbol{U}\,\boldsymbol{\alpha} \wedge \boldsymbol{\alpha} \neq \boldsymbol{I}^i$$

Slide 5 – Aplicações Lineares

$f : E \to E'$ é aplicação linear (sobre $\mathbb{K}$) $\iff$

$$\iff \begin{cases} \boldsymbol{f(u+v) = f(u) + f(v)} \\ \boldsymbol{f(\alpha\, u) = \alpha\, f(u)} \end{cases} \qquad \begin{cases} \forall\, \{\boldsymbol{u}, \boldsymbol{v}\} \in \boldsymbol{E} \\ \forall\, \boldsymbol{\alpha} \in \mathbb{K} \end{cases}$$

1 Extenção das condições

$$\boldsymbol{f(\alpha\, u + \beta\, v) = \alpha\, f(u) + \beta\, f(v)} \quad \begin{cases} \forall\, \{\boldsymbol{\alpha}, \boldsymbol{\beta}\} \in \mathbb{K} \\ \forall\, \{\boldsymbol{u}, \boldsymbol{v}\} \in \boldsymbol{E} \end{cases}$$

2 Categorização

Homotetia

$$\boldsymbol{f} : \begin{cases} \boldsymbol{E \to E'} & \forall\, \boldsymbol{w} \in \boldsymbol{E} \\ \boldsymbol{f(w) = \beta\, w} & \forall\, \boldsymbol{\beta} \in \mathbb{K} \end{cases}$$

f é uma aplicação homotetia de razão β

Aplicação Nula

$$f:\begin{cases} E \to E \\ f(u) = 0_E \end{cases} \qquad \forall\, u \in E$$

Uma aplicação homotetia de razão 0

Aplicação Identidade

$$\mathrm{id}_E \coloneqq f:\begin{cases} E \to E \\ f(u) = u \end{cases} \qquad \forall\, u \in E$$

Derivada

$$D \coloneqq f:\begin{cases} \mathbb{R}_n[x] \to \mathbb{R}_n[x] \\ D\left(\sum_{k=0}^{n} a_k\, x^k\right) = \sum_{k=1}^{n} k\, a_k\, x^{k-1} \end{cases}$$

3 Inclusão do termo nulo

$$f(0_E) = 0_{E'}$$

$$f(u) + 0_{E'} = f(u) = f(u + 0_E) = f(u) + f(0_E) \implies 0_{E'} = f(0_E)$$

Exemplo 1

$$\boldsymbol{f(-u) = -f(u) \quad \forall\, u \in E}$$

$$\implies 0_{E'} = f(u) - f(u) = f(u) + f(-u) = f(u + (-u)) = f(0_E)$$

Operações com aplicações

4 Aplicação Soma

$$\boldsymbol{f + g : \begin{cases} E \to E' \\ (f+g)(u) = f(u) + g(u) \end{cases}} \qquad \begin{array}{l} \forall\, \boldsymbol{f : E \to E'} \\ \forall\, \boldsymbol{g : E \to E'} \\ \forall\, \boldsymbol{u \in E} \end{array}$$

5 Aplicação Produto

$$\boldsymbol{\alpha\, f : \begin{cases} E \to E' \\ (\alpha\, f)(u) = \alpha\, f(u) \end{cases}} \qquad \begin{array}{l} \forall\, \boldsymbol{f : E \to E'} \\ \forall\, \boldsymbol{\alpha} \in \mathbb{K} \\ \forall\, \boldsymbol{u \in E} \end{array}$$

6 Aplicação Composta

$$\boldsymbol{g \circ f : \begin{cases} A \to C \\ (g \circ f)(a) = g(f(a)) \end{cases}} \qquad \begin{array}{l} \forall\, \boldsymbol{f : A \to B} \\ \forall\, \boldsymbol{g : B \to C} \\ \forall\, \boldsymbol{a \in A} \end{array}$$

obs: Tambem designada por "g após f"

7 Potencia de Expoente k

$$\boldsymbol{f^k : \begin{cases} A \to A \\ f^k = \begin{cases} \mathrm{id}_A & : k = 0 \\ f^{k-1} \circ f & : k \in \mathbb{N} \end{cases} \end{cases}}$$

8 Imagem

$$\boldsymbol{\mathrm{Im}\, F = \{f(a) : a \in A\} \subseteq B}$$

9 Núcleo

$$\boldsymbol{\mathrm{Nuc}\, f = \mathrm{Ker}\, f = \{i \in E : f(u) = 0_{E'}\}}$$

$\mathrm{Nuc}\, f \neq \emptyset$

$f(0_E) = 0'_E \implies 0_E \in \mathrm{Nuc}\, f$

Slide 6 – Valores e Vetores Próprios

1 Endomorfismo

Seja E um espaço vetorial sobre $\mathbb{K}$. Chamamos endomorfismo de E a qualquer aplicação linear de E em E.

Slide 7 – Produto Interno, Externo e Misto

1 Produto Interno

$$\boldsymbol{a|b = \sum_{k=1}^{n} a_1\, b_1 = a\, b}$$

Propriedades

- $a|b = b|a$
- $(a+b)|c = a|c + b|c$
- $a|a = 0 \iff a = 0_{\mathbb{R}^n}$
- $(\alpha a)|b = \alpha(a|b)$
- $a|a \geq 0$

2 Norma

$$\boldsymbol{\|u\| = \sqrt{u|u}}$$

Propriedades

- $\|u\| \geq 0$
- $||\alpha\, u|| = |\alpha|\|u\|$
- $\|u\| = 0 \iff u = 0_{\mathbb{R}^n}$

(i) Vetor unitário (versor)

$$\boldsymbol{\hat{u} = u/\|u\|}$$

(ii) Desigualdade de Schwarz

$$|\boldsymbol{u}|\boldsymbol{v}| \leq \|\boldsymbol{u}\|\,\|\boldsymbol{v}\|$$

(iii) Desigualdede triangular

$$\|\boldsymbol{u}+\boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$$

3 Angulo dentre vetores

$$\measuredangle(\boldsymbol{u},\boldsymbol{v}) = \textbf{arccos}\,\frac{\boldsymbol{u}|\boldsymbol{v}}{\|\boldsymbol{u}\|\,\|\boldsymbol{v}\|}$$

Demonstração

$$|u|v| \leq \|u\|\,\|v\| : u \neq 0_{\mathbb{R}^n} \neq v \implies \frac{|u|v|}{\|u\|\,\|v\|} \leq 1 \implies$$
$$\implies -1 \leq \frac{u|v}{\|u\|\,\|v\|} \leq 1 \implies \cos\measuredangle(u,v) = \frac{u|v}{\|u\|\,\|v\|} \implies$$
$$\implies \measuredangle(u,v) = \arccos\frac{u|v}{\|u\|\,\|v\|}$$

Propríedades

- $\measuredangle(\boldsymbol{u},\boldsymbol{v}) = \measuredangle(\boldsymbol{v},\boldsymbol{u})$
- $\boldsymbol{u}|\boldsymbol{v} = \|\boldsymbol{u}\|\|\boldsymbol{v}\|\measuredangle(\boldsymbol{u},\boldsymbol{v})$

Exemplo 1

$$\measuredangle((\mathbf{1},\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0},\mathbf{1}))$$

$$= \arccos\frac{(1,0,0)|(0,0,1)}{\|(1,0,0)\|\|(0,0,1)\|} = \arccos(0) = \pi/2$$

Exemplo 2

$$\measuredangle(\boldsymbol{u},\boldsymbol{v}) : \{\boldsymbol{u},\boldsymbol{v}\} \subset \mathbb{R}^{\mathbf{3}} \backslash \{\mathbf{0}\} \wedge \boldsymbol{u} = \boldsymbol{\alpha}\,\boldsymbol{v} : \boldsymbol{\alpha} \in \mathbb{R}$$

$$\measuredangle(u,v) = \arccos\frac{u|v}{\|u\|\|v\|} = \arccos\frac{\alpha\,v|v}{\|\alpha\,v\|\|v\|} = \arccos\frac{\alpha}{|\alpha|}\frac{v|v}{\|v\|^2} = \arccos\pm1 = \begin{cases}0\\ \pi\end{cases}$$

3.1 Ortogonalidade

$$\measuredangle(\boldsymbol{u},\boldsymbol{v}) = \mathbf{0} \iff \boldsymbol{u}|\boldsymbol{v} = \mathbf{0}$$

(i) Proprieades

$$\measuredangle(\boldsymbol{u},\boldsymbol{v}) = \mathbf{0} \iff \|\boldsymbol{u}+\boldsymbol{v}\|^{\mathbf{2}} = \|\boldsymbol{u}\|^{\mathbf{2}} + \|\boldsymbol{v}\|^{\mathbf{2}}$$

(ii) Sequencia Ortogonal

$$\boldsymbol{u} \subset \mathbb{R}^{n} : \boldsymbol{u}_i | \boldsymbol{u}_j = 0 \quad \forall\, i \neq j$$

(iii) Sequencia Ortonormada

$$\boldsymbol{u} \subset \mathbb{R}^{n} : \boldsymbol{u}_i | \boldsymbol{u}_j = \begin{cases} 0 : & i \neq j \\ 1 : & i = j \end{cases}$$

(iv) Base ortonormada

$$\left\{ \begin{array}{l} \boldsymbol{u} | \boldsymbol{v} = \sum_{i=1}^{n} \boldsymbol{\alpha}_i \, \boldsymbol{\beta}_i \\ \| \boldsymbol{u} \| = \sqrt{\sum_{i=1}^{n} \boldsymbol{\alpha}_i^2} \\ \boldsymbol{\alpha}_i = \boldsymbol{u} | \boldsymbol{e}_i \quad \forall\, i \iff \\ \quad \iff \boldsymbol{u} = \sum_{i=1}^{n} (\boldsymbol{u} | \boldsymbol{e}_i)\, \boldsymbol{e}_i \end{array} \right\} : \left\{ \begin{array}{ll} \quad \boldsymbol{u} = \boldsymbol{e}\, \boldsymbol{\alpha} & \wedge \\ \wedge\, \boldsymbol{v} = \boldsymbol{e}\, \boldsymbol{\beta} & \wedge \\ \wedge\, \boldsymbol{e} \text{ base ortonormada} & \wedge \\ \wedge\, \boldsymbol{e} \subset \mathbb{R}^{n} & \end{array} \right\}$$

Exemplo 3

Determinar a sequencia de coodenadas do vetor v na base $\mathcal{B}$

$$\boldsymbol{v = (a, b, c)}$$
$$\boldsymbol{\mathcal{B} = \left(\frac{(2,1,0)}{\sqrt{5}}, \frac{(-1,2,0)}{\sqrt{5}}, (0,0,1)\right)}$$

$$\left\{\begin{array}{l} b_1|b_1 = 4/5 + 1/5 = 1 \\ b_2|b_2 = 1/5 + 4/5 = 1 \\ b_3|b_3 = 1 \\ b_1|b_2 = -2/5 + 2/5 = 0 \\ b_1|b_3 = 0 \\ b_2|b_3 = 0 \end{array}\right\} ; (v|b_1, v|b_2, v|b_3) = \left(\frac{2\,a + b}{\sqrt{5}}, \frac{2\,b - a}{\sqrt{5}}, c\right)$$

Proposição 1

$$\boldsymbol{\left\{\begin{array}{l} \quad u \text{ é seq. orto.} \quad \wedge \\ \wedge\, u_i \neq 0 \quad \forall\, i \end{array}\right\} = \left\{\begin{array}{l} \quad u_i | u_j = 0 \quad \forall\, i \neq j \quad \wedge \\ \wedge\, u_i \neq 0 \quad \forall\, i \end{array}\right\} \Longrightarrow}$$
$$\boldsymbol{\Longrightarrow \left\{\begin{array}{l} \quad u \text{ é l.i.} \quad \wedge \\ \wedge\, \#u \leq n \end{array}\right\} =}$$
$$\boldsymbol{= \left\{\begin{array}{l} \quad \nexists\, \alpha \in \mathbb{R}^n \backslash \{I_i\} : u_i = \alpha\, u \quad \forall\, i \quad \wedge \\ \wedge\, \#u \leq n \end{array}\right\} :}$$
$$\boldsymbol{\left\{\begin{array}{l} \quad u \subset \mathbb{R}^n \qquad\qquad \wedge \\ \wedge\, u_i \neq 0_{\mathbb{R}^n} \quad \forall\, i \end{array}\right\}}$$

Exemplo 4

encontre uma base ortornomada para $\mathbb{R}^3$ usando u

$$\boldsymbol{u = ((2,1,0),(-2,4,0),(0,0,2))}$$

$$\left\{\begin{array}{ll} u_1|u_2 = -4+4 = 0 & \land \\ \land\, u_1|u_3 = 0 & \land \\ \land\, u_2|u_3 = 0 & \end{array}\right\} \implies \left\{\begin{array}{ll} u \text{ é l.i.} & \land \\ \land\, \#u = 3 = \dim \mathbb{R}^3 & \end{array}\right\} \implies$$

$$\implies \left(\left(\frac{u_1}{\|u_1\|}\right),\left(\frac{u_2}{\|u_2\|}\right),\left(\frac{u_3}{\|u_3\|}\right)\right) = \left(\left(\frac{u_1}{\sqrt{5}}\right),\left(\frac{u_2}{\sqrt{20}}\right),\left(\frac{u_3}{2}\right)\right)$$

4 Processo de Ortogonalização de Gram-Schmidt

$$\left\{\begin{array}{ll} \boldsymbol{v_i = u_i - \sum_{j=2}^{i} \frac{u_i|v_j}{\|v_j\|^2} v_j} & \land \\ \land\, \boldsymbol{u} \textbf{ é base de } \mathbb{R}^{\boldsymbol{n}} & \end{array}\right\} \implies \boldsymbol{v} \textbf{ é base ortog. de } \mathbb{R}^{\boldsymbol{n}}$$

Objetivo: Processo para encontrar uma base ortogonal a partir de uma base arbritraria de qualquer subespaço não nulo de $\mathbb{R}^n$

Exemplo 5

Encontre a base ortogonal para o subespaço $F \subset \mathbb{R}^n$ a partir de u

$$\boldsymbol{u = \big((1,1,1),(1,3,2)\big)}$$

$$v = \left((1,1,1),(1,3,2) - \frac{(1,3,2)|(1,1,1)}{\|(1,1,1)\|}(1,1,1)\right) =$$
$$= \left((1,1,1),(1,3,2) - \frac{6}{\sqrt{3}^2}(1,1,1)\right) = \big((1,1,1),(-1,1,0)\big)$$

Exemplo 6

Encontre uma base ortogonal para $\mathbb{R}^3$ usando a base do exemplo anterior

$$v = \left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} - \frac{(0,0,1)|(1,1,1)}{\|(1,1,1)\|}\begin{bmatrix}1\\1\\1\end{bmatrix} - \frac{(0,0,1)|(-1,1,0)}{\|(-1,1,0)\|}\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) =$$
$$= \left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} - \frac{0}{\|(-1,1,0)\|}\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) =$$
$$= \big((1,1,1),(-1,1,0),(-1/3,-1/3,2/3)\big)$$

5 Complemento Ortogonal

$$\boldsymbol{F^\perp = \big\{u \in \mathbb{R}^3 : u|v = 0 \quad \forall\, v \in F\big\}}$$

Proposição 2

$$\boldsymbol{F} \textbf{ subespaço de } \mathbb{R}^{\boldsymbol{n}} \implies \boldsymbol{F}^{\perp} \textbf{ sybespaço de } \mathbb{R}^{\boldsymbol{n}}$$

Exemplo 7

(i)

$$\boldsymbol{F^{\perp} : F = \{(0,0,0)\}}$$

$u|(0,0,0) = 0 \quad \forall\, u \implies$
$\implies F^{\perp} = \mathbb{R}^3$

(ii)

$$\boldsymbol{F^{\perp} : F = \mathbb{R}^3}$$

$u|v = 0 \quad \forall\, v \in \mathbb{R}^3 \implies$
$\implies F^{\perp} = \{(0,0,0)\}$

(iii)

$$\boldsymbol{F^{\perp} :}$$

$$\boldsymbol{F = \left\{\begin{matrix} (x,y,z) \in \mathbb{R}^3 : \\ x - 2\,y + 3\,z = 0 \end{matrix}\right\}}$$

$u|(x,y,z) = 0 = x - 2\,y + 3\,z \implies$
$\implies F^{\perp} = \{(1,-2,3)\}$

Proposição 3

$$\boldsymbol{w|u_i = 0 \quad \forall\, i \implies w \in \langle u \rangle^{\perp} : \Big\{ w \in \mathbb{R}^n u \subset \mathbb{R}^n}$$

Exemplo 8

$$\boldsymbol{F^\perp : F = \{(x, y, z) \in \mathbb{R}^3 : x - 2\,y + 3\,z = 0\}}$$

$F = \{(x, y, z) \in \mathbb{R}^3 : x - 2\,y + 3\,z = 0\} = \{(2\,y - 3\,z, y, z) : \{y, z\} \subset \mathbb{R}\} =$
$= \langle (2, 1, 0), (-3, 0, 1) \implies$
$\implies F^\perp = \{(x, y, z) \in \mathbb{R}^3 : (x, y, z)|(2, 1, 0) = 0 \wedge (x, y, z)|(-3, 0, 1) = 0\} =$
$= \{(x, y, z) \in \mathbb{R}^3 : y = -2\,x \wedge z = 3\,x\} = \langle (1, -2, 3) \rangle$

Proposição 4

$$\boldsymbol{F \oplus F^\perp = \mathbb{R}^3 \wedge \dim \mathbb{R}^3 = \dim F + \dim F^\perp}$$

Proposição 5

$\boldsymbol{F^\perp = \mathcal{L}(A)}$ **:**
$\boldsymbol{F}$ **subespaço de** $\boldsymbol{\mathbb{R}^n}$ **das soluções**
de equações lineares homogêneo $\boldsymbol{AX = 0}$

6 Projeção Ortogonal

$$\boldsymbol{u = \mathrm{Proj}_F\, u + \mathrm{Proj}_{F^\perp}\, u : \begin{cases} \mathrm{Proj}_F\ (u) \in F \\ \mathrm{Proj}_{F^\perp}(u) \in F^\perp \end{cases}}$$

Slide 8 – Reta e Plano

Elementos

1 Espaço

Equação vetorial

$$\boldsymbol{\mathcal{E}sp_m \subset \mathbb{R}^n : \left\{\begin{array}{ll} P = A + \sum_{i=1}^{m} \lambda_i U_i & \wedge \\ \wedge \{P, A\} \subset \mathcal{E}sp_m & \wedge \\ \wedge \{i, n, m\} \subset \mathbb{N} & \wedge \\ \wedge\, m \leq n & \wedge \\ \wedge \{\lambda, U_i\} \subset \mathbb{R}^n & \end{array}\right\}}$$

Equações paramétricas

$$\boldsymbol{P_i = A_i + \sum_{j=1}^{m} \lambda_{j,i} U_{j,i}}$$

Exemplos

- $m = 0 \implies$ ponto
- $m = 1 \implies$ reta
- $m = 2 \implies$ plano

2 Reta

Equação vetorial

$$\boldsymbol{\mathcal{R}eta_m \subset \mathcal{E}sp_m : \begin{Bmatrix} P = A + \lambda U & \wedge \\ \wedge \{P, A\} \in \mathcal{R}eta_m & \end{Bmatrix}}$$

Definem uma reta:

- 2 Pontos
- 1 Ponto e 1 vetor

Vetor diretor: Vetor não nulo parapelo a reta

Equações cartezianas

(i) Equações paramétricas

$$\boldsymbol{P_i = A_i + \lambda U_i \quad \forall\, i \leq n}$$

(ii) Equações normais

$$\boldsymbol{\left.\begin{matrix} P_i = A_i + \lambda U_i & \wedge \\ \wedge\, P_j = A_j + \lambda U_j & \end{matrix}\right\} \implies}$$
$$\boldsymbol{\implies \lambda = \frac{P_i - A_i}{U_i} = \frac{P_j - A_j}{U_j} \begin{cases} \{i, j\} \leq n & \wedge \\ \wedge \{U_i, U_j\} \neq 0 & \end{cases}}$$

(iii) Equações reduzidas

$$\boldsymbol{P_i = A'_i + P_j\,U'_i : \begin{Bmatrix} \{i,j\} \subset \mathbb{N} & \wedge \\ \wedge\,\{i,j\} \le n & \wedge \\ \wedge\, i \ne j & \wedge \\ \wedge\, U'_i = U_i/U_j & \wedge \\ \wedge\, A'_i = A_i - A_j\,U'_i & \wedge \\ \wedge\, U_j \ne 0 \end{Bmatrix}}$$

$$\left.\begin{matrix} P_i = A_i + \lambda\,U_i & \wedge \\ \wedge\,\lambda = (P_j - A_j)/U_j : U_j \ne 0 \end{matrix}\right\} \Longrightarrow$$
$$\Longrightarrow\; P_i = A_i + U_i\,(P_j - A_j)/U_j =$$
$$= A'_i + U'_i\,P_j : \begin{cases} U'_i = U_i/U_j & \wedge \\ \wedge\, A'_i = A_i - A_j\,U'_i & \wedge \\ \wedge\, U_j \ne 0 \end{cases}$$

Reduzida pois possui uma equação a menos comparando com as equações cartezianas

3 Plano

Equação vetorial

$$\boldsymbol{\mathcal{P}lano_m \subset \mathcal{E}sp_m : \left\{ \{P, A\} \subset \mathcal{P}lano_m \wedge n \ge 2 \;\right\}}$$

Definição 1 ponto e 2 vetores não paralelos

$$\boldsymbol{P = A + \sum_{j=1}^{2} \lambda_j U_j}$$

Definição 1 ponto e 1 vetor perpendicular ao plano

$$\boldsymbol{\overrightarrow{AP}|(\lambda_1 \times \lambda_2) = |P - A, \lambda_1, \lambda_2| = \overrightarrow{AP}|\lambda' = \\ = A\,\lambda' + d = 0 \\ \left\{\begin{matrix} d \in \mathbb{R} & \wedge \\ \wedge\, \lambda' \in \mathbb{R}^n : \lambda' \perp \mathcal{P}lano & \end{matrix}\right\}}$$

Definem um plano:

- 3 Pontos
- 1 Ponto e 2 vetores não paralelos
- 1 Ponto e 1 Vetor

Vetores diretores: Vetores não nulos e não colineares paralelos ao plano.

Equações cartesianas

(i) Equações paramétricas

$$\boldsymbol{P_i = A_i + \sum_{j=1}^{2} \lambda_{j,i} U_{j,i}}$$

(ii) **Equação geral**

$$\boldsymbol{A + \sum_{i=1}^{n} \lambda_i P_i = 0 : \begin{cases} A \in \mathbb{R} & \wedge \\ \wedge\, \lambda \in \mathbb{R}^n \end{cases}}$$

Distâncias

4 Distancia dentre 2 espaços

$$\mathbf{d}(\mathcal{E}_1, \mathcal{E}_2) = \mathbf{min}(||\overrightarrow{e_1\, e_2}||) : \begin{Bmatrix} \mathcal{E}_1 \in \mathcal{E}sp_m & \wedge \\ \wedge\, \mathcal{E}_2 \in \mathcal{E}sp_n & \wedge \\ \wedge\, e_1 \in \mathcal{E}_1 & \wedge \\ \wedge\, e_2 \in \mathcal{E}_2 & \end{Bmatrix}$$

4.1 Distancia entre 1 Ponto e 1 Reta

$$\mathbf{d}(P, \mathcal{R}eta) = \left\|\overrightarrow{A\,P}\right\| \mathbf{sin}(\theta) = \left\|\overrightarrow{A\,P}\right\| \|u\| \,\mathbf{sin}\,\theta / \|u\| =$$
$$= \left\|\overrightarrow{A\,P} \times u\right\| / \|u\| : \theta = \measuredangle\left(u, \overrightarrow{A\,P}\right)$$

4.2 Distancia dentre 1 ponto e 1 plano

$$\mathbf{d}(P, \mathcal{P}lano) = \left\|\overrightarrow{A\,P}\right\| |\mathbf{cos}(\theta)| = \left\|\overrightarrow{A\,P}\right\| \|w\| |\mathbf{cos}(\theta)| / \|w\| =$$
$$= \left|\overrightarrow{A\,P} | w\right| / \|w\|$$

Exemplo 1

Conclua que $\mathcal{R}eta$ é extritamente paralela ao $\mathcal{P}lano$

$$\boldsymbol{\mathcal{R}eta = \{(x,y,z) \in \mathbb{R}^3 : x = 2 \wedge 2\,y - 6 = z\}}$$
$$\boldsymbol{\{(1,0,-1),(2,2,0),(1,1,1)\} \subset \mathcal{P}lano}$$

(i)

$$\mathcal{R}eta \| \mathcal{P}lano \implies u|v = 0 : \begin{Bmatrix} u \in \mathcal{R}eta & \wedge \\ \wedge\, v \in \mathcal{P}lano & \end{Bmatrix} \implies$$
$$\implies \left(\begin{bmatrix}2\\3\\0\end{bmatrix} - \begin{bmatrix}2\\0\\-6\end{bmatrix}\right) \middle| \left(\left(\begin{bmatrix}2\\2\\0\end{bmatrix} - \begin{bmatrix}1\\0\\-1\end{bmatrix}\right) \times \left(\begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}1\\0\\-1\end{bmatrix}\right)\right) =$$
$$= \left(\begin{bmatrix}0\\3\\6\end{bmatrix}\right) \middle| \left(\left(\begin{bmatrix}1\\2\\1\end{bmatrix}\right) \times \left(\begin{bmatrix}0\\1\\2\end{bmatrix}\right)\right) =$$
$$= \left(\begin{bmatrix}0\\3\\6\end{bmatrix}\right) \middle| \left(\begin{bmatrix}3\\-2\\1\end{bmatrix}\right) =$$
$$= -6 + 6 = 0$$

(ii)

$$\mathcal{R}eta \not\subseteq \mathcal{P}lano \implies \nexists\, p : p \in \mathcal{R}eta \wedge p \in \mathcal{P}lano \implies$$