ALGA – Testes Resoluções.tex

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Conteúdo

b

Teste 1 –

Questão 1

$$\begin{bmatrix} 0 & 1 & 0 & 3 \\ -1 & 2 & 0 & 0 \\ 1 & -1 & 0 & 3 \end{bmatrix} \in \mathcal{M}_{3\times4}(\mathbb{R})$$
$$B : B^T_{(2)} = (4, 2, 3, -1) \wedge \exists BA$$

(i)

$BA \in \mathcal{M}_{4\times4}$

(ii)

$(AB)^T_{(2,3)} = B^T_2\, A^{T\,(3)} = 1 * 4 + (-1) * 2 + 0 * 3 + 3 * (-1) = -1$

(iii)

$r(A) = 3$

Questão 2

$$\begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{1} \\ \mathbf{-1} & \mathbf{3} & \mathbf{-1} \end{bmatrix}$$

(i)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow[\substack{l_1+=-2l_2 \\ l_2+=-l1}]{} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} -1 & 3 & -1 \\ 2 & -4 & 2 \end{bmatrix} \xrightarrow[l_2 < l_2/2l_2 < - > l_1]{} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

(v)

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow[l_2+=-l_1]{} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \end{bmatrix}$$

Questão 3

$$\begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{-2} & \mathbf{0} \\ \mathbf{1} & \mathbf{4} & \mathbf{0} & \mathbf{0} \end{vmatrix}$$

$$= (-1)(-1)^{1+4} \begin{vmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \\ 1 & 4 & 0 \end{vmatrix} = (-2)(-1)^{2+3} \begin{vmatrix} -1 & 1 \\ 1 & 4 \end{vmatrix} = 2((-1)*4 - 1*1) = -10$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$$B^{-1} = \frac{\operatorname{adj} B}{\det B} = (\det B)^{-1} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 2 \end{bmatrix}^T = -1 \begin{bmatrix} 2 & 1 & 2 \\ -1 & 0 & -1 \\ 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -2 \\ 1 & 0 & 1 \\ -3 & 1 & -2 \end{bmatrix}$$

(i)

$$\left(\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & 1 \\ -3 & 1 & -2 \end{bmatrix} \right)_{(2,2)} = 1*1 + 2*0 + 1*1 = 2 \neq 1$$

$$\begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & 1 \\ -3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}\} \in \boldsymbol{\mathcal{M}}_{\boldsymbol{n\times n}}(\mathbb{R})$$

(ii)

$$A\,B = A\,C \land \det A \neq 0 \implies \exists A^{-1} \land A^{-1}\,A\,B = A^{-1}\,A\,C \implies B = C$$

(iii)

$$A^2 = A\,A = I \implies \det(A)\det(A) = \det(I) \implies \det(A)^2 = 1 \implies |\det(A)| = 1$$

(iv)

$$\implies \det B = \det(C^{-1}\,A\,C) = \det(C^{-1})\ \det(A)\ \det(C) = \det(A)$$

Questão 4

$$\boldsymbol{A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \in \mathcal{M}_3(\mathbb{R}) \land \det A = k \neq 0}$$

(i)

$\det(A/2) = (1/2)^3 \det A = k/6$

(ii)

$$\begin{vmatrix} -a2\,b-c \\ d-2\,ef \\ -g2\,h-i \end{vmatrix} = (-1)(-1)\begin{vmatrix} a-2\,bc \\ d-2\,ef \\ g-2\,hi \end{vmatrix} = \begin{vmatrix} a & d & g \\ -2\,b & -2\,e & -2\,h \\ c & f & i \end{vmatrix} = (-2)\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = (-2)\begin{vmatrix} a & b \\ d & e \\ g & h \end{vmatrix}$$

Questão 5

$$\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-1} & \mathbf{0} & | & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} & | & \mathbf{1} \\ \mathbf{-1} & \mathbf{-2} & \boldsymbol{\alpha} \mathbf{+1} & \mathbf{1} & | & \mathbf{1} \\ \mathbf{2} & \mathbf{3} & \mathbf{-1} & \boldsymbol{\alpha} \mathbf{-1} & | & \boldsymbol{\beta} \mathbf{-3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 & | & -1 \\ 0 & 1 & 0 & 2 & | & 1 \\ -1 & -2 & \alpha+1 & 1 & | & 1 \\ 2 & 3 & -1 & \alpha-1 & | & \beta-3 \end{bmatrix} \xrightarrow[l_3 += l_1 l_4 += -l_1 l_4 += -l_2]{} \begin{bmatrix} 1 & 2 & -1 & 0 & | \\ 0 & 1 & 0 & 2 & | \\ 0 & 0 & \alpha & 1 & | \\ 1 & 0 & 0 & \alpha-3 & | \end{bmatrix}$$

(i)

$$\begin{bmatrix} 1 & 2 & -1 & 0 & | & -1 \\ 0 & 1 & 0 & 2 & | & 1 \\ 0 & 0 & 0 & 1 & | & 0 \\ 1 & 0 & 0 & 0-3 & | & 0-3 \end{bmatrix} \xrightarrow[l_4 += 3\, l_3 l_2 += -2\, l_3 l_1 += -l_4 l_1 += -2\, l_2 l_1 <-- l_1 l_1]{}$$

(ii)

$$\begin{bmatrix} 1 & 2 & -1 & 0 & | & -1 \\ 0 & 1 & 0 & 2 & | & 1 \\ 0 & 0 & -1 & 1 & | & 0 \\ 1 & 0 & 0 & -4 & | & -3 \end{bmatrix} \xrightarrow[\substack{l_2 += -2\, l_3 \\ l_4 += 4\, l_3 \\ l_1 += -2\, l_2 \\ l_4 += -l_1 \\ l_3 += l_4 l_2 += -2\, l_4 l_1 += 5\, l_4 l_4 <-> l_3}]{} \begin{bmatrix} 1 & 0 & 0 & 0 & | \\ 0 & 1 & 0 & 0 & | \\ 0 & 0 & 1 & 0 & | \\ 0 & 0 & 0 & 1 & | \end{bmatrix}$$

Questão 6

$$\begin{bmatrix} 1 & 2 & -1 & 0 & | & -1 \\ 0 & 1 & 0 & 2 & | & 1 \\ 0 & 0 & 0 & 1 & | & 0 \\ 1 & 0 & 0 & -3 & | & \beta-3 \end{bmatrix}$$

Questão 10

$$\boldsymbol{A \xrightarrow[l_2<->l_3]{} A_1 \xrightarrow[l_1+=-2\,l_3]{} A_2 \xrightarrow[2\,l_2]{} B}$$
$$\boldsymbol{C \xrightarrow[2\,l_2]{} C_1 \xrightarrow[l_2+=-2\,l_3]{} B}$$

(i)

$$E_3\,E_2\,E_1\,A = E_3\,E_2\,E_1 = B$$

(ii)

$$E_3\,E_2\,E_1\,A = B = I \therefore A^{-1} = E_3\,E_2\,E_1$$

(iii)

$$A^T = \left(E_1^{-1}\,E_2^{-1}\,E_3^{-1}\,B\right)^T = (E_3^{-1})^T\,(E_2^{-1})^T\,(E_1^{-1})^T$$

(iv)

Exame 0 –