Ficha 2 – Soluções

Para efeitos de notação, consideremos as seguintes aplicações:

$$T_{pol}: [0,+\infty[\times [0,2\pi] \to \mathbb{R}^2$$

$$T_{pol}(r,\theta) = (r\cos(\theta)\,, r\ \sin(\theta));$$

$$\tilde{T}_{pol}: [0,+\infty[\times [-\pi,\pi] \to \mathbb{R}^2$$

$$\tilde{T}_{pol}(r,\theta) = (r\cos(\theta)\,, r\ \sin(\theta));$$

$$T_{cil}: [0,+\infty[\times [0,2\pi] \times \mathbb{R} \to \mathbb{R}^3$$

$$T_{cil}(r,\theta,z) = (r\cos(\theta)\,, r\ \sin(\theta)\,, z);$$

$$T_{esf}: [0,+\infty[\times [0,2\pi] \times [0,\pi] \to \mathbb{R}^3$$

$$T_{esf}(\rho,\theta,\varphi) = (\rho\sin(\varphi)\cos(\theta)\,, \rho\sin(\varphi)\sin(\theta)\,, \rho\cos(\varphi));$$

Sejam A, B conjuntos e $f: A \to B$ uma aplicação. Dado $C \subseteq A$, relembramos que

$$f(C) = \{f(c): c \in C\}$$

e que, dado $D \subseteq B$,

$$f^{-1}(D) = \{a \in A: f(a) \in D\}.$$

1. Seja D o conjunto do enunciado.
$$T_{pol}^{-1}(D) = \left\{(r,\theta) \in [0,+\infty[\times [0,2\pi]: 0 < r \leq 1 \wedge \theta \in \left(\left[0,\frac{\pi}{6}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]\right)\right\}.$$
Outra opção é
$$\tilde{T}_{pol}^{-1}(D) = \left\{(r,\theta) \in [0,+\infty[\times [-\pi,\pi]: 0 < r \leq 1 \wedge -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{6}\right\}.$$

2. Seja D o conjunto do enunciado.
   a. $T_{pol}(D) = \{(x,y) \in \mathbb{R}^2: x \leq 0 \wedge y = 0\}$.
   b. $T_{pol}(D) = \left\{(x,y) \in \mathbb{R}^2: 1 < x^2 + y^2 \leq 9 \wedge y \leq 0 \wedge y < -\sqrt{3}\,x\right\}$.

3.
   a. $r\sin(\theta) = 3$.
   b. $r = 3$.
   c. $r = 4\sin(\theta)$.
   d. $\theta = \frac{\pi}{4} \vee\ \theta = \frac{5\pi}{4}$.
   e. $r^2\cos(2\theta) = 4$.
   f. $r^2 = \sin(2\theta)$.

4.
   a. $x = 4$.
   b. $x = -1 + \frac{1}{4}y^2$.
   c. $x^2 + (y+2)^2 = 4$.
   d. $y = 2x$.

e. $y = x \wedge x \leq 0$.

5.

a. $D_f = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4 \wedge y > 0\}$.

b. $T_{pol}^{-1}(D_f) = \{(r,\theta) \in [0,+\infty[\times [0,2\pi] : 0 < r \leq 2 \wedge \theta \in]0,\pi[\}$.

6. Seja D o sólido indicado em cada alínea.

a. $T_{cil}^{-1}(D) = \{(r,\theta,z) \in [0,+\infty[\times [0,2\pi] \times \mathbb{R} : r \leq 2 \wedge 0 \leq \theta \leq \pi \wedge r^2 \leq z \leq 8 - r^2\}$.

b. $T_{cil}^{-1}(D) = \{(r,\theta,z) \in [0,+\infty[\times [0,2\pi] \times \mathbb{R} : r \leq 1 \wedge r - 1 \leq z \leq 1 - r^2\}$.

c. $T_{cil}^{-1}(D) = \left\{(r,\theta,z) \in [0,+\infty[\times [0,2\pi] \times \mathbb{R} : r \leq 1 \wedge 1 - \sqrt{1 - r^2} \leq z \leq r\right\}$.

7.

a. $D_f = \{(x,y,z) \in \mathbb{R}^3 : xy > 0 \wedge x^2 + y^2 < z \wedge z \leq 3\}$.

b. $T_{cil}^{-1}(D_f) = \left\{(r,\theta,z) \in [0,+\infty[\times [0,2\pi] \times \mathbb{R} : 0 < r \leq \sqrt{3} \wedge \theta \in \left(]0,\frac{\pi}{2}[\cup]\pi,\frac{3\pi}{2}[\right) \wedge r^2 < z \leq 3\right\}$.

8. Seja D o sólido indicado em cada alínea.

a. $T_{esf}^{-1}(D) = \left\{(\rho,\theta,\varphi) \in [0,+\infty[\times [0,2\pi] \times [0,\pi] : 1 \leq \rho < \sqrt{2} \wedge 0 \leq \theta < \frac{\pi}{2} \wedge 0 \leq \varphi \leq \frac{\pi}{2}\right\}$.

b. $T_{cil}^{-1}(D) = \left\{(r,\theta,z) \in [0,+\infty[\times [0,2\pi] \times \mathbb{R} : 0 < z < 2 \wedge r \geq \sqrt{2}\, z\right\}$.

c. $T_{cil}^{-1}(D) = \left\{(r,\theta,z) \in [0,+\infty[\times [0,2\pi] \times \mathbb{R} : r \leq \sqrt{3} \wedge 0 \leq \theta \leq \pi \wedge r \leq z \leq \sqrt{6 - r^2}\right\}$.

9.

a. $T_{esf}^{-1}(V) = \left\{(\rho,\theta,\varphi) \in [0,+\infty[\times [0,2\pi] \times [0,\pi] : \rho \leq 1 \wedge \frac{\pi}{4} \leq \varphi \leq \frac{\pi}{3}\right\}$.

b. $T_{esf}^{-1}(V) = \left\{(\rho,\theta,\varphi) \in [0,+\infty[\times [0,2\pi] \times [0,\pi] : \rho \leq 1 \wedge 0 \leq \varphi \leq \frac{\pi}{4}\right\}$.

10. Seja D o conjunto do enunciado.

a. $T_{esf}(D) = \left\{(x,y,z) \in \mathbb{R}^3 : z = -\sqrt{x^2 + y^2}\right\}$.

b. $T_{esf}(D) = \left\{(x,y,z) \in \mathbb{R}^3 : z = \sqrt{x^2 + y^2}\right\}$.

c. $T_{esf}(D) = \left\{(x,y,z) \in \mathbb{R}^3 : y = \frac{\sqrt{3}}{3} x \wedge x \geq 0\right\}$.