AM3C – Exam 2023.1.3 Resolution

Felipe B. Pinto 71951 – EQB

11 de dezembro de 2024

Conteúdo

Grupo I

Questão 1

A equação diferencial linear de primeira ordem

$$\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}\boldsymbol{x}} + \frac{\mathbf{1}}{\mathbf{4}}\,\boldsymbol{x}\,\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{4}}\,\boldsymbol{x}^{\mathbf{3}}$$

Com acondição $y(0) = -4$ tem como solução:

Resposta

$$y = \frac{c_0}{\varphi(x)} + \frac{1}{\varphi(x)} \int \left(\frac{1}{4}\,x^3\right) \varphi(x)\,\mathrm{d}x =$$

Using (1.3)

$$= \frac{c_0}{(c_1\,e^{x^2/8})} + \frac{1}{(c_1\,e^{x^2/8})} \int \left(\frac{1}{4}\,x^3\right)\left(c_1\,e^{x^2/8}\right)\mathrm{d}x =$$

$$= \frac{c_0}{(c_1\,e^{x^2/8})} + \frac{1}{e^{x^2/8}} \int \left(\frac{1}{4}\,x^3\right)\left(e^{x^2/8}\right)\mathrm{d}x =$$

Using (1.4)

$$= c_2\,e^{-x^2/8} + \frac{1}{e^{x^2/8}}\left(\left(x^2-8\right)e^{x^2/8}\right) =$$

$$= c_2\,e^{-x^2/8} + x^2 - 8 = \tag{1.1}$$

Using (1.2)

$$= 4\,e^{-x^2/8} + x^2 - 8$$

$$c_2 = c_0/c_1$$

Using (1.1)

$$y(0) = c_2\,e^{-0^2/8} + 0^2 - 8 = c_2 - 8 = -4 \implies c_2 = 4 \tag{1.2}$$

$$\varphi(x) = \exp\left(\int \frac{1}{4}\,x\,\mathrm{d}x\right) = \exp\left(\frac{1}{4}\left(\frac{x^2}{2}+c\right)\right) = \exp\left(\frac{c}{4}\right)\exp\left(\frac{x^2}{8}\right) = c_1\,e^{\frac{x^2}{8}}; \tag{1.3}$$

$$c_1 = e^{c/4}$$

$$P\left(\left(\frac{1}{4}\,x^3\right)\left(e^{x^2/8}\right)\right) = P\left(\left(x^2\right)\left(e^{x^2/8}\,\frac{x}{4}\right)\right) = P\left(\left(x^2\right)\left(e^{x^2/8}\right)'\right) =$$

$$= x^2\,P\left(\left(e^{x^2/8}\right)'\right) - P\left(P\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{x^2/8}\right)\right)\frac{\mathrm{d}x^2}{\mathrm{d}x}\right) =$$

$$= x^2\,e^{x^2/8} - P\left(e^{x^2/8}\,2\,x\right) = x^2\,e^{x^2/8} - 8\,P\left(e^{x^2/8}\,x/4\right) =$$

$$= \left(x^2-8\right)e^{x^2/8} \tag{1.4}$$

Questão 2

A equação diferencial

$$\mathbf{3\,x\,y^2\ dx + 4\,x^2\,y\ dy = 0}$$

admite um fator integrante da forma $\varphi(x,y) = x\,y^k$, em que k é uma constante real. Encontre k

---

Resposta

$$k : \varphi(x,y) = x\,y^k \implies$$

$$\implies (x\,y^k)\,3\,x\,y^2\ \mathrm{d}x + (x\,y^k)\,4\,x^2\,y\ \mathrm{d}y = 0 \implies$$

$$\implies \frac{\partial}{\partial y}((x\,y^k)\,3\,x\,y^2) = \frac{\partial}{\partial y}(3\,x^2\,y^{2+k}) = (2+k)\big(3\,x^2\,y^{1+k}\big) =$$

$$= \frac{\partial}{\partial x}((x\,y^k)\,4\,x^2\,y) = \frac{\partial}{\partial x}(4\,x^3\,y^{1+k}) = 3\big(4\,x^2\,y^{1+k}\big) \implies$$

$$\implies k = (12-6)/3 = 2$$