QUANTITATIVE ECONOMICS - SPRING 2018
Answer any FIVE questions.

1. Suppose you have the nonlinear regression

$$y_i = \alpha(1+\beta)^{(2x_i-1)} + u_i, \quad E[u_i|x_i] = 0$$

where u_i are zero mean IID error term and IID data $\{(x_i, y_i)\}_{i=1}^{n}$. Describe in detail how you could estimate α and β?

2. Prove that R^2 associated with a restricted least squares estimator is never larger than that associated with the the unrestricted least squares estimator.

3. Consider the following data generating process

$$Y_t = c + \phi_1 Y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

where ε_t is a white noise process. Assuming $0 < \phi_1 < 1$ and $0 < \theta_1 < 1$ answer the following:

   (a) Derive the expected value of this process.
   (b) Derive the variance of this process.
   (c) Derive the covariance of this process for $j = 1, 2, 3$.
   (d) Derive the autocorrelation function of this process for $j = 1, 2, 3$.

4. Consider two processes y_t and x_t where we have $t = 1, 2, \ldots, T$ observations on each. Suppose that y_t represents a 10 year interest rate and x_t represents a three month interest rate. Suppose we are interested in eventually forecasting y_t with the help of both past values of y as well as past values of x. With this information, answer the following questions:

   (a) Write down an ARMA(1,1) which also includes a lagged value for x.
   (b) How do you know how many lags to include for y? How do you know how many lags to include for ε (the error term)?
   (c) How do you know how many lags to include for x?
   (d) Write down the $h - step$ ahead value for y (y_{t+h}) that you put down in part (a).
   (e) For $h = 1$, construct the forecast value of y ($\widehat{y}_{t+h|t}$).

5. A researcher investigating the incidence of teenage knife crime has the following data for each of 35 cities for 2008:

$$\begin{aligned} K &= \text{number of knife crimes per 1,000 population in 2008} \\ N &= \text{number of teenagers per 1,000 population living in social deprivation in 2008} \end{aligned}$$

The researcher hypothesis that the relationship between K and N is given by

$$K = \beta_1 + \beta_2 N + u$$

where u is a disturbance term that satisfies the usual regression model assumptions. However, knife crime tends to be under-reported, with the degree of under-reporting worst in the most heavily afflicted boroughs, so that

$$R = K + w$$

where R =number of reported knife crimes per 1,000 population in 2008 and w is a random variable with $E(w) < 0$ and $cov(w, K) < 0$. w may be assumed to be distributed independently of u. Note that $cov(w, K) < 0$ implies $cov(w, N) < 0$. Derive analytically the sign of the bias in the estimator of β_2 if the researcher regresses R on N using ordinary least squares.

6. You are given the regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

You want to test the null hypothesis

$$\begin{aligned} H_0 &: \beta_1 \beta_3^2 = \beta_2 \\ H_A &: \beta_1 \beta_3^2 \neq \beta_2 \end{aligned}$$

Show how you would construct the test statistic.

7. Consider the regression $Y = X\beta + u$ where X is $n \times k$ matrix and β is a $k \times 1$ vector. $E(u_i) = 0$; $E(u_i^2) = \sigma^2$ and $E(u_i u_j) = 0$ if $i \neq j$. Let $\widehat{\beta}$ denotes the least squares estimator of β.

   (a) Show that $\widehat{\beta}$ is conditionally unbiased.

   (b) Derive the conditional variance of $\widehat{\beta}$.

   (c) Show that $\widehat{\beta}$ is a consistent estimator. (Hint: Show that $Var(\widehat{\beta}) \to 0$ as $n \to \infty$)

   (d) Prove that $\widehat{\beta}$ is **BLUE**.

8. Consider observations (Y_{it}, X_{it}) from the linear panel data model

$$Y_{it} = \alpha_i + \beta_1 X_{it} + u_{it}, \qquad i = 1, \ldots, N, t = 1, \ldots, T.$$

where α_i is an unobserved individual-specific fixed effects, X_{it} is a vector of exogenous regressors and u_{it} are zero mean $i.i.d$ innovations. Describe how you would estimate β_1?