QUANTITATIVE ECONOMICS - FALL 2021
Answer any FIVE questions.

1. Suppose you have the nonlinear regression

$$y_i = \alpha(1+\beta)^{(3x_i-1)} + u_i, \quad E[u_i|x_i] = 0$$

where u_i are zero mean IID error term and IID data $\{(x_i, y_i)\}$ for $i = 1, ..., n$. Describe in detail how you could estimate α and β?

2. Prove that R^2 associated with a restricted least squares estimator is never larger than that associated with the the unrestricted least squares estimator.

3. Consider the following data generating process

$$Y_t = c + \phi_1 Y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

where ε_t is a white noise process. Assuming $0 < \phi_1 < 1$ and $0 < \theta_1 < 1$ answer the following:

   (a) Derive the expected value of this process.
   (b) Derive the variance of this process.
   (c) Derive the covariance of this process for $j = 1, 2, 3$.
   (d) Derive the autocorrelation function of this process for $j = 1, 2, 3$.

4. Consider two processes y_t and x_t where we have $t = 1, 2, \ldots, T$ observations on each. Suppose that y_t represents a 10 year interest rate and x_t represents a three month interest rate. Suppose we are interested in eventually forecasting y_t with the help of both past values of y as well as past values of x. With this information, answer the following questions:

   (a) Write down an ARMA(1,1) which also includes a lagged value for x.
   (b) How do you know how many lags to include for y? How do you know how many lags to include for ε (the error term)?
   (c) How do you know how many lags to include for x?
   (d) Write down the $h - step$ ahead value for y (y_{t+h}) that you put down in part (a).
   (e) For $h = 1$, construct the forecast value of y ($\widehat{y}_{t+h|t}$).

5. Suppose that you are asked to conduct a study to determine whether smaller class sizes lead to improved student performance of fourth graders.

   (a) Design a randomized experiment that captures the causal link between the class size and student performance? Be specific.
   (b) In your designed randomized experiment model, show that the observed difference in the dependent variable average is equal to the treatment effect and the potential selection bias.
   (c) Explain how randomized experiment can solve the selection bias problem.
   (d) More realistically, suppose you can collect observational data on several thousand fourth graders in a given state. You can obtain the size of their fourth-grade class and a standardized test score taken at the end of fourth grade. Why might you expect a negative correlation between class size and test score? Would a negative correlation necessarily show that smaller class sizes cause better performance? Explain.

6. You are given the regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

You want to test the null hypothesis

$$\begin{aligned} H_0 &: \beta_1\beta_3 = \beta_2^2 \\ H_A &: \beta_1\beta_3 \neq \beta_2^2 \end{aligned}$$

Show how you would construct the test statistic.

7. We want to estimate the causal impact of the variable S_i on the variable Y_i, using an instrument variable (IV) Z_i in the following model with covariates:

$$Y_i = \alpha' X_i + \rho s_i + \eta_i$$

   (a) Write down the first stage and the reduced form equations for the IV model.
   (b) Show that the IV estimator in your model can be written as a function of two coefficients from the first stage and reduced form equations.
   (c) For a Two-Stage Least Squares model, derive the reduced form equation using the first stage equation and the above the causal relation of interest.
   (d) Explain the exclusion restriction and the strong first stage assumptions for a using IVs.

8. In 1985, neither Florida nor Georgia had laws banning open alcohol containers in vehicle passenger compartments. By 1990, Florida had passed such a law, but Georgia had not. Suppose you can collect random samples of the driving-age population in both states, for 1985 and 1990. Let arrest be a binary variable equal to unity if a person was arrested for drunk driving during the year.

   (a) Using a diff-in-diff identification strategy, write down a linear probability model (LPM) that allows you to test whether the open container law reduced the probability of being arrested for drunk driving. Which coefficient in your model measures the effect of the law?
   (b) What is the interpretation of the coefficient of interest in your LPM model?
   (c) How will the interpretation be different if you use a Logit model instead of the LPM?
   (d) What are the advantages and disadvantages of the LPM when compared to other models with limited dependent variables?