HOWARD UNIVERSITY
DEPARTMENT OF ECONOMICS

CODE NUMBER---------- **TOTAL NUMBER OF PAGES----------**

DATE----------

COMPREHENSIVE EXAMINATION: Spring 2021 Microeconomic Theory Ph.D.

EXAMINERS:

1. **Dr. Omari H. Swinton, Chairperson**
2. **Dr. Deniz Baglan**
3. **Dr. Alexander Henke**

1. **The examination is scheduled between the hours: 9:30 a.m-1.00 pm**

   ALL STUDENTS ARE TO BE ON CAMERA BY 9:25 a.m.
2. **YOU ARE REQUIRED TO ANSWER ONLY FIVE (5) QUESTIONS.**

   Any additional questions answered over the required number from each category will NOT receive credit.
3. **Correct answers to questions NOT asked will receive NO credit.**
4. **Be sure to write the Code Number assigned to you in the TOP LEFT HAND CORNER OF THIS SHEET AND ON EACH ANSWER SHEET. DO NOT WRITE YOUR NAME ON ANY SHEET OF THE EXAMINATION.**
5. **Begin each question on a new page. Number each page used in sequence. Write only on one side of the paper.**
6. **Write clearly and illustrate your answers with graphs whenever and wherever possible.**
7. **USE ONLY BLACK INK PENS.**
8. **At the end of the examination, please indicate the total number of pages being submitted in the space provided in the TOP RIGHT HAND CORNER of this sheet.**
9. **PLEASE SUBMIT A PICTURE OF EACH PAGE TO GPRITCHETT@HOWARD.EDU AND CONFIRM THAT THE PICUTRE WAS RECEIVED BEFORE EXITING THE EXAM.**

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1. **Bring your pens, pencils, calculators and rulers.**
2. **No briefcases, cell phones, book bags or sacks, no handbags larger than 10 x 6 of any form are to be near you when taking the exam.**
3. **No books, notes or other study material are to be near you when taking the exam.**
4. **You can only used an unopened pack of paper that is opened on camera.**
5. **You must show a video of your sitting area before you can be approved to take the exam. This will confirm that the testing area is clear. Your video must remain on the entire time. Turn off any virtual background.**
6. **Use *private* Zoom chat to ask questions of the proctor.**
7. **During the Examination there is to be no communication between or amongst students or anyone for any purpose. All questions must be directed to and channeled through the faculty member conducting the examination.**
8. **Students are not expected to leave their chairs before completing their examination and turning it in to the proctor.**
9. **NO FOOD OR SMOKING is permitted during the exam.**
10. **The student may have a drink with them during the exam.**
11. **If you get disconnected for any reason, please call the proctor. Upon reentry to the test, you must verify that your area is clear again. In the event that you cannot reconnect, please contact the Associate Chair of the department to monitor the remainder of the exam via phone.**
12. **EXAMINATION ZOOM INFORMATION WILL ONLY BE GIVEN TO STUDENTS WHO ARE REGISTERED.**

Revised 1/25/2020

CODE NUMBER_____

STUDENTS: PLEASE CIRCLE ONLY THE QUESTIONS ANSWERED AND PROVIDE THE PAGE NUMBERS.

QUESTIONS	PAGE NUMBERS
1.	
2.	
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1.Let $U(x,y) = xy^4$. Find the following:

a. the Marshallian demand functions for x and y.
b. the indirect utility function.
c. the compensated (Hicksian) demand functions for x and y.
d. the expenditure function.
e. Use your answers to a and d to derive the Hicksian demand functions for x and y. Compare these to your answers to c. Explain what you are doing.
f. Use your answers to b and c to derive the Marshallian demand functions. Compare these to your answers for a. Explain what you are doing.
g. Write the Slutsky equation for good x.

2.A seller's production function is $q = f(k,l) = (kl)^{0.25}$, $w = v = 1$, and the competitive output price is P.

a. Does the production function exhibit constant, increasing, or diminishing returns to scale?
b. For part (b) only, suppose k is fixed at $k_1 = 16$ and labor is variable. Find the short run cost function $C_{SR}(q)$.

For all parts going forward, assume we are in the long run and all inputs are variable.

c. Find the long run cost function $C(q)$.
d. Find conditional demand for labor.
e. Find the profit-maximizing q.
f. Find unconditional demand for labor.
g. Find the profit function.

3. Suppose the production possibility frontier for an economy that produces one publicgood (y) and oneprivate good (x) is given bySuppose we have a representative consumer with a utility function $U(x,y) = x^4y$,industry x with a production function $x = f(k_x, l_x) = \left(\sqrt{k_x} + \sqrt{l_x}\right)^2$, and industry y with a production function $y = f(k_y, l_y) = \sqrt{k_y} + \sqrt{l_y}$. Total capital and labor endowments are as follows: $l_x + l_y = 1; k_x + k_y = 1$.

a. Find the RTS for industries x and y.
b. Find the condition for *technical efficiency* using the RTS from each industry.
c. Combine your answer from part (b) with the given labor and capital endowments to construct the PPF.
d. Using your answer from part (c), find the Rate of Product Transformation (RPT).
e. Find the representative consumer's Marginal Rate of Substitution (MRS).
f. Find the efficient x and y.

4. An auctioneer holds a second-price auction for two bidders, Ann (A) and Bonnie (B), who have independent private values of the good θ_A and θ_B.If a bidder wins, her payoff is her valueθ minus the price she pays, and if she loses, her payoff is 0. The valuesareindependently and identically distributed, but otherwise you don't need to know the specific distributions to solve the problem. Ann and Bonnie's respective strategies are to bid some value $b_i(\theta_i)$, that is, bid given their privately-known value (type).
   a. Explain what a second price auction is, who wins given some pair of bids b_A and b_B, and what the winner pays.
   b. Why is a strategy where Ann bids above her own value θ_A weakly dominated by a strategy where she bids her value?
   c. Why is a strategy where Ann bids below her own value θ_A weakly dominated by a strategy where she bids her value?
   d. Applying the ideas from (b) and (c) to both Ann and Bonnie, what is the Weakly Dominant Strategy Equilibrium for this game?
   e. Suppose the good had one true value for both bidders equal to the average of θ_A and θ_B (signals that are still i.i.d.); hence, the good's true value has a *common component*. Suppose Ann knows Bonnie is going to bid her own evaluation θ_B no matter what, but like normal, Ann doesn't know θ_B. Explain why bidding θ_A is now a strictly dominate*d* strategy for Ann.

5. Duopolist firms 1 and 2 compete on price in a market with some product differentiation. Firms set their own prices p_1 and p_2 respectively. Demand for duopolist$i$ is $q_i = 1 - p_i + 0.5p_j$. Neither firm has any cost, so a firm's payoff is simply its own revenue.
   a. Find the best response functions for each firm in the static game.
   b. Find the unique Nash equilibrium prices in the static game.
   c. Are prices strategic complements or strategic substitutes? Briefly explain.
   d. Does Firm 1 want Firm 2 to increase or decreasep_2? Don't explain.

For parts (e) and (f), modify the game as follows: Firm 1 sets p_1, Firm 2 observesp_1, *then* Firm 2 sets p_2, then goods are sold and payoffs are determined as normal.
   e. Either by using your answer from part (f) or with intuition, note whether prices will be higher or lower for Firm 1 and then for Firm 2compared to part (b).
   f. Find the new unique subgame perfect Nash equilibrium prices.

6. Consider a Bertrand duopoly where two firms (1 and 2) with zero costs wish to maximize their own revenue. Demand can be written as $q = 10 - p$, where p is the prevailing market price. Firms simultaneously set p_1 and p_2; if $p_1 = p_2$, they split the market and each obtains a payoff $0.5(10 - p)p$. If one firm sets a lower price (undercuts the other), that firm takes the market and gets a payoff of $(10 - p)p$ while the other firm gets a payoff of 0. When it is relevant, the discount factor is δ.
   a. Find the Nash Equilibrium for this static game.
   b. If both parties could write a binding contract that set prices, what is the most profitable symmetric price they could set? Label this price p_M.
   c. Suppose the game is repeated 10 times. Write the outcome (prices) for all SPNE.
   d. Suppose the game is repeated indefinitely. Construct a *grim trigger* set of strategies that induce each player to play p_M indefinitely.
   e. What is the lowest δ can be where the solution from part (d) works?

7. A seller sells a good to a prospective buyer. The buyer values the good at θq, where θ is his (privately known) marginal utility of quality and q is the good's quality. It is common

knowledge that θ is high ($\theta = 2$) with probability $\frac{1}{4}$ and θ is low ($\theta = 1$) with probability $\frac{3}{4}$. The monopolist incurs a cost based on quality $c(q) = \frac{q^2}{2}$, so that his profit is $p - \frac{q^2}{2}$. The buyer can reject an offer (not buy anything) and get a payoff of 0, or he can buy a good and get a payoff of $\theta q - p$. The seller offers a menu consisting of $\{p_1, q_1, p_2, q_2\}$, where the subscript means the price and quality is meant for the seller of that type (θ), and the buyer picks which good she wants.

a. Suppose, for part (a) only, the seller observes θ directly and can offer a single type of good based on the buyer's type $\{p_i, q_i\}$, $i \in \{1,2\}$. Construct the optimal price/quality combination when $\theta = 1$ and when $\theta = 2$.
b. Suppose the seller asks the buyer what his type is, assumes he answers honestly, and offers a contract like in part (a). Who will lie, and why?
c. Construct the optimal contract where everyone buys their appropriate good.
d. Whose good has an inefficient level of quality? Is it too high or low? Why does the seller do this?

8. A monopolist's cost function is $C(q) = 1 + \frac{q^2}{2}$. The firm faces a market demand curve given by $q = 10 - p$.
   a. Calculate the profit-maximizing price-quantity combination for the monopolist. Also calculate the monopolist's profits, producer surplus, and consumer surplus.
   b. What output level would be produced by this industry under perfect competition?
   c. Calculate consumer surplus and producer surplus in case (b).
   d. Calculuate the deadweight loss of monopoly.