

**HOWARD UNIVERSITY  
DEPARTMENT OF ECONOMICS**

**CODE NUMBER**\_\_\_\_\_

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**COMPREHENSIVE EXAMINATION: Fall 2023 Microeconomic Theory**

**Ph.D. EXAMINERS:**

1. Dr. Omari H. Swinton, Chairperson
2. Dr. Amirhossein Amini Behbahani
3. Dr. Haydar Kurban

1. The examination is scheduled between the hours: 9:30 a.m-1.00 pm  
**ALL STUDENTS ARE TO BE SEATED BY 9:15 a.m.**

2. **YOU ARE REQUIRED TO ANSWER ONLY FIVE (5) QUESTIONS.**

Any additional questions answered over the required number from each category will NOT receive credit.

3. Correct answers to questions NOT asked will receive NO credit.

4. Be sure to write the Code Number assigned to you in the TOP LEFT HAND CORNER OF THIS SHEET AND ON EACH ANSWER SHEET. DO NOT WRITE YOUR NAME ON ANY SHEET OF THE EXAMINATION.

5. Begin each question on a new page. Number each page used in sequence. Write only on one side of the paper.

6. Write clearly and illustrate your answers with graphs whenever and wherever possible.

7. **USE ONLY BLACK INK PENS.**

8. At the end of the examination, please indicate the total number of pages being submitted in the space provided in the TOP RIGHT HAND CORNER of this sheet.

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- 1. Bring your pens, pencils, calculators and rulers.**
- 2. No briefcases, book bags or sacks, no handbags larger than 10 x 6 of any form are to be brought into the examination room.**
- 3. No books, notes or other study material are to be brought into the examination room.**
- 4. During the Examination there is to be no communication between or amongst students for any purpose. All questions must be directed to and channeled through the faculty member conducting the examination.**
- 5. Only the scrap paper provided by the proctor is to be used for the examination. Scrap paper should bear the code number assigned to each student, and be handed over to the proctor along with the examination.**
- 6. Students are not expected to leave the examination room before completing their examination and turning it in to the proctor.**
- 7. NO FOOD OR SMOKING is permitted in the examination room.**
- 8. It is the student's responsibility to remove any coffee or water containers taken into the examination room.**
- 9. NO CELL PHONES ARE ALLOWED.**
- 10. EXAMINATION RESULTS WILL ONLY BE GIVEN TO STUDENTS WHO ARE REGISTERED.**

**CODE NUMBER**\_\_\_\_\_

**STUDENTS: PLEASE CIRCLE ONLY THE QUESTIONS ANSWERED AND  
PROVIDE THE PAGE NUMBERS.**

<b>QUESTIONS</b>	<b>PAGE NUMBERS</b>
<b>1.</b>	
<b>2.</b>	
<b>3.</b>	
<b>4.</b>	
<b>5.</b>	
<b>6.</b>	
<b>7.</b>	
<b>8.</b>	

Writing out some acronyms:

NE is Nash Equilibrium

SPNE is Subgame Perfect Nash Equilibrium

PBE is Perfect Bayesian Equilibrium

1. Let  $U(x, y) = \sqrt{x} + \sqrt{y}$ . Find the following:
- the Marshallian demand functions for x and y.
  - the indirect utility function.
  - the compensated (Hicksian) demand functions for x and y.
  - the expenditure function.
  - Explain, in words, how you could have derived Marshallian demand in a different way.
  - Explain, in words, how you could have derived Hicksian demand in a different way.
  - Write the Slutsky equation for good x and show both sides of the equation are equal.
  - Suppose there is a per-unit tax imposed on good x and the government collects revenue R from this tax. If the government instead directly collected the amount R from the consumer's budget (I), would the consumer be better off, worse off, or just as well off, and why?

2. Small sellers operate in a competitive market. Each seller has a production function  $q = f(k, l) = (lk)^{0.25}$  and face competitive input prices  $w = v = 1$ . In addition to production costs, each firm faces a fixed cost of 1 just for existing (compliance costs, etc.). The only way to avoid that fixed cost is to exit the industry. The demand curve for the entire market can be described as  $P = \frac{100}{Q}$ . **In the short run**, capital for each firm is fixed at  $k = 1$ , there are 40 firms, and there is no entry or exit.

- Find a firm's **short-run** cost function. Remember to include all costs.
- Find a firm's **short-run** supply curve (not the market supply curve, yet).
- Find the **short-run** equilibrium quantity and price.

In the **long run**, capital is variable, and so is the number of firms (N).

- Find a firm's **long-run** cost function  $C(q)$ .
- Find a firm's **long-run** supply curve (again, individual firm).
- For some number of firms N, find equilibrium price and quantity in the **long run**.
- Using the zero-profit condition, find the **long-run** equilibrium number of firms N. You don't have to calculate anything else.

3. Suppose the production possibility frontier for an economy that produces one public good (y) and one private good (x) is given by

$$x^2 + 10y^2 = 44$$

This economy is populated by 100 identical individuals, each with a utility function of the form

$$U = \sqrt{x_i} + \sqrt{y}$$

where  $x_i$  is the individual's share of private good production ( $=x/100$ ). Notice that the public good is nonexclusive and that everyone benefits equally from its level of production.

- a. If the market for  $x$  and  $y$  were perfectly competitive, what levels of those goods would be produced? What would the typical individual's utility be in this situation?
  - b. What are the optimal production levels for  $x$  and  $y$ ? What would the typical individual's utility level be?
4. An auctioneer holds a second-price auction for two bidders, Ann (A) and Bonnie (B), who have independent private values of the good  $\theta_A$  and  $\theta_B$ . If a bidder wins, her payoff is her value  $\theta$  minus the price she pays, and if she loses, her payoff is 0. The values are independently and identically distributed, but otherwise you don't need to know the specific distributions to solve the problem. Ann and Bonnie's respective strategies are to bid some value  $b_i(\theta_i)$ , that is, bid given their privately-known value (type).
  - a. Explain what a second price auction is, who wins given some pair of bids  $b_A$  and  $b_B$ , and what the winner pays.
  - b. Why is a strategy where Ann bids above her own value  $\theta_A$  weakly dominated by a strategy where she bids her value?
  - c. Why is a strategy where Ann bids below her own value  $\theta_A$  weakly dominated by a strategy where she bids her value?
  - d. Applying the ideas from (b) and (c) to both Ann and Bonnie, what is the Weakly Dominant Strategy Equilibrium for this game?
  - e. Suppose the good had one true value for both bidders equal to the average of  $\theta_A$  and  $\theta_B$  (signals that are still i.i.d.); hence, the good's true value has a *common component*. Suppose Ann knows Bonnie is going to bid her own evaluation  $\theta_B$  no matter what, but like normal, Ann doesn't know  $\theta_B$ . Explain why bidding  $\theta_A$  is now a strictly dominated strategy for Ann.
5. Duopolist firms 1 and 2 set prices  $p_1$  and  $p_2$ , and demand for firm  $i \in \{1,2\}$  is  $q_i = 5 - p_i + 0.5p_2$ , and costs for firm  $i \in \{1,2\}$  is  $c(q_i) = q_i$ .
  - a. Find the best response functions for each firm in the static game.
  - b. Find the unique Nash equilibrium prices in the static game.
  - c. Find the prices that would be formed by a cartel.
  - d. Suppose the static game is repeated 100 times. Find all SPNE.
  - e. Suppose the game is repeated indefinitely. Describe a set of strategies which form a SPNE where both firms play cartel prices.
  - f. How low can the discount factor  $\delta$  be?

6. A law firm hires a new associate and (initially) puts her in charge of filing paperwork. It is common knowledge that the associate is talented with probability 0.6 and not talented with probability 0.4, and the associate observes her type directly. Filing paperwork for  $t$  hours per day costs the associate  $t$  if she is talented and  $2t$  if she is not. The firm observes  $t$  and then makes the decision to either make her partner or fire her ("up or out"). The payoff for the associate of being fired is zero minus the cost of paperwork, and the payoff of being made partner is 10 minus the cost of paperwork. The payoff for the firm does not depend on the paperwork; it is 0 for firing her in any event, 5 for making a talented lawyer a partner, and -20 for making her partner if she is not talented. Let  $\mu_t$  represent the firm's updated belief that the associate is talented given she puts in  $t$  hours of work per day.
  - a. Describe the unique outcome ( $t$ , partner or fire) of all pooling PBE for this game, along with the belief on the equilibrium path. Then describe more generally what the beliefs have to be off the equilibrium path.
  - b. In any separating PBE, if the associate isn't talented, how many hours of paperwork  $t$  does she do per day?
  - c. What are the range of possible values for  $t$  for the talented associate that can be supported in a separating PBE?
  - d. Find the unique PBE that survives the intuitive criterion (equilibrium dominance). Is it separating or pooling?
  
7. A seller sells a good to a prospective buyer. The buyer values the good at  $\theta q$ , where  $\theta$  is his (privately known) marginal utility of quality and  $q$  is the good's quality. It is common knowledge that  $\theta$  is high ( $\theta = 2$ ) with probability  $\frac{1}{4}$  and  $\theta$  is low ( $\theta = 1$ ) with probability  $\frac{3}{4}$ . The monopolist incurs a cost based on quality  $c(q) = \frac{q^2}{2}$ , so that his profit is  $p - \frac{q^2}{2}$ . The buyer can reject an offer (not buy anything) and get a payoff of 0, or he can buy a good and get a payoff of  $\theta q - p$ . The seller offers a menu consisting of  $\{p_1, q_1, p_2, q_2\}$ , where the subscript means the price and quality is meant for the seller of that type ( $\theta$ ), and the buyer picks which good she wants.
  - a. Suppose, for part (a) only, the seller observes  $\theta$  directly and can offer a single type of good based on the buyer's type  $\{p_i, q_i\}$ ,  $i \in \{1, 2\}$ . Construct the optimal price/quality combination when  $\theta = 1$  and when  $\theta = 2$ .
  - b. Suppose the seller asks the buyer what his type is, assumes he answers honestly, and offers a contract like in part (a). Who will lie, and why?
  - c. Construct the optimal contract where everyone buys their appropriate good.
  - d. Whose good has an inefficient level of quality? Is it too high or low? Why does the seller do this?
  
8. A monopolist's cost function is  $C(q) = \frac{q^2}{2}$ . The firm faces a market demand curve given by  $q = 16 - p$ .
  - a. Calculate the profit-maximizing price-quantity combination for the monopolist. Also calculate the monopolist's profits, producer surplus, and consumer surplus.
  - b. What output level would be produced by this industry under perfect competition?
  - c. Calculate consumer surplus and producer surplus in case (b).
  - d. Calculate the deadweight loss of monopoly.
  - e. Describe first degree price discrimination. If the monopolist could do this, would efficiency increase or decrease? Producer surplus? Consumer surplus?