Example State-Dependent Pricing 2

1 Intoduction

- Some comment

2 Model

- Production function
$$y_t(h) = Z_t a_t(h) \ell_t(h) \tag{2.1}$$
- Each firm h chooses prices $\{p_t\}_t$ in order to maximize its market value
$$\mathbb{E}_t \sum_{\tau=0}^{\infty} D_{t,t+\tau} \Pi_{t+\tau}(h) \tag{2.2}$$
where $D_{t,t+\tau}$ is the nominal stochastic discount factor of the agent and $\Pi_{t+\tau}(h)$ are the nominal profits in period t given by
$$\Pi_t(h) = p_t(h) y_t(h) - W_t \ell_t(h) - \xi_t(h) \mathbb{1}\{p_t(h) \neq p_{t-1}(h)\} \tag{2.3}$$
- Value function for the firm
$$\begin{aligned} V_t\left(a_t(h), \frac{p_{t-1}(h)}{P_t}, \xi_t;\ \cdot\ \right) = \max_p \Bigg\{ \Pi^R\left(a_t(h), \frac{p_{t-1}(h)}{P_t}, \cdot\right) - \mathbb{1}\{p \neq p_{t-1}(h)\}\xi + \\ + \mathbb{E}_t\left[D^R_{t,t+1} V_{t+1}\left(a_{t+1}(h), \frac{p}{P_{t+1}}, \xi_{t+1}, \cdot\right)\right]\Bigg\} \end{aligned} \tag{2.4}$$
- Rewriting the problem
$$v(a, \tilde{p}_{-1};\ \cdot\ ) = \int_\xi \max\Big\{ V^A(a, \tilde{p}_{-1};\ \cdot\ ) - \xi w(\cdot),\ V^N(a, \tilde{p}_{-1};\ \cdot\ )\Big\} dH(\xi)$$
where
$$\begin{aligned} V^A(a, \tilde{p}_{-1};\ \cdot\ ) &= \max_{\tilde{p}} \left\{ \Pi^R\left(a, \tilde{p}, \cdot\right) + \mathbb{E}\Big[D^R(\cdot,\cdot) v\Big(a, \tilde{p}\pi^{-1}_{t+1};\ \cdot\ \Big)\Big]\right\} \\ V^N(a, \tilde{p}_{-1};\ \cdot\ ) &= \Pi^R\left(a, \tilde{p}_{-1}, \cdot\right) + \mathbb{E}\Big[D^R(\cdot,\cdot) v\Big(a', \tilde{p}_{-1}\pi^{-1}_{t+1};\ \cdot\ \Big)\Big] \end{aligned} \tag{2.5}$$
The firm will choose to pay the fixed cost iff $V^A - \xi \geq V^N$. Hence, for each individual state $a, \tilde{p}_{-1}$ there is a unique threshold which makes the firm indifferent between these two options
$$\tilde{\xi}(a, \tilde{p};) = \frac{V^A(a, \tilde{p}) - V^N(a, \tilde{p})}{w}$$
- The firm value function V is therefore given by
$$v(a, \tilde{p}_{-1};\ \cdot\ ) = \int_0^{\xi(a, \tilde{p}_{-1})} \Big[V^A(a, \tilde{p}_{-1};\ \cdot\ ) - \xi w(\cdot)\Big] d\xi + \Big[1 - H\Big(\tilde{\xi}(a, \tilde{p}_{-1};)\Big)\Big] V^N(a, \tilde{p}_{-1};\ \cdot\ ) \tag{2.6}$$

2.1 Household

2.2 Equilibrium

Equilibrium. *A recursive competitive equilibrium is a set of value functions* $\left\{v, V^A, V^N\right\}$*, policies* $\{\tilde{p}, \xi\}$ *for the firm and household* $\left\{C(), N()\right\}$*, and wage* $w(\cdot)$ *such that*

1. *Firm optimization*
   Taking $w(), Y()$ *as given the value function solves the Bellman equation and the* $\{\tilde{p}, \xi\}$ *are the associated policies*
2. *Household optimization*

$$R_t \mathbb{E}\left\{\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{P_t}{P_{t+1}}\right\} = 1, \qquad N^{1/\varphi} = \frac{1}{\chi} C^{-\sigma} w(\cdot)$$

3. *Market clearing*

- *Labor market*

$$\left(\frac{1}{\chi} C^{-\sigma} w\right)^{\varphi} = \int \left[\frac{\tilde{p}(a, \tilde{p}_{-1})^{-\epsilon} Y}{a} + \left(\int^{\xi()} \zeta dH(\zeta)\right) w\right] d\mu \tag{2.7}$$

- *Goods market*

$$\begin{aligned} C_t = Y_t &= \left(\int_0^1 y(h)^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\int_0^1 y(a, \tilde{p}_{-1}) d\mu\right)^{\frac{\epsilon}{\epsilon-1}} \end{aligned} \tag{2.8}$$

where $y(a, \tilde{p}_{-1}) = \left(\tilde{p}(a, \tilde{p}_{-1})\right)^{-\epsilon} Y$

4. *Law of motion Distribution*

2.3 Computation

Compute Steady State

1. Guess a value for the wage w^*
2. Given w^* compute the firm's value function by iterating on Bellman equation. *Note that* Y *can be suppressed from the stationary Bellman because it is a multiplicative constant.*
3. Using firm's decision rules, compute the invariant distribution
4. Compute aggregate supply using the invariant distribution

$$\frac{C}{Y} = \left(\int \tilde{p}(a, \tilde{p}_{-1})^{1-\epsilon} d\mu\right)^{\frac{\epsilon}{\epsilon-1}}$$

   If < 1 increase w otherwise decrease w (*Check on code*)

2.4 How to do the perturbation of policy

Policy outside steady state must satisfy

$$x'(z,x;X_t) = \underset{x'}{\operatorname{argmax}} \left\{ U(z,x,x';X_t) + \beta \ \mathrm{itp}\Big[V_{E,t}, V_E^*\Big](x') \right\} \tag{2.9}$$

where

$$\mathrm{itp}\Big[V_{E,t}^*, V_E^*\Big](\tilde{x}) = V_E^*(\tilde{x}) + \frac{\tilde{x}-\bar{x}_i}{\bar{x}_{i+1}-\bar{x}_i}\Big(V_{E,t}[i+1] - V_{E,t}^*(\bar{x}_{i+1})\Big) + \frac{\bar{x}_{i+1}-\tilde{x}}{\bar{x}_{i+1}-\bar{x}_i}\Big(V_{E,t}[i] - V_{E,t}^*(\bar{x}_i)\Big) \tag{2.10}$$

so that in steady state the whole expression reduces to simply $V^*(\tilde{x})$.

Taking *foc*

$$0 = U_{x'}\Big(z,x,x'(z,x;X_t);X_t\Big) + \beta\left[\frac{dV_E^*(x'(z,x;X_t))}{dx} + \frac{dV_{E,t}[i+1]-dV_{E,t}[i]}{\bar{x}_{i+1}-\bar{x}_i}\right] \tag{dvdc}$$

which at steady-state satisfy

$$0 = U_{x'}\Big(z,x,x^*(z,x);X^*\Big) + \beta\left[\frac{dV_E^*\Big(x^*(z,x)\Big)}{dx}\right]$$

while the *soc* at steady state values

$$U_{x'x'}\Big(z,x,x^*(z,x);X^*\Big) + \beta\frac{d^2V^*\Big(x^*(z,x)\Big)}{dx^2} \tag{d2vdc2}$$

Hence

$$\frac{dx'\Big(z,x;X_t\Big)}{dX_t} = -\frac{1}{\mathrm{d2vdc2}}\Big(\mathrm{d2vdc2.der}\Big) \tag{2.11}$$