

Example State-Dependent Pricing 2

1 INTRODUCTION

- Some comment

2 MODEL

- Production function

$$y_t(h) = Z_t a_t(h) \ell_t(h) \quad (2.1)$$

- Each firm h chooses prices $\{p_t\}_t$ in order to maximize its market value

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} D_{t,t+\tau} \Pi_{t+\tau}(h) \quad (2.2)$$

where $D_{t,t+\tau}$ is the nominal stochastic discount factor of the agent and $\Pi_{t+\tau}(h)$ are the nominal profits in period t given by

$$\Pi_t(h) = p_t(h) y_t(h) - W_t \ell_t(h) - \xi_t(h) \mathbb{1}\{p_t(h) \neq p_{t-1}(h)\} \quad (2.3)$$

- Value function for the firm

$$\begin{aligned} V_t \left(a_t(h), \frac{p_{t-1}(h)}{P_t}, \xi_t; \cdot \right) = \max_p \left\{ \Pi^R \left(a_t(h), \frac{p_{t-1}(h)}{P_t}, \cdot \right) - \mathbb{1}\{p \neq p_{t-1}(h)\} \xi + \right. \\ \left. + \mathbb{E}_t \left[D_{t,t+1}^R V_{t+1} \left(a_{t+1}(h), \frac{p}{P_{t+1}}, \xi_{t+1}, \cdot \right) \right] \right\} \end{aligned} \quad (2.4)$$

- Rewriting the problem

$$v(a, \tilde{p}_{-1}; \cdot) = \int_{\xi} \max \left\{ V^A(a, \tilde{p}_{-1}; \cdot) - \xi w(\cdot), V^N(a, \tilde{p}_{-1}; \cdot) \right\} dH(\xi)$$

where

$$\begin{aligned} V^A(a, \tilde{p}_{-1}; \cdot) &= \max_{\tilde{p}} \left\{ \Pi^R(a, \tilde{p}, \cdot) + \mathbb{E} \left[D^R(\cdot, \cdot) v(a, \tilde{p} \pi_{t+1}^{-1}; \cdot) \right] \right\} \\ V^N(a, \tilde{p}_{-1}; \cdot) &= \Pi^R(a, \tilde{p}_{-1}, \cdot) + \mathbb{E} \left[D^R(\cdot, \cdot) v(a', \tilde{p}_{-1} \pi_{t+1}^{-1}; \cdot) \right] \end{aligned} \quad (2.5)$$

The firm will choose to pay the fixed cost iff $V^A - \xi \geq V^N$. Hence, for each individual state a, \tilde{p}_{-1} there is a unique threshold which makes the firm indifferent between these two options

$$\tilde{\xi}(a, \tilde{p}; \cdot) = \frac{V^A(a, \tilde{p}) - V^N(a, \tilde{p})}{w}$$

- The firm value function V is therefore given by

$$v(a, \tilde{p}_{-1}; \cdot) = \int_0^{\tilde{\xi}(a, \tilde{p}_{-1})} \left[V^A(a, \tilde{p}_{-1}; \cdot) - \xi w(\cdot) \right] d\xi + \left[1 - H(\tilde{\xi}(a, \tilde{p}_{-1})) \right] V^N(a, \tilde{p}_{-1}; \cdot) \quad (2.6)$$

2.1 HOUSEHOLD

2.2 EQUILIBRIUM

Equilibrium. A recursive competitive equilibrium is a set of value functions $\{v, V^A, V^N\}$, policies $\{\tilde{p}, \xi\}$ for the firm and household $\{C(), N()\}$, and wage $w(\cdot)$ such that

1. Firm optimization

Taking $w(), Y()$ as given the value function solves the Bellman equation and the $\{\tilde{p}, \xi\}$ are the associated policies

2. Household optimization

$$R_t \mathbb{E} \left\{ \beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{P_t}{P_{t+1}} \right\} = 1, \quad N^{1/\varphi} = \frac{1}{\chi} C^{-\sigma} w(\cdot)$$

3. Market clearing

• Labor market

$$\left(\frac{1}{\chi} C^{-\sigma} w \right)^\varphi = \int \left[\frac{\tilde{p}(a, \tilde{p}_{-1})^{-\epsilon} Y}{a} + \left(\int^{\xi(\cdot)} \zeta dH(\zeta) \right) w \right] d\mu \quad (2.7)$$

• Goods market

$$\begin{aligned} C_t = Y_t &= \left(\int_0^1 y(h)^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\int_0^1 y(a, \tilde{p}_{-1}) d\mu \right)^{\frac{\epsilon}{\epsilon-1}} \end{aligned} \quad (2.8)$$

where $y(a, \tilde{p}_{-1}) = \left(\tilde{p}(a, \tilde{p}_{-1}) \right)^{-\epsilon} Y$

4. Law of motion Distribution

2.3 COMPUTATION

Compute Steady State

1. Guess a value for the wage w^*
2. Given w^* compute the firm's value function by iterating on Bellman equation. *Note that Y can be suppressed from the stationary Bellman because it is a multiplicative constant.*
3. Using firm's decision rules, compute the invariant distribution
4. Compute aggregate supply using the invariant distribution

$$\frac{C}{Y} = \left(\int \tilde{p}(a, \tilde{p}_{-1})^{1-\epsilon} d\mu \right)^{\frac{\epsilon}{\epsilon-1}}$$

If < 1 increase w otherwise decrease w ([Check on code](#))

2.4 PERTURBING THE VALUE FUNCTION

$$\text{itp}[V_{E,t}^*, V_E^*](\tilde{x}) = V_E^*(\tilde{x}) + \frac{\tilde{x} - \bar{x}_i}{\bar{x}_{i+1} - \bar{x}_i} (V_{E,t}[i+1] - V_{E,t}^*(\bar{x}_{i+1})) + \frac{\bar{x}_{i+1} - \tilde{x}}{\bar{x}_{i+1} - \bar{x}_i} (V_{E,t}[i] - V_{E,t}^*(\bar{x}_i)) \quad (2.9)$$

2.5 HOW TO DO THE PERTURBATION OF POLICY

Policy outside steady state must satisfy

$$x'(z, x; X_t) = \underset{x'}{\operatorname{argmax}} \left\{ U(z, x, x'; X_t) + \beta \text{itp}[V_{E,t}, V_E^*](x') \right\} \quad (2.10)$$

where

so that in steady state the whole expression reduces to simply $V^*(\tilde{x})$.

Taking *foc*

$$0 = U_{x'}(z, x, x'(z, x; X_t); X_t) + \beta \left[\frac{dV_E^*(x'(z, x; X_t))}{dx} + \frac{dV_{E,t}[i+1] - dV_{E,t}[i]}{\bar{x}_{i+1} - \bar{x}_i} \right] \quad (\text{dvdc})$$

which at steady-state satisfy

$$0 = U_{x'}(z, x, x^*(z, x); X^*) + \beta \left[\frac{dV_E^*(x^*(z, x))}{dx} \right]$$

while the *soc* at steady state values

$$U_{x'x'}(z, x, x^*(z, x); X^*) + \beta \frac{d^2 V^*(x^*(z, x))}{dx^2} \quad (\text{d2vdc2})$$

Hence

$$\frac{dx'(z, x; X_t)}{dX_t} = -\frac{1}{\text{d2vdc2}} (\text{d2vdc2.der}) \quad (2.11)$$