Job Ladder and Business Cycles*

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Abstract

I study the aggregate implications of job-to-job flows in a Heterogeneous Agents New Keynesian model. Workers search both off and on-the-job and cannot directly insure against the earnings risk stemming from labor market frictions. The state of the economy includes the distribution of workers over wealth, labor earnings and jobs, with employment dynamics giving rise to a job ladder where employed workers slowly move towards more productive jobs through job-to-job transitions, while occasionally being thrown back into unemployment. Fluctuations in labor market transitions over the cycle have supply and demand-side implications, as job-to-job transitions both reallocate labor towards more productive jobs and drive workers' labor earnings. In the wake of an adverse financial shock, firms reduce hiring, causing unemployment to increase and job-to-job transitions to decline. Essentially, the job ladder 'stops working'. This leaves wages stagnant, triggering demand-side induced consumption and output losses. The accumulation of workers at low-productivity jobs leads to a gradual slowdown in aggregate labor productivity. When calibrated to the US Great Recession unemployment dynamics, the model response matches the slow recovery and missing disinflation of that period.

JEL Codes: D31, D52, E21, E24, E31, E32.

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1 Introduction

The transmission of aggregate shocks to overall production and consumption depends heavily on the mechanics of the labor market, where search and matching frictions shape both the supply and demand side responses. On the supply side, search frictions give rise to unemployment and allow for good (productive) jobs to coexist alongside bad (unproductive) ones. Hence the supply of goods is constrained both by unemployment, which restricts the overall amount of labor used in production, and by the misallocation of employed workers, which affects the level of aggregate productivity. On the demand side, the workers' employment history is an important factor determining their earnings. Unemployment spells have long lasting impact on earnings, while job-to-job transitions drive earnings growth of employed workers. Since labor income is the primary source of workers' overall disposable income and accounts for a significant portion of their income risk, these events directly affects workers' consumption expenditures and precautionary savings decisions. Moreover, these supply and demand effects are dependent on labor market flows (i.e., unemployment to employment and job-to-job flows), which fluctuate in response to aggregate shocks.

To study how these supply and demand forces play out in equilibrium and over the cycle, I develop a Heterogeneous Agents New Keynesian (HANK) model with search and matching frictions. Workers are risk averse and search both off and on-the-job for vacancies posted by firms. Worker–firm matches are heterogeneous in productivity and firms are allowed to counter outside offers received by the worker, like in the sequential auction framework of Postel-Vinay and Robin (2002). This gives rise to a *job ladder*: workers are employed in a range of match productivities in equilibrium, which they ascend through job-to-job transitions while occasionally falling back to unemployment. Workers are subject to the income risk stemming from climbing of falling off the ladder and can only self-insure by saving on a risk-free government bond. The remaining blocks of the model closely follow the New Keynesian tradition. The output of the worker–firm match, which I denote by "labor services", is an input to the production of monopolistically competitive retailers. Retailers produce specialized goods by combining labor services and intermediate material goods and face nominal rigidities. A final good producer combines the varieties produced by retailers to produce the final good, that can be used for final consumption or as an input into production (materials). A government runs an unemployment insurance program, and monetary policy follows a Taylor rule.

Market incompleteness and the job ladder render the cross-sectional distribution of workers over match productivities, earnings, and wealth a state of the economy. Unemployment to employment flows and job-to-job transitions are endogenous and respond to aggregate shocks, moving workers along the ladder with implications for labor earnings and aggregate productivity. In the presence of nominal frictions, these demand and supply forces interact to determine the equilibrium in the labor and goods markets. I calibrate the stationary equilibrium to the US economy. To discipline the job ladder I use information on labor market flows and on *frictional* wage dispersion measures.[1] Labor earnings dynamics are endogenous as they depend on workers' search outcomes and the assumed distribution of firm productivity. These in turn generate a distribution of labor earnings growth characterized by negative skewness and excess kurtosis, consistent with the higher-order moments from the Social Security Administration data documented by Guvenen et al. (2016). Moreover, I evaluate the model's implications against the empirical literature on earnings and consumption dynamics following a job displacement.

My main quantitative exercise focus on the U.S. Great Recession, which I model as a shock to the discount rate of firms (a shortcut for worsening financial frictions). The higher discount reduces firm entry, causes unemployment to increase, and decreases job-to-job transitions, causing the job ladder to essentially 'stop working' (Moscarini & Postel-Vinay, 2016a). Consumption and productivity suffer a persistent and sharp drop, while inflation falls only temporarily and by a moderate amount, consistent with the data.[2] In the model inflation behavior is linked to the failure of the ladder, the effects of which change from initially disinflationary to inflationary later in transition. In the initial periods following the shock, consumption falls sharply in response to the reduction in future income, which (because of price rigidity) leads to a large contraction of marginal costs, hence, on inflation. As time passes, however, the decline in job-to-job transitions slows down worker reallocation up the ladder causing labor productivity to fall. This effect is persistent—in particular, more persistent than unemployment—and puts upward pressure on marginal costs at longer horizons, which partially constrains the initial fall in inflation.[3]

The remainder of the paper explores the demand and supply-side channels operating through the job ladder. Turning to the supply-side effects first, I study a counterfactual equilibrium where unemployment and labor earnings move as in the baseline, but where the productivity effects of the ladder are turned off. Compared to the full model, the recovery in the counterfactual is much faster and inflationary pressures arise much sooner. The rationale for this result is simple. The reduction in labor mobility during a recession not only increases unemployment, but also leaves employed workers stuck at low-productivity jobs. The distribution of employment across jobs is a slow-moving state that impairs production even after the direct effects of the shock have died

[1]Frictional wage dispersion is understood as dispersion in wages among ex-ante similar workers. While the baseline search model falls short of generating the magnitude of frictional wage dispersion there is in the data (Hornstein, Krusell, & Violante, 2011), models that include on-the-job search—in particular, with Bertrand competition between firm like it is the case here— can match this evidence (Papp, 2013).

[2]The joint behavior of unemployment and inflation during the Great Recession, in particular the small disinflation in face of a large and persistent unemployment, came to be known as *missing disinflation puzzle*. I discuss why it was viewed as missing in Section 6.

[3]The productivity consequences of job-to-job transitions were first raised by Barlevy (2002), who named it the *sullying effect* of recessions. This contrasts with the so-called "cleansing effect" of recessions, according to which recessions may increase labor productivity through the destruction of the least productive jobs.

out, which delays the return of the economy to steady state. The unemployment-employment margin is part of this reallocation process but by itself offers only a restricted view of the state of the labor market, and misses the movements among employed workers, whose dynamics are as (if not more) important to determine the slackness in the labor market. This point is also highlighted by Moscarini and Postel-Vinay (2019), which I further discuss in the literature review.

To understand the ladder's demand-side implications, I conduct a series of exercises aimed to shed light on the forces driving the consumption response. In the first exercise, I decompose the consumption impulse response function into the contribution of different variables entering the household problem (wages, interest rate, transfer, labor flows, etc).[4] Similar to what others have found (Auclert, Rognlie, & Straub, 2018; Kaplan, Moll, & Violante, 2018), I show that changes in disposable income are the main driver of the consumption response. In my setup, however, those income changes can materialize through (i) changes in aggregate components like wages, dividends, and lump-sum transfers that affect the *current income* of all workers, and (ii) changes in labor market transition rates that move the expectation (as well as higher-order moments) of the *labor income growth* distribution. As it turns out, the bulk of the aggregate consumption response to the financial shock can be attributed to fluctuations in labor market transition rates, therefore to changes in the income growth distribution. In particular, this channel operates mostly through changes in the contact rate of employed workers and not through the job finding rate of the unemployed.

Second, I look at the time-zero cross-sectional consumption response. Interestingly, I find that workers who reduce their consumption the most are the *non hand-to-mouth* located at the lower rungs of the ladder (mainly the unemployed and recently hired employed workers). This result contrasts with the existing HANK literature, which has thus far mainly emphasized the role played by *constrained hand-to-mouth* agents in the transmission of aggregate shocks to consumption. The reason for this difference is the following. Low-paying workers standing on the first rungs of the ladder are the ones most impact by the shock, as their wage growth depends on competition among firms for their labor services. The collapse of the job ladder therefore decreases workers' expected future labor earnings growth, even though it doesn't affect their current disposable earnings. While *hand-to-mouth* workers have a high marginal propensity to consume (MPC) out of transitory income changes, they are not sensitive to what happens to their future earnings distribution. Conversely, unconstrained workers respond to the decline in expected future labor earnings by adjusting their consumption expenditure plans.

The rest of the chapter proceeds as follows. Section 2 reviews some of the empirical literature documenting the importance of job-to-job flows for productivity and earnings growth, and relates

[4] Specifically, the exercise computes the partial equilibrium consumption in which some variables in the worker's problem adjust as in equilibrium, while others are kept fixed at their steady-state level. See Kaplan, Moll, and Violante (2018) for a similar decomposition.

this paper to the (scarce) literature including job ladder into a business cycle model. Section 3 outlines the model and defines the equilibrium. Section 4 and 5 explains the calibration strategy and evaluate the model's implications for earnings and consumption dynamics in the stationary equilibrium. Section 6 presents the results for the Great Recession exercise, while Section 7 unpacks the demands and supply effects of the ladder. Section 8 concludes.

2 Literature Review

Job-to-job flows are abundant in the data and represent over half of new hires each month.[5] Besides its contribution to overall flows, job-to-job transitions constitute a major source of productivity and earnings growth, making them important for the transmission of aggregate shocks more generally. In this section, I start by discussing some of the empirical evidence on the (cyclical) job ladder and its consequences for worker allocation and earnings. Later, I describe how this paper connects to the literature.

The defining characteristic of the job ladder is that "workers agree on a common ranking of available jobs which they aspire to climb through job search, while being occasionally thrown back into unemployment".[6] Whenever given the opportunity, workers tend to move toward "better jobs". Therefore, a robust implication of the ladder is that higher ranked firms should be more successful in attracting and retaining workers. Bagger and Lentz (2019) use this insight to rank firms in Danish matched employer-employee data by the fraction of their hires filled by workers coming from other jobs, as opposed to unemployment. They show that firms' position in this "poaching rank" is stable over time and positively correlated with the firm's value added per worker, suggesting that firms high up in the ladder are also more productive. Looking over the cycle, Crane, Hyatt, and Murray (2019) rank firms by productivity using matched employer-employee US data and find that the firm productivity distribution shifts down in recessions. Also looking at the US, Haltiwanger et al. (2018) documents the presence of a robust wage ladder with highly procyclical net flows from low to high-wage firms. In the context of the Great Recession, Haltiwanger et al. (2018) and Moscarini and Postel-Vinay (2016) show that the job ladder all but stopped working.

As for the impact of job-to-job transitions on earnings, there is extensive empirical evidence documenting that workers experience wage increases when they undergo a job-to-job transition.[7]

[5] Job-to-job transition probabilities fluctuate around 2.4%, an order of magnitude smaller than the job finding probabilities, but since the measure of employed worker is also much bigger than the measure of unemployed agents, gross flows are similar.

[6] Citation from Moscarini and Postel-Vinay (2017).

[7] See, for example Topel and Ward (1992), Hyatt and McEntarfer (2012), Moscarini and Postel-Vinay (2017), Hahn et al. (2017). Gertler, Huckfeldt, and Trigari (2018) estimates the average wage changes of job changers is about plus 4.5%. The average hides lots of heterogeneity, with conditional wage changes equal to plus 30% for workers realizing

Just as important, even employed workers who do not switch jobs may still benefit from outside offers, as those can be used to increase their wages at their current jobs. As evidence of the latter mechanism, Moscarini and Postel-Vinay (2017) find, using longitudinal microdata from the Survey of Income and Program Participation (SIPP), that earnings growth covaries with "predicted" job-to-job transitions even among workers who do not actually experience one. The "predicted" rate means to capture how likely it is for a worker to undergo a job-to-job transitions based on effective transitions experienced by observationally similar workers. The authors interpret the positive correlation as evidence of worker's gaining surplus via outside offers, as they would in a sequential auction model like that of Postel-Vinay and Robin (2002).

Next, I discuss how this paper relates to the literature. By featuring risk averse workers making consumption and savings decisions in an environment with search frictions and on-the-job search, this paper relates to Lise (2012). His partial equilibrium analysis is the building block of the demand-side of my model, as the regular income fluctuation problem in the traditional heterogenous agent incomplete markets model. This paper also relates to the extensive labor literature studying cyclical movements in labor market flows. Papers in this literature tend to feature workers with linear-utility and do not address the impact of the job ladder on aggregate variables outside the labor market (see Menzio and Shi (2011), Robin (2011), Lise and Robin (2017), Moscarini and Postel-Vinay (2018)). In the few cases where labor market frictions are incorporated into business cycle frameworks with consumption decisions and nominal rigidities, models tend to abstract from job-to-job flows (see **christianoetal16**).

An exception is the work of Moscarini and Postel-Vinay (2019), which heavily motivates this paper. They are the first to introduce a job ladder into a DSGE New-Keynesian model and study the aggregate responses to productivity, preference, and monetary shocks. Backed by their previous empirical work uncovering a positive relation between job-to-job transitions and wage inflation,[8] the authors use the model as a laboratory to test the predictive power of labor market flows on future inflation. While I share their motivation to study the role of the job ladder over business cycles, this paper differs from theirs in two respects. First, I examine economy's response to an adverse financial shock and show that the job ladder helps accounting for the aggregate behavior during and after the Great Recession, an exercise they do not consider. Second, on the modeling side, I assume that labor earnings risk is uninsurable. I show that this assumption affects the transmission of aggregate shocks to consumption, with workers reducing their consumption expenditures when the job ladder breaks down. This work also relates to Faccini and Melosi (2019). The authors empirically evaluate a simpler version of Moscarini and Postel-Vinay (2019) model for the US during the post-Great Recession period, but focus mainly on the missing inflation following the recession instead of the missing disinflation during the recession, which is the main

wage gains and minus 23% for workers realizing wage losses.

[8] See Moscarini and Postel-Vinay (2017).

focus of this paper.

This paper also contributes to the burgeoning literature on Heterogenous Agent New Keynesian (HANK) models by adding realistic labor market flows to this framework.[9] Den Haan, Rendahl, and Riegler (2017), Gornemann, Kuester, and Nakajima (2016) and Kekre (2019) also study HANK models with labor market frictions, but none considers that the employed also face search frictions through on-the-job search. In an analytically tractable HANK model with unemployment, Ravn and Sterk (2018) highlight that the precautionary savings response to *countercyclical* unemployment risk amplifies the consumption response to shocks compared to a complete market economy. This result contrasts with the dampening in consumption I find in response to the financial shock. There are two main differences between the model I develop here and their analysis. First, the cyclicality of the earnings risk in this paper is much more complex and takes into account wage fluctuations while employed, as well as unemployment risk.[10] Moreover, the model features a full distribution of marginal propensities to consume (MPCs), introducing a *redistribution channel* (**auclert19**) to any aggregate shock that unevenly affect workers.

3 Model

In this section, I lay out the Heterogeneous Agent New Keynesian (HANK) model I use to study the aggregate implications of labor market flows.

Goods, Technology, Agents Time is continuous. There are three vertically integrated sectors in the economy, each producing a different type of good that can be used either as an input by other sectors or consumed.[11]

At the bottom of this supply chain, *labor intermediaries* hire workers in a frictional labor market. Technology is linear in labor, with a unit of labor mapping to z units of labor services (thought as an intermediate input), which is then sold in a competitive market at price φ_t. Productivity z is specific to the worker–firm match and is drawn at origination from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0, 1]$.

A measure one of *retailers* indexed by $j \in [0, 1]$ lies above the intermediate sector. Each retailer produces a *specialized input* $\tilde{Y}_j$ with a constant returns to scale technology in two inputs: labor ser-

[9]The recent literature that incorporates micro heterogeneity into New Keynesian models of the macroeconomy include among others Guerrieri and Lorenzoni (2017), Bayer et al. (2019), McKay and Reis (2016), **auclert19**, McKay, Nakamura, and Steinsson (2016), Ravn and Sterk (2017), Auclert and Rognlie (2018), Kaplan, Moll, and Violante (2018).

[10]Ravn and Sterk (2018) also feature *aggregate* wage fluctuations that impact the cyclicality of earnings risk in their model. My point here refers to the piece-rate wage changes induced by the job ladder, which introduces a complex mapping between labor market flows and workers' labor income process that varies over the cycle.

[11]See **christianoetal16** and Moscarini and Postel-Vinay (2019) for a similar supply-side structure.

vices and materials.[12] The specialized inputs are then aggregated by a competitive representative firm to produce the final good $\tilde{Y}_t$.

The economy is populated by a continuum of *ex-ante* identical risk averse workers indexed by $i \in [0, 1]$. Labor market risk makes workers heterogeneous in their employment status, labor income, and wealth. A government issues debt and taxes labor income in order to finance government expenditures and an unemployment insurance program. I start by describing the worker's problem.

Workers Workers receive utility flow u from consuming c_{it} and do not value leisure. Preferences are time-separable, and the future is discounted at rate ρ

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt, \tag{1}$$

where the expectation reflects individual-level uncertainty in labor income.

An unemployed worker receives unemployment insurance (UI) benefits in the amount of $b \times \varphi_t$. An employed worker in a match of productivity z receives as a wage $y \times \varphi_t$, where the piece-rate $y \leq z$ depends on the worker's history in the labor market. I delay the discussion on the piece-rate wage determination for later.

Workers receive lump-sum dividends in the amount of d_{it}, save through a riskless government bond at flow real rate r_t, and are subject to a no-borrowing constraint. Wealth a_{it} evolves according to

$$\begin{aligned} \dot{a}_{it} &= (1 - \tau)\varphi_t \left(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it} \right) + r_t a_{it} + d_{it} - c_{it} - \tau^0_t, \\ a_{it} &\geq 0 \end{aligned} \tag{2}$$

where $\mathbb{1}^u_{it}$ is an indicator for unemployment status, τ^0_t is a government lump-sum transfer and τ is a proportional tax. The distribution of dividends across workers is a crucial determinant of the aggregate consumption response in HANK models (e.g., Bilbiie, 2018; Broer, Hansen, and Krusell, 2018; Werning, 2015). I follow Kaplan, Moll, and Violante (2018) and distribute profits in proportion to individuals' labor income

$$d_{it} = \frac{\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it}}{\int \left(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it} \right) di} D_t, \tag{3}$$

where D_t denotes aggregate profits.[13]

[12] Materials are converted one-for-one from the final good. I discuss the importance of materials in the retailer's problem. See Christiano, Trabandt, and Walentin (2010) for a standard New Keynesian model with materials.

[13] Aggregate profits include profits earned both by monopolistically competitive firms and labor intermediaries.

Workers maximize their lifetime utility given in (1) subject to the wealth accumulation process in (2), the labor income process $\{\mathbb{1}^u_{it}, y_{it}\}_{t\geq 0}$, dividends payouts $\{d_{it}\}_{t\geq 0}$, and paths of $\{r_t, \varphi_t, \tau^0_t\}_{t\geq 0}$, which they take as given. In Appendix A.2, I write the Hamilton–Jacobi–Bellman equation associated with the household problem and discuss the impact of the job ladder on consumption and savings decisions, following the insights from Lise (2012). At steady state, the recursive solution to this problem consists of value functions and consumption decision rules for the unemployed and the employed worker $\{c^u(a), c^e(a, y)\}$.[14] The worker's consumption policy function together with labor market transition rates and wage contracts induce a stationary distribution over wealth, labor income, and match productivities $\Psi(a, y, z)$. With a slight abuse of notation, I denote marginal distributions by Ψ as well. Outside steady state, distributions and policies are time varying and described by a Kolmogorov forward and a Hamilton–Jacobi–Bellman equations. I indicate that dependence when necessary by adding a t subscript to equilibrium variables.

Search Frictions in the Labor Market The labor market features search frictions. Labor intermediaries post vacancies v_t to match with workers. Employed and unemployed workers search for open job vacancies. The searching effort of unemployed workers is normalized to one, while employed workers search with lower intensity s_e. Combined, they produce a search effort of

$$\mathcal{S}_t = u_t + s_e(1 - u_t). \tag{4}$$

Effective job market tightness is therefore

$$\theta_t = \frac{v_t}{u_t + s_e(1 - u_t)}. \tag{5}$$

The flow of meetings at time t is given a by constant returns to scale matching function $\mathcal{M}(v_t, \mathcal{S}_t)$. Define $\lambda_t := \frac{\mathcal{M}(v_t,\mathcal{S}_t)}{\mathcal{S}_t}$ as the rate at which an unemployed worker meets a vacancy, while employed workers contact outside firms at a rate $\lambda_{et} = s_e\lambda_t$. A vacancy contacts a worker with intensity $q_t := \lambda_t/\theta_t$. Once a worker and firm meet, the firm makes a wage offer (details below) that may

Rewriting the worker's budget constraint under this profit distribution rule, we get

$$\dot{a}_{it} = \left((1-\tau)\varphi_t + \frac{D_t}{\int \left(\mathbb{1}^u_{it} + (1 - \mathbb{1}^u_{it})y_{it}\right) di} \right) \left(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it} \right) + r_t a_{it} - c_{it} - \tau^0_t$$

Hence, distributing profits in proportion to labor earnings neutralizes the redistribution effects by making all workers equally exposed to its fluctuations. Overall dividends D_t and price of labor services φ_t enter in the same way in the budget constraint by multiplying the idiosyncratic worker labor market state $(\mathbb{1}^u_{it}, y_{it})$.

[14]Note that policy functions depend on wealth and the piece rate wage only. The attentive reader may notice the lack of match productivity z in the worker's state space, that, even if not a direct payoff relevant variable, still contains information about future labor income distribution. As I discuss below, the worker does not observe the productivity of its current match, making income and wealth the only state variables in the worker problem.

or may not be accepted by the worker. Finally, all matches are subject to a destruction shock at an exogenous flow probability δ.

Wage Contract Firms are restricted to offer workers piece-rate wage contracts that can be renegotiated only if the worker receives a better outside offer.[15] When the firm contacts a worker, it observes the worker's employment status and incumbent match productivity in case the worker is already employed. In contrast, workers are uninformed about their match productivity, but learn about it from labor market transitions and wage offers—I discuss this assumption in the next section. In what follows, I describe wage offers to employed and unemployed workers.

Employed Worker Consider a worker employed at a match of productivity z who contacts an outside firm with which the match productivity draw is z'. The two firms Bertrand compete for the worker's services over piece-rate wage contracts, with the more productive firm winning the bidding for the worker.

First, let me consider the case where $z' > z$; that is, when the poacher is more productive than the incumbent firm. The incumbent's maximum wage offer is to promise the worker the whole output flow of the match—i.e., offer a piece-rate $y = z$. The poaching firm z' attracts the worker by outbidding incumbent's piece-rate wage offer by ϵ, which results in the worker moving to firm z' at a piece-rate wage of $z + \epsilon$. In the solution of the model, I take ϵ to be an arbitrarily small number.[16]

Now, suppose instead that $z' < z$. The competition between the two firms has the worker staying with the incumbent, but the wage contract can still be renegotiated if the poaching firm's maximum wage offer is above the worker's current piece-rate (i.e., if $z' > y$). In this case, the worker's piece-rate wage from the incumbent firm increases to $z' + \epsilon$.

Unemployed Worker Upon meeting an unemployed worker, I assume that the firm makes a piece-rate offer of $\underline{z}$; that is, the firm offers the unemployed the full production of the least productive firm. In the calibration, I choose the unemployment insurance replacement rate b to be equal to $\underline{z}$, so firms effectively offer the unemployment insurance rate to unemployed workers.

[15]Note that piece-rates are usually defined in terms of a share of the match output flow, so if the match produces X, a piece-rate p would entail a wage of pX with $p \leq 1$. In the presentation here, I instead define the piece-rate in terms of the price of labor services. So the wage of a worker in match z with piece-rate of y is $y\varphi_t$, with the restriction $y \leq z$. See Bagger et al. (2014) for an implementation of piece rate version of the sequential auction framework in a standard labor market model that abstracts from incomplete markets and consumption and savings decisions.

[16]Note that this assumption departs from Postel-Vinay and Robin (2002) as the more productive firm attracts the worker by matching the *wage offer* of the less productive firm, as opposed to matching the worker's value of staying at the incumbent firm under the maximum wage offer. The same assumption is made by Graber and Lise (2015) and is intended to keep the problem tractable in the presence of a non-degenerate wealth distribution on the worker side.

In the description above, I have treated the worker's acceptance decision as given. In particular, I implicitly assume that (i) the unemployed worker accepts the initial wage offer coming from any firm, and (ii) the employed worker always moves/stays in the firm offering the highest wage. While (ii) is a natural assumption in the current setup, where more productive matches also offer higher wages, it is not clear that (i) would hold without any additional assumptions. In what follows, I discuss the unemployed worker's reservation strategy in the presence of such wage contracts.

Worker's Reservation Strategy While firms offer the same initial wage contract to workers coming out of unemployment, the unemployed workers' value of meeting a vacancy increases with the productivity of the match. This is because being hired by a firm with greater productivity implies a better (in the first-order sense) distribution of future wages.[17]

Because the unemployed search intensity is greater that of the employed ($\lambda > \lambda_e$), there is an *option value* associated with waiting to meet more productive firms. The value of remaining in unemployment and waiting for better matches versus accepting an offer at a match of productivity z will depend on the worker's assets, leading to a reservation productivity policy that depends on wealth.

The extent to which search decisions depend on worker's wealth is certainly an important question.[18] My main interest here, however, is not to analyze how incomplete markets impact search decisions but instead to study how a "realistic" model of the labor market transmits aggregate shocks to consumption. Therefore, I simplify workers' reservation decisions by assuming that the worker never gets to observe the productivity z of its own match.[19] This transforms the reservation decision of the unemployed into a trivial one: by making all offers coming out of unemployment identical— meaning that all firms offer the same wage, so they all look the same to the unemployed worker—they are either all accepted or all rejected. Since being employed entails a higher present value of earnings than being unemployed, all offers will be all accepted by the worker.

Making the productivity a hidden state adds a learning/filtering dimension to the worker's problem, who still gets to observe his wage history in the labor market. I describe this problem in Appendix A.1. Next I turn to the supply side of the economy.

[17]To see this, consider the future path of wages for a recently hired worker at matches of productivity z_1, z_2, with $z_1 > z_2$, in the circumstance where he meets an outside firm of productivity $z_3 \in [z_1, z_2]$. If employed at firm z_1, the worker switches jobs and his piece-rate wage changes to z_1. If employed at firm z_2, however, the worker stays in the firm and the wage increases to $z_3 > z_1$.

[18]For examples of papers that study this, see Lentz and Tranæs (2005) and Eeckhout and Sepahsalari (2018).

[19]A simpler way to eliminate the option value would be to assume that the search intensity is the same for the employed and unemployed, $s_e = 1$. This, however, would preclude the model from matching the small flow of employer to employer transitions relative to unemployment to employment. But, as I show in the experiments, it is the slow reallocation along the ladder that generates long-lasting impacts of misallocation—one of the main points of the paper.

Final Good Producer A competitive representative final good producer aggregates a continuum of specialized inputs, $\tilde{Y}_{j,t}$, using the technology

$$\tilde{Y}_t = \left(\int_0^1 \tilde{Y}_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $\epsilon > 0$ is the elasticity of substitution across goods. The firm's first-order condition for the jth input is

$$\tilde{Y}_{j,t}(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} \tilde{Y}_t, \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{7}$$

Retailers The jth input good in (6) is produced by a *retailer*, who is a monopolist in the product market. Following Basu (1995) and Nakamura and Steinsson (2010), each retailer produces their specialized good by combining materials $M_{j,t}$ and labor services $N^e_{j,t}$ according to the production function

$$\tilde{Y}_{j,t} = M_{j,t}^{\gamma} (Z_t N^e_{j,t})^{1-\gamma}, \tag{8}$$

where Z_t is an aggregate productivity component. Materials are converted one-for-one from the final good $\tilde{Y}_t$ in (6), so each retailer effectively uses the output of all other retailers as input to production. Retailers buy labor services at the competitive price φ_t and materials for the real price of one.

Cost minimization implies a common marginal cost across all retailers, given by

$$m_t = \left(\frac{1}{\gamma} \right)^{\gamma} \left(\frac{\varphi_t / Z_t}{1-\gamma} \right)^{1-\gamma}. \tag{9}$$

Cost minimization also implies that the relative price of labor services and materials inputs must be equal to the ratio of their marginal productivities

$$\frac{\varphi_t / Z_t}{1} \frac{\gamma}{1-\gamma} = \frac{M_{jt}}{Z_t N_{jt}}. \tag{10}$$

Each retailer must also choose a price $P_{j,t}$ to maximize profits subject to demand curve (7) and price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the firm's rate of price change $\dot{P}_{j,t}/P_{j,t}$ and expressed as a fraction of gross output $\tilde{Y}_t$ as

$$\Theta_t \left(\frac{\dot{P}_{j,t}}{P_{j,t}} \right) = \frac{\theta}{2} \left(\frac{\dot{P}_{j,t}}{P_{j,t}} \right)^2 \tilde{Y}_t, \tag{11}$$

where $\theta > 0$.[20] Therefore, each retailer chooses $\{P_{j,t}\}_{t\geq 0}$ to maximize

$$\int_0^\infty e^{-\int_0^t r_s ds} \left\{ \tilde{\Pi}_t(P_{j,t}) - \Theta_t\left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right) \right\} dt,$$

where retailers discount profits at the real rate $\{r_t\}_{t\geq 0}$ and

$$\tilde{\Pi}_t(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} - m_t\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t$$

are flow profits before price adjustment costs.

In a symmetric equilibrium, all firms choose the same price $P_{j,t} = P_t$ and produce the same amount of goods $\tilde{Y}_{j,t} = \tilde{Y}_t$. Moreover, as shown in Kaplan, Moll, and Violante (2018), the quadratic price adjustment costs in continuous-time setting yields a simple equation characterizing the evolution of aggregate inflation $\pi_t := \dot{P}_t / P_t$

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\varepsilon}{\theta}\left(m_t - m^*\right) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon}. \tag{12}$$

Equation (12) is the New Keynesian Phillips curve, which can also be represented in present-value form as

$$\pi_t = \frac{\varepsilon}{\theta}\int_t^\infty e^{-\int_t^s r_\tau d\tau}\frac{\tilde{Y}_s}{\tilde{Y}_t}\left(m_s - m^*\right) ds. \tag{13}$$

The presence of materials adds a flexible factor input into production, which allows output to change immediately (at time-0) upon aggregate shocks. To see this, substitute (10) into (8) and (6) evaluated at the symmetric equilibrium. This gives an aggregate *restriction* between aggregate production $\tilde{Y}_t$, marginal costs m_t and labor services N_t^e

$$\tilde{Y}_t = (m_t \gamma)^{\frac{\gamma}{1-\gamma}} Z_t N_t^e. \tag{14}$$

So production changes in equilibrium if (i) productivity Z changes, (ii) marginal costs m changes or (iii) labor services inputs N^e change. Market clearing in the market for labor services—see equilibrium definition in Section 3.1—imposes that all the supply of labor service must be employed by retailers. This is a stock (state variable), however, so it cannot adjust at the impact of an aggregate shock—retailers, while individually allowed to reduce their usage of labor services, cannot do so in the aggregate immediately following the shock. Labor service competitive price φ_0 must therefore adjust to make retailers willing to hire the labor service stock in the economy.

[20] I follow Hagedorn, Manovskii, and Mitman (2019) in assuming that price adjustment costs are "virtual", meaning that they affect optimal choices but do not cause real resources to be expended. That is why pricing costs do not appear in the goods market clearing condition in the definition of equilibrium.

As φ_0 changes to clear the labor market, retailers adjust their materials-labor ratio according to (10), which leads to production to adjust.

Labor Intermediaries A firm in the intermediate sector can post a vacancy at a flow cost of κ^f, expressed in units of the consumption good. Upon meeting a worker, the firm must pay an additional fixed screening/training cost to learn the match productivity and start producing.[21] This cost is allowed to depend on the employment status of the worker, but different from the vacancy cost, it does not expend any real resources and does not show up in any budget constraint.[22]

Firms discount their profit flow at the rate $r_t + \chi_t$, where χ_t is a (exogenous) spread between the return of vacancy posting investments and the risk-free rate.[23] Let $\mathcal{J}_t(z, y)$ denote the expected present discounted value of dividends for a firm with match productivity z currently offering the worker a piece-rate contract y. The firm's value function is defined recursively in Appendix A.3.

The measure of vacancies v_t is pinned down in equilibrium by the following free-entry condition

$$\begin{aligned}\frac{\kappa^f}{q_t} = \int \Bigg\{ &\frac{u_t}{u_t + s_e(1-u_t)} \left[\mathcal{J}_t(z, \underline{z}) - \tilde{\kappa}^{su}\right] + \\ &+ \frac{s_e(1-u_t)}{u_t + s_e(1-u_t)} \left[\int_{\underline{z}}^{z} \mathcal{J}_t(z, z') \frac{d\Psi_t(z')}{1-u_t} - \tilde{\kappa}^{se} \right] \Bigg\} d\Gamma(z),\end{aligned} \tag{15}$$

which equates the expected flow cost of hiring a worker, $\frac{\kappa^f}{q_t}$, to the expected value of a match. The latter accounts for the probability of meeting an unemployed worker, an event which the firm values by $\mathcal{J}(z, \underline{z})$, and the probability of meeting an employed worker matched with a firm of productivity z', which has a value of $\mathcal{J}(z, z')$ if $z > z'$ and otherwise, is zero.

The distribution of workers in the labor market—the measure u_t of unemployed workers and the distribution $\Psi_t(z)$ of employed workers—affects firms' incentives by changing their expectations of the type of worker they will encounter. At the same time, the distribution of employment depends on the measure of vacancies posted through the market tightness.

Monetary Authority The monetary authority sets the nominal interest rate on nominal government bonds i_t according to a Taylor rule

$$i_t = \bar{r} + \phi_\pi \pi_t + \epsilon_t, \tag{16}$$

[21] As suggested by Pissarides (2009) and exploited by **christianoetal16** in an estimated model without OJS, screening costs raise amplification of unemployment fluctuations to aggregate shocks by insulating hiring costs from vacancy congestion coming from the matching function.

[22] These costs can be thought of as utility costs associated with the training/screening of workers. See Moscarini and Postel-Vinay (2019) for a similar assumption.

[23] At steady state, I set χ to zero. Outside steady state, I interpret shocks to χ_t as a reduced form financial shock.

where $\phi > 1$ and ϵ_t is a monetary policy shock. Given inflation and the nominal interest rate, the real return on the government bonds r_t is determined by the Fisher equation $r^b = i_t - \pi_t$.

Government The government issues *real* bonds of infinitesimal maturity B^g_t, with positive values denoting debt. This is the only savings instrument available to workers.

The government taxes workers' labor income at rate τ and uses this revenue to finance unemployment insurance, government expenditures G_t, and real rate payments on its debt. The government fiscal policy must satisfy the sequence of budget constraints

$$\dot{B}^g_t = r_t B^g_t + G_t + u_t(1-\tau)\varphi_t b - \tau\varphi_t \int y d\Psi_t(y) - \tau^0_t, \text{ for all } t. \tag{17}$$

At steady state, lump-sum transfers τ^0 are set to zero. Outside steady state, I let lump-sum transfers be the fiscal instrument that adjusts in order to keep government debt B^g_t constant at the steady-state level.[24]

3.1 Equilibrium

The rich worker heterogeneity over jobs, earnings and wealth shows up in the equilibrium definition below only through a small number of *functions* that integrate workers' decisions and states over its distribution.[25] For example, while the consumption of workers varies across earnings and wealth, equilibrium conditions only depend on an *aggregate consumption function*

$$\mathcal{C}_t := \int c_t(a,y) d\Psi_t(a,y)$$

where the time index t subsumes the dependency of policies and distributions on the whole sequence of equilibrium prices and quantities entering the worker's problem.[26] In a similar way, the *aggregate labor services supply*

$$\mathcal{N}^e_t := \int z d\Psi_t(z)$$

is all that enters the market clearing of labor services. Notwithstanding all the complexity involved in *evaluating* those functions,[27] they still constitute a mapping from *aggregate* sequences of equilibrium prices and quantities (like real rate) into other *aggregate* sequences (like consump-

[24]See Kaplan and Violante (2018) for a discussion on the importance of fiscal adjustment in HANK models.

[25]See Auclert et al. (2019) for this insight, who call these functions by *heterogeneous-agent block*.

[26]Specifically, the worker cares about the evolution of $\{r_t, \varphi_t, d_t, \tau_t, \tau^0_t, \lambda_t, \lambda_{et}\}_{t\geq 0}$.

[27]Aggregate consumption at time t, for instance, is the summation of consumption decisions $c_t(a,y)$, itself a function of the whole sequence of prices, labor market transitions and fiscal policy, across wealth and earnings distribution, the evolution of which depends on the consumption decisions and labor market transitions up to time t.

tion), which in turn must satisfy certain equilibrium conditions.[28] This observation is the basis of the numerical algorithm used to solve the model – see Appendix C for details. I now turn to the equilibrium definition.

Definition 1 *(Equilibium) Given an initial government debt B^g, an initial distribution Ψ_0 over wealth, labor income and match productivity, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, an general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N_t^e, M_t, u_t, v_t, D_t\}_{t\geq 0}$, labor market transition rates $\{\lambda_t, \lambda_{et}\}_{t\geq 0}$, government policies $\{G_t, B_t^g, \tau_t, \tau_t^0, i_t\}_{t\geq 0}$, labor income process $\{\mathbb{1}_{it}^u, y_{it}\}_{i\in[0,1], t\geq 0}$, worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t, \mathcal{N}_t^e\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that workers optimize, firms optimize, monetary and fiscal policy follow their rules, the labor income process is the result of labor market transitions and wage-setting, worker aggregate functions and distributions are consistent with labor market transition rates and worker's decision rules,*

- *the free-entry condition (15) holds,*
- *and all markets clear:*
  - *asset market*
$$\mathcal{A}_t = B_t^g$$
  - *labor services market*
$$N_t^e = \mathcal{N}_t^e$$
  - *goods market*
$$\mathcal{C}_t + G_t + \kappa^f v_t = Y_t = \tilde{Y}_t - M_t$$

4 Calibration

I calibrate the model at a monthly frequency. The calibration strategy is divided into four main steps. First, I calibrate the labor market transition rates to match estimated flows and choose the firm productivity distribution to match the dispersion in the residual wage distribution. Second, I choose the vacancy costs and the relative importance of screening versus flow costs. Third, I use the overall amount of liquidity, which in the economy takes the form of government bonds, to directly target average MPC in the data. Finally, I calibrate the parameters of the production and monetary side to standard values used in the New Keynesian literature. The full list of parameter values and targeted moments is given in Table 1.

Labor Market (Transitions and Productivity) I assume a standard Cobb–Douglas matching function $\mathcal{M}(v, \mathcal{S}) = v^\alpha \mathcal{S}^{1-\alpha}$, with $\alpha = 0.5$, as in Moscarini and Postel-Vinay (2018). I target a

[28] Even though continuous time perfect-foresight transition equilibrium objects consists of *real valued functions* $X : [0, \infty) \to \mathbb{R}$ and not really *sequences* $Y : \mathbb{N} \to \mathbb{R}$, I use sequences when describing those in the text since this agrees with the more commonly used discrete time convention.

job finding rate λ of 0.45, which implies a monthly job finding probability of $1 - exp(\lambda) = 0.36$. I set δ to match the monthly probability of transitioning from employment to unemployment. These two flows imply a steady-state rate of $\frac{\delta}{\delta+\lambda} = 5\%$. The relative search efficiency of employed worker s_e is set so the steady-state monthly job-to-job transition rate equals 2.4%.

The productivity distribution Γ is assumed to be an affine transformation of Beta distribution; that is, a match productivity $z = c_0 + c_1 X$, where $X \sim \text{Beta}(\beta^1, \beta^2)$. This reduces the distribution of firm productivity to four parameters $(c_0, c_1, \beta^1, \beta^2)$ which I calibrate as follows. I fix $c_0 = 0.3$, since it is just a normalization. I use the evidence on the frictional wage dispersion to pin down the remaining parameters (c_1, β^1, β^2). In the data this is understood as the dispersion in wages after eliminating all variation due to observable and unobservables individual characteristics (e.g experience, education, marital status, gender, ability, etc).[29] The dispersion left in the residual is the correct empirical counterpart for the *overall* wage dispersion in the model, where workers are ex-ante identical and wage differentials arise from different labor market histories. At the chose calibration ($c_1 = 2.61, \beta^1 = 1, \beta^2 = 10.0$) log differences between the 50/10 and 90/10 percentiles of the wage distribution are 0.64 and 1.10 respectively, in line with the estimates reported by Lemieux (2006) (Figure 1A) and Autor, Katz, and Kearney (2008) (Figure 8).Finally, I set $b = \underline{z}$ so the unemployed earns as much as a recently employed agent. This delivers a UI replacement rate of approximately 50%, which is within the range of values used in the literature.

Labor Market (Vacancy Costs) The canonical search and matching model fails to match the cyclical volatility in the job finding rate—a point initially noted by Shimer (2005). The same difficulty is also present in a model with on-the-job search—see Moscarini and Postel-Vinay (2018) for a detailed comparison of the canonical model versus a model with on-the-job search. Since one of my objectives is to study the impact of labor market fluctuations on consumption, it is crucial to get fluctuations in unemployment and earnings risks that approximate those in the data.

I achieve this by resorting to high fixed screening costs. Specifically, I need three restrictions to pin down the values of vacancy posting κ^f and screening costs $(\tilde{\kappa}^{se}, \tilde{\kappa}^{su})$. The targeted job finding rate $\lambda = 0.45$ imposes the first restriction—through the matching function, this implies a steady-state level of market tightness θ that must be consistent with the free-entry condition. I impose two additional restrictions by (i) making the firm indifferent between hiring an employed worker and hiring an unemployed worker at steady state [30] and (ii) making screening costs 90% of the total hiring cost. The fixed cost's share of total cost is in line with **christianoetal16**, who estimate

[29] That is why the literature also refers to this measure as *residual* wage dispersion.

[30] In terms of the values defined before, this restriction writes as

$$\int \left[\mathcal{J}(z, \underline{z}) - \kappa^u\right] d\Gamma(z) = \int \left\{ \left[\int_{\underline{z}}^{z} \mathcal{J}(z, z') \frac{d\Psi(z')}{1-u} - \tilde{\kappa}^{se} \right] \right\} d\Gamma(z).$$

Table 1: List of parameter values and targeted moments

Variable		Value	Target
Labor market			
$\mathcal{M}$	matching function	$v^{0.5}\mathcal{S}^{0.5}$	—
δ	destruction rate	0.024	—
λ	job finding prob.	0.412	unemployment of 5%
s_e	employed search intensity	0.127	ee transition of 0.024
b	replacement rate	$\underline{z}$	UI replacement rate of 50%
$z = c_0 + c_1 X$	productivity grid	$(0.30, 2.61)$	*residual wage dispersion*
$X \sim \text{Beta}(\beta^1, \beta^2)$		$(1.0, 10.0)$	$p^{50}/p^{10}, p^{90}/p^{10} = (0.64, 1.10)$[1]
$\kappa^f, \kappa^{su}, \kappa^{se}$	vacancy costs	$0.34, 3.4, 1.0$	see text
Preferences and Liquidity			
ρ	discount rate	$0.08/12.0$	$r^{ann} = 0.02$
$u(\bullet)$	utility function	$\log(\bullet)$	—
B^g/Y^{ann}		≈ 0.30	*target quarterly MPC of 0.25*[2]
Retailers, Final Good and Government			
γ	material share	0.50	share of materials in gross output
ϵ	elasticity of substitution	10.0	—
ϵ/θ	slope of Phillips curve	0.0067	price rigidity of 12 months
τ	tax rate	0.25	$G/Y \approx 0.20$
ϕ_π	Taylor rule coefficient	1.50	—

[1] Lemieux (2006) and Autor, Katz, and Kearney (2008).
[2] Johnson, Parker, and Souleles (2006); Parker et al. (2013) report quarterly MPC estimates around $[0.15, 0.30]$.

this to be 94%.

To understand the rationale behind (i), suppose I did not make the screening costs dependent on employment status. As the value of meeting an unemployed worker is greater than that of meeting an employed worker, firms would be more willing to post vacancies whenever unemployment is high because these are periods when firms face a higher probability of meeting an unemployed worker. This force, which is quite powerful in the model, accelerates transitions back to steady state and reduces the unemployment response to shocks. Hence, having the screening cost depend on the employment status of workers and satisfying restriction (i) mitigates this effect.

Liquidity and Preferences I assume that steady-state inflation is equal to zero and that the steady-state real interest rate equals 2%. Workers have log utility over consumption, and their annual discount rate is 8%. As discussed in Kaplan, Moll, and Violante (2018), one-asset HANK models feature a tension between matching the high observed aggregate wealth-to-output ratio and generating a large average MPC, as in the data. If the model is calibrated to target the former, it implies small MPCs; if we directly target the MPCs in the data, the model must feature a low aggregate wealth. Given the importance of the MPCs to the demand response to aggregate shocks, as outlined by Auclert, Rognlie, and Straub (2018), I set B^g to directly target MPCs. Specifically, I target an average quarterly MPC out of a \$500 unexpected transfer of 0.25. The estimate lies within the range of values reported by Johnson, Parker, and Souleles (2006); Parker et al. (2013). This target yields a government debt B^g in the amount of 28% of annual GDP.

Production The elasticity of substitution for the inputs produced by retailers ϵ is set to 10. The input share of materials γ is set to 0.5, which lies in the interval of values considered by Nakamura and Steinsson (2010). I set the price adjustment cost θ coefficient to 1500, so the slope of the Phillips curve is given by 0.0067. The Phillips curve under Rotemberg or Calvo price rigidities has the same log-linear representation, so we can map the slope of the Rotemberg Phillips curve to the implied Calvo parameter determining the time between price changes. In that case, the slope of 0.0067 implies prices change once every 12 months, which is close to the Bayesian estimates from Smets and Wouters (2007) and Christiano, Eichenbaum, and Trabandt (2014).[31]

Fiscal and Monetary Policy I set the labor income tax to 25%. Government expenditures are determined residually from the government budget constraint and amounts to around 20% of GDP. The Taylor rule coefficient is set to 1.5.

5 Earnings, Consumption and Job Ladder

In the standard heterogenous-agent setting earnings dynamics are driven by an *exogenous* labor productivity process, usually calibrated to match some key moments of the earnings change distribution. In the current environment, labor earnings are instead a function of labor market transition rates, the distribution of firm productivity and firm's competition for employed workers as described in Section 3. The previous section discussed how to calibrate these parameters using information on labor market flows and measures of frictional wage dispersion, but did not address its implication for earnings dynamics. I explore this in this section.

[31]This mapping is given by

$$\frac{\epsilon}{\theta} = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha},$$

where β is the household discount factor, and $1-\alpha$ denotes the probability with which the firm gets to reset prices in the month. Setting $\beta = exp(-12.0r)$, $\epsilon/\theta = 0.0067$ which leads to $\alpha = 0.92$, meaning an expected price rigidity of $(1-\alpha)^{-1} \approx 12$ months.

Table 2: Moments of earnings change distribution

Moment	Data	Model
$Var[\Delta\tilde{y}^A]$	0.260	0.140
$Skew[\Delta\tilde{y}^A]$	-1.07	-0.721
$Kurt[\Delta\tilde{y}^A]$	14.93	5.907
Fraction $\|\Delta\tilde{y}^A\| < 0.05$	0.310	0.337
Fraction $\|\Delta\tilde{y}^A\| < 0.10$	0.490	0.434
Fraction $\|\Delta\tilde{y}^A\| < 0.20$	0.670	0.578
Fraction $\|\Delta\tilde{y}^A\| < 0.50$	0.830	0.838

Notes: **Data:** Moments of the one-year log labor earnings changes distribution computed from the Master Earnings File of the Social Security Administration data (values are taken from Guvenen et al. (2016)). **Model:** Moments of the one-year log labor earnings changes distribution computed by simulating a panel of 100,000 workers in the stationary equilibrium of the model. The measure of labor earnings in the model and data excludes unemployment insurance payments collected by workers.

First, I look at the high-order moments of labor earnings growth distribution. I show that the job ladder structure, although parsimonious, generates the negative skewness and excess kurtosis in the distribution of annual earnings changes that are documented by Guvenen et al. (2016) using the U.S. Social Security Administration data. Second, I discuss how the model performs with respect to the empirical evidence on wage and consumption dynamics following a job displacement.

5.1 Earnings Dynamics

Let the economy rest at its stationary equilibrium. I follow the literature and look at the distribution of annual earnings changes. For worker i with piece-rate wage $\{y_{is}\}_{s\in[t,t+12]}$ during year t, yearly labor earnings are given by

$$y_{it}^A \equiv \varphi \int_t^{t+12} y_{is} ds.$$

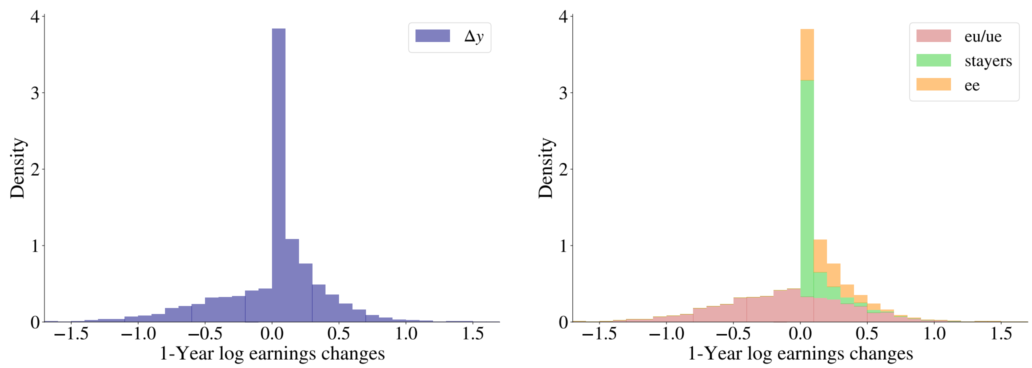
Let $\tilde{y}_{it}^A \equiv \log y_{it}^A$ denote annual log earnings of individual i, so changes in annual log earnings are given by $\Delta\tilde{y}_{it}^A \equiv \tilde{y}_{i,t+1}^A - \tilde{y}_{it}^A$.[32] To compute this distribution, I simulate a panel of workers and record their earnings as well as their movements in the labor market.

Table 2 reports some moments of the simulated data along with estimates by Guvenen et al. (2016) from administrative Social Security data.[33] The model successfully generates two key facts of the data: a strong negative skewness and kurtosis (in the case of kurtosis, the model value falls

[32]Notice that the measure of labor earnings ignores unemployment insurance payments. This ensures that the moments computed in the model are comparable to data from Guvenen et al. (2016) whose measure of earnings include only wages, salaries, bonuses.

[33]Figure 2 Panel (A) has the model-implied histogram of log earnings changes.

Figure 1: Model implied histogram of earnings changes



(A) Unconditional (B) Conditional

Notes: **Panel (A) Unconditional:** model implied histogram of one-year log earnings changes. **Panel (B) Conditional:** same histogram, but with colors to indicate different groups of workers depending on their labor market experience in years $t, t+1$. The legends are defined as follow: "stayers" corresponds to workers employed *at the same* firm throughout years t and $t+1$; "ee" corresponds to workers employed throughout years t and $t+1$ that experience at least one job-to-job transition; "ue,eu" includes any worker that has experienced least one unemployment spell during either year t or $t+1$.

short of the estimates in the data, but is still above what would be expected from a normal distribution). The fraction of small earnings changes in the model (less than 5%, 10% and 20%) are also close to the data. These facts on higher-order moments—which are also highlighted by Guvenen et al. (2016)—are a natural consequence of the job ladder structure. The wage received by workers is fixed within the contract and grows only with the arrival of outside offers (which are infrequent but sizable, contributing to excess kurtosis), while occasional unemployment shocks lead to large earnings losses (contributing to the negative skewness).[34] Notwithstanding the model's ability to generate those facts, the magnitude of the shocks is much smaller than what we observe in the data—the variance of log earnings changes $Var[\Delta\tilde{y}^A]$ is only half of that in the data, with kurtosis also falling short. This is somewhat expected as the data compounds the influence of factors beyond the job ladder such as idiosyncratic productivity (human capital). In the model, wage dispersion and earnings growth are solely due to search frictions, so we should not expect it to capture all the risk contained in the data.

A natural follow-up question is how does earnings changes vary with the type of transitions workers experience in the labor market. Figure 2, Panel (B), shows exactly that. It plots the distribution of earnings changes for three different groups of workers: "stayers" corresponds to workers employed *at the same* firm throughout years t and $t+1$; "ee" corresponds to workers employed throughout years t and $t+1$ that experience at least one job-to-job transition; "ue,eu" includes any

[34] The ability of a job ladder model to reproduce the high-order moments documented by Guvenen et al. (2016) is also highlighted by Hubmer (2018).

worker that has experienced least one unemployment spell during either year t or $t+1$. In terms of the likelihood of those events note that 46% undergo at least one "eu" or "ue" type of transition during the two year period, Among those who remain employed throughout the period (54% of workers), 53% experience a job-to-job transition. The left tail of the earnings changes distribution comes from workers transition through unemployment and is the result of both (i) lack of earnings during unemployment spell and (ii) the low re-entry wages. Workers who do not suffer an unemployment spell experience positive earnings growth, but the gains are higher for workers who experience a job-to-job transition. Carrillo-Tudela, Visschers, and Wiczer (2019) uses the SIPP to investigate the same relationship between the distribution of earnings changes and worker mobility and find similar patterns.[35]

5.2 Earnings and Consumption upon a Job Loss

I now turn to the model's predictions on earnings and consumption dynamics following a job loss event. These nontargeted moments are important to assess the magnitude of the downside earnings risk and worker's ability to smooth consumption in face of such events.

I follow the methodology of Saporta-Eksten (2014). The author uses the 1999-2009 biennial waves of Panel Study of Income Dynamics (PSID) to document the dynamics of wages and consumption around a job displacement.[36] The main regression specification is

$$\log w_{it}^{A} = \alpha_0 + \sum_{k\geq -2}^{10} \delta_k D_{it}^{k} + u_{it} \tag{18}$$

where the outcome variable w_{it}^{A} is the annual wage rate of individual i in year t (total labor earnings divided by total hours), and D_{it}^{k} is a set of dummy variables used to indicate a worker in his kth year before, during or after a job loss.[37] [38] The coefficients δ_k capture the wage losses of workers who were displaced k years ago (or will be displace in $-k$ years) relative to workers who have not experienced displacement at that time. These losses are estimated for the two years preceding

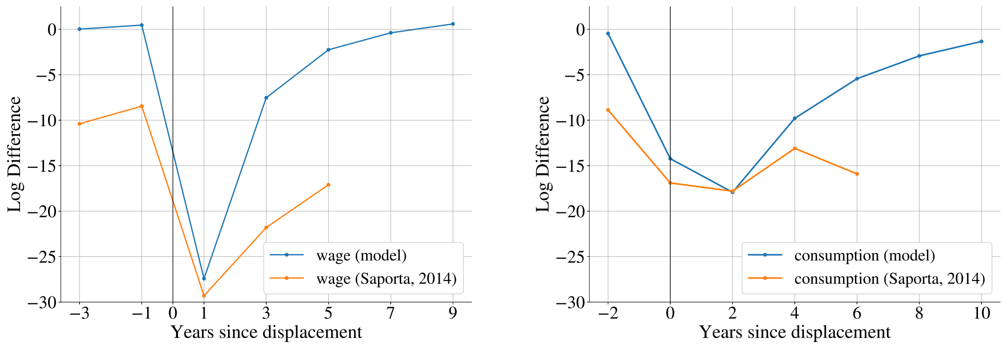
[35]See Figure 1 in their paper for the conditional distribution of earnings changes in the data. The main difference between the model and the data is that a fraction of earnings changes following job-to-job transitions, or no transitions are associated with earnings losses, which the model cannot generate by construction.

[36]While there is a extensive literature exploring the effects of job displacement on wages and earnings (see Jacobson, LaLonde, & Sullivan, 1993; Stevens, 1997), there are fewer studies that also explore the consumption response. This has to due with the difficulty of coming up with panel data with information on both consumption and employment. Starting in 1999, the PSID began to collect data on a wide variety of consumption categories, offering an unique view of the consumption dynamics around a job loss.

[37]The original specification also includes a set of worker's observable characteristics and time fixed effects. Since in the model all workers are ex-ante identical and the simulated data comes from stationary equilibrium, I omit those terms.

[38]A job loss is defined as in Saporta-Eksten (2014). A job loser is an individual unemployed at the time of the interview due to involuntary unemployment of firm closure. Notice that in the model all transitions to unemployment are involuntary.

Figure 2: Effects of job displacement on wages and consumption



(A) Wages (B) Consumption

Notes: Coefficients $\{\delta_k\}$ from the distributed lag regression (18) for log wages (**Panel A**) and consumption (**Panel B**). The blue line corresponds to estimates using model simulated data. The orange line corresponds to estimates from Saporta-Eksten (2014) using PSID for the years 1999-2009. The PSID is conducted in biannual waves, with earnings data collected for the year previous to the interview, while consumption is reported for the interview year (see text for more details on the timing). The sample includes all non-SEO male heads of households, 24-65 years, hourly wages above 0.5 the state minimum wage, with a minimum of 80 annual hours of employment. The sample in the model simulated data follows as close as possible the sample selection and the timing restrictions.

the displacement ($k = -2$), the year of job loss ($k = 0$), and ten years following the displacement ($k = 2, 4, ..., 10$).

I run regression (18) on data simulated from the model.[39] Figure 2, Panel A reports the estimation results for wage dynamics on data simulated by the model together with numbers reported by Saporta-Eksten (2014). Panel B shows the coefficients from the regression with log consumption as the dependent variable. Overall, the model implied behavior of wages and consumption following a job loss is comparable to those in the data.

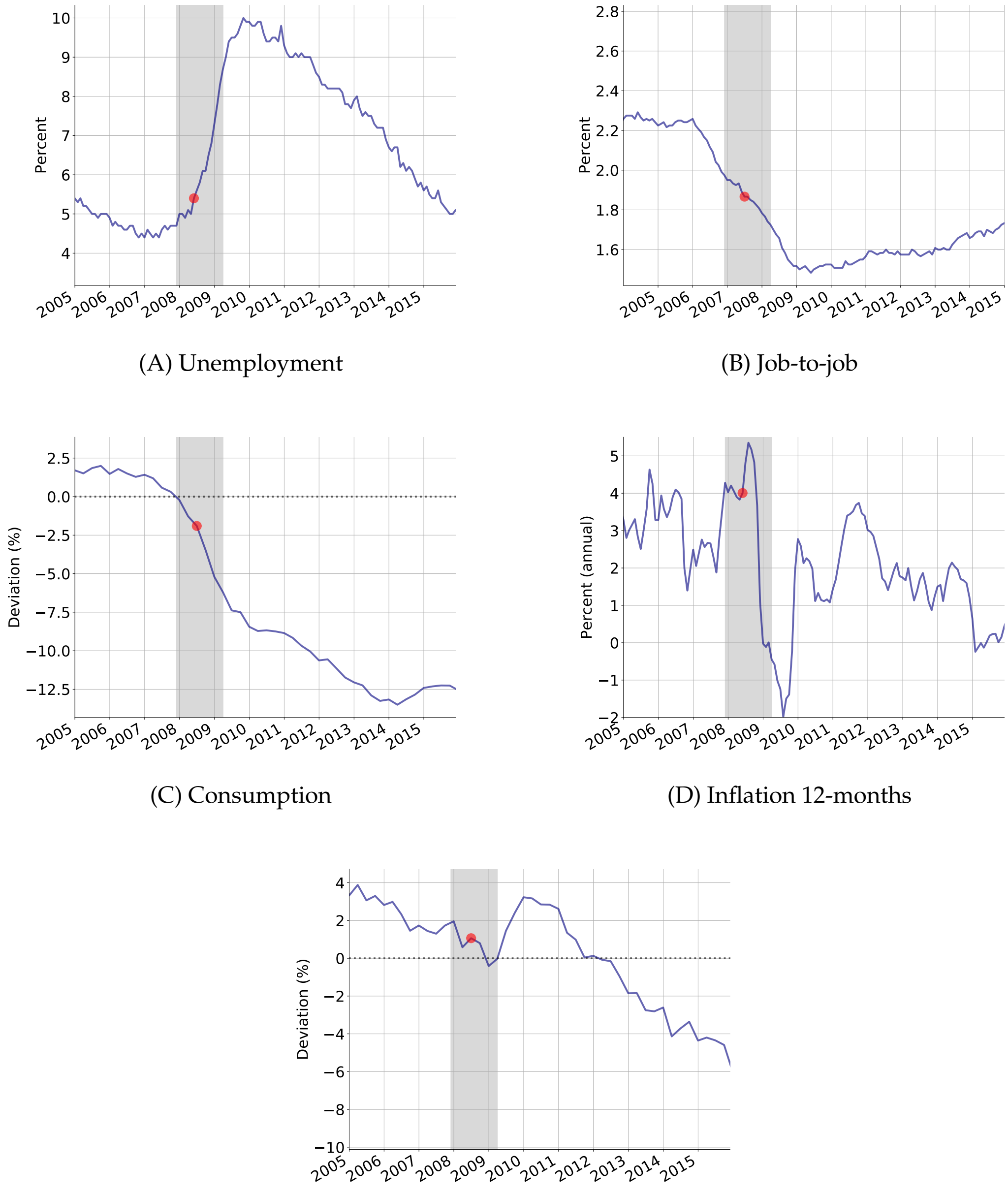
6 Results

In what follows, I conduct and analyze the main quantitative exercise of the paper: an adverse (reduced form) financial shock aimed to capture labor market movements during the Great Recession (GR). I consider the perfect-foresight solution to an unanticipated aggregate shock, starting from the steady state with no aggregate risk ("MIT shocks"). The numerical implementation is discussed in Appendix C.

Great Recession Figure 3 shows the behavior of some aggregate variables during and after the GR. From the last quarter of 2007 until the second quarter of 2009, the US experienced a severe

[39]Details on the simulation and adjustments of the data are discussed on Appendix B.

Figure 3: Great Recession aggregate series



(A) Unemployment

(B) Job-to-job

(C) Consumption

(D) Inflation 12-months

(E) Labor productivity

Notes: This figure plots the behavior of US aggregates for the GR period. Consumption and labor productivity are log-linearly detrended. Inflation corresponds to year-over0year changes in price level. All other variables are in levels. The red dot marks the second quarter of 2008, which I will use as the time-0 steady state when comparing model IRFs to the data. See Appendix B for data sources.

economic downturn: unemployment rate more than doubled, reaching 10 percent, job-to-job transitions fell by 0.6 percentage points and consumption dropped by almost 4%. Recovery has been really slow. Unemployment took 6 years to go back to its steady-state level, while job-to-job transitions failed to do so to this date. Figure 3, Panel (C), which plots log-deviations of consumption from a linear trend estimated from 1984, shows that consumption growth during the recovery has not been high enough to close the negative gap opened during the GR. Despite the depth of the downturn, inflation only fell modestly – with the exception of last quarter of 2008, when prices fell by 6%, inflation has fluctuated in the range of 1-3% for most of the recovery. The limited amount of disinflation in face of the large contraction in economic activity was seen as puzzling.[40] In particular, inflation behavior is surprising if viewed thought lens of the Phillips curve, here thought both as an empirical and theoretical relation connecting real variables, like unemployment, marginal cost or other measure of "slackness", to inflation. Coibion and Gorodnichenko (2013) make this point by showing that a Phillips curve relating inflation and unemployment estimated from 1960 to 2007 consistently underpredicts inflation by 2-3% in the years following the GR. This fact is usually referred to as *missing disinflation*.

Labor productivity, Figure 3, Panel (E), starts to decrease sometime before the Great Recession, features short-lived spike in 2009/2010, only to slow down again around 2012. The slowdown in labor productivity, also highlighted by Christiano, Eichenbaum, and Trabandt (2014), Reifschneider, Wascher, and Wilcox (2015) and Fernald et al. (2017), is often cited as contributing to the slow recovery following the recession. The causes behind it are a matter of debate. One view, considers that the productivity behavior could be a direct result of the crisis, which led firms to reduce their productivity-enhancing investments.[41] A second view, articulated by Fernald et al. (2017), considers the fall to be unrelated to the factors leading to the GR and simply the result of poor luck (i.e., of exogenous negative shocks to TFP). As I discuss next, the job ladder provides an alternative (complementary) explanation that ties the fall in labor productivity to the slowdown in labor reallocation.

Financial Shock In what follows, I hit the economy with a reduced form financial shock calibrated to target unemployment dynamics during the Great Recession.[42] While I do not model financial frictions explicitly, I consider a shock that transmits through the economy in manner similar to that of a financial shock. Specifically, I shock the spread χ_t in the discount rate of labor intermediaries. The shock raises the required rate of return for their vacancy-posting investment

[40] Hall (2011), for instance, argues that popular DSGE models based on the simple New Keynesian Phillips curve "cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack".

[41] An example of such is Anzoategui et al. (2019), who develop a model of R&D and technology adoption. In this environment, the fall in TFP becomes an endogenous outcome of a financial shock.

[42] Although the fundamental cause of the GR is still a matter of debate, it is clear that a shock to the financial sector played a crucial role.

decisions, directly reducing firms' incentives to enter the labor market. In a similar exercise, likewise trying to understand the GR, Christiano, Eichenbaum, and Trabandt (2014) model a financial shock as a "wedge" to the household intertemporal Euler equation for capital investment, which drives a spread between the rate of return of capital and the risk-free rate.[43] In my model, investment occurs through vacancy creation: firms must expend resources to post vacancies, which can lead to the creation of a worker–firm match providing a long-lived profit stream to the firm. The financial shock then raises the required rate of return for this investment, as would the investment wedge in a model with capital.[44]

Figure 4 shows the impulse response to a increase in the spread of labor intermediaries. The shock is calibrated to target unemployment dynamics during the Great Recession.[45] The shock directly affects vacancy-posting incentives by reducing the value of a match for the firms. Through the free-entry condition (15), vacancies collapse, making unemployment surge (Panel (A)) and job-to-job transitions fall (Panel (B)). In equilibrium, unemployment increases by 5 percentage points, consumption falls 10% at the trough, and labor productivity – measured as output divided by the measure of employed workers – falls by 6%. The overall behavior predicted by the model is similar to that during the Great Recession. Figure 4 also shows the behavior of marginal costs and inflation. The model predicts a sharp initial drop of marginal costs. Inflation, however, falls only momentarily and quickly reverts above steady state.

What explains these results? The fall in job-to-job transitions keeps employed workers stuck at the lower rungs of the productivity ladder. This misallocation in the employment distribution explains the aggregate labor productivity movements in Panel (B), which fall even though total factor productivity Z_t has not changed.[46] The effects of misallocation are persistent and prevail even after the unemployment rate returns to its steady-state value. Similar to an adverse tech-

[43]More generally, this shock relates to the investment wedges from business cycle accounting literature explored by Chari, Kehoe, and McGrattan (2007), who show that popular theories of financial frictions, such as Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999), manifest themselves as wedges to investment Euler equation.

[44]Versions of the search and matching model in which firms' discount factor fluctuates in response to aggregate shocks have been recently explored by Hall (2017), Kehoe, Midrigan, and Pastorino (2017) and Borovicka and Borovickova (2018). Time-varying discount rates considerably increase the model's unemployment volatility compared with the risk-neutral textbook search and matching model. In these examples, however, the firm's discount rate varies endogenously in response to technological shock or a credit tightening shock. Here, I consider exogenous variations in the wedge χ and interpret those as standing for a financial shock.

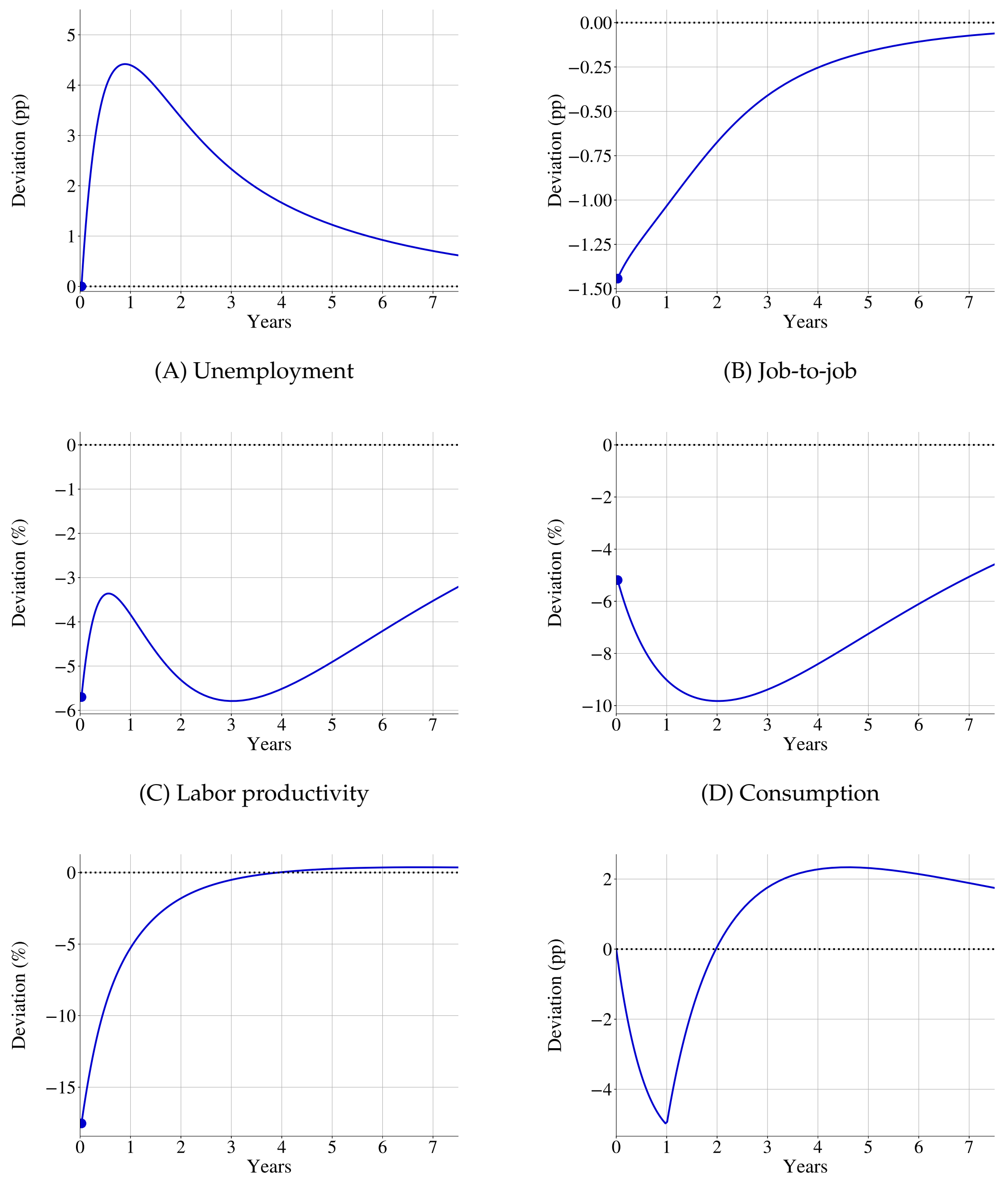
[45]I consider paths for χ_t of the form

$$\begin{cases} \chi_0 & \text{if } t < \bar{T} \\ \chi_0 \exp(-\chi_1 t) & \text{if } t > \bar{T} \end{cases} \tag{19}$$

I explore different combinations of $\bar{T}, \chi_0, \chi_1$ and choose the one that more closely matches the unemployment dynamics during the GR. Getting the persistence of unemployment is particularly hard, since the misallocation induced by the shock is itself a force that pushes unemployment back to steady state. See calibration section for an explanation of this point.

[46]The labor productivity measure captures changes both in materials input usage and to the average match productivity of employed workers. Using the production function of retailers, one can show that model implied labor

Figure 4: Aggregate responses to a spread shock.



(A) Unemployment

(B) Job-to-job

(C) Labor productivity

(D) Consumption

(E) Marginal costs

(F) Inflation 12-months

Notes: Model response to the financial shock defined in the text. Inflation 12-month denote the year-over-year change in the price level. Unemployment and job-to-job transitions are in percentage point deviations. All other variables are plotted in log deviations from steady-state.

nological shock, the misallocation exerts upward pressures on marginal costs, which explains the inflationary pressures during the recovery.

At the moment of the shock, however, the supply of labor services has not yet changed.[47] So the response over initial periods is mainly driven by a fall in aggregate demand that responds to the lower future incomes and higher real interest rates. Since the supply of labor services takes time to adjust, most of the initial reaction occurs via the usage of material inputs, driving down price of labor services and of marginal costs.[48] This does not result in a major disinflation because inflation depends on the whole discounted sum of future marginal costs – recall equation (13). Higher future marginal costs during the recovery therefore prevent inflation from falling too much at the outset. Several other papers offered related explanations for the missing disinflation.[49] Similar to those, I relate the missing disinflation to a fall in productivity. But in my case, the fall in labor productivity comes from the slowdown in employment reallocation in the labor market.

7 Unpacking the Mechanism

In the introduction, I've argued that the job ladder structure leads to both supply and demand-side consequences: the distribution of employed workers moves in response to aggregate shocks and drives aggregate labor productivity, while workers' demand for consumption reacts to earnings changes induced by labor market flows.

In what follows, I try to evaluate the relevance of each component by comparing the response of the full model to counterfactuals aimed to expose the impact of the job ladder on demand and supply. Section 7.1 investigates the supply consequences of the job ladder, while Section 7.2 explores the demand/consumption block.

productivity is given by

$$\frac{Y_t}{1-u_t} = (1-\gamma mc_t)(m_c\gamma)^{\frac{\gamma}{1-\gamma}} Z_t \frac{\mathcal{N}^e_t}{1-u_t}.$$

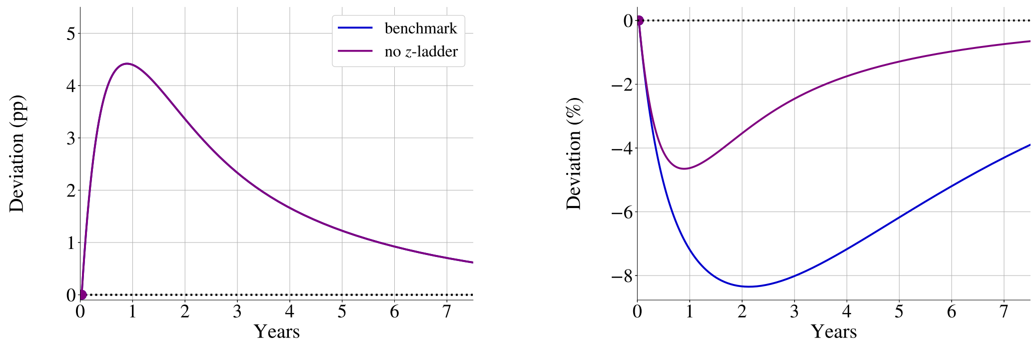
So labor productivity can fall either due to (i) fall in TFP component Z_t; (ii) decline in marginal costs, which induces a decline in materials; (ii) decline in the average match productivity of employed workers $\mathcal{N}^e_t/1-u_t$. Since $\mathcal{N}^e$ is a state variable in the model, the initial drop in labor productivity comes entirely through a reduction in materials. Along the recovery, marginal costs rise *above* steady state, so the labor productivity fall is entirely due to the lower average match productivity of employed workers.

[47] Remember that the supply of labor services is given by $\int z \, d\Psi_t(z)$. At $t = 0$, the distribution Ψ_0 is a state variable so labor services are equal to their steady-state value.

[48] The dynamic of the response is similar to the response of new shocks explored in Christiano (2010) and Barsky and Sims (2011). As explained in Christiano (2010): "News that technology will worsen in the future creates the expectation that future inflation will be high and this leads an inflation forecast targeting monetary authority to increase the real rate of interest. This policy reaction creates an immediate contraction in the economy which reduce marginal costs."

[49] See Christiano, Eichenbaum, and Trabandt (2014) and Anzoategui et al. (2019) for explanations that rely on the slowdown on productivity growth, and Del Negro, Giannoni, and Schorfheide (2015) for an explanation that does not rely on supply-side considerations, but on monetary policy instead.

Figure 5: Aggregate responses to a spread shock. Left panel: unemployment. Right panel: labor services.



(A) Unemployment

(B) Labor services

Notes: The blue line corresponds to the *benchmark* model response to the adverse financial shock. The purple line corresponds to the *exogenous-*Λ *equilibrium* response to the same shock. In this counterfactual, the supply of labor services varies only with the measure of employment, ignoring the distribution of workers across the z-ladder.

7.1 Aggregate Productivity Effects of the Job Ladder

What are the supply-side effects induced by worker reallocation in the job ladder? To answer this question, I will consider a different notion of equilibrium, which I denote as *exogenous-*Λ *equilibrium*. The definition is analogous to the original equilibrium, except for the following modifications: I drop the free-entry condition (15) and treat both the supply of labor services $\mathcal{N}_t^e$ and workers' income process $\{\mathbb{1}_{it}^u, y_{it}\}$ as exogenous.[50] The full definition is laid out in the appendix.

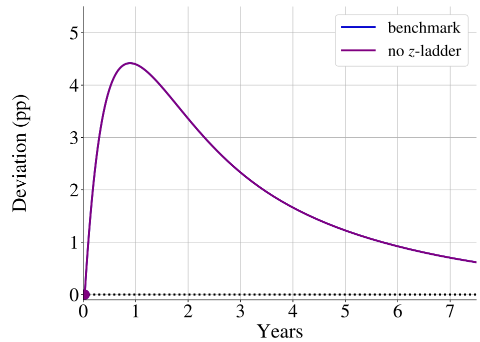
To isolate the productivity effects coming from the job ladder, I compare the benchmark response with the exogenous-Λ equilibrium in which workers face the same equilibrium labor income processes $\{\mathbb{1}_{it}^u, y_{it}\}_{t\geq 0}$, but where the supply of labor services $\mathcal{N}_t^e$ varies only with the measure of employed workers, according to $(1-u_t)\mathcal{N}^{e,SS}$. This counterfactual neutralizes the impact of the job ladder on supply of labor services by treating all employed workers as equally productive, while keeping the job ladder implications for labor earnings unchanged.[51]

Figure 5 plots the evolution of labor market stocks for the benchmark and the counterfactual equilibrium. Panel (A) shows that unemployment fluctuations are the same in the two economies, as expected. Panel (B) plots the overall supply of labor services $\mathcal{N}_t^e$. In the counterfactual equi-

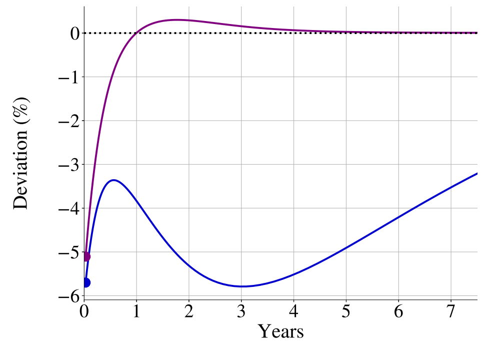
[50]Importantly, I do not impose that the exogenous paths/processes $\{\mathcal{N}_t^e, \mathbb{1}_{it}^u, y_{it}\}$ must be the outcome of a feasible path of transition rates Λ_t.

[51]An alternative way to answer this would be write down a model without on-the-job search and compute the economy's response to the same underlying shocks. There are a couple of difficulties with this strategy though. First, it is not clear how to incorporate the benchmark earnings process into a model without a job ladder. Second, the model without on-the-job search would feature a different response for variables in and out of the labor market, making the comparison of variables like inflation and consumption less transparent.

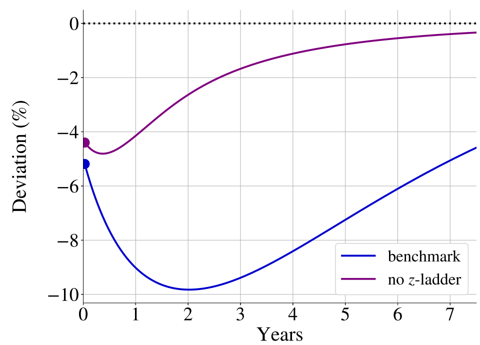
Figure 6: Aggregate responses to a spread shock.



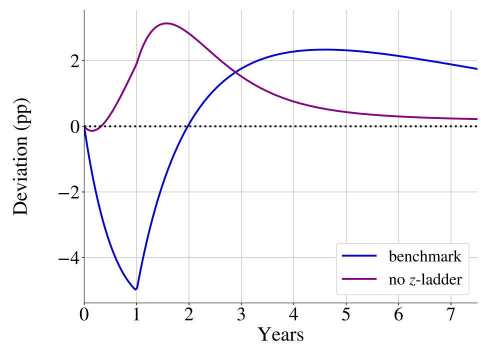
(A) Unemployment



(B) Labor productivity



(C) Consumption



(D) Inflation 12-months

Notes: The blue line corresponds to the *benchmark* model response to the adverse financial shock. The purple line corresponds to the *exogenous*-Λ *equilibrium* response to the same shock. In this counterfactual, the supply of labor services varies only with the measure of employment, ignoring the distribution of workers across the z-ladder.

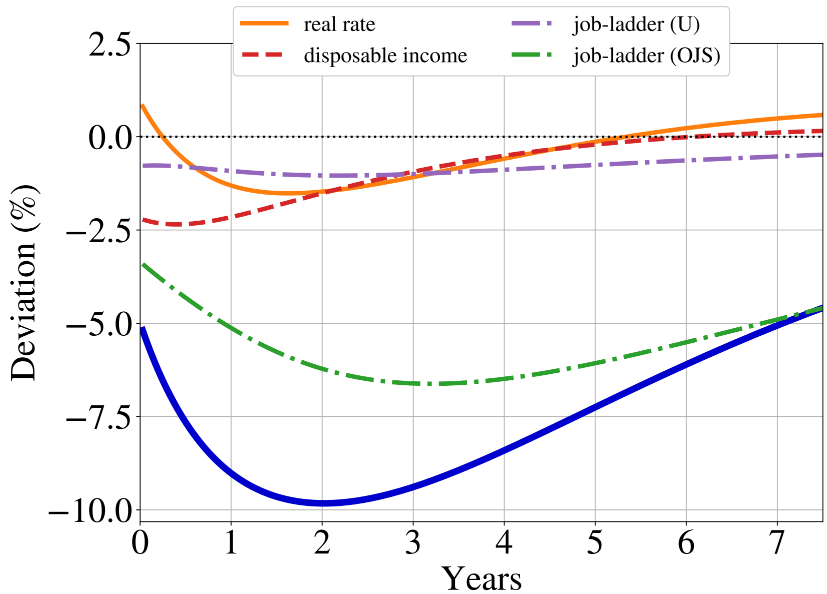
librium (purple line), labor services mirror the movements in unemployment. In the benchmark (blue line), the stock of labor services suffer a larger and more persistent decline than unemployment, reflecting the misallocation that occurs among employed workers.

Figure 6 shows that this difference matters tremendously for the response of other aggregates. Consumption in the counterfactual economy is much less persistent and quickly recovers toward steady state. The counterfactual also predicts inflation throughout the whole transition, with measured labor productivity rising above steady state instead of falling.

7.2 Aggregate Demand Effects of the Job Ladder

Understanding the Consumption Response Those in favor of adding incomplete markets to the study of business cycles often defend that view by noticing that the consumption behavior of in-

Figure 7: Consumption response decomposition



Notes: The blue line corresponds to the equilibrium consumption response to the financial shock. Other lines correspond to counterfactuals consumption responses that allow for some equilibrium variables entering (20) to adjust as in equilibrium, while the remaining variables are kept at their steady-state values. Real rate refers to the case where only the real rate $\{r_t\}_{t\geq 0}$ adjusts. Disposable income refers to the case where price of labor services, lump-sum profits, taxes and transfers $\{\varphi_t, d_t, \tau_t, \tau_t^0\}_{t\geq 0}$ adjust. Job ladder (U) refers to the case where the job-finding rate $\{\lambda_t\}_{t\geq 0}$ adjusts. Job Ladder (OJS) refers to the case where the on-the-job contact rate $\{\lambda_{et}\}_{t\geq 0}$ adjusts.

complete markets households is much more aligned with empirical evidence than the behavior of the representative-agent (complete markets) consumer (see Kaplan & Violante, 2018). This different behavior in turn has been shown to matter for how monetary and fiscal shocks are transmitted to consumption, with the overall message being that household consumption in heterogeneous agent models is more sensitive to income and less sensitive to interest rates than in representative agent models.

In a standard HANK model (with no frictions in the labor market), the income channel operates through changes in competitive prices (like wage) and quantities (hours, dividends) that directly determine workers current disposable income.The frictional labor market adds another channel: fluctuations to transition rates impact workers' expected *future* labor income path and higher order moments of the labor income process. In particular, recessions increase the duration of unemployment and dampen the expected wage growth of employed workers. In what follows, I study the role of this new channel for the consumption response following the financial shock.

The aggregate consumption function $\mathcal{C}_t$ is constructed by integrating workers' optimal consumption response $\{c_{it}\}_{i\in[0,1],t\geq 0}$, which is a function of the sequence of equilibrium prices, quantities and labor market transition rates. I make this dependence explicit by expressing aggregate consumption as a direct function of these equilibrium paths

$$\mathcal{C}_t(\{r_s, \varphi_s, d_s, \tau_s, \tau_s^0, \lambda_s, \lambda_{es}\}_{s\geq 0}) := \int_i c_{it} di \tag{20}$$

To evaluate the impact of the different channels, I compute the partial equilibrium consumption response to paths that let some variables adjust as in equilibrium while keeping others at their steady-state value. In particular, I divide variables entering the worker's problem into three groups: (i) the real rate (r); (ii) the competitive price of labor services, dividends and government transfers (φ, d, τ_0), which I jointly refer below by *disposable income*; (iii) labor market transition rates – in other words, the job finding rate (λ) and on-the-job contact rate (λ_e).[52]

Totally differentiating (20), we can write the change in consumption at date t, denoted by $d\mathcal{C}_t$, as

$$d\mathcal{C}_t = \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial r_\tau} dr_\tau d\tau + \sum_{i\in(\varphi,d,\tau_0)} \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial i_\tau} di_\tau d\tau + \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \lambda_\tau} d\lambda_\tau d\tau + \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \lambda_{e\tau}} d\lambda_{e\tau} d\tau \tag{21}$$

Figure 7 plots this decomposition together with the equilibrium consumption response (blue line). In line with what others have found, consumption response is driven mainly by changes in income (both current and future) rather than changes in the real rate. Among the variables affecting workers' income, changes in the on-the-job contact rate, λ_e, account for most of the response, especially at longer horizons. Changes in the price of labor services, dividends and government transfers constitute the second most relevant channel, while the job finding rate accounts for a small fraction of the overall consumption adjustment.[53] The contribution of worker contact rate λ_e to overall consumption response highlights the importance of going beyond unemployment and incorporating job-to-job transitions if one wants to understand the impact of shocks that significantly move labor market flows.

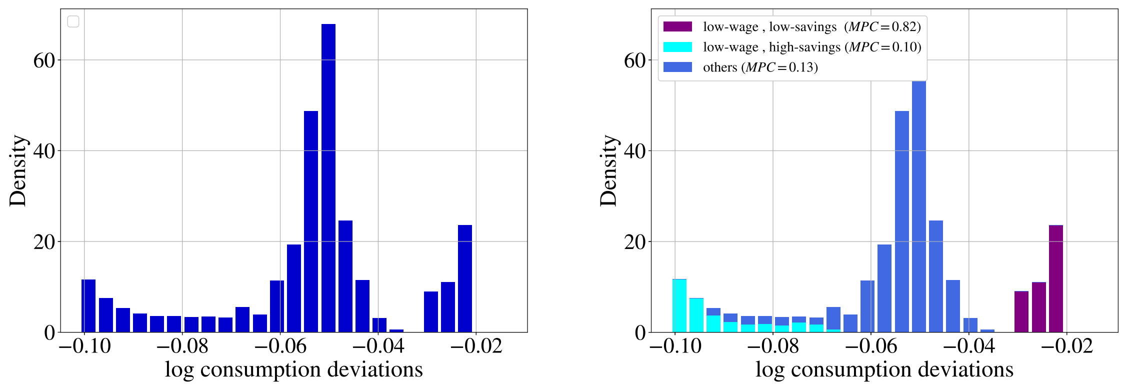
Consumption Response Across the Distribution The aggregate consumption response hides a significant amount of heterogeneity across the worker's distribution. Figure 8, Panel (A) highlights this. It features the distribution of time-zero consumption log-deviations from steady-state upon the financial shock (i.e., the consumption adjustment that takes place immediately upon impact of the shock). While aggregate consumption falls by approximately 8%, the cross-sectional consumption response shows a significant dispersion, with percentage changes ranging from -2%

[52] In the context of a monetary policy shock, Kaplan, Moll, and Violante (2018) distinguish between direct (real rate) and indirect (general equilibrium) effects. In my exercise, all variables entering the worker's problem are indirect general equilibrium effects.

[53] As I show below, while unemployed workers are the sensitive to the fall in the job finding probability, they represent a small fraction of the population, so their reaction contributes little to overall consumption fluctuation.

to -10%.

Figure 8: Histogram for time-0 log-deviations of consumption

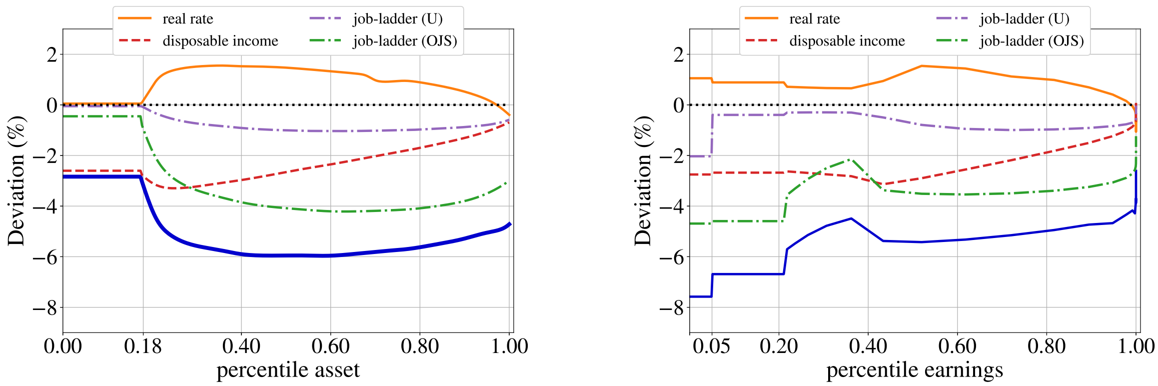


Notes: This figure explores the time-0 consumption reaction to the financial shock. The left panel features the histogram of time-0 consumption log-deviation from steady state for the cross-section of workers. The right panel splits this distribution in three different groups defined by their joint wage earnings and wealth holdings: *low-wage, low-savings* corresponds to workers with zero wealth earning wages below the 40*th* percentile; *low-wage, high-savings* corresponds to workers with savings above the 40*th* percentile and wages below the 40*th* percentile; *others* corresponds to all oher workers. In terms of their MPCs, the quarterly marginal propensity to consume out a $500 lump-sum transfer for each group is 0.82, 0.10 and 0.13 respectively.

To explain the dispersion in responses, I examine how the initial consumption drop varies along the wealth (Figure 9, Panel A) and wage earnings distribution (Figure 9, Panel B). Each panel plots the overall consumption drop (blue line) along with the decompositions at each point of the distribution. Consider first the consumption responses across the wealth distribution. While consumption response is relatively flat over most of the distribution (ranging from 3 to 6%), its decomposition is far from uniform. The fall in consumption for workers with zero wealth (the initial flat section of the figure) is almost entirely due to the drop in disposable income (red line). As we move towards higher percentiles, the response to changes in disposable income dampens (consumption falls by less) while workers become more reactive to the changes in the real rate. These observations are consistent with Kaplan and Violante (2018), who report similar decompositions for a monetary shock.[54] The response to labor market rates (green and purple lines)—the new element here—is U-shaped in the wealth distribution. Workers at the borrowing constraint have a low sensitivity to changes in labor market flows, while workers in the middle of the distribution react markedly to it. Hence, even workers who have a large buffer-stock of savings and who are

[54]In particular, Kaplan and Violante (2018) state the result in terms of direct and indirect effects. The direct effects of the monetary policy shock are those stemming from changes in the real rate alone; that is, those that operate even in the absence of any change in household disposable labor income. The indirect effect is the change in consumption coming from the movements in household income that arise in general equilibrium, which mostly operate through an increase in labor demand. They show, in the context of their two-asset HANK model, that most of the consumption response to monetary policy shock comes from the indirect effects.

Figure 9: Consumption decomposition across the wealth distribution



Notes: This figure plots the consumption decomposition across worker distribution. The left panel plots time-zero consumption percentage deviation from steady state along the wealth distribution; the right panel does the same exercise for the earnings distribution. Other lines correspond to counterfactuals consumption responses that allow for some equilibrium variables entering (20) to adjust as in equilibrium, while the remaining variables are kept at their steady-state values. Real rate refers to the case where only the real rate $\{r_t\}_{t\geq 0}$ adjusts. Disposable income refers to the case where price of labor services, lump-sum profits, taxes and transfers $\{\varphi_t, d_t, \tau_t, \tau_t^0\}_{t\geq 0}$ adjust. Job ladder (U) refers to the case where the job-finding rate $\{\lambda_t\}_{t\geq 0}$ adjusts. Job Ladder (OJS) refers to the case where the on-the-job contact rate $\{\lambda_{et}\}_{t\geq 0}$ adjusts.

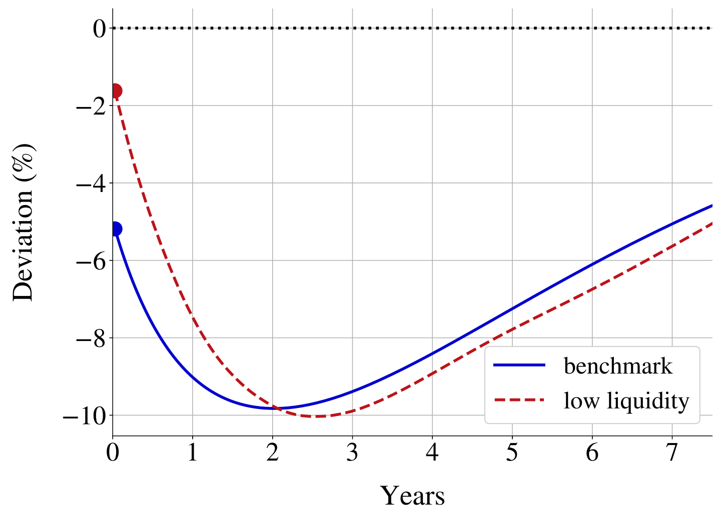
well insured to changes in disposable income react strongly to movements in labor market rates.

Panel B shows repeats the exercise, but now for the wage distribution. The flat portions in the graph, from 0 to 5% and from 5% to 20%, represent the mass of unemployed and recently employed workers respectively. Consumption expenditures falls mostly for unemployed workers and decrease as we move along the income distribution. Turning to the decompositions, changes in labor market rates account for the most of the unemployed worker reaction. As we move right in the distribution (starting around 40^{th} percentile), workers become less sensitive to changes in disposable income ("current income" line increases), but are still very affected by changes in labor market rates (the "job-ladder (OJS)" green line is basically flat over most percentile 40 to 90). As before, the effect of the interest rate is positive for most workers, except at the very high percentiles.

The joint inspection of Panel A and B of Figure 9 suggests that workers whose consumption fall the most are those with mid to high levels of savings holding low-wage jobs (equivalently, located at lower rungs of the ladder) at the moment of the shock. To verify this conjecture, I condition the distribution of consumption log deviations in Figure 8 on worker's wealth and wage. I focus on the subset of workers receiving low-wages at the moment of the shock and split them into two different groups: (i) low and high savings.[55] Panel B of Figure 8 confirms the suspicion that

[55] The thresholds for savings and earnings are discussed in the legend of Figure 8.

Figure 10: Share of HtM and the aggregate consumption response



Notes: The blue line corresponds to the response to the financial shock in the *benchmark* calibration. The red dashed line corresponds to the response to the financial shock in the *low-liquidity* calibration, which features a higher share of HtM individuals.

low-wage high-savings individuals are the ones cutting consumption the most upon the shock's impact. In contrast, workers from the low-wage low-savings group are among those cutting consumption the least. This occurs despite the fact that the MPC of the latter group is much higher than of the former ($0.82 > 0.10$).

The fact that low-MPC workers (conditional on their position in the ladder) reduce their consumption by more in response to the negative financial shock contrasts with the kind of analysis one come across in HANK papers, which usually highlights high-MPC individuals as the driving force behind consumption response.[56] The apparent contradiction has to due with the fact that I consider labor market frictions (in particular, with a long job ladder structure), and—as we already noted in the decomposition of aggregate consumption—the type of income changes that are brought by fluctuations in labor market flows.

To better understand this result, note that the consumption behavior of low-wage workers is driven by their expectation of earnings growth. Their low position in the ladder imply that the expectation of *future* wage increases brought by on-the-job search dominates the potential earnings losses from a job loss. Facing an increasing expected income path, low-wage workers want to raise their current consumption relative to their current income in order to achieve a smoother consumption path. Their ability to do so hinges on their savings, and is smaller the closer workers are to being constrained. A shock that persistently reduces overall labor market tightness hurts

[56]See **auclert19**; Kaplan and Violante (2018) for examples of this.

workers by reducing their whole path of expected *future income*, as contacts between workers and vacancies are tapered off. It does nothing, however, to workers' *current disposable income*.[57] High-MPC individuals react strongly to changes in their current disposable income, but their sensitivity to changes in future income is much lower as they are either constrained or find themselves close to the constraint.

Overall, this suggests that the presence of HtM individuals should dampen the initial consumption response. The share of HtM in the economy is directly related to the overall amount of liquidity available for households to self-insure against earnings risk. Hence, in order to verify the influence of HtM behavior on the consumption response, I solve the economy response to the same financial shock under a low-liquidity calibration. [58] Figure 10 plots the consumption response on both cases. Indeed, the larger the share of HtM agents, smaller are the initial consumption decrease to a financial shock.

8 Conclusion

The shutdown of the job ladder during recessions has demand and supply implication that go way beyond the equilibrium in the labor market. As Moscarini and Postel-Vinay (2017) puts it, the *cyclical job ladder* shapes business cycles. In this paper, I developed a Heterogeneous Agents New Keynesian (HANK) model with search frictions in the labor market. Workers search on-the-job as well as through unemployment, which gives rise to a job ladder structure. The job ladder plays a critical role in transmitting aggregate shocks to aggregate labor productivity and consumption demand—the allocation of workers over the ladder partially determines production at any given point in time, while workers' labor income and consumption expenditures varies with the intensity of labor market flows. An adverse financial shock calibrated to mimic the dynamics around the Great Recession generates both the missing disinflation and slow recovery.

[57]In equilibrium, the changes in labor market are accompanied by changes in other variables like the price of labor services, profits and government transfers that do affect workers' disposable income. But, as we showed in Figure 7, those have only a small impact on the consumption response to the financial shock, which is driven by changes in labor market flows.

[58]This is obtained by reducing the amount of government bonds available in the economy, therefore reducing the mean available for households to self-insure against earnings risk.

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A Model derivation

A.1 Filtering problem

I start by writing down the process for productivity and piece-rate wage $\{z_t, y_t\}$ for the worker. For the derivations in this section, I consider the economy to be at steady state, so transitions rates are not indexed by t. Productivity z_t is specific to the worker-firm match and is drawn at origination from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0, 1]$. The type of contract offered by firms and labor market transitions determine the evolution of piece-rate y_t. Let $(0, 0)$ stand in for the status of unemployed agent and $X = \{0\} \cup [\underline{z}, \bar{z}]$, so the state space for the Markov process $\{z_t, y_t\}$ is X^2. In what follows I describe this process in recursive notation, letting $\cdot^*$ denote the new state.

The rate at which worker leaves state (z, y) to a new state (z^*, y^*) depend only on the employment status and the type of transition: workers leave unemployment state $(0, 0)$ with intensity λ, employed workers contact other firms with intensity λ_e and suffer exogenous destruction shocks with intensity δ. Upon any of those events, the distribution of the worker's new state (z^*, y^*) is given by a *stochastic kernel funtion* $T_i : X^2 \times X^2 \to \mathbb{R}$, where $i \in \{ue, ee, eu\}$ indexes the different type of transitions.

The kernel when finding a job from unemployment T_{ue} or receiving a match destruction shock T_{eu} do not depend on the current state (z, y). I write then as

$$T_{ue}(z^*, y^*) = \gamma(z^*)\delta(y^* - \underline{z}) \tag{A.1}$$

$$T_{eu}(z^*, y^*) = \delta(z^* - 0)\delta(y^* - 0) \tag{A.2}$$

where $\delta(\cdot)$ is a Dirac delta function.[59] The match productivity of a worker moving out of unemployment is drawn from exogenous distribution Γ and its piece-rate wage is $\underline{z}$ no matter which firm he goes to. An employed worker that receives a destruction shock moves to unemployment state $(0, 0) \in S$.

The stochastic kernel for an employed worker T_{ee} is more complicated as it depends on the worker's current state (z, y). Remember from the discussion in the main text that an employed

[59] The Dirac delta can be loosely thought of as a object with the following properties

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} g(x)\delta(x)\, dx = g(0)$$

I use the Dirac delta in the derivation whenever the worker transition is deterministic — for instance, when the worker looses his job he transitions to state $(0, 0)$ with certainty.

worker with state (z, y) who receives an offer from outside firm will: (i) with probability $\Gamma(y)$ discard the offer since it is smaller than its current wage; (ii) with probability $\gamma(y')$ receive an wage offer of $y' \in (y, z)$ which is matched by its current firm, who offers $y' + \epsilon$ to the worker; (iii) with probability $1 - \Gamma(z)$ the worker meets a firm $z^* > z$ which poaches the worker by offering $z + \epsilon > z$, the maximum wage offer of the incumbent. So, taking $\epsilon \downarrow 0$, I write

$$T_{ee}(z^*, y^*|z, y) = \begin{cases} \gamma(z^*) \times \delta(y^* - z) & \text{for } z^* > z, \\ \gamma(y^*) \times \delta(z^* - z) & \text{for } y^* \in (y, z) \\ \Gamma(y) \times \delta(y^* - y) \times \delta(z^* - z) & \text{ow} \end{cases}$$

Integrating out firm productivity z^* from stochastic kernel T_i, we recover the conditional density f_i for piece-rate wage y^*

$$f_{ue}(y^*) = \delta(y^* - \underset{\sim}{z}), \quad f_{eu}(y^*) = \delta(y^* - 0)$$

$$f_{ee}(y^*|z, y) = \begin{cases} \bar{\Gamma}(z) \times \delta(y^* - z) & \text{for } y^* = z \\ \gamma(y^*) & \text{for } y^* \in (y, z) \\ \Gamma(y) \times \delta(y^* - y) & \text{for } y^* = y \end{cases}$$

which ultimately is the distribution workers care about when deciding how much to consume. Importantly, f_i is a function of current match productivity z, which the worker is uninformed about.

Therefore, when making its consumption/savings decisions he must hold beliefs about z to evaluate the probability distribution for the future piece-rate wages f_i. Let Φ be worker's belief distribution regarding the firm's productivity, with ϕ denoting the (generalized) density function.[60] This distribution is a function of the whole history of job transition and wage offers experienced by the worker. Fortunately, Bayes's rule gives us a way to update this distribution in response to a new signal, so we can treat this problem recursively.[61] Using the same notation as before, let ϕ denote the pre-transition belief density and ϕ^* the updated belief following a labor market event (transition or a wage gain inside the firm). Again, transitions in and out of unemployment involve simple updates that are independent of the previous belief

$$\phi^*_{ue}(z^*) = \gamma(z^*) \tag{A.3}$$

$$\phi^*_{eu}(z^*) = \delta(z^* - 0) \tag{A.4}$$

[60] I use the generalized classification because some densities will be degenerate.

[61] The derivation in this section draws upon Hansen (2007).

where δ is again the Dirac delta function. An unemployed worker who meets a firm holds as belief the exogenous

For an employed worker who meets an outside firm, the updated ϕ^* density function given new wage offer y^* and transition status is determined by Bayes's rule according to

$$\phi^*_{ee}(z^*) = \begin{cases} \dfrac{\int_{z<z^*} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int \left[\int_{z<z^*} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker switch jobs} \\ \\ \dfrac{\int_{z^*=z} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int \left[\int_{z^*=z} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker does not switch jobs} \end{cases} \tag{A.5}$$

Note that an employed worker gets to observe two signals: whether the highest wage offer came from the incumbent or the poacher — in the former, he realizes that $z^* > z$, while if he stays in the same match $z^* = z$ — and the new piece-rate offer y^*. The filtering problem can thus be thought as a substituting the original Markov process $\{z_t, y_t\}$ by a new one where the hidden match productivity z is replaced by a distribution Φ over possible values, the evolution of which is determined by equations (A.3) to (A.5).

In this case, the conditional density for piece-rate wages y^* becomes a *compound lottery*

$$\bar{f}_i(y^*|y, \phi) = \int f_i(y^*|z, y) d\Phi(z) \tag{A.6}$$

The following proposition show that distribution $\Phi(z)$ is fully characterized by the current piece-rate wage.

Proposition

The belief ϕ for an unemployed is degenerate at $z = 0$. Piece-rate densities in case of a job destruction and job finding from unemployment are independent from z and agree with the full information case, i.e. $\bar{f}_{ue} = f_{ue}$ and $\bar{f}_{eu} = f_{eu}$.

The belief ϕ for an employed worker is a function of piece-rate wage y only

$$\phi(z; y) = \frac{\gamma(z)}{\bar{\Gamma}(y)} \text{ for } z > y \tag{A.7}$$

with the condition piece-rate density in case of job-to-job transition given by

$$\bar{f}_{ee}(y^*|y) = \int f_{ee}(y^*|z, y) \times \frac{\gamma(z)}{\bar{\Gamma}(y)} dz \tag{A.8}$$

Proof: Conditional density in the case of transitions in and out of unemployment f_{ue}, f_{eu} follow directly from the discussion in the text.

For the employed worker, the proof simply apply Bayes rule for each possible transitions.

coming from unemployment When the worker is hired from unemployment he receives wage $y = \underline{z}$ and holds belief $\phi = \gamma$ equal to exogenous distribution of match productivity. Note that this satisfies (A.7).

employed worker with job transition Consider an employed worker with belief distribution Φ and piece-rate y who contacts an outside firm. Suppose that as an outcome of this contact, the worker receives an offer $y_1(+\epsilon)$ from the outside firm, while the incumbent offer is y_1. The worker accepts the offer from the poacher and his belief over productivity z^* of the new match is given by (A.5)

$$\begin{aligned}
\phi^*(z^*) &= \frac{\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z, y)d\Phi(z)}{\int \left[\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z, y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z)d\Phi(z)}{\int \int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z)d\Phi(z)dz^*} \\
&= \frac{\phi(y_1)\gamma(z^*)}{\phi(y_1) \int \mathbb{1}\{z^* > y_1\}\gamma(z^*)dz^*} \\
&= \frac{\gamma(z^*)}{\bar{\Gamma}(y_1)} \text{ for } z^* > y_1
\end{aligned}$$

where the second line substitutes T_{ee}, the third integrates with respect to z^*.

employed worker with wage increase in the firm Consider an employed worker with belief distribution Φ and piece-rate wage y. Suppose a poaching firm comes along with the following outcome: the incumbent firm offers a wage increase $y_2(+\epsilon)$ above the poacher's offer of y_2. The worker stays in the incumbent under a higher wage and its belief evolves as

$$\begin{aligned}
\phi^*(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z, y)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z, y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z=z^*\}} \mathbb{1}\{z > y_2\}\gamma(y_2)\delta(z^* - z)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \mathbb{1}\{z > y_2\}\gamma(y_2)\delta(z^* - z)d\Phi(z)\right] dz^*} \\
&= \frac{\gamma(y_2)\phi(z^*)\mathbb{1}\{z^* > y_2\}}{\gamma(y_2) \int \mathbb{1}\{z^* > y_2\}\phi(z^*)dz^*} \\
&= \frac{\phi(z^*)}{\bar{\Phi}(y_2)} \text{ for } z^* > y_2
\end{aligned}$$

employed worker with discarded wage offer Consider an employed worker with belief distribution Φ and piece-rate wage y. Suppose a poaching firm comes along and offer a wage smaller than the current piece-rate y, which does induce a counteroffer from the incumbent, i.e. $y^* = y$. Applying

Bayes rule one more time,

$$\begin{aligned}\phi(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y|z, y)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y|z, y)d\Phi(z)\right] dz^*} \\ &= \frac{\int_{\{z=z^*\}} \Gamma(y)\delta(y^* - y)\delta(z^* - z)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \Gamma(y)\delta(y^* - y)\delta(z^* - z)d\Phi(z)\right] dz^*} \\ &= \frac{\phi(z^*)}{\int \phi(z^*)dz^*} = \phi(z^*)\end{aligned}$$

which is the expected result as the signal does not reveal any new information regarding the productivity of the current match.

conclusion Whenever the worker moves from jobs or when he finds a job from unemployment, his belief ϕ satisfies (A.7). When he receives a wage increase by the incumbent, the updated density ϕ^* a function of the previous belief ϕ, which does not satisfy (A.7) for a generic ϕ. Note however that if we assume that ϕ if of form (A.7), then we get

$$\phi^*(z^*) = \frac{\phi(z^*)}{\bar{\Phi}(y_2)} = \frac{\gamma(z^*)/\bar{\Gamma}(y)}{\bar{\Gamma}(y_2)/\bar{\Gamma}(y)} = \frac{\gamma(z^*)}{\bar{\Gamma}(y_2)} = \phi(z; y_2)$$

Since, worker arrives at a firm either by job-to-job transition or from unemployment, ϕ must be of form (A.7). This proves the result. □

Comments on Wage Setting I end this section by discussing the wage contracts.

First, note that the specification adopted satisfies worker and firm's *individual rationality* as both parts always prefer following the contract to dissolving the match. Second, match origination and job-to-job transitions are efficient with workers always moving toward more productive matches.

However, contracts are not optimally designed.[62] First, since workers are risk averse, firms would be willing to offer some insurance against aggregate fluctuations in the price of labor services. Moreover, different from the current contract has firms overpaying workers when poaching. Since expected future earnings are increasing in the match productivity, more productive firms could in principle poach workers from less productive firms by offering less than the incumbent's maximum wage offer. By how much workers value smoother income paths and the timing of payments depends on their wealth, which makes the optimal contract a function of worker's assets. Not only I regard wage contracts conditional on workers' asset holdings a poor representation of

[62]See Lentz (2014) for the derivation of the optimal contracts in a environment with risk averse workers and where firms are allowed to make counter offers.

reality, implementing a wealth dependent wage would greatly complicate the determination of wages.

A.2 Worker Problem – Recursive Formulation

I present household's Hamilton-Jacobi-Bellman (HJB) in this section. I focus on the stationary versions of these equations. Let $V^u(a), V(a,y)$ denote the optimal value of unemployed, employed worker's original problem — see the description on the main text — starting from initial level of assets a and, in the case of employed worker, from earnings y. Furthermore, let $s^u(a,c), s(a,y,c)$ denote the savings of employed and unemployed worker with assets a, labor earnings y (b for the unemployed) when he consumes a flow c

$$s^u(a,c) := (1-\tau)\varphi b + ra + d(b) - c, \qquad s(a,y,c) := (1-\tau)\varphi y + ra + d(y) - c$$

where I have already incorporated the fact that dividends are distributed in proportion to labor earnings. The HJB is thus given by

$$\Big(\rho + \lambda_u\Big) V^u(a) = \max_c \Big\{ u(c) + \partial_b V^u(a)\, s^u(a;c) \Big\} + \lambda_u V(a, \underline{z}) \tag{A.9}$$

$$\begin{aligned} \rho V(a,y) = &\max_c \Big\{ u(c) + \partial_a V(a,y)\, s(a,y;c) \Big\} + \delta\big[V^u(b) - V(a,y)\big] \\ &+ \lambda_e \int_y \Big[\int_y^z \big[V(a,y^*) - V(a,y)\big]\, \mathrm{d}\Gamma(y^*) + \bar{\Gamma}(z)\big[V(a,z) - V(a,y)\big] \Big] \phi(z;y)\, \mathrm{d}z \end{aligned} \tag{A.10}$$

where $\phi(z;y)$ is the household belief regarding the current match productivity. Remembering the definition of $\bar{f}$ in (A.8) we can rewrite the HJB of the employed as

$$\begin{aligned} \rho V(a,y) = &\max_c \Big\{ u(c) + \partial_a V(a,y)\, s(a,y;c) \Big\} + \delta\big[V^u(a) - V(a,y)\big] \\ &+ \lambda_e \int_y \big[V(a,y^*) - V(a,y)\big] \bar{f}(y^*|y) dy^* \end{aligned} \tag{A.11}$$

Insights from Lise (2012) Lise (2012) also develops a model of on-the-job search in which risk averse workers decide how much to save. He derives an Euler equation describing how consumption growth depends on the preference fundamentals and labor market transitions rates. I follow his derivation in what follows.

Differentiating the value of employment (A.11) with respect to assets we have

$$(\rho - r)V_a(a,y) = V_{aa}(a,y)s(a,y,c) + \delta\big[V_a^u(a) - V_a(a,y)\big] + \lambda_e \int_y \big[V_a(a,y^*) - V_a(a,y)\big] \bar{f}(y^*|y) dy^*$$

Substituting the foc $u'(c(a,y)) = V_a(a,y)$

$$(\rho - r)u'(c(a,y)) = u''(c(a,y))c_a(a,y)s(a,y,c) + \delta\left[u'(c^u(a)) - u'(c(a,y))\right] \\ + \lambda_e \int_y \left[u'(c(a,y^*)) - u'(c(a,y))\right]\bar{f}(y^*|y)dy^*$$

which is the Euler equation for employed workers describing how consumption evolves between job transitions. We can express it in a more standard form by: (i) dividing everything by $u'(c(a,y))$; (ii) noting that $c_a(a,y)s(a,y,c)dt = dc$, i.e. the change in consumption absent any change to the worker's employment status. In that case, we get

$$\frac{dc}{c(a,y)} = \frac{1}{\gamma}\left(r - \rho + \delta\left(\frac{u'(c^u(a))}{u'(c(a,y))} - 1\right) - \lambda_e \int_y \left(1 - \frac{u'(c(a,y^*))}{u'(c(a,y))}\right)\bar{f}(y^*|y)dy^*\right)dt \tag{A.12}$$

where γ is the coefficient of relative risk aversion. Equation (A.12) illustrate the impact of labor market frictions λ, δ on the worker's desire for saving or dissaving.

- The $r - \rho$ term contrast the rate of return on savings versus the discount rate and drives the usual intertemporal substitution savings motive.
- The term that appears multiplying δ induces precautionary savings.[63] The effect is stronger the larger is the consumption decrease upon an job loss, that is the larger is the difference between consumption of employed worker $c(a,y)$ and the unemployed worker with same assets $c^u(a)$.
- The term multiplying the intensity at which wages grow while on the job (through job-to-job transitions and matched outside offer), captured by λ_e, induces additional *impatience* over and above discount rate ρ. In particular, this force is stronger the higher is the integral that multiplies it. This term is bigger (i) the smaller are the worker earnings y, that is the lower he stands in the wage ladder; (ii) the smaller is $u'(c(a,y^*))/u'(c(a,y)$, that is the more worker consumption increases when moving from y to y^*.

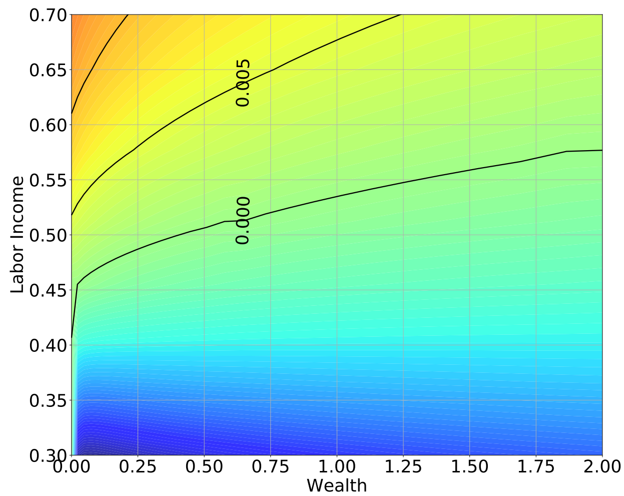
Figure 11 plots dc/c for different points of the state space for an employed worker. For any given level of assets, worker with low earnings tend to dissave, as we would expect from the in-

[63] Some caution in the use of the term "precautionary savings" is warranted. At loosely level, the empirical literature usually associates changes in intertemporal consumption behavior driven by changes in labor-income risk as due to "precautionary savings". Theoretically, this was first made formal by Kimball (1990) who showed that additional savings induced by moving from a non-stochastic future labor income to a stochastic future income of *equal mean* depended on the third derivative of utility, which he labelled as *prudence*. This result, however, relies on a very specific change in risk and it is not clear how it generalizes for different sources of risk.

The job loss risk considered here is a first-order change in risk — the possibility of going through unemployment deteriorates the distribution of future labor income via first-order stochastic dominance. As highlighted by Eeckhoudt and Schlesinger (2008): "If one considers the increased risk in future labor income to be a higher probability of unemployment, then one cannot use theoretical conclusions based upon prudence." As the author shows, even a model with quadratic utility generates a precautionary demand for saving under first-degree risk increases.

tertemporal substitution and λ_e terms in (A.12), while workers with high earnings tend to save, as their precautionary savings motive coming from job loss term becomes more relevant. Moreover, differences in savings behavior are most stark at low level of assets. This is also consistent with the comments above, as the strength of labor market transitions δ, λ_e depended on consumption changes upon those events. The wealthier the worker is, less sensitive is his consumption to these labor market events, which makes their consumption growth depend mainly on intertemporal substitution.

Figure 11: Consumption policy: dc/c



A.3 Intermediate firm Problem – Recursive Formulation

Consider a firm with productivity z under a piece-rate wage contract of y. The value of a match to the firm $\mathcal{J}_t(z, y)$ satisfies the following HJB

$$\begin{aligned}\Big(r_t + \chi_t\Big)\mathcal{J}_t(z, y) = \varphi_t(z - y) + \lambda_{et} \int_y^z \Big[\mathcal{J}_t(z, z') - \mathcal{J}_t(z, y)\Big] d\Gamma(z') \\ + (\lambda_{et}\bar{\Gamma}(z) + \delta)[0 - \mathcal{J}_t(z, y)] + \partial_t \mathcal{J}_t\end{aligned} \tag{A.13}$$

The firm discount profit flows at rate $(r_t + \chi_t)$. With intensity λ_{et}, the worker meets an outside firms. If productivity z' of the poaching firm is between the current piece-rate y and the productivity of the match z, the firm makes a counteroffer z' to the worker who stays in the incumbent firm. This event changes the firm value to $\mathcal{J}(z, z')$. If the productivity z' of the poaching firm is above the productivity of the match, i.e. $z' > z$, the firms looses its worker and the match is dissolved, which leaves the firm with value of 0. The same happens if the match is hit by a destruction shock

δ. Finally, the value of the match changes with calendar time t by $\partial_t \mathcal{J}_t$.

We can interpret this HJB equation by treating the function $\mathcal{J}$ as the value of an asset with flow dividends $\{\varphi_t(z - y_t)\}$, where y is indexed by time to reflect that it evolves in the history of the match. [64] The returns in this asset comes from two sources. The first is the flow dividends $\varphi_t(z - y_t)$. The second comes from capital and losses, which in the current context incorporate all right hand-side terms after dividends in (A.13). The asset looses value whenever the firm has to renegotiate the contract y, or the match is destroyed. The asset also appreciate/depreciate depending on the evolution of aggregate variables, which is captured by $\partial_t \mathcal{J}_t$. The HJB equation then states that return on this asset – right-hand side of (A.13) – must be equal to the required rate of return – $(r_t + \chi_t)\mathcal{J}$.

In order to derive some properties of the value of the firm, let me consider a stationary environment where $\lambda_{et}, r_t, \chi_t, \varphi_t$ do not vary with time. In this case, (A.13) simplifies to

$$\Big((r + \chi) + \delta + \lambda_e \bar{\Gamma}(z)\Big)\mathcal{J}(z, y) = \varphi(z - y) + \lambda_{et} \int_y^z \Big[\mathcal{J}(z, z') - \mathcal{J}(z, y)\Big] d\Gamma(z') \tag{A.14}$$

The derivative of $\mathcal{J}$ with respect to piece-rate y is

$$\begin{aligned}(r + \delta + \lambda_e \bar{\Gamma}(y))\mathcal{J}_y(z, y) - \lambda_e \gamma(y)\mathcal{J}(z, y) &= (-\varphi - \lambda_e \gamma(y)\mathcal{J}(z, y)) \\ \mathcal{J}_y(z, y) &= -\frac{\varphi}{(r + \delta + \lambda_e \bar{\Gamma}(y))\mathcal{J}_y(z, y)}.\end{aligned}$$

Doing to same for productivity z, I can write

$$\mathcal{J}_z(z, y) = -\frac{\varphi + \lambda_e \mathcal{J}(z, z)}{\Big(r + \delta + \lambda_e \bar{\Gamma}(y)\Big)}.$$

Hence, I conclude $\mathcal{J}_z > 0, \mathcal{J}_y < 0$, that is the value of the match is decreasing in piece-rate wage and increasing in match productivity.

A.4 Exogenous-Λ equilibrium

I spell out the full definition of the exogenous-Λ equilibrium.

Definition 2 *(Exogenous-$\boldsymbol{\Lambda}$) Given an initial government debt B^g, an initial distribution Ψ_0 over assets and labor income, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, exogenous labor income process $\{\mathbb{1}^u_{it}, y_{it}\}_{i\in[0,1],t\geq 0}$ and an exogenous path of labor services supply $\{\mathcal{N}^e_t\}_{t\geq 0}$, a general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N^e_t, M_t, u_t, D_t\}_{t\geq 0}$, government policies $\{G_t, B^g_t, T_t, \tau_t, \tau^0_t, i_t\}_{t\geq 0}$,*

[64] See Acemoglu (2007), Chapter 7 for a similar argument.

worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that households optimize, firms optimize, monetary and fiscal policy follow their rules, the worker aggregates and distribution are consistent with the worker's decision rules and exogenous process for income, and all markets clear

- *Asset market clearing*
$$\mathcal{A}_t = B_t^g$$
- *Labor services market clearing*
$$N_t^e = \mathcal{N}_t^e$$
- *Goods market clearing*
$$\mathcal{C}_t + G_t = Y_t$$

A.5 Complete Market Family

The complete market version of the model follow Merz (1995) in adopting a representative family construct, which allows for perfect consumption insurance. The family is composed by a continuum of workers who are either employed or unemployed. At time t, a measure u_t of its workers is unemployed and receives unemployment insurance in the amount of $b\varphi_t$ from the government. The distribution of employed workers inside a family is given $\Psi_t(z,y)/(1-u_t)$, where again z denotes the productivity of the match and y the piece-rate contract earned by employed workers. The family pools all income earned by workers in the form of unemployment insurance and wages. Additionally, the firm receive profits D_t from its ownership of firms. The family then decides on consumption C_t to members and saves through government bonds at rate of return r_t. The problem of the family is then

$$\max_{\{C_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(C_t) dt$$
$$\text{S.t. } \dot{A}_t = r_t A_t + (1-\tau_t)\varphi_t \left(\int y d\Psi_t(y) + bu_t \right) + D_t - C_t + \tau_t^0$$

where τ_t^0 are lump-sum transfers from the government.

B Data

Consumption is given by real personal consumption expenditures (GDPC), inflation is the PCE deflator (PCECTPI). Both are produced by the Bureau of Economic Analysis (BEA) at quarterly frequency. For labor productivity, I use Nonfarm Business Sector Real Output Per Hour of All Persons from the Bureau of Labor Statistics (BLS). Job-to-job transitions data comes from Fallick and Fleischman (2004).

B.1 Job Loss simulation

Saporta-Eksten (2014) uses the 1999-2009 biennial waves of Panel Study of Income Dynamics (PSID) to document the dynamics of wages and consumption around a job displacement. To make the results comparable to the empirical estimates, I apply the same treatment to model simulated data as the author does to actual PSID data. First, while the model is simulated continuously in time, I use the (annually aggregated) measures for every other year in the regression to replicate the way data is collected in the PSID. A job loser is an individual who reports being unemployed at the time of the interview (assumed to take place between March and May). The timing of variables is also important. In the PSID, earnings variables used to compute wage rate refer to the year *prior to each survey*. Information on consumption expenditures are mostly reported for "typical week or month", usually understood as reflecting consumption in the first quarter of the survey year. So for the sample year t earnings and hours data refers to $t-1$, while consumption refers to (annualized) first quarter of year t.

C Numerical Implementation

The numerical solution adapts Auclert et al. (2019) method for solving nonlinear perfect-foresight transitions to a continuous time setting.

The perfect-foresight equilibrium defined in Section **??** can be framed in the form of a *functional equation*. Let X be a the space of real-valued functions $x : [0, \infty) \mapsto \mathbb{R}$. Equilibrium restrictions form an operator $\mathcal{H} : X^n \to X^n$ for $n \in \mathbb{N}$ and an equilibrium is a set of real-values functions $y^* \in X^n$ such that $\mathcal{H}(y^*) = \mathbf{0}$. For instance, the real rate path $\{r_t\}_{t\geq 0} \in X$ is one of the n dimensions of the equilibrium vector y^*, while the asset market $\{\mathcal{A}_t - B_t^g\}_{t\geq 0}$ is one of the n dimensions (say $i \leq n$) of the image of y^* under $\mathcal{H}$, that is $\mathcal{H}(y^*)$. In equilibrium, the restriction that asset markets must clear is equivalent to the statement that $\mathcal{H}_i(y^*)(t) = 0$ for all $t \geq 0$.

Solving this in a computer involves discretizing and truncating the time dimension, in which case the X turns into $\mathbb{R}^K$ for some finite K and $\mathcal{H}$ becomes a nonlinear system of equations $H : \mathbb{R}^{nK} \to \mathbb{R}^{nK}$. So solving for the equilibrium is equivalent to solving a root-finding problem of a conventional (although potentially big) nonlinear system of equations. Spreading time points effectively and reducing dimension n by substituting equilibrium conditions makes solving this problem possible.

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