From Micro to Macro Hysteresis: Long-Run Effects of Monetary Policy*

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Abstract

We develop a Heterogeneous-Agent New-Keynesian model with a three-state frictional labor market consistent with the empirical evidence that (i) low-skilled workers are more exposed to the business cycle, (ii) displacement leads to long-lasting earnings losses, and (iii) unemployment is a stepping stone toward exit from the labor force. In this environment, a transient contractionary monetary policy shock induces a very persistent reduction in labor force participation and labor productivity, especially among workers at the bottom of the skill distribution. Despite the negative hysteresis on output, the model does not give rise to protracted deflation because diminished productivity and participation create inflationary pressures that offset deflationary forces caused by depressed economic activity.

JEL Codes: E21, E24, E31, E32, E52, J24, J64

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1 Introduction

The traditional view of macroeconomic dynamics is that aggregate time series can be decomposed into a long-run component (the trend, or the deterministic steady state) and an orthogonal short-run component (the business cycle) which fluctuates around the trend. Quantitative DSGE models used for research and policy analysis fit into this description and, consistently with this view, routinely assume that transitory shocks have no long-term effects on aggregates.[1] An alternative view of business cycles is that of *macroeconomic hysteresis*. According to this interpretation, the economy's long-run dynamics are not driven by an exogenous trend, but are instead a function of the entire history of shocks hitting the economy. Under this hypothesis, transitory shocks have permanent effects on the level of economic activity (see Cerra et al., 2023, for a recent survey of macro hysteresis).[2]

The hysteresis view of aggregate fluctuations can be traced back to Okun (1973) (and later Tobin, 1980), who argued that recessions could, through erosion of human capital among the labor force, leave potentially permanent scars on the economy. In a similar fashion, Blanchard and Summers (1986) used the hysteresis view to describe the experience of European labor markets during the 1970s and 1980s, when the unemployment rate seemed to have permanently settled at a higher level after a series of negative cyclical shocks.[3] After losing center stage for about two decades, the conjecture that transitory shocks can lead to permanent, or at least very persistent, effects on aggregates reemerged with the Great Recession, after which the US and Euro area economies suffered a slow and protracted recovery.[4]

Whereas this earlier work lacked well-identified empirical evidence on hysteresis and the channels through which they arise, there is now a sizable and growing body of work—at both the micro and macro level—backing up the idea that short-run fluctuations can lead to nearly permanent effects on aggregates. At the micro level, we have accumulated

[1]In models where exogenous TFP growth drives the trend and TFP is subject to permanent shocks, an innovation to productivity has an impact on the growth rate, but other shocks do not.

[2]This idea is also linked to the notion of macroeconomic *resilience* to shocks (the opposite of hysteresis) articulated in Brunnermeier (2021).

[3]Ljungqvist and Sargent (1998) developed a model to microfound such unemployment hysteresis where the interaction between a generous welfare state and skill decay during nonemployment plays a central role.

[4]Reifschneider et al. (2015) highlight that the estimates of long-run growth had been systematically revised downward following the Great Recession.

considerable evidence of negative hysteresis of recessions on individual labor market outcomes. This work includes findings of long-lasting impacts on earnings and participation from aggregate labor market conditions upon college graduation (Rothstein, 2023), local business cycle fluctuations (Yagan, 2019; Cajner et al., 2021), and job displacements (Davis and Von Wachter, 2011; Guvenen et al., 2017). There is also suggestive evidence that these scarring effects are felt disproportionately among disadvantaged groups of workers. For instance, Yagan (2019) reports highly uneven impacts of the Great Recession on employment and earnings, with low-wage workers suffering the most. Cajner et al. (2021) find that labor force participation of young (ages 16 to 24) and black workers exhibit a much larger and more persistent response to local business cycle fluctuation compared to prime-age and white workers.

At the macro level, much of the available evidence of negative hysteresis focuses on monetary policy shocks, as they constitute a well-identified example of transitory demand shocks. Blanchard et al. (2015) analyze more than 20 international recessions driven by large contractionary monetary policy shocks. They find that 2/3 of these episodes are associated with a permanently lower output level, and some of them even with permanently lower output growth. Applying local projections IV techniques on long panel data for over a century for 17 countries, Jordà et al. (2020) uncover very long-lasting effects of monetary policy shocks. Ma and Zimmermann (2023) show that monetary policy, through its impact on innovation activity, affects the productive capacity of the economy in the long term. Furlanetto et al. (2021) use local projection methods to identify generic demand shocks as those innovations that lead output and inflation to comove positively in the short run. They conclude that a subset of these shocks have also an impact in the long run and are quantitatively important in the US, in particular when the Great Recession is included in the sample.

In this paper, we connect these two pieces of literature by developing a macroeconomic hysteresis model built on the micro evidence that job losses lead to persistently lower individual earnings through a combination of skill decay and abandonment of the labor force. We then use the model to investigate whether the long-term negative effects of recessions on individual job prospects, which our model directly targets, carry over to the overall economy. In other words, we examine whether the *micro hysteresis* sources we feed into the model give rise to *macro hysteresis* in response to transitory aggregate shocks. In line with much of the macroeconomic empirical literature discussed above, we focus on the aggregate economy response to a short-lived contractionary monetary policy shock.

Our model merges the standard New Keynesian heterogeneous-agent incomplete-markets framework with a three-state labor market featuring search frictions and endogenous labor supply at the extensive margin. Labor market frictions prevent full employment: some employed individuals who want to keep working are forcefully separated; others searching for a job don't find it. Crucially, separation and job-finding rates depend on individual skill levels: in our calibration, workers at the bottom of the distribution are both less likely to find a job when searching for one, and more likely to lose it when employed, as in the data. Workers' productivity process depends on their labor market status: skills tend to grow during employment through returns to experience, but gradually depreciate when the worker is not employed. Workers also make labor supply decisions by choosing whether or not to participate in the labor market at the prevailing equilibrium wage.

Each of these three model ingredients—i.e., labor market frictions, skill dynamics, and participation decisions—is disciplined by micro evidence. We estimate the dependence of job-finding and separation rates on workers' skill levels from the Basic Monthly Current Population Survey (CPS) merged with the Annual Social and Economic Supplement (ASEC), following the approach of Heathcote et al. (2020). We calibrate skill losses during nonemployment to match the large and persistent earnings losses upon displacement documented by Topel (1990), Jacobson et al. (1993), and Davis and Von Wachter (2011). Finally, to get participation dynamics that resemble the data, we target what Hobijn and Şahin (2021) call the *attachment wedge*, i.e., the difference between the unemployment-to-nonparticipation (un) and the employment-to-nonparticipation (en) flows in the data. The fact that workers are much more likely to drop out of participation during unemployment than employment spells ($un >> en$) creates, mechanically, a downward pressure on the participation rate during downturns when the pool of unemployed workers rises sharply.

Our main experiment studies the long-run effects of a transitory unanticipated contractionary monetary policy shock that reduces total labor income by 1 percent in the first year following the shock. In the short run, the shock causes both an increase in job-separation and a decline in job-finding rates. As workers flow into and remain stuck in unemployment, their skills depreciate, making job opportunities less likely to arrive and wages upon re-employment less attractive. Ten years after the shock, long after the surprise to the monetary policy rule has died out, participation and labor productivity are still depressed by 0.06 ppt and 0.11%, respectively. Together, these two components add up to a 0.20% reduction in total labor income or 1/5th of the first year impact. Thus a temporary shortfall of aggregate demand in our model disrupts aggregate supply over the long run.[5] Importantly, this average effect hides a substantial heterogeneity along the skill distribution—the scarring of labor income for workers in the lowest skill quartile a decade after the shock is almost ten times the aggregate effect. Hysteresis, we find, operates disproportionately through low-wage workers.

Interestingly, despite the long shadow cast on output, the shock generates only a mild and short-lived reduction in inflation. A decomposition of inflation dynamics shows that the reduction in labor productivity, fall in participation, and selection of workers into employment all contribute positively to inflation, helping to explain why inflation remains stable despite the persistently depressed levels of output.

Related literature Our paper is related to several strands in the literature. First, our emphasis on the joint dynamics of unemployment and labor force participation relates to the recent literature extending general equilibrium business cycles models to incorporate frictional labor markets and an endogenous participation margin. Contributions in this literature include Galí et al. (2012), Shimer (2013), Krusell et al. (2017), Christiano et al. (2021) and Cairó et al. (2022).[6] None of these papers studies hysteresis at the macro level.

Our paper also relates to the small but growing literature where macro hysteresis originate from the labor market.[7] Chang et al. (2002) develop a model where skill accumulation through past work experience (i.e., learning-by-doing) gives rise propagation mechanism through labor productivity that resembles our channel. Galí (2022) incorporates an insider-outsider model of the labor market within a New Keynesian framework and shows that the inefficiently high wage arising in equilibrium can be a source of macro hysteresis. Abbritti et al. (2021) develop a similar logic in a model with downward wage rigidity and endogenous growth. Acharya et al. (2022) analyze the impact of monetary

[5]Eventually, all labor market variables return to their steady state, so the shock does not have permanent effects in the very long run in our model. But since the adverse effects of the shock survive far after the shock itself has already died out, we treat these long-lasting negative effects of transitory shocks as evidence of macro hysteresis within the model.

[6]Like Krusell et al. (2017) and Cairó et al. (2022), we also focus on matching labor market worker flows as well as their implied employment, unemployment and participation dynamics.

[7]A parallel literature explores the impact of business cycles on long-term growth through innovation and technological change (e.g. Comin and Gertler, 2006; Bianchi et al., 2019; Fornaro and Wolf, 2020; Gaillard and Wangner, 2023).

policy in a search and matching model where skill depreciation of unemployed workers can lead to steady-state multiplicity.

With respect to this body of work, our contribution is twofold. First, we develop our insights within a state-of-the-art framework that combines elements of heterogeneous-agent models with elements of New Keynesian models. This class of HANK models is becoming a workhorse for quantitative fiscal and monetary policy analysis. Relative to the representative-agent framework, the heterogeneity makes the mapping between the model and the relevant cross-sectional evidence much easier to draw.[8] Second, we highlight the role of skill losses upon displacement and participation decisions as complementary sources of macro hysteresis.

Finally, our paper heavily builds upon the empirical literature documenting scarring effects of recessions on individual labor market trajectories, especially among low-wage workers. Aaronson et al. (2019) and Cajner et al. (2017) show that low-wage workers are more exposed to aggregate fluctuations. Kahn (2010) and Rothstein (2023) uncover evidence of persistently depressed labor market outcomes for individuals who enter the labor market in recessions. Davis and Von Wachter (2011) show that long-term earnings losses are worse when job displacement occurs in a recession. Guvenen et al. (2017) and Athey et al. (2023) find that such earnings losses are most severe at the bottom of the distribution because they lead to disattachment from the labor force. Yagan (2019), Rinz (2022) and Hershbein and Stuart (2020) all present evidence of strong persistence in local labor market outcomes in the aftermath of the Great Recession. They conclude that human capital decay is an important mechanism generating negative hysteresis on labor earnings, with stronger impacts for low-wage workers. Furlanetto et al. (2021) document that negative hysteresis propagate almost exclusively through lower employment and labor force participation, and that these effects are especially strong at the bottom of the wage distribution. Finally, Lepetit (2023) provide evidence that, in response to demand shocks, the slope of the inflation-output relationship is much flatter at long-horizons than at short ones, consistently with our model's prediction.

The rest of the paper is organized as follows. Section 2 outlines the model. Section 3

[8] In this aspect, our framework shares many of the same ingredients with Krusell et al. (2017), who also develop a heterogeneous-agent model with search frictions and endogenous participation. Their focus is on matching the behavior of gross worker flows. Relative to their analysis, our paper makes the following two improvements. First, we extend our analysis to a general equilibrium environment with nominal wage rigidities, whereas theirs is done in partial equilibrium. Next, we rely on a more extensive set of micro evidence to calibrate the model's labor market frictions and skill dynamics, which we show to matter for the quantitative aggregate implications of a monetary policy shock.

describes its parameterization. Section 4 discusses the results of our model's simulations. Section 5 concludes and examines the implications of our findings for the optimal design of monetary policy.

2 Model

The structure of the model follows closely the framework we have developed in previous work (Alves and Violante, 2024).

2.1 Households

Time is continuous and indexed by t. The economy is populated by a continuum of infinitely-lived households (or individuals) with measure 1 who discount the future at rate $\rho > 0$.

Individuals can be in one of three mutually exclusive labor market states s_t: employed and earning labor income, ($s_t = e$), unemployed and searching for a job ($s_t = u$), outside the labor force, ($s_t = n$). Among the unemployed, we distinguish between those who are eligible ($u = u_1$) and not eligible ($u = u_0$) for unemployment insurance (UI) benefits. Workers gain eligibility only if they are laid off from work. They then lose it at some constant rate which reflects benefit duration. Among those out of the labor force, we distinguish between "active" non-participants ($n = n_1$) and "passive" non-participants ($n = n_0$). The former ones can, at a lower rate than the unemployed, find jobs and enter employment, while the latter ones cannot.[9]

Households derive utility from consumption c_t, and incur disutility from the effort cost κ^s associated to being in labor market status s (the extensive margin) and from the effort cost of working h_t hours (the intensive margin). We specify the following functional form for period utility

$$\mathrm{u}^s\left(c_t, h_t\right) = \log c_t - \psi \frac{h_t^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} - \kappa^s \tag{1}$$

where $\sigma > 0$ is the Frisch elasticity of labor supply. We assume that $\kappa^e > \kappa^u > \kappa^n \geq 0$.

[9]This differentiation captures the heterogeneity in the pool of non-participants (Hall and Kudlyak, 2019) where some individuals are able and willing to work, while others are unable to accept any job offer (e.g., because they are sick) or are discouraged from searching further.

	e	u_1	u_0	n_1	n_0
e	$\ddots$	λ^{eu}_{zt}	$\times$	$\blacktriangleright$	η^{en_0}
u_1	$\lambda^{ue}_{zt} \cdot \triangleright$	$\ddots$	$\eta^{u_1 u_0}$	$\blacktriangleright$	η^{un_0}
u_0	$\lambda^{ue}_{zt} \cdot \triangleright$	$\times$	$\ddots$	$\blacktriangleright$	η^{un_0}
n_1	$\lambda^{ne}_{zt} \cdot \triangleright$	$\times$	$\blacktriangleright$	$\ddots$	$\eta^{n_1 n_0}$
n_0	$\times$	$\times$	$\times$	$\eta^{n_0 n_1}$	$\ddots$

Table 1: Transition matrix across the 5 employment states. The $\times$ symbol means that transition cannot happen. The $\blacktriangleright$ symbol means that an endogenous participation decision moves the individual in that state. The $\triangleright$ symbol means that an endogenous job acceptance decision moves the individual into employment. $\lambda^{ss'}_{zt}$ and $\eta^{ss'}$ are exogenous Poisson rates. The diagonal dots stand for the negative of the sum of all the other entries on that line.

Each individual is endowed with efficiency units of labor (or skills) z evolving according to a Ornstein-Uhlenbeck diffusion process which depends on labor market status s_t:

$$d \log z_t = \Big\{ -\rho_z \log z_t + \mathbb{I}_{\{s_t=e\}}\, \delta^+_z - \mathbb{I}_{\{s_t \neq e\}}\, \delta^-_z \Big\} dt + \sigma_z d\mathcal{W}_t \tag{2}$$

When workers are employed ($s_t = e$), skills drift up at rate $\delta^+_z > 0$, and when they are not employed ($s_t = u, n$) they drift down at rate $\delta^-_z < 0$. The parameter $\rho_z > 0$ measures the degree of mean reversion in skill dynamics, the standard deviation σ_z determines uncertainty about future realizations, and $\mathcal{W}_t$ is a Wiener process.

Every period individuals can transition across states through a combination of exogenous Poisson rates and optimal mobility decisions. Table 1 describes all the possible transitions and their endogenous/exogenous nature.

At any date t, employed and unemployed workers can decide to quit the labor force and enter active non-participation (rows 1, 2, 3 of Table 1). Similarly, an active non-participant can choose to re-enter the labor force as an unemployed ineligible for UI (row 4). Employed workers who decide to remain attached can still be laid off, and thus move from e to u at an exogenous rate λ^{eu}_{zt} which depends on the worker's skill level z (row 1). Unemployed workers who choose to remain in the labor force draw an employment opportunity at an exogenous rate λ^{ue}_{zt} and decide whether to accept it or not (rows 2 and

3).[10] UI benefits can expire at rate $\eta^{u_1 u_0}$ and an eligible unemployed becomes ineligible (row 2). Also active participants receive job opportunities at rate λ_{zt}^{ne} and decide whether to accept them or not (row 4). All workers can exogenously move into passive nonparticipation at rate η^{s,n_0} (rows 1, 2, 3, 4). At rate $\eta^{n_0 n_1}$, passive nonparticipants become active again (row 5).

Employed individuals earn labor income $w_t h_t z_t$, where w_t is the real wage per effective hour. Eligible unemployed receive benefits $b(z_t)$. We let UI benefits be a function of current worker productivity z_t, as a proxy for actual replacement rates. Both types of income are taxed at a proportional rate $\mathfrak{t}$. Every household is entitled to a lump-sum transfer ϕ. Households can save through a financial asset a_t with rate of return r_t, but cannot borrow.

Household problem The vector (s, a, z) fully summarizes the individual state variables. The dynamic problem solved by the household at time t is a mix of an optimal control problem, the choice of $c_t > 0$, and two optimal stopping problems: a continuous one, the participation decision $\mathfrak{p}_t^s \in \{0, 1\}$, and one arising at random Poisson jump times, the job acceptance decision $\mathfrak{f}_t^s \in \{0, 1\}$. The stochastic nature of the problem is due to both the Poisson arrival rates that determine transitions across labor market states, and the diffusion that describes the evolution of skills z_t. Conditional on these realizations, wealth evolves deterministically. Let $v_t^s(a, z)$ be the value at date t of an individual with employment state s, wealth a, and productivity z.

Consider the problem of the active non-participant (n_1):

$$
\begin{aligned}
v_0^{n_1}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}, \tau^*} \quad & \mathbb{E}_0 \Bigg[\int_0^{\tau^{\min}} e^{-\rho t} \mathfrak{u}^n (c_t, h_t)\, dt + \mathbb{I}_{\{\tau^{\min} = \tau^e\}} e^{-\rho \tau^e} \max \left\{ v_{\tau^e}^e (a_{\tau^e}, z_{\tau^e}), v_{\tau^e}^{n_1} (a_{\tau^e}, z_{\tau^e}) \right\} \\
& + \mathbb{I}_{\{\tau^{\min} = \tau^*\}} e^{-\rho \tau^*} \left(v_{\tau^*}^{u_0} (a_{\tau^*}, z_{\tau^*}) - \vartheta \right) + \mathbb{I}_{\{\tau^{\min} = \tau^{n_0}\}} e^{-\rho \tau^{n_0}} v_{\tau^{n_0}}^{n_0} (a_{\tau^{n_0}}, z_{\tau^{n_0}}) \Bigg] \\
s.t. & \\
c_t + \dot{a}_t \quad = & \quad r_t a_t + \phi \qquad (3) \\
a_t \quad \geq & \quad 0
\end{aligned}
$$

[10] The unemployed ineliglible for UI always accept job offers because in equilibrium there is a unique wage per effective hours, and if they did not want to work, they would choose non-participation where the fixed cost κ^s is lower. Eligible unemployed instead may turn down job opportunities if UI benefits are generous enough.

Active non participants receive job opportunities at rate λ_{zt}^{ne}, with τ^{e} being the first arrival time of this event. Conditional on receiving this job offer, they choose whether to accept it $\left(\mathfrak{f}_t^{n_1} = 1\right)$ or not $\left(\mathfrak{f}_t^{n_1} = 0\right)$. At every instant, the non-participant also chooses whether to remain unattached $\left(\mathfrak{p}_t^{n_1} = 0\right)$ or re-enter the labor force $\left(\mathfrak{p}_t^{n_1} = 1\right)$, in which case they become unemployed without UI benefits ($u = u_0$). We assume that re-entering the labor force involves a small fixed switching cost ϑ.[11] The optimal stopping time τ^* represents the first instant in which the choice $\mathfrak{p}_t^{n_1}$ switches from 0 to 1. Finally, at rate $\eta^{n_1 n_0}$ (with τ^{n_0} being the first arrival rate of this shock) active non-participants become passive non-participants. The conditional expectation reflects the uncertainty in transition rates and skill dynamics. In addition to the participation and job acceptance decisions, at every instant the worker chooses its consumption flow c_t. The last two lines of this problem state the budget constraint (in real terms) and the borrowing limit.

Problems for passive nonparticipants, ineligible unemployed, eligible unemployed, and employed workers are analogous, and described in detail in Appendix A.

2.2 Firms

Final-goods producers. A competitive representative final-good producer aggregates a continuum of intermediate inputs indexed by $j \in [0, 1]$ with technology

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} \tag{4}$$

where $\nu > 0$ is the elasticity of substitution across inputs. This firm takes prices as given and solves

$$\max_{\{y_{jt}\}} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \tag{5}$$

subject to (4). Cost minimization implies that demand for intermediate good j at price p_{jt} is

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\nu} Y_t, \quad \text{where} \quad P_t = \left(\int_0^1 p_{jt}^{1-\nu} dj \right)^{\frac{1}{1-\nu}} \tag{6}$$

[11] The presence of a small switching cost is mostly a technical assumption to avoid "chattering", i.e. infinitely fast switching between n_1 and u_0, in the optimal solution of the problem. For all other participation decisions, this problem does not arise because switching back can only occur upon the realization of Poisson shocks.

is the price of the final good and the numeraire of the economy.

Intermediate-goods producers. A continuum of measure one of monopolistically competitive firms produce the intermediate goods using labor. Production requires hiring labor on a continuum of tasks indexed by $k \in [0, 1]$. Each firm j hires labor services (efficiency-weighted hours) ℓ_{jkt} on every task k, combines them into a final labor input ℓ_{jt} using a Dixit-Stiglitz aggregator with elasticity of substitution ε, and produces the intermediate good according to the linear technology $y_{jt} = \alpha \ell_{jt}$. Every period firms face a fixed operating cost χ expressed in terms of final good. At every date t, these firms take the task-specific wage as given, and maximize profits by solving

$$\begin{aligned} \max_{p_{jt}, \{\ell_{jkt}\}_k} \quad & \left(\frac{p_{jt}}{P_t}\right) y_{jt} - \int_0^1 w_{kt} \ell_{jkt} dk - \chi \\ s.t. \quad & \\ y_{jt} \quad &= \quad \alpha \ell_{jt} \\ \ell_{jt} \quad &= \quad \left[\int_0^1 \ell_{jkt}^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ y_{jt} \quad &= \quad \left(\frac{p_{jt}}{P_t}\right)^{-\nu} Y_t \end{aligned} \tag{7}$$

where w_{kt} is the real wage on task k. Cost minimization yields the relative demand of labor for task k

$$\ell_{jkt} = \left(\frac{w_{kt}}{w_t}\right)^{-\varepsilon} \ell_{jt}, \tag{8}$$

where w_t is the Dixit-Stiglitz real aggregate wage index $w_t = \left[\int_0^1 w_{kt}^{1-\varepsilon} dk\right]^{\frac{1}{1-\varepsilon}}$ that satisfies $\int_0^1 w_{kt} \ell_{jkt} dk = w_t \ell_{jt}$. The profit-maximizing price setting decision yields the standard expression whereby the relative price equals a markup over the marginal cost of production

$$\frac{p_{jt}}{P_t} = \frac{\nu}{\nu - 1} \left(\frac{w_t}{\alpha}\right). \tag{9}$$

In a symmetric equilibrium with $p_{jt} = P_t$, all firms produce the same amount $y_{jt} = Y_t$ with labor $\ell_{jt} = \ell_t = \alpha^{-1} Y_t$.

From the assumption of constant returns to scale in production, imposing $p_{jt} = P_t$ in (9) implies that the equilibrium aggregate real wage per effective hour is constant over time. As a consequence price inflation equals wage inflation and the real wage is constant.

Finally, the real aggregate profits of the production sector are

$$\Pi_t = Y_t - w_t \ell_t - \chi. \tag{10}$$

Every period, profits are paid as dividends to the mutual fund that owns all intermediate producers.

2.3 Wage Setting

This block of the model adapts the wage setting mechanism of Erceg et al. (2000) —i.e., the standard New Keynesian *sticky wage* model— to an heterogeneous-agent economy. We follow closely the approach of Auclert et al. (2018, 2020), with the needed modifications due to our continuous time formulation and the presence of the extensive margin in labor supply.

Every worker i at date t supplies hours on each task k. The *nominal* wage ω_{kt} per effective hour worked on task k is set by a union that represents all workers on that particular task. By adhering to the union, each employed worker agrees to supply, at that wage, the same number of hours h_{kt} to producers. The problem of each union k is:

$$\begin{aligned}
\max_{\{\omega_{kt}\}_{t\geq 0}} \quad & \int_0^\infty e^{-\rho t}\left[\int_{s_{it}=e} \mathrm{u}^e\left(c_{it}, h_{it}\right) di - \frac{\Theta}{2}\left(\frac{\dot{\omega}_{kt}}{\omega_{kt}} - \pi^*\right)^2\right] dt \qquad (11)\\
s.t. \quad & \\
h_{it} \quad = \quad & \int_0^1 h_{kt} dk \\
c_{it} + \dot{a}_{it} \quad = \quad & r_t a_{it} + (1 - \mathfrak{t}) \frac{1}{P_t} z_{it} \int_0^1 \omega_{kt} h_{kt} dk + \phi \\
h_{kt} \int_{s_{it}=e} z_{it} di \quad = \quad & \ell_{kt} = \left(\frac{\omega_{kt}}{\omega_t}\right)^{-\varepsilon} \ell_t
\end{aligned}$$

At every date t, the union sets the nominal wage ω_{kt} in order to maximize welfare of its current members (all individuals employed at date t) subject to a Rotemberg-style quadratic costs of adjusting the nominal wage, in utility terms, with scaling parameter Θ. Let inflation be denoted by $\pi_t = \dot{P}_t / P_t$. This cost is expressed in terms of deviations of nominal wage growth from the central bank's inflation target, the deterministic steady-state trend inflation rate π^*. The first constraint faced by the union states that total hours worked by an employed worker equal the sum of hours worked on each task. The second constraint is the budget constraint of employed workers. The third one states that contractual effective hours required by the union from its workers must equal the firm's demand for task k effective labor, ℓ_{kt}.[12] Because each task-specific union is "small" (there is a continuum of tasks) the impact of a union's wage on individual income or firm's employment is negligible. As a result, the union takes as given all individual decisions and the firm's labor demand curves for their task.[13]

In a symmetric equilibrium where all unions charge the same nominal wage $\omega_{kt} = \omega_t$, the amount of labor demanded for all tasks is the same $\ell_{kt} = \ell_t$, and, since unions represent the same set of workers, the number of hours worked on each task is equalized $h_{kt} = h_t$. Combining this with the production function of intermediate-goods producers we arrive at an aggregate production function

$$Y_t = \alpha \left(\int_{s_{it}=e} z_{it} di \right) h_t = \alpha Z_t^e H_t \tag{12}$$

where $Z_t^e = \left(\frac{1}{E_t} \int_{s_{it}=e} z_{it} di \right)$ denotes average labor productivity among the employed, E_t is aggregate employment, and $H_t = h_t E_t$ is aggregate hours worked.

The solution to the unions' wage setting problem yields the wage Phillips curve

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \frac{\epsilon}{\Theta} H_t \left[\psi h_t^{\frac{1}{\sigma}} - \left(\frac{\epsilon - 1}{\epsilon} \right) (1 - \mathrm{t}) w_t \left(\frac{1}{E_t} \int_{s_{it}=e} \frac{1}{c_{it}} z_{it} di \right) \right] \tag{13}$$

where π_t is aggregate (wage and price) inflation rate. See Alves and Violante (2024) for a detailed derivation.

The term in the square brackets of equation (13) captures unions' incentives to raise or decrease nominal wages. When the marginal disutility of an extra hour of work exceeds the productivity-weighted marginal utility generated by the (markup-augmented after-tax) income derived from this additional hour of work, unions will push up nominal wages to reduce labor demand and close the gap between these two margins. Another useful interpretation of the term in brackets relates to the notion of the labor wedge, as

[12] Note that the right hand side of this latter constraint equals (8).

[13] Huo and Ríos-Rull (2020) criticize the RANK model featuring nominal wage rigidity because, in the equilibrium of that model, workers may end up being forced to supply hours against their will (thus violating the principle of voluntary exchange) and would be better off not working. They suggest a resolution based on a different equilibrium concept. We propose different solution: in our model, unions offer all workers an employment contract that specifies a non-negotiable pair of wages and hours, but workers can always voluntarily choose not to participate in it and remain non-employed.

discussed in Dávila and Schaab (2023). Defining the aggregate labor wedge as

$$H_t\left[\left(\frac{\epsilon-1}{\epsilon}\right)(1-\mathfrak{t})w_t\left(\frac{1}{E_t}\int_{s_{it}=e}\frac{1}{c_{it}}z_{it}di\right)-\psi h_t^{\frac{1}{\sigma}}\right],$$

we conclude that unions increase (decrease) their nominal wages whenever the aggregate labor wedge is negative (positive), that is, whenever the measured gains from asking its members to work an additional hour stands below (above) the marginal disutility of an extra hour of work.

2.4 Mutual Fund

A competitive risk-neutral mutual fund owns all intermediate good firms and holds all debt issued by the government.[14] Let X_t^m denote the shares of the intermediate good producers held by the mutual fund, q_t the unit share price, Π_t per-share dividends (or profits), B_t^m the amount of government bonds held by the fund, and r_t^b the real interest rate on government bonds. In Appendix B, we show that the equilibrium must satisfy the following no-arbitrage condition

$$r_t = \frac{\Pi_t + \dot{q}_t}{q_t} = r_t^b, \tag{14}$$

which holds at every t, except when a shock hits the economy.[15] The value of the fund, denoted by A_t, is given by $A_t = q_t X_t^m + B_t^m$.

2.5 Government

Let G_t be the units of the final goods purchased by the government (fiscal authority) at time t, ϕ lump-sum transfers, b UI benefits, $\mathfrak{t}$ the labor income tax, and $B_t^g > 0$ outstanding real government debt. The government faces the following intertemporal budget constraint:

$$G_t + \phi + (1-\mathfrak{t})\int_{s_{it}=u^1} b(z_{it})di + r_t^b B_t^g = \mathfrak{t}w_t h_t\int_{s_{it}=e} z_{it}di + \dot{B}_t^g \tag{15}$$

[14] The set up in this section follows closely Alves et al. (2020).

[15] In this case, the price q_t features a jump.

Outside of steady-state, we assume that the government follows the passive fiscal policy rule:

$$G_t = G^* - \beta_B(B_t^g - B^*), \quad \beta_B > 0 \tag{16}$$

where the superscript * denotes steady-state values. Thus, following an aggregate shock debt adjusts to satisfy the government budget constraint, and government expenditures respond to deviations of debt from its steady-state level to keep debt from growing too quickly.

2.6 Monetary Authority

The monetary authority sets the nominal interest rate $\imath$ according to a rule that reacts to deviations of inflation from its targets with some inertia

$$\frac{d\imath_t}{dt} = -\beta_\imath\Big(\imath_t - \imath^* - \beta_\pi(\pi_t - \pi^*)\Big) \tag{17}$$

We let $\imath^*$ denote the steady-state nominal rate and $\pi_t = \dot{P}_t/P_t$ the aggregate inflation rate at date t. The coefficients β_π capture the strength of the policy response to deviations of inflation from target π^*. The coefficient $\beta_\imath$ captures the degree of interest rate smoothing. The nominal interest rate and the real interest rate on government bonds r_t^b are linked through the Fisher equation $r_t^b = i_t - \pi_t$.

2.7 Equilibrium

An equilibrium for this economy is defined as time paths for household consumption decisions $\{c_t^s(a,z)\}_{t\geq 0}$ for $s \in \{e, u_0, u_1, n_0, n_1\}$, participation and job offer acceptance decisions $\{\mathfrak{p}_t^s(a,z), \mathfrak{f}_t^s(a,z)\}_{t\geq 0}$ for all s, unions' nominal wage setting $\{\omega_{kt}\}_{t\geq 0}$ for all labor types k, intermediate producers' hiring decisions $\{\ell_{kt}\}_{t\geq 0}$ for all k, mutual fund allocations between equity and government bonds $\{X_t^m, B_t^m\}_{t\geq 0}$, real rates of return on the mutual fund and on government bonds $\{r_t, r_t^b\}_{t\geq 0}$, firms' share price $\{q_t\}_{t\geq 0}$, fiscal variables (taxes, transfers, UI benefits, expenditures and debt) $\{\mathfrak{t}, \phi, b(z), G_t, B_t^g\}_{t\geq 0}$, nominal interest rates $\{i_t\}_{t\geq 0}$, aggregate output, consumption, profits, contractual hours worked, and inflation $\{Y_t, C_t, \Pi_t, h_t, \pi_t\}_{t\geq 0}$, and measures of households $\{\mu_t^s(a,z)\}_{t\geq 0}$ for all s such that at every t: (i) households optimize; (ii) final good and intermediate good producers solve (5) and (7), respectively; (iii) unions solve (11) and inflation satisfies the Phillips curve in (13) (iv) the mutual fund maximizes profits; (v) the government budget constraint (15) holds; (vi) the fiscal and monetary authorities follow their policy rules (16) and (17); (vii) the sequence of distributions satisfies aggregate consistency conditions, and (viii) all good and asset markets clear.

Besides the continuum of intermediate goods' and labor varieties markets, there are five other markets in our economy: the intermediate firms' shares market, the government bond market, the mutual fund shares market, the final good market, and the labor market. The first three markets clear when, respectively

$$\begin{aligned} X_t^m &= 1 \\ B_t^m &= B_t^g \\ \sum_{s\in\{e,u,n\}} \int a_t d\mu_t^s &= q_t + B_t^g \end{aligned}$$

where, without loss of generality, we normalized the measure of firms' shares to 1. These market clearing conditions, together with the no-arbitrage condition (14) and the definition of firm profits (10), determine firm share prices, real interest rates, and aggregate profits. The final goods market clears when

$$Y_t = C_t + G_t + \chi.$$

The labor market is frictional with workers in one of the three labor market states: employment, unemployment and non-participation.

A stationary equilibrium is a particular case of our definition where—absent aggregate shocks—all decisions, prices, aggregate variables, and distributions are time invariant.

3 Parameterization

Preferences. The discount rate ρ is set to target a ratio of mean wealth to annual earnings of 0.56, corresponding to the amount of liquid wealth immediately available for consumption smoothing among US households (Kaplan and Violante, 2022). This choice allows the model to match a sizable quarterly aggregate marginal propensity to consume of 0.10 without adding illiquid assets or preference heterogeneity. We set $\gamma = 1$ (log-utility over consumption expenditures) and $\sigma = 1$ (quadratic disutility of hours worked).

Working entails a variable and a fixed cost. The variable disutility parameter ψ is set so that there is no inflationary pressure beyond trend inflation in steady state. The

Parameter		Value	Target
Preferences			
Discount rate	ρ	0.0060	Liquid wealth to annual earnings (0.56)
Risk aversion	γ	1.00	External
Labor supply elasticity	σ	1.00	External
Utility weight on hours	ψ	0.8579	No wage inflationary pressures at SS
Disutility of working	κ^e	0.9147	Sensitivity of *en* flows
Disutility of searching	κ^u	0.0379	30 minutes per day searching
Disutility of nonparticipation	κ^n	0	Normalization
Productivity process			
Skill mean reversion	ρ_z	-0.0017	External
Skill drift while employed	δ^+	0.0016	Normalization of average skill level to 1
Skill drift while non-employed	δ^-	-0.0262	10-Year earnings losses from displacement (15%)
Skill diffusion	σ_z	0.0288	P90-P50 hourly wage ratio (3)
Labor market frictions			
Job-separation rate out of E	$\{\lambda_i^{eu}\}_{i=1}^3$	$\{0.008, 0.051, -2.490\}$	Average labor market flows
Job-finding rate out of U	$\{\lambda_i^{ue}\}_{i=1}^3$	$\{0.375, -0.229, -6.123\}$	Average labor market flows
Job-finding rate out of N	$\{\lambda_i^{ne}\}_{i=1}^3$	$\{0.214, -0.131, -6.123\}$	Average labor market flows
Passive nonparticipation rate during E	η^{en_0}	0.007	Average labor market flows
Passive nonparticipation rate during U/N	$\eta^{un_0}, \eta^{n_1n_0}$	0.099	Average labor market flows
Passive nonparticipation exit rate	$\eta^{n_0n_1}$	0.339	Average labor market flows
Elasticity of job-finding rates to hours	$-$	15.00	Sensitivity of *ue* flows to MP shock
Elasticity of job-separation rates to hours	$-$	5.00	Sensitivity of *eu* flows to MP shock
Taxes and transfers			
UI replacement rate	$\bar{b}$	0.50	External
UI expiration rate	$\eta^{u_1u_0}$	0.167	Average duration of UI (6 months)
Lump-sum transfer	ϕ	0.055	6% of annual average earnings
Labor tax rate	$\mathfrak{t}$	0.2	External
Technology and Price/Wage Setting			
Firm productivity	α	1.38	Normalization
Firm fixed cost	χ	0.12	Steady-state real rate of 2%
Price/Wage markups	ν, ε	10	External
Wage adjustment cost	Θ	6667	Slope of wage Phillips curve (0.015 quarterly)
Fiscal and Monetary Policy			
Trend inflation	π^*	2%	Fed inflation target
Taylor rule persistence	$\beta_\imath$	0.07	Response of *u* to MP shock
Taylor rule reaction to inflation	β_π	2.25	External
Government expenditures response to debt	β_B	0.10	External

Table 2: Parameter values. The corresponding targeted moments are discussed in the main text. The model period is one month.

fixed disutility of work κ^e is set to match the sensitivity of *en* flows as discussed below. The disutility cost of searching κ^u is set to match the observation that job-seekers spend less than 30 minutes per day searching (Faberman et al., 2017). The flow utility of non-participation κ^n is normalized to zero.[16]

Productivity dynamics. The mean reversion parameter ρ_z is set to -0.0017, corresponding to an annual autocorrelation of $\exp(-12 \times \rho_z) = 0.98$. The negative drift δ^- is set to

[16] The switching cost ϑ is set to a very small number to make the optimal stopping problem well behaved.

match the evidence on earnings losses upon displacement from Davis and Von Wachter (2011). Specifically, we target the estimate that laid-off workers still earn on average 15% less than their control group 10 years after separation. As a normalization, we set the positive drift δ^+ so that the average skill level of the employed is 1. Finally, we choose the standard deviation σ_z to match a 90-50 wage ratio of 3, the value for the 2019 CPS (Heathcote et al., 2023).[17]

Labor market frictions. The estimation and calibration of the labor market frictions is based on Alves and Violante (2024). We leave the detailed discussion to that paper, but provide a short summary of our strategy here.

Going back to the transition matrix in Table 1, the model features seven rates to calibrate. The separation rate λ_{zt}^{eu}, the job-finding rates for unemployed λ_{zt}^{ue}, and the job-finding rate for active non-participants λ_{zt}^{ne} vary with time t and are allowed to depend on the worker's skill level z. In the steady state, we model their dependence on worker's skill level z as

$$\lambda^{ss'}(z) = \lambda_0^{ss'} + \lambda_1^{ss'} \exp(\lambda_2^{ss'} z). \tag{18}$$

We choose the coefficients in (18) in two steps. In the first step, we use data on transition rates across the workers' wage distribution to get an estimate of $\lambda_0^{ss'}, \lambda_1^{ss'}, \lambda_2^{ss'}$ for eu, ue and ne.[18] These estimated coefficients determine the "shape" of transition rates along worker's skill level. In the second step, which takes place during the calibration, we rescale the first-stage $\lambda_0^{ss'}, \lambda_1^{ss'}$ coefficients to target average worker flows eu, ue and ne measured from the CPS.

The exogenous $\eta^{ss'}$ rates to and from the passive nonparticipant state don't depend on time nor on worker's skill z. These are set as follows. We set the transition rate from employment to passive non-employment η^{en_0} to match the average level and sensitivity of the en flows in response to a monetary policy shock.[19] For the transition rate between

[17] We target the 90-50 ratio because earnings variation at the top of the distribution is more directly associated with productivity variation, which is what we aim to measure, compared to the rest of the distribution where the extensive margin of labor supply plays a bigger role.

[18] We don't use ne transitions directly in our estimation because the job acceptance decisions from nonparticipants create a wedge between job-finding rates out of non-participation $\lambda^{ne}(z)$ (our object of interest) and the observed ne flows (our empirical measure). Instead, we impose that the job-finding rate out of nonparticipation shares the same shape as the job-finding rate out of unemployment.

[19] In the model, both κ^e and η^{en_0} are potential sources of en flows. Increasing the utility cost of working κ^e raises the likelihood that a worker decides to leave employment to non-participation after a negative skill shock. Similarly, increasing the transition rate to passive non-participation η^{en_0} mechanically induces

unemployment and passive non-employment η^{un_0}, we set it to match the average *un* flow.[20] Finally, we choose the transition rate from passive to active nonparticipation $\eta^{n_0 n_1}$ to match the flows out of participation, since workers have to be active nonparticipants before becoming jobseekers.

Taxes and transfers. We assume that unemployment benefits are given by $b(z_{it}) = \bar{b}\, w_t h_t z_{it}$, and set the UI replacement rate $\bar{b}$ to 0.5 of individual earnings. We set the rate $\eta^{u_1 u_0}$ to 0.167 to reflect an average UI benefits duration of 6 months. The proportional tax rate $\mathfrak{t}$ is set to 0.2 and the lump-sum transfer ϕ is set to match 6% of average earnings in steady-state (Alves and Violante, 2024). The amount of government debt is set to equal 1/4 of total equity (2019 Flow of Funds, Table B.101.h Balance Sheet of Households). Government expenditures are set residually to satisfy the budget constraint in steady state.

Production and price setting. Firm productivity α is set so that the after-tax hourly wage per efficiency unit in steady state is normalized to 1. The fixed operating cost χ affects the value of equity and, therefore, the size of the aggregate supply of liquid wealth. We set χ so that, given the household demand curve, the annual real interest rate that clears the asset market is 2%.

Both elasticities of substitution across labor types (ε) and across intermediate goods (ν) are set to 10 which implies wage and price markups around 10 percent. The nominal wage adjustment cost Θ is set to match a slope of the structural wage Phillips curve (the semi-elasticity of inflation to deviations of marginal rate of substitution from the real wage) of 0.015 quarterly as recently estimated by Del Negro et al. (2020).

Monetary and fiscal policy. We set steady-state (trend) inflation rate π^* at 2%. In our Inflation Targeting (IT) rule (17), we set the reaction coefficient on deviations of inflation from its trend to $\beta_\pi = 2.25$. The interest rate smoothing parameter is set to $\beta_\iota = 0.07$ to

a flow towards passive non-participation. However, these two sources of *en* transitions hold very distinct implications for the sensitivity of *en* to the monetary policy shock. If we rely solely on the disutility κ^e to match the average flows, we find a counterfactually strong positive *en* response following a contractionary monetary policy shock, as workers adjust their labor supply to counteract the negative wealth effects of the shock. If we rely solely on forced transitions η^{en_0} instead, then all movements from employment to non-participation are exogenous and we find no *en* response following a monetary policy shock. To discipline the relative importance of endogenous and exogenous *en* flows, we thus use the estimated *en* response in Graves et al. (2023).

[20] We set $\eta^{n_1 n_0} = \eta^{un_0}$ which corresponds to the assumption that all non-employed workers transition into inactive nonparticipation at the same rate.

match the empirical persistence of the deviations of the unemployment rate after a monetary policy shock, as estimated by Graves et al. (2023). Namely, in the data unemployment returns to its pre-shock value after 4-5 years (see Figure C1). Finally, in the fiscal rule (16), we set $\beta_B = 0.1$.

Cyclicality of frictions. We model out of steady-state fluctuations of labor market frictions (separation and job finding rates) in a mechanical way. Specifically, we make the entire job-finding and separation rates functions $\lambda_t^{eu}(z)$, $\lambda_t^{ne}(z)$ and $\lambda_t^{ue}(z)$ fluctuate in proportion to changes in the average hours per worker. This approach allows us to capture the heterogeneous fluctuations in job finding rates and separation rates over the business cycle across skill levels without complicating the model further.

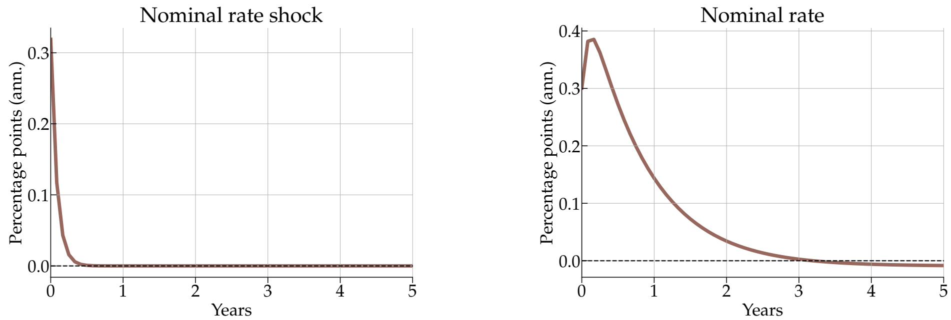

Figure 1: Left panel: monetary policy shock. Right panel: Taylor rule (17) implied path for the nominal interest rate.

4 Results

The results are organized as follows. In section 4.1, we compare our model against recent empirical estimates of the effect of monetary policy surprises on labor-market variables. As we show, our calibration captures well the estimated responses of workers' stocks and flows to a monetary policy shock, including the response of the flows along the participation margin. In section 4.2, we focus on the long-run impact (10 years after the shock) of a transitory monetary policy contractionary shock in the aggregate and along the workers' skill distribution. In subsection 4.2.1, we compute the long-run impact of the shock on aggregate labor earnings and explore its drivers along the skill distribution. In section 4.2.2, we use the Phillips curve (13) to investigate the short and long-run dynamics of inflation. Overall, we find that the micro-level sources of scarring present in the labor market do spill over to the macro economy, with a transitory contractionary monetary policy shock leading to long-lasting negative effects on aggregate earnings but not on inflation.

4.1 Monetary Policy Transmission in a Frictional Labor Market

We study the impulse response to a (unanticipated) extremely transitory negative shock to the Taylor rule (17). In what follows, we compare the model's impulse response to a monetary policy shock to the Graves et al. (2023) empirical impulse response estimated using a high-frequency identification strategy around policy announcements and Fed Chairs' speeches. Their results are reproduced in Appendix C for reference.

To ease the comparison with Graves et al. (2023), the size of the monetary shock in this section is chosen to match their estimated peak effect for the unemployment rate of 0.20 percentage points. The time paths for the shock and the nominal rate are illustrated

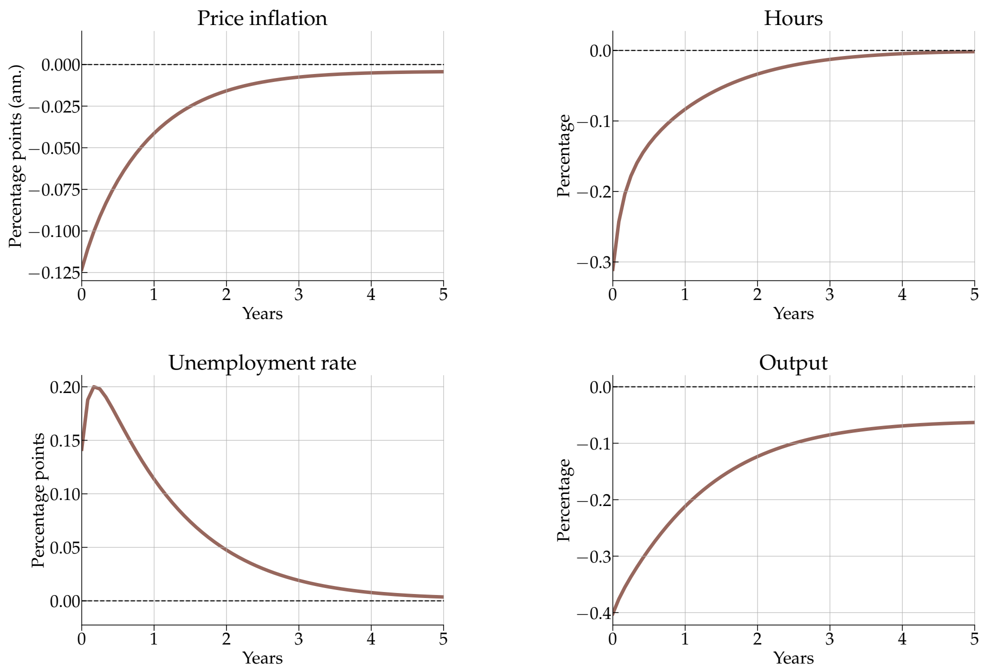

Figure 2: Model's IRFs to a contractionary monetary policy shock.

in Figure 1. This target implies a shock of 30 basis points that after a quarter is almost completely reabsorbed. The deviations of the nominal rate, plotted in the right-hand side panel, persist for longer due to the inertial reaction embedded in the rule (17).[21]

Impulse-response functions to a monetary policy shock. Figure 2 plots the impulse-response functions (IRFs) for inflation, hours worked, unemployment rate, and output. As expected, the unexpected spike in nominal rates leads to a recession: unemployment rises, hours worked and output fall, and so does inflation. Note, however, that even though inflation, hours worked, and unemployment revert quite quickly to their pre-shock values, output is much slower to recover and remains depressed five years after the monetary shock.[22]

Figure 3 displays the IRFs of a number of labor market stock and flow variables to this surprise increase in the policy rate. We begin with the stocks. The response of the unemployment rate, participation rate and employment to population ratio are all consistent

[21] Recall that the inertial parameter has been chosen to match the estimated persistence of unemployment rate deviations to the identified monetary policy shock. The internal propagation mechanism of the model generates persistence that goes well beyond the mechanical one due to the inertial Taylor rule.

[22] We discuss the long-run scars on output and earnings, as well as their sources, in the next section.

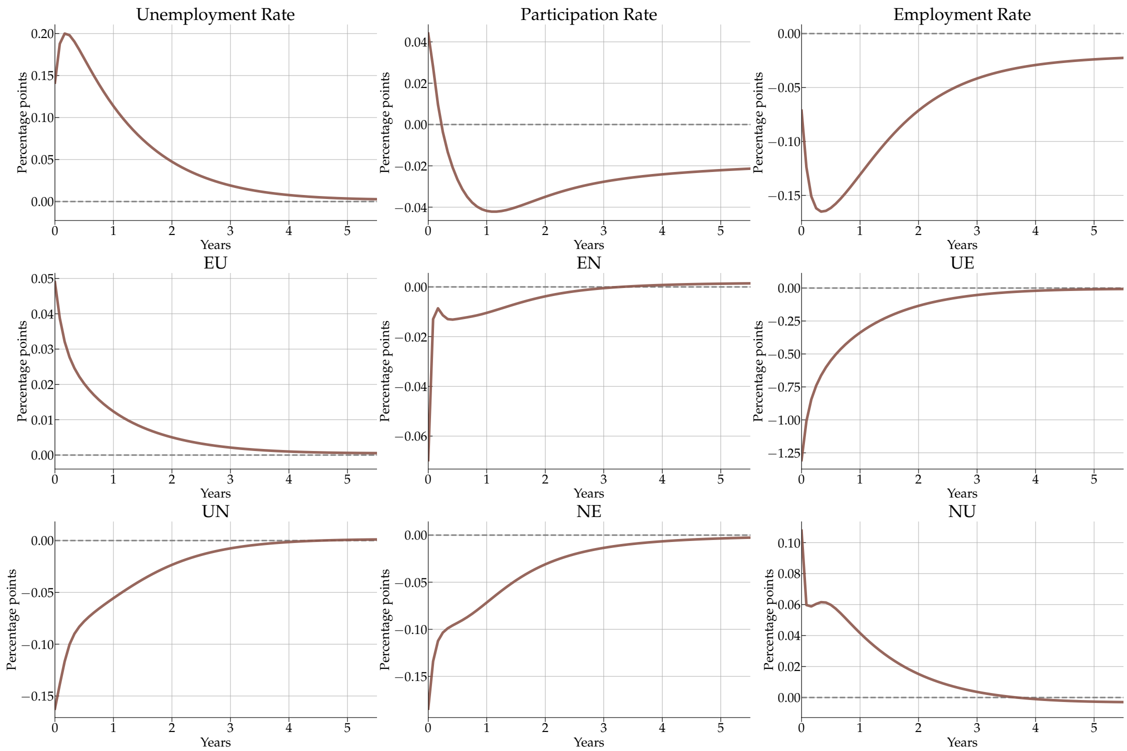

Figure 3: Model's labor market stocks and flows IRFs to a contractionary monetary policy shock. E denotes employment, U unemployment and N nonparticipation.

with the estimated VAR responses in Graves et al. (2023) (see their Figure 5 reproduced in Figure C1). In line with their results, the unemployment rate reacts sooner and displays the strongest response among all labor market stocks. In contrast, the response in labor-force participation is weaker and takes more time to materialize: its trough is around one-fifth of the peak in unemployment and occurs roughly a year later. As in the data, the dynamics of participation are very persistent: participation is still depressed five years after the shock, when the unemployment rate has already converged back to its steady state.

Turning to labor-market flows, all six flows move in the same direction as the estimates of Graves et al. (2023) (see their Figure 6 reproduced in the Figure C2). Upon a monetary contraction, unemployment inflows (*eu* and *nu*) rise, unemployment outflows (*ue* and *un*) fall, and flows between employment and non-participation (*en* and *ne*) also decline. The response of *ue* and *eu* flows are mechanical, given the way we model fluctuations in labor-market frictions.[23] Other flows are mediated by individual labor-supply reactions.

[23] Recall that job-finding and separation rates are proportional to hours worked. Since hours are procycli-

Importantly, the model reproduces the negative response of participation exit flows (*en* and *un* flows) which is responsible for the initial (countercyclical) increase in labor-force attachment.[24]

Our model is thus not only consistent with the empirical response of employment, unemployment, and nonparticipation but also with the dynamics of the underlying flows between the three labor-market states. Matching the dynamics of labor-market flows is important as these provide additional information about the relative role of labor-market frictions versus workers' labor-supply decisions in observed fluctuations of labor-market stocks.[25] For instance, the labor-market flows' reaction is crucial to understanding the weak negative response of labor-force participation to a contractionary monetary policy shock. As discussed in Elsby et al. (2015) and Graves et al. (2023), the procyclical movement in labor force participation exit rates (*en* and *un*) tends to push participation up in recessions. Working against this force, fluctuations in the *eu* and *ue* rates, which are determined mostly by fluctuations in labor-market frictions, exert a strong negative pressure on participation during downturns. Even though these flows do not affect participation directly, they induce a sharp countercyclical response of the unemployment rate. As unemployed workers are more likely than employed ones to drop out of the labor force—i.e., the *un* flow is, on average, much larger than the *en* flow—a large increase in the unemployment pool exerts, over time, downward pressure on participation. The effect of these two countervailing forces shows up on the model implied dynamics of the participation rate, as depicted in Figure 3. At impact, the upward pressure from the *un* and *en* responses dominates, causing participation to increase initially. This effect dissipates quickly and, less than a year after the shock, the labor-force participation rate falls below trend, where it remains persistently depressed for the entire plotted horizon.

cal with respect to monetary shocks, the chosen elasticities of frictions to hours guarantee that *ue* and *eu* flows respond negatively and positively to the shock.

[24] Apart from an initial spike on *en* flows, the model also does a good job quantitatively at matching the magnitude of the flow response out of a monetary contraction compared to the estimated VAR responses in Graves et al. (2023) (see their Figure 6 reproduced in Figure C2).

[25] We are not the first to tackle the task of developing a labor-market model consistent with the joint dynamics of labor-market stocks and flows. In a standard representative-agent model, Cairó et al. (2022) show that the opportunity cost of employment needs to be significantly more procyclical than the returns to work in order for the baseline model to match the procyclicality of participation flows *en* and *un*. Working with a heterogeneous-agent model similar to ours, Krusell et al. (2017) highlight the importance of wealth heterogeneity and composition effects to explain the cyclicality of labor-market flows. Looking specifically at the worker flows dynamics conditional on a monetary policy shock, Graves et al. (2023) also appeal to wealth effects in order to justify of the procyclical reaction of *en* and *un* flows.

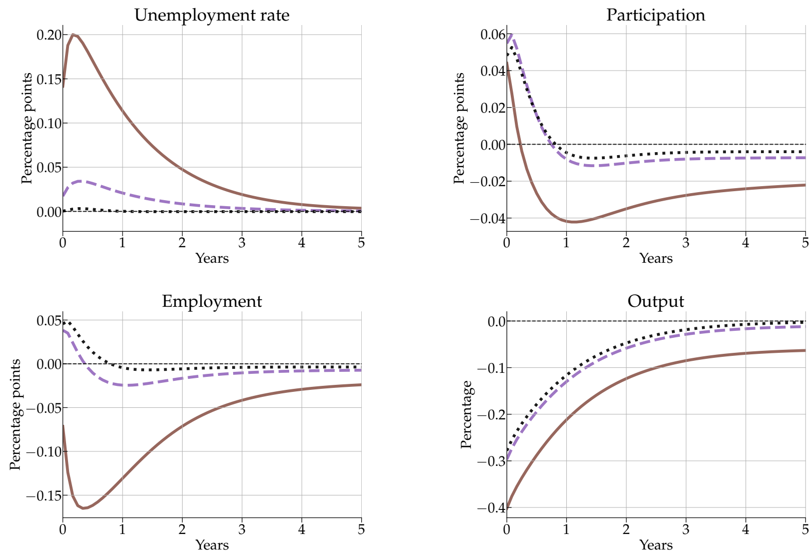

Figure 4: Counterfactual exploring the importance of fluctuations in job-finding and separation rates. Solid line: baseline model IRFs. Dotted line: counterfactual IRFs with both job-finding and separation rates fixed at their steady-state level. Dashed line: counterfactual IRFs with job-finding rate fixed at its steady-state level, but job-separation rates varying as in baseline.

Importance of job-finding and separation rates. Matching the cyclicality in job finding and separation rates is thus crucial to the model's success in generating the right dynamics of labor force participation and the other stocks. Next, we assess the relative importance of these two margins. We do so by computing an IRF to the same monetary policy shock, while holding the job finding and separation rates fixed at their steady-state levels, i.e., we set the elasticities of job finding and separation rates (first jointly, then separately) to hours worked to zero. Figure 4 shows the outcome of this counterfactual exercise.

When we fixed both transition rates at their steady-state levels (dotted line), unemployment, participation and output display very different dynamics compared to the baseline case (solid line). The unemployment rate shows almost no response to the shock, while participation features a strong counterfactual positive response without any significant decline below steady state thereafter. In addition, output falls less on impact than in the baseline and, importantly, it displays no long-lasting scarring effects. This result highlights that, besides being important for the response of labor market stocks, fluctuations in the job-finding and separation rates are also the driving force behind the macro hysteresis.

Between job finding and separation rates, which one contributes the most to the response of labor market variables in our baseline calibration? The dashed line in Figure 4 computes the IRF when the job finding rate is kept constant and the separation rate is allowed to move with hours worked. Thus, the difference between the dotted (where both transition rates are kept fixed) and the dashed line (where job finding rate only is held fixed) measures the role of fluctuations in the job separation rates. For all the variables in the figure the dashed line is very close to the dotted line, indicating that it is fluctuations in the job-finding rate, through their impact on the unemployment pool, which are the main driving force of the response of the real economy to a contractionary monetary policy shock.

4.2 Long-Run Effects of Monetary Policy

In this section, we explore the possibility that a transient contractionary monetary policy shock can lead to long-run negative effects on our economy, i.e., we check whether our model can generate negative hysteresis at the macro level. To better gauge the magnitude of the long-run effects, the simulations in this section are computed under a monetary policy shock that reduces total labor income by 1 percent over the first year. We start by looking at the behavior of labor earnings to the monetary policy contraction.

4.2.1 Long-Run Labor Earnings

Aggregate labor income in our model $W_t \equiv w\ell_t$ can be written as

$$W_t = wZ^e_t\, h_t(1 - u_t)\; LFPR_t \tag{19}$$

where w is the constant real wage, Z^e_t is average labor productivity, h_t is average hours of employed workers, u_t is the unemployment rate, and $LFPR_t$ is the labor-force participation rate.[26] In what follows, we separate terms in (19) into three, reflecting the different channels through which the monetary policy shock affect earnings. The first is labor productivity Z^e_t, which is driven both by the individual skill dynamics and the selection of workers into employment. The second term in the combined effect of hours worked and the employment rate $h_t(1 - u_t)$. Since the unemployment rate is essentially determined by the job-finding and separation rates, which, in turn, are a function of hours worked,

[26]To arrive at this decomposition, start by noting that total labor income $w\ell_t = w\int_{s_{it}=e} z_{it}h_t di$ can be expressed as $wZ^e_t h_t E_t$. Next use that total employment $E_t = (1 - u_t)LFPR_t$, which delivers expression (19).

this term can be thought of as capturing changes in earnings driven by short-run fluctuations in firms' labor demand. The third term is simply the labor force participation $LFPR_t$.

Figure 5 shows the response of total labor income (and its decomposition into the three channels) one year and ten years after the monetary policy shock. To analyze the differential effects of the shock on low and high-wage workers, we also plot the earnings responses separately for the top and bottom quartiles of the workers' skill distribution.[27] Comparing the responses in the first year following the shock, we find that total labor income falls roughly twice more at the bottom quartile than it does at the top. Through the labor income decomposition, we see that the reaction of hours and the unemployment rate (the darkest region of the bars) is main force pushing down income in the aggregate and across the skill distribution.[28]

Ten years after the shock, aggregate labor income is still depressed by 0.2% (one fifth of the first-year decline). The income scarring is particularly acute at the bottom, with total labor income for the lowest quartile barely recovering from the first-year decline—for this group, ten-year ahead labor earnings are still 1.5% below steady-state, a reduction in earnings fifteen times larger than at the top quartile. The drivers of these long-run losses are also very different from those operating in the short-run. While short-run earnings losses are driven mostly by hours and unemployment dynamics, long-run earnings are depressed through a combination of weaker labor-force participation and productivity. Interestingly, participation falls only for the bottom of the skill distribution—as low-skilled workers go through unemployment they also become more likely to persistently exit the labor force.

Figure 6 offers another way to visualize what we have just discussed. The figure plots IRFs for total labor income and its three components over the first ten years after the shock. The solid line denotes outcomes for the whole population, while the dashed and dotted lines show outcomes for the bottom and top quartiles of the skill distribution respectively. The IRFs for the quartiles confirm our previous observation that the long-run impact of the shock is much stronger at the bottom of the skill distribution.

[27] Clearly, equation (19) holds at the group level as well, e.g., across all workers in the bottom quartile of the skill distribution at every t.

[28] Participation shows a different dynamic at the bottom and the top, with a rise in participation at the bottom quartile moderating earnings losses for that group. However, these are small compared to the changes induced by movements in hours and unemployment.

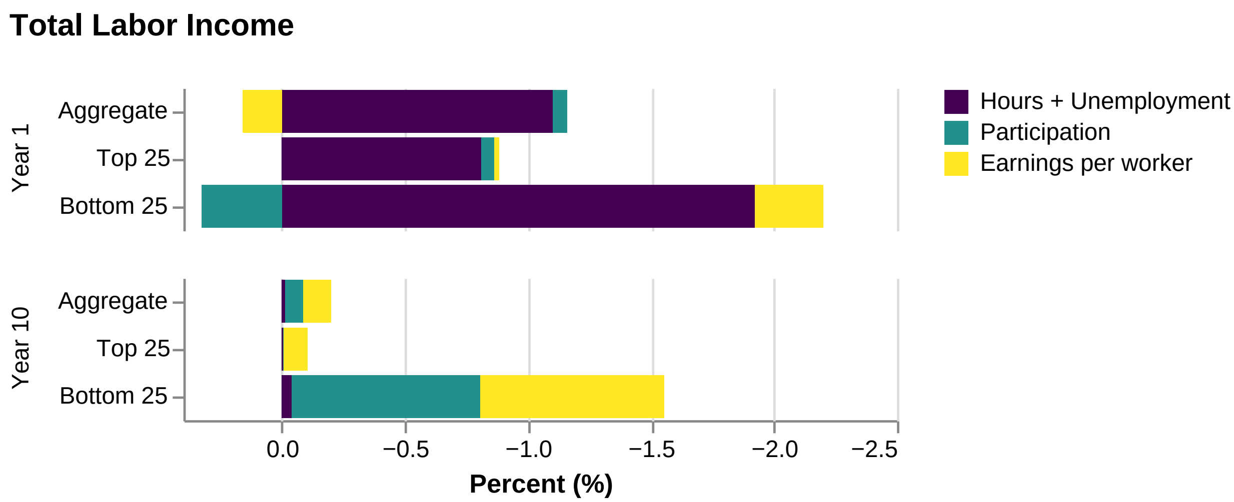

Figure 5: Decomposition of the response of total labor income for the whole population, the population with skills in the top quartile (Top 25) and the population with skills in the bottom quartile (Bottom 25), at two different horizons (years 1 and 10 after the shock). See the discussion of equation (19) for a description of the three components. The size of the contractionary monetary policy shock is normalized so that total earnings drop by 1% over the first year following the shock.

4.2.2 Long-Run Inflation

The Phillips curve we derived in equation (13) reveals that inflationary pressures are associated with an aggregate notion of the labor wedge. Log-linearizing the labor wedge around the steady state and substituting the result back into our wage Phillips curve, we obtain the following expression for the dynamics of inflation:

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \kappa^w \Big[(\xi d \log Y_t + d \log C_t) - \xi d \log LFPR_t \\ - (\xi + 1) d \log Z_t^e + (d \log \tilde{C}_t^e - d \log C_t) \Big] \tag{20}$$

where Y_t is aggregate output, C_t is aggregate consumption, $LFPR_t$ is the labor force participation rate, Z_t^e is the average labor productivity, and $\tilde{C}_t^e$ is the productivity-weighted consumption of employed workers. Log-deviations of X from steady state are denoted by $d \log X$. See Appendix D for a detailed derivation.

Drivers of inflation dynamics. Expression (20) identifies four drivers of inflation dynamics in our model. The first term $(\xi d \log Y_t + d \log C_t)$ combines movements in aggre-

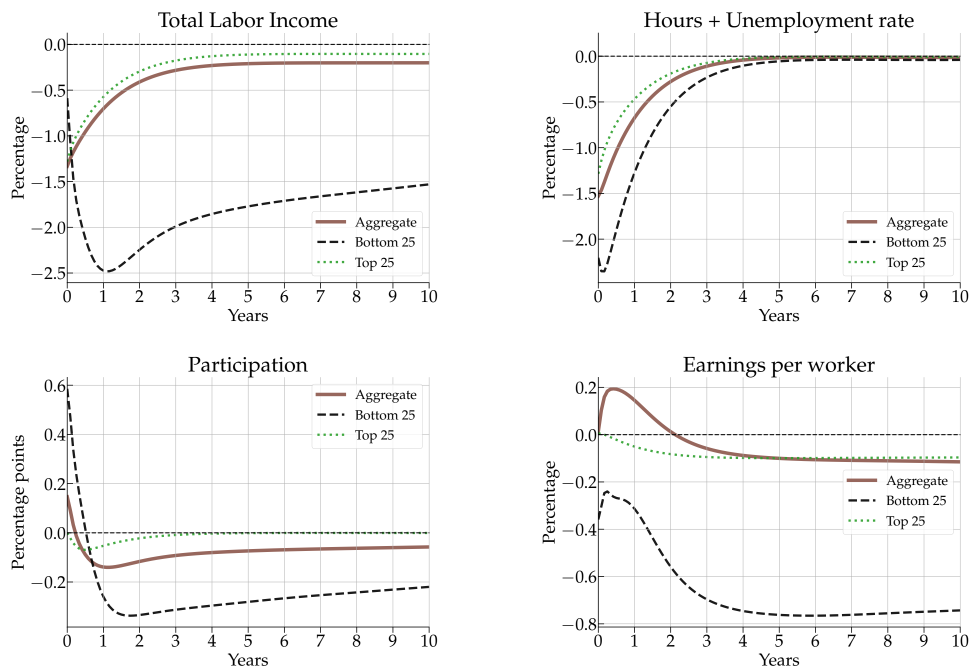

Figure 6: IRFs to a contractionary monetary policy shock of total labor income and its three components. Solid line: aggregate. Dashed line: bottom quartile of the population by skill. Dotted line: top quartile of the population by skill. The size of the contractionary monetary policy shock is normalized so that total earnings drop by 1% over the first year following the shock.

gate output Y_t and household total consumption C_t, and is equivalent to the marginal rate of substitution between leisure and consumption of an "as-if" representative-agent with log utlity over consumption and inverse Frisch elasticity ξ.[29] This term is procyclical and leads to deflationary pressures upon a contractionary monetary policy shock.

The other three components are germane to our heterogeneous-agent model with endogenous participation and state-dependent skill dynamics. In contrast to the first term, they all create inflationary pressures following a contractionary monetary policy shock. The second term is driven by movements in labor-force participation $d\log LFPR_t$. Intuitively, a reduction in participation adds inflationary pressures by reducing the supply of potential workers. Average labor productivity $d\log Z_t^e$, which shows up in the third term,

[29]Imagine an economy with linear production technology on hours $Y_t = H_t$ and a representative agent with utility over consumption and hours worked given by $U(C, H) = \log(C) - \psi\frac{H^{1+\xi}}{1+\xi}$. In this case, the marginal rate of substitution between leisure and consumption, already using the production technology to substitute hours for output, is given by $MRS_t = \psi Y_t^{\xi} C_t$, or in log-linear deviations from steady state, $d\log MRS_t = (\xi d\log Y_t + d\log C_t)$.

has a similar effect: a decrease in average labor productivity following a monetary contraction adds inflationary pressures as workers need to supply more hours to produce the same amount of the final good. The fourth term ($d\log \tilde{C}^e_t - d\log C_t$) denotes the gap between productivity-weighted consumption of the employed and total consumption. This component captures the idea that the labor union, when setting nominal wages, cares only about the marginal utility of union "insiders." In a recession driven by a contractionary monetary policy (or a demand) shock, this term is positive, making unions less willing to lower nominal wages in response to a reduction in the demand for their labor task.[30] This extra degree of nominal wage rigidity induced by union's behavior is reminiscent of the classical insider-outsider model (Galí, 2022).

Decomposition of inflation dynamics. Figure 7 shows the decomposition of inflation dynamics in the four terms in equation (20). As we previewed above, in response to a contractionary monetary policy shock, the first term in the decomposition is deflationary, while the remaining three terms are all inflationary. Quantitatively, the inflationary pressures coming from the last three terms are quite strong.[31] In particular, even though output and consumption in the long run remain depressed—which, through the first term, exerts a persistent deflationary pressure—inflation returns to target very quickly as the other terms keep pushing inflation up.

5 Conclusions

We have developed a model where the transmission mechanism that impresses such long memory to the macroeconomy operates through the labor market, according to Okun (1973) hypothesis. During economic downturns, many workers are displaced from their jobs. As they spend time unemployed, they are subject to large and persistent skill losses which lead some of them to transition into nonparticipation. Nonparticipation tends to be a long-lasting state that fuels further skill deterioration and crystallizes disattachment from the labor force. This vicious circle is the reason why even a very transitory demand shock, such as a contractionary monetary policy shock, leads to a long-run decline in labor

[30] To see this, remember that high-wages workers are less likely to lose their jobs in recessions, making the employment pool during contractions skewed towards high-skilled workers. This composition effect explains why the consumption of employed workers $\tilde{C}^e_t$ falls less than total consumption C^e_t during downturns.

[31] The inflationary effect of productivity is smaller in the short run because of the positive selection effect in labor-force participation.

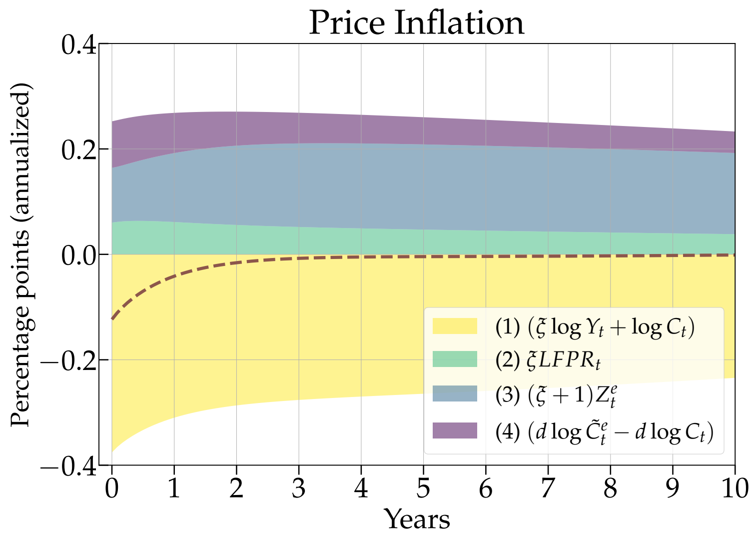

Figure 7: Decomposition of the response of inflation to a contractionary monetary policy shock at different horizons. The dashed line is model's aggregate price inflation. The four shaded areas correspond to the four terms in equation (20). The size of the contractionary monetary policy shock is normalized so that total earnings drop by 1% over the first year following the shock.

productivity, labor-force participation, and output. These chronic effects do not extend to persistent deflation because lower productivity and deficient labor supply represent inflationary forces that counteract the protracted decline in aggregate demand.

Going forward, there are at least three interrelated issues that we have not tackled in this paper. First, because of the way we numerically solve for the equilibrium dynamics of the model, our environment features both scarring effects of negative temporary shocks and *uplifting* effects of positive transitory ones. An expansionary demand shock that pulls more people into employment allows them to gradually build their productivity and reinforces attachment to the labor force. Whether hysteresis is only negative or also positive is an empirical question that remains to be settled.[32]

Second, the model has implications for the optimal conduct of monetary policy. In light of our results that in the model short-lived negative demand shocks leave persistent scars on output, but they do not necessarily generate deflationary pressures, an inflation-focused central bank may do too little for the economy (Galí, 2022). Putting excessive weight on inflation (or deflation) would lead a central bank to stop responding to the shock too quickly thus allowing it to persistently damage the real economy. Optimal

[32] Bluedorn and Leigh (2019) find some empirical support for the positive hysteresis hypothesis based on long-run revisions of professional forecasters when positive news about the labor market is released.

monetary policy should be more aggressive early on to moderate the increase in unemployment, which is the driving source of the hysteresis in participation and productivity. In addition, according to our model, a rule that responds to output (or participation) is more suitable than one that responds to unemployment because it incorporates deviations of productivity and labor-force participation from their efficient (flexible price) level. In addition, even once the post-shock hysteresis have taken place and the damage to the real economy has occurred, our model implies that keeping monetary policy expansionary for a while can fully revert the underutilization of labor without a surge in inflation.

Finally, we have shown that hysteresis is much more severe at the bottom of the skill distribution. The reason is that it is low-wage workers who are marginally attached to the labor force and those for whom earnings losses upon displacement are the largest (Athey et al., 2023; Cajner et al., 2017; Guvenen et al., 2017; Yagan, 2019). Since the new framework of the Fed (Powell, 2020) reinterprets its full employment mandate as *broad-based and inclusive*, policymakers should be especially aware of the long-run effects, both positive and negative, that untamed shocks can have on the more disadvantaged groups.

Appendix

A Household Optimization Problems

We presented the problem of the active nonparticipant in the main text. Here we describe all others.

Consider the problem of the passive non-participant (n_0):

$$
\begin{aligned}
v_0^{n_0}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}} \quad & \mathbb{E}_0 \int_0^{\tau^{n_1}} e^{-\rho t} \mathfrak{u}^n(c_t, h_t)\, dt + e^{-\rho \tau^{n_1}} v_{\tau^{n_1}}^{n_1}(a_{\tau^{n_1}}, z_{\tau^{n_1}}) \\
s.t. \quad & \\
c_t + \dot{a}_t \quad & = \quad r_t a_t + \phi \\
a_t \quad & \geq \quad 0
\end{aligned}
\tag{A1}
$$

Passive non-participants do not receive any job offers. At rate η_1, with τ^{n_1} being the first arrival rate of this event, they become active non-participants (state n_1). The conditional expectation reflects the uncertainty in transition rates and skill dynamics. In addition to the participation decision $\mathfrak{p}_t^{n_0}$, at every instant the worker chooses its consumption flow c_t. The last two lines of this problem state the budget constraint (in real terms) and the borrowing limit.

The problem of an unemployed household who is not eligible for UI benefits is:

$$
\begin{aligned}
v_0^{u_0}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}, \tau^*} \quad & \mathbb{E}_0 \Bigg[\int_0^{\tau^{\min}} e^{-\rho t} \mathfrak{u}^u(c_t, h_t)\, dt + \mathbb{I}_{\{\tau^{\min} = \tau^e\}} e^{-\rho \tau^e} v_{\tau^e}^e(a_{\tau^e}, z_{\tau^e}) \\
& + \mathbb{I}_{\{\tau^{\min} = \tau^*\}} e^{-\rho \tau^*} v_{\tau^*}^{n_1}(a_{\tau^*}, z_{\tau^*}) + \mathbb{I}_{\{\tau^{\min} = \tau^{n_0}\}} e^{-\rho \tau^{n_0}} v_{\tau^{n_0}}^{n_0}(a_{\tau^{n_0}}, z_{\tau^{n_0}}) \Bigg] \\
s.t. \quad & \\
c_t + \dot{a}_t \quad & = \quad r_t a_t + \phi \\
a_t \quad & \geq \quad 0
\end{aligned}
\tag{A2}
$$

Ineligible unemployed workers receive a job offer at rate λ_{zt}^{ue} (with τ^e being the first arrival time of this event) and always take it. At any time τ^* during the unemployment spell, the individual can exit the labor force $\left(\mathfrak{p}_t^u = 0\right)$. Finally, at rate η_0 (with τ^{n_0} being the first arrival rate of this shock) they can become passive non-participants.

The problem of an unemployed household who is eligible for UI benefits is:

$$
\begin{aligned}
v_0^{u_1}(a_0,z_0) = \max_{\{c_t\}_{t\geq 0},\tau^*} \quad & \mathbb{E}_0\Bigg[\int_0^{\tau^{\min}} e^{-\rho t}\mathfrak{u}^u(c_t,h_t)\,dt + \mathbb{I}_{\{\tau^{\min}=\tau^e\}}e^{-\rho\tau^e}\max\left\{v_{\tau^e}^{e}(a_{\tau^e},z_{\tau^e}),v_{\tau^e}^{u_1}(a_{\tau^e},z_{\tau^e})\right\} \\
& +\mathbb{I}_{\{\tau^{\min}=\tau^*\}}e^{-\rho\tau^*}v_{\tau^*}^{n_1}(a_{\tau^*},z_{\tau^*}) + \mathbb{I}_{\{\tau^{\min}=\tau^{u_0}\}}e^{-\rho\tau^{u_1}}v_{\tau^{u_0}}^{u_0}(a_{\tau^{u_0}},z_{\tau^{u_0}}) \\
& +\mathbb{I}_{\{\tau^{\min}=\tau^{n_0}\}}e^{-\rho\tau^{n_0}}v_{\tau^{n_0}}^{n_0}(a_{\tau^{n_0}},z_{\tau^{n_0}})\Bigg] \\
s.t. \quad & \\
c_t+\dot{a}_t \quad = \quad & r_t a_t + (1-\mathfrak{t})b(z_t)+\phi \qquad\qquad \text{(A3)} \\
a_t \quad \geq \quad & 0
\end{aligned}
$$

Besides receiving job opportunities and choosing whether to take them, choosing to drop out of the labor force, and exogenously switching to passive non-participant status, the eligible unemployed could lose its entitlement to UI benefit at rate $\eta^{u_1u_0}$, with τ^{u_0} being the first arrival time of this event.

Finally, the problem of the employed household is:

$$
\begin{aligned}
v_0^{e}(a,z) = \max_{\{c_t\}_{t\geq 0},\tau^*} \quad & \mathbb{E}_0\Bigg[\int_0^{\tau^{\min}} e^{-\rho t}\mathfrak{u}^e(c_t,h_t)\,dt + \mathbb{I}_{\{\tau^{\min}=\tau^u\}}e^{-\rho\tau^u}v_{\tau^u}^{u_1}(a_{\tau^u},z_{\tau^u}) \qquad \text{(A4)} \\
& +\mathbb{I}_{\{\tau^{\min}=\tau^*\}}e^{-\rho\tau^*}v_{\tau^*}^{n_1}(a_{\tau^*},z_{\tau^*}) + \mathbb{I}_{\{\tau^{\min}=\tau^{n_0}\}}e^{-\rho\tau^{n_0}}v_{\tau^{n_0}}^{n_0}(a_{\tau^{n_0}},z_{\tau^{n_0}})\Bigg] \\
s.t. \quad & \\
c_t+\dot{a}_t \quad = \quad & r_t a_t + (1-\mathfrak{t})\,w_t z_t h_t+\phi \\
a_t \quad \geq \quad & 0
\end{aligned}
$$

Employed workers (e) can be displaced at rate λ_{zt}^{eu}, in which case they become eligible for UI benefits $(u=u_1)$. Let τ^u be the first arrival time of this Poisson shock. At every instant τ^*, the employed worker can choose to quit the labor force $\left(\mathfrak{p}_t^e=0\right)$.[33] In addition, an employed worker can exogenously switch to passive non-participant status at rate η^{en_0}, with τ^{n_0} being the first arrival time of this event.

Each problem (including the one for the active nonparticipant in the main text) can be expressed recursively as a Hamilton-Jacobi-Bellman Quasi-Variational Inequality. This equation, in turn, can be discretized and solved. See Alves and Violante (2024) for details.

[33] Quitting into unemployment is never optimal, because the worker would not receive UI benefits, and would pay a higher disutility cost κ for the opportunity to be re-employed at the same wage.

B Problem of the Mutual Fund

The problem of the mutual fund, which takes prices as given, entails choosing the optimal portfolio composition between bonds and equity:

$$
\begin{aligned}
r_t A_t\left(X^m, B^m\right) = \max_{\dot{X}_t^m, \dot{B}_t^m} \quad & \Pi_t X_t^m - q_t \dot{X}_t^m + r_t^b B_t^m - \dot{B}_t^m \\
& + \partial_X A_t\left(X^m, B^m\right) \dot{X}^m + \partial_B A_t\left(X^m, B^m\right) \dot{B}_t^m + \partial_t A_t\left(X^m, B^m\right)
\end{aligned}
\tag{B1}
$$

with first-order conditions with respect to $\dot{X}_t^m$ and $\dot{B}_t^m$

$$
\begin{aligned}
q_t &= \partial_X A_t\left(X^m, B^m\right) \\
1 &= \partial_B A_t\left(X^m, B^m\right)
\end{aligned}
$$

Substituting these first order conditions into ($B1$) and exploiting the linear homogeneity of the problem which implies that $A_t = q_t X_t^m + B_t^m$, we arrive at

$$
r_t\left(q_t X_t^m + B_t^m\right) = \Pi_t X_t^m + r_t^b B_t^m + \dot{q}_t X^m.
$$

By matching coefficients on equity and bonds, we obtain

$$
r_t = \frac{\Pi_t + \dot{q}_t}{q_t} = r_t^b \tag{B2}
$$

which determines the real return on the household financial asset a_t (wealth invested in the mutual fund), and establishes a no-arbitrage condition between government bonds and firm equity which holds at every t, except when a shock hits the economy, in which case the price q_t features a jump.

C Empirical Estimates from Graves et al. (2023)

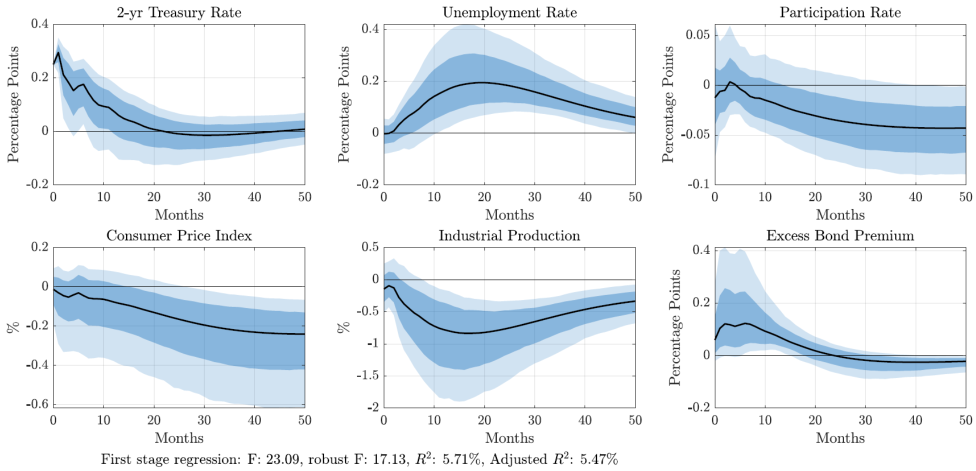

Figure C1: Response of aggregate variables to a monetary policy shock. Estimated impulse responses to a 25bp monetary policy tightening shock in the baseline VAR. Solid black lines report impulse response functions, while light- and dark-blue-shaded regions report bootstrapped 68% and 90% standard error bands. See Graves et al. (2023) for more details.

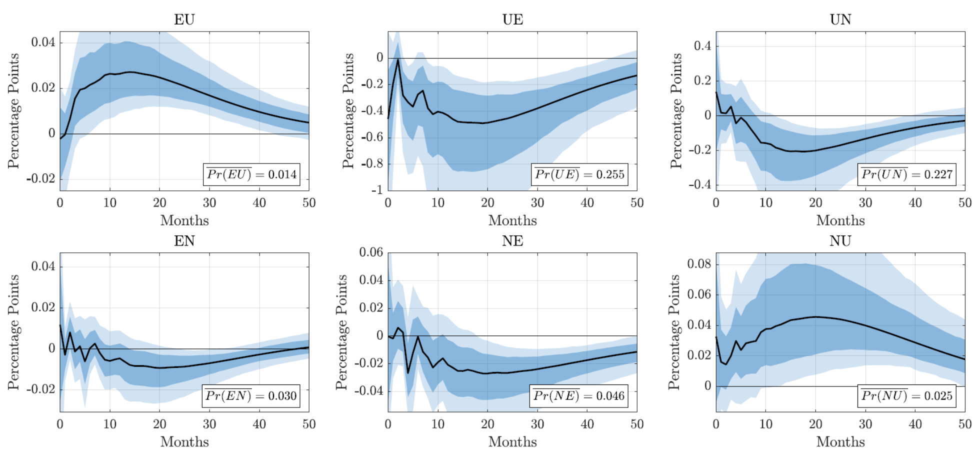

Figure C2: Response of labor market flows to a monetary policy shock. Estimated impulse responses to a 25bp monetary policy tightening shock, computed by appending the given labor market flow variable to the baseline VAR from Figure C1. Solid black lines report impulse response functions while light- and dark-blue-shaded regions report bootstrapped 68% and 90% standard error bands. See Graves et al. (2023) for more details.

D Log-linear Approximation of the Labor Wedge

We start with the wage Phillips curve in equation (13), which we write as

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \frac{\epsilon}{\Theta}(H_t)\left[\psi h_t^{\frac{1}{\sigma}} - \left(\frac{\epsilon-1}{\epsilon}\right)(1-\mathfrak{t})w\, Z_t^e\, (\tilde{C}_t^e)^{-1}\right]$$

where π_t is aggregate (wage and price) inflation rate, H_t aggregate hours, h_t average hours per worker, Z_t^e average productivity among the employed, and $\tilde{C}_t^e$ is the *virtual aggregate consumption of the employed* implicitly defined by the following equation

$$\frac{1}{\tilde{C}_t^e} = \int_{s_{it}=e} \frac{1}{c_{it}}\left(\frac{z_{it}}{\int_{s_{it}=e} z_{it} di}\right) di.$$

We now take a log-linear approximation around the steady state of the equation's right-hand side, obtaining

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \kappa^w\left[\sigma^{-1} d\log h_t - d\log Z_t^e + d\log \tilde{C}_t^e\right] \tag{D1}$$

where $\kappa^w \equiv \frac{\epsilon}{\Theta}(H)\,\psi h^{\frac{1}{\sigma}}$. Using the aggregate production function (12), we can write

$$d\log Y_t = d\log h_t + d\log(1-u_t) + d\log LFPR_t + d\log Z_t^e.$$

Under the assumption that the unemployment rate is proportional to average hours worked h_t, which is approximately true in our model given the imposed relation between labor market frictions and h_t, we can write

$$d\log Y_t = (1+\varepsilon_{e,h})d\log h_t + d\log LFPR_t + d\log Z_t^e$$

where $\varepsilon_{e,h}$ is the elasticity of the $(1-u_t)$ to hours h_t. Using this to substitute out hours worked from (D1) and arrive at:

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \kappa^w\Bigg[\frac{\sigma^{-1}}{1+\varepsilon_{e,h}}\left(d\log Y_t - d\log LFPR_t - d\log Z_t^e\right) \\ - d\log Z_t^e + d\log \tilde{C}_t^e\Bigg] \tag{D2}$$

If we let

$$\xi \equiv \frac{\sigma^{-1}}{1+\varepsilon_{e,h}}$$

and collect terms, we can re-express (D2) as

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \kappa^w \Big[\xi d \log Y_t - \xi d \log LFPR_t - (\xi + 1) d \log Z_t^e + d \log \tilde{C}_t^e \Big]$$

Finally, add and subtract log deviations in aggregate consumption $d \log C_t$ to obtain

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \kappa^w \Big[\xi d \log Y_t + d \log C_t - \xi d \log LFPR_t - (\xi + 1) d \log Z_t^e + (d \log \tilde{C}_t^e - d \log C_t) \Big]$$

which is equation (20) in the main text.

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