Some Like It Hot: Monetary Policy Under Okun's Hypothesis*

Felipe Alves   Giovanni L. Violante

November 9, 2024

Abstract

The current monetary policy framework of the Fed intends to be more 'inclusive' by running the economy hot for longer during expansions. The logic of this new strategy rests on Okun's (1973) hypothesis that sustaining a 'high-pressure economy' persistently improves labor market outcomes of low-wage workers by fostering labor force attachment and earnings growth. To evaluate this conjecture, we develop a Heterogeneous Agent New Keynesian framework with a three-state frictional model of the labor market where low-skilled workers are more exposed to the business cycle and recessions have a long-lasting effect on their labor force participation and earnings, in line with the evidence. Under a canonical Inflation Targeting rule, the ZLB generates a deflationary bias and severely amplifies the persistent scars of recessions at the bottom of the income distribution. The new Lower-for-Longer strategy is a successful antidote to the ZLB-driven hysteresis and leads to notable earnings gains for low-wage workers. If pursued aggressively, however, it reverts the inflation bias from negative to positive. Since policymakers might prioritize differently inflation relative to inclusion, we conclude by quantifying the inflation-inclusion trade-off implied by various monetary policy rules.

JEL Codes: E21, E24, E31, E32, E52, J24, J64

Keywords: Distribution, Earnings Losses Upon Displacement, Hysteresis, Inflation, Labor Market Frictions, Monetary Policy, Participation, Okun, Unemployment, Upward Mobility, Zero Lower Bound.

---

*Alves: Bank of Canada, felipe.a.alves0@gmail.com. Violante: Princeton University, CEPR, IFS, IZA and NBER, violante@princeton.edu. We thank Edouard Challe, Marco Del Negro, Andres Drenik, Keshav Dogra, Jordi Gali, Mark Gertler, Greg Kaplan, Ben Moll, Juanpa Nicolini, and many workshop participants at the Atlanta Fed, Bank of Canada, Bank of Chile, Bank of Portugal, 2023 Barcelona Summer Forum, Canadian Economic Association, Danmarks Nationalbank, Dartmouth, ECB, 2023 IARIW–Bank of Italy conference (Naples), Maryland, National Bank of Belgium, 2023 NBER Summer Institute, OFCE-Sciences Po-PSE Workshop, Oxford, Penn, Richmond Fed, 2023 SED (Cartagena), St. Louis Fed, Wharton, Workshop on Micro Data Meet Macro Models (Lund), Workshop on Heterogeneity in Macroeconomics (Rennes), Yale.

1 Introduction

Monetary policy regimes are neither permanent nor timeless. As the economy evolves and new information about its functioning emerges, monetary rules must adapt to changing macroeconomic conditions and the expanding body of knowledge. All central banks are fully aware of this reality and regularly reassess their conduct. The most recent strategic review conducted by the US Federal Reserve, and concluded in 2020, culminated in a substantially revised monetary policy framework. The objective of this paper is to assess the aggregate and distributional implications of this regime shift.

The Fed's review contained two major novelties. The first one was motivated by the Zero Lower Bound (ZLB) on nominal rates which, in a low 'natural' rate environment, can dramatically constrain monetary policy's ability to support the economy during downturns. Under these circumstances, *the result is worse economic outcomes in terms of both employment and price stability, with the costs of such outcomes likely falling hardest on those least able to bear them* (Powell, 2020). To address this concern, the Federal Reserve adopted a more flexible approach to its price stability mandate, shifting from a rigid 2 percent inflation target to an average inflation framework. Specifically, after periods of persistently low inflation—such as those resulting from extended episodes at the ZLB—the Fed *will likely aim to achieve inflation moderately above 2 percent for some time* (Powell, 2020). Importantly, this approach was designed to be asymmetric: while the Fed would pursue inflation above 2 percent following periods where inflation remained below target, it would not actively seek inflation below 2 percent after inflationary episodes (Clarida, 2022).

The second novelty is a reinterpretation of the maximum employment mandate—the other pillar of the Fed's mission—as a *broad-based and inclusive goal*. This policy shift stemmed from FedListens, a series of nationwide events where the Fed engaged with a diverse array of organizations to better understand how monetary policy impacts people's livelihoods. In the foreword of the project's final report (Federal Reserve System, 2020), Chair Powell notes that: *One clear takeaway from these events was the importance of sustaining a strong job market, particularly for people from low- and moderate-income communities. Everyone deserves the opportunity to participate fully in our society and in our economy.*[1] In alignment with this objective, the new framework stipulates that monetary policy is no longer informed by deviations, but only by shortfalls of employment from its max-

[1] Interestingly, the European Central Bank's (ECB) most recent monetary policy strategy review reveals that, while price stability remains its sole official objective, the ECB now acknowledges more explicitly the indirect impact of its policy decisions on aspects of social progress, such as wealth and income inequality (Ioannidis et al., 2021).

imum level, adding another element of asymmetry in the policy response to aggregate disturbances.

Jointly, these two shifts in the conduct of monetary policy indicate that interest rates would be kept *lower for longer* following contractions in order to let the economy *run hot for longer* during recoveries. The new strategy marked a sharp departure from the Fed's traditional 'preemptive restraint' approach—whereby the Fed would raise rates early on during recoveries out of fear of fueling inflation—and reflected the broad consensus in the aftermath of the Great Recession that the Phillips curve had flattened, leading to the belief that a robust labor market could be sustained without causing an outbreak in inflation. Low-wage workers, who are severely hurt by downturns and stand to gain the most from extended and vigorous recoveries, are meant to especially benefit from this new strategy.

The hypothesis that a hot economy benefits workers at the bottom of the distribution is not new. In his classic article entitled *Upward Mobility in a High Pressure Economy* (Okun, 1973), Arthur Okun conjectured that a 'high-pressure' economy could persistently improve the economic circumstances of more disadvantaged workers by giving them opportunities to find steady employment, strengthen their skills, and gradually climb the job ladder. While Okun recognized explicitly that running a 'high-pressure' economy carries the risk of propelling inflationary pressures, he also thought that foregoing these benefits accruing to traditionally disadvantaged groups *should be carefully reckoned as a high cost of accepting slack as an insurance policy against inflation* (page 244). Policymakers, according to Okun, thus face a stark *inflation-inclusion trade-off*.

The post-pandemic experience in the US illustrates this trade-off clearly. Unlike its approach during the recovery from the Great Recession, when the Fed raised rates preemptively at the first signs of a tightening labor market, it waited significantly longer to raise rates in the post-pandemic recovery. Although inflation continued to rise during this time—arguably for a variety of other reasons—employment rates and earnings for low-educated and minority workers reached historic highs and, for the first time in decades, the economy also experienced a notable compression in the earnings distribution (Autor et al., 2023).

The aim of this paper is to develop a quantitative framework to assess the implications of this historic regime shift in monetary policy for labor market outcomes across the entire distribution of workers. The primary challenge we confront is how to formalize and articulate Okun's hypothesis. At the time of his writing, Okun presented only tentative empirical evidence to bolster his conjecture that a high-pressure economy would primarily benefit low-wage workers. Nearly fifty years later, our understanding of the employment and earnings dynamics of workers has significantly deepened, with numerous findings corroborating this insight. First, the employment outcomes of low-income groups are more cyclically sensitive than those of the broader population. During economic downturns, these groups experience disproportionately higher unemployment, while in recovery, their unemployment rates decline more significantly due to increased job-finding rates and reduced job-separation rates. Second, unemployed workers are far more likely to exit the labor force compared to their employed counterparts ($UN >> EN$), regardless of the phase of the cycle. This disparity in labor force attachment between employed and unemployed workers ("attachment wedge") makes the size of the unemployment pool a crucial driver of procyclical fluctuations in participation—a force often referred to as the 'participation cycle.' Since low-wage workers are particularly vulnerable to unemployment during recessions, this force is especially pronounced among them. Third, earnings losses resulting from job displacement are substantial and enduring, especially at the lower end of the income distribution, where they more frequently lead to a persistent exit from the labor force. These three mechanisms—uneven exposure to aggregate fluctuations, the labor force attachment wedge, and long-lasting displacement effects—form the foundation of how we concretely think about Okun's hypothesis.

To study the aggregate and distributional effects of alternative monetary policy regimes under Okun's hypothesis, we integrate these three channels into a heterogeneous-agent incomplete-markets New-Keynesian (HANK) framework featuring a three-state frictional labor market model. Workers can be either employed, unemployed, or out of the labor force. Job-finding and job-separation rates, which move workers between employment and unemployment, are allowed to depend on individual skill levels. Low-skilled workers face higher separation and lower job-finding rates, leading to higher unemployment and stronger exposure to business cycle fluctuations at the bottom of the distribution. Labor market frictions and participation decisions give rise to a large attachment wedge and thus a strong 'participation cycle', just as in the data. Finally, we make the evolution of workers' skills contingent on their individual labor market status. Specifically, workers' productivity typically increases while employed but gradually diminishes during periods of nonemployment. When combined with the attachment wedge, this skill depreciation during nonemployment contributes to generate significant and enduring earnings losses following displacement. Together, these three factors make running a hot economy particularly beneficial to low-income groups.

The rest of the model closely follows the New Keynesian literature with nominal wage rigidities. A wage Phillips curve links inflation to fluctuations in hours per worker, a fiscal authority taxes labor and provides unemployment insurance, and a monetary authority sets the nominal rate subject to a Zero Lower Bound (ZLB). Demand and supply shocks are the source of business cycles. The model is parameterized to match a wide set of macro and micro facts that give credibility to the strength of the three Okun's channels.

We begin our analysis by revisiting the deleterious effects of the ZLB under Inflation Targeting (IT) through the lenses of our model. In agreement with much of the existing literature, we find that the ZLB produces, in the ergodic distribution, small biases in inflation (-20bp) and unemployment (+0.55 ppt). Unlike the previous research that has typically concentrated on the aggregate effects of the ZLB, we uncover that this aggregate bias conceals a significantly greater cost for those at the lower end of the distribution. For instance, whereas the negative bias in total labor earnings is just 2% in the top tercile of the skill distribution, it escalates to 12% in its bottom tercile. This heterogeneity is largely driven by an endogenous labor market hysteresis mechanism that is particularly acute at the bottom: a temporary shock that pushes the economy into the ZLB creates a vicious cycle of prolonged unemployment, diminished labor force attachment, and skill erosion, leaving low-skilled workers with lasting scars that extend well beyond the ZLB period. As the costs of the ZLB fall hardest on low-wage workers, the gap between high- and low-skill workers' labor market outcomes widens relative to the non-stochastic steady state.

These severe shortcomings of IT open the door to alternative, potentially more effective policy regimes, like the Lower-for-Longer (LfL) strategy embedded in the 2020 monetary policy framework. We ask whether this alternative rule that promises to run the economy hot for longer during expansions can, possibly at the cost of higher inflation, foster an economy with more robust labor force attachment, enduring earnings growth, and stronger upward mobility among low-skilled workers. In other words, we use our model to test whether the renewed monetary policy framework gives rise to an inflation-inclusion trade-off along the lines of Okun's hypothesis.

We evaluate the effect of the LfL framework through two counterfactuals. First, we ask how the recovery from the US economy from the Great Recession would have differed had the Fed followed a LfL strategy. This historical episode offers an ideal laboratory for this counterfactual because of the depth of the recession, the extended duration of the ZLB, and the preemptive monetary tightening that took place upon the very first signs of a tight labor market. Second, we contrast the model's aggregate and distributional outcomes in the ergodic distribution under IT and LfL strategies, separating the roles played the asymmetric Average Inflation Targeting (AIT) and Shortfall (SR) components of LfL. Taken together, our two counterfactuals yield four additional findings.

First, we find that the LfL strategy successfully reverses the contractionary labor market biases that emerge under IT, generating sizable gains at the lower end of the wage distribution. For example, relative to the ergodic distribution under IT, the LfL strategy achieves gains in participation, labor productivity, and earnings for the bottom tercile of the skill distribution of 3.5 ppts, 6.5%, and 17%, respectively. These strong gains at the bottom contribute to an overall reduction in labor market inequality, with the earnings gap falling by 14%. Inflation, the other side of the trade-off, averages 33 bp above the target.

Our second message pertains to the contribution of each of the two components of the LfL strategy. We find that AIT and SR generate similar benefits to average labor market outcomes of low-skilled workers. However, the gains under each rule are realized at different points of the business cycle. The AIT component primarily reduces the likelihood and severity of ZLB episodes, lessening the long-lasting harmful effects of recessions on low-wage workers. In contrast, the SR component operates mostly by letting the economy run hot during recoveries. Doing so enables low-wage workers to partially recover from the losses incurred during past recessions. Regardless of the phase of the cycle in which these gains take place, the improvements in participation and earnings for low-wage workers are remarkably resilient, enduring even when the economy inevitably returns to a state of increased slack.

Our third set of results looks beyond the cross-section and explores the impact of alternative rules along workers' life-cycle earnings profiles, in the spirit of Okun's original upward mobility conjecture. We find that recessions under IT have large and long-lasting effects on workers' earnings, particularly for those starting their careers at the bottom of the skill distribution. Labor market hysteresis thus not only decreases earnings in the cross-section, but also flattens workers' life-cycle earnings profiles. For instance, the expected earnings at age 35 (55) in the ergodic distribution under IT for a worker whose initial skills fall at the 10th percentile are 9% (14%) depressed relative to the stationary equilibrium. Shifting from an IT rule to a LfL strategy fully closes this life-cycle earnings gap.

Finally, since policymakers may have different attitudes towards inclusion vs inflation, we leverage our model to trace out the menu of choices available under the LfL approach. These Okun's cones map the cost, in terms of additional inflation, required to achieve specific average improvements in labor market outcomes across the distribution. For example, we establish that a hawkish central bank interested only in closing the deflationary bias induced by the ZLB, but with zero tolerance for any additional inflation, would already boost participation by 1.5ppts and earnings by 8% at the bottom tercile of the skill distribution.

1.1 Related Literature and Contribution

Our paper stands at the intersection of several key areas in macroeconomics.

First, our formalization of Okun's hypothesis through the exposure, attachment, and persistence channels rests on solid empirical evidence from micro data. Cajner et al. (2017) and Aaronson et al. (2019) document that low-income groups are more exposed than the average to cyclical fluctuations, while Graves et al. (2023) documents similar patterns for well-identified monetary policy shocks. Hobijn and Şahin (2021) provide evidence on the 'participation cycle' mechanism which pushes unemployed workers out of the labor force, and show it is more pronounced for low-skill ones. Davis and Von Wachter (2011) show that earnings losses upon job displacement are sizable and long-lasting, especially in recessions. Guvenen et al. (2017), Yagan (2019), and Athey et al. (2023) illustrate that this phenomenon is particularly severe at the bottom of the distribution because it is associated with more frequent exit from the labor force. We also leverage this body of work to calibrate the strength of each of these three channels.

From a modelling perspective, our paper presents two novelties relative to the growing literature that employs Heterogeneous Agent New Keynesian (HANK) models to study aggregate fluctuations and stabilization policies (see Auclert et al., 2024, for a recent survey). First, we augment the canonical framework to incorporate labor market frictions and a participation decision along the lines of Krusell et al. (2017). Second, within this framework we give formal content to Okun (1973) conjecture.

Our paper also relates to a long-standing body of work on labor market hysteresis which goes back at least to Blanchard and Summers (1986) who adopted the hysteresis view to describe the differential experience of European labor markets during the 1970s and 1980s relative to the US (see Cerra et al., 2023, for a recent survey). We contribute to this literature by providing a microfounded mechanism, operating via the labor market, through which transitory aggregate shocks can have enduring effects on the macroeconomy. A recent literature, fueled by long shadow casted by the the Great Recession, argues that monetary policy can have long-run repercussions on the economy (Comin and Gertler, 2006; Jordà et al., 2020; Fornaro and Wolf, 2020; Ma and Zimmermann, 2023). While these papers emphasize the long-run effects of monetary policy on productivity through capital and R&D investment, the human capital channel is what is central to our framework—workers become more productive when employed continuously and suffer skill losses when long-term unemployed or outside the labor force.[2] Furlanetto et al. (2023) find strong evidence of precisely this transmission mechanism on US time series.

Fueled by the redefinition of the maximum employment mandate of the Fed, a number of contemporaneous papers have started to investigate the impact of monetary policy on racial income and wealth gaps (Bartscher et al., 2021; Bergman et al., 2020; Lee et al., 2021; Nakajima, 2022; Cairó and Lipton, 2023). Our approach differs from this literature along several dimensions. First, we chose to focus on a more comprehensive notion of skills and inequality whose sources can be both unobservable (e.g., innate abilities, specialized knowledge) and observable (e.g., race, gender, education). Second, these papers contain at most one of the three Okun's channels we model (the uneven exposure), but abstract from human capital losses from non-employment and from the labor force attachment margin which are crucial to the hysteresis. Third, most of these papers concentrate on the impact of monetary policy *shocks* to standard Taylor rules on racial inequality, while we think that it is more consequential to study how alternative monetary policy *rules* in reaction to aggregate shocks can generate durable gains for low-income groups. Focusing on sytematic rules also allows us to quantify the inflation-inclusion trade-off faced by policymakers.

Two recent papers explore the macroeconomic impact of asymmetric monetary policy rules, but they do it within representative agent models. Bianchi et al. (2021) study how an asymmetric rule with respect to inflation gaps can correct the deflationary bias caused by the ZLB. Bundick and Petrosky-Nadeau (2021) analyze the impact of employment shortfall-based rules on business cycle dynamics. Our interest lies in assessing how inclusive these rules are with respect to the low-wage segment of the labor force. This requires moving beyond the representative agent and modelling new transmission mechanisms, which we do here.

Finally, as Fernández-Villaverde et al. (2023), we are interested in investigating the implications of an occasionally binding ZLB in an incomplete-market economy with heterogeneous agents. Their emphasis is on how the frequency of ZLB episodes can influence aggregate precautionary saving and, in turn, the natural real rate and the effectiveness of monetary policy. Our main insight, instead, is that, paired with strict Inflation Targeting, the ZLB aggravates negative labor market hysteresis opening the door to more fruitful monetary policy strategies.

[2] In Alves and Violante (2023), we use the setup with labor market hysteresis developed in this paper to study the long-run effects of a transitory monetary policy shock.

The rest of the paper is organized as follows. Section 2 outlines the model. Section 3 discusses its parameterization. Section 4 analyzes the economy's dynamics under an Inflation Targeting rule and highlights the negative hysteresis effects of the ZLB. Section 5 evaluates aggregate and distributional outcomes under the Lower-for-Longer strategy. Section 6 concludes.

2 Model

It is useful to begin with an overview of the model, starting from its real side. Individuals consume, save and can be either employed, unemployed, or out of the labor force. One way to depict the structure of the labor market is to envision three separate "islands" (Lucas and Prescott, 1978; Alvarez and Shimer, 2011). Some transitions across islands are exogenous and some are endogenous. Workers endogenously choose whether to participate to the labor market. Job finding opportunities for unemployed, and at a lower rate for non-participants, arise exogenously, but workers choose whether to accept them or not. Layoffs are exogenous. Individuals' efficiency units of labor (skills) are subject to uninsurable idiosyncratic shocks whose distribution depends on labor market status. Both job finding and job separation rates are indexed by skill level.

Monopolistically competitive intermediate good producers with flexible prices take wages as given and demand the profit-maximizing amount of labor. A competitive final good sector packages the intermediate goods into a final good, the numeraire of the economy, and sells it to households. Households trade shares of a mutual fund which owns claims to the firms' profits and holds government bonds. The fiscal authority finances expenditures and transfers by levying taxes on households and issuing debt.

The nominal side of the model follows closely the New Keynesian tradition. Every worker on the employment island adheres to labor unions which set nominal wages subject to adjustment frictions which give rise to a wage Phillips curve. The central bank (monetary authority) sets the nominal interest rate to stabilize the economy in response to aggregate demand and supply shocks.

2.1 Households

Demographics. Time is continuous and indexed by t. The economy is populated by a continuum of households (or individuals) with measure 1 who discount the future at rate $\tilde{\rho} > 0$, and face mortality rate ϱ. Let $\rho = \tilde{\rho} + \varrho$ be the effective discount rate.

Labor market status. At any date t, individuals can be in one of three mutually exclusive labor market states s_t: employed and earning labor income ($s_t = e$), unemployed and searching for a job ($s_t = u$), and non-participant, or outside the labor force, ($s_t = n$). Among the unemployed, we distinguish between eligible ($u = u_1$) and not eligible ($u = u_0$) for unemployment insurance (UI) benefits. Workers gain eligibility when they enter the unemployment pool due to an exogenous separation, and they lose it at some constant rate which reflects benefit duration. Among those out of the labor force, we distinguish between "active" non-participants ($n = n_1$) and "passive" non-participants ($n = n_0$). The former still receive job offers and thus can transition back into employment, though at a lower rate than the unemployed. The latter, instead, do not engage in any search and hence do not transition into employment. This differentiation is meant to capture the fact that the pool of non-participants is heterogeneous (Hall and Kudlyak, 2019) with some individuals able and willing to work while others are unable and not searching at all for a job (e.g., because they are sick, heavily involved in household care, or discouraged by the failure of previous job search).

Labor productivity. Each individual is endowed with efficiency units of labor (or skills) z evolving according to a Ornstein-Uhlenbeck diffusion process which depends on labor market status s_t

$$d \log z_t = \Big\{ -\gamma_z \log z_t + \mathbb{I}_{\{s_t=e\}}\, \delta_z^+ - \mathbb{I}_{\{s_t \neq e\}}\, \delta_z^- \Big\} dt + \sigma_z d\mathcal{W}_t \tag{1}$$

as in Ljungqvist and Sargent (1998), Kehoe et al. (2019) and Braxton and Herkenhoff (2021). When workers are employed ($s_t = e$), skills drift up at rate $\delta_z^+ > 0$, and when they are not employed ($s_t = u, n$) they drift down at rate $\delta_z^- < 0$. The parameter $\gamma_z > 0$ measures the degree of mean reversion in skill dynamics, σ_z determines uncertainty about future realizations, and $\mathcal{W}_t$ is a Wiener process. Upon death, workers are replaced by an offspring with log skill drawn from a Normal distribution with mean $\bar{z}_0$ and variance $\sigma_{z_0}^2$.

Labor market transitions. Every period individuals can transition between employment states through a combination of exogenous Poisson rates and optimal mobility decisions. Table 1 describes all the possible transitions and their endogenous/exogenous nature. Employed and unemployed workers can choose to quit the labor force and enter active non-participation (rows 1, 2, 3 of Table 1). Similarly, an active non-participant can choose to re-enter the labor force as unemployed ineligible for UI (row 4). Employed workers who decide to remain attached can still be laid off, and thus move from e to u at an exogenous rate λ_{zt}^{eu} which depends on the worker's skill level z (row 1). Unemployed workers draw a job opportunity at an exogenous rate λ_{zt}^{ue} and choose whether to accept it or not

	e	u_1	u_0	n_1	n_0
e	$\ddots$	λ_{zt}^{eu}	$\times$	$\blacktriangleright$	η^{en_0}
u_1	$\lambda_{zt}^{ue} \cdot \triangleright$	$\ddots$	$\lambda_z^{u_1 u_0}$	$\blacktriangleright$	η^{un_0}
u_0	$\lambda_{zt}^{ue} \cdot \triangleright$	$\times$	$\ddots$	$\blacktriangleright$	η^{un_0}
n_1	$\lambda_{zt}^{ne} \cdot \triangleright$	$\times$	$\blacktriangleright$	$\ddots$	$\eta^{n_1 n_0}$
n_0	$\times$	$\times$	$\times$	$\eta^{n_0 n_1}$	$\ddots$

Table 1: Transition matrix across the 5 employment states. The $\times$ symbol means that transition cannot happen. The $\blacktriangleright$ symbol means that an endogenous participation decision moves the individual in that state. The $\triangleright$ symbol means that an endogenous job acceptance decision moves the individual into employment. $\lambda_{zt}^{ss'}$ and $\eta^{ss'}$ are exogenous Poisson rates capturing labor market frictions and nonparticipation shocks, respectively. The diagonal dots stand for the negative of the sum of all the other entries on that line.

(rows 2 and 3).[3] Upon expiration of UI benefits, at rate $\lambda_z^{u_1 u_0}$, eligible unemployed become ineligible (row 2). Also active participants receive job opportunities at rate λ_{zt}^{ne} and decide whether to accept them or not (row 4). All workers can exogenously transition into passive nonparticipation at rate η^{sn_0} (rows 1, 2, 3, 4). At rate $\eta^{n_0 n_1}$, passive nonparticipants become active again (row 5).

Employed individuals earn labor income $w_t h_t z_t$, where w_t is the real wage per effective hour, and eligible unemployed receive benefits $b(z_t)$. We let UI benefits be a function of current worker productivity z_t, as a proxy for actual replacement rates. Both types of income are taxed at a proportional rate τ. Every household is entitled to a lump-sum transfer ϕ_t.

Preferences. Households derive utility from consumption c_t, suffer disutility from the effort cost κ^s associated to being in labor market status s (the extensive margin) and, if employed, from the effort cost of working h_t hours (the intensive margin). We specify the following functional form for period utility

$$\mathrm{u}^s\left(c_t, h_t; z_t\right) = \log\left(c_t - \psi z_t \frac{h_t^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}\right) - \kappa^s \tag{2}$$

[3] The unemployed ineligible for UI always accept job offers because in equilibrium there is a unique wage per effective hours, and if they did not want to work, they would choose non-participation. Eligible unemployed instead may turn down job opportunities if UI benefits are generous enough.

where $\sigma > 0$ is the Frisch elasticity of labor supply, and $\kappa^e > \kappa^u > \kappa^n \geq 0$. Note that these preferences induce wealth effects on the extensive margin (labor force participation), but not on the intensive margin (hours worked) of labor supply of employed workers. In addition, because the disutility of hours is scaled by individual productivity z_t, optimal hours worked are the same for every employed individual. This property is useful to isolate the sources of inefficiency arising from nominal wage rigidity, as we explain in Section 2.7.[4]

Saving instruments. Households can save through a financial asset a_t with rate of return r_t, and face a zero unsecured credit limit. Newborn workers enter the economy with zero wealth holdings. Perfect annuity markets insure workers against survival risk, so that the net wealth of the deceased is redistributed to surviving households in proportion to their own net wealth.[5]

Household problem

The dynamic problem solved by the household at time t is a mix of an optimal control problem, the choice of $c_t > 0$, and two optimal stopping problems: a continuous one, the participation decision $\mathfrak{p}_t^s \in \{0,1\}$, and one arising at random Poisson jump times, the job acceptance decision $\mathfrak{j}_t^s \in \{0,1\}$. The stochastic nature of the problem is due to both the Poisson arrival rates that determine transitions across labor market states, and the diffusion that determines the evolution of skills z_t. Conditional on these realizations, wealth evolves deterministically. Let $v_t^s(a,z)$ be the value at date t of an individual with employment state s, wealth a, and productivity z.

Consider first the problem of the passive non-participant (n_0):

$$
\begin{aligned}
v_0^{n_0}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}} \quad & \mathbb{E}_0 \int_0^{\mathfrak{t}^{n_1}} e^{-\rho t} \mathfrak{u}^n(c_t, h_t)\, dt + e^{-\rho \mathfrak{t}^{n_1}} v_{\mathfrak{t}^{n_1}}^{n_1}(a_{\mathfrak{t}^{n_1}}, z_{\mathfrak{t}^{n_1}}) \qquad (3) \\
s.t. \quad & \\
c_t + \dot{a}_t &= r_t a_t + \phi_t \\
a_t &\geq 0
\end{aligned}
$$

Passive non-participants do not receive any job opportunity. At rate $\eta^{n_0 n_1}$, with $\mathfrak{t}^{n_1}$ being

[4] We have also solved the model with a specification of period utility that is separable between consumption and hours worked, and results of all our numerical experiments are qualitatively robust and quantitatively similar. This set of results is available upon request.

[5] We fold this adjustment directly into the rate of return r_t, which should therefore be interpreted as inclusive of the rate of return θ from annuity contracts.

the first arrival rate of this event, they become active non-participants and enter employment status n_1. Alternatively, one can think of non-participants disposable income ϕ_t as being (at least partly) the result from home production activity.[6] The conditional expectation reflects the uncertainty in transition rates and in the evolution of skill dynamics. In addition to the participation decision $\mathfrak{p}_t^{n_0}$, at every instant the worker chooses its consumption flow c_t. The last two lines of this problem state the budget constraint (in real terms) and the borrowing limit.

The problem of the active non-participant (n_1) is:

$$
\begin{aligned}
v_0^{n_1}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}, \mathfrak{t}^*} \quad & \mathbb{E}_0 \Bigg[\int_0^{\mathfrak{t}^{\min}} e^{-\rho t} \mathfrak{u}^n (c_t, h_t)\, dt + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^e\}} e^{-\rho \mathfrak{t}^e} \max \left\{ v_{\mathfrak{t}^e}^e (a_{\mathfrak{t}^e}, z_{\mathfrak{t}^e}), v_{\mathfrak{t}^e}^{n_1} (a_{\mathfrak{t}^e}, z_{\mathfrak{t}^e}) \right\} \\
& + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^*\}} e^{-\rho \mathfrak{t}^*} \left(v_{\mathfrak{t}^*}^{u_0} (a_{\mathfrak{t}^*}, z_{\mathfrak{t}^*}) - \xi \right) + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^{n_0}\}} e^{-\rho \mathfrak{t}^{n_0}} v_{\mathfrak{t}^{n_0}}^{n_0} (a_{\mathfrak{t}^{n_0}}, z_{\mathfrak{t}^{n_0}}) \Bigg] \\
s.t. \quad & \\
c_t + \dot{a}_t \quad = \quad & r_t a_t + \phi_t \qquad (4) \\
a_t \quad \geq \quad & 0
\end{aligned}
$$

Active non participants receive job opportunities at rate λ_{zt}^{ne}, with $\mathfrak{t}^e$ being the first arrival time of this event. Conditional on receiving this job offer, they decide whether to accept it or not. At every instant, the non-participant chooses whether to remain unattached $\left(\mathfrak{p}_t^{n_1} = 0\right)$ or re-enter the labor force $\left(\mathfrak{p}_t^{n_1} = 1\right)$, in which case they become unemployed, but are not eligible for UI benefits ($u = u_0$). We assume that re-entering the labor force involves a small fixed switching cost ξ.[7] The optimal stopping time $\mathfrak{t}^*$ represents the first instant in which the choice $\mathfrak{p}_t^{n_1}$ switches from 0 to 1. Finally, at rate $\eta^{n_1 n_0}$ (with $\mathfrak{t}^{n_0}$ being the first arrival rate of this shock) active non-participants become passive non-participants.

[6]Nothing in our analysis would change in that case, except that the total income gains under an alternative monetary policy rule that raises participation would have to be adjusted for the loss in home production.

[7]The presence of a small switching cost is mostly a technical assumption to avoid "chattering," i.e., infinitely fast switching between n_1 and u_0, in the optimal solution of the problem. For all other participation decisions, this problem does not arise because switching back can only occur upon the realization of Poisson shocks.

The problem of an unemployed household who is not eligible for UI benefits is:

$$
\begin{aligned}
v_0^{u_0}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}, \mathfrak{t}^*} \quad & \mathbb{E}_0 \Bigg[\int_0^{\mathfrak{t}^{\min}} e^{-\rho t} \mathfrak{u}^u (c_t, h_t)\, dt + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^e\}} e^{-\rho \mathfrak{t}^e} v_{\mathfrak{t}^e}^e (a_{\mathfrak{t}^e}, z_{\mathfrak{t}^e}) \\
& + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^*\}} e^{-\rho \mathfrak{t}^*} v_{\mathfrak{t}^*}^{n_1} (a_{\mathfrak{t}^*}, z_{\mathfrak{t}^*}) + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^{n_0}\}} e^{-\rho \mathfrak{t}^{n_0}} v_{\mathfrak{t}^{n_0}}^{n_0} (a_{\mathfrak{t}^{n_0}}, z_{\mathfrak{t}^{n_0}}) \Bigg] \\
s.t. \quad & \\
c_t + \dot{a}_t \quad = \quad & r_t a_t + \phi_t \qquad (5) \\
a_t \quad \geq \quad & 0
\end{aligned}
$$

Ineligible unemployed workers receive a job opportunity at rate λ_{zt}^{ue} (with $\mathfrak{t}^e$ being the first arrival time of this event) and always take it. At any time $\mathfrak{t}^*$ during the unemployment spell, the individual can quit the labor force $\left(\mathfrak{p}_t^u = 0\right)$. Finally, at rate $\eta^{n_1 n_0}$ (with $\mathfrak{t}^{n_0}$ being the first arrival rate of this shock) they can become passive non-participants.

The problem of an unemployed household who is eligible for UI benefits is:

$$
\begin{aligned}
v_0^{u_1}(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}, \mathfrak{t}^*} \quad & \mathbb{E}_0 \Bigg[\int_0^{\mathfrak{t}^{\min}} e^{-\rho t} \mathfrak{u}^u (c_t, h_t)\, dt + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^e\}} e^{-\rho \mathfrak{t}^e} \max \left\{ v_{\mathfrak{t}^e}^e (a_{\mathfrak{t}^e}, z_{\mathfrak{t}^e}), v_{\mathfrak{t}^e}^{u_1} (a_{\mathfrak{t}^e}, z_{\mathfrak{t}^e}) \right\} \\
& + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^*\}} e^{-\rho \mathfrak{t}^*} v_{\mathfrak{t}^*}^{n_1} (a_{\mathfrak{t}^*}, z_{\mathfrak{t}^*}) + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^{u_0}\}} e^{-\rho \mathfrak{t}^{u_1}} v_{\mathfrak{t}^{u_0}}^{u_0} (a_{\mathfrak{t}^{u_0}}, z_{\mathfrak{t}^{u_0}}) \\
& + \mathbb{I}_{\{\mathfrak{t}^{\min} = \mathfrak{t}^{n_0}\}} e^{-\rho \mathfrak{t}^{n_0}} v_{\mathfrak{t}^{n_0}}^{n_0} (a_{\mathfrak{t}^{n_0}}, z_{\mathfrak{t}^{n_0}}) \Bigg] \\
s.t. \quad & \\
c_t + \dot{a}_t \quad = \quad & r_t a_t + (1 - \tau) b_t (z_t) + \phi_t \qquad (6) \\
a_t \quad \geq \quad & 0
\end{aligned}
$$

Besides receiving job opportunities and choosing whether to take them, choosing to drop out of the labor force, and exogenously switching to passive non-participant status, the eligible unemployed could lose its entitlement to UI benefit at rate $\lambda_z^{u_1 u_0}$, with $\mathfrak{t}^{u_0}$ being the first arrival time of this event.

Finally, the problem of the employed household is:

$$v_0^e(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}, \mathfrak{t}^*} \quad \mathbb{E}_0\left[\int_0^{\mathfrak{t}^{\min}} e^{-\rho t}\mathfrak{u}^e(c_t, h_t)\,dt + \mathbb{I}_{\{\mathfrak{t}^{\min}=\mathfrak{t}^u\}} e^{-\rho \mathfrak{t}^u} v_{\mathfrak{t}^u}^{u_1}(a_{\mathfrak{t}^u}, z_{\mathfrak{t}^u}) \right. \tag{7}$$

$$\left. + \mathbb{I}_{\{\mathfrak{t}^{\min}=\mathfrak{t}^*\}} e^{-\rho \mathfrak{t}^*} v_{\mathfrak{t}^*}^{n_1}(a_{\mathfrak{t}^*}, z_{\mathfrak{t}^*}) + \mathbb{I}_{\{\mathfrak{t}^{\min}=\mathfrak{t}^{n_0}\}} e^{-\rho \mathfrak{t}^{n_0}} v_{\mathfrak{t}^{n_0}}^{n_0}(a_{\mathfrak{t}^{n_0}}, z_{\mathfrak{t}^{n_0}})\right]$$

$$\begin{aligned} & s.t. \\ c_t + \dot{a}_t &= r_t a_t + (1-\tau) w_t z_t h_t + \phi_t \\ a_t &\geq 0 \end{aligned}$$

Employed workers (e) can be laid-off at rate λ_{zt}^{eu}, in which case they become eligible for UI benefits ($u = u_1$). Let $\mathfrak{t}^u$ be the first arrival time of this Poisson shock. At every instant $\mathfrak{t}^*$, the employed worker can choose to quit the labor force $\left(\mathfrak{p}_t^e = 0\right)$.[8] In addition, an employed worker can exogenously switch to passive non-participant status at rate η^{en_0}, with $\mathfrak{t}^{n_0}$ being the first arrival time of this event.

Each of these five problems can be expressed recursively as a Hamilton-Jacobi-Bellman Quasi-Variational Inequality (HJBQVI) which can, in turn, be appropriately discretized to numerically solve the household problem. See Appendix A.

Figure 1 plots the participation region as well as wealth and skill dynamics as a function of workers' individual state (a, z). The positive slope of the participation boundary indicates that workers choose to participate when they are sufficiently productive and sufficiently wealth-poor. Employed households (left panel) accumulate skills, which moves them away from the participation threshold. For a given skill level, wealth accumulation moves them closer to the threshold but, as clear from the plot, employed workers' target wealth level is well inside the participation region. A bad sequence of negative productivity shocks can still induce workers to drop out of the labor force, however. For unemployed workers (middle panel), the opposite forces are at work: wealth decumulation for consumption smoothing keeps them away from the boundary, but skill decay eventually pushes them out of the labor force. Also, recall that both employed and non-employed workers can also drop out participation exogenously through the η^{en_0} and η^{un_0} shocks.

[8] Quitting into unemployment is never optimal, because the worker would not receive UI benefits, and would pay a higher disutility cost κ^u for the opportunity to be re-employed at the same wage.

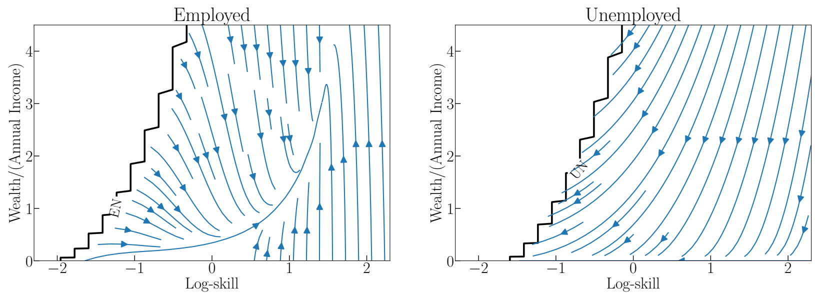


Figure 1: Participation decision for employed, unemployed without UI, and active nonparticipant worker represented in the state space of productivity and savings. The arrows indicate skill and wealth dynamics in the absence of skill shocks.

2.2 Firms

Final-goods producers. A competitive representative final-good producer aggregates a continuum of intermediate inputs indexed by $j \in [0,1]$ into aggregate output Y_t using a constant returns to scale (CRS) technology with constant elasticity of substitution across inputs $\nu > 0$. Let P_t be the price of the final good and the numeraire of the economy, and let $\pi_t = \dot{P}_t / P_t$ denote the inflation rate.

Intermediate goods producers. A continuum of measure one of monopolistically competitive firms produce the intermediate goods. Production requires hiring labor on a continuum of tasks indexed by $k \in [0,1]$. Each firm j hires labor services (efficiency-weighted hours) ℓ_{jkt} on every task k by taking the task-specific wage w_{kt} as given, combines them into a labor input ℓ_{jt} using a Dixit-Stiglitz aggregator with elasticity of substitution $\varepsilon > 0$, and produces the intermediate good according to the linear technology $y_{jt} = \alpha \ell_{jt}$.[9] We allow the government to subsidize labor hired by these firms at rate τ^{ℓ}, with the subsidy partially financed by a lump sum tax ϕ^{ℓ} levied on the same firms, in order to offset the steady-state monopolistic distortion to the quantity produced.

The optimal price setting decisions of firms yields that the relative price p_{jt}/P_t equals a markup over the marginal cost of production, in turn equal to the ratio between the after-subsidy real aggregate wage index $w_t = \left[\int_0^1 w_{kt}^{1-\varepsilon} dk\right]^{\frac{1}{1-\varepsilon}}$ and labor productivity. Because of the CRS technology, in absence of shocks to markups ν and productivity α, imposing a symmetric equilibrium ($p_{jt} = P_t$) implies that the equilibrium aggregate real wage per

[9] An alternative structure, which gives rise to the same allocations, would be to assume that there is a competitive labor intermediary that hires task-specific labor services for all tasks $k \in [0,1]$ and packages into a CES aggregate labor input sold to firms.

effective hour is constant over time, i.e.

$$w_t = \alpha \tag{8}$$

and thus, price inflation equals wage inflation at every t. Appendix B contains the full statement of the optimization problems of final and intermediate goods producers, and their solution.

2.3 Wage Setting

This block of the model adapts the wage setting mechanism of Erceg et al. (2000) —i.e., the standard New Keynesian *sticky wage* model— to an heterogeneous-agent economy. We follow closely the approach of Auclert et al. (2018, 2020a), with the necessary modifications due to our continuous time formulation and the presence of the extensive margin in labor supply.

The *nominal* wage ω_{kt} per effective hour worked on task k is set by a union that represents all workers on that particular task. By adhering to the union, each employed worker agrees to supply, at that wage, the same number of hours h_{kt} to intermediate good producers.[10] This number equals whichever total amount of effective hours is demanded by the firms for that task, ℓ_{kt}, divided uniformly across all employees' efficiency units, or $h_{kt} \int_{s_{it}=e} z_{it} di = \ell_{kt}$.[11] The union sets the nominal wage in order to maximize the welfare of its current members (all individuals employed at date t) subject to a Rotemberg-style quadratic costs of adjusting the nominal wage, in utility terms, with scaling parameter $\Theta \geq 0$. This cost is expressed in terms of deviations of nominal wage growth from the central bank's inflation target, the deterministic steady-state trend inflation rate π^*.[12]

In Appendix C we show that the solution to this problem yields the wage Phillips

[10] As detailed in Appendix C, to offset the steady-state hours distortion generated by the union monopoly power, the government sets a wage subsidy fully financed by a lump sum tax levied on households.

[11] Huo and Ríos-Rull (2020) raise a valid criticism to the RANK model featuring nominal wage rigidity. In that model, along the equilibrium path workers may end up being forced to supply hours against their will, violating the principle of voluntary exchange. They propose a resolution based on a different equilibrium concept. Here, we have a different solution: in our model, unions propose all workers an employment contract that specifies a non-negotiable pair of wage and hours, but workers can always voluntarily choose not to participate in it and, in fact, in equilibrium some do and quit employment.

[12] Our interpretation of this adjustment cost technology is therefore that wage setters can freely index nominal wage growth to π^*. They understand that inflation in the long-run always converges to that value, and they can take advantage of this information in making their wage setting plans. It is only costly for them to set a value for nominal wage growth which deviates from it. A consequence of this assumption is that changes in the inflation target have no real effects, unless the ZLB binds.

curve

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \frac{\epsilon}{\Theta}\left[h_t \int_{s_{it}=e} \partial_c u^e(c_{it}, h_t; z_{it}) z_{it} di\right]\left(\psi h_t^{\frac{1}{\sigma}} - (1-\tau)\alpha\right). \tag{9}$$

The right hand side of equation (9) indicates that whenever the marginal disutility of an extra hour exceeds the marginal utility of an extra unit of after-tax real wage income for households, each union will push up its nominal wage in order to reduce labor demand for their own type. If prices were rigid, the upward pressure on nominal wages would translate in higher real wages and a fall in labor demand. In our environment with flexible prices, and hence a constant real wage, this reduction in labor demand occurs through the monetary policy tightening in response to inflation.

We now return briefly to our preference specification (2). Note that steady-state hours per worker determined by (9), $h^* \equiv \psi^{-1}[(1-\tau)\alpha]^{\sigma}$, are equal to what each employed worker would choose if wages were set competitively in the employment island. Therefore, the union's imposition that all workers supply the same hours is consistent with worker optimization.[13]

2.4 Mutual Fund

Households wealth is invested in shares of a competitive risk-neutral mutual fund.[14] The fund owns all intermediate good firms and invests in real debt issued by the government as well as in a nominal bond in zero net supply. Holding real and nominal bonds carries a liquidity premium (or convenience yield) $\bar{\iota}$ which, as we will explain in Section 2.7, is a source of aggregate shocks. Let X_t^f denote the shares of the intermediate good producers held by the mutual fund, q_t the unit share price, Π_t per-share dividends (or profits), B_t^f the amount of real government bonds held by the fund, r_t^b the real interest rate on government bonds, M_t^f real value of the nominal bond, and r_t^m the real rate of return on the nominal bond. Let A_t be the real value of the fund and $\tilde{r}_t$ the total real rate of return of the fund.

In Appendix D, we solve the problem of the fund and show its value satisfies $A_t = q_t X_t^f + B_t^f + M_t^f$. In addition,

$$\tilde{r}_t = \frac{\Pi_t + \dot{q}_t}{q_t} = r_t^b + \bar{\iota} = r_t^m + \bar{\iota}. \tag{10}$$

These relationships determine the real return on the household financial asset a_t (wealth

[13]Under preferences displaying income effects (Auclert et al., 2018), instead, the union's equal-hour restriction gives rise to an additional distributional distortion both in and out-of steady state, relative to its representative agent counterpart.

[14]The set up in this section follows closely Alves et al. (2020).

invested in the mutual fund), and establish a no-arbitrage condition between nominal bonds, real bonds and firm equity which holds at every t, except when a shock hits the economy, in which case the price q_t features a jump. Note that, because of the absence of trading frictions or short-sale constraints, the mutual fund is willing to absorb any amount of each asset. We conclude by noting that, because of the annuity markets, the total rate of return on saving to households is $r_t = \tilde{r}_t + \varrho$.

2.5 Fiscal Authority

The fiscal authority faces the following intertemporal budget constraint:

$$G_t + \Psi_t^{\ell} + \phi_t + (1-\tau)\int_{s_{it}=u^1} b_t(z_{it})di + r_t^b B_t^g = \tau w_t h_t \int_{s_{it}=e} z_{it} di + \dot{B}_t^g \tag{11}$$

where $\Psi_t^{\ell} = \tau^{\ell} w_t \ell_t - \phi^{\ell}$ is the net subsidy which undoes the monopolistic distortions of intermediate producers. Finally, government purchases of the final good G_t outside of steady-state follow the policy rule:

$$G_t = G^* - \beta_B(B_t^g - B^*), \quad \beta_B > 0 \tag{12}$$

where the superscript * denotes steady-state values. Thus, following an aggregate shock, debt adjusts to satisfy the government budget constraint, and government expenditures respond to deviations of debt from its steady-state level to keep debt from growing or shrinking too quickly.

2.6 Monetary Authority

In our baseline, the monetary authority sets the nominal interest rate according to an Inflation Targeting (IT) rule that reacts to deviations of inflation and unemployment rate from their targets with some inertia. If we let i_t denote the shadow policy instrument not subject to the ZLB and i^* the steady-state nominal rate, then the IT rule is defined as

$$\frac{di_t}{dt} = \begin{cases} -\beta_i(i_t - i^* - \mathcal{R}_t) & \text{if } i_t > 0 \\ \max\left\{ -\beta_i(i_t - i^* - \mathcal{R}_t),\ 0\right\} & \text{if } i_t = 0 \end{cases}$$

where the reaction function to inflation and unemployment is

$$\mathcal{R}_t = \beta_\pi(\pi_t - \pi^*) + \beta_u(u_t - u^*). \tag{13}$$

The coefficients $\beta_\pi > 1$ and $\beta_u \leq 0$ capture the strength of the policy response to deviations of inflation from target π^* and of unemployment from its steady-state value u^*, while $\beta_i > 0$ measures the degree of interest rate smoothing. The monetary authority is constrained by a zero lower bound (ZLB) on nominal rates which forces i_t to be weakly above zero at all times.

Finally, a standard Fisher equation states that the real financial return on the nominal bond must equal the nominal policy rate minus inflation at every t, or $r_t^m = i_t - \pi_t$.

2.7 Sources of Aggregate Fluctuations

Aggregate dynamics in our economy are driven by two shocks denoted by ζ_t^k, labeled demand ($k = d$) and supply ($k = s$) shocks. The demand shock ζ^d is a perturbation to the bonds' liquidity premium $\bar{\iota}$ which shows up as a time-varying wedge between the rate of return on equity and the return on bonds in the no-arbitrage equation (10)

$$\tilde{r}_t = \frac{\Pi_t + \dot{q}_t}{q_t} = r_t^b + \bar{\iota} + \zeta_t^d = r_t^m + \bar{\iota} + \zeta_t^d. \tag{14}$$

An increase in ζ^d reduces the value of equity relative to bonds and raises the rate of return to household wealth. These two forces contract household demand for consumption of the final good and firms' demand for labor to produce it. Facing lower demand for their labor task, unions lower nominal wages which pushes down inflation. Thus, output, hours, and inflation comove positively under this shock, which is why we label it a demand shock.

The supply shock ζ^s is a disturbance to the union's desired markup $\varepsilon/(\varepsilon - 1)$. As clear from the wage Phillips curve derivation (C9), the shock shows up as a wedge in (9), whose linearized version (recall that the real wage is constant) becomes

$$\rho(\pi_t - \pi^*) - \dot{\pi}_t = \theta_w\left[\frac{1}{\sigma}(\log h_t - \log h^*) + \zeta_t^s\right]. \tag{15}$$

where $\theta_w \equiv \frac{\epsilon}{\Theta}\left[\int_{s_{it}=e} \partial_c u^e(c_{it}, h_t; z_{it}) z_{it} di\right] \psi(h^*)^{\frac{1+\sigma}{\sigma}}$. A positive shock to unions' desired markup ($\zeta_t^s > 0$) raises nominal wages, which puts upward pressure on inflation. A central bank following (13) reacts to the inflation by raising real rates, which lowers household demand for consumption of the final good and thus firm demand for hours. Thus, output and hours comove negatively with inflation under this shock, which is why we label it a supply shock.

Our two disturbances are akin, respectively, to a "risk-premium" shock and a "cost-push" shock, both common in the literature on estimated New Keynesian DSGE models, and routinely found to contribute significantly to the overall fluctuations of total hours worked in the US economy (see, e.g. Smets and Wouters, 2007; Galí et al., 2012).

It only remains to specify how labor market frictions $\lambda_{zt}^{ss'}$ adjust out of steady state. Because employment and hours per worker are tightly correlated in the data, we posit that job offer arrival and separation rates shift with fluctuations in average hours per worker h_t. Specifically, we assume that

$$\log\left(\lambda_{zt}^{ss'}\right) = \log\left(\lambda_z^{*\,ss'}\right) + \vartheta^{ss'} \log\left(\frac{h_t}{h^*}\right), \quad \text{for } ss' \in \{eu, ue, ne\} \tag{16}$$

where $\vartheta^{ss'}$ determines the elasticity with respect to hours.[15]

To understand the distortions caused by aggregate fluctuations in our model, and, thus, the potential scope for monetary stabilization policy, it is useful to consider a benchmark version of our economy where nominal wages are flexible and set competitively. This configuration corresponds to a limit case of our economy where the wage Phillips curve is vertical ($\theta_w \to \infty$) and wage markup shocks are absent ($\zeta_t^s = 0$ for all t). In this benchmark, employed workers are always on their labor supply curves for hours, which specifies a constant number of hours worked h^* independently of the realization of demand shocks. Since h^* coincides with hours worked at our non-stochastic steady state, this condition also translates into a roughly constant unemployment rate, equal to its steady-state value.[16]

[15] Under this specification, the ratio $\lambda_{zt}^{ss'}/\lambda_z^{*\,ss'}$ is the same for all z, i.e. aggregate shocks shift frictions proportionately along the workers' skill distribution. This proportional shift, however, implies changes to unemployment that are *not* proportional in level. To understand this property, consider the expression for steady-state unemployment in a simple 2-state model: $u_z = \frac{\lambda_z^{eu}}{\lambda_z^{eu}+\lambda_z^{ue}}$. Differentiating with respect to the log of the separation rate $\frac{\partial u_z}{\partial(\lambda_z^{eu}/\lambda_z^{eu})}$ (and similarly when we differentiate with respect to the job finding rate) yields $u_z(1-u_z)$. As long as the unemployment rate is below 0.5 (an upper bound that is never reached for any plausible model calibration), the response of unemployment rate to a given percentage change in frictions is increasing in the level of steady-state unemployment u_z. In the data, and in the calibrated model, low z workers have higher unemployment.

[16] Unemployment in our model, as it is the case in the data, moves primarily in response to fluctuations in job-separation and job-finding rates. A constant level of hours leads, through (16), to a constant job-separation and finding rate and, thus, constant unemployment.

By targeting steady-state inflation and unemployment, the Inflation Targeting rule (13) acts to minimize deviations of the economy from this benchmark allocation.[17] However, the presence of the ZLB generates another dimension to the central bank's stabilization objectives. As we show below, recessions that trigger the ZLB in our model generate long-lasting scars on labor force attachment and productivity, especially among workers at the bottom of the distribution. In this scenario, a policy that deviates from strict inflation and unemployment stabilization during expansions (e.g., a policy that lets the economy run hot during expansions) has the benefit of allowing workers to re-enter participation, regain employment and recuperate the earnings losses they endured in the recession.

2.8 Equilibrium

We now formally state the definition of a perfect foresight equilibrium for our economy, which is the relevant one for all our experiments given our solution method outlined in Section 2.9.[18]

For any given initial distribution of households $\mu_0^s(a,z)$ and shock realization ζ_0^k, a perfect foresight equilibrium is defined as time paths for household consumption, participation and job offer acceptance decisions $\{c_t^s(a,z), \mathfrak{p}_t^s(a,z), \mathfrak{j}_t^s(a,z)\}_{t\geq 0}$, unions' nominal wage setting $\{\omega_{kt}\}_{t\geq 0}$, intermediate producers' hiring decisions $\{\ell_{kt}\}_{t\geq 0}$, mutual fund allocations $\left\{X_t^f, B_t^f, M_t^f\right\}_{t\geq 0}$, real rates of return on the mutual fund, real and nominal government bonds $\left\{\tilde{r}_t, r_t^b, r_t^m\right\}_{t\geq 0}$, firms' share price $\{q_t\}_{t\geq 0}$, fiscal variables (transfers, UI benefits, expenditures, and debt) $\left\{\phi_t, b_t(z), G_t, B_t^g\right\}_{t\geq 0}$, nominal interest rates $\{i_t\}_{t\geq 0}$, aggregate output, consumption, profits, contractual hours worked, and inflation $\{Y_t, C_t, \Pi_t, h_t, \pi_t\}_{t\geq 0}$, and measures of households $\{\mu_t^s(a,z)\}_{t\geq 0}$ such that at every t: (i) households solve problems (3)-(7); (ii) final good and intermediate good producers solve (B2) and (B4), respectively; (iii) unions solve (C6) and inflation satisfies the Phillips curve in (9) (iv) the mutual fund solves (D1); (v) the government budget constraint (11) holds; (vi) the fiscal and monetary authorities follow their policy rules (12) and (13); (vii) aggregate profits are given by (B8); (viii) the sequence of distributions satisfies aggregate consistency conditions, (ix) asset markets clear, and (x) all goods markets clear.

There are four asset markets in our economy: the intermediate firms' shares market,

[17] In response to a demand shock, movements in inflation and unemployment elicit the same response from the inflation targeting rule, which pushes the economy towards steady state. In response to a supply shock instead, this rule trades off stabilizing unemployment versus minimizing inflation deviations from target.

[18] A stationary equilibrium (or steady-state) is a particular case of our definition where all decisions, prices, aggregate variables, and distributions are constant over time.

the two government bond markets, and the mutual fund shares market which clear when, respectively,

$$\begin{aligned} X_t^f &= 1 \\ B_t^f &= B_t^g \\ M_t^f &= 0 \\ \sum_{s\in\{e,u,n\}} \int a_t d\mu_t^s &= A_t = q_t + B_t^g \end{aligned}$$

where, without loss of generality, we normalized the measure of firms' shares to 1. These equations, together with the no-arbitrage conditions (10), determine firm share prices and real interest rates. Market clearing in the continuum of intermediate goods' markets requires

$$y_{jt} = Y_t, \text{ for all } j$$

and the final good market clears by Walras law. Given the set of employed workers, average hours per worker h_t required by final good producers is determined by

$$Y_t = \alpha \int_{s_{it}=e} h_t z_{it} di.$$

Lastly, we note that the labor market is frictional and the evolution of the distribution of employed, unemployed, and non participants is implicit in the KFEs for μ_t^s.[19]

2.9 Numerical Solution, Simulation and Asymmetric Rules

We solve for the stationary equilibrium of our model using standard continuous-time finite-difference methods as described in Achdou et al. (2017) and compute the economy's linearized response to aggregate shocks using the sequence-space approach of Auclert et al. (2021). To simulate the ergodic distribution of our economy, we extend the procedure in Boppart et al. (2017) to incorporate an occasionally binding ZLB, as in Holden (2016). This procedure maintains the model's linearity with respect to aggregate shocks, but allows for nonlinearities coming from the ZLB. Finally, we compute our policy counterfactuals following Hebden and Winkler (2024) and McKay and Wolf (2023). The idea is to use monetary policy "news" shocks, computed under the baseline IT rule, to construct

[19]We report the KFE for the distributions μ^s in Appendix E.

the economy's counterfactual trajectory under alternative asymmetric monetary policy rules. We leave the details of the solution method to Appendix F.

3 Parameterization

We organize our discussion of the parameterization in three parts. Section 3.1 explains our calibration strategy for parameters which determine the model's stationary equilibrium, and Section 3.2 does it for parameters which determine out-of-steady-state dynamics. Section 3.3 validates our calibration along a series of micro and macro untargeted moments.

Overall, our calibration targets a wide set of moments of the earnings and employment distribution in order to discipline the strength of Okun's channels (exposure, attachment, and persistence). Specifically, the model replicates (i) the level and cyclicality of labor market stocks and flows, (ii) the dynamics of unemployment and participation at the bottom of the skill distribution, and (iii) statistics of earnings losses upon displacement and earnings growth over the life cycle. Table 2 summarizes the model parameter values. Table 3 reports data and model's values of the targeted moments.

3.1 Parameters Determining the Steady State

Some parameters are set externally, and others internally to match a set of targeted moments. Generally, the internally calibrated parameters affect all targeted moments, but some moments are more informative for a subset of the parameters than others. Our presentation and discussion of the calibration strategy reflects this logic. The model period is set to one month.

Demographics and Preferences. We set the monthly mortality rate ϱ so that workers' average lifespan is 36 years (25 to 60). The Frisch elasticity on the intensive margin of labor supply is set to $\sigma = 1$. Working entails a variable and a fixed cost. The variable disutility parameter ψ is set so that $h^* = 1$ satisfies (9) in steady state. The flow utility of non-participation κ^n is normalized to zero. The fixed costs of searching κ^u and working κ^e , which regulate workers' desire to participate in the labor market, are set jointly with frictional parameters to match labor market moments as we discuss below.[20]

The effective discount rate ρ is set to target a ratio of mean wealth to annual earnings of 0.56 under a real interest rate of 3%. This corresponds to the amount of liquid wealth among US households immediately available for consumption smoothing (Kaplan and

[20] The switching cost ξ is set to a very small number to make the optimal stopping problem well behaved.

Steady State		
Parameter		Value
Demographics and Preferences		
Death rate	ϱ	$1/432$
Effective discount rate	ρ	0.0053
Labor supply elasticity	σ	1.00
Utility weight on hours	ψ	1.00
Disutility of nonparticipation	κ^n	0
Disutility of searching	κ^u	0.33
Disutility of working	κ^e	1.05
Productivity process		
Mean of initial skill distribution	$\bar{z}_0$	$\log(0.68) - \sigma_{z_0}^2/2$
S.D. of initial skill distribution	σ_{z_0}	0.50
Skill mean reversion	γ_z	0.0017
Skill drift while employed	δ_z^+	0.0024
Skill drift while non-employed	δ_z^-	0.0214
Skill diffusion coefficient	σ_z	0.0467
Labor market frictions		
Job-separation rate out of E	λ_z^{eu}	Appx. G
Job-finding rate out of U	λ_z^{ue}	Appx. G
Job-finding rate out of N	λ_z^{ne}	Appx. G
Passive nonparticipation rate during E	η^{en_0}	0.0075
Passive nonparticipation rate during U/N	$\eta^{un_0}, \eta^{n_1 n_0}$	0.0750
Passive nonparticipation exit rate	$\eta^{n_0 n_1}$	0.2500
Technology		
Firm productivity	α	1.25
Elast. of subst. between goods	ν	10
Elast. of subst. between tasks	ε	10
Liquidity premium	ι	0.001
Taxes, transfers and expenditures		
Government debt	B^g	1.69
UI replacement rate	$\bar{b}$	0.50
UI expiration rate	$\lambda^{u_1 u_0}$	0.167
Labor income tax rate	τ	0.20
Lump-sum transfer	ϕ	0.068
Government expenditures	G	0.17
Intermediate firms labor subsidy	τ^ℓ	0.10
Intermediate firms net subsidy	Ψ^ℓ	0.001

Out of Steady State		
Parameter		Value
Monetary and Fiscal Policy		
Trend inflation	π^*	$0.02/12$
Taylor rule persistence	β_i	0.07
Taylor rule reaction to inflation	β_π	2.25
Taylor rule reaction to unemployment rate	β_u	-0.15
Government expenditures response to debt	β_B	0.10
Phillips Curve		
Slope of the wage Phillips curve	θ_w	0.01
Aggregate Fluctuations		
Demand shock drift	γ_d	0.0200
Demand shock diffusion	σ_d	0.0001
Supply shock drift	γ_s	0.0200
Supply shock diffusion	σ_s	0.0008
Elasticity of (λ_z^{eu}) to hours	ϑ^{eu}	-11.00
Elasticity of ($\lambda_z^{ue}, \lambda_z^{ne}$) to hours	$\vartheta^{ue}, \vartheta^{ne}$	24.00

Table 2: Model parameter values expressed in monthly frequency. See Section 3 in the main text for a discussion of parameter choices and targets.

Violante, 2022). In equilibrium, total savings by households must be equal to the value of the mutual fund, which includes the value of government bonds and firm equity. We set government debt B^g to be 1/4 of total equity (2019 Flow of Funds, Table B.101.h Balance Sheet of Households) and adjust the value of the net subsidy Ψ^ℓ, which determines aggregate profits, so that the value of equity held by the fund, plus the value of bonds, equals workers' demand for savings.[21]

Skill Dynamics. The mean of the initial skill distribution $\bar{z}_0$ is set so that the average skill in levels $\exp\left(\bar{z}_0 + 0.50\sigma_{z_0}^2\right)$ equals 0.68, reflecting the average wage of age group 23-27 relative to the pool of all workers in the 2019 Current Population Survey (CPS) (Heathcote et al., 2023). Its dispersion σ_{z_0} is set to match the group's P90-P50 hourly wage ratio of 2.00.

[21] An alternative way of interpreting this calibration strategy is to say is that we choose Ψ^ℓ and B^g so that, given the household demand curve, the annual real interest rate that clears the asset market is 3%.

The skill diffusion σ_z is set to match the P90-P50 wage ratio for the 2019 CPS.[22] The mean reversion γ_z is set to 0.0017, corresponding to an annual autocorrelation of $\exp(-12\times\gamma_z) = 0.98$, while the positive δ_z^+ and negative δ_z^- drifts are chosen to match (i) the average worker log earnings growth between ages 25 and 60, and (ii) the average earnings losses of laid-off workers 10 years after separation as computed in Davis and Von Wachter (2011).[23] Table 3 shows that our calibration does a good job matching these targets with earnings losses 10-years after displacement falling at the low end of the range of estimates in Davis and Von Wachter (2011). Like in the data, earnings losses in the model are cyclical and larger during recessions.

Labor Market Frictions, Nonparticipation Shocks and Participation Costs. We use flows computed from the Basic Monthly CPS, in the aggregate and across the weekly earnings distribution (our proxy for skills), to discipline search frictions, passive nonparticipation shocks, and participation costs in the model. See Appendix G for a description of the data and more details on our calibration strategy. Below, we discuss the driving parameters behind each of the flows in the model and highlight how we use the data on transitions to pin them down.

EU and UE flows for each skill level z are linked to the job-separation λ_z^{eu} and job-finding λ_z^{ue} rates: all EU transitions are forced separations from employment, while UE transitions are the outcome of an accepted job offer out of unemployment. Given this tight relation between search frictions and the measured flows, we calibrate λ_z^{eu} and λ_z^{ue} to match the EU and UE flows across workers' skill distribution.

EN and UN flows occur either because workers suddenly find working or searching too costly given their skills (e.g., following a bad skill shock) or because of an exogenous passive nonparticipation shock η^{sn_0} for $s = e, u$. We use information along the skill distribution to distinguish between these two motives. Specifically, we set η^{en_0} and η^{un_0} to match the empirical transition rates among high-skilled workers—who, otherwise, rarely exit the labor force in the model—and rely on the average EN and UN to inform the fixed costs κ^e and κ^u regulating participation decisions.[24]

[22] We target the P90-P50 ratio—here and for the initial skill dispersion—because wage variation at the top of the distribution is more directly tied with skill variation, whereas the extensive margin of labor supply plays a bigger role at the bottom of the distribution.

[23] We compute average earnings losses upon displacement in the model by comparing the earnings profile of a random sample of employed workers displaced at time zero against the a control group who does not separate. As in Davis and Von Wachter (2011), we restrict our sample in the treatment and control groups to high-tenure workers (i.e., workers with expected tenure at the job of at least 3 years), condition the inclusion in the control group to workers that remain employed for the first year of the simulation, and keep workers with zero earnings.

[24] As for $\eta^{n_1 n_0}$, we assume that nonparticipants and unemployed workers transition into inactive nonpar-

Finally, NE and NU flows arise from workers in nonparticipation who would like to move back into employment. Besides the parameters determining workers' incentives to participate, two other parameters regulate workers' ability to rejoin employment: the exit rate from passive nonparticipation $\eta^{n_0 n_1}$ and the job-finding out nonparticipation λ_z^{ne}.[25]

The model matches the average flows reasonably well (see Table 3). In particular, the model generates a large attachment wedge between unemployed and employed workers (i.e., $EN >> UN$), which, as we argue in the introduction, is essential to capture the strong 'participation cycle' present in the data.

Technology. Firm productivity α is set to normalize after-tax hourly wage per efficiency units $(1-\tau)w^*$ to 1 in steady state. The elasticities of substitution across labor types (ε) and across intermediate goods (ν) are set to 10, which imply wage and price markups around 10 percent. Finally, we set the mutual fund's "liquidity premium" of holding bonds ι to 10bp so that the steady-state (annual) nominal interest rate $i^* = r^m + \pi^*$ is equal to 1%.[26]

Steady-State Fiscal Policy. We assume that unemployment benefits are given by $b_t(z_{it}) = \bar{b}\ w_t h_t z_{it}$, and set the UI replacement rate $\bar{b}$ to 0.5 of individual earnings. We make $\lambda_z^{u_1 u_0}$ constant across skill z and equal to 0.167 to reflect an average UI benefits duration of 6 months. The proportional tax rate τ is set to 0.2 and the lump-sum transfer ϕ is calibrated to match 6% of average earnings in steady-state.[27] The subsidy to labor hired by intermediate firms is set to $\tau^{\ell} = 1/\nu$ in order to undo the distortion caused by monopolistic competition. Government expenditures are set residually to satisfy the government budget constraint in steady state.

ticipation at the same rate, i.e., we set $\eta^{n_1 n_0}$ equal to $\eta^{u n_0}$.

[25]As we discuss in Appendix G, participation decisions and time aggregation create a wedge between the job-finding out of nonparticipation λ_z^{ne} and NE flows across the skill distribution. Since this wedge can vary across the skill distribution, we don't rely on NE to capture the shape of λ_z^{ne}. Instead, we assume that λ_z^{ne} shares the same shape across z as λ_z^{eu}, i.e., $\lambda_{z_1}^{ne}/\lambda_{z_2}^{ne} = \lambda_{z_1}^{ue}/\lambda_{z_2}^{ue}$ for any skill levels z_1 and z_2.

[26]The main role of the "liquidity premium" parameter is to vary how frequently the ZLB binds in our simulations without affecting the calibration of the stationary equilibrium.

[27]This number is obtained by dividing Government Social Benefits by Wages and Salaries. Transfers are computed as: Workers' compensation, SNAP, Supplemental security income, Refundable tax credits, Temporary disability insurance, Workers' compensation, Family assistance, General assistance, Energy assistance, Employment and training, Other benefits, and 0.4*Medicaid (Table 3.12 of NIPA). Wages and salaries are taken from Table 2.1 of NIPA for 2019. The share of Medicaid expenditures that are effective transfers to households (0.4) is obtained from Finkelstein et al. (2019).

Table 3: Targeted moments.

Stationary Equilibrium	Data	Model		Data	Model
Liquid wealth to annual earnings[1]	0.560	0.5756	Employment[6]	0.764	0.7316
Lump-sum transfer to total earnings[2]	0.060	0.0549	Unemployment rate[6]	0.055	0.0621
90-50 wage ratio (entrants)[3]	2.000	1.9534	EU[6]	0.013	0.0131
90-50 wage ratio (all workers)[3]	3.000	3.0424	UE[6]	0.248	0.2603
55/25 log earnings difference[4]	0.700	0.6840	NE[6]	0.069	0.0227
10-Year earnings losses upon displacement[5]	[-0.15,-0.10]	-0.0963	EN[6]	0.017	0.0095
			UN[6]	0.133	0.0900
			NU[6]	0.027	0.0273
Out of Steady State	**Data**	**Model**		**Data**	**Model**
Standard deviation of total hours	3.45	3.43	$Cov(E_t, H_t)/Var(H_t)$	0.70	0.71
Inflation and unemployment correlation	-0.32	-0.34	$SD(UE_t)/SD(EU_t)$	1.15	1.14

Stationary Equilibrium. [1]Kaplan and Violante (2022); [2]Authors calculations from Table 3.12 and 2.1 of NIPA for 2019; [3]Heathcote, Perri, Violante, and Zhang (2023); [4]Difference in mean log income at ages 55 and 25 for the US as reported in the Global Repository of Income Dynamics (GRID); [5]Davis and Von Wachter (2011) 10-year ahead earnings losses as a percent of predisplacement annual earnings during recessions (-0.15) and expansions (-0.10); [6]Employment, unemployment, labor force participation and all six gross worker flows are computed directly from the CPS (see Appendix G for a detailed description of our data and sample selection).

Out of Steady State. Data is quarterly from 1989 to 2019. Total hours H and hours per worker h are computed from the BLS-CES. To calibrate the correlation of inflation and unemployment and inflation, we consider data from 1995 to 2019. UE and EU flows are quarterly averages of monthly data computed directly from the CPS. All series are in log and are detrended using a linear trend. The model based statistics are simulated at quarterly frequency to match the frequency of the data. See Appendix F for a description of the simulation procedure.

3.2 Parameters Determining Out of Steady State Dynamics

Monetary and Fiscal Policy. We assume a steady-state (trend) inflation rate π^* of 2%. We set the interest rate smoothing to $\beta_i = 0.07$, corresponding to a quarterly persistence of 0.81. Values for the coefficient on inflation $\beta_\pi = 2.25$ and the coefficient on the unemployment gap $\beta_u = -0.15$ are consistent with estimates from Bayer et al. (2020). We set β_B in our fiscal rule (12) to 0.1 as in Auclert et al. (2020b).

Slope of the Phillips curve. The slope of the linearized Phillips curve θ_w is set to match the peak response of inflation to high-frequency identified monetary policy shocks as reported in McKay and Wolf (2022), i.e., approximately 30bp for a 100bp change in the nominal rate.

Aggregate Fluctuations. The aggregate demand and supply shocks ζ_t^d and ζ_t^s both follow an Ornstein-Uhlenbeck diffusion process

$$d\zeta_t^k = -\gamma_k \zeta_t^k dt + \sigma_k d\mathcal{W}_t^k, \tag{17}$$

where $d\mathcal{W}_t^k$ is a standard Wiener process. For both shocks we set γ_k to 0.02, corresponding to a monthly persistence of 0.98. The values of the demand and supply diffusion coefficients (σ_d, σ_s) are set to match the correlation between unemployment rate and inflation (-0.32) and the quarterly standard deviation of total hours $H_t = E_t \times h_t$ (3.45). The elasticities of frictions to hours ($\vartheta^{eu}, \vartheta^{ue}$) are chosen to replicate the share of the variance in total hours coming from fluctuations in the extensive margin of employment (70%) and the relative volatility of *eu* and *ue* flows (1.15).[28] The bottom of Table 3 reports the model's performance along these business cycle targeted moments.

3.3 Untargeted Labor Market Moments

In this section we explore the ability of our model to match some other important, but untargeted, empirical patterns of workers' earnings and employment dynamics. The model and data moments are shown in Table H1 in the Appendix.

Marginal Propensity to Earn. Our parameterization is consistent with recent micro evidence on the impact of lottery wins on worker's labor supply with an annual marginal propensity to earn (the dollar reduction in earnings for an additional dollar of non-earned income) around -0.02 (Golosov et al., 2021).

Participation and Unemployment for Low-skilled Workers Low-skilled workers in our model experience lower participation and higher unemployment rates, as in the data. Participation and unemployment rates among workers at the bottom tercile of the skill distribution are, respectively, 41% and 12% in the model against 43% and 15% in our data.

Earnings Dynamics. Individual earnings in our model arise from the interplay between an exogenous process for skill dynamics (which depends on workers' employment status), job finding and separation rates (which depends on workers' skills), and an endogenous participation decision (which depends both on workers' wealth and skill levels). This rich interaction allows our model to jointly match the large cross-sectional dispersion of log-earnings changes (0.437 in the model against 0.51 in the data) and the rise of earnings inequality over the life cycle (from 0.53 at age 25 to 0.977 at age 55 in the model, against 0.595 to 0.905 in the data, respectively).[29] Moreover, we can also reproduce, at least qualitatively, the higher-order moments of the (log) earning growth distribution—a

[28] Keeping with the steady-state calibration strategy, we set the elasticity of λ_{zt}^{ne} with respect to hours equal to the elasticity of λ_{zt}^{ue}, or $\vartheta^{ne} = \vartheta^{ue}$.

[29] As demonstrated by Heathcote et al. (2010), the traditional linear Gaussian model for earnings dynamics struggles to jointly match these two facts.

negative skewness and a high kurtosis—even though these patterns are not as stark as in the data.

Earnings Losses Across the Distribution. The earnings losses due to long nonemployment spells (full-year of nonemployment) in the model are also comparable to evidence documented by Guvenen et al. (2017) across the worker recent earnings distribution. Comparing future earnings of workers experiencing a full-year of nonemployment against a control group that is employed at least part of the year, we find 10-year ahead losses of 45% (27%) at the 25th (75th) percentile of the recent earnings distribution, close to the 50% (30%) losses found by (Guvenen et al., 2017, Figure 1).[30] Moreover, consistent with what the authors find, these larger earnings losses at the bottom are mostly due to a stronger adjustment at the extensive margin. In the model, a full-nonemployment in an year is associated with a 18 ppt higher nonemployment rate 10 years ahead at the 25th percentile of the recent earnings distribution, compared to an increase of only 3 ppt at the 75th percentile.

Business Cycle Properties of Stocks and Flows. The volatility and cyclicality of employment, unemployment, and labor force participation match very well the business cycle propreties in the data.[31] Notably, our calibration also yields the right cyclicality for all six labor market flows.

4 Economic Outcomes Under Inflation Targeting

We now analyze the aggregate and distributional outcomes of our economy when monetary policy follows a traditional Inflation Targeting (IT) rule as specified in equation (13). We proceed in three steps. We first study aggregate and distributional impacts of small demand and supply shocks for which the ZLB is not binding (Section 4.1). Next, we turn to the case of a large negative demand shock that triggers the ZLB (Section 4.2). Last, we simulate the model under demand and supply shocks to recover the ZLB induced biases in the ergodic distribution (Section 4.3).

Jointly, the key message of this section is that the presence of Okun's channels (exposure, attachment, and persistence) give rise to a form of negative labor market hysteresis for low-skill workers that is severely amplified by the ZLB when the monetary authority follows an IT rule.

[30] For this exercise, we follow Guvenen et al. (2017) in defining a worker as nonemployed in a year t if their annual earnings fall below a minimum earnings threshold.

[31] Employment fluctuations in our model are, just as in the data, a combination of a strong countercyclical reaction in the unemployment rate together with a weaker, but procycylical, response in participation.

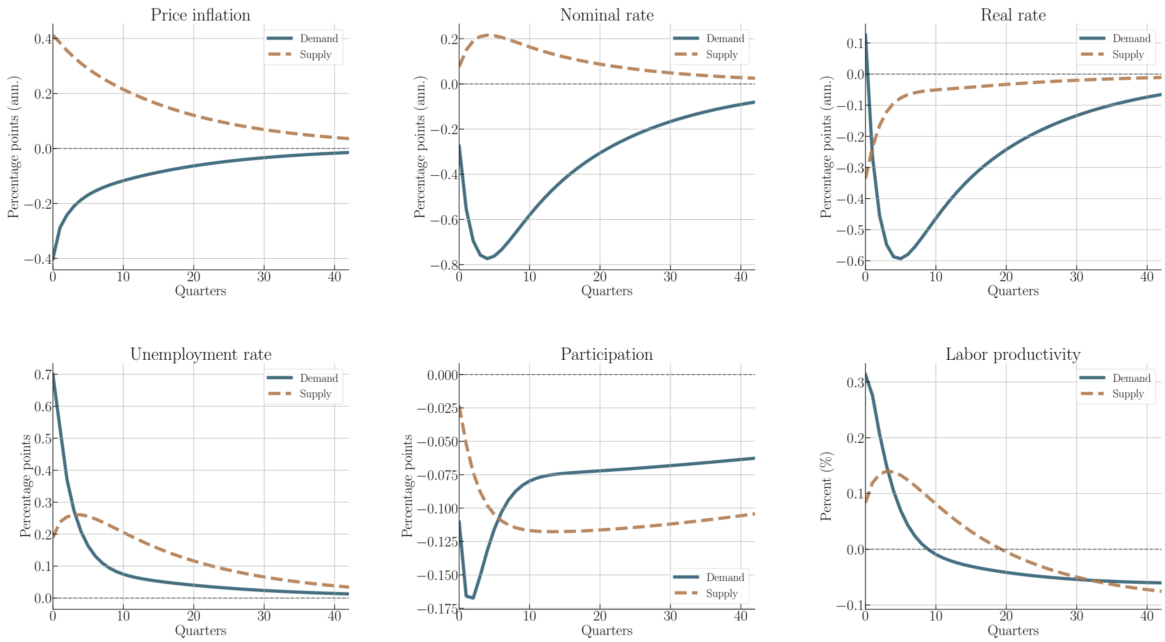


Figure 2: Impulse response of aggregate variables to a demand and supply shocks.

4.1 Shocks that Do Not Trigger the ZLB

4.1.1 Aggregate Implications

The impulse response functions (IRFs) to aggregate demand and supply contractionary shocks are illustrated in Figure 2. The responses of inflation, unemployment and nominal rates are all standard. Importantly, their responses die out as the driving demand and supply shocks dissipate from the economy. In contrast, participation and labor productivity remain depressed for much longer.

The mechanism behind this hysteresis is that an increase in unemployment, even if transitory, triggers adverse long-lasting effects on productivity and participation. First, skill losses during non-employment recover only slowly when workers regain their jobs and drive down labor productivity persistently.[32] Second, lower skills also make job opportunities less likely to arrive and push down potential wages, driving marginal workers out of participation. It is through these two channels that hysteresis arises in our model.

[32] Labor productivity displays a short-lived increase due to a selection effect—as low-productivity workers are more exposed to layoffs, a spike in separations shifts the composition of employed workers toward high-productivity individuals.

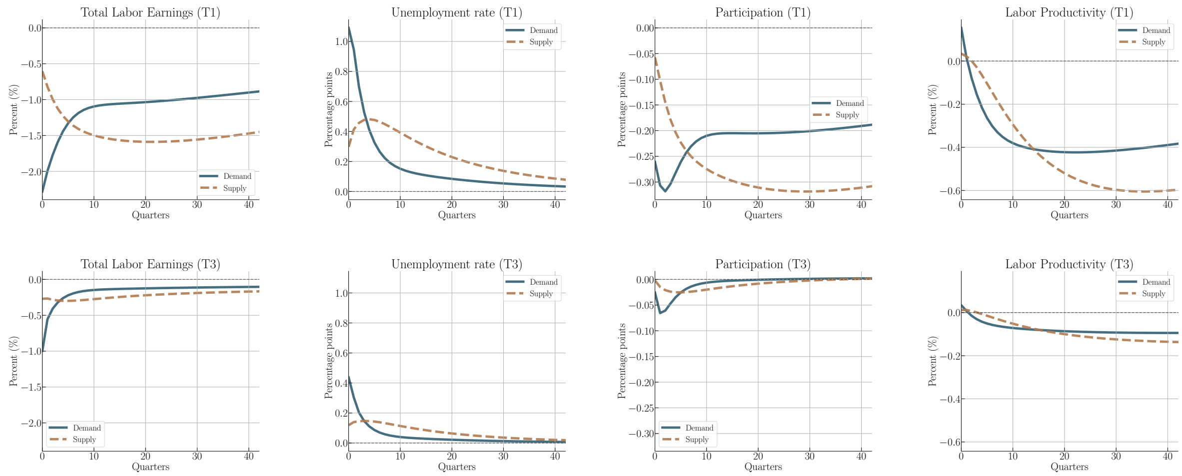


Figure 3: Impulse response of labor market outcomes for the bottom (T1) and top (T3) skill terciles to a demand and supply shocks.

4.1.2 Distributional Implications

To highlight the heterogeneous effects across the distribution, Figure 3 plots the impact of the shocks at the bottom and top terciles of the skill distribution. We choose terciles because they roughly correspond to the share of workers with a high-school degree or less, those with some college education, and those with a college degree or more.[33]

The first column of Figure 3 shows that the impact of recessions on total earnings is very different at the bottom and top of workers' skill distribution. Take the case of the demand shock, for instance. The reduction in earnings at the bottom (first row) is not only more prominent than at the top (second row), but also significantly more persistent—10 years after the shock, total earnings at the bottom are still depressed by roughly half of the initial year impact, while earnings at the top have all but recovered.

To better understand the sources of this earnings gap response, we rely on a simple decomposition of total labor earnings for each group g

$$W_t^g = wh_t \times (1 - u_t^g) \times P_t^g \times Z_t^{e,g} \tag{18}$$

where $(1 - u_t^g)$ is the employment rate, P_t^g is labor force participation, and $Z_t^{e,g}$ is average labor productivity at time t, all three conditional on skill group g. Intuitively, each component captures one of the Okun's channels of exposure, attachment, and persistence.

[33] As we showed in the calibration, our model does a good job at capturing the participation and unemployment rates for the bottom tercile of the skill distribution.

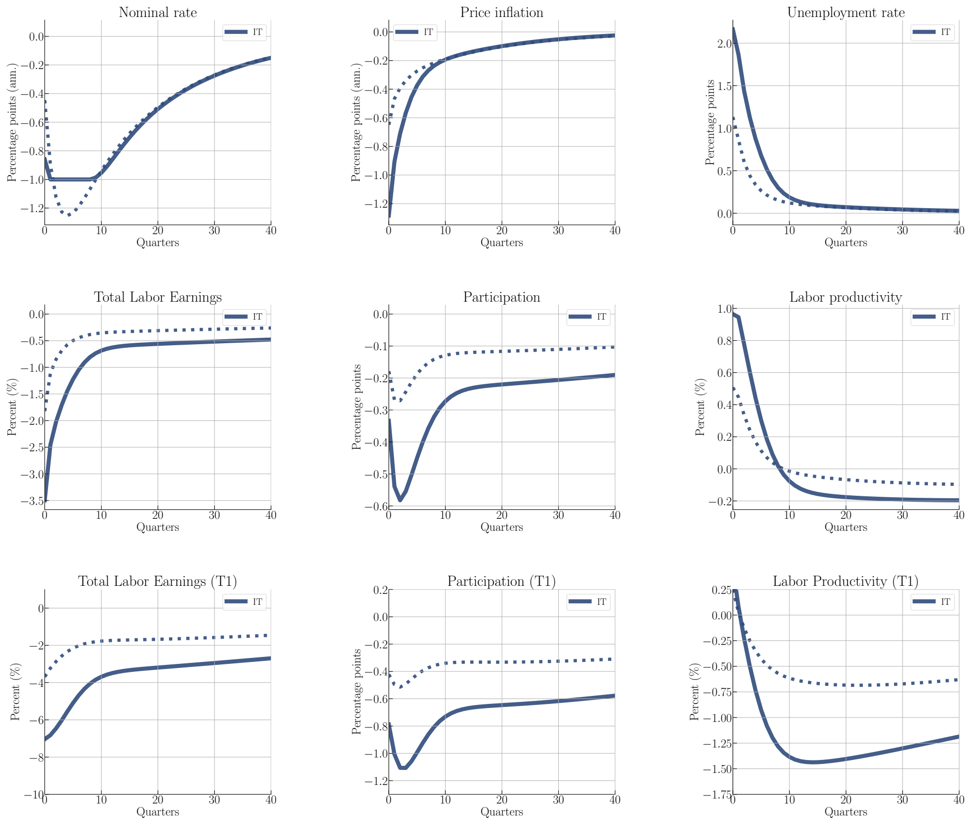


Figure 4: Impulse response to a large demand shock that triggers the ZLB constraint.

Consistent with the dynamics of total earnings, the responses of all three channels at the top of the distribution are small and tend to die out relatively quickly—labor productivity displays a persistent decline even at the top of the distribution, but notice that this effect is relatively small compared to the bottom. In contrast, low-skilled workers are 2-3 times more exposed to the rise in unemployment. Through the mechanism we described above, this stronger rise in unemployment at the bottom triggers larger losses in attachment and productivity, which, over time, reinforce each other and explain why we find much more persistent earnings losses for this group. In fact, weaker productivity and participation account for almost all of the long-term (40 quarters ahead) decline in earnings at the bottom, with each channel explaining roughly half of the total reduction. Hysteresis, therefore, occurs disproportionately through low-wage workers.

4.2 Shocks that Trigger the ZLB

The solid line in Figure 4 plots the economy's response to a one-time large negative demand shock that triggers the ZLB for around 10 quarters. The dashed line plots the econ-

Table 4: Business Cycle Statistics of the Ergodic Distribution Under Alternative Rules

	Inflation Targeting		Lower for Longer		Shortfall		AIT		Higher IT	
Business Cycle Features										
Boom frequency	0.059		0.238		0.229		0.095		0.071	
Recession frequency	0.165		0.101		0.132		0.101		0.103	
ZLB frequency	0.143		0.057		0.084		0.062		0.053	
Shadow rate at the ZLB	-0.66		-0.21		-0.62		-0.19		-0.13	
Statistics	$\mathbb{E}$	std	$\mathbb{E}$	std	$\mathbb{E}$	std	$\mathbb{E}$	std	$\mathbb{E}$	std
Output	-2.54	2.70	1.06	1.79	-0.13	2.63	-0.10	1.27	-0.44	1.35
Price inflation	-0.19	1.15	0.33	0.55	0.22	0.86	0.11	0.54	0.33	0.74
Total Hours	-1.96	3.43	0.83	2.21	-0.08	3.28	-0.07	1.55	-0.34	1.70
Labor productivity	-0.58	1.06	0.23	0.70	-0.04	1.00	-0.02	0.49	-0.10	0.53
Unemployment rate	0.55	1.70	-0.23	1.08	0.03	1.61	0.02	0.76	0.10	0.84
Participation	-0.89	0.76	0.38	0.54	-0.03	0.76	-0.03	0.39	-0.15	0.39
T1 Total Labor Earnings	-12.17	9.99	5.20	7.13	-0.47	10.06	-0.39	5.20	-2.11	5.14
T3 Total Labor Earnings	-1.79	2.13	0.74	1.38	-0.10	2.05	-0.07	0.96	-0.31	1.06

Note. "Recession frequency" denotes the share of time in the simulation where the aggregate unemployment rate is at least 1.0 ppt above its non-stochastic steady state. "Boom frequency" denotes the share of time in the simulation where the aggregate unemployment rate is at least 1.0 ppt below its non-stochastic steady state. "Shadow rate" denotes the gap between the nominal rate under the ZLB and the rate monetary authority would want to set if it was not constrained. T1 and T3 refer to average outcomes for the bottom and top terciles of the worker's skill distribution. Expected values ($\mathbb{E}$) are expressed in terms of deviations from the non-stochastic steady state. Inflation, unemployment and participation are denoted in percentage points deviations. All other variables are in percent deviations.

omy's response abstracting from the constraint on nominal rates. The difference between the two lines thus captures the impact of the ZLB on aggregate and distributional outcomes.

Similar to other studies on the deleterious effects of the ZLB (e.g., Williams, 2009; Kiley and Roberts, 2017; Bernanke et al., 2019), our model also predicts worse outcomes in terms of both aggregate unemployment and inflation when the economy is at the constraint. Compared to this earlier body of work, the presence of labor market hysteresis in our environment yields a lot of endogenous persistence to the negative effects of the ZLB, particularly at the bottom of the skill distribution (third row of Figure 4). Earnings, labor force attachment and skills losses for this group are all deeply accentuated by the ZLB. More than just accentuating their losses, the ZLB continues to impair their recovery long after the constraint ceases to be binding, 30-40 quarters following the shock. Crucially, these large negative biases in earnings and participation don't elicit a correction from the monetary authority since unemployment and inflation, the two variables entering the IT rule, revert much faster to their steady-state values.

4.3 Ergodic Distribution

The previous section highlighted the long-lasting effects of a single visit to the ZLB under an IT rule. In this section, we look at what happens when the economy is repeatedly hit

Table 5: Total Labor Earnings Biases and Its Sources under Alternative Rules

	Inflation Targeting		Lower for Longer		Shortfall		AIT		Higher IT	
Statistics	$\mathbb{E}$	(%)	$\mathbb{E}$	(%)	$\mathbb{E}$	(%)	$\mathbb{E}$	(%)	$\mathbb{E}$	(%)
T1 Total Labor Earnings	-12.17	—	5.20	—	-0.47	—	-0.39	—	-2.11	—
T1 Unemployment rate	1.10	10	-0.47	10	0.04	11	0.04	13	0.19	10
T1 Participation rate	-2.52	50	1.09	51	-0.09	45	-0.07	46	-0.44	50
T1 Labor productivity	-4.54	37	1.92	37	-0.20	43	-0.15	38	-0.78	37
T3 Total Labor Earnings	-1.79	—	0.74	—	-0.10	—	-0.07	—	-0.31	—
T3 Unemployment rate	0.31	18	-0.13	18	0.02	20	0.01	14	0.05	19
T3 Participation rate	-0.03	2	0.01	1	-0.01	10	-0.00	0	-0.01	3
T3 Labor productivity	-1.22	68	0.50	68	-0.06	60	-0.04	57	-0.21	67

Note. Expected values ($\mathbb{E}$) are expressed in terms of deviations from the non-stochastic steady state. Unemployment and participation are denoted in percentage points deviations. All other variables are in percent deviations. The (%) measures the contribution to the earnings bias attributable to the corresponding variable.

by aggregate shocks and undergoes multiple of these episodes. To do this, we compute the model's ergodic distribution. Table 4 summarizes our results for both aggregate and distributional outcomes.

The presence of the ZLB skews the ergodic distribution toward negative outcomes: the share of time spent in recessions (defined as periods when unemployment is 1 pp above the non-stochastic steady state) is 16.5%, much larger than the 6% spent in expansions (periods when unemployment is 1 pp below the non-stochastic steady state). This asymmetry results in a deflationary bias of nearly 20bp and a sizable negative bias for aggregate labor market outcomes: the unemployment rate is 0.55ppt higher, and the labor force participation rate is 0.9ppt lower. What is striking is how unequal the effect of the ZLB is across the skill distribution. For example, total labor earnings in the bottom tercile display a negative bias that exceeds 12% compared to less than 2% for the top tercile.[34]

Why do we observe such a large negative bias in labor market outcomes of low-skilled workers if monetary policy is constrained during a relatively small share (14%) of time? As we pointed out in our earlier analysis of the IRFs, the deleterious effects of the ZLB at the bottom of the distribution persist far beyond the period where the constraint is binding. To appreciate this point even further, consider the following simple back-of-the-envelope calculation: if all the negative bias in total earnings at the bottom tercile in Table 4 came purely from stronger recessions during ZLB episodes—and not from persistent negative effects that outlast these episodes—total labor income at the bottom should fall by roughly 86% (0.12/0.14). Instead, our simulations show that total labor income

[34] We note that our model features both positive hysteresis (uplifting effects) following expansions and negative ones (scarring effects) following recessions, which, absent the ZLB, would tend to offset each other and lead to virtually no bias in either aggregate or distributional outcomes in the ergodic distribution (in fact exactly so, under our linear solution method). It is the ZLB, and its amplifying effect on negative labor market hysteresis, that gives rise to a negative bias.

at the bottom tercile only falls by 30% on average during ZLB episodes, implying that earnings also remain depressed even when the ZLB is not binding.

We can also explore the low-skilled earnings bias in terms of the contribution of Okun's channels using our total earnings decomposition (18) from before. Table 5 summarizes the average bias on each channel at the top and bottom tercile, as well as their contribution to the observed total earnings losses (e.g., the unemployment rate at the bottom tercile under IT has a negative bias of 1.1 ppt, which accounts for 10% of the total earnings bias of -12%). Similar to what we found for the IRFs, the large negative earnings bias at the bottom arises mostly from lower participation (50%) and productivity (37%).

Taking stock, the main insight of this section is that a monetary policy rule that does not act to mitigate the ZLB opens the door to enduring scars on labor force participation and productivity. Furthermore, this force operates disproportionately on low-skilled, marginally attached workers. Alternative monetary policy rules that reduce the negative effects of the ZLB or let the economy run hot during recoveries would push against this hysteresis and, as a result, have the potential to improve labor market outcomes of low-wage workers persistently. The monetary policy framework introduced by the Fed in 2020, which promises to keep rates Lower for Longer following recessions, has precisely this flavor.

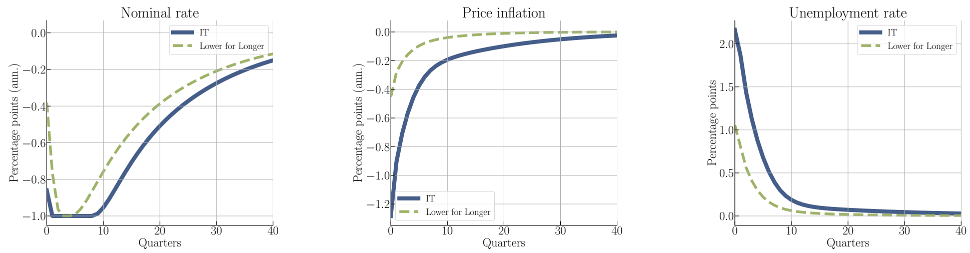
5 Economic Outcomes Under Lower for Longer

We model the Lower for Longer (LfL) strategy by changing the Taylor rule reaction function $\mathcal{R}_t$ in (13) to

$$\mathcal{R}_t = 2.25(\pi_t - \pi^*) + 5.00(\pi_t^{MA} - \pi^*)^- - 0.15(u_t - u^*)^+, \tag{19}$$

where π_t^{MA} is the exponential moving average of past inflation $\int_0^\infty (1/48)\ e^{-(1/48)\tau}\pi_{t-\tau}d\tau$ with a smoothing factor $1/48$ to target a window of 4 years, x^+ is the shorthand for $\max\{x, 0\}$, and x^- for $\min\{x, 0\}$. The addition of the (negative) past inflation average π^{MA} into (19) is meant to capture the (asymmetric) Average Inflation Targeting (AIT) component of the new framework. Mechanically, following periods where inflation falls persistently below 2 percent, the average inflation term in the rule creates a downward pressure on nominal rates, which persists until average inflation returns to target. The Shortfall (SR) component is captured by making the response to unemployment fluctuations asymmetric. Instead of reacting to all unemployment deviations, as in the baseline IT, the LfL rule (19) predicts that the monetary authority should not raise rates whenever unemployment

(a) Demand-driven recession



(b) Supply-driven expansion

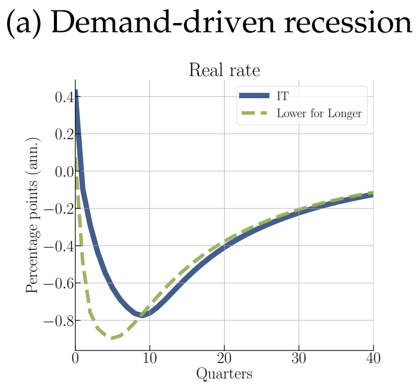
Figure 5: Impulse response under IT and LfL.

goes below steady state.

The LfL rule has two main goals. First, it aims to better handle the ZLB and alleviate some of its most harmful consequences. Second, it seeks to promote a strong labor market during recoveries, allowing low-skilled workers to re-enter participation, find jobs, and, over time, improve their skills. If successful in these two objectives, this rule can act as an ‘antidote’ to ZLB’s negative hysteresis we found under IT. As we previewed in the Introduction, however, running a LfL strategy also carries the risk of pushing inflation above target.[35]

The rest of this section is focused in measuring this trade-off. We first analyze how the LfL strategy changes the IRFs to demand and supply shocks (Section 5.1). Second, we study the counterfactual where we ask how the recovery from the Great Recession would have developed had the Fed followed a LfL strategy early in the downturn (Section 5.2). Third, we study the impact of the LfL rule on aggregate and distributional outcomes in the ergodic distribution of our model (Section 5.3).

[35]Strictly speaking, the inflation cost is a concern only under a demand-driven expansion since, in the case of supply-driven expansions, inflation is below target under IT. In this scenario, pushing for a stronger response in the labor market moves the economy closer to price stability. In terms of policy rules, however, what matters is the average trade-off faced by monetary policy over the business cycle. Our calibration—as the data—displays a positive correlation between hours and inflation, meaning that the monetary authority is more likely to face inflationary pressures during expansions.



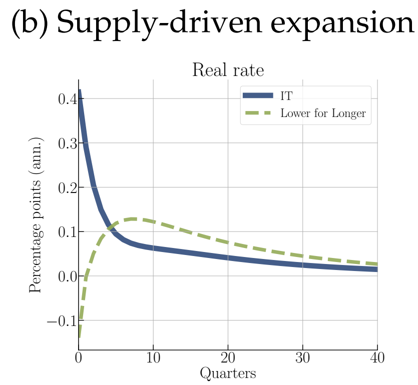


Figure 6: Real rate response under IT and LfL.

5.1 IRFs Under Lower-for-Longer

Figure 5 presents the model's impulse responses under the IT and LfL strategies. To highlight the differences between the two policy rules, we focus on the IRFs to a large negative demand shock (a demand-driven recession) and a positive supply shock (a supply-driven expansion).[36] The LfL rule significantly alters the economy's response to these two types of shocks. In the case of a demand-driven recession, the LfL strategy greatly helps to stabilize unemployment and inflation, whose responses become less than half of those under IT. As a result, the economy also spends less time at the ZLB, which binds only for a few quarters under LfL. In a supply-driven expansion, the economy undergoes a much stronger expansion under a LfL strategy. Unemployment falls 3 times more than under IT, while inflation, which falls significantly under IT, displays a small positive response under LfL.

Somewhat counterintuitively, nominal rates are actualy higher under the LfL strategy. This has to due with the Taylor rule's feedback response to inflation and unemployment. As the LfL rule reduces unemployment and increases inflation in equilibrium, the rule's automatic reaction to these variables pushes up nominal rates above what they would have been without the commitment to keep rates lower *given* the same inflation and unemployment levels. Under both shocks, however, the LfL approach delivers a lower path for real rates (Figure 6).

To sum up, adopting a LfL strategy can promote weaker recessions and more robust expansions in response to shocks. This suggests that the new strategy can dramatically alter the "shape" of business cycles. To understand whether the rule can also foster in-

[36] The IRFs of other shocks, like demand-driven expansions and supply-driven recessions, are shown in the appendix I.

clusion for low-wage workers and, if so, at what costs in terms of inflation, we need to evaluate the consequences of this strategy over a sufficiently long horizon under a realistic sequence of aggregate shocks. The following two sections do precisely this.

5.2 Great Recession Under Lower-for-Longer

How would the US labor market and inflation dynamics have looked like during the Great Recession and its recovery had the Fed followed a LfL strategy? The Great Recession is an ideal laboratory—the extended duration of the ZLB, the persistently below-target inflation and the sluggish recovery of participation and earnings, especially among low-wage workers, were central to the 2020 framework revision and are all "failures" that the new framework was meant to correct.

To generate this counterfactual, we first use the model's impulse responses together with unemployment and inflation data from 1990:01 to 2019:12 to filter the sequence of demand and supply innovations that exactly replicate, under our model with a baseline IT rule, the observed data. Figure I2 in the Appendix reports the filtered shock series, while the implied evolution of aggregate and distributional variables are shown in Figures 7 and 8. Beyond matching inflation and unemployment dynamics, which we do by construction, the simulation also captures other important dimensions of Great Recession and its recovery like the prolonged period at ZLB, the rise in labor productivity around 2008-2010, and the persistent reduction to participation, particularly at the bottom of the skill distribution, .

We then use the sequence of filtered shocks to simulate a counterfactual economy where the Fed follows the LfL monetary policy rule. We also present the economy's dynamics under a counterfactual where the Fed follows a rule that includes the AIT component of the LfL framework, but keeps the reaction to unemployment symmetric as in the baseline IT rule. The difference between LfL and AIT shows the impact of the Shortfall (SR) component (see Table H2 in the Appendix).

The commitment to keep rates lower in response to the low inflation significantly reduces the costs during the outset of the Great Recession when the economy first hit the ZLB in 2008: inflation doesn't fall as much as it does under IT and unemployment comes down faster from its peak. As a result, the economy exits the ZLB sooner with beneficial impacts on participation and earnings. Starting from 2016, once the expansion is well under way, the AIT component of the rule becomes moot (past inflation, under LfL, is averaging around 2% at that point) and the SR component takes over. The SR pushes down unemployment and pulls up participation and earnings: at the end of the sample

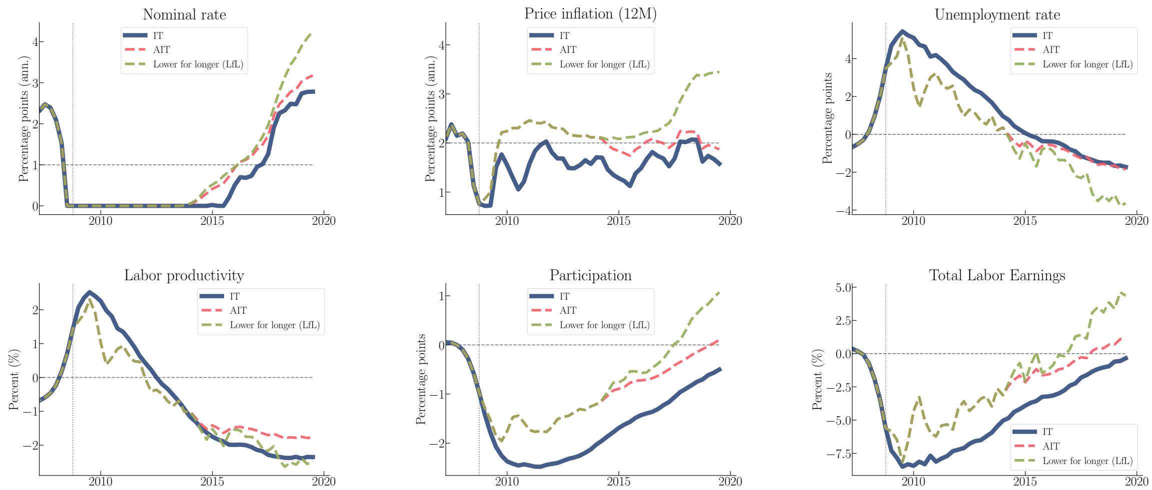


Figure 7: Great Recession dynamics under IT, asymmetric AIT, and LfL rule. See Table H2 in the Appendix for the specification of the different monetary policy rules.

period, under the LfL rule, unemployment is 2 ppts lower, participation 1.5 ppt higher, and earnings are 5 ppt higher compared to the economy under IT.

Figure 8 shows the impact of the LfL rule along the workers' skill distribution. Compared to the top tercile, which is well insulated from the cycle, the fluctuations in earnings at the bottom tercile are much more pronounced. As in the aggregate, the benefits of the LfL rule to this group of workers accrue both in the downturn and in the recovery. At the onset of the recession, the rule prevents the sharp and persistent decline in labor productivity and labor force participation. Later in the recovery, it reinforces and amplifies the gains allowing workers to fully recoup their earnings losses.[37]

The cost of the new strategy shows up clearly on inflation: while inflation remained 0.5 ppts below target throughout the recovery under IT, under the LfL rule, inflation is on average 0.7 ppts above it. Noticeably, this bias is caused mostly by the SR component during the last two years of the boom.

Altogether, the Great Recession counterfactual suggests that the LfL strategy would have delivered a major boost to low-wage workers with only a modest deviation of inflation from target, except late in the recovery. The next question we address is whether these short-run gains of a LfL strategy can be sustained over the long run, or whether they tend to vanish once the economy invariably returns to a state of greater slack.

[37] It might seem puzzling that, under LfL, by 2020 labor productivity of each tercile has fully recovered, whereas aggregate labor productivity remains depressed (Figure 7). The reason is that the LfL strategy shifts the composition of employment toward low-skill workers who are less productive than the rest. This selection channel imparts more countercyclicality to aggregate productivity.

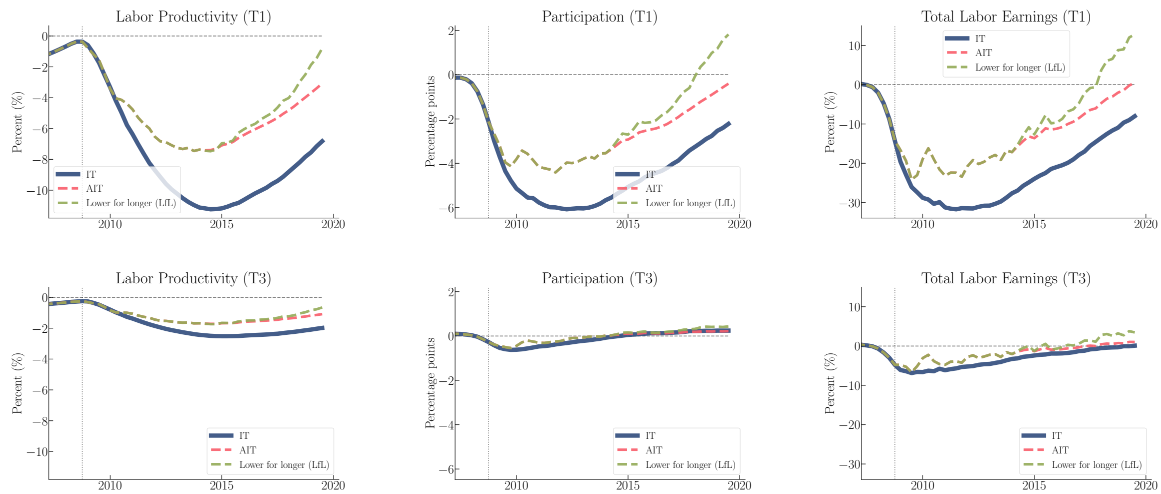
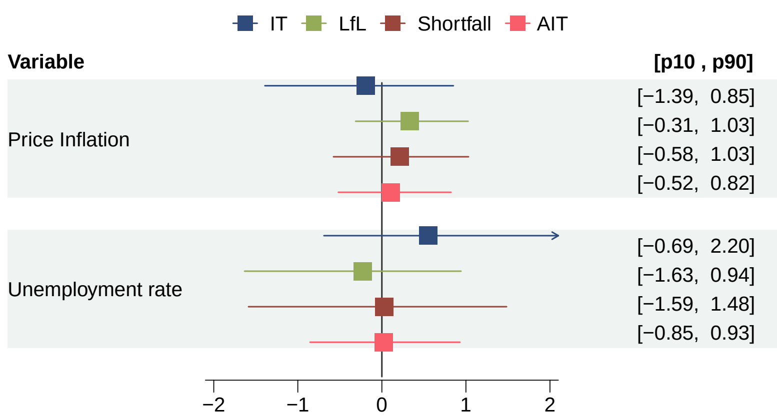


Figure 8: Great Recession dynamics under IT, asymmetric AIT, and LfL rule. See Table H2 in the Appendix for the specification of the different monetary policy rules.

5.3 Ergodic Distribution Under Lower-for-Longer



Notes: The lines plot the distance between the 10^{th} and the 90^{th} percentiles across the ergodic distribution. The square denotes the average.

Figure 9: Inflation and unemployment ergodic distribution under alternative monetary policy rules.

Table 4 reports the model's simulated ergodic distribution results under the LfL strategy. Overall, our conclusions from the Great Recession counterfactual seem to survive in the long-run simulation. Relative to IT, the LfL rule lowers the ZLB frequency (from 14% to 6%), reduces their intensity (the average shadow rate goes from -66bp to -21bp), and increases the share of expansions in the simulation (from 6% under IT to 24%). These effects also show up in the ergodic distribution of inflation and unemployment, as shown in Figure 9, which depicts the 10th and 90th percentiles of the distribution of these variables under various monetary policy rules. The distribution under IT (blue line) highlights the asymmetric effects of the ZLB: we get a long left tail in the ergodic distribution of inflation and a stretched right tail in the unemployment distribution. Shifting to a LfL strategy (green line) better stabilizes the economy at the ZLB, effectively reducing much of the mass at the left (right) tail of the inflation (unemployment) distribution. In addition, it succeeds at running the economy hot during expansions, as depicted by the stretched left tail of the distribution of unemployment and the (modest) longer right tail of the inflation distribution. Consequently, the negative inflation and positive unemployment biases—indicated by the squares—we get under IT are fully reversed with an LfL rule, under which unemployment bias is negative (-0.23 ppt) and inflation averages slightly above target (33 bp).

Turning to the impact of the rule on other labor market variables, Table 4 shows that, relative to IT, the LfL rule raises participation by 1.3 ppt and strengthens labor productivity by 0.8 ppt. As in the Great Recession counterfactual, labor market gains are much stronger at the bottom skill tercile. Overall, total labor earnings at the bottom tercile increase by over 17% (5.20+12.17) through a combination of stronger labor force attachment (1.09+2.52 ppt) and higher labor productivity (1.92 +4.54%). As a result, labor market inequality is reduced, with unemployment, participation, and earnings gaps between the top and bottom skill terciles falling by 1.2 ppt, 3.5 ppt, and 14%, respectively.

5.3.1 The Role Played by the AIT and Shortfall Components

What are the separate contributions of the AIT and SR components to the gains of the LfL strategy? Table 4 shows that the two policies, when implemented individually, have a similar impact on average outcomes. Moreover, both rules reduce the frequency of ZLB episodes (which binds 8% of the time under SR, and 6% of the time under AIT), significantly dampen the negative labor market biases relative to IT, and push inflation slightly above the target (22bp the SR, and 11bp the AIT). However, they vary a lot in terms of their predicted share of booms and recessions, as well as in their implied volatilities.

To understand these differences, compare the 10th and 90th percentiles of unemployment under the IT (blue line), AIT (pink line) and SR (brown line) in Figure 9. It is clear that the AIT's gains come mainly from avoiding extremely low inflation and high unem-

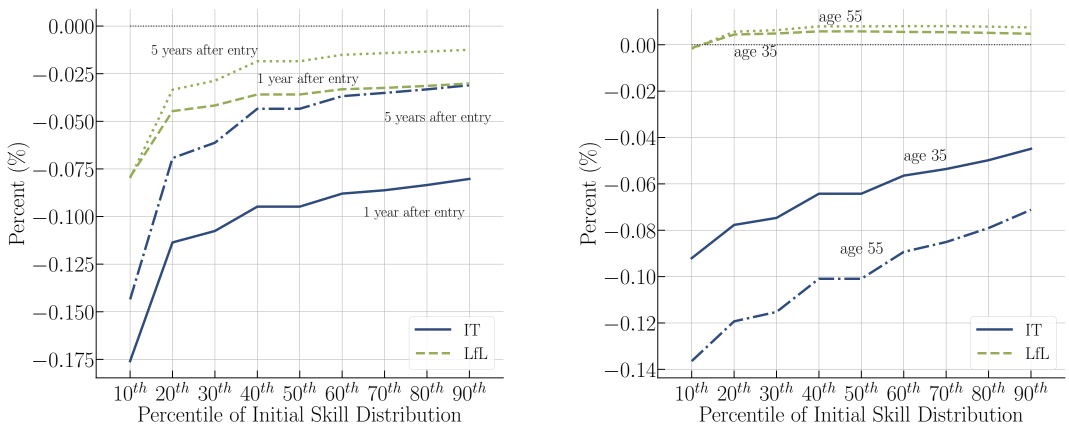


Figure 10: **Panel (A)** Earnings losses from joining the labor market in a recession by deciles of the initial skill distribution. **Panel (B)** Earnings gaps over the life-cycle in the ergodic distribution by deciles of the initial skill distribution.

ployment outcomes, i.e., the benefits associated with the AIT rule occur primarily during recessions. Coherently, Table 4 reports that the AIT rule reduces the share of recessions in the simulation from 16.5% under IT to 10%, lifts the shadow rate at the ZLB from -66bp to -19bp, and curtails the volatility of inflation and real outcomes by a factor of 2. In contrast, the SR rule realizes most of its gains during expansions, as it is clear from its ability to generate periods of very low unemployment rates. In line with this observation, Table 4 reports that the SR rule increases the share of booms in the simulation from 6% under IT to 23%, whereas its impact during downturns, as indicated by the change in the share of recessions and the shadow rate at the ZLB, is much more modest.

The AIT and SR components of the new framework thus seem to operate at very different points of the business cycle: the AIT shrinks the size and frequency of negative hysteresis brought by the ZLB, while the SR component overheats the economy during expansions. The LfL strategy, which combines the AIT and SR components, realizes gains across the cycle.

5.3.2 Life-Cycle Earnings Mobility

In his 1974 article, Okun stressed an improvement in upward mobility as one of the major benefits of running a high-pressure economy—through stronger attachment and faster skill growth, a hot labor market could deliver a steeper life-cycle earnings profile, especially among low-skilled workers. In this section, we use the life-cycle structure of our model to explore this additional dimension of Okun's conjecture.

As we did when looking at the results in the aggregate or along the cross-section, we start by documenting the effect of recessions and the ZLB on workers' life-cycle outcomes under IT. A large literature finds that joining the job market during recessions can have significant and persistent effects on labor market outcomes of young workers, particularly among less advantaged groups (see von Wachter, 2020, for a summary of the empirical evidence). To see how well the model captures this fact, the left panel in Figure 10 plots the effect of entering the labor market in a recession on log-annual earnings by workers' initial position in the distribution of skills. Specifically, the lines denote the difference in log-annual earnings, 1 and 5 years following entry, between joining the labor market in the stationary equilibrium versus during a demand-driven recession like the one in Figure 5. Consistent with the empirical evidence, the model predicts large and persistent losses from joining the labor market in a recession. Also, losses seem disproportionately concentrated among workers below the 30th percentile of the initial skill distribution.

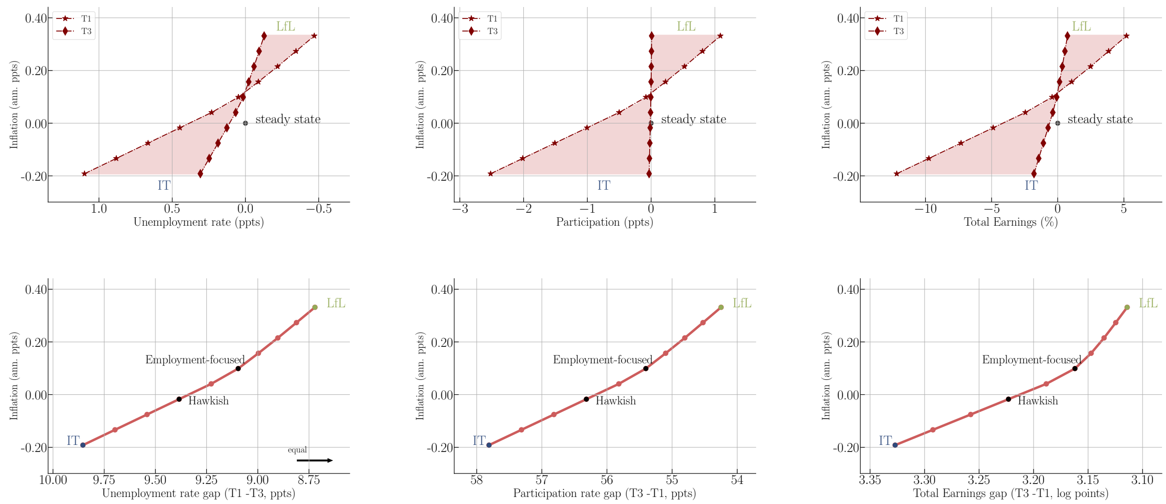
Recessions are also damaging for workers at other points of the life cycle. To highlight this, the right panel of Figure 10 plots the gap in workers' life-cycle earnings at age 35 and 55 averaged across the ergodic distribution. For instance, a worker who joins the labor market at the 50th percentile of the initial skill distribution features, at age 55, annual earnings that are 10% on average below his/her expected earnings at the same age but in the stationary equilibrium. Therefore, the repeated experience of recessions contributes to a substantial reduction in workers' life-cycle earnings.

Shifting from an IT to a LfL strategy significantly curtails workers' earnings losses. By lessening the impact of recessions, the LfL reduces earnings losses from joining in a recession across the whole skill distribution, particularly for the bottom percentiles (left panel of Figure 10). The LfL also successfully closes the negative earnings gap at ages 35 and 55 that arises under IT (right panel of Figure 10).

5.3.3 Okun's Cones: A Menu for Policymakers

Up to now, we have analyzed and quantified the inflation-inclusion trade-off exclusively for a plausible, but specific, parameterization of the LfL rule. In this section, we try to assess the entire menu of outcomes available to a policymaker interested in exploring some version of the LfL strategy. Ultimately, this menu expresses the additional average inflation required to achieve a certain reduction to the gap of a particular labor market variable (e.g., the T3-T1 participation gap).

For this purpose, we repeat the LfL simulations of Section 5.3 while varying the reaction coefficients of the LfL policy rule between the values taken under IT and those under



Notes: The cones' lower and upper limits reproduce the outcomes under baseline IT and the parameterized Lower for Longer rule reported in Table 4.

Figure 11: Okun's cones and inflation-inclusion trade-off.

our main parameterization of the LfL rule. For each parameterization, we track the implied average biases for unemployment, participation, and total earnings at the bottom and top terciles of the skill distribution. This mapping—which, because of its shape, we denote by "Okun's cones"—is plotted in the first row of Figure 11. The figures in the second row summarize the results in terms of the top-bottom tercile gaps in unemployment, participation, and earnings.

First, consider a more hawkish central bank interested in the LfL strategy only as a way to close the deflationary bias induced by the ZLB (outcomes corresponding to zero on the Y axis). Implementing this allocation will already deliver sizable gains in labor market outcomes for workers in the bottom tercile. Compared to an IT rule, even this conservative version of the LfL rule shrinks unemployment, participation, and earnings gaps by 0.50ppt, 1.5ppt, and 10 log points, respectively. A more employment-focused central bank that wishes to fully offset the detrimental effect of the ZLB on the labor market outcomes of the low-skilled workers (outcomes corresponding to zero on the X axis) would instead have to tolerate a positive, but small, inflation bias of 10 bps. Finally, a more inclusion-focused policymaker who wishes to go beyond this point and push for even greater inclusion, can do so at the cost of a larger inflation bias.

5.4 Higher Inflation Target

Another option available to central banks wanting to reduce the frequency and intensity of the ZLB is to raise the inflation target π^*. Because nominal rates move one-to-one with inflation at the steady state, increasing the inflation target reduces the chances of hitting the constraint and should thus also help to alleviate the negative hysteresis effects of the ZLB.

In this section, we explore the impact of a higher inflation target π^{**} on the economy's ergodic distribution.[38] The last column in Table 4 reports the results for a higher inflation target (Higher IT) generating the same inflationary bias of the LfL approach. While the Higher IT reduces ZLB frequency and increases the shadow rate as much as the LfL strategy, it promotes much smaller gains to labor market outcomes of low-wage workers—compared to a positive bias of 5.2% for total earnings at the bottom tercile under LfL, there is still a -2.3% negative bias under a Higher IT policy. In fact, the results under Higher IT are dominated even by the AIT rule alone, which produces higher labor market gains at smaller costs of inflation and less volatility overall.

6 Conclusions

In this paper we have formalized Arthur Okun's conjecture that a high-pressure economy would lead to a persistent uplifting of labor market trajectories for low-wage workers, possibly at the cost of higher inflation. This step required extending the canonical HANK framework with a rich model of the labor market where heterogeneous workers choose their degree of attachment to the labor force, are unequally exposed to aggregate fluctuations, and upgrade or downgrade their skills depending on their employment status. In this world, recessions are especially detrimental to low-skilled labor through a vicious cycle of job loss, exit to non-participation, and skill deterioration which imparts persistent scars on their lifetime earnings. Within this framework, we have analyzed the stabilization role of monetary policy. We have found that the ZLB severely amplifies these negative hysteresis at the bottom of the income distribution under a strict Inflation Targeting regime. Shifting to a Lower-for-Longer strategy disproportionately benefits low-wage workers by ameliorating the negative effects of the ZLB and promoting a hot economy during recoveries. Because policymakers might have different preferences for inflation vs. inclusion, we use our framework to quantify the inclusion-inflation frontier implied by the Lower-for-Longer strategy.

[38] In the simulation of the higher IT, we assume that unions adjust wages to the new inflation target π^{**}. Thus, the ZLB is the only reason that the higher inflation target is not neutral.

In exploring this tradeoff, we have made a modelling effort to achieve a credible quantification of the labor market gains of inclusive stabilization policy across the distribution. We instead purposefully kept the cost-side of this tradeoff much closer to the canonical HANK framework. A natural next step in this research agenda would be to enrich further this less developed block of the model. One possible direction would be to acknowledge that an especially tight labor market could lead to a steepening of the Phillips curve (Benigno and Eggertsson, 2023) and that persistent deviations of inflation from the target may lead to de-anchoring of inflation expectations (Carvalho et al., 2023). Another direction would be to recognize that the costs of inflation are uneven across the distribution of households and integrate the many channels through which they operate (e.g., via heterogeneity in consumption baskets, nominal net positions, and nominal wage rigidity) into our setup. Jaravel (2021), Olivi et al. (2023), Del Canto et al. (2023), Afrouzi et al. (2024), and Guerreiro et al. (2024) are recent examples of studies that try to quantify the relative role of these channels.

In HANK models, the clear cut distinction between stabilization and redistribution/social insurance policies invoked half a century ago by the neoclassical synthesis inevitably vanishes (Sargent, 2024). In this class of economies, any redistribution or social insurance policy reshapes aggregate fluctuations and any stabilization policy is redistributive to some extent. Our paper is a stark exemplification of this latter link. Much research is still needed to further refine our understanding of the connection between these two dimensions of macroeconomic policy which, traditionally, have been analyzed in isolation.

References

Aaronson, Stephanie R, Mary C Daly, William L Wascher, and David W Wilcox (2019), "Okun revisited: Who benefits most from a strong economy?" *Brookings Papers on Economic Activity*, 2019(1), 333–404.

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll (2017), "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," Technical Report w23732, National Bureau of Economic Research, Cambridge, MA, doi: 10.3386/w23732.

Adjemian, Stephane, and Michel Juillard (2013), "Stochastic Extended Path Approach," .

Afrouzi, Hassan, Andres Blanco, Andres Drenik, and Erik Hurst (2024), "A Theory of How Workers Keep Up with Inflation," Technical report, Mimeo, Columbia University.

Alvarez, Fernando, and Robert Shimer (2011), "Search and Rest Unemployment," *Econometrica*, 79(1), 75–122.

Alves, Felipe, Greg Kaplan, Benjamin Moll, and Giovanni L Violante (2020), "A Further Look at the propagation of Monetary Policy Shocks in HANK," *Journal of Money, Credit and Banking*, 52(S2), 521–559.

Alves, Felipe, and Giovanni L Violante (2023), "From Micro to Macro Hysteresis: Long-Run Effects of Monetary Policy," .

Athey, Susan, Lisa K. Simon, Oskar N. Skans, Johan Vikstrom, and Yaroslav Yakymovych (2023), "The Heterogeneous Earnings Impact of Job Loss Across Workers, Establishments, and Markets," Working paper, Stanford University.

Auclert, Adrien, Bence Bardóczy, and Matthew Rognlie (2020a), "MPCs, MPEs and Multipliers: A Trilemma for New Keynesian Models," National Bureau of Economic Research.

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021), "Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models," *Econometrica*, 89(5), 2375–2408.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub (2018), "The Intertemporal Keynesian Cross," Working paper, Stanford University.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub (2020b), "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model," Technical Report w26647, National Bureau of Economic Research.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub (2024), "Fiscal and Monetary Policy with Heterogeneous Agents," Technical report, National Bureau of Economic Research.

Autor, David, Arindrajit Dube, and Annie McGrew (2023), "The unexpected compression: Competition at work in the low wage labor market," Working paper, MIT.

Bartscher, Alina K, Moritz Kuhn, Moritz Schularick, and Paul Wachtel (2021), "Monetary policy and racial inequality," CEPR Discussion Paper No. DP15734.

Bayer, Christian, Benjamin Born, and Ralph Luetticke (2020), "Shocks, Frictions, and Inequality in US Business Cycles," .

Benigno, Pierpaolo, and Gauti B. Eggertsson (2023), "It's Baaack: The Surge in Inflation in the

2020s and the Return of the Non-linear Phillips Curve," Technical report, National Bureau of Economic Research.

Bergman, Nittai, David A Matsa, and Michael Weber (2020), "Heterogeneous labor market effects of monetary policy," *Chicago Booth Research Paper*, (21-02).

Bernanke, Ben S., Michael T. Kiley, and John M. Roberts (2019), "Monetary Policy Strategies for a Low-Rate Environment," 2019(009).

Bianchi, Francesco, Leonardo Melosi, and Matthias Rottner (2021), "Hitting the elusive inflation target," *Journal of Monetary Economics*, 124, 107–122.

Blanchard, Olivier J, and Lawrence H Summers (1986), "Hysteresis and the European Unemployment Problem," *NBER Macroeconomics Annual*, 1, 15–78.

Boppart, Timo, Per Krusell, and Kurt Mitman (2017), "Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative," 54.

Braxton, and Kyle Herkenhoff (2021), "Changing Income Risk across the US Skill Distribution: Evidence from a Generalized Kalman Filter," 107.

Bundick, Brent, and Nicolas Petrosky-Nadeau (2021), "From Deviations to Shortfalls: The Effects of the FOMC's New Employment Objective," Working paper, Federal Reserve Bank of San Francisco.

Cairó, Isabel, and Avi Lipton (2023), "Labor Market Discrimination and the Racial Unemployment Gap: Can Monetary Policy Make a Difference?" Finance and Economics Discussion Series, Federal Reserve Board 2023-065.

Cajner, Tomaz, Tyler Radler, David Ratner, and Ivan Vidangos (2017), "Racial gaps in labor market outcomes in the last four decades and over the business cycle," FED Working Paper.

Carvalho, Carlos, Stefano Eusepi, Emanuel Moench, and Bruce Preston (2023), "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, 15(1), 1–47.

Cerra, Valerie, Antonio Fatás, and Sweta C Saxena (2023), "Hysteresis and Business Cycles," *Journal of Economic Literature*, 61(1), 181–225.

Clarida, Richard (2022), "The Federal Reserve's New Framework: Context and Consequences," FED Working Paper.

Comin, Diego, and Mark Gertler (2006), "Medium-term Business Cycles," *American Economic Review*, 96(3), 523–551.

Davis, Steven J., and Till Von Wachter (2011), "Recessions and the Costs of Job Loss," *Brookings Papers on Economic Activity*, 2011(2), 1–72.

Del Canto, Felipe N, John R Grigsby, Eric Qian, and Conor Walsh (2023), "Are Inflationary Shocks Regressive? A Feasible Set Approach," National Bureau of Economic Research.

Elsby, Michael WL, Bart Hobijn, and Ayşegül Şahin (2015), "On the importance of the participation margin for labor market fluctuations," *Journal of Monetary Economics*, 72, 64–82.

Erceg, Christopher J, Dale W Henderson, and Andrew T Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46(2), 281–313.

Federal Reserve System (2020), "FedListens: Perspectives from the Public," Federal Reserve Board.

Fernández-Villaverde, Jesús, Joël Marbet, Galo Nuño, and Omar Rachedi (2023), “Inequality and the Zero Lower Bound,” Working Paper 31282, National Bureau of Economic Research.

Finkelstein, Amy, Nathaniel Hendren, and Erzo FP Luttmer (2019), “The value of medicaid: Interpreting results from the oregon health insurance experiment,” *Journal of Political Economy*, 127(6), 2836–2874.

Fornaro, Luca, and Martin Wolf (2020), “The scars of supply shocks,” CEPR Discussion Paper No. DP15423.

Furlanetto, Francesco, Antoine Lepetit, Ørjan Robstad, Juan Rubio Ramírez, and Pål Ulvedal (2023), “Estimating Hysteresis Effects,” *American Economic Journal: Macroeconomics*.

Galí, Jordi, Frank Smets, and Rafael Wouters (2012), “Unemployment in an Estimated New Keynesian Model,” 32.

Golosov, Mikhail, Michael Graber, Magne Mogstad, and David Novgorodsky (2021), “How Americans respond to idiosyncratic and exogenous changes in household wealth and unearned income,” Technical report, National Bureau of Economic Research.

Graves, Sebastian, Christopher Huckfeldt, and Eric T Swanson (2023), “The Labor Demand an Labor Supply Channels of Monetary Policy,” Working Paper, Federal Reserve Board.

Guerreiro, Joao, Jonathon Hazell, Chen Lian, and Christina Patterson (2024), “Why Do Workers Dislike Inflation? Wage Erosion and Conflict Costs,” Technical report, National Bureau of Economic Research.

Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song (2017), “Heterogeneous scarring effects of full-year nonemployment,” *American Economic Review*, 107(5), 369–373.

Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song (2021), “What do data on millions of US workers reveal about lifecycle earnings dynamics?” *Econometrica*, 89(5), 2303–2339.

Hall, Robert E., and Marianna Kudlyak (2019), “Job-finding and job-losing: A comprehensive model of heterogeneous individual labor-market dynamics,” National Bureau of Economic Research.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante (2010), “Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006,” *Review of Economic Dynamics*, 13(1), 15–51.

Heathcote, Jonathan, Fabrizio Perri, Giovanni L. Violante, and Lichen Zhang (2023), “More unequal we stand? Inequality in the United States from the Great Recession to the Covid Pandemic,” *Review of Economic Dynamics*, 50, 235–266.

Hebden, James, and Fabian Winkler (2024), “Computation of Policy Counterfactuals in Sequence Space,” Working paper, Federal Reserve Board.

Hobijn, Bart, and Ayşegül Şahin (2021), “Maximum Employment and the Participation Cycle,” National Bureau of Economic Research.

Holden, Tom D (2016), “Computation of solutions to dynamic models with occasionally binding constraints,” Working paper, University of Surrey.

Holden, Tom D (2023), “Existence and uniqueness of solutions to dynamic models with occasionally binding constraints,” Working paper, University of Surrey.

Huo, Zhen, and José-Víctor Ríos-Rull (2020), "Sticky wage models and labor supply constraints," *American Economic Journal: Macroeconomics*, 12(3), 284–318.

Ioannidis, Michael, Sarah Jane Hlásková, and Chiara Zilioli (2021), "The mandate of the ECB: Legal considerations in the ECB's monetary policy strategy review," *ECB Occasional Paper*, (2021-276).

Jaravel, Xavier (2021), "Inflation Inequality: Measurement, Causes, and Policy Implications," *Annual Review of Economics*, 13(1), 599–629.

Jordà, Òscar, Sanjay R Singh, and Alan M Taylor (2020), "The long-run effects of monetary policy," National Bureau of Economic Research.

Kaplan, Greg, and Giovanni L Violante (2022), "The marginal propensity to consume in heterogeneous agent models," *Annual Review of Economics*, 14, 747–775.

Kehoe, Patrick J., Virgiliu Midrigan, and Elena Pastorino (2019), "Debt Constraints and Employment," *Journal of Political Economy*, 127(4), 1926–1991.

Kiley, Michael T., and John M. Roberts (2017), "Monetary Policy in a Low Interest Rate World," *Brookings Papers on Economic Activity*, 48(1 (Spring), 317–396.

Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin (2017), "Gross Worker Flows over the Business Cycle," *American Economic Review*, 107(11), 3447–76.

Laibson, David, Peter Maxted, and Benjamin Moll (2021), "Present bias amplifies the household balance-sheet channels of macroeconomic policy," National Bureau of Economic Research.

Lee, Munseob, Claudia Macaluso, and Felipe Schwartzman (2021), "Minority Unemployment, Inflation, and Monetary Policy," Federal Reserve Bank of Richmond.

Ljungqvist, Lars, and Thomas J. Sargent (1998), "The European Unemployment Dilemma," *Journal of Political Economy*, 106(3), 514–550.

Lucas, Robert E, and Edward C Prescott (1978), "Equilibrium Search and Unemployment," in *Uncertainty in Economics*, 515–540, Elsevier.

Ma, Yueran, and Kaspar Zimmermann (2023), "Monetary Policy and Innovation," National Bureau of Economic Research.

McKay, Alisdair, and Johannes F Wieland (2021), "Lumpy durable consumption demand and the limited ammunition of monetary policy," *Econometrica*, 89(6), 2717–2749.

McKay, Alisdair, and Christian K Wolf (2022), "Optimal Policy Rules in HANK," 49.

McKay, Alisdair, and Christian K Wolf (2023), "What Can Time-Series Regressions Tell Us About Policy Counterfactuals?" .

Nakajima, Makoto (2022), "Monetary Policy with Racial Inequality," Federal Reserve Bank of Philadelphia.

Okun, Arthur M (1973), "Upward mobility in a high-pressure economy," *Brookings Papers on Economic Activity*, 1973(1), 207–261.

Olivi, Alan, Vincent Sterk, and Dajana Xhani (2023), "Optimal Monetary Policy during a Cost of Living Crisis," Technical report, Mimeo, UCL.

Pfauti, Oliver (2024), "The Inflation Attention Threshold and Inflation Surges," Technical report.

Powell, Jerome H. (2020), "New Economic Challenges and the Fed's Monetary Policy Review," Technical report, Remarks at the jackson Hole Symposium sponsored by the Federal Reserve Bank of Kansas City.

Sargent, Tomas J. (2024), "HAOK and HANK Models," in Sofia Bauducco, Andres Fernandez, and Giovanni L. Violante, editors, *Heterogeneity in Macroeconomics: Implications for Monetary Policy*, Bank of Chile.

Smets, Frank, and Rafael Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

von Wachter, Till (2020), "The Persistent Effects of Initial Labor Market Conditions for Young Adults and Their Sources," *Journal of Economic Perspectives*, 34(4), 168–94.

Weber, Michael, Bernardo Candia, Tiziano Ropele, Rodrigo Lluberas, Serafin Frache, Brent H Meyer, Saten Kumar, Yuriy Gorodnichenko, Dimitris Georgarakos, Olivier Coibion, Geoff Kenny, and Jorge Ponce (2023), "Tell Me Something I Don't Already Know: Learning in Low and High-Inflation Settings," Working Paper 31485, National Bureau of Economic Research.

Williams (2009), "Heeding Daedalus: Optimal Inflation and the Zero Lower Bound," .

Yagan, Danny (2019), "Employment Hysteresis from the Great Recession," *Journal of Political Economy*, 127(5), 2505–2558.

Appendix

A Recursive Formulation of the Household Problem

In this section we show how the individual problems can be expressed recursively as a Hamilton-Jacobi-Bellman Quasi-Variational Inequality (HJBQVI). We follow Laibson et al. (2021) and their the notation with the max operator which we find intuitive. We use the symbol $\mathcal{Z}^s$ to denote the infinitesimal generator of the stochastic process defined in (1), transformed in levels, in employment state s. Then, $\mathcal{Z}^s v_t = \mu^s(z)\partial_z v_t^s + \sigma(z)\partial_{zz} v_t^s$, with $\mu^s(z) = \left(-\gamma_z \log z + \mathbb{I}_{\{s_t=e\}}\, \delta_z^+ - \mathbb{I}_{\{s_t \neq e\}}\, \delta_z^- + \frac{\sigma_z^2}{2}\right) z$ and $\sigma(z) = \sigma_z z$.

The problem of the employed households in (7) at time t can be written as:

$$
\begin{aligned}
\rho v_t^e = \max\ \Big\{ & \max_{c_t} \left\{ \mathrm{u}^e\left(c_t, h_t; z_t\right) + \partial_a v_t^e \left(r_t a_t + \left(1 - \tau_t\right) w_t z_t h_t + \phi_t - c_t\right)\right\} + \lambda_{zt}^{eu}\left(v_t^{u_1} - v_t^e\right) \\
& + \eta^{en_0}(v_t^{n_0} - v_t^e) + \mathcal{Z}^e v_t^e + \partial_t v_t^e, \rho v_t^{n_1} \Big\}
\end{aligned}
$$

The problem of the passive non-participant in (3) is:

$$
\begin{aligned}
\rho v_t^{n_0} = \max_{c_t}\ & \left\{ \mathrm{u}^n\left(c_t\right) + \partial_a v_t^{n_0}\left(r_t a_t + \phi_t - c_t\right)\right\} + \eta^{n_0 n_1}\left(v_t^{n_1} - v_t^{n_0}\right) \\
& + \mathcal{Z}^{n_0} v_t^{n_0} + \partial_t v_t^{n_0}
\end{aligned}
$$

The problem of the active non-participant in (4) is:

$$
\begin{aligned}
\rho v_t^{n_1} = \max\ \Big\{ & \max_{c_t}\left\{ \mathrm{u}^n\left(c_t\right) + \partial_a v_t^{n_1}\left(r_t a_t + \phi_t - c_t\right)\right\} + \lambda_{zt}^{ne} \max\left\{v_t^e - v_t^{n_1}, 0\right\} + \eta^{n_1 n_0}\left(v_t^{n_0} - v_t^{n_1}\right) \\
& + \mathcal{Z}^{n_1} v_t^{n_1} + \partial_t v_t^{n_1}, \rho v_t^{u_0} - \xi \Big\}
\end{aligned}
$$

The problem of the non-eligible unemployed in (5) becomes:

$$
\begin{aligned}
\rho v_t^{u_0} = \max\ \Big\{ & \max_{c_t}\left\{ \mathrm{u}^u\left(c_t\right) + \partial_a v_t^{u_0}\left(r_t a_t + \phi_t - c_t\right)\right\} + \lambda_{zt}^{ue} v_t^e + \eta^{un_0}\left(v_t^{n_0} - v_t^{u_0}\right) \\
& + \mathcal{Z}^{u_0} v_t^{u_0} + \partial_t v_t^{u_0}, \rho v_t^{n_1} \Big\}
\end{aligned}
$$

The problem of the eligible unemployed in (5) becomes:

$$\rho v_t^{u_1} = \max \left\{ \max_{c_t} \left\{ \mathfrak{u}^u (c_t) + \partial_a v_t^{u_1} \left(r_t a_t + \phi_t + b(z_t) - c_t \right) \right\} + \lambda_{zt}^{ue} \max \left\{ v_t^e - v_t^{u_1}, 0 \right\} \right.$$
$$\left. + \lambda_z^{u_1 u_0} \left(v_t^{u_0} - v_t^{u_1} \right) + \eta^{u n_0} \left(v_t^{n_0} - v_t^{u_1} \right) + \mathcal{Z}^{u_1} v_t^{u_1} + \partial_t v_t^{u_1}, \rho v_t^{n_1} \right\}$$

The four HJBQVI's above, jointly with the five first-order conditions at every t

$$\partial_c \mathfrak{u}^s (c_t, h_t; z_t) = \partial_a v_t^s \quad s \in \{e, u_0, u_1, n_0, n_1\}$$

can be solved for value functions $\left\{ v_t^s (a, z) \right\}_{t \geq 0}$, consumption decision rules $\left\{ c_t^s (a, z) \right\}_{t \geq 0}$ for $s \in \{e, u_0, u_1, n_0, n_1\}$, participation rules $\left\{ \mathfrak{p}_t^s (a, z) \right\}_{t \geq 0}$ for $s \in \{e, u_0, u_1, n_1\}$, and job acceptance rules $\left\{ \mathfrak{j}_t^s (a, z) \right\}_{t \geq 0}$ for $s \in \{u_1, n_1\}$.

B Problem of the Final and Intermediate Goods Producers

B.1 Final good sector

A competitive representative final-good producer aggregates a continuum of intermediate inputs indexed by $j \in [0, 1]$ with CRS technology

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\nu - 1}{\nu}} dj \right)^{\frac{\nu}{\nu - 1}} \tag{B1}$$

where $\nu > 0$ is the elasticity of substitution across inputs. This firm takes prices as given and solves

$$\max_{\{y_{jt}\}} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \tag{B2}$$

subject to (B1). Cost minimization implies that demand for intermediate good j at price p_{jt} is

$$y_{jt}(p_{jt}) = \left(\frac{p_{jt}}{P_t} \right)^{-\nu} Y_t, \quad \text{where} \quad P_t = \left(\int_0^1 p_{jt}^{1-\nu} dj \right)^{\frac{1}{1-\nu}} \tag{B3}$$

is the price of the final good and the numeraire of the economy.

B.2 Intermediate goods sector

At every date t, the intermediate-good producing firms, indexed by j take the task-specific wage w_{kt} as given, and solve

$$\begin{aligned}
\max_{p_{jt},\{\ell_{jkt}\}_k} \quad & \left(\frac{p_{jt}}{P_t}\right) y_{jt} - (1-\tau^\ell)\int_0^1 w_{kt}\ell_{jkt}dk - \phi^\ell \qquad \text{(B4)}\\
s.t. & \\
y_{jt} &= \alpha \ell_{jt}\\
\ell_{jt} &= \left[\int_0^1 \ell_{jkt}^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}\\
y_{jt} &= \left(\frac{p_{jt}}{P_t}\right)^{-\nu} Y_t
\end{aligned}$$

where w_{kt} is the real wage on task k, τ^ℓ is a wage subsidy to producers partially financed by the lump sum tax ϕ^ℓ that is set to undo the steady-state monopolistic distortion to production. Cost minimization yields the relative demand of labor for task k

$$\ell_{jkt} = \left(\frac{w_{kt}}{w_t}\right)^{-\varepsilon} \ell_{jt}, \qquad \text{(B5)}$$

where w_t is the Dixit-Stiglitz real aggregate wage index $w_t = \left[\int_0^1 w_{kt}^{1-\varepsilon} dk\right]^{\frac{1}{1-\varepsilon}}$ that satisfies $\int_0^1 w_{kt}\ell_{jkt}dk = w_t\ell_{jt}$. The profit-maximizing price setting decision yields the standard expression whereby the relative price equals a markup over the after-subsidy marginal cost of production

$$\frac{p_{jt}}{P_t} = \left(\frac{\nu}{\nu-1}\right)\frac{(1-\tau^\ell)w_t}{\alpha} \qquad \text{(B6)}$$

This price corresponds an optimal quantity sold and, in turn, to a total amount of labor demanded to produce it which, in the symmetric equilibrium with $p_{jt} = P_t$, satisfies $\ell_{jt} = \ell_t$ with

$$\ell_t = \left(\frac{1}{\alpha}\right) Y_t.$$

Imposing $p_{jt} = P_t$ and setting $\tau^\ell = 1/\nu$ in (B6) implies that the equilibrium aggregate real wage per effective hour is equal to the marginal product of labor, or

$$w_t = \alpha. \qquad \text{(B7)}$$

Finally, we note that because of the CRS assumption and the result that the wage rate equals the marginal product of labor, aggregate profits of the intermediate sector are equal to the net subsidy $\Psi_t^\ell = \tau^\ell w_t \ell_t - \phi^\ell$, thus in equilibrium

$$\Pi_t = \Psi_t^\ell. \tag{B8}$$

As long the tax only partially finances the subsidy, profits will be strictly positive in equilibrium. Profits are paid to the mutual fund which owns all firms as dividends.

C Derivation of the Wage Phillips Curve

The problem of the union representing workers on task k is:

$$\begin{aligned}
& \max_{\{\omega_{kt}\}_{t\geq 0}} \int_0^\infty e^{-\rho t}\left[\int_{s_{it}=e} u^e\left(c_{it}, h_{it}; z_{it}\right) di - \frac{\Theta}{2}\left(\frac{\dot{\omega}_{kt}}{\omega_{kt}} - \pi^*\right)^2\right] dt && \text{(C1)}\\
& s.t. && \text{(C2)}\\
h_{it} &= \int_0^1 h_{kt} dk && \text{(C3)}\\
c_{it} + \dot{a}_{it} &= r_t a_{it} + \left(1 - \tau + \tau^s\right) y_{it} + \phi_t - \phi_{it}^s && \text{(C4)}\\
y_{it} &= \frac{1}{P_t} z_{it} \int_0^1 \omega_{kt} h_{kt} dk && \text{(C5)}\\
h_{kt} &= \left(\frac{\omega_{kt}}{\omega_t}\right)^{-\varepsilon} h_t && \text{(C6)}
\end{aligned}$$

The first constraint faced by the union states that total hours worked by an employed worker equal the sum of hour worked on each task. The second constraint is the budget constraint of employed workers. Note that, symmetrically with the problem of the intermediate producers, we have introduced a wage subsidy τ^s financed by the lump-sum tax ϕ_{it}^s in order to offset the distortion in steady state hours and production caused by the unions' monopoly power. The third line is simply the definition of pre-tax real labor income. The fourth one states that contractual hours worked required by the union from all workers must satisfy firm's demand for effective labor on task k, ℓ_{kt}. This constraint is equivalent to (B5) because $\ell_{kt} = \left(\int_{s_{it}=e} z_{it} di\right) h_{kt}$ and $\ell_t = \left(\int_{s_{it}=e} z_{it} di\right) h_t$, and because all firms j demand the same amount of labor in a symmetric equilibrium.

Since each task-specific union is "small" (there is a continuum of tasks), the impact of a union's wage on an individual income and the firm's employment is negligible. In addition, the union takes the lump sum tax ϕ_{it}^s as given. As a result, the union takes as given all individual decisions embedded in the budget constraint, and the firm's labor demand curves for each task.

We can write this problem recursively as

$$\begin{aligned}
\rho J_t(\omega_{kt}) &= \max_{\pi_{kt}} \int_{s_{it}=e} \mathrm{u}^e(c_{it}, h_{it}; z_{it})\, di - \frac{\Theta}{2}\left(\pi_{kt} - \pi^*\right)^2 + \partial_\omega J_t(\omega_{kt})\,\omega_{kt}\pi_{kt} + \partial_t J_t(\omega_{kt}) \\
& s.t. \\
c_{it} &= r_t a_{it} + \left(1 - \tau + \tau^s\right) y_{it} + \phi_t - \phi_{it}^s - \dot{a}_{it} \\
y_{it} &= \frac{1}{p_t} z_{it} \int_0^1 \omega_{kt}\left[\left(\frac{\omega_{kt}}{\omega_t}\right)^{-\varepsilon} h_t\right] dk \\
h_{it} &= \int_0^1 \left[\left(\frac{\omega_{kt}}{\omega_t}\right)^{-\varepsilon} h_t\right] dk
\end{aligned}$$

The first order condition with respect to π_{kt} and the envelope condition are, respectively:

$$\begin{aligned}
\partial_\omega J_t(\omega_{kt})\,\omega_{kt} &= \Theta\left(\pi_{kt} - \pi^*\right) \\
\left(\rho - \pi_{kt}\right)\partial_\omega J_t(\omega_{kt}) &= \int_{s_{it}=e} \left\{\partial_c \mathrm{u}^e(c_{it}, h_{it}; z_{it})\left[\left(1 - \tau + \tau^s\right)\partial_\omega y_{it}\right] di + \partial_h \mathrm{u}^e(c_{it}, h_{it}; z_{it})\,\partial_\omega h_{it}\right\} di \\
& + \partial_{\omega\omega} J_t(\omega_{kt})\,\omega_{kt}\pi_{kt} + \partial_{\omega t} J_t(\omega_{kt})
\end{aligned}$$

where

$$\begin{aligned}
\partial_\omega y_{it} &= \frac{1}{P_t} z_{it}\omega_t^{\varepsilon} h_t (1 - \varepsilon) \int_0^1 \omega_{kt}^{-\varepsilon} dk \\
\partial_\omega h_{it} &= -\varepsilon h_t \omega_t^{\varepsilon} \int_0^1 \omega_{kt}^{-(1+\varepsilon)} dk
\end{aligned}$$

and note that we did not differentiate neither consumption nor participation/job acceptance decisions with respect to ω_{kt} because of the assumption of a 'small union' which cannot affect any individual decisions.

Imposing symmetry $(\omega_{tk} = \omega_t)$, we obtain

$$\begin{aligned}
\partial_\omega y_{it} &= \frac{1}{P_t} z_{it} h_t (1 - \varepsilon) \\
\partial_\omega h_{it} &= -\varepsilon h_t \omega_t^{-1}
\end{aligned} \tag{C7}$$

and

$$
\begin{aligned}
\partial_\omega J_t(\omega_t) &= \frac{\Theta(\pi_t - \pi^*)}{\omega_t} \\
(\rho - \pi_t)\,\partial_\omega J_t(\omega_t) &= \int_{s_{it}=e} \left\{ \partial_c u^e(c_{it}, h_{it}; z_{it}) \left[(1-\tau+\tau^s)\,\partial_\omega y_{it} \right] di + \partial_h u^e(c_{it}, h_{it}; z_{it})\,\partial_\omega h_{it} \right\} di \\
&\quad + \partial_{\omega\omega} J_t(\omega_t)\,\omega_t \pi_t + \partial_\omega J_t(\omega_t)
\end{aligned}
$$

Substituting (C7) into the envelope condition, we obtain

$$
\begin{aligned}
(\rho - \pi_t)\frac{\Theta(\pi_t - \pi^*)}{\omega_t} &= (1-\tau+\tau^s)\frac{1}{P_t} h_t (1-\varepsilon) \int_{s_{it}=e} \partial_c u^e(c_{it}, h_{it}; z_{it})\, z_{it} di \\
&\quad - \varepsilon \frac{h_t}{\omega_t} \int_{s_{it}=e} \partial_h u^e(c_{it}, h_{it}; z_{it})\, di + \partial_{\omega\omega} J_t(\omega_t)\,\omega_t \pi_t + \partial_{\omega t} J_t(\omega_t).
\end{aligned}
$$

Differentiating the first-order condition (C7) with respect to time yields

$$
\partial_{\omega t} J_t(\omega_t) + \partial_{\omega\omega} J_t(\omega_t)\,\dot{\omega}_t = \frac{\Theta \dot{\pi}_t}{\omega_t} - \frac{\Theta(\pi_t - \pi^*)}{\omega_t}\left(\frac{\dot{\omega}_t}{\omega_t}\right) \tag{C8}
$$

Substituting (C8) into the envelope condition yields

$$
\begin{aligned}
(\rho - \pi_t)\frac{\Theta(\pi_t - \pi^*)}{\omega_t} &= (1-\tau+\tau^s)\frac{1}{P_t} h_t (1-\varepsilon) \int_{s_{it}=e} \partial_c u^e(c_{it}, h_{it}; z_{it})\, z_{it} di - \varepsilon \frac{h_t}{\omega_t} \int_{s_{it}=e} \partial_h u^e(c_{it}, h_{it}; z_{it}) \\
&\quad + \frac{\Theta \dot{\pi}_t}{\omega_t} - \frac{\Theta(\pi_t - \pi^*)}{\omega_t}\pi_t
\end{aligned}
$$

Multiply both sides by ω_t/Θ to arrive at

$$
\begin{aligned}
(\rho - \pi_t)(\pi_t - \pi^*) &= \frac{\omega_t}{\Theta}(1-\tau+\tau^s)\frac{1}{P_t} h_t (1-\varepsilon) \int_{s_{it}=e} \partial_c u^e(c_{it}, h_{it}; z_{it})\, z_{it} di - \frac{\varepsilon}{\Theta} h_t \int_{s_{it}=e} \partial_h u^e(c_{it}, h_{it}; z_{it} \\
&\quad + \dot{\pi}_t - (\pi_t - \pi^*)\pi_t
\end{aligned}
$$

Simplifying this expression and rearranging, we obtain

$$
\rho(\pi_t - \pi^*) = \frac{\varepsilon}{\Theta} h_t \left[-\int_{s_{it}=e} \partial_h u^e(c_{it}, h_t; z_{it})\, di - \left(\frac{\varepsilon - 1}{\varepsilon}\right)(1-\tau+\tau^s)\, w_t \int_{s_{it}=e} \partial_c u^e(c_{it}, h_{it}; z_{it})\, z_{it} di \right] + \dot{\pi}_t \tag{C9}
$$

To fully undo the union's distortion to hours worked in steady state, we set $\tau^s = (1-\tau)/(\varepsilon - 1)$. We also assume that this subsidy is funded by the lump-sum tax, i.e. $\phi^s_{it} = \tau^s z_{it} w_t h_{it}$. As a result, in equilibrium it does not appear in either the household budget constraint nor in the government budget constraint, which is why we have omitted it altogether from the main text. After this substitution, and after setting $w_t = \alpha$ from (B7), we arrive at

$$\rho\left(\pi_t - \pi^*\right) = \frac{\varepsilon}{\Theta} h_t \left[- \int_{s_{it}=e} \partial_h u^e\left(c_{it}, h_t; z_{it}\right) di - (1-\tau)\,\alpha \int_{s_{it}=e} \partial_c u^e\left(c_{it}, h_{it}; z_{it}\right) z_{it} di \right] + \dot{\pi}_t$$

which equals the expression for the Phillips curve (9) in the main text, once we use our functional form for u^e in (2).

To obtain the linearized expression for the Phillips curve in (15), note that when linearizing the term in round brackets in (9) around the steady-state we have exploited the fact that $\psi(h^*)^{\frac{1}{\sigma}+1} = (1-\tau)\alpha h^*$.

D Problem of the Mutual Fund

Recall that the fund owns all intermediate good firms and holds real debt issued by the government and a nominal bond in zero net supply which carry a liquidity premium $\bar{\iota}$. Let X_t^f denote holdings of shares of the intermediate good producers, q_t the unit share price, Π_t per-share profits, B_t^f holdings of real government bonds, r_t^b the real interest rate on government bonds, M_t^f holdings of nominal bond in real terms, and r_t^m the real rate of return on the nominal bond. Let A_t be the real value of the fund. The problem of the mutual fund, which takes prices as given, entails choosing the optimal portfolio composition between equity, real bonds and nominal bonds:

$$\begin{aligned}
\tilde{r}_t A_t\left(X^f, B^f, M^f\right) = \max_{\dot{X}^f, \dot{B}^f,, \dot{M}^f} \quad & \Pi_t X^f - q_t \dot{X}^f + (r_t^b + \bar{\iota}) B^f - \dot{B}^f + (r_t^m + \bar{\iota}) M^f - \dot{M}^f \\
& + \partial_X A_t\left(X^f, B^f, M^f\right) \dot{X}^f + \partial_B A_t\left(X^f, B^f, M^f\right) \dot{B}^f \\
& + \partial_M A_t\left(X^f, B^f, M^f\right) \dot{M}^f + \partial_t A_t\left(X^f, B^f, M^f\right)
\end{aligned} \tag{D1}$$

with first-order conditions with respect to $\dot{X}^f$, $\dot{B}^f$, and $\dot{M}^f$

$$\begin{aligned}
q_t &= \partial_X A_t\left(X^f, B^f, M^f\right) \\
1 &= \partial_B A_t\left(X^f, B^f, M^f\right) \\
1 &= \partial_M A_t\left(X^f, B^f, M^f\right)
\end{aligned}$$

These first-order conditions, together with the linear homogeneity of the problem imply that $A_t\left(X^f, B^f, M^f\right) = q_t X^f + B^f + M^f$. Substituting this expression for A_t and the first order conditions into (D1), we arrive at

$$\tilde{r}_t \left(q_t X_t^f + B_t^f + M_t^f \right) = (\Pi_t + \dot{q}_t) X_t^f + (r_t^b + \bar{\iota}) B_t^f + (r_t^m + \bar{\iota}) M_t^f$$

Finally, by matching coefficients on equity and bonds, we obtain

$$\tilde{r}_t = \frac{\Pi_t + \dot{q}_t}{q_t} = r_t^b + \bar{\iota} = r_t^m + \bar{\iota}. \tag{D2}$$

which is the expression in equation (10) in the main text.

E Kolmogorov-Forward Equations

[TBC]

F Numerical Solution Method

Stationary Equilibrium and Impulse Response Functions. We solve for the stationary equilibrium using standard continuous-time finite-difference methods as described in Achdou et al. (2017).[39] This defines the steady state X^* for any variable of interest X which can be aggregate (e.g., unemployment) or distributional (e.g., total earnings at bottom tercile of skill distribution) variable. Out of steady state, we discretize our continuous-time economy in T monthly time steps and solve for the economy's first-order response to aggregate shocks using the sequence-space approach of Auclert et al. (2021). The most costly step in their method is the computation of the sequence-space Jacobian for the heterogeneous-agent block. For this step, we rely on their fake-news algorithm. We then write the sequence-space representation of the equilibrium of our economy, as described in Section 2.8. For any endogenous variable X and shock $k \in \{d, s, m\}$, we then solve for the general equilibrium Jacobians $\mathbf{G}^{x,k}$. These are $(T \times T)$ matrices mapping the impulse response to an unit innovation of aggregate shocks $\epsilon_t^k \equiv \sigma_k[\mathcal{W}_t^k - \mathcal{W}_{t-1}^k]$ in (17) to the equilibrium impulse responses of equilibrium variables, $d\mathbf{X}^k \equiv (X_1 - X^*, \ldots, X_T - X^*)'$.

Model Simulation and the ZLB. To simulate our economy, we adapt the procedure described in Boppart et al. (2017) in order to incorporate the ZLB as in Holden (2016, 2023).

[39] As the authors highlight, these methods generalize to stopping time problems such as the participation choice in our setup.

The idea is to exploit the linearity of the economy's response to aggregate shocks, while allowing for nonlinearities coming from an occasionally binding aggregate constraint, like the ZLB.

The simulation is implemented recursively, starting from $t = 0$ until T^{sim}. At each time t, we are interested in the full projected path for some variable X, $\mathbb{E}_t[X_{t+h}]$ for $h = 0, \ldots, T$. In what follows, we use boldface $\mathbf{X}_{(t)}$ to denote the expected time path of variable X at time t, i.e., $\mathbf{X}_{(t)} = (\mathbb{E}_t[X_{t+0}], \ldots, \mathbb{E}_t[X_{t+T}])'$. Also, we use $\mathcal{F}$ to represent the forward-shift operator, i.e., applying $\mathcal{F}$ to $\mathbf{X}_{(t)}$ delivers $\mathcal{F}\mathbf{X}_{(t)} = (\mathbb{E}_t[X_{t+1}], \ldots, \mathbb{E}_t[X_{t+T+1}])'$.

Absent the ZLB and with linearity, time-t projections are given by the moving-average (MA) process

$$\mathbf{X}_{(t)} = \sum_{s=0}^{T-1} \sum_{k=d,s} \left(\mathcal{F}^s d\mathbf{X}^k\right)\epsilon^k_{t-s}. \tag{F1}$$

which coincides with the expressions in Boppart et al. (2017) and Auclert et al. (2021). Alternatively, we can get a recursive representation of $\mathbf{X}_{(t)}$ by decomposing (F1) into period $t-1$ projections $\mathcal{F}\mathbf{X}_{(t-1)}$ and the impulse responses $d\mathbf{X}^k$ to time-t surprises

$$\mathbf{X}_{(t)} = \mathcal{F}\mathbf{X}_{(t-1)} + \sum_{k=d,s} d\mathbf{X}^k \epsilon^k_t. \tag{F2}$$

The recursive formulation in (F2) is very useful in the presence of the ZLB, as the probability of the ZLB binding or not in the future depends, in general, on the entire sequence of past shocks hitting the economy. In formulation (F2) this information is fully summarized in past projections $\mathcal{F}\mathbf{X}_{(t-1)}$.

With the ZLB, we must check and enforce that the projected path of nominal rates, $\mathbb{E}_t[\imath_{t+h}]$, does not violate the ZLB today or at any point in the future ($h \geq 0$). We start by representing the baseline monetary policy rule (13) as

$$\mathcal{A}\mathbf{y}_{(t)} + \mathbf{Z}^m_{(t)} = \mathbf{0}$$

where $\mathbf{y}_{(t)}$ collects the time path of variables entering the policy rule (e.g., interest rate, inflation, and unemployment), $\mathcal{A}$ is the linear map summarizing the policy rule (13) absent the ZLB, and $\mathbf{Z}^m_{(t)}$ denotes the set of "news" shocks to the policy rule at all future horizons. A policy shock at horizon h, for instance, captures a deviation from the policy rule $\mathcal{A}$ at time $t+h$ announced at time t. The idea is to use positive "news" shocks $\mathbf{Z}^m_{(t)} \geq 0$ to the interest rate path whenever the interest rate path set according to $\mathcal{A}$ would otherwise violate the ZLB constraint. In terms of our notation, this problem can be written as the following complementarity slackness condition on the paths of interest rates $\boldsymbol{\imath}_{(t)}$ and monetary policy shocks $\mathbf{Z}^m_{(t)}$

$$\boldsymbol{\imath}_{(t)} \cdot \mathbf{Z}^m_{(t)} = 0, \qquad \boldsymbol{\imath}_{(t)} \geq 0, \qquad \mathbf{Z}^m_{(t)} \geq 0. \tag{F3}$$

What makes the problem above complicated is that the nominal rate path is itself a function of the monetary "news" shock $\mathbf{Z}^m_{(t)}$, as these shocks affect the interest rate path both directly and indirectly through the feedback response to inflation and unemployment. Importantly, however, we know the impact that monetary "news" at any horizon have in interest rates: these are given by the columns of general equilibrium Jacobian $\mathbf{G}^{\imath,m}$ computed in the first step. Appealing again to linearity, we can write the project interest rate path as

$$\boldsymbol{\imath}_{(t)} = \mathcal{F}\boldsymbol{\imath}_{(t-1)} + \sum_{k=d,s} d\mathbf{X}^k \epsilon^k_t + \mathbf{G}^{\imath,m}\left(\mathbf{Z}^m_{(t)} - \mathcal{F}\mathbf{Z}^m_{(t-1)}\right). \tag{F4}$$

Substituting (F4) into the complementary slackness (F3), we arrive at a linear complementarity problem (LCP) in $\mathbf{Z}^m_{(t)}$ of the form $\mathbf{Z}^m_{(t)} \geq 0$, $Q\mathbf{Z}^m_{(t)} + q \geq 0$, and $\left(Q\mathbf{Z}^m_{(t)} + q\right) \cdot \mathbf{Z}^m_{(t)} = \mathbf{0}$ with

$$\begin{aligned} Q &= \mathbf{G}^{\imath,m} \\ q &= \mathcal{F}\boldsymbol{\imath}_{(t-1)} + \sum_{k=d,s} d\mathbf{X}^k \epsilon^k_t - \mathbf{G}^{\imath,m}\mathcal{F}\mathbf{Z}^m_{(t-1)}. \end{aligned}$$

Using a LPC solver to find $\mathbf{Z}^m_{(t)}$, we can then update projections of other variables X in the same way as we did for the policy rate $\boldsymbol{\imath}_{(t)}$

$$\mathbf{X}_{(t)} = \mathcal{F}\mathbf{X}_{(t-1)} + \sum_{k=d,s} d\mathbf{X}^k \epsilon^k_t + \mathbf{G}^{x,m}\left(\mathbf{Z}^m_{(t)} - \mathcal{F}\mathbf{Z}^m_{(t-1)}\right).$$

Computing Monetary Policy Counterfactuals. Our strategy for simulating the economy under the counterfactual monetary policy rules follows Hebden and Winkler (2024) and McKay and Wolf (2023). The basic idea is to use the previous section projections computed under IT, $\mathbf{X}^{IT}$, together with the impulse responses to monetary news shocks also under IT, $\mathbf{G}^{\cdot,m}$, to obtain the economy's counterfactual trajectory under any alternative monetary policy rule, $\mathbf{X}^{alt}$. The method again leverages the monetary "news" shocks to the policy rule. In particular, we choose shocks $\tilde{\mathbf{Z}}^m_{(t)}$ so that the IT news-perturbed nominal rate path, $\tilde{\boldsymbol{\imath}}^{IT}_{(t)}$ computed as in (F4) using shocks $\tilde{\mathbf{Z}}^m_{(t)}$, mimics exactly the interest rate path that would arise under the counterfactual regime $\boldsymbol{\imath}^{alt}_{(t)}$.[40] As remaining equilibrium conditions depend on policy only through the current and expected future path of nominal rates, the IT news-perturbed equilibrium allocations $\tilde{\mathbf{X}}^{IT}$ must coincide with the equilibrium allocations under the alternative monetary policy rule $\mathbf{X}^{alt}$. As McKay and Wolf (2023) put it "it does not matter whether the policy rate path comes about due to systematic conduct of policy or due to policy shocks."

Ergodic Distribution. The previous section described how to perform the simulation given a sequence of demand and supply shocks. The model moments reported for the ergodic distribution are computed from averages across 250 simulated samples. In each sample, we initialize the model at the non-stochastic steady state and simulate the economy for 160 quarters (40 years). We discard the first 10 years of each sample and compute moments on the last 30 years of the simulated data.

Filtering in the Great Recession Exercise. For the Great Recession exercise, conducted in Section 5.2, we need to find the sequence of innovations that replicate, in our model under a baseline IT rule, the observed U.S. unemployment and inflation dynamics. To do so we adopt the filtering procedure discussed in McKay and Wieland (2021). The benefit of their approach relative to more common Kalman filter algorithm is that it only requires information on the impulse response functions, which we have from above.

We start the filtering back in 1990:$Q$1 assuming that the economy starts from its non-stochastic steady-state, and proceed recursively. Abstracting from the ZLB, we can use (**??**) to recover the demand and supply series of innovations $\{\epsilon^d_t, \epsilon^s_t\}_{t\geq 1990Q1}$ that are consistent with the observed values of unemployment and inflation. At t = 1990:$Q$1, this just requires inverting the impact matrix of the impulse response function to find the shocks that move unemployment and inflation in the model as much as in the data. For t > 1990:$Q$1, we can use the same logic to find the shocks that explain the "surprise" movements in inflation and unemployment, i.e., the difference between current realizations and previous period projections $(X_t - \mathbb{E}_{t-1}[X_t])$.

To deal with the ZLB, we rely on monetary news shocks as before. Specifically, at each point t, we verify whether the projected path of nominal rates under the filtered shocks violates the ZLB. If it doesn't, we proceed to $t+1$. If it does, we compute the monetary news shocks to the path of nominal rates that ensure the constraint is satisfied along the projection, i.e., $\mathbb{E}_t[\imath_{t+h}] \geq 0$. Because the monetary news shocks also affect the economy's

[40] Hebden and Winkler (2024) detail how to compute these monetary "news" surprises for a counterfactual with an asymmetric policy rule like the Lower for Longer strategy we consider in our application.

response at time t, we need to solve for the news, demand and supply shocks jointly. An iterative algorithm works well in practice.

Limitations of our Method. A shortcoming of our simulations and policy counterfactuals is that these are computed—much like other papers in this literature (e.g., Williams, 2009; Kiley and Roberts, 2017; Bernanke et al., 2019)—under the assumption of perfect foresight, i.e., at each point in time agents in our model form expectations under the assumption that the economy won't be hit with any further aggregate shocks.

In particular, this implies that agents don't internalize that ZLB might bind in the future when it is not binding today, nor do they internalize, in normal periods, that the shortfall rule will not tighten aggressively if the economy were to enter a boom. Instead, agents always expect, throughout our simulation, that inflation will converge back to the target in the long run—inflation expectations are well "anchored"—even though (as we show) the presence of the ZLB or an asymmetric response in the monetary policy rule can push average inflation to deviate systematically from the target.

This point is crucial for the assessment of the inflation-inclusion trade-off. The worry is that by ignoring future uncertainty, we may be providing a too favorable view of the inflation-inclusion trade-off compared to a fully rational expectation solution of our economy under the Lower for Longer strategy. While computing our model's fully rational expectation solution is unfeasible, we offer two additional results that lessen this concern. First, we extend our solution method to take into account future aggregate uncertainty along the lines of the stochastic extended path discussed in Adjemian and Juillard (2013). As it turns out, incorporating aggregate uncertainty up to 8 quarters ahead has a negligible effect on the bias under IT and LfL approach. Qualitatively, the results align with our previous intuition: the possibility of a binding ZLB in the future lowers the inflation bias under IT, while the promise to run the economy hot raises the inflation bias under LfL. However, the magnitude of these changes to inflation and unemployment are tiny (smaller than 1bp) and don't change our previous findings. Second, we look at the percentage of time when inflation rises significantly above the target in our simulations. Weber et al. (2023) and Pfauti (2024) argue that inflation expectations are sticky, and agents only become more attentive and update their expectations when inflation significantly deviates relative to the status quo. In our simulations, however, inflation rarely crosses these attention thresholds and remains below 4%, the threshold estimated by Pfauti (2024), almost the entire simulation (99% of the time).

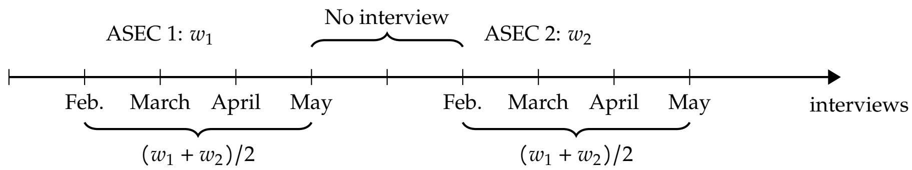


Figure G1: Graphical description of weekly earnings inference for individuals in our sample.

G Data on Worker Flows

G.1 Sample and Measure of Skills

We measure worker stocks and flows directly from the Basic Monthly Current population Survey (CPS) from 1989 to 2019. Our sample includes individuals between 25 and 59 years old and exclude self-employed, government employees and unpaid family workers. This sample selection leaves us with a sample of 18,697,072 workers, or 47,215 individuals per month on average. This is our primary sample to compute average worker stocks and flows.

To estimate how the dependence of flows on workers' skill levels z_{it}, we merge the CPS with the Annual Social and Economic Supplement (ASEC). The ASEC, which is conducted as a supplement of the March CPS, asks each individual additional questions about past earnings and weeks worked. We use this information to construct a measure of individual weekly earnings, which serves as our proxy for workers' skills. Every individual in our sample is interviewed at most twice in the ASEC supplement. For each interview the individual participates, we compute weekly real earnings by dividing annual nominal labor earnings [INCWAGE] by the product of number of annual weeks worked [WKSWORK1] and the Consumer Price Index (CPI-U) for that year. If individuals are interviewed only once, we use the weekly earnings of that interview for the whole period we observe that individual. If individuals are interviewed twice, we use the average of weekly earnings from the two interviews. We express skills z_{it} in relative terms by taking the ratio of individual weekly earnings to average weekly earnings computed for that year. We further drop all individuals who report positive weeks worked but no earnings as well as all entries that are assigned zero weight in the survey. After this step, we are left with a sample of 3,933,752 workers/months, or 21,264 individuals per month on average.[41] Figure G1 summarizes our approach to assign a weekly earnings, hence

[41] Because of the rotating nature of the CPS, in which households are interviewed for four months, then left aside for eight months, and finally interviewed again for four months, this leaves many individuals in the Basic Monthly CPS out of our merged sample because they are not part of any ASEC supplement. More

Data

	E	U	N
E	-	0.01265380	0.01699532
U	0.24649601	-	0.18987506
N	0.06965465	0.03964243	-

Data (deNUN)

	E	U	N
E	-	0.01265380	0.01699532
U	0.24649601	-	0.13261037
N	0.06960832	0.02669908	-

Table G1: Average labor market flows computed from the CPS. Raw (left table) and corrected (right table).

skill, level to individuals in the sample.

G.2 Measuring Worker Flows

We measure worker flows by looking at month-to-month changes in individual employment status [EMPSTAT]. A well known concern in this literature is the misclassification of workers' labor market status, especially between unemployment and nonparticipation. We follow the correction approach of Elsby et al. (2015) and re-code some sequences of labor market states to eliminate high-frequency reversals between unemployment and nonparticipation states.[42] The raw and corrected average worker flows are reported in Table G1.

To measure flows across the workers' skill distribution, we divide individuals into deciles based on the distribution of skills in the pooled time sample. For each quantile q and employment status s, we then measure flows by looking at the share of workers that transition to a different employment state s' from t to $t+1$. For instance, to compute EU flows for quantile q, we compute the share of workers whose skills fall in quantile q that are employed at time t and move to unemployment at time $t+1$. Figure G2 plots the average worker flows as a function of skill level.

The EU flow is strongly declining in skills: from a relative skill level of 1/2 to w (form half to twice the observed weekly earnings of an average worker), the job losing probability falls by almost a factor of two. The UE and NE flows are much flatter over the same region, trending down only for very low-skill levels. We also observe a strong dependence of EN and UN flows on skills, with workers at the bottom deciles of the distribution being far more likely to exit the labor force than workers at the top deciles. Overall, workers at the bottom of the skill distribution are more likely drop out of participation (EN and UN rates are larger at the bottom deciles) and, conditionally on participating in the la-

specifically, workers interviewed from July to November are not in our sample.

[42] Examples of the latter are recorded sequences 'UUNU' reclassified as 'UUUU' or 'NNUN' reclassified as 'NNNN', and so on.

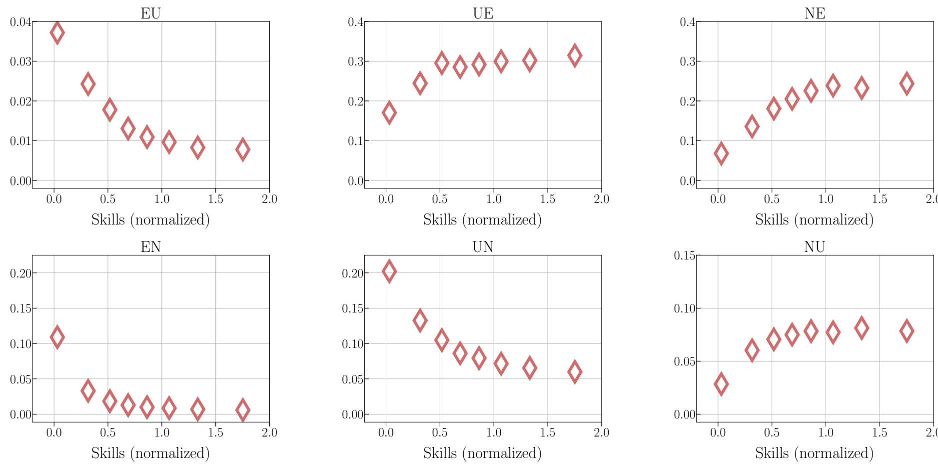


Figure G2: Labor market average transition rates as a function of workers' skill (weekly earnings) levels. The average skill level is normalized to 1 in our sample.

bor market, experience less stable employment (EU is high and UE is low at the bottom deciles).

G.3 Estimating Job-separation and Finding across the Skill Distribution

In the model, we make no restrictions on the dependence of $\lambda_z^{eu}, \lambda_z^{ue}, \lambda_z^{ne}$ on the worker's skill level z. To make progress in our estimation, we parameterize search frictions $\lambda_z^{ss'}$ according to the following functional form

$$\lambda^{ss'}(z) = \lambda_0^{ss'} + \lambda_1^{ss'} \exp(\lambda_2^{ss'} z) \qquad ss' \in \{eu, ue, ne\}. \tag{G1}$$

To get estimates for the parameters in (G1) for $ss' \in \{eu, ue, ne\}$ we proceed in two steps. First, we use the data on EU and UE flows by weekly earnings from Figure G2 to get an initial estimate for $\tilde{\lambda}_0^{ss'}, \tilde{\lambda}_1^{ss'}$, and $\tilde{\lambda}_2^{ss'}$ for $ss' \in \{eu, ue\}$. Specifically, we search for the values of $\lambda_0^{ss'}, \lambda_1^{ss'}, \lambda_2^{ss'}$ that minimize the quadratic distance between $\lambda^{ss'}(z)$, defined as in (G1), and the observed EU and UE transition probabilities at the empirical skill deciles. For the job-finding rate out of NE, λ_z^{ne}, we assume the same shape as the job-finding rate out of unemployment—i.e., we set $\tilde{\lambda}_i^{ne} = \tilde{\lambda}_i^{ue}$ for $i = 0, 1, 2$. These first-stage estimates are meant to capture the "shape" of transition rates along workers' skill level.

The second step, which takes place in the calibration stage, adjusts the "level" of frictions to match our empirical target for average flows. Specifically, we posit that model parameters are $\lambda_i^{ss'} = c^{ss'} \tilde{\lambda}_i^{ss'}$ for $i = 0, 1$ and $\lambda_2^{ss'} = \tilde{\lambda}_2^{ss'}$ for $ss' \in \{eu, ue, ne\}$, where the scaling coefficients $c^{ss'}$ serve to target average EU, UE and NE flows.

We don't use NE transitions across the skill distribution to inform the shape of $\lambda^{ne}(z)$ because a number of factors lead to a wedge between job-finding rates out of active nonparticipation (the object we want to parameterize) and NE flows (the object we actually observe in the data). Time aggregation (a measured NE transition can actually be a within period NU+UE), the presence of a stock of passive nonparticipants (who don't receive job offers but add to the measure of nonparticipants), or simply the acceptance decision of nonparticipants are all factors that can open up this wedge. Given that these factors can change systematically along the skill distribution, we prefer to rely on the UE flows to capture the shape of job-finding frictions.[43]

[43] UE transitions also suffer from a similar issue when not all job offers from unemployment are accepted. This is less of a concern in our exercise because unemployed workers almost always accept job offers in our model (all job offers are the same, so unemployed workers don't have an option value of waiting for better offers). The only case when an unemployed worker can reject a job offer in our model is when the worker would be better off leaving the labor force but chooses to remain unemployed to collect their unemployment insurance benefits. Given that UI duration is not long in our calibration, the importance of this wedge to our estimation of job-finding rate λ_z^{ue} is quantitatively small.

H Additional Tables

Table H1: Untargeted moments.

Stationary Equilibrium

	Data	**Model**
Marginal propensity to earn[1]	-0.023	-0.02
Unemployment rate bottom tercile[4]	0.150	0.122
Participation rate bottom tercile[4]	0.430	0.410
Cross-sectional earnings variance (age 25)[2]	0.595	0.530
Cross-sectional earnings variance (age 55)[2]	0.905	0.977
Standard dev. of one-year earnings change[2]	0.510	0.437
Skewness of one-year earnings change[2]	-1.070	-0.320
Kurtosis of one-year earnings change[2]	14.930	7.548
10-Year earnings losses full-year non-emp. (p25)[3]	-0.500	-0.452
10-Year earnings losses full-year non-emp. (p75)[3]	-0.300	-0.272

Out of Steady State (Standard Deviation)

	Data	**Model**
Employment	2.81	2.61
Labor Force Participation	1.38	0.99
Unemployment rate	29.36	27.61

Out of Steady State (Correlations with total hours)

	Data	**Model**
Employment	0.98	0.99
Labor Force Participation	0.76	0.78
Unemployment rate	-0.91	-0.97
EU	-0.82	-0.91
UE	0.88	0.91
NE	0.84	0.81
EN	0.51	0.76
UN	0.78	0.94
NU	-0.73	-0.84

Stationary Equilibrium. [1]Golosov, Graber, Mogstad, and Novgorodsky (2021); [2]Guvenen, Karahan, Ozkan, and Song (2021); [3]Guvenen, Karahan, Ozkan, and Song (2017); [4]Unemployment and participation rates across skill distribution are computed directly from the CPS using earnings data from the March ASEC Supplement (see Appendix G for a detailed description of our data and sample selection).

Out of Steady State. Data is quarterly from 1989 to 2019. Total hours and hours per worker are computed from the BLS-CES. Labor market stocks and flows are quarterly averages of monthly data computed directly from the CPS (see Appendix G for a detailed description of our data and sample selection). All series are in log and are detrended using a linear trend.

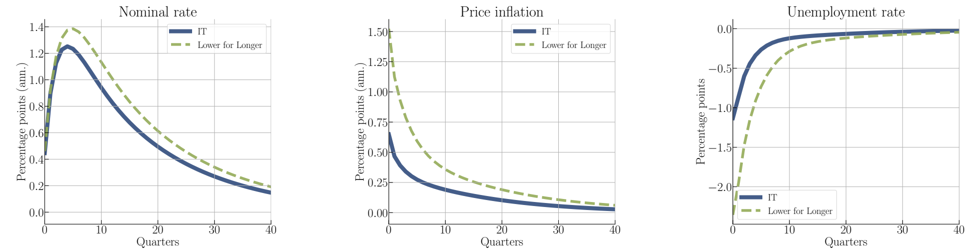
For all rules $\mathcal{R}_t$,	$\frac{d\imath_t}{dt} = \begin{cases} -0.07(\imath_t - i^* - \mathcal{R}_t) & \text{if } \imath_t > 0 \\ \max\left\{-0.07(\imath_t - i^* - \mathcal{R}_t), 0\right\} & \text{if } \imath_t = 0 \end{cases}$
Baseline Inflation Target (IT)	$\mathcal{R}_t = 2.25(\pi_t - \pi^*) - 0.15(u_t - u^*)$
Average Inflation Target (AIT)	$\mathcal{R}_t = 2.25(\pi_t - \pi^*) + 5.00(\pi_t^{MA} - \pi^*)^- - 0.15(u_t - u^*)$
Shortfall Rule (SR)	$\mathcal{R}_t = 2.25(\pi_t - \pi^*) - 0.15(u_t - u^*)^+$
Lower for Longer (LfL)	$\mathcal{R}_t = 2.25(\pi_t - \pi^*) + 5.00(\pi_t^{MA} - \pi^*)^- - 0.15(u_t - u^*)^+$
Higher IT	$\mathcal{R}_t = 2.25(\pi_t - \pi^{**}) - 0.15(u_t - u^*)$

Table H2: Monetary policy rules considered in our counterfactual experiments. X^+ is the shorthand for $\max\{X, 0\}$, while X^- stands for $\min\{X, 0\}$. π_t^{MA} is the exponential moving average of past inflation $\int_0^\infty (1/48)\ e^{-(1/48)\tau}(\pi_{t-\tau} - \pi^*)d\tau$ with a smoothing factor $1/48$ to target a window of 4 years.

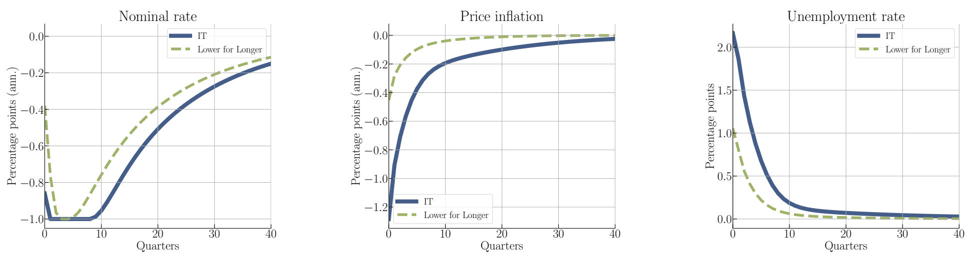
I Additional Figures

I.1 IRFs under Lower-for-Longer

(a) Demand-driven expansion



(b) Demand-driven recession



(c) Supply-driven expansion

(c) Supply-driven recession

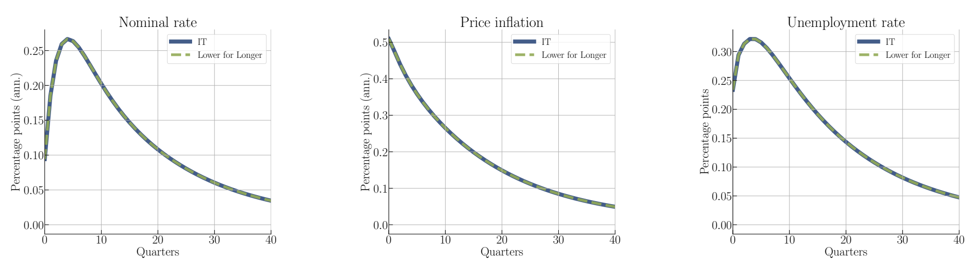


Figure I1: Impulse response under IT and LfL.

I.2 Great Recession

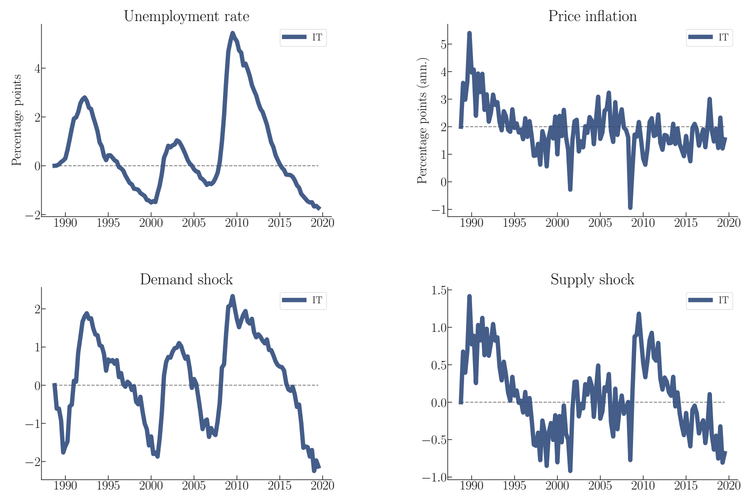


Figure I2: Price inflation is headline PCE inflation (Fred Series: PCEPILFE). The unemployment rate measure is computed directly from the CPS (see Appendix G for a detailed description of our data and sample selection).