The Job Ladder in HANK: Inspecting the Mechanism

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Abstract

I build an incomplete markets Heterogeneous Agent New Keynesian (HANK) model with labor market frictions and on-the-job search. Heterogeneity in productivity across jobs gives rise to a job ladder. Workers slowly move toward more productive and better-paying jobs through job-to-job transitions, but occasionally lose their jobs and fall down the ladder. I use this novel setting to study the transmission of standard New-Keynesian and labor market shocks, emphasizing the distinct roles played by the job ladder structure and the incomplete markets assumption. The job ladder component is a powerful internal propagation mechanism that amplifies and extends recessions. Incomplete markets, in turn, amplify shocks that trigger a strong reallocation along the ladder—such as those originating in the labor block of the model—but dampen shocks with a strong intertemporal substitution component—such as standard "demand and supply" New-Keynesian shocks. Quantitatively, the impact of these two factors can be quite substantial, with the ladder or incomplete markets more than doubling the change in consumption in some cases.

JEL Codes: D31, D52, E12, E21, E24, E32.

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1 Introduction

Search and matching frictions are pervasive in labor markets. It takes time for the worker to find a job and some jobs may be more desirable than others. As a result, frictional labor markets can give rise to a job ladder, i.e., a turnover process where some workers successfully move to more productive and better-paying jobs, while others lose their high-paying jobs and are forced to restart their careers at lower, less productive, "rungs" of the ladder. As the pace of this worker reallocation picks up during expansions and cools down during recessions, the ladder also acts as an important channel for the transmission of macroeconomic shocks to aggregate productivity, wages and consumption—a propagation mechanism Moscarini and Postel-Vinay (2017) calls the *cyclical job ladder*.

In this paper, I incorporate a canonical job ladder model into a standard incomplete markets heterogeneous agents New Keynesian (HANK) model to study the interaction of these two ingredients in the transmission and propagation of aggregate shocks. By enabling the joint study of aggregate and distributional effects within a unified framework, HANK models have been steadily incorporated to central banks toolkits as policymarkers let go of the fiction that stabilization and redistributive policies can be studied in isolation from each other (Sargent, 2024). Most of the HANK literature, however, still features a very stylized labor market block with no reference to unemployment or labor market turnover, despite the fact that those remain a central focus of the policy discussion.

In the model, search is random and workers search both off- and on-the-job for vacancies posted by firms. Jobs are ex-post heterogeneous in productivity, and firms Bertrand-compete for employed workers by making 'take it or leave it' wage offers, like in the Postel-Vinay and Robin (2002)'s sequential auction protocol. Between-firm competition drives individual wage growth and reallocates labor toward more productive jobs, giving rise to a job ladder in wages and productivity. As labor markets tighten during expansions, unemployed workers find jobs more easily, and employed workers reallocate to more productive and better-paying jobs. The opposite happens during recessions, when workers get stuck in unemployment and at the bottom low-paying rungs of the ladder. Overall, productivity and wage dynamics are endogenous and determined by the pace of workers' turnover along the ladder.[1] Finally, price rigidities in the New Keynesian tradition render aggregate demand forces—salient in the model thanks to the realistic large marginal propensities to consume (MPCs)—important for the equilibrium.

[1]This distinguishes the current setup from the majority of the models in the HANK literature, where labor productivity (and earnings) are tipically modeled as an exogenous and time-invariant process. Multiple papers add endogenous income risk through unemployment, but very few allow for worker turnover in a full job ladder. I cite the relevant references in the literature review.

I then use the model to study the propagation of four different sources of aggregate shocks: (1) a shock to the workers' time preference, (2) a shock to firms' desired markups, (3) a shock to job-separations, and (4) a shock to the vacancy-posting cost of labor intermediaries. The first two shocks are standard New-Keynesian "demand and supply shocks." The other two shocks are perturbations to the labor market block of the model. The job-separation shock (3) captures the increase in employment to unemployment flows during recessions and represents an outward shift of the Beveridge curve. The shock to the vacancy-posting cost (4) affects the free-entry condition for labor intermediaries and moves the economy along the Beveridge curve.[2] In all four cases, a contractionary shock that raises unemployment also shifts the distribution of employed workers to the bottom, low-productivity and low-paying, rungs of the ladder. Unlike unemployment, which goes back relatively quickly to steady state, the distribution of employed workers along the ladder is a slow-moving endogenous state that takes much longer to recover after shocks. The job ladder thus acts as a powerful internal propagation mechanism, which continues to hinder productivity, earnings, and consumption recovery, even after unemployment returns to normal.

I rely on a series of counterfactuals aimed at disentangling the contributions of the job ladder and incomplete markets—the model's two novel ingredients—for the transmission of aggregate shocks to the economy. In the first exercise, I "shut off" the *cyclical* aspect of the job ladder by exploring a counterfactual where worker turnover between jobs does not decline in response to a contractionary shock that raises unemployment. Specifically, I consider a counterfactual where the rate at which employed workers receive outside offers from other firms remains constant during contractions, a scenario I refer to as the *acyclical* ladder equilibrium in contrast to the *cyclical* ladder suggested by Moscarini and Postel-Vinay (2017). Despite job-to-job transitions affecting only a small share of workers in each period, fluctuations in on-the-job search significantly amplify the effect of aggregate shocks on consumption. For example, in response to a shock that raises firms' vacancy-posting costs and hampers job creation, I find that the consumption decline in the baseline model is five to six times larger than the reduction observed in the acyclical ladder counterfactual.[3]

In the second exercise, I compare the incomplete markets heterogeneous agent (HANK) model with a complete-markets representative-agent (RANK) benchmark. Interestingly, I find that the effect of incomplete markets on aggregates depends on the type of shock hitting the economy. In response to standard New-Keynesian demand and supply shocks, RANK

[2] Alternatively, the vacancy-posting cost shock leads to a negative correlation between vacancies and unemployment, while vacancies and unemployment are positively correlated under the job-separation shock.

[3] While I highlight the ladder as a force that propagates and extends recessions, I also show that it can have the opposite effect, i.e., the ladder can also speed up the economy's recovery if the shock causing the recession also leads worker turnover to increase instead of decline. The job-separation shock, which leads to a outward shift of the Beveridge curve, provides a clear example of this possibility.

exhibits sharper contractions than HANK. In response to the job-separation and vacancy-posting shocks, however, it is HANK that displays deeper and more prolonged recessions. The differences between RANK and HANK can be pretty substantial, with consumption in HANK falling 40% less in response to a markup shock but 2.5 times more in response to a job-separation shock. A PE-GE decomposition of the consumption response along the lines of Kaplan and Violante (2018) hints at the sources of these large differences. Consistent with the existing literature, I find that consumption in HANK is less sensitive to shifts in the workers' time preference or changes in the real interest rate—the intertemporal substitution channel—but reacts much more strongly than RANK to transitory movements in income—the income channel. These PE discrepancies highlight that HANK can either amplify or dampen the impact of an aggregate shock on consumption, depending on which of the two channels contributes the most to the propagation of the shock to consumption. In line with this intuition, shocks that transmit mainly through labor markets by shifting workers along the job ladder, such as the job-separation and vacancy-posting cost shocks, are amplified in HANK. In contrast, shocks that feature a strong intertemporal substitution channel, whether directly through preferences (like the time preference shock) or indirectly through large changes in the real interest rate (as for a markup shock), are dampened.[4]

Related Literature. This paper contributes to the burgeoning literature on Heterogeneous Agent New Keynesian (HANK) models (see Auclert, Rognlie, and Straub, 2024a, for a recent survey of this literature). Den Haan, Rendahl, and Riegler (2017), Gornemann, Kuester, and Nakajima (2016), Ravn and Sterk (2018), Kekre (2021), Lee (2021), Bardóczy (2022), Graves (2025) all study versions of HANK models with labor market frictions, but assume away on-the-job search. Relative to this literature, I highlight three main contributions. First, I show that even a simple job ladder already goes a long way in capturing some of the key stylized facts of income dynamics (e.g., wage losses from displacement and higher-order moments of the earnings growth distribution).[5] Given the central role of income risk for the amplification and propagation of business cycle shocks in HANK models (Acharya and Dogra, 2019; Bilbiie, Primiceri, and Tambalotti, 2022), it is important to have a model where these dynamics can arise endogenously. Second, I complement the existing discussion

[4] A very revealing neutrality case in the HANK literature is Werning (2015), whose baseline HANK model delivers the same response as RANK to a monetary policy shock. In his benchmark, the stronger income channel in HANK is exactly offset by the its weaker sensitivity to real interest rates, leading to the same consumption response as RANK in equilibrium. Besides the fact that I looking at different sources of aggregate shocks, the model here also deviates from Werning's neutrality case in important ways: (i) the ladder structure gives rise to heterogeneous exposures to aggregate fluctuations and countercyclical income risk, and (ii) the economy features a positive supply of government bonds with a deficit-financed fiscal policy in response to shocks.

[5] Hubmer (2018) also explores the ability of a job ladder model to fit the higher order moments documented by Guvenen, Karahan, Ozkan, and Song (2016), but does it in a partial equilibrium environment.

of transmission channels of aggregate shocks in HANK models by highlighting on-the-job search as a quantitatively important channel of transmission and a force capable of generating hump-shaped consumption dynamics (Alves, Kaplan, Moll, and Violante, 2020; Auclert, Rognlie, and Straub, 2020). Finally, my exercise comparing the responses in RANK and HANK provides yet another example where HANK is shown to have very different implications from the standard RANK model (Kaplan and Violante, 2018; Auclert, Rognlie, and Straub, 2024b).

More specifically, this paper fits into a growing literature of papers merging job ladder into standard New-Keynesian models. As such, I draw heavily on the seminal work by Moscarini and Postel-Vinay (2023), who were the first to develop a tractable setup merging these two literatures and use it to study the inflationary consequences of cyclical misallocation of workers along the ladder. This paper builds upon their work by extending their setup to an incomplete markets heterogeneous-agent (HANK) setting.[6] One of HANK's key strengths is its ability to generate realistic large MPCs out of unexpected transitory income changes (Kaplan and Violante, 2021). This marks a significant improvement over the complete markets representative-agent framework underlying the standard RANK model, which assumes permanent-income behavior and exhibits unrealistically low MPCs. By offering a more accurate depiction of individual consumption behavior, HANK offers a much more credible environment to study the transmission of fluctuations in worker mobility to the demand-side of the economy.[7]

This paper also relates to the work by Faccini and Melosi (2023), Birinci, See, Karahan, and Mercan (2024) and Darougheh, Faccini, Melosi, and Villa (2024). Using a stylized version of Moscarini and Postel-Vinay (2023) model, Faccini and Melosi (2023) derive a measure of interfirm wage competition from data on unemployment-to-employment and job-to-job transition rates, which they show can help explain the U.S. 'missing inflation' of the second half of the 2010s. Building on my setup, Birinci, See, Karahan, and Mercan (2024) develop a quantitative state-of-the-art HANK model to study both positive and normative implications of fluctuations in job-to-job transitions. On the positive side, they revisit Faccini and Melosi (2023) earlier analysis on the impact of job-to-job transitions on inflation dynamics over the last two U.S. recoveries (Great Recession and COVID-19). On the normative side, they study the properties of augmented Taylor rule that responds to job-to-job transitions on

[6] Lise (2012) and Graber and Lise (2015) offer an early analysis of the standard income fluctuation problem in the presence of on-the-job search, but do not extend the model to general equilibrium or introduce New-Keynesian frictions.

[7] Individually, large MPCs can replicate the fact that workers significantly cut their consumption expenditures in response to wage losses following displacement (Saporta-Eksten, 2014). In the aggregate, the average MPC is tightly linked to the strength of the Keynesian multiplier (Bilbiie, 2020) and the transmission mechanism of aggregate shocks to consumption (Auclert, Rognlie, and Straub, 2024b).

top of movements in unemployment and inflation. Finally, Darougheh, Faccini, Melosi, and Villa (2024) add endogenous on-the-job search to the incomplete markets model to study the effects of a tax reform in Denmark on employment and wage dynamics along the income distribution. Relative to this body of work, I conduct a detailed analysis of how the interaction between the job ladder structure and incomplete markets—two key building blocks of modern macroeconomics—shapes the propagation of aggregate shocks to productivity, earnings, and consumption.

Outline. The rest of the paper proceeds as follows. Section 2 outlines the model and defines the equilibrium. Section 3 discusses the calibration and the models' impulse response functions (IRFs) to aggregate shocks. Section 4 inspects the interaction of the job ladder and incomplete markets for the transmission of aggregate shocks, carefully highlighting the contribution of each ingredient. The last section concludes.

2 Model

Goods, Technology, and Agents. Time is continuous. There are three vertically integrated sectors in the economy, each producing a different type of good that can be used either as an input by other sectors or consumed.[8]

At the bottom of this supply chain, labor intermediaries hire workers in a frictional labor market by posting vacancies v_t. Unemployed and employed workers search for open job vacancies, with the unemployed searching at exogenously higher intensity than the employed $1 = s_u > s_e$. The flow of meetings at time t is given by a constant returns-to-scale matching function $\mathcal{M}(v_t, \mathcal{S}_t)$, where $\mathcal{S}_t \equiv 1 \times u_t + s_e \times (1 - u_t)$ denotes the total search effort of unemployed and employed workers. This defines the rate at which unemployed and employed workers meet with a vacancy, given by $\lambda_t \equiv {}^{\mathcal{M}(v_t,\mathcal{S}_t)}/_{\mathcal{S}_t}$ and $\lambda_{et} \equiv s_e \lambda_t$, respectively. A vacancy contacts a worker with intensity $q_t \equiv {}^{\mathcal{M}(v_t,\mathcal{S}_t)}/_{v_t}$. All matches are subject to a destruction shock at an exogenous flow rate δ_t which can vary over time. A worker–firm match produces z units of labor services, which is then sold in a competitive market at price φ_t. Match productivity z is fixed within the match but drawn from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0, 1]$ at the origination of the match. Letting $\Omega_t(z)$ be the distribution of employment across matches at time t, total production of labor services at time t is then given by $\int_{\underline{z}}^{\bar{z}} z d\Omega_t(z)$, whereas employment rate is given by $1 - u_t = \int_{\underline{z}}^{\bar{z}} d\Omega_t(z)$. Above the labor sector, there is a measure one of retailers indexed by $j \in [0, 1]$. Each retailer produces

[8] See Christiano, Eichenbaum, and Trabandt (2016) and Moscarini and Postel-Vinay (2023) for a similar supply-side structure.

a specialized input $\tilde{Y}_{j,t}$ with a constant returns-to-scale technology in labor services and intermediate inputs denoted by materials. The specialized inputs are then aggregated by a competitive representative firm to produce the final good $\tilde{Y}_t$.

Finally, the economy is populated by a continuum of ex-ante identical risk-averse workers indexed by $i \in [0,1]$, whose problem I turn to next.

Workers. Workers receive utility flow from consuming c_{it} and do not value leisure. Preferences are time-separable, and the future is discounted at rate ρ subject to a time preference shock μ_t

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \mu_t u(c_{it}) dt, \tag{1}$$

where the expectation reflects individual-level uncertainty with respect to movements along the job ladder. Workers at any time t are either unemployed ($\mathbb{1}^u_{it} = 1$) or employed ($\mathbb{1}^u_{it} = 0$). Unemployed workers receive $\varphi_t \times b$ in unemployment insurance (UI) benefits, while an employed worker at the "rung" y of the ladder earns $\varphi_t \times y$ in wages. Defining $\ell_{it} \equiv \left(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it}\right)$ as the workers' position in the ladder, we can summarize labor earnings as $\varphi_t \times \ell_{it}$. Besides collecting UI or wages, workers also receive lump-sum dividends d_{it} from firms and lump-sum transfers t_{it} from the government. Both profits and transfers are distributed in proportion to workers' position in the job ladder ℓ_{it}, that is

$$d_{it} = \frac{D_t}{\int_0^1 \ell_{it} di} \ell_{it}, \qquad t_{it} = T_t \ell_{it}$$

where $D_t = D^R_t + D^I_t$ denotes aggregate profits made by retailers and intermediaries, and T_t is level-shifter of government transfers. Under these distribution rules for profits and transfers, we can summarize workers' pre-tax income as $w_t \ell_{it}$, where

$$w_t = \varphi_t + \frac{D_t}{\int_0^1 \ell_{it} di} + T_t, \tag{2}$$

stands for the price "per unit of the ladder." It is well-known in the HANK literature that the responses of wages, profits, and transfers, as well as workers' exposure to their fluctuations, matters tremendously for the consumption response to aggregate shocks (Auclert, 2019; Patterson, 2023). Assuming that these income sources are all distributed proportionally to workers' position in the ladder sidestep these distributional issues by reducing workers' earnings dynamics to the evolution of two variables only: (i) the price "per unit of the ladder" w_t and (ii) workers' individual dynamics along the ladder ℓ_{it}.[9]

[9] As I discuss in specification of the fiscal policy, I further assume that transfers adjust to maintain the out-of-steady-state price w_t constant. Under this assumption, workers' dynamics along the job ladder—the new

Workers save through a riskless government bond at flow real rate r_t subject to a no-borrowing constraint. Wealth a_{it} thus evolves according to

$$\begin{aligned} \dot{a}_{it} &= (1-\tau_t) w_t \ell_{it} + r_t a_{it} - c_{it} \\ a_{it} &\geq 0, \end{aligned} \tag{3}$$

where τ_t is a proportional labor tax levied by the government.[10]

Workers maximize their lifetime utility given in (1) subject to the wealth accumulation process in (3), paths of real rate $\{r_t\}_{t\geq 0}$, labor taxes $\{\tau_t\}_{t\geq 0}$, price $\{w_t\}_{t\geq 0}$, and the process for workers' position in the job ladder $\{\ell_{it}\}_{t\geq 0}$.

Wage Determination. The determination of piece-rates y follows the Postel-Vinay and Robin (2002)'s sequential auction protocol, as implemented in Graber and Lise (2015). Consider a worker employed at a match of productivity z under a piece-rate contract y who gets contacted by a firm of productivity z' (I consider the unemployed case below). Firms get to observe each other's match productivities and engage in Bertrand competition in piece-rate contracts for the services of the worker.

First, imagine the situation where the poacher is more productive than the incumbent firm ($z' > z$). The incumbent's maximum wage offer is to promise the worker the whole output flow of the match, i.e., offer the worker a new contract with a piece-rate equal to the productivity of the match $y^* = z$. The poaching firm z' can thus attract the worker by offering a piece-rate wage contract of $y^* = z + \epsilon$, with $\epsilon \to 0$. In this scenario, the worker transitions from z to z' at a piece-rate wage contract equal to the productivity of the incumbent firm.[11]

Alternatively, consider the situation where the incumbent is more productive than the poacher ($z' \leq z$). In this case, the incumbent ends up retaining the worker. However, the threat from the poaching firm might still allow the worker to renegotiate its piece-wage contract within the firm. This can happen as long as the productivity of the outside firm is greater than the workers' current piece-rate contract with the incumbent ($z' > y$). If that is the case, I assume that the incumbent offers to cover the maximum piece-rate offer from the poacher, i.e., it offers the worker a piece-rate $y^* = z' + \epsilon$ with $\epsilon \to 0$. If $z' \leq y$ instead, the outside offer gets discarded by the worker.

ingredient of our model—becomes the single source of (pre-tax) earnings fluctuations in the model.

[10] Notice that the labor tax τ_t and lump-sum transfers T_t enter the household budget constraint in the same way, so we could, in principle, have written the household problem with only one of them. As I make clear in the discussion of fiscal policy, it is useful to separate the two because each instrument fulfils a different purpose.

[11] The assumption that the more productive firm matches the wage contract of the least productive firm—as opposed to the worker's value of remaining at the incumbent under the maximum wage offer—departs from Postel-Vinay and Robin (2002). This is taken from Graber and Lise (2015) and is made for tractability, as matching offers in terms of utility to the worker would result in wage offers that depend on workers' asset holdings.

Finally, if the firm meets an unemployed worker, I assume that the firm makes a piece-rate wage offer equal to the output flow of the least productive match, i.e., the firm offers $y^* = \underline{z}$. In the calibration, I choose the unemployment insurance replacement rate b to be equal to $\underline{z}$, so firms effectively offer the unemployed the value they get from unemployment insurance.

While firms are able to observe both their and their competitors match productivities, I assume that workers only get to observe the wage offers. This assumption greatly simplifies the unemployed job-acceptance decision by making all initial wage offers the same independently of the productivity of the workers' first match out of unemployment. On the other hand, the assumption implies that employed workers must now form beliefs about the productivity of match z in order to forecast the evolution of their piece-rate wages y_{it}. However, given my assumption on the wage process and match formation, the filtering problem turns out to be very tractable with workers' beliefs $\phi(z; y)$ about the current match productivity z depending only on their current piece-rate wage y. For a description of the worker's filtering problem, I refer the reader to Appendix A.

Workers' Value Function. Having specified the process for piece-rate wages y_{it}, we are ready to write down the workers' value function. Let $V_t(a, y)$ denote the optimal value of worker's problem starting from an initial level of assets a and piece-rate y, where $y = 0$ index the unemployment state. Notice that the workers' state does not include the match productivity z, consistent with our assumption that workers' only observe their wages.

The Hamilton–Jacobi–Bellman (HJB) for an unemployed worker satisfies

$$\begin{aligned}\rho V_t(a,0) - \partial_t V_t(a,0) = &\max_c \left\{ \mu_t u(c) + \partial_a V_t(a,0) \left[(1-\tau_t) w_t b + r_t a - c\right] \right\} \\ &+ \lambda_t \Big(V_t(a,\underline{z}) - V_t(a,0) \Big),\end{aligned} \tag{4}$$

where λ_t is the job finding-rate and $V_t(a, \underline{z})$ is the value of a employed worker with piece-rate wage $\underline{z}$. Employed workers' value function in turn evolves as

$$
\begin{aligned}
\rho V_t(a,y) - \partial_t V_t(a,y) = \max_c \; & \Big\{ \mu_t u(c) + \partial_a V_t(a,y)\,[(1-\tau_t)w_t y + r_t a - c] \Big\} \\
& + \delta_t \Big(V_t(a,0) - V_t(a,y) \Big) + \lambda_{et} \int_y^{\bar{z}} \Big[\\
& \qquad \Gamma(y) \Big(V_t(a,y) - V_t(a,y) \Big) + \\
& \qquad \int_y^z \Big(V_t(a,z') - V_t(a,y) \Big)\, \mathrm{d}\Gamma(z') + \\
& \qquad \bar{\Gamma}(z) \Big(V_t(a,z) - V_t(a,y) \Big) \\
& \Big] \phi(z;y)\, \mathrm{d}z.
\end{aligned}
\tag{5}
$$

At rate δ_t the match is dissolved and the worker "falls back" into unemployment. At rate λ_{et} the worker meets an outside firm, in which case the piece-rate wage y evolves as follows. If the worker matches with a firm less productive than the worker's current piece-rate wage ($z' < y$), the offer is discarded (line 3). If the worker matches with a firm less productive than the current match but more productive than the worker's current piece-rate wage ($z' \in [y,z]$), between-firm competition for the worker leads to a renegotiation of the piece-rate wage at the incumbent firm to z' (line 4). If the worker matches with a firm more productive than the current match ($z' > z$), the worker gets poached by the more productive firm at a piece-rate equal to the current match's productivity z (line 5). As workers don't know the current match's productivity, they take expectations over the different scenarios according to their belief $\phi(z;y)$.

Next, I turn to the supply side of the economy, starting from the problem of the final good producer.

Final Good Producer. A competitive representative final good producer aggregates a continuum of specialized inputs, $\tilde{Y}_{j,t}$, using the technology

$$
\tilde{Y}_t = \left(\int_0^1 \tilde{Y}_{j,t}^{\frac{\vartheta_t - 1}{\vartheta_t}} \, dj \right)^{\frac{\vartheta_t}{\vartheta_t - 1}}, \tag{6}
$$

where $\vartheta_t > 0$ is the elasticity of substitution across goods. The firm's demand for the jth input is given by

$$
\tilde{Y}_{j,t}(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} \right)^{-\vartheta_t} \tilde{Y}_t, \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\vartheta_t} dj \right)^{\frac{1}{1-\vartheta_t}}. \tag{7}
$$

Retailers. The jth input good in (6) is produced by a retailer, who is a monopolist in the product market. Following Basu (1995) and Nakamura and Steinsson (2010), each retailer produces their specialized good by combining materials $M_{j,t}$ and labor services $N^e_{j,t}$ according to the production function

$$\tilde{Y}_{j,t} = M^{\gamma}_{j,t}(\bar{Z}N^e_{j,t})^{1-\gamma}, \tag{8}$$

where $\bar{Z}$ is an aggregate productivity component. Materials are converted one-for-one from the final good $\tilde{Y}_t$ in (6), so each retailer effectively uses the output of all other retailers to produce its own good.[12] Labor services are sold at the competitive price φ_t, while materials are sold at a unity price. Cost minimization implies a common material to labor ratio and marginal cost across all retailers, given respectively by

$$\frac{M_{j,t}}{N^e_{j,t}} = \frac{\gamma}{1-\gamma}\varphi_t, \qquad m_t = \left(\frac{1}{\gamma}\right)^{\gamma}\left(\frac{\varphi_t/\bar{Z}}{1-\gamma}\right)^{1-\gamma}. \tag{9}$$

Each retailer also chooses a price $P_{j,t}$ to maximize profits subject to the demand curve (7) and price adjustment costs Θ_t

$$\Theta_t\left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right) = \frac{\theta}{2}\left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right)^2\tilde{Y}_t, \tag{10}$$

as in Rotemberg (1982).[13] Therefore, the problem of each retailer consists in choosing $\{P_{j,t}\}_{t\geq 0}$ to maximize expected discounted profits

$$\mathbb{E}_0\int_0^{\infty} e^{-\int_0^t r_s ds}\left\{\left(\frac{P_{j,t}}{P_t} - m_t\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\vartheta_t}\tilde{Y}_t - \Theta_t\left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right)\right\}dt,$$

In a symmetric equilibrium, all firms choose the same price $P_{j,t} = P_t$ and produce the same amount of goods $\tilde{Y}_t$ by employing labor services N^e_t and materials M_t at the same ratio. As such, retailer profits are given by $D^R_t = (1-m_t)\tilde{Y}_t - \Theta_t$. As shown in Kaplan, Moll, and Violante (2018), this pricing problem of the firm yields a simple New Keynesian Phillips curve characterizing the evolution of aggregate inflation $\pi_t \equiv \dot{P}_t/P_t$ as a function of marginal costs

$$\left(r_t - \frac{\dot{\tilde{Y}}_t}{\tilde{Y}_t}\right)\pi_t - \dot{\pi}_t = \frac{\vartheta_t}{\theta}\left(m_t - m^*_t\right), \tag{11}$$

[12]See Christiano, Trabandt, and Walentin (2010) for a discussion of the role of materials in a standard New-Keynesian model.

[13]I assume that price adjustment costs are "virtual," meaning that they affect optimal choices but do not cause real resources to be expended. That is why pricing costs do not appear in the goods market clearing condition below.

where $1/m_t^* = \left(\frac{\vartheta_t - 1}{\vartheta_t}\right)^{-1}$ denotes firms' desired markup. Shocks to the elasticity of substitution ϑ_t changes firms' desired markup and act as a standard cost-push supply shock.

Labor Intermediaries. Labor intermediaries can post vacancies at a exogenous flow cost of κ_t^f, expressed in units of the consumption good. Upon meeting a worker, the firm must pay an additional fixed screening cost to start producing. This screening cost $\tilde{\kappa}^{si}$ is allowed to depend on the employment status $i \in \{e, u\}$ of the worker contacted by the firm, but, different from the vacancy-posting cost, it does not expend any real resources, so it does not show up in goods market clearing condition.[14]

Let $\mathcal{J}_t(z, y)$ denote the expected present discounted value of dividends for a firm with match productivity z currently offering a piece-rate contract y to its worker. The value $\mathcal{J}_t(z, y)$ satisfies the following HJB

$$\begin{aligned} r_t \mathcal{J}_t(z,y) - \partial_t \mathcal{J}_t(z,y) = \varphi_t(z-y) + \delta_t \Big(0 - \mathcal{J}_t(z,y)\Big) + \lambda_{et} \Big[& \\ \Gamma(y)\Big(\mathcal{J}_t(z,y) - \mathcal{J}_t(z,y)\Big) + & \\ \int_y^z \Big(\mathcal{J}_t(z,z') - \mathcal{J}_t(z,y)\Big) d\Gamma(z') + & \\ \bar{\Gamma}(z)\Big(0 - \mathcal{J}_t(z,y)\Big)\Big]. & \end{aligned} \tag{12}$$

At rate δ_t the match is dissolved. At rate λ_{et} the worker meets with an outside firm, in which case the continuation value of the match is the counterpart of the one for the employed worker described in (5). If the poaching firm's productivity z' is less than the current piece-rate y, the offer is discarded leaving the firm with the same continuation value $\mathcal{J}_t(z, y)$ (line 2). If the poaching firm's productivity z' is between the current piece-rate y and the productivity of the match z, then the firm retains the worker at a continuation value of $\mathcal{J}_t(z, z')$ (line 3). If the productivity of the poaching firm is above the productivity of the match, i.e., $z' > z$, then the match is dissolved leaving the firm with a continuation value of zero (line 4).

Vacancies are pin down by a free-entry condition. This condition equates the expected flow cost of hiring a worker (κ_t^f/q_t) to the firms' expected value from a match, which, in turn, depends on whether the firm meets with an unemployed versus an employed worker. Upon meeting with an unemployed worker, the firm pays a screening cost $\tilde{\kappa}^{su}$ and hires the worker at the piece-rate $\underline{z}$. Taking expectations with respect to the productivity of the new match

[14]These costs can be thought of as utility costs associated with the training and/or the screening of workers. As discussed in Pissarides (2009), screening costs raise amplification of unemployment fluctuations to aggregate shocks by insulating hiring costs from vacancy congestion coming from the matching function.

$z \sim \Gamma$, the expected value of meeting with an unemployed worker is

$$E\mathcal{J}^U = \int_{\underline{z}}^{\bar{z}} \left[\mathcal{J}_t(z,\underline{z}) - \tilde{\kappa}^{su}\right] d\Gamma(z).$$

Different from meeting an unemployed worker, if the firm contacts an employed workers it only gets to form a match if it can draw a productivity z greater than the productivity of the workers' current match z'. If successful, the firm pays the screening cost $\tilde{\kappa}^{se}$ and poaches the worker at a piece-rate z' according to our wage determination rule. In expectation, the value of meeting with an employed worker is thus

$$E\mathcal{J}^E = \int_{\underline{z}}^{\bar{z}} \left\{ \int_{\underline{z}}^{z} \left[\mathcal{J}_t(z,z') - \tilde{\kappa}^{se}\right] \frac{d\Omega_t(z')}{1-u_t} \right\} d\Gamma(z).$$

Weighting by the probabilities of meeting an unemployed and employed worker, the free-entry condition writes as

$$\frac{\kappa_t^f}{q_t} = \frac{u_t}{u_t + s_e(1-u_t)} E\mathcal{J}^U + \frac{s_e(1-u_t)}{u_t + s_e(1-u_t)} E\mathcal{J}^E. \tag{13}$$

Finally, using $\Omega_t(z,y)$ to denote the distribution of workers over match productivities z and piece-rate wages y, we can write intermediaries profits as $D_t^I = \varphi_t \int (z-y) d\Omega_t(z,y)$.

Fiscal Policy. The government issues real bonds of infinitesimal maturity B_t, provides (proportional) lump-sum transfers $t_{it} = T_t \ell_{it}$ to workers, taxes workers' total earnings at rate τ_t, finances the unemployment insurance benefits, and pays interest on its debt $r_t B_t$.

In steady state, transfers T_t are set to zero and, as a result, do not appear in either workers' or the government's budget constraints. Outside the steady state, T_t is used to stabilize the price w_t faced by workers. Specifically, T_t adjusts according to:

$$w \equiv \varphi + \frac{D}{\int_0^1 \ell_i di} = \varphi_t + \frac{D_t}{\int_0^1 \ell_{it} di} + T_t,$$

ensuring a constant price $w_t = w$ independently of variations in profits (D_t) and the price of labor services (φ_t).[15]

The labor income tax τ adjust residually to finance the UI and the interest rate payments on the government debt in steady state. Outside the steady state, labor income taxes τ_t

[15] By making the price faced by workers constant, this assumption renders workers' dynamics along the job ladder—the new ingredient of our model—the single source of (pre-tax) earnings fluctuations.

adjusts according to

$$\tau_t - \tau = \phi_B \frac{(B_t - B)}{\left(w \int_0^1 \ell_{it} di\right)}$$

to ensure that government debt B_t returns to its steady-state level B in the long-run.[16]

Under these choices for government taxes and transfers, the evolution of government bonds is governed by:

$$\dot{B}_t = r_t B_t + T_t \int_0^1 \ell_{it} di + \varphi_t b u_t - \tau_t w \int_0^1 \ell_{it} di. \tag{14}$$

Monetary Policy. The monetary authority sets the nominal interest rate on nominal government bonds i_t according to a Taylor rule reacting to inflation only

$$i_t = \bar{r} + \phi_\pi \pi_t. \tag{15}$$

Given inflation and the nominal interest rate, the real return on the government bonds r_t is determined by the Fisher equation, $r_t = i_t - \pi_t$. I now turn to the definition of equilibrium.

2.1 Equilibrium

Definition 1 (Equilibrium)**.** For a given initial distribution of workers and a time path for shocks $\{\mu_t, \vartheta_t, \delta_t, \kappa_t^f\}_{t \geq 0}$, a general equilibrium is a path for aggregate prices and quantities $\{r_t, \varphi_t, m_t, \pi_t, \tilde{Y}_t, N_t^e, M_t, D_t, v_t\}_{t \geq 0}$, government policies $\{B_t, \tau_t, T_t, i_t\}_{t \geq 0}$, labor market rates $\{\lambda_t, \lambda_{et}, q_t\}_{t \geq 0}$, such that workers and firms optimize, monetary and fiscal policy follow their rules, the free-entry condition for vacancies holds, the workers' labor income process is the outcome of labor market transitions and the wage-setting protocol, distributions are consistent with labor market transition rates and worker's decision rules, and all markets clear.

There are three markets in this economy: the market for government bonds, the market for labor services, and the final good market. The market for bonds and the final good clears when

$$\int a_{it} di = B_t,$$
$$\int c_{it} di + M_t + \kappa_t^f v_t = \tilde{Y}_t.$$

[16] See Alves, Kaplan, Moll, and Violante (2020) for a discussion of the role of government fiscal rule for the consumption response in the context of HANK models.

The labor market clears when the demand for labor services from retailers equals the supply from labor intermediaries

$$N_t^e = \int_{\underline{z}}^{\bar{z}} z d\Omega_t(z).$$

3 Parameterization, Earnings Dynamics and IRFs

This section is divided in three subsections. Subsection 3.1 discusses the parameterization of the model. Subsection 3.2 computes some relevant moments of the stationary distribution of earnings, and compare those to the data. Subsection 3.3 presents the model's response to aggregate shocks.

Table 1: List of parameter values and targeted moments

Variable		Value	Target
Labor market			
$\mathcal{M}$	matching function	$v^{0.5}\mathcal{S}^{0.5}$	Literature
δ	destruction rate	0.024	$EU = 2.4\%$
s_e	employed search intensity	0.258	$EE = 2.4\%$
κ^f	flow cost	0.34	$UE = 36\%$
$\tilde{\kappa}^{su}$	screening costs unemp.	3.4	Screening/total hiring costs = 90%
$\tilde{\kappa}^{se}$	screening costs emp.	1.0	$E\mathcal{J}^U = E\mathcal{J}^E$
β	productivity grid	10.0	Residual wage dispersion[A]
c_1	productivity grid	2.61	Residual wage dispersion[A]
Preferences			
$u(\bullet)$	utility function	$\log(\bullet)$	—
ρ	discount rate	0.08/12	$r^{ann} = 0.02$
Monetary and Fiscal Policy			
ϕ_π	Taylor rule coeff.	1.25	Literature
b	UI replacement rate	$\underline{z}$	Literature
τ	labor tax rate	0.022	Gov. budget constraint
ϕ_B	labor tax reaction	0.02	Gov. debt half-life of 3 years
B/Y^{ann}	government debt	0.28	$\mathbb{E}[MPC_i] = 0.25$[B]
Retailers and Final Good			
γ	material share	0.50	Literature
ϑ	elasticity of substitution	10.0	Literature
ϑ/θ	slope of Phillips curve	0.001	Literature

[A] Firm productivity distribution is used to target a 90-10 and 50-10 log wage gaps for the residual wage distribution of 1.10 and 0.64, respectively. Values are taken from Autor, Katz, and Kearney (2008, Figure 8).
[B] Kaplan and Violante (2021) places the average quarterly MPC on non-durable goods and services out of windfalls of \$500-\$1,000 between 15% and 25%.

3.1 Model Parameterization

The calibration of the model is divided into six main steps. First, I calibrate the labor market transition rates to match the estimated flows. Second, I choose the (flow and screening) vacancy costs. Third, I calibrate the firm productivity distribution so that the wage dispersion in the model matches the residual wage dispersion in the data. Fourth, I set the discount rate and the supply of government bonds so that the model can deliver a large average MPC at the targeted real interest rate. Fifth, I select the parameters on the production and monetary side to standard values used in the New-Keynesian literature. The full list of parameter values is displayed in Table 1.

Labor Market Frictions. I calibrate the model at a monthly frequency. I assume a standard Cobb–Douglas matching function $\mathcal{M}(v, \mathcal{S}) = v^{\alpha}\mathcal{S}^{1-\alpha}$, with an elasticity with respect to vacancies of $\alpha = 0.5$, as in Moscarini and Postel-Vinay (2018). I target a job-finding rate λ of 0.45 corresponding to a monthly unemployment-to-employment transition probability of $1 - exp(\lambda) = 0.36$, and set $\delta = 0.024$ to match the monthly probability of transitioning from employment to unemployment. Together, the two rates imply a steady-state unemployment rate of 5%. The relative search efficiency of employed worker s_e is chosen to obtain a monthly job-to-job transition rate of 2.4% in the steady state.

Vacancy Costs. The vacancy cost parameters are chosen to satisfy the free-entry condition (13) with the ratio of screening cost to total hiring costs ratio at 90%, in line with the estimates in Christiano, Eichenbaum, and Trabandt (2016).[17] These two restrictions are enough to pin down the flow cost κ^f and the total expected screening costs (in utils units), but they can't separately identify screening costs for employed and unemployed workers. For that, I impose the additional restriction that labor intermediaries are indifferent between meeting employed and unemployed workers in the steady state, i.e., $E\mathcal{J}^U = E\mathcal{J}^E$.

This assumption deviates from the rest of the literature, which typically assumes a single fixed screening cost. Since firms have to pay much higher wages to attract employed workers, a common fixed cost would make firms more willing to post vacancies whenever unemployment is high, dampening the unemployment response to aggregate shocks as discussed in Moscarini and Postel-Vinay (2018). In my experience, this composition effect of the search pool can be quite strong, making it hard for the model to generate reasonable unemployment fluctuations in response to aggregate shocks.[18] Having larger screening costs

[17]The constant returns to scale matching function allows us to substitute the vacancy filling rate q_t with $q_t = \lambda_t^{-\frac{1-\alpha}{\alpha}}$, expressing the free-entry condition (13) in terms of the λ_t.

[18]Of course, one could always choose the size of shock to induce the desired response in labor market tightness

for the unemployed so that firms are indifferent between meeting unemployed and employed workers in the steady state $E\mathcal{J}^U = E\mathcal{J}^E$ removes the dampening force and allows the model to generate a strong response of unemployment to aggregate shocks. Notice, however, that the model still predicts that shifts to the distribution of employed workers across matches $\Omega_t(z)$ feeds into firms' incentives to post vacancies, a point I return to later when discussing the effect of a job-separation shock.

Firm Productivity Distribution. The productivity distribution Γ is assumed to be an affine transformation of a Beta distribution; that is, a match productivity $z = c_0 + c_1 X$, where $X \sim \text{Beta}(1, \beta)$. I fix c_0 to 0.3 just as a normalization. Next, I choose the scale and shape parameter (c_1, β) to match a 50–10 and 90–10 log wage gaps of 0.64 and 1.10, respectively, in line with the estimates in Autor, Katz, and Kearney (2008, Figure 8) for the U.S. *residual* wage distribution—the remaining wage dispersion after controlling for individual characteristics such as experience, education, marital status, and gender.

Preferences and Liquidity. Workers have log utility over consumption. I adjust the discount rate of workers and the level of government bonds to target an average quarterly MPC of 0.25 with a real interest rate of 2%. This strategy delivers an annual discount rate of 8%, and a government debt to annual output of 28%.[19]

Fiscal and Monetary Policy. Unemployment insurance guarantees the unemployed the same wage income as a recently employed worker (i.e., $b = \underline{z}$), which corresponds to a replacement rate of approximately 50%, within the range of the values used in the literature. The steady-state labor income tax is set residually from the government budget constraint, while the sensitivity ϕ_B to government debt is set to 0.02, implying a half-life of roughly 3 years to the deviations of government debt from steady-state level B. Steady-state inflation is set to zero. The Taylor rule coefficient on deviations of inflation from steady state ϕ_π is equal to 1.25.

Production. The elasticity of substitution for the inputs produced by retailers ϑ is set to 10. The input share of materials γ is set to 0.5, which lies in the interval of values considered in

and thus unemployment. However, as my results in Section 4.2 highlight, the effect of incomplete markets on consumption depends heavily on the response of income vis a vis other intertemporal substitution channels. As such, it is important to get a (realistic) sizable response in labor market tightness—the main driver of workers' income—to aggregate shocks so that the model displays a reasonable transmission of aggregate shocks to consumption.

[19] As discussed in Kaplan and Violante (2021), one-asset HANK models feature a tension between matching the observed high levels of wealth together with the large estimated MPCs. Even though that tension is also present here, I note that the resulting liquidity for my calibration targeting MPCs is not too far from the *liquid wealth* calibration in Kaplan and Violante (2021) of 0.56.

Table 2: Moments of earnings growth distribution

Moment	Data	Model
$Var[\Delta\tilde{y}^A]$	0.260	0.140
$Skew[\Delta\tilde{y}^A]$	-1.07	-0.721
$Kurt[\Delta\tilde{y}^A]$	14.93	5.907
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.05$	0.310	0.337
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.10$	0.490	0.434
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.20$	0.670	0.578
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.50$	0.830	0.838

Notes: $\tilde{y}^A_{it} \equiv \log y^A_{it}$ denotes the level of annual log earnings, with (log-)growth rates written as $\Delta\tilde{y}^A_{it} \equiv \tilde{y}^A_{i,t+1} - \tilde{y}^A_{it}$. **Data:** Moments of the one-year log labor earnings changes distribution computed from the Master Earnings File of the Social Security Administration data (values taken from Guvenen, Karahan, Ozkan, and Song, 2016). **Model:** Moments of the one-year log labor earnings changes distribution computed by simulating a panel of 100,000 workers in the stationary equilibrium of the model.

Nakamura and Steinsson (2010). I set the price adjustment cost θ coefficient, so the slope of the Phillips curve is given by 0.001 monthly (or approximately 0.01 quarterly), a value within the range of estimates obtained by Del Negro, Lenza, Primiceri, and Tambalotti (2020).

3.2 Earnings and Consumption Dynamics in the Stationary Equilibrium

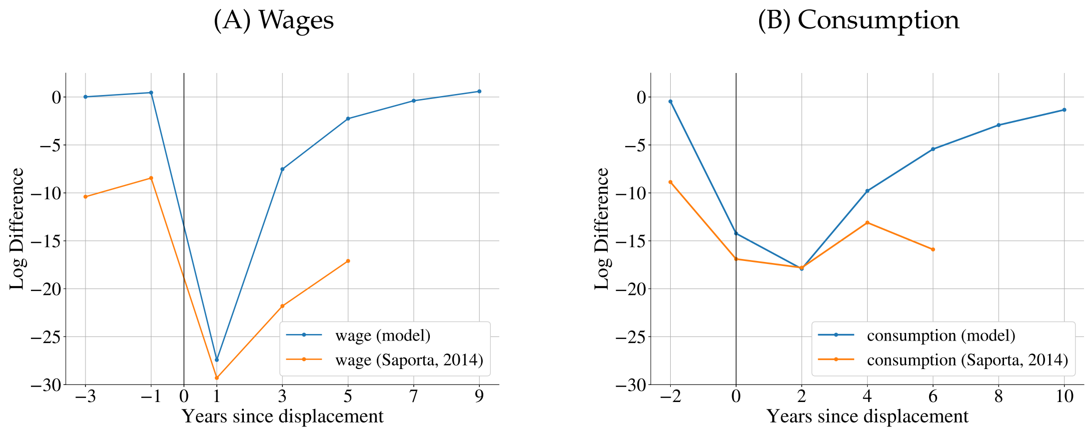
Before turning to the model's aggregate response to shocks, I present some non-targeted moments of the earnings and consumption dynamics at the stationary equilibrium. As I show, the job ladder structure of the model, although parsimonious, successfully captures (at least qualitatively) the higher-order moments of the earnings growth distribution (Guvenen, Karahan, Ozkan, and Song, 2016) and the large and persistent losses upon displacement (Saporta-Eksten, 2014).

Earnings Growth Distribution. For a worker i with piece-rate wage $\{y_{is}\}_{s\in[t,t+12]}$ throughout year t, yearly labor earnings are defined as $y^A_{it} \equiv w\int_t^{t+12} y_{is}ds$.[20] Table 2 reports some moments of the earnings growth distribution for the model simulated data along with estimates from Guvenen, Karahan, Ozkan, and Song (2016).

Overall, the distribution of earnings growth in the model captures the main facts of the data, at least qualitatively. Consistent with the data, the models earnings growth distribution exhibits strong negative skewness and a kurtosis exceeding that of a normal distribution. These deviations from normality naturally arise from the models job ladder structure. Em-

[20] This measure of labor earnings ignores unemployment insurance payments. This ensures that the moments computed in the model are comparable to data from Guvenen, Karahan, Ozkan, and Song (2016), whose measure of earnings include wages, salaries and bonuses.

Figure 1: Effects of job displacement on wages and consumption



Notes: Coefficients $\{\gamma_k\}$ from the distributed lag regression (16) for log wages (**Panel A**) and consumption (**Panel B**). The blue line corresponds to estimates using model simulated data. The orange line corresponds to estimates from Saporta-Eksten (2014) using PSID for the years 1999–2009. The PSID is conducted in biannual waves with earnings data collected for the year previous to the interview, while consumption is reported for the interview year. The sample includes all non-SEO male heads of households, 24–65 years, hourly wages above 0.5 the state minimum wage, with a minimum of 80 annual hours of employment. In the data, a job loser is an individual who reports being unemployed at the time of the interviews (taken between March and May). I construct the model sample to best reflect this sample selection and timing of recorded variables.

ployed workers who don't contact other firms remain on a fixed piece-rate wage, which explains why the model's distribution features a large mass of workers that experience no or very small earnings changes. In contrast, workers who are hit with a separation shock or renegotiate their wages following an outside offer can face large earnings changes, helping to explain the excess kurtosis. Moreover, job losses lead to significant and persistent earnings declines, explaining the negative skewness observed in the distribution. One aspect of the data that the model fails to capture is the volatility of earnings changes, which are roughly twice as high than in the model. However, since the model contains a single source of earnings uncertainty, i.e., the job ladder, this shortfall is largely anticipated.

Job Displacement Effect on Wages and Consumption. I compare the model's earnings and consumption dynamics following job displacement with the evidence presented by Saporta-Eksten (2014).[21] I follow his main distributed-lag specification and estimate:

$$Y_{it} = \alpha_0 + \sum_{k \geq -2}^{10} \gamma_k D_{it}^k + u_{it}, \tag{16}$$

[21]While there is an extensive literature exploring the effects of job displacement on wages and earnings (see Jacobson, LaLonde, and Sullivan, 1993; Stevens, 1997; Davis and von Wachter, 2011), there are fewer studies extending its implications for the response of consumption expenditures. Saporta-Eksten (2014) uses the consumption expenditure information on the 1999-2009 waves of the Panel Study of Income Dynamics (PSID) to study the dynamics of both wages and consumption around job loss.

where the outcome variable Y_{it} denotes workers' i time t log annual wage rate (total labor earnings divided by total hours) or log consumption. The variables $\{D_{it}^k\}_{k=-2}^{10}$ are a set of dummy indicators for whether worker i is in their kth year before, during, or after a job loss. The coefficients γ_k capture the wage (or consumption) losses of displaced workers relative to non-displaced workers over time. Following Saporta-Eksten (2014), I estimate the losses for two years preceding the displacement ($k = -2$), the year of job loss ($k = 0$), and ten years following the displacement ($k = 2, 4, ..., 10$).

Figure 1A presents the estimation results for the wage dynamics using model-simulated data alongside the estimates reported by Saporta-Eksten (2014). The model replicates the magnitude of wage losses following displacement, with wages in both the model and data declining by approximately 30 log points in the year of job displacement. However, the model struggles to generate the observed persistence of these losses. Five years after displacement, wages in the model are only 2.5 log points below the control group, whereas the data show significantly larger wage gaps.[22] Figure 1B shows the results for log consumption. Displaced workers significantly cut their consumption expenditures in response to wage losses following displacement, consistent with the model's large MPCs.

3.3 Aggregate Shocks

I now turn to the aggregate dynamics of the model. The aggregate shocks $\{\mu_t, \vartheta_t, \delta_t, \kappa_t^f\}$ are assumed to follow a mean-reversion process described by:

$$\begin{aligned} d\log\mu_t &= -\rho^\mu \log\mu_t dt + d\varepsilon_{\mu,t}, \\ d\log\vartheta_t &= -\rho^\vartheta(\log\vartheta_t - \log\vartheta)dt + d\varepsilon_{\vartheta,t}, \\ d\delta_t &= -\rho^\delta(\delta_t - \delta)dt + d\varepsilon_{\delta,t}, \\ d\log\kappa_t^f &= -\rho^\kappa(\log\kappa_t^f - \log\kappa^f)dt + d\varepsilon_{\kappa,t}, \end{aligned}$$

where $d\varepsilon_{\mu,t}$, $d\varepsilon_{\vartheta,t}$, $d\varepsilon_{\delta,t}$ and $d\varepsilon_{\kappa,t}$ are unexpected "MIT" shocks. I solve the model using the sequence-space linearization method developed by Boppart, Krusell, and Mitman (2018); Auclert, Bardóczy, Rognlie, and Straub (2021).[23] The solution method involves finding the nonlinear deterministic transition paths in response to "small" one-time unexpected shocks at time 0 to each source of aggregate shock. Due to certainty equivalence, the resulting transition paths are equivalent to the model's linear impulse response functions (IRFs).

[22] In order to capture the long-lasting effects of job displacement on wages, it is important to combine the ladder structure with other ingredients such as human capital accumulation (Krolikowski, 2017) and heterogeneous separation rates (Jarosch, 2021).

[23] While I write and describe the economy in continuous time, I compute the models' solution to aggregate shocks using standard discrete-time methods applied to the discretized version of my continuous-time economy.

Figures 2–5 in Appendix C plot the transition dynamics to the four shocks in the model. The shocks are all assumed to revert back to steady state at a rate of 0.05 and are normalized to generate a 1 percent decline in consumption. In all four cases, the increase in unemployment is accompanied by a reduction in average labor productivity as workers get stuck at (or fall to) the bottom low-productivity rungs of the ladder. Different from unemployment, which returns to steady state within 3 to 4 years following the shocks, the decline of labor productivity is much more persistent lasting at least 7 years.

The time preference and markup shocks transmit like standard New-Keynesian "demand and supply" shocks. A negative time preference shock ($d\varepsilon_{\mu,0} < 0$) lowers inflation and raises unemployment, while a positive shock to firms' desired markups ($d\varepsilon_{\vartheta,0} < 0$) raises both inflation and unemployment. A positive shock to intermediaries vacancy-posting cost ($d\varepsilon_{\kappa,0} > 0$) raises the expected costs of meeting with a worker, which in turn causes labor market tightness to decline in order to restore the free-entry condition (13). As a result, we get lower firm vacancy creation and higher unemployment. A positive shock to job-separation ($d\varepsilon_{\delta,0} > 0$) also raises unemployment, but does so by increasing the inflow of workers from employment into unemployment instead of reducing the unemployment outflows. Unlike other shocks, the job-separation shock leads to an increase (after a initial temporary decline) in job-to-job transitions. As workers fall to the bottom of the ladder and start to rejoin employment, the distribution of employed workers shifts toward lower-productivity matches. This makes workers easier to poach, increasing the expected value of vacancies since intermediaries are now more likely to encounter employed workers at low-productivity matches. As a result, vacancy creation increases, the labor market tightens and, together with it, on-the-job search activity peaks up. Curiously, both the vacancy-posting cost and the job-separation shocks lead to only little movement in inflation, which goes from positive to negative later in the transition.

Comparing the New-Keynesian demand and supply shocks with the shocks that originate in the labor market reveals two additional important points. First, even though all four shocks are normalized to generate the same 1% decline in consumption, labor market shocks (e.g., job-separation and vacancy-posting cost shocks) trigger a much stronger response in unemployment, labor productivity, and earnings. Earnings, for example, respond approximately twice as much following job-separation and vacancy-posting shocks compared to time preference and markup shocks. Second, consumption shows hump-shaped dynamics in response to shocks to the labor market block. While these "humps" are ubiquitous in estimated consumption impulse responses to identified aggregate shocks, models tipically cannot generate this behavior unless they resort to some type of adjustment friction such as habit formation or sticky information.[24]

4 Job Ladder and Incomplete Markets: Inspecting the Mechanism

The key novelty of our analysis is to integrate the job ladder framework from Moscarini and Postel-Vinay (2023) with a incomplete markets heterogeneous agents (HANK) demand block. In this section, I try to disentangle the role played by each one of these two ingredients.[25]

Subsection 4.1 focuses on the job ladder component, particularly on the contribution of the cyclical turnover across the ladder in response to shocks. In the spirit of Moscarini and Postel-Vinay (2017) conjecture of a *cyclical job ladder* that shapes and propagates business cycles, I find that fluctuations to on-the-job search activity are a powerful source of internal propagation of shocks, especially for the dynamics of labor productivity and consumption. Subsection 4.2 examines the role of incomplete markets by comparing the responses in the incomplete markets (HANK) model to those of the complete-market representative-agent (RANK) model. The results reveal stark differences between the two models, with HANK amplifying the consumption response to labor market shocks but dampening the impact of standard New-Keynesian "demand and supply" shocks.

4.1 The Cyclical Job Ladder as a Transmission Mechanism

What role does the *cyclical* job ladder play in the transmission of aggregate shocks to the economy? To answer this question, I consider a counterfactual equilibrium that treats labor market frictions $\{\lambda_t, \lambda_{et}\}_{t\geq 0}$ as exogenous.[26] Specifically, the equilibrium I am interested in is one where, in response to aggregate shocks, the path of job-finding rate $\{\lambda_t\}_{t\geq 0}$ moves exactly as in the baseline model, but the rate at which employed workers receive outside offers $\{\lambda_{et}\}_{t\geq 0}$ remains constant at their steady-state level. I denote this alternative equilibrium by *acyclical ladder* equilibrium, in contrast with the baseline where the reallocation over the ladder varies in response to aggregate shocks.

Figures 6-9 plot the responses to the four shocks in the cyclical and acyclical ladder equilibria. I first discuss the responses to the time preference, markup and vacancy-posting cost shocks as they all share similar patterns. By construction, unemployment features the

[24] As discussed by Auclert, Rognlie, and Straub (2020), HANK models should strive to match both the microeconomic evidence on MPCs as well as the macroeconomic evidence of hump-shaped impulse responses to aggregate shocks. By successfully fitting both micro- and macro-level evidence, the interaction of the job ladder with market incompleteness emerges as a compelling alternative to other frictions commonly explored in the literature.

[25] New-Keynesian price rigidities, the third ingredient to the model, are important to allow demand-side forces play a role in the equilibrium determination.

[26] The counterfactual is defined as in 2.1, except that I replace the free-entry condition with the assumption that frictions $\{\lambda_t, \lambda_{et}\}_{t\geq 0}$ are exogenous.

exact same dynamics in the two models. Job-to-job transitions, labor productivity and income behave very differently though. The three variables fall heavily in the baseline model as the reduction in firm entry lowers labor market tightness and leads to a reduction in worker reallocation over the ladder. In contrast, job-to-job transitions barely fall in the acyclical ladder equilibrium, leading to a slight increase to labor productivity as employed workers keep climbing the ladder at the same pace as in the stationary equilibrium. Consumption also falls by more and is more persistent in the cyclical ladder equilibrium. This amplification can be quite substantial, as in the case of the shock to the vacancy-posting cost, where consumption falls five times more in the baseline than in the acyclical ladder equilibrium. Inflation, by contrast, seems to respond similarly in the cyclical and acyclical ladder equilibria. This is because consumption and productivity responses in the cyclical ladder equilibrium push inflation in opposite directions: weaker consumption puts downward pressure in inflation, while lower productivity acts as an inflationary force. Overall, the results to these three shocks suggest that the job ladder, or more specifically, its cyclical component regulated by fluctuations to $\{\lambda_{et}\}_{t\geq 0}$, is a powerful source of internal propagation giving rise to a lot of endogenous persistence in response to shocks.[27]

Interestingly, Figure 8 shows that we get the opposite patterns in response to a job-separation shock: labor productivity, consumption, and income all decline more in the acyclical ladder equilibrium than in the baseline. The difference has to due with the response of labor market tightness—and thus the path of $\{\lambda_{et}\}_{t\geq 0}$—which increases following a shock to separations. As I discussed in Section 3.3, a shock to job separations, which captures an outward shift to the Beveridge curve, changes the distribution of employed workers toward lower-productivity matches which stimulates job creation. So, different from the previous cases where a reduction to on-the-job search activity deepened the recession, the increase in worker turnover in response to the separation shock actually speeds up workers' reallocation across the ladder, accelerating the recovery relative to the acyclical ladder counterfactual.

4.2 Incomplete vs Complete Markets

As I highlighted in the Introduction, incomplete markets lead to very different (and more realistic) individual consumption behavior than the one implied by the representative-agent

[27]Our amplification results regarding movements to on-the-job search activity are reminiscent of those highlighted by Faberman, Mueller, ahin, and Topa (2022). Though similar, the two results are derived from slightly different exercises. I arrive at this result by comparing a model with *exogenous* on-the-job search effort but time-varying on-the-job outside offers arrival rates $\{\lambda_{et}\}_{t\geq 0}$ with an acyclical ladder counterfactual where $\{\lambda_{et}\}_{t\geq 0}$ do not fall in response to shocks. Using novel survey evidence, Faberman, Mueller, ahin, and Topa (2022) document that employed search effort is itself highly elastic, falling significantly during recessions. As they show, this strong *endogenous* reaction of search activity among the employed creates another (complementary) source of amplification to shocks.

complete markets. Whether or not these differences to *individual* consumption translates into different *aggregate* consumption dynamics in response to shocks depends heavily on the source and distributional implications of the underlying shock. The literature features examples that go from exact equivalence (like Werning, 2015, monetary policy shock) all the way to very strong non-equivalence (like Auclert, Rognlie, and Straub, 2024b, deficit-financed government expenditure shock) between RANK and HANK.[28] In this section, I add to this literature by comparing incomplete and complete markets structures in an economy where individual income dynamics are endogenous and driven by workers' reallocation along the job ladder. Aside from the differences introduced by the ad-hoc wage setting rule, the RANK version of the model can be seen as a version of Moscarini and Postel-Vinay (2023) model.[29]

Figures 10-13 plot the IRFs for all four shocks in the model under incomplete and complete-markets. Different from above, the effect of incomplete markets depends heavily on the source of the shock. In response to standard New-Keynesian demand and supply shocks, HANK exhibits milder contractions than RANK: employment, productivity and consumption all fall more in the RANK than in HANK. In contrast, shocks that originate in the labor market, such as the job-separation and vacancy-posting shocks, lead to significantly more prolonged contractions and slower recoveries in HANK than in RANK. Looking at consumption one year after the shock, I find that the consumption decline in HANK in response to labor market shocks is amplified by a factor of two to three. Notably, RANK also loses the hump-shaped consumption dynamics that we highlighted before.

To better understand the consumption differences between the two models, I turn to a GE-PE decomposition following Kaplan and Violante (2018). The idea is to decompose the total differences into a component due to different paths of real rates and income (GE discrepancy) and a component due to different consumption sensitivities in HANK and RANK to the same movements in these variables (PE discrepancy). To express this decomposition, I first make explicit the dependence of the consumption function $C_t^m\left(\{\Theta_s^m\}_{s\geq 0}\right)$ on the vector of inputs entering the household problem $\{\Theta_s^m\}_{s\geq 0}$, where $m \in \{HA, RA\}$ indexes the two versions of the model. This vector includes the path of the time preference shock $\{\mu_s^m\}_{s\geq 0}$, real interest rate $\{r_s^m\}_{s\geq 0}$, job-separation shock $\{\delta_s^m\}_{s\geq 0}$, job-finding rate $\{\lambda_s^m\}_{s\geq 0}$, on-the-job search rate $\{\lambda_{es}^m\}_{s\geq 0}$, and labor taxes $\{\tau_s^m\}_{s\geq 0}$.[30] Totally differentiating the consumption function in each

[28] The notion of equivalence is drawn from Kaplan and Violante (2018). The authors say that two models are *strongly* equivalent when they feature the same IRFs and "transmission mechanisms." If the models display the same IRFs, but the transmission mechanisms are different, they are said to be *weakly* equivalent. The decomposition in the main text offers one way to evaluate the transmission mechanism in response to a shock.

[29] The complete-market representative-agent (RANK) version of our model is detailed in Section B.

[30] In principle, the aggregate consumption in each model also depends on the path of $\{w_s^m\}_{s\geq 0}$. Remember, however, that we made the price w_t constant in our baseline fiscal policy specification.

model with respect to their own vector of inputs leads to

$$dC_t^{HA} = \sum_{j=1}^{J} \int_{s=0}^{\infty} \frac{\partial C_t^{HA}}{\partial \Theta_{js}} d\Theta_{js}^{HA} ds, \qquad dC_t^{RA} = \sum_{j=1}^{J} \int_{s=0}^{\infty} \frac{\partial C_t^{RA}}{\partial \Theta_{js}} d\Theta_{js}^{RA} ds, \tag{17}$$

for all $t \geq 0$. Letting

$$dC_t^{HA,RA} = \sum_{j=1}^{J} \int_{s=0}^{\infty} \frac{\partial C_t^{HA}}{\partial \Theta_{js}} d\Theta_{js}^{RA} ds,$$

denote the (counterfactual) consumption response in HANK to the equilibrium prices in RANK, we can decompose the consumption differences between HANK and RANK as

$$\begin{aligned} dC_t^{HA} - dC_t^{RA} = & \underbrace{\sum_{j=1}^{J} \int_{s=0}^{\infty} \frac{\partial C_t^{HA}}{\partial \Theta_{js}} (d\Theta_{js}^{HA} - d\Theta_{js}^{RA}) ds}_{\text{GE discrepancy}} \\ & + \underbrace{\sum_{j=1}^{J} \int_{s=0}^{\infty} \left(\frac{\partial C_t^{HA}}{\partial \Theta_{js}} - \frac{\partial C_t^{RA}}{\partial \Theta_{js}} \right) d\Theta_{js}^{RA} ds}_{\text{PE discrepancy}} . \end{aligned} \tag{18}$$

A negative value for the PE or GE components indicates that the component amplifies the response in HANK compared to RANK, or, alternatively, that the component contributes to consumption falling more in HANK relative to RANK.

Panels 14A-14B and Panels 16A-16B plot the HANK and RANK consumption decompositions (17), i.e., the decompositions to their own vector inputs, for the time preference and job-separation shocks, respectively.[31] For each case, I separate the terms of (17) into two main channels: (i) an intertemporal substitution channel due to movements in the real interest rate and the time preference shock $\{r_s, \mu_s\}_{s\geq 0}$, and (ii) an income channel due to changes in workers' after-tax earnings arising from movements in $\{\delta_s, \lambda_s, \lambda_{es}, \tau_s\}_{s\geq 0}$. In RANK (Panels 14B and Panel 16B), the direct intertemporal substitution channel due to fluctuations in the real rate or the time preference shock accounts for virtually all of the consumption response. In HANK (Panels 14A and 16A), the income channel plays a much bigger role, especially in response to the job-separation shock.

These different "transmission mechanisms" in RANK and HANK are well-documented in the literature and stem from the significantly higher MPCs in HANK compared to RANK (Kaplan and Violante, 2021; Auclert, Rognlie, and Straub, 2024a).[32]

[31]The case for the markup and vacancy-posting shocks are similar and are presented in Figures 15 and 17.

[32]The higher MPCs in HANK amplifies the consumption sensitivity to transitory income changes while it dampens the response to time preference shock or movements to the real interest rate, shifting the transmission of shocks away from intertemporal substitution and toward income channels.

One issue with the decompositions (17) is that they are "contaminated" by the fact that equilibrium prices are also different in RANK and HANK. To isolate the PE from GE forces in the total consumption response differences, Panels 14C and 16C show the results for our PE-GE decomposition (18). Both PE and GE components are large in absolute value and move in the same direction in response to shocks, indicating that GE forces in the model work to amplify and propagate the PE discrepancies. Consistent with our results that HANK amplifies the consumption response under the job-separation shock ($dC_t^{HA} < dC_t^{RA}$) and dampens it under the time preference shock ($dC^{HA} > dC^{RA}$), the PE component is negative in the former but positive in the latter case.

To understand the sources of the different PE responses, Panels 14D and 16D separate the PE discrepancies into the contributions coming from the income and intertemporal substitution channels. In line with our previous discussion about the different "transmission mechanisms" in HANK and RANK, the PE discrepancy from the income channel is always negative (reflecting that the income channel exerts greater downward pressure in consumption in HANK than in RANK) while the PE discrepancy from intertemporal substitution channel is always positive (reflecting that the direct effect of the time preference shock or movements to the real rate causes consumption to fall more in RANK than HANK). Therefore, whether the total PE component is negative or positive depends on which of the two channels, income or intertemporal substitution, play a more dominant role in the transmission of the shock to consumption. In cases where income reacts strongly to the shock, like in response to the job-separation shock, the negative PE component from income channel dominates the positive PE from intertemporal forces, leading to a negative total PE component and a stronger response in HANK than RANK. In cases where the intertemporal substitution forces remain dominant, like in response to a time preference shock, the PE component is positive and contributes to consumption falling less in HANK than RANK.

5 Conclusion

This paper integrates the tractable job ladder from Moscarini and Postel-Vinay (2023) with an incomplete markets heterogeneous agent (HANK) model. Incorporating the job ladder structure into the HANK framework delivers several interesting results: (1) it generates endogenously realistic individual earnings dynamics, (2) adds a powerful internal propagation mechanism that amplifies and propagates aggregate shocks with pro-cyclical worker turnover, (3) generates hump-shaped consumption dynamics in response to shocks, and (4) amplifies the effects of shocks relative to RANK when the income channel is strong, such as with the shocks originating in the labor market block. Despite integrating two key building blocks of modern macroeconomics, the model is no more difficult to solve and or calibrate than a standard HANK model.

While I've considered a set of standard New-Keynesian and labor market shocks to illustrate the interaction of the job ladder and HANK, the model can also be used to study the effect of labor market policies (e.g., unemployment insurance) or standard fiscal and monetary policy. To the latter two, the job ladder adds two potentially interesting channels. On the supply side, it allows pure demand shocks to directly affect labor productivity, an implication that finds empirical support in the estimated effects from monetary policy shocks in Christiano, Eichenbaum, and Evans (2005) and Baqaee, Farhi, and Sangani (2021). On the demand side, it features a different source of profits (those accruing to the labor service firms), which, in the case of an expansionary shock, are slowly captured by workers as they attract outside offers and bid up their wages within the match. To the extent that workers have larger MPCs than the "capitalists" receiving the profits, this redistribution channel can operate to further stimulate the demand response to fiscal and monetary shocks.

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A Workers' Filtering Problem

I start by writing down the process for productivity and piece-rate wage $\{z_t, y_t\}$ for the worker. Letting $(0,0)$ stand in for the unemployment state, the state space for the Markov process $\{z_t, y_t\}$ is X^2, where I define $X \equiv \{0\} \cup [\underline{z}, \bar{z}]$. In what follows, I drop the t subscript and describe this process in recursive notation, letting $\cdot^*$ denote the updated state. The rate at which workers leave state (z, y) to a new state (z^*, y^*) depends on the type of labor market event they go through: unemployed workers find a job with intensity λ; employed workers contact other firms with intensity λ_e and suffer exogenous destruction shocks with intensity δ. Upon any of those events, the distribution of the worker's new state (z^*, y^*) given their current state (z, y) is described by a *stochastic kernel function* $T_i : X^2 \times X^2 \to \mathbb{R}$, where $i \in \{ue, ee, eu\}$ indexes the different transitions.

First, consider the transitions between employment and unemployment. The kernel functions in these cases are given by

$$T_{ue}(z^*, y^*) = \gamma(z^*)\delta(y^* - \underline{z})$$
$$T_{eu}(z^*, y^*) = \delta(z^* - 0)\delta(y^* - 0),$$

where $\delta(\cdot)$ is a Dirac delta function.[33] The match productivity of a worker moving out of unemployment is drawn from exogenous distribution Γ and its piece-rate wage is $\underline{z}$ no matter which firm they go to. An employed worker that receives a destruction shock moves to unemployment state, indexed by $(0,0)$. Notice that the kernels for transitions between unemployment and employment do not depend on the workers' current state (z, y).

The stochastic kernel for an employed worker upon meeting an outside firm (T_{ee}) is more complicated. Remember from the discussion in the main text that an employed worker with state (z, y) who receives an offer from outside firm will: (i) with probability $\Gamma(y)$ discard the offer since it is smaller than its current wage; (ii) with probability $\Gamma(z) - \Gamma(y)$ renegotiate its wage with the incumbent to $y' \in (y, z)$; and (iii) with probability $1 - \Gamma(z)$ meet a firm $z^* > z$ that poaches the worker by offering the maximum wage offer of the incumbent z. I

[33] The Dirac delta can be loosely thought of as a object with the following properties

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}, \qquad \int_{-\infty}^{\infty} \delta(x)\, dx = 1, \qquad \int_{-\infty}^{\infty} g(x)\delta(x)\, dx = g(0).$$

I use the Dirac delta in the derivation whenever the worker transition is deterministic—for instance, when the worker loses their job, they transition to state $(0,0)$ with certainty.

summarize these alternatives by writing

$$T_{ee}(z^*, y^*|z, y) = \begin{cases} \Gamma(y) \times \delta(y^* - y) \times \delta(z^* - z) & \text{if case (i)} \\ \gamma(y^*) \times \delta(z^* - z) & \text{if case (ii)} \\ \gamma(z^*) \times \delta(y^* - z) & \text{if case (iii)} \end{cases}$$

Integrating out firm productivity z^* from stochastic kernel T_i, we recover the conditional density f_i for piece-rate wage y^*, the object workers ultimately care about when making their consumption and savings decisions.[34] Again, conditional densities for finding a job from unemployment (f_{ue}) or losing one's job (f_{eu}) do not depend on the current state and are given by

$$f_{ue}(y^*) = \delta(y^* - \underline{z}), \quad f_{eu}(y^*) = \delta(y^* - 0).$$

But for an employed worker, the conditional density for piece-rate y^* is a function of current match productivity z

$$f_{ee}(y^*|z, y) = \begin{cases} \Gamma(y) \times \delta(y^* - y) & \text{if case (i)} \\ \gamma(y^*) & \text{if case (ii)} \\ \bar{\Gamma}(z) \times \delta(y^* - z) & \text{if case (iii)} \end{cases}$$

The worker is uninformed about z, however. So in order to evaluate the probability distribution for its future piece-rate wages y^*, the worker must then hold beliefs about z. Let Φ be the worker's belief distribution regarding the incumbent firm productivity, with ϕ denoting the (generalized) density function. In this case, the workers' conditional density for the evolution of piece-rate wages $f_i(y^*)$ is substituted by a *compound lottery* of the form

$$\bar{f}_i(y^*|\Phi, y) = \int f_i(y^*|z, y) d\Phi(z), \quad \text{for } i \in \{ue, ee, ue\}. \tag{A.1}$$

The filtering problem can thus be thought as a substituting the original Markov process $\{z_t, y_t\}$ with $\{\Phi_t, y_t\}$, where Φ_t denotes worker's time t belief about productivity z. In principle, the problem is complicated by the fact that belief Φ_t depends on the worker's entire history of transitions and wage offers. The following proposition shows that the belief $\Phi_t(z)$ is fully characterized from information of workers' current piece-rate wage y_t only, making it a simple object to track.

Proposition 1. *The belief density function ϕ for an unemployed worker is degenerate at $z = 0$. The belief density function ϕ for an employed worker depends only on worker's current piece-rate wage y,*

[34] Notice that workers care about the productivity of the match only insofar it helps them to predict the evolution of its piece-rate wage contracts.

and is given by

$$\phi(z;y) = \frac{\gamma(z)}{\bar{\Gamma}(y)} \text{ for } z > y. \tag{A.2}$$

Conditional densities for ue and eu transition don't depend on current beliefs and are given by $\bar{f}_{ue} = f_{ue}$ *and* $\bar{f}_{eu} = f_{eu}$*. The conditional piece-rate density for the employed in the event of an outside offer is given by*

$$\bar{f}_{ee}(y^*|y) = \int_y^{\bar{z}} f_{ee}(y^*|z,y) \times \frac{\gamma(z)}{\bar{\Gamma}(y)} dz. \tag{A.3}$$

Proof. I start by considering conditional densities $\bar{f}_i$ and updated beliefs Φ_i^* for each labor market transition events. The final step confirms that the belief function in (A.2) is in fact preserved upon these events.

Unemployment to Employment (*ue*). An unemployed worker who meets a firm holds as belief the exogenous firm productivity distribution $\phi_{ue}^*(z^*) = \gamma(z^*)$, which satisfies (A.2) for $y = \underline{z}$. Since f_{ue} does not depend on z, $\bar{f}_{ue} = f_{ue}$ follows directly from the definition in (A.1).

Employment to Unemployment (*eu*). An employed worker hit with a match destruction shock update its belief to reflect its unemployment status, that is $\phi_{eu}^*(z^*) = \delta(z^* - 0)$. Similar to the case above, because f_{eu} does not depend on z, $\bar{f}_{ue} = f_{ue}$ follows directly from the definition in (A.1).

Employed worker (*ee*). Now consider an employed worker with belief Φ. If belief Φ has the form (A.2), conditional density $\bar{f}_{ee}$ in (A.3) follows directly from the definition in (A.1). Upon meeting an outside firm, the updated density function ϕ^* is determined by Bayes's rule according to

$$\phi_{ee}^*(z^*) = \begin{cases} \dfrac{\int_{\{z^*=z\}} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int_{\{z^* \geq y^*\}} \left[\int_{\{z^*=z\}} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker does not switch jobs} \\[2em] \dfrac{\int_{\{z<z^*\}} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int_{\{z^* \geq y^*\}} \left[\int_{\{z<z^*\}} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker switch jobs} \end{cases} \tag{A.4}$$

Note that an employed worker gets to observe two signals: the new piece-rate offer y^* and whether the highest wage offer came from the incumbent or the poacher. In the case the offer comes from the poacher, the worker realizes that the poacher is more productive than the incumbent, that is $z^* > z$. Otherwise, the worker conditions its expectation on $z^* = z$, as productivity he remains in the same match. I cover cases (i)-(iii) below.

Employed worker with discarded wage offer— Consider an employed worker with piece-rate wage y. Suppose a poaching firm comes along and offers a wage smaller than y, which does induce a counteroffer from the incumbent, i.e., $y^* = y$. Applying Bayes rule (A.4), I get

$$\begin{aligned}\phi(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y|z, y) d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y|z, y) d\Phi(z)\right] dz^*} \\ &= \frac{\Gamma(y)\delta(y^* - y) \int_{\{z=z^*\}} \delta(z^* - z) d\Phi(z)}{\Gamma(y)\delta(y^* - y) \int \left[\int_{\{z=z^*\}} \delta(z^* - z) d\Phi(z)\right] dz^*} \\ &= \frac{\phi(z^*)}{\int \phi(z^*) dz^*} = \phi(z^*).\end{aligned}$$

which agrees with the previous belief. This makes sense as the signal does not reveal any new information regarding the productivity of the current match.

Employed worker with wage increase in the job— Consider an employed worker with piece-rate y who contacts an outside firm. Suppose that, as a result of this contact, the incumbent firm offers a wage increase $y_2(+\epsilon)$ above the poacher's offer of y_2. The worker stays in the incumbent under a higher wage and their belief evolves as

$$\begin{aligned}\phi^*(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z, y) d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z, y) d\Phi(z)\right] dz^*} \\ &= \frac{\int_{\{z=z^*\}} \gamma(y_2)\delta(z^* - z) d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \gamma(y_2)\delta(z^* - z) d\Phi(z)\right] dz^*} \\ &= \frac{\gamma(y_2)\phi(z^*)}{\gamma(y_2) \int \mathbb{1}\{z^* > y_2\}\phi(z^*) dz^*} \\ &= \frac{\phi(z^*)}{\overline{\Phi}(y_2)} \quad \text{for } z^* > y_2,\end{aligned}$$

where the second line substitutes T_{ee}, the third integrates with respect to z^*.

Employed worker with job transition— Consider an employed worker with piece-rate y who contacts an outside firm. Suppose that, as a result of this contact, the worker receives an offer $y_1(+\epsilon)$ from the outside firm, while the incumbent offer is y_1. The worker accepts the offer from the poacher and their belief over productivity of the new match z^* is given by

$$\begin{aligned}\phi^*(z^*) &= \frac{\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z, y) d\Phi(z)}{\int \left[\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z, y) d\Phi(z)\right] dz^*} \\ &= \frac{\int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z) d\Phi(z)}{\int \int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z) d\Phi(z) dz^*} \\ &= \frac{\phi(y_1)\gamma(z^*)}{\phi(y_1) \int \mathbb{1}\{z^* > y_1\}\gamma(z^*) dz^*} \\ &= \frac{\gamma(z^*)}{\overline{\Gamma}(y_1)} \quad \text{for } z^* > y_1,\end{aligned}$$

where the second line substitutes T_{ee}, the third integrates with respect to z^*.

Preservation of Beliefs. Losing the job resets beliefs to those of an unemployed worker. Employed workers arrive at a firm either by job-to-job transition or from unemployment. Whenever this happens, it is easy to check that their beliefs Φ satisfy condition (A.2). This is not the case whenever workers receive a wage increase within the match, as the updated belief Φ^* in this scenario is still a function of the previously held belief Φ (as discussed in the case *Employed worker with wage increase in the firm*). However, if we assume that Φ is of form (A.2) (either because workers had previously made the transition from unemployment or from another job), we get that the updated belief Φ^* following a wage increase in the job is given by

$$\phi^*(z^*) = \frac{\phi(z^*)}{\bar{\Phi}(y_2)} = \frac{\gamma(z^*)/\bar{\Gamma}(y)}{\bar{\Gamma}(y_2)/\bar{\Gamma}(y)} = \frac{\gamma(z^*)}{\bar{\Gamma}(y_2)},$$

which also satisfies (A.2).

This shows that the belief function in (A.2) is indeed preserved across labor market transitions, concluding the proof. □

B Complete-Market Representative-Agent case

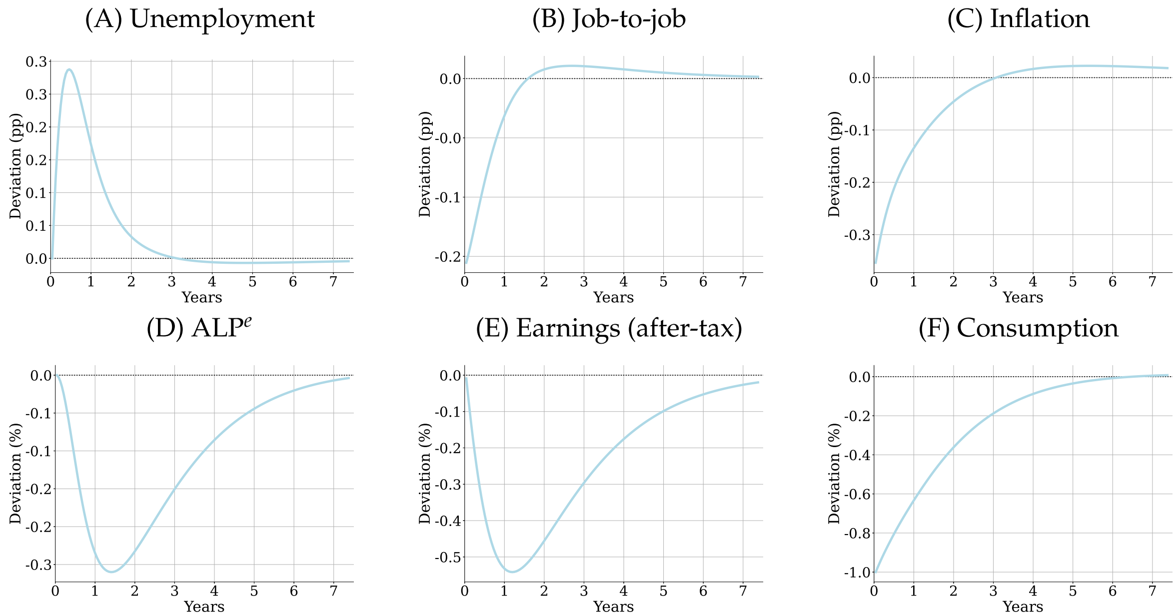
The complete-market representative-agent version of the model follows the representative family construct of Merz (1995). The family is composed of a continuum $i \in [0,1]$ of workers distruted along the job ladder according to $\{\ell_{it}\}_{i\in[0,1]}$. Profits, transfers and taxes are as in the incomplete-market heterogeneous agent economy, so the family's after-tax income is decribed by $(1-\tau_t)w_t \int_0^1 \ell_{it} di$ where w_t is defined as in (2). The family pools all income earned by workers, decides how much to save through government bonds at rate of return r_t, and how much to give each one of its members as consumption C_t. The problem of the family is then described by

$$\max_{\{C_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t}\mu_t u(C_t)dt$$

$$\text{S.t. } \dot{A}_t = r_t A_t + (1-\tau_t)w_t \int_0^1 \ell_{it} di - C_t.$$

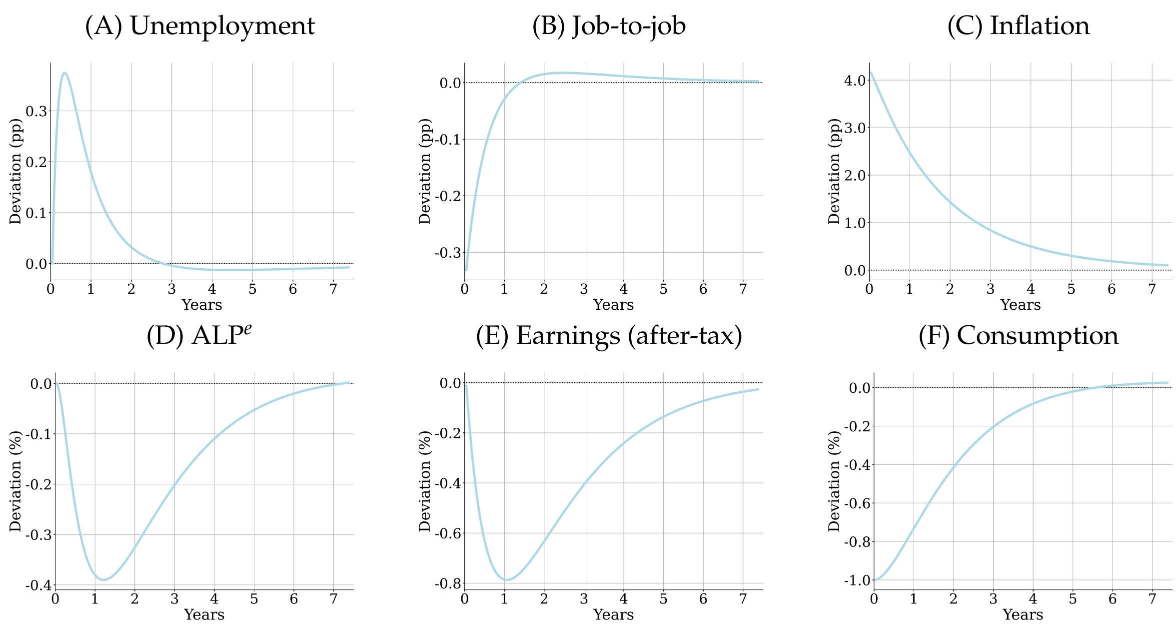
C Impulse Responses to Shocks and Counterfactuals

Figure 2: Time Preference Shock



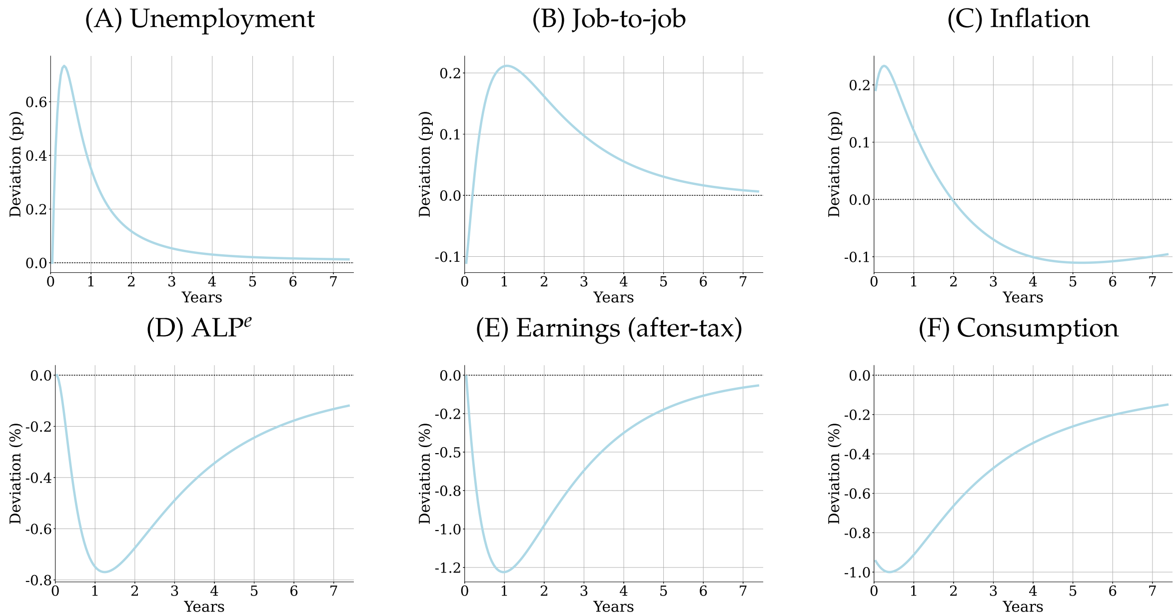
Notes: Response to a negative time preference shock normalized to generate a 1% decline in consumption. ALP^e denotes average labor productivity $\frac{1}{1-u_t}\int_{\underline{z}}^{\bar{z}} z d\Omega_t(z)$, and earnings denotes after-tax total earnings $(1-\tau_t)w\int_0^1 \ell_{it} di$. Unemployment, job-to-job transitions and (annualized) inflation are in percentage point deviations from steady state. ALP^e, earnings, and consumption are denoted in log-deviations from steady state.

Figure 3: Markup Shock



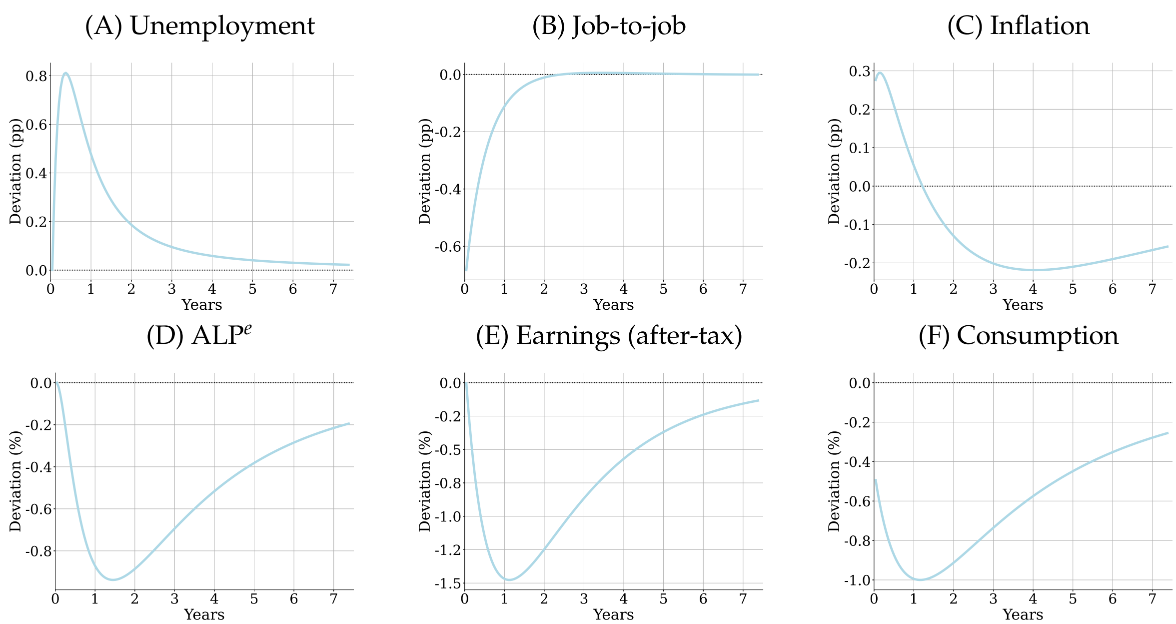
Notes: Response to a positive markup shock normalized to generate a 1% decline in consumption. ALP^e denotes average labor productivity $\frac{1}{1-u_t}\int_{\underline{z}}^{\bar{z}} z d\Omega_t(z)$, and earnings denotes after-tax total earnings $(1-\tau_t)w\int_0^1 \ell_{it} di$. Unemployment, job-to-job transitions and (annualized) inflation are in percentage point deviations from steady state. ALP^e, earnings, and consumption are denoted in log-deviations from steady state.

Figure 4: Job Separation Shock



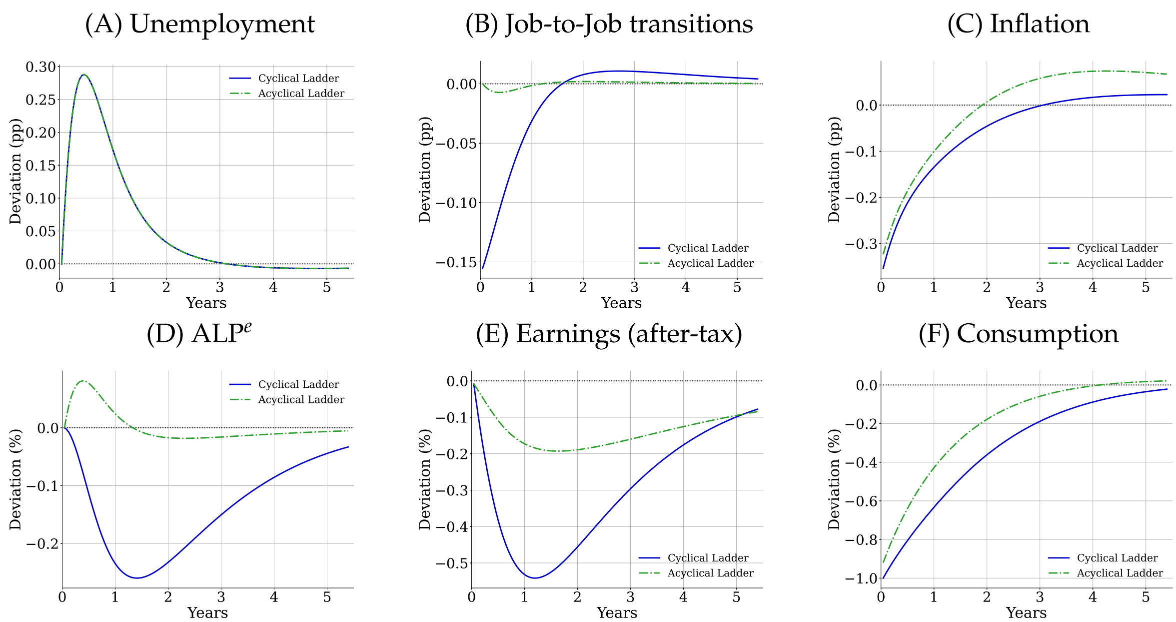
Notes: Response to a positive job-separation shock normalized to generate a 1% decline in consumption. ALP^e denotes average labor productivity $\frac{1}{1-u_t}\int_{\underline{z}}^{\bar{z}} z d\Omega_t(z)$, and earnings denotes after-tax total earnings $(1-\tau_t)w\int_0^1 \ell_{it} di$. Unemployment, job-to-job transitions and (annualized) inflation are in percentage point deviations from steady state. ALP^e, earnings, and consumption are denoted in log-deviations from steady state.

Figure 5: Vacancy Posting Shock



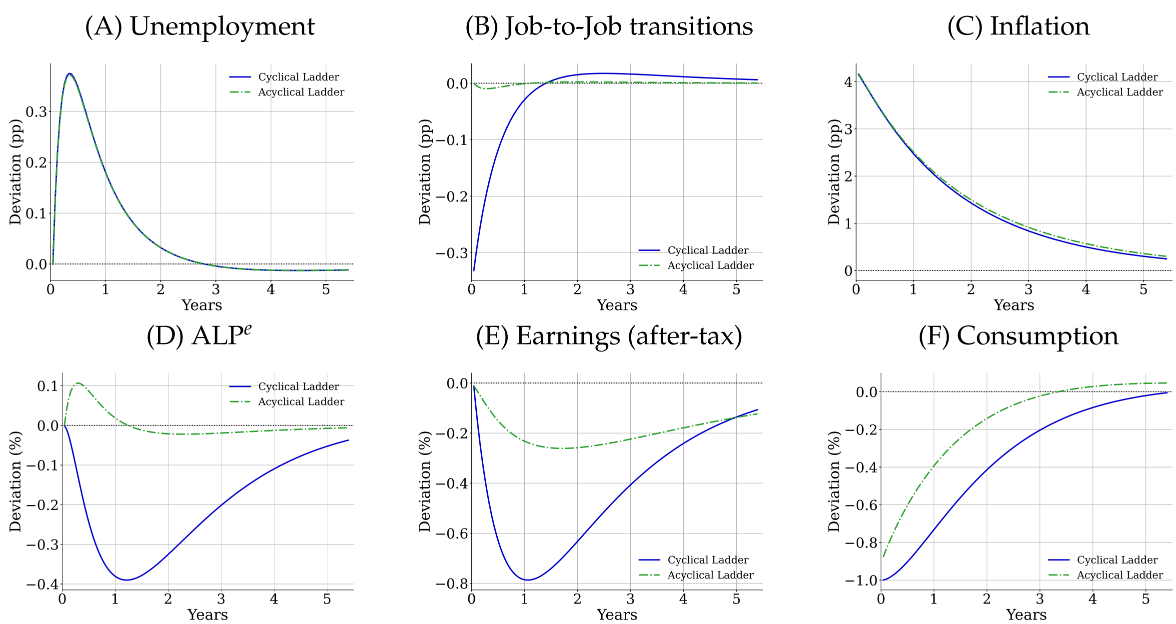
Notes: Response to a positive shock to vacancy-posting cost normalized to generate a 1% decline in consumption. ALP^e denotes average labor productivity $\frac{1}{1-u_t}\int_{\underline{z}}^{\bar{z}} z d\Omega_t(z)$, and earnings denotes after-tax total earnings $(1-\tau_t)w\int_0^1 \ell_{it} di$. Unemployment, job-to-job transitions and (annualized) inflation are in percentage point deviations from steady state. ALP^e, earnings, and consumption are denoted in log-deviations from steady state.

Figure 6: Cyclical vs Acyclical Ladder (Time Preference Shock)



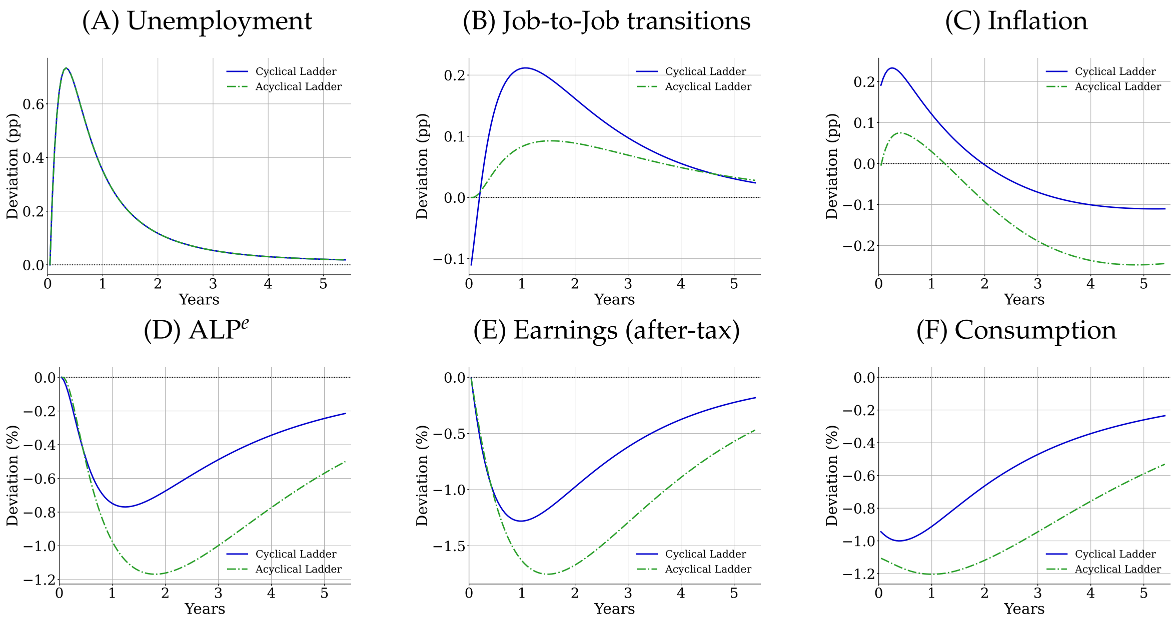
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.1 for details on the acyclical ladder counterfactual.

Figure 7: Cyclical vs Acyclical Ladder (Markup Shock)



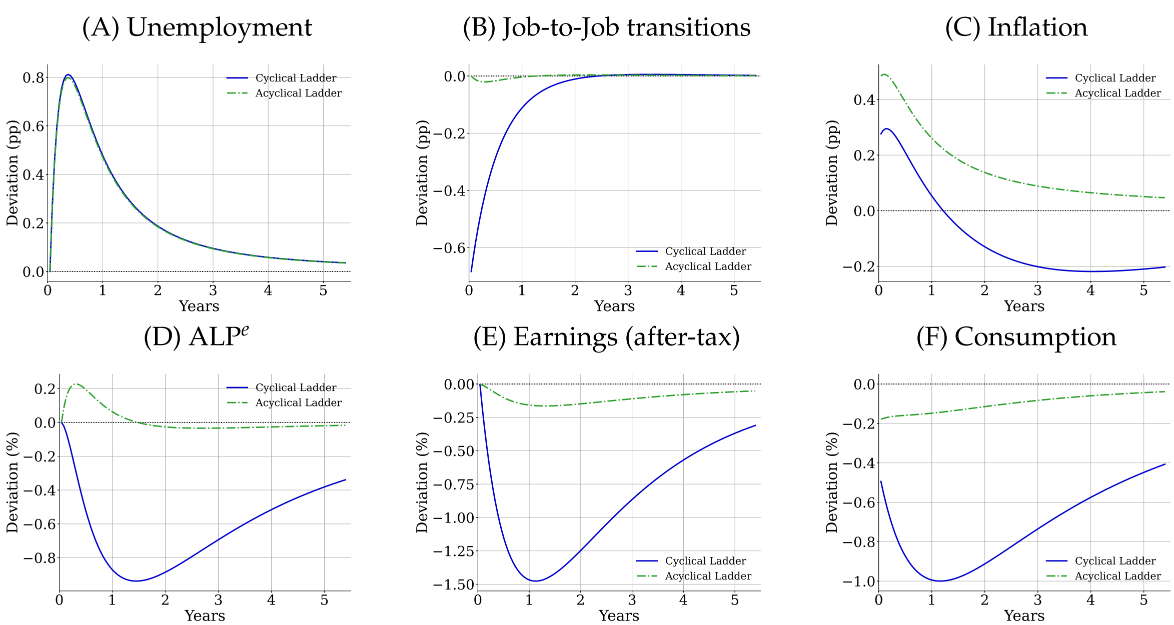
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.1 for details on the acyclical ladder counterfactual.

Figure 8: Cyclical vs Acyclical Ladder (Job Separation Shock)



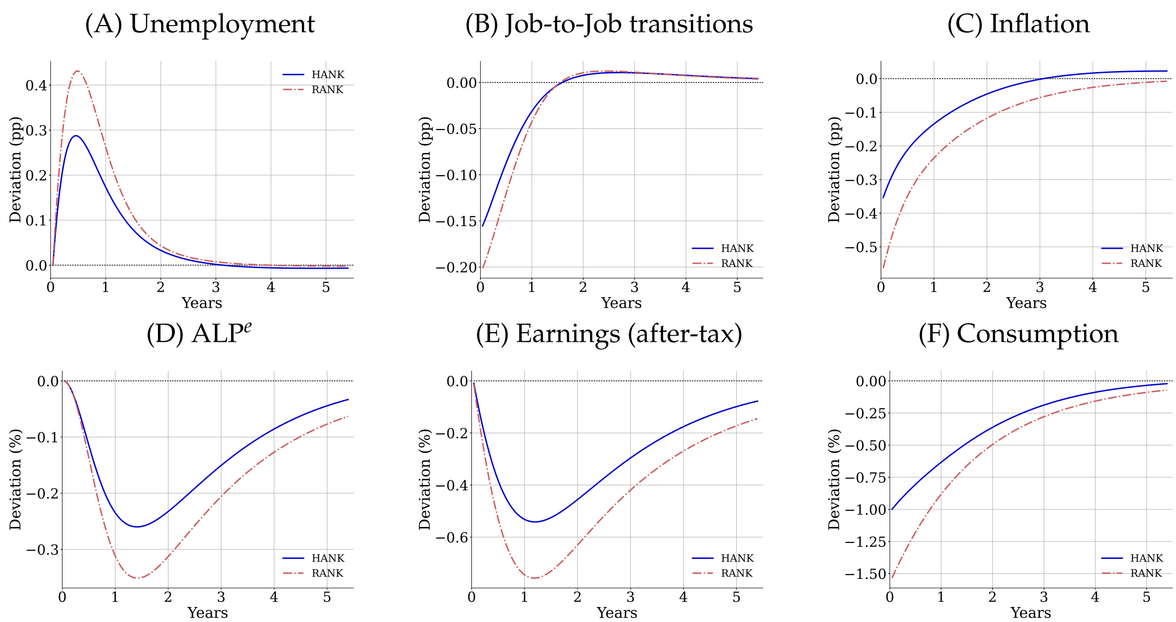
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.1 for details on the acyclical ladder counterfactual.

Figure 9: Cyclical vs Acyclical Ladder (Vacancy Posting Shock)



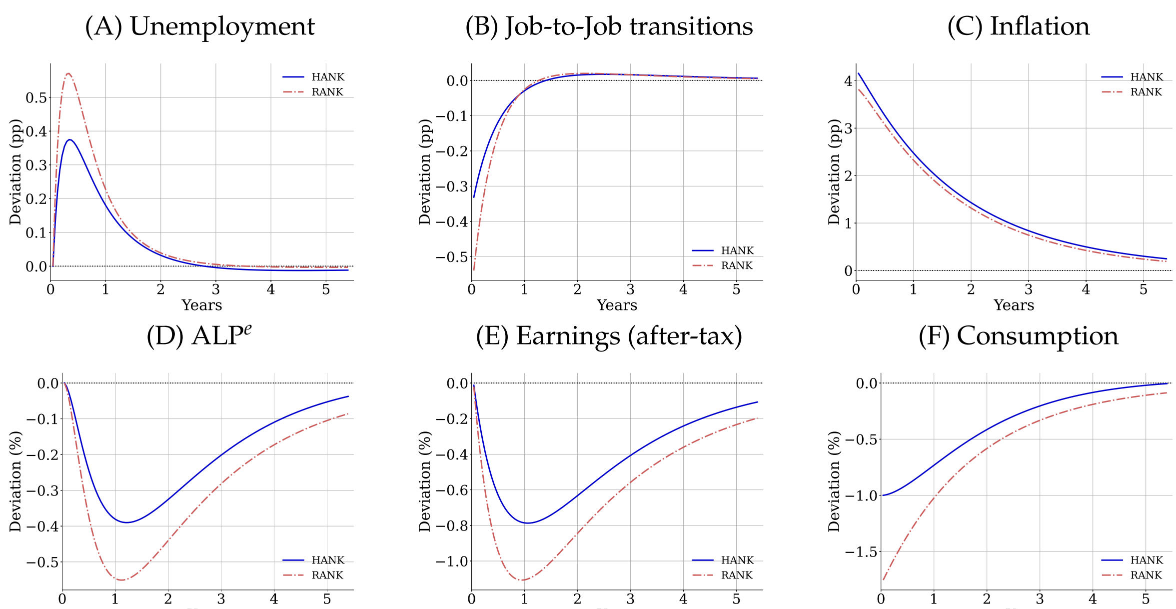
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.1 for details on the acyclical ladder counterfactual.

Figure 10: HANK vs RANK (Time Preference Shock)



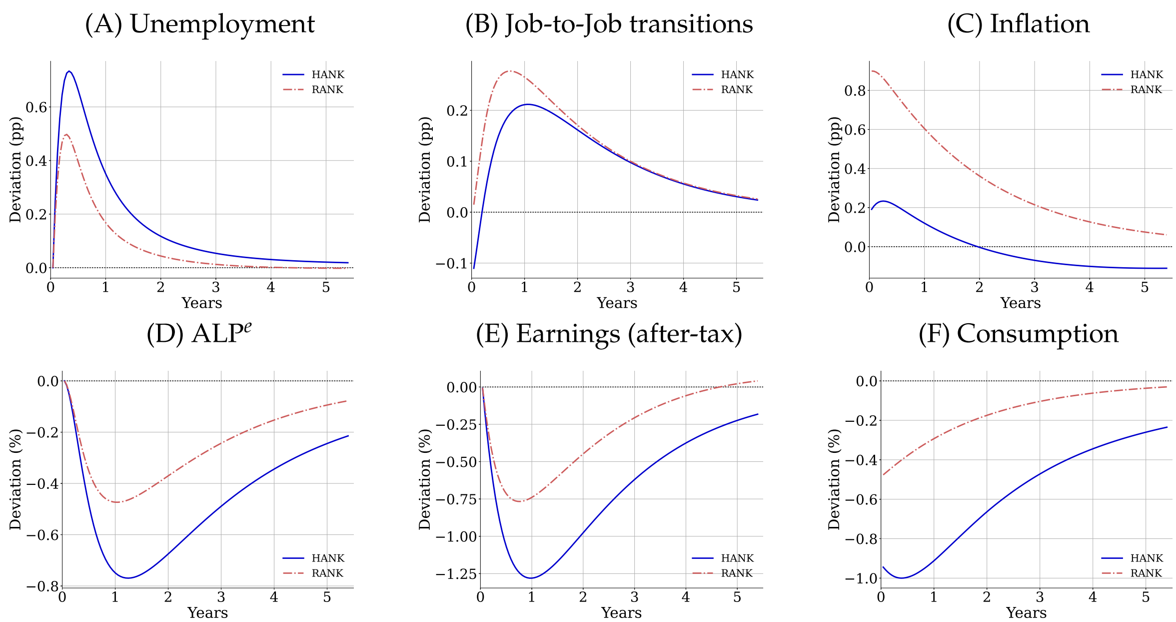
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.2 for details on the RANK counterfactual.

Figure 11: HANK vs RANK (Markup Shock)



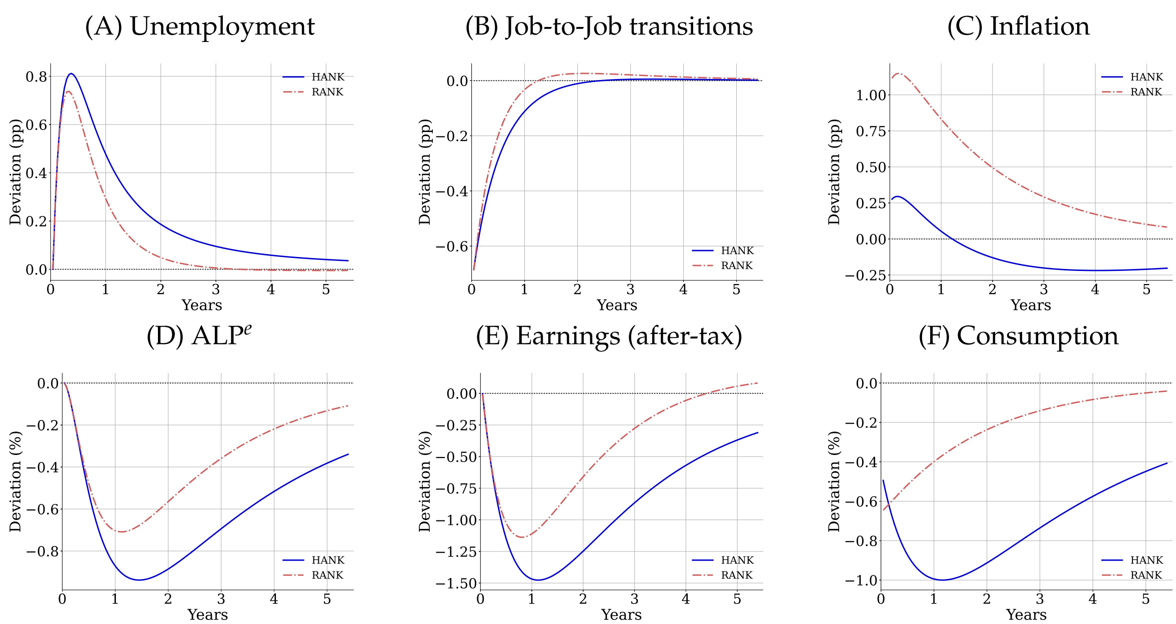
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.2 for details on the RANK counterfactual.

Figure 12: HANK vs RANK (Job Separation Shock)



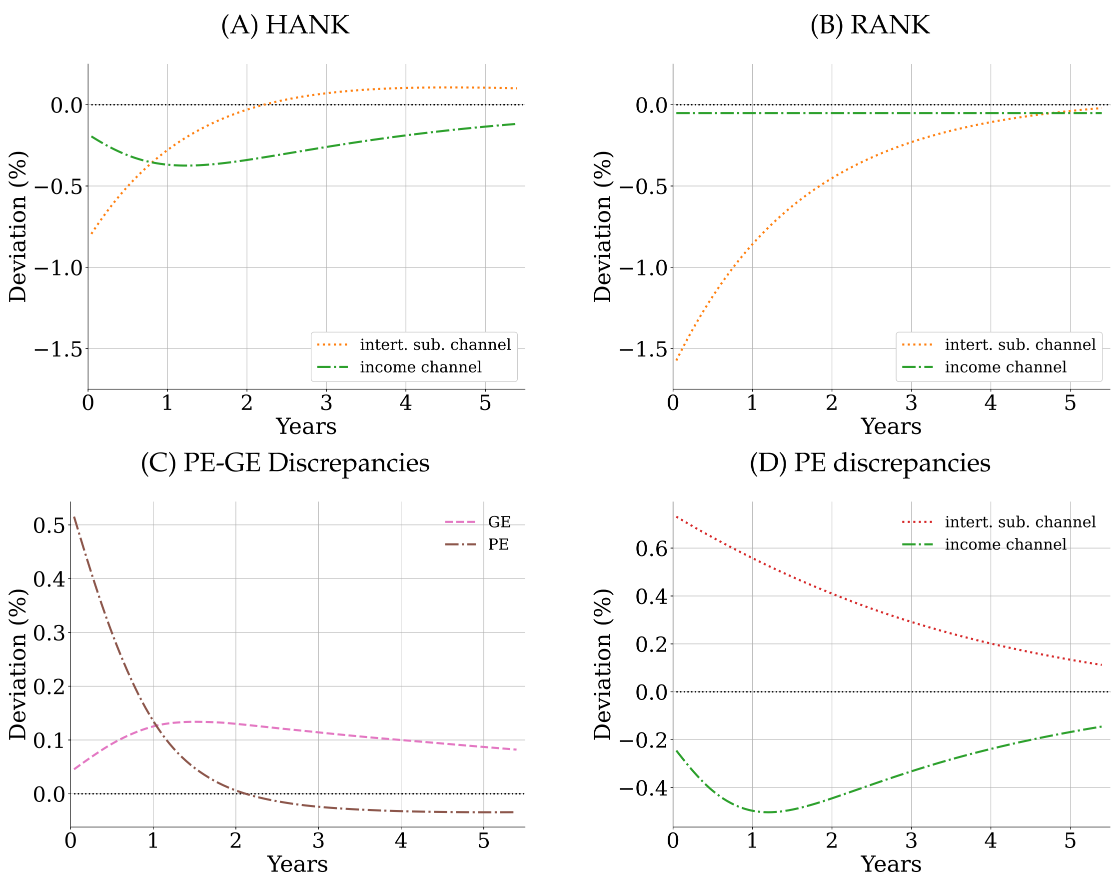
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.2 for details on the RANK counterfactual.

Figure 13: HANK vs RANK (Vacancy Posting Shock)



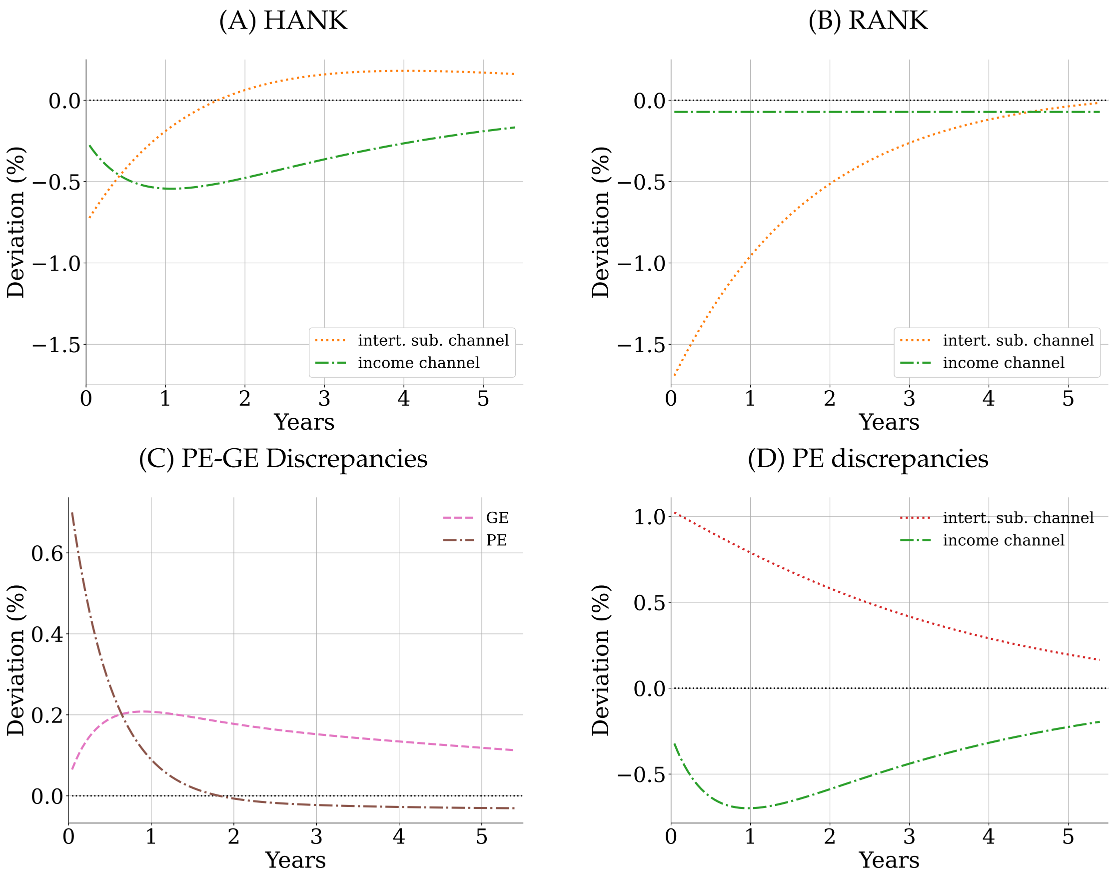
Notes: Shock is normalized to generate a 1% decline in consumption. See Section 4.2 for details on the RANK counterfactual.

Figure 14: Consumption Decomposition (Time Preference Shock)



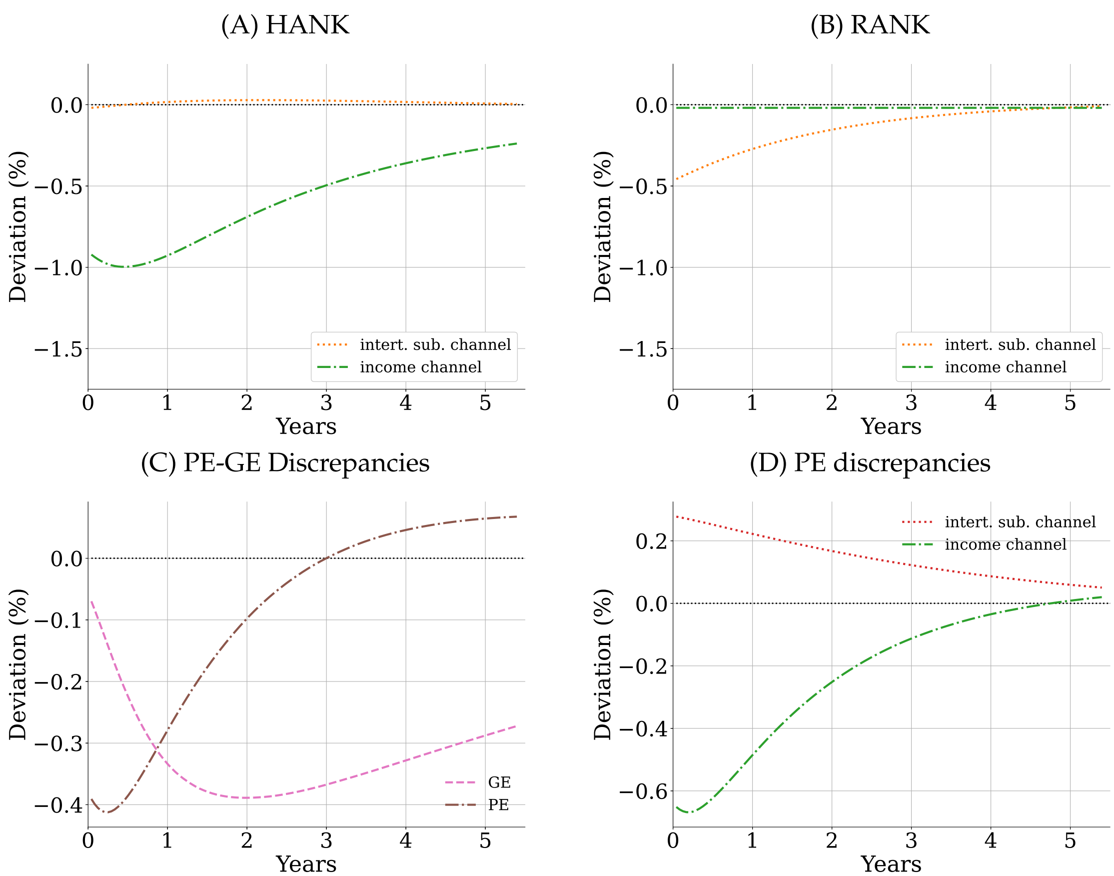
Notes: Shock is normalized to generate a 1% decline in consumption in HANK. **Panel A and B:** HANK and RANK consumption decomposition following (17). The 'intertemporal substitution channel' denotes the consumption response to movements in the real interest rate $\{r_s\}_{s\geq 0}$ and preferecen shock $\{\mu_s\}_{s\geq 0}$; the 'income channel' denotes consumption response to movements in the job-separation shock $\{\delta_s\}_{s\geq 0}$, job-finding $\{\lambda_s\}_{s\geq 0}$, on-the-job search $\{\lambda_{es}\}_{s\geq 0}$, and labor taxes $\{\tau_s\}_{s\geq 0}$. **Panel C and D:** PE-GE discrepancies following decomposition (18).

Figure 15: Consumption Decomposition (Markup Shock)



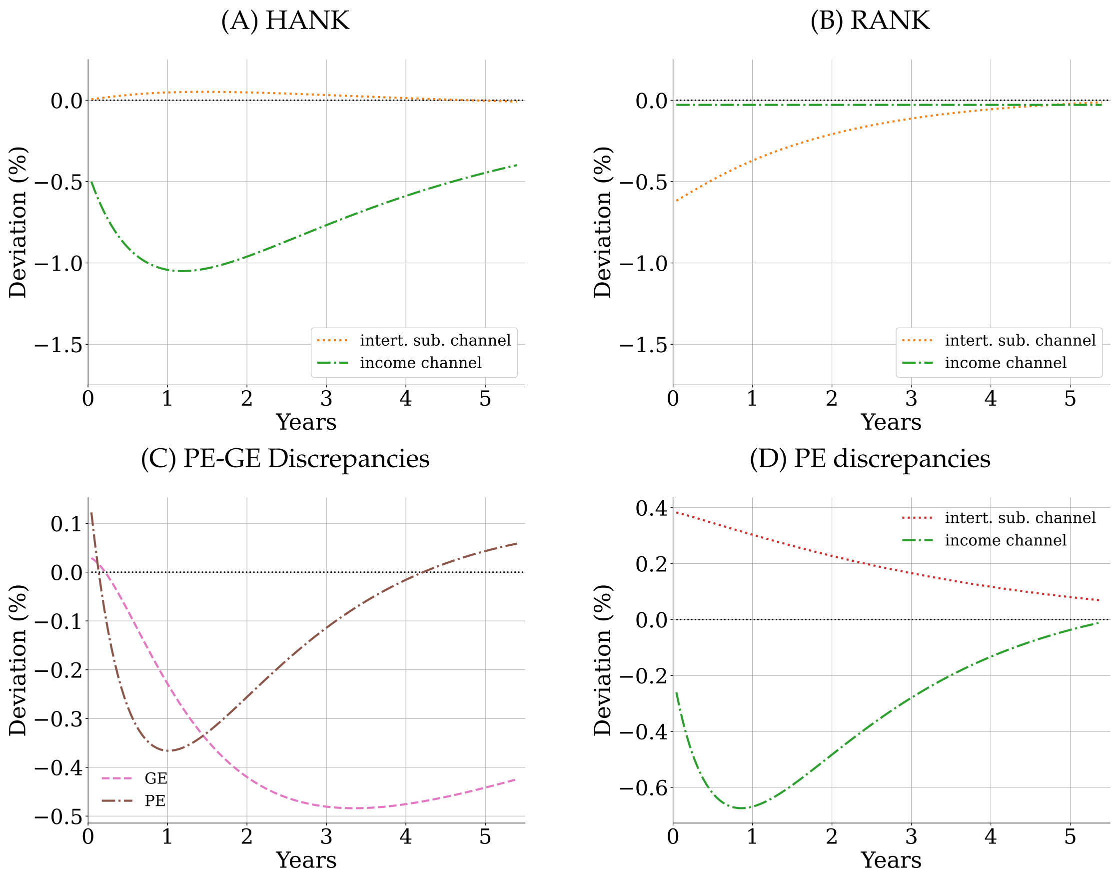
Notes: Shock is normalized to generate a 1% decline in consumption in HANK. **Panel A and B:** HANK and RANK consumption decomposition following (17). The 'intertemporal substitution channel' denotes the consumption response to movements in the real interest rate $\{r_s\}_{s\geq 0}$ and preferecen shock $\{\mu_s\}_{s\geq 0}$; the 'income channel' denotes consumption response to movements in the job-separation shock $\{\delta_s\}_{s\geq 0}$, job-finding $\{\lambda_s\}_{s\geq 0}$, on-the-job search $\{\lambda_{es}\}_{s\geq 0}$, and labor taxes $\{\tau_s\}_{s\geq 0}$. **Panel C and D:** PE-GE discrepancies decomposition following (18).

Figure 16: Consumption Decomposition (Job Separation Shock)



Notes: Shock is normalized to generate a 1% decline in consumption in HANK. **Panel A and B:** HANK and RANK consumption decomposition following (17). The 'intertemporal substitution channel' denotes the consumption response to movements in the real interest rate $\{r_s\}_{s\geq 0}$ and preferecen shock $\{\mu_s\}_{s\geq 0}$; the 'income channel' denotes consumption response to movements in the job-separation shock $\{\delta_s\}_{s\geq 0}$, job-finding $\{\lambda_s\}_{s\geq 0}$, on-the-job search $\{\lambda_{es}\}_{s\geq 0}$, and labor taxes $\{\tau_s\}_{s\geq 0}$. **Panel C and D:** PE-GE discrepancies decomposition following (18).

Figure 17: Consumption Decomposition (Vacancy Posting Shock)



Notes: Shock is normalized to generate a 1% decline in consumption in HANK. **Panel A and B:** HANK and RANK consumption decomposition following (17). The 'intertemporal substitution channel' denotes the consumption response to movements in the real interest rate $\{r_s\}_{s\geq 0}$ and preferecen shock $\{\mu_s\}_{s\geq 0}$; the 'income channel' denotes consumption response to movements in the job-separation shock $\{\delta_s\}_{s\geq 0}$, job-finding $\{\lambda_s\}_{s\geq 0}$, on-the-job search $\{\lambda_{es}\}_{s\geq 0}$, and labor taxes $\{\tau_s\}_{s\geq 0}$. **Panel C and D:** PE-GE discrepancies decomposition following (18).