1.1.1. Seven students go on vacations. They decide that each will send a postcard to three of the others. Is it possible that every student receives postcards from precisely the three to whom he sent postcards?

1.1.2. a. Prove that for every even number $n \geq 4$ there exists a graph with n vertices, all of which have degree 3, without using Theorem 1.1.2.

b. Prove that for every odd number $n \geq 5$ there exists a graph with $n+1$ vertices such that exactly n vertices have degree 3.

1.1.3. Prove that for every number $n \geq 5$ there exists a graph with n vertices, all of which have degree 4.

1.1.5. Show that in a graph the number of vertices of odd degree is even.

1.1.9. Draw a graph with degree sequence

a) (4, 3, 2, 2, 1)

b) (4, 3, 3, 3, 1).

1.1.11. Draw a graph with eight vertices, four of which have degree 4 and four of which have degree 3.

1.2.1. Find a graph with five vertices and with exactly
   a. one cycle
   b. three cycles
   c. six cycles.

1.2.2. Find a graph with five vertices and exactly 22 cycles.

1.2.3. Find a graph with five vertices and exactly 13 cycles.

1.2.4. Find a graph G with six vertices and seven edges such that G does not contain a subgraph isomorphic to C_4. (There are several solutions.)

1.2.5. Find a graph G with $p = 6$ and $q = 12$ such that G has no subgraph isomorphic to K_4.

[redacted]

1.2.7. Prove that the two graphs in Figure 1.2.6 are isomorphic.

1.2.8. Consider the graphs in Figure 1.2.4. Are any two of them isomorphic? Prove that your answer is correct.

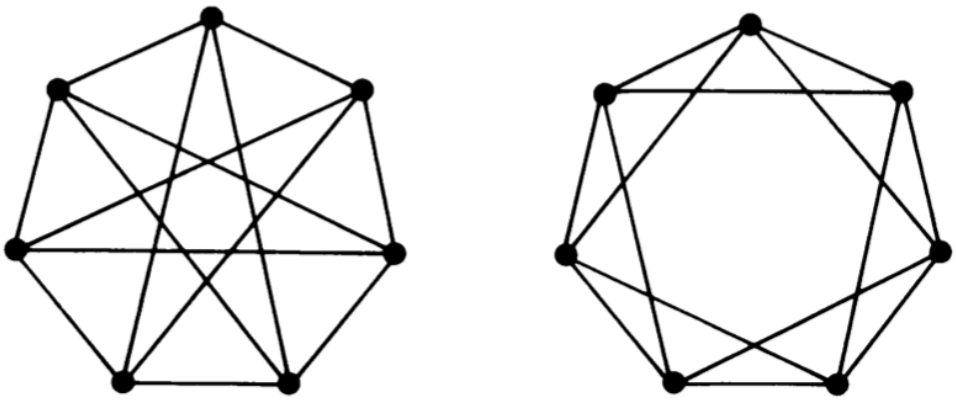

Figure 1.2.6

1.2.9. Are any two of the graphs in Figure 1.2.7 isomorphic? Prove that your answer is correct.

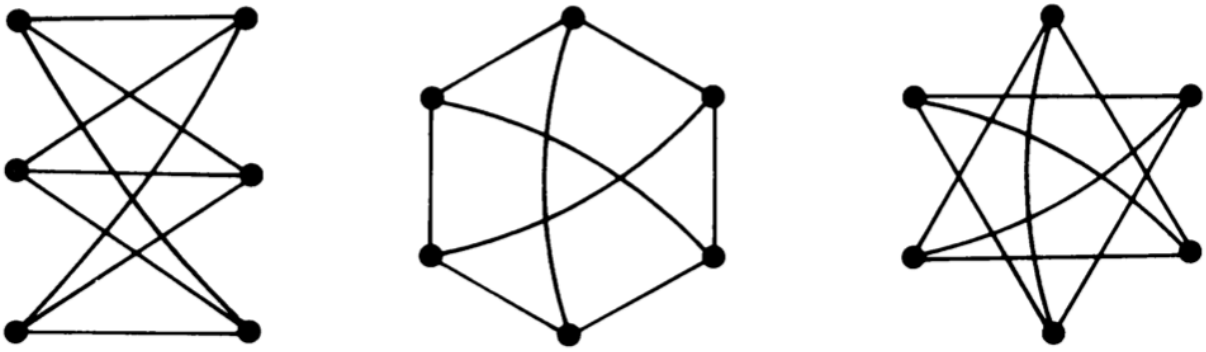

Figure 1.2.4

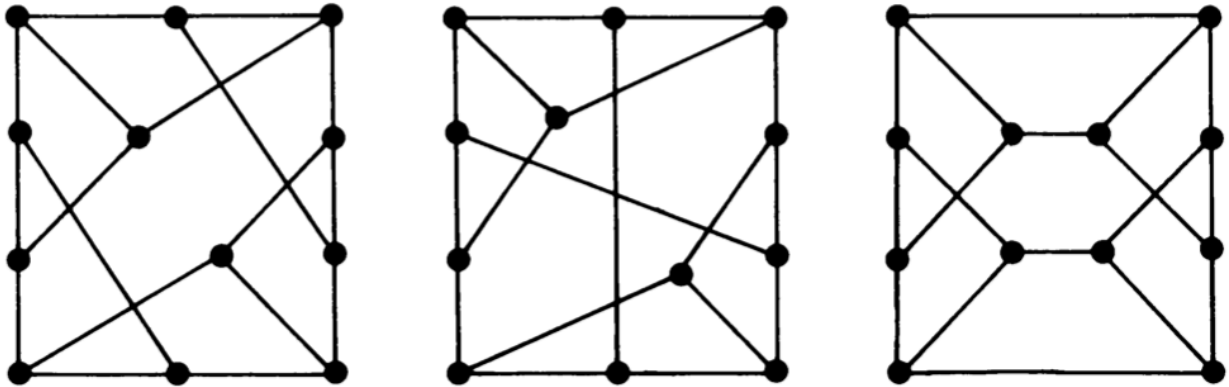

Figure 1.2.7

1.3.1. Let T be a tree with vertices of degree only 3 or 1. If T has 10 vertices of degree 3, how many vertices of degree 1 are in T?

1.3.2. Let G be a connected graph with n vertices and n edges. How many cycles does G have?