FX_calibration

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1 FX Triangle Calibration

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```
In [1]: # Imports
        import numpy as np
        import numpy.random as npr
        import pandas as pd
        import matplotlib.pyplot as plt
        import seaborn
        %matplotlib inline
```

Next we are simulation two GBM with initial parameters S_0 and $S_{0\_}$ growing at the same rates

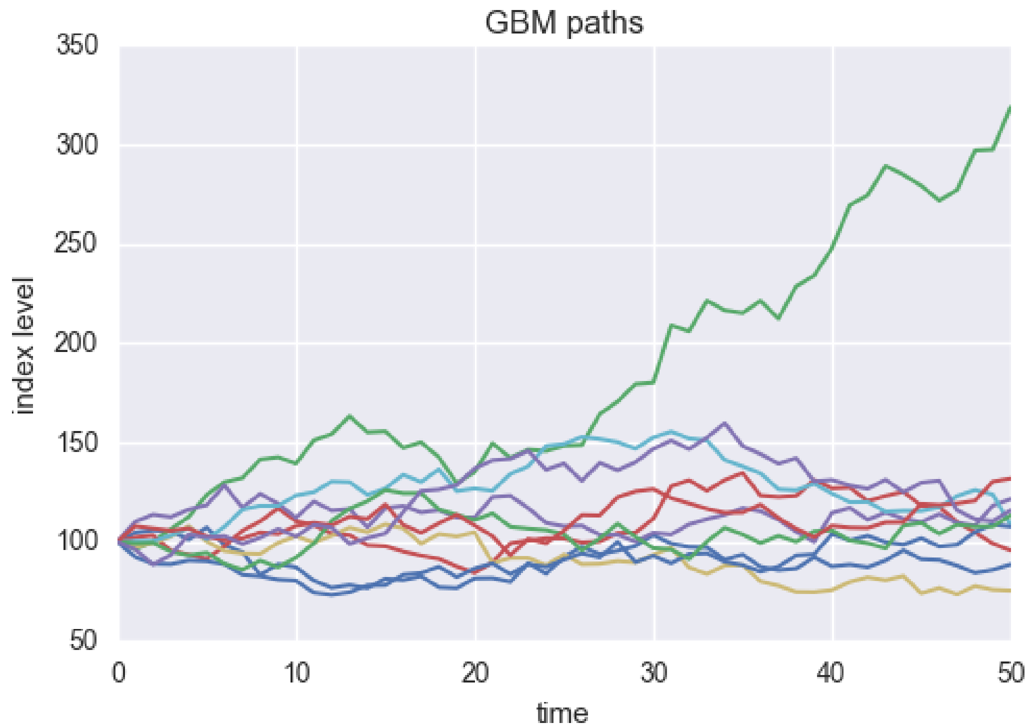
```
In [2]: S0, S0_ = 100, 100 # initial value
        r = 0.05  # constant short rate
        sigma = 0.25  # constant volatility
        T = 2.0  # time in years
        M = 50 # maturity
        I = 100  # number of random draws

        dt = T / M
        S_1 = np.zeros((M + 1, I))
        S_2 = np.zeros((M + 1, I))

        S_1[0] = S0
        S_2[0] = S0_

        for t in range(1, M + 1):
            S_1[t] = S_1[t - 1] * np.exp((r - 0.5 * sigma ** 2) * dt
                    + sigma * np.sqrt(dt) * npr.standard_normal(I))
            S_2[t] = S_2[t - 1] * np.exp((r - 0.5 * sigma ** 2) * dt
                    + sigma * np.sqrt(dt) * npr.standard_normal(I))
```

Plotting the GBM

```
In [3]: plt.plot(S_1[:, :10], lw=1.5)
        plt.xlabel('time')
```

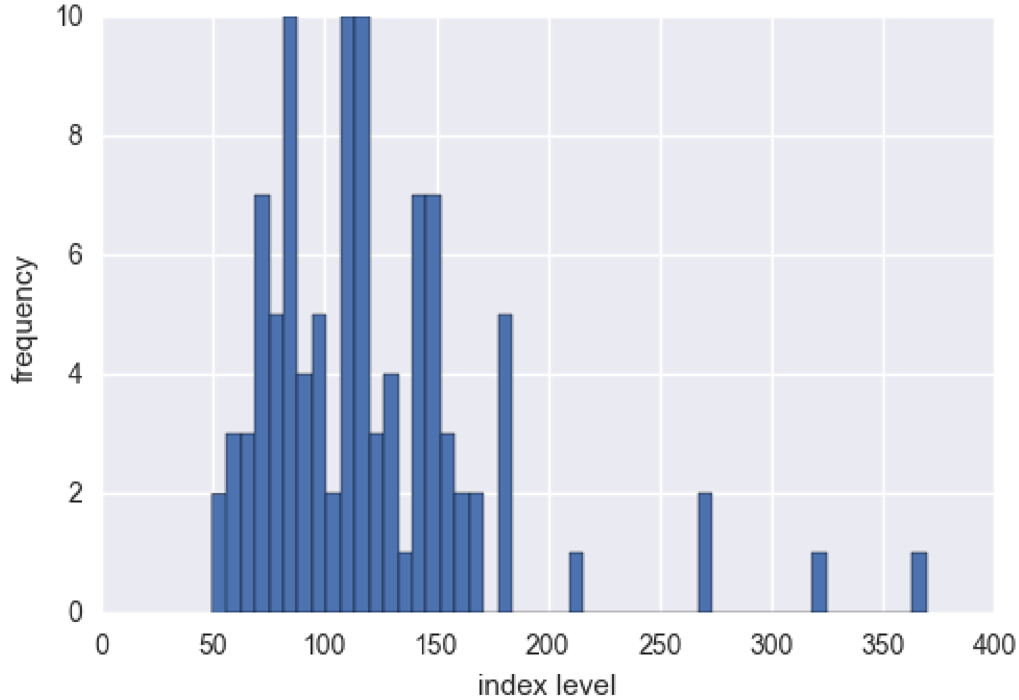
```
plt.ylabel('index level')
plt.grid(True)
plt.title("GBM paths")
# tag: gbm_dt_paths
# title: Simulated geometric Brownian motion paths
# size: 60
```

```
Out[3]: <matplotlib.text.Text at 0x116d245d0>
```



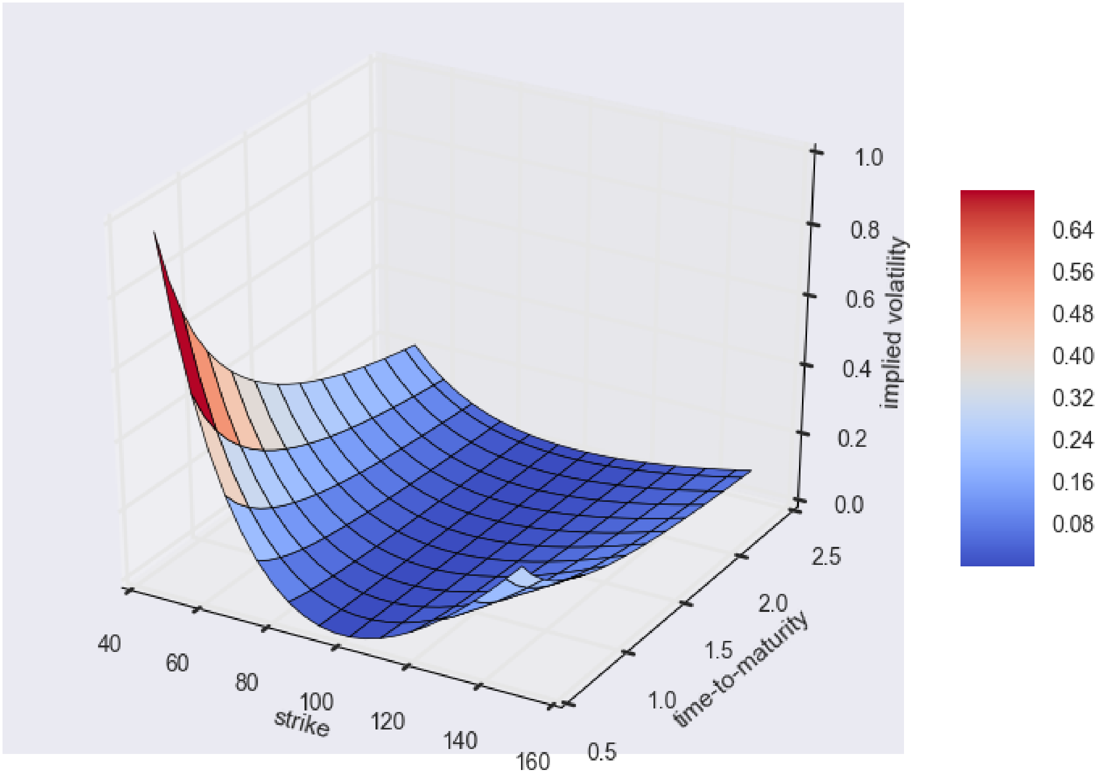
Simulating geometric Brownian motion at maturity

```
In [4]: plt.hist(S_1[-1], bins=50)
        plt.xlabel('index level')
        plt.ylabel('frequency')
        plt.grid(True)
        # tag: gbm_dt_hist
        # title: Simulated geometric Brownian motion at maturity
        # size: 60
```




```
In [5]: strike = np.linspace(50, 150, 24)
        ttm = np.linspace(0.5, 2.5, 24)
        strike, ttm = np.meshgrid(strike, ttm)


        iv = (strike - 100) ** 2 / (100 * strike) / ttm
          # generate fake implied volatilities

In [6]: from mpl_toolkits.mplot3d import Axes3D

        fig = plt.figure(figsize=(9, 6))
        ax = fig.gca(projection='3d')

        surf = ax.plot_surface(strike, ttm, iv, rstride=2, cstride=2,
                               cmap=plt.cm.coolwarm, linewidth=0.5,
                               antialiased=True)

        ax.set_xlabel('strike')
        ax.set_ylabel('time-to-maturity')
        ax.set_zlabel('implied volatility')

        fig.colorbar(surf, shrink=0.5, aspect=5)
        # tag: matplotlib_17
        # title: 3d surface plot for (fake) implied volatilities
        # size: 70
```

Out[6]: <matplotlib.colorbar.Colorbar at 0x117b15fd0>



2 Simulation of FX

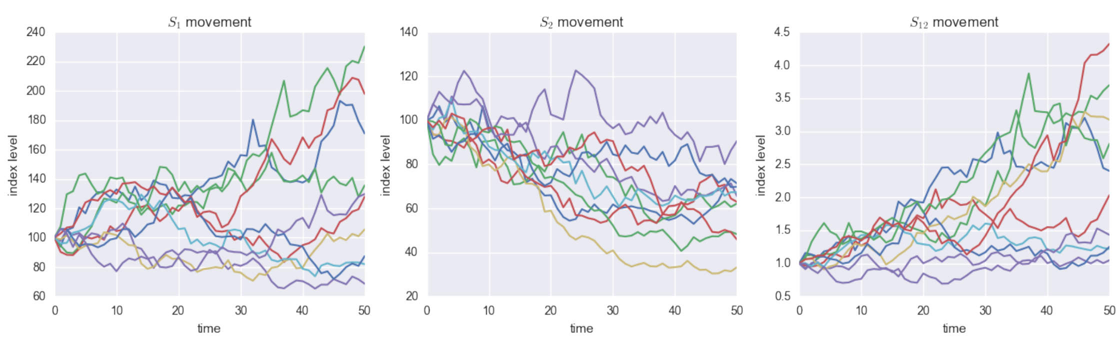
Here we simulate paths for the FX triangle S_1, S_2 and S_{12}

In [7]:
```
# Data simulation
S0, S0_, S0__ = 100.0, 100.0, 1.0 # initial value
r1, r2, rd = 0.5, 0.8, 0.6  # constant short rate
sigma1, sigma2 = 0.25, 0.25  # constant volatility
T = 2.0  # time in years
M = 50 # maturity
I = 100  # number of random draws

dt = T / M
S_1 = np.zeros((M + 1, I))
S_2 = np.zeros((M + 1, I))

S_1[0] = S0
S_2[0] = S0_

for t in range(1, M + 1):
```

```
            S_1[t] = S_1[t - 1] * np.exp((rd - r1 ) * dt
                    + sigma1 * np.sqrt(dt) * npr.standard_normal(I))
            S_2[t] = S_2[t - 1] * np.exp((rd - r2) * dt
                    + sigma2 * np.sqrt(dt) * npr.standard_normal(I))

        S_12 = S_1 / S_2
```

Here we plot the paths obtained via the Log-Euler scheme for S_1, S_2 and S_{12} :

```
In [8]: f, (ax1, ax2, ax3) = plt.subplots(1, 3, sharey=False)
        f.set_size_inches(16, 4)

        ax1.plot(S_1[:, :10], lw=1.5)
        ax1.set_xlabel('time')
        ax1.set_ylabel('index level')
        ax1.set_title('$S_1$ movement')
        ax1.grid(True)

        ax2.plot(S_2[:, :10], lw=1.5)
        ax2.set_xlabel('time')
        ax2.set_ylabel('index level')
        ax2.set_title('$S_2$ movement')
        ax2.grid(True)

        ax3.plot(S_12[:, :10], lw=1.5)
        ax3.set_xlabel('time')
        ax3.set_ylabel('index level')
        ax3.set_title('$S_{12}$ movement')
        ax3.grid(True)
```



2.1 Implementation of The Algorithm

Here we implement the particle method. We first define our delta dirac kernel: $f_n(x) = \dfrac{n}{\pi(1+(nx)^2)}$ for $n = 100$

```
In [9]: from scipy.interpolate import interp1d
        # Approximation of Delta dirac function
        def delta(x):
            n = 100
            return n / (1 + (n * x) ** 2) / np.pi
```

2.1.1 Local in cross volatility model

We are taking the case for constant volatility $\sigma\_1 = \sigma\_2\$and(\rho = \frac{\sigma_1^2+\sigma_2^2-\sigma_{12}^2}{2\sigma_1\sigma_2}$ as described in the paper. $a = \sigma_1^2 + \sigma_2^2$ and $b = -2\sigma_1\sigma_2$

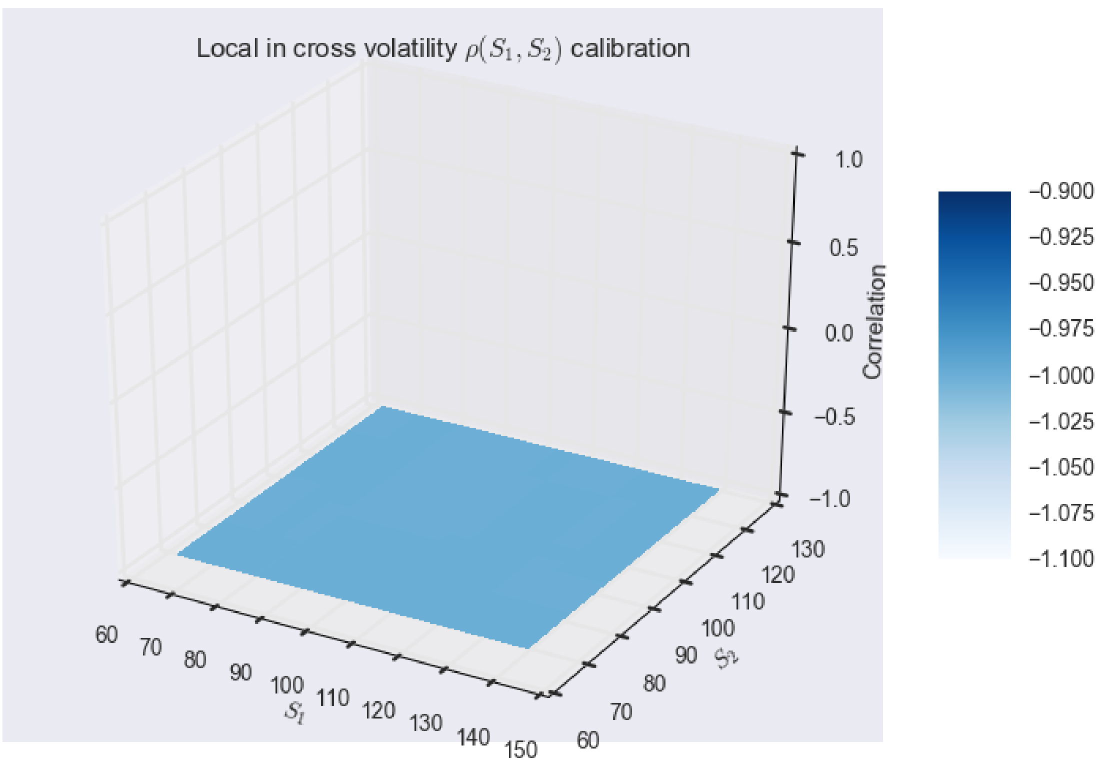
```
In [10]: # Definition of Parameters
         k = 1
         sigma_12 = 0.5
         sigma1, sigma2 = 0.25, 0.25
         a = sigma1 ** 2 + sigma2 ** 2
         b = -2 * sigma1 * sigma2

         N = 10 # grid
         grilha = np.linspace(0,100, N)
```

```
In [11]: # interpolation formula
         def interp(p, f):
             x = p
             y = f
             f2 = interp1d(x, y, kind='cubic')
             return f2
```

```
In [12]: # Particle Method
         def calibrate(S_1, S_2, sigma1, sigma2, a, b, grilha):
             ind = 0
             rho = (sigma1**2 + sigma2**2 - sigma_12**2) / (2 * sigma1 * sigma2)
             Enum = np.zeros((N, M+1))
             Eden = np.zeros((N, M+1))
             f = np.zeros((N, M+1))
             for p in grilha:
                 Enum[ind] = np.sum((S_2*(sigma1**2 + sigma2**2 + 2*(a/b)*sigma1*si
                 Eden[ind] = np.sum((S_2*(sigma1*sigma2)/b) * delta(S_1/S_2 - p), a
                 f[ind] = (Enum[ind] - sigma_12**2)/(2*Eden[ind])
                 ind+=1

             f2 = []
             for t in range(1, M + 1):
                 f2.append(interp(grilha, f[:, t]))
             return f2
```

```
In [13]: flist = calibrate(S_1, S_2, sigma1, sigma2, a, b, grilha)
         t = 10 # time where to calculate rho
```

```
s1, s2 = np.meshgrid(S_1[t], S_2[t])
z = (1/b) * (flist[t](s1/s2) - a)

fig = plt.figure(figsize=(9, 6))
ax = fig.gca(projection='3d')

surf = ax.plot_surface(s1, s2, z, rstride=1, cstride=1,
                       cmap=plt.cm.Blues, linewidth=0,
                       antialiased=False)

ax.set_xlabel(r'$S_1$')
ax.set_ylabel(r'$S_2$')
ax.set_zlabel(r'Correlation')
ax.set_zlim([-1,1])
ax.set_title(r'Local in cross volatility $\rho(S_1,S_2)$ calibration')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/constant_local_in_cross_vol.pdf", bbox_inches='tight')
```

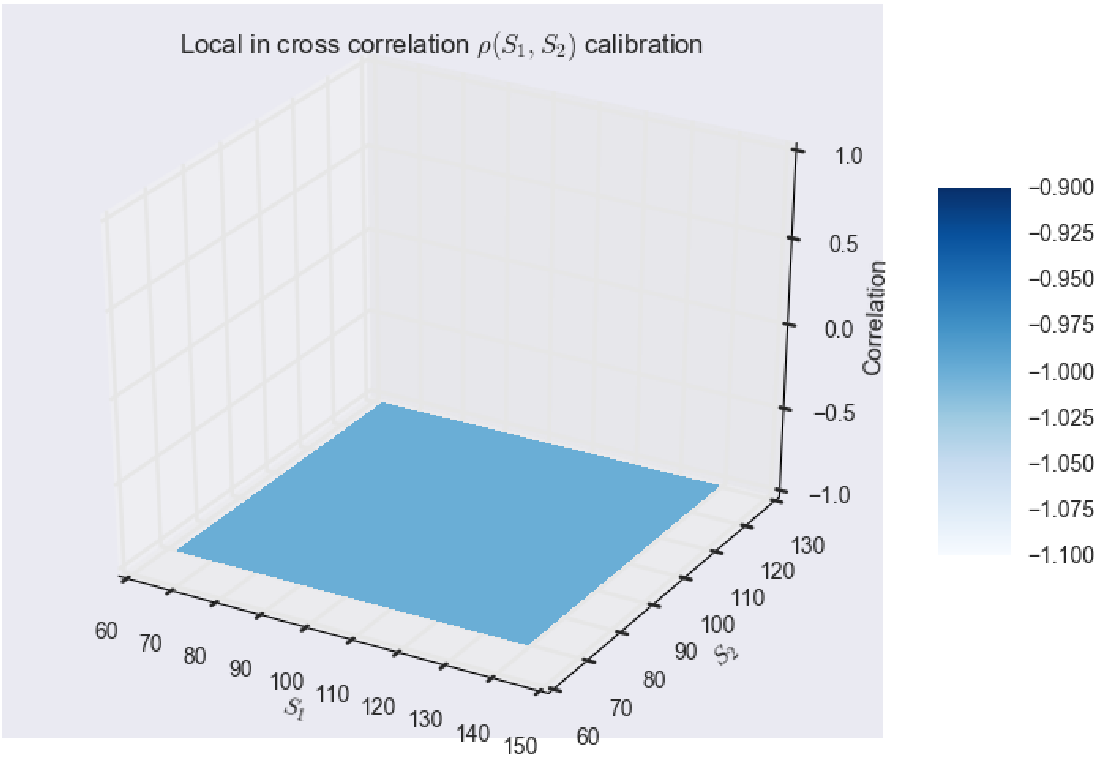


We see a costant correlation as expected with value $\rho = \dfrac{\sigma_1^2 + \sigma_2^2 - \sigma_{12}^2}{2\sigma_1\sigma_2} = -1$ as expected.

2.1.2 Local in cross correlation model

We are taking $a = 0$ and $b = 1$

In [14]:
```
a,b = 0, 1
flist = calibrate(S_1, S_2, sigma1, sigma2, a, b, grilha)
t = 10
s1, s2 = np.meshgrid(S_1[t], S_2[t])
z = (1/b) * (flist[t](s1/s2) - a)

fig = plt.figure(figsize=(9, 6))
ax = fig.gca(projection='3d')

surf = ax.plot_surface(s1, s2, z, rstride=1, cstride=1,
                       cmap=plt.cm.Blues, linewidth=0,
                       antialiased=False)

ax.set_xlabel(r'$S_1$')
ax.set_ylabel(r'$S_2$')
ax.set_zlabel(r'Correlation')
ax.set_zlim([-1,1])
ax.set_title(r'Local in cross correlation $\rho(S_1,S_2)$ calibration')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/constant_local_in_cross_corr.pdf", bbox_inches='tight'
```

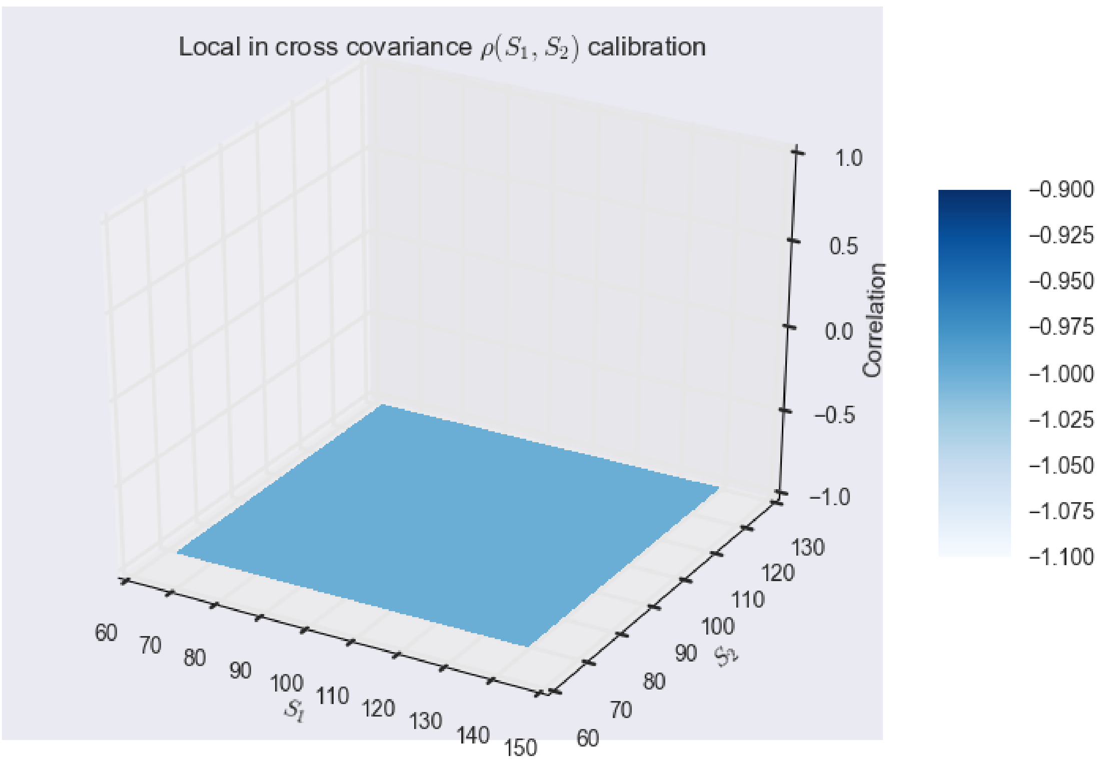


We see a costant correlation as expected with value $\rho = \dfrac{\sigma_1^2 + \sigma_2^2 - \sigma_{12}^2}{2\sigma_1\sigma_2} = -1$ as expected.

2.1.3 Local in cross covariance model

We are taking $a = 0$ and $b = \sigma_1\sigma_2$

```
In [15]: a, b = 0, sigma1 * sigma2
         flist = calibrate(S_1, S_2, sigma1, sigma2, a, b, grilha)
         t = 10
         s1, s2 = np.meshgrid(S_1[t], S_2[t])
         z = (1/b) * (flist[t](s1/s2) - a)

         fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')

         surf = ax.plot_surface(s1, s2, z, rstride=1, cstride=1,
                                cmap=plt.cm.Blues, linewidth=0,
                                antialiased=False)

         ax.set_xlabel(r'$S_1$')
         ax.set_ylabel(r'$S_2$')
         ax.set_zlabel(r'Correlation')
         ax.set_zlim([-1,1])
         ax.set_title(r'Local in cross covariance $\rho(S_1,S_2)$ calibration')

         fig.colorbar(surf, shrink=0.5, aspect=5)
         fig.savefig("images/constant_local_in_cross_cov.pdf", bbox_inches='tight')
```



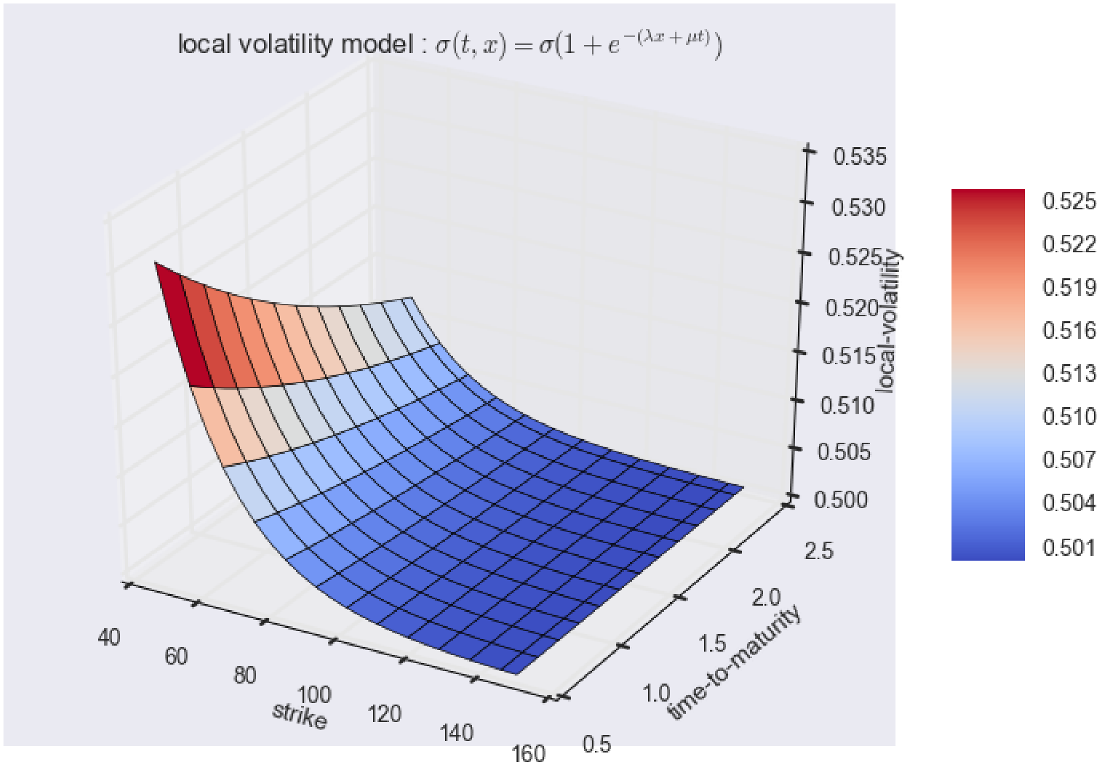
We see a costant correlation as expected with value $\rho = \frac{\sigma_1^2 + \sigma_2^2 - \sigma_{12}^2}{2\sigma_1\sigma_2} = -1$ as expected.

3 Local Volatility Model

Here we discuss the model with exponential volatilities of the form:

$$\sigma(t,x) = \sigma(1 + e^{-(\lambda x + \mu x)}).$$

```
In [16]: strike = np.linspace(50, 150, 24)
         ttm = np.linspace(0.5, 2.5, 24)
         strike, ttm = np.meshgrid(strike, ttm)


         iv = 0.50 * (1 + np.exp(- 0.05 * strike - 0.5 * ttm))
           # generate fake implied volatilities

         from mpl_toolkits.mplot3d import Axes3D

         fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')

         surf = ax.plot_surface(strike, ttm, iv, rstride=2, cstride=2,
                                cmap=plt.cm.coolwarm, linewidth=0.5,
                                antialiased=True)

         ax.set_xlabel('strike')
         ax.set_ylabel('time-to-maturity')
         ax.set_zlabel('local-volatility')
         ax.set_title(r'local volatility model : $\sigma(t,x) = \sigma (1 + e^{-(\l

         fig.colorbar(surf, shrink=0.5, aspect=5)
         fig.savefig("images/exponential_volatility.pdf", bbox_inches='tight')
         # tag: matplotlib_17
         # title: 3d surface plot for (fake) implied volatilities
         # size: 70
```



For the Triangle model :

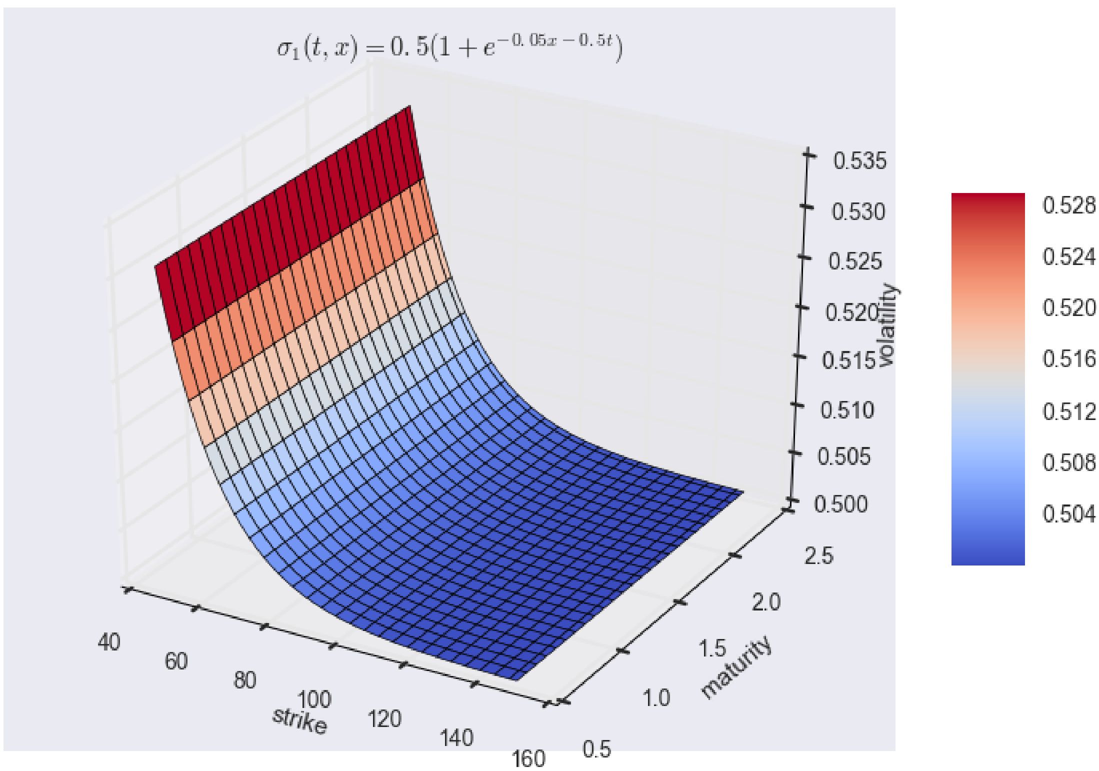
$$\begin{aligned} dS_t^1 &= (r_t^d - r_t^1)S_t^1\,dt + \sigma_1(t,S_t^1)S_t^1\,dW_t^1 \\ dS_t^2 &= (r_t^d - r_t^2)S_t^2\,dt + \sigma_2(t,S_t^2)S_t^2\,dW_t^2 \\ d\left\langle W^1, W^2 \right\rangle_t &= \rho(t,S_t^1,S_t^2)\,dt \end{aligned} \tag{1}$$

We start by simulating S_1, S_2 and S_{12} We will now plot the functions $\sigma_1, sigma_2, \sigma_{12}$ and ρ that we will use to compare with the calibration method

3.1 Plot for $\sigma_1(t,x)$

```
In [17]: fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')

         def sigma1(t,x):
             aux = 0.50 * (1 + np.exp(- 0.05 * x - 0.5 * t))
             return np.minimum(np.maximum(aux, -1), 1)

         x = np.linspace(50, 150, 50)
         t = np.linspace(0.5, 2.5, 50)
         s1, s2 = np.meshgrid(x, t)
```

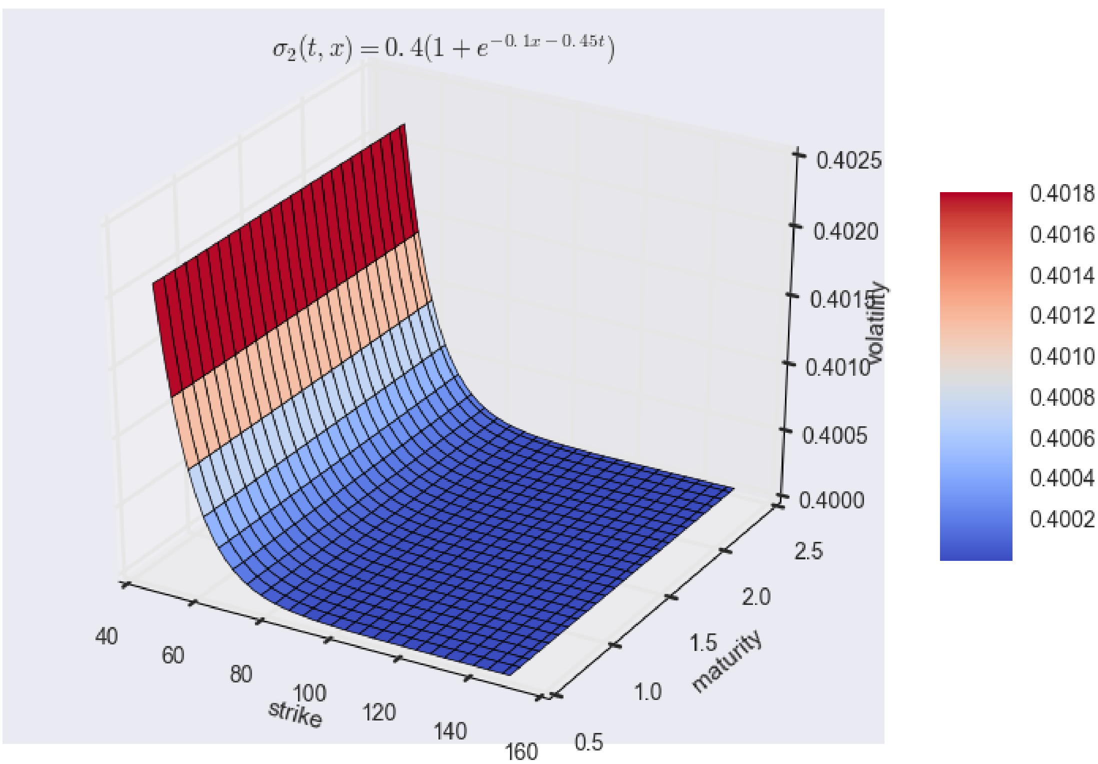
```
z = sigma1(t, x)

surf = ax.plot_surface(s1, s2, z, rstride=2, cstride=2,
                       cmap=plt.cm.coolwarm, linewidth=0.5,
                       antialiased=True)

ax.set_title(r'$\sigma_1(t,x) =  0.5 (1 + e^{- 0.05  x - 0.5  t})$')
ax.set_xlabel('strike')
ax.set_ylabel('maturity')
ax.set_zlabel(r'volatility')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/sigma1_exp.pdf", bbox_inches='tight')
```



3.2 Plot for $\sigma_2(t, x)$

```
In [18]: fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')

         def sigma2(t,x):
             aux = 0.40 * (1 + np.exp(- 0.1 * x - 0.45 * t))
             return np.minimum(np.maximum(aux, -1), 1)
```

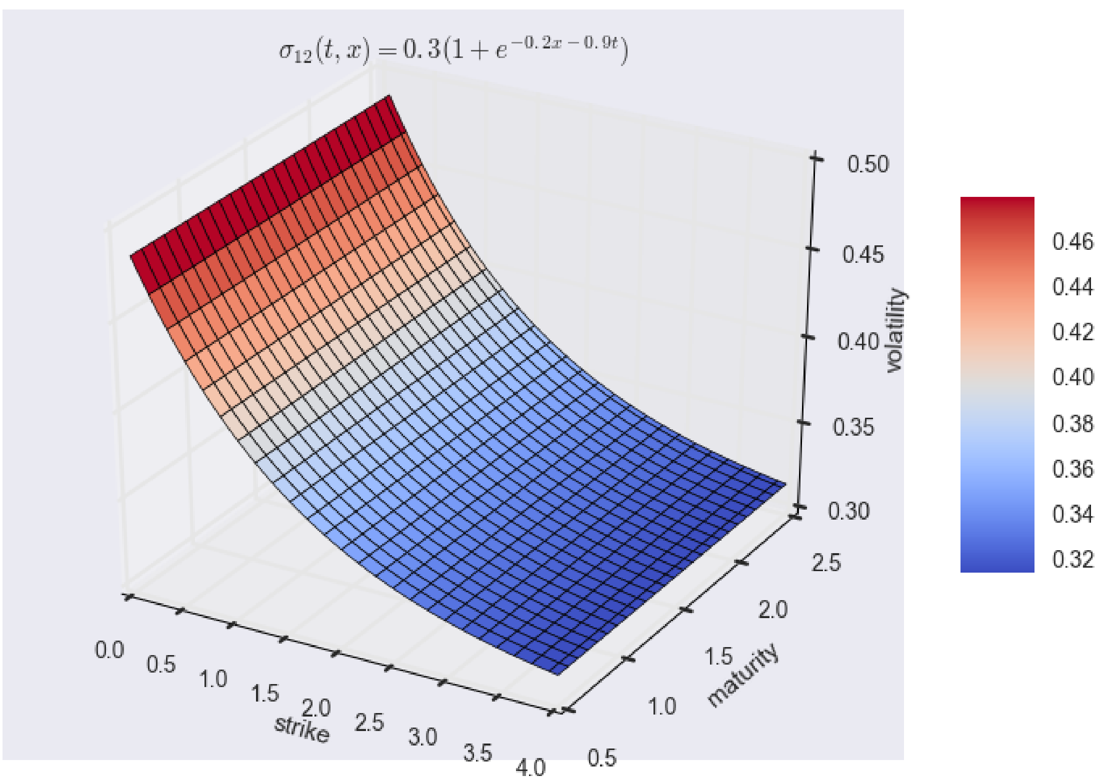
```
x = np.linspace(50, 150, 50)
t = np.linspace(0.5, 2.5, 50)
s1, s2 = np.meshgrid(x, t)
z = sigma2(t, x)

surf = ax.plot_surface(s1, s2, z, rstride=2, cstride=2,
                       cmap=plt.cm.coolwarm, linewidth=0.5,
                       antialiased=True)

ax.set_title(r'$\sigma_2(t,x) =  0.4 (1 + e^{- 0.1  x - 0.45  t})$')
ax.set_xlabel('strike')
ax.set_ylabel('maturity')
ax.set_zlabel(r'volatility')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/sigma2_exp.pdf", bbox_inches='tight')
```



3.3 Plot for $\sigma_{12}(t,x)$

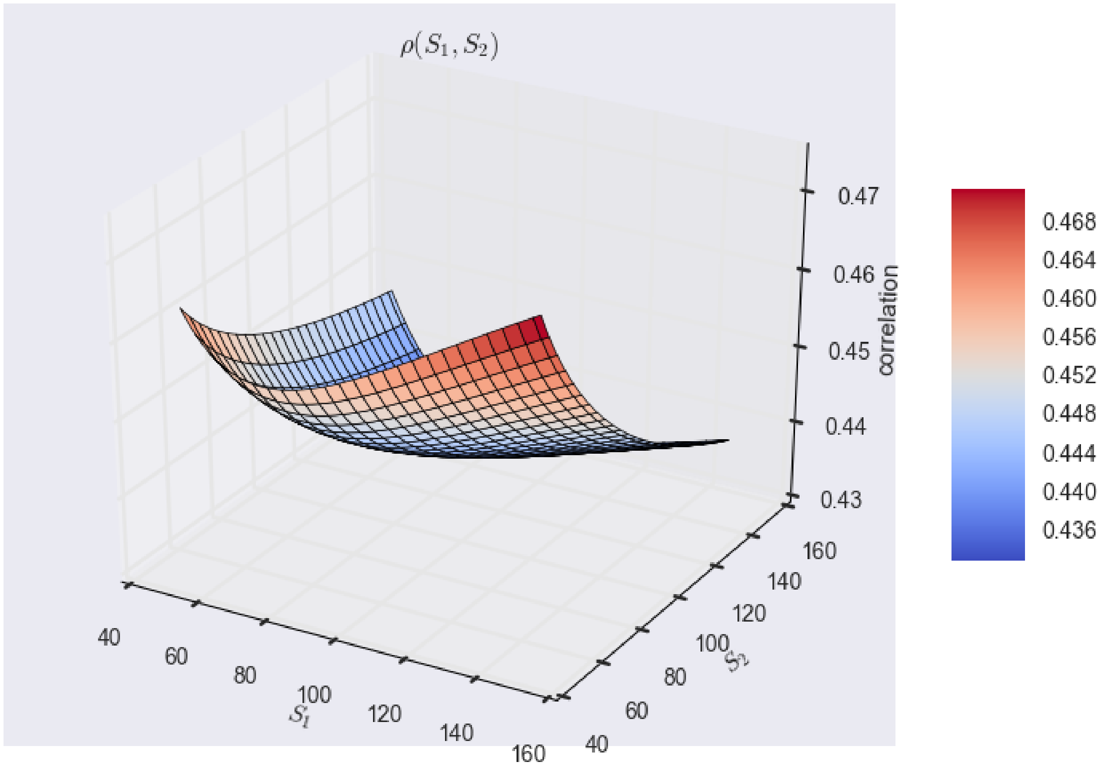
```
In [19]: fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')
```

```python
def sigma12(t,x):
    aux = 0.30 * (1 + np.exp(- 0.2 * x - 0.9 * t))
    return np.minimum(np.maximum(aux, -1), 1)

x = np.linspace(0.1, 4, 50)
t = np.linspace(0.5, 2.5, 50)
s1, s2 = np.meshgrid(x, t)
z = sigma12(t, x)

surf = ax.plot_surface(s1, s2, z, rstride=2, cstride=2,
                       cmap=plt.cm.coolwarm, linewidth=0.5,
                       antialiased=True)

ax.set_title(r'$\sigma_{12}(t,x) =  0.3 (1 + e^{- 0.2  x - 0.9  t})$')
ax.set_xlabel('strike')
ax.set_ylabel('maturity')
ax.set_zlabel(r'volatility')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/sigma12_exp.pdf", bbox_inches='tight')
```



3.4 Plot for $\rho(S_1, S_2)$

```
In [20]: fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')

         def ro(t,x1,x2):
             aux = (sigma1(t,x1)**2 + sigma2(t,x2)**2 - sigma12(t,x1/x2)**2) / (2 *
             return np.minimum(np.maximum(aux, -1), 1)

         t = 20
         ttm = (M - t) * T / M
         x = np.linspace(50, 150, 50)
         y = np.linspace(50, 150, 50)
         s1, s2 = np.meshgrid(x, y)
         z = ro(ttm, s1, s2)

         surf = ax.plot_surface(s1, s2, z, rstride=2, cstride=2,
                                cmap=plt.cm.coolwarm, linewidth=0.5,
                                antialiased=True)

         ax.set_title(r'$\rho(S_1, S_2)$')
         ax.set_xlabel(r'$S_1$')
         ax.set_ylabel(r'$S_2$')
         ax.set_zlabel('correlation')

         fig.colorbar(surf, shrink=0.5, aspect=5)
         fig.savefig("images/rho_exp.pdf", bbox_inches='tight')
```



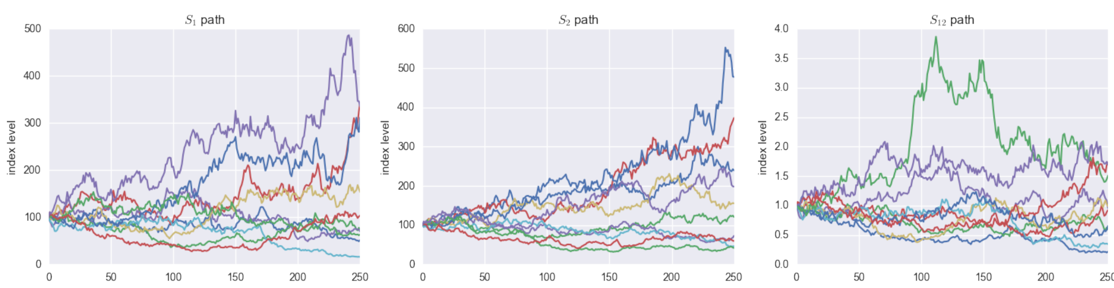
3.5 Path Simulation: S_1^t, S_2^t, S_{12}^t

In [21]:
```
# Data simulation
S0, S0_, S0__ = 100.0, 100.0, 1.0 # initial value
r1, r2, rd = 0.5, 0.4, 0.6  # constant short rate

T = 2.5  # time in years
M = 250 # maturity
I = 5000  # number of random draws

dt = T / M
S_1 = np.zeros((M + 1, I))
S_2 = np.zeros((M + 1, I))
S_12 = np.zeros((M + 1, I))

S_1[0] = S0
S_2[0] = S0_
S_12[0] = S0__

for t in range(1, M + 1):
    ttm = (M - t) * T / M
    rh = ro(ttm, S_1[t - 1], S_2[t - 1])
    W1 = npr.standard_normal(I)
```

```
W2 = rh * W1 + np.sqrt(1-rh**2) * npr.standard_normal(I)

S_1[t] = S_1[t - 1] * np.exp((rd-r1) * dt + sigma1(ttm, S_1[t - 1]) *
S_2[t] = S_2[t - 1] * np.exp((rd-r2) * dt + sigma2(ttm, S_2[t - 1]) *
S_12[t] = S_12[t - 1] * np.exp((r2-r1) * dt + sigma1(ttm, S_1[t - 1])
```

3.6 Plot of the trajectories

In [22]:
```
fig, (ax1, ax2, ax3) = plt.subplots(1, 3, sharey=False)
fig.set_size_inches(18, 4)

ax1.plot(S_1[:, :10], lw=1.5)
ax1.set_xlabel('time')
ax1.set_ylabel('index level')
ax1.set_title('$S_1$ path')
ax1.grid(True)

ax2.plot(S_2[:, :10], lw=1.5)
ax2.set_xlabel('time')
ax2.set_ylabel('index level')
ax2.set_title('$S_2$ path')
ax2.grid(True)

ax3.plot(S_12[:, :10], lw=1.5)
ax3.set_xlabel('time')
ax3.set_ylabel('index level')
ax3.set_title('$S_{12}$ path')
ax3.grid(True)
fig.savefig("images/exp_paths.pdf", bbox_inches='tight')
```



3.7 Particle Method Implementation

Here we improve our old algorithm to apply it to non constant volatilities

In [23]:
```
# Algorithm
N_g = 500 # grid
```

```
grilha = np.linspace(0.01, 4, N_g)

def particle_method(S_1, S_2, S_12, sigma1, sigma2, sigma12, a, b, grilha)
    ind = 0
    Enum = np.zeros((N_g, M+1))
    Eden = np.zeros((N_g, M+1))
    f = np.zeros((N_g, M+1))
    for p in grilha:
        ttm = (M - ind) * T / M
        Enum[ind] = np.sum((S_2*(sigma1(ttm, S_1)**2 + \
                             sigma2(ttm, S_2)**2 + 2*(a(ttm, S_1, S_2)
                             *sigma1(ttm, S_1)*sigma2(ttm, S_2)))  \
                       * delta(S_1/S_2 - p), axis = 1) / np.sum(S_2 *
        Eden[ind] = np.sum((S_2*(sigma1(ttm, S_1)*sigma2(ttm, S_2))/b(ttm,
                       delta(S_1/S_2 - p), axis = 1)/np.sum(S_2 * delt
        f[ind] = (Enum[ind] - sigma12(ttm, p)**2 * np.ones(M+1))/(2*Eden[i
        ind+=1

    f2 = []
    for time in range(1, M + 1):
        f2.append(interp(grilha, f[:, time]))
    return f2
```

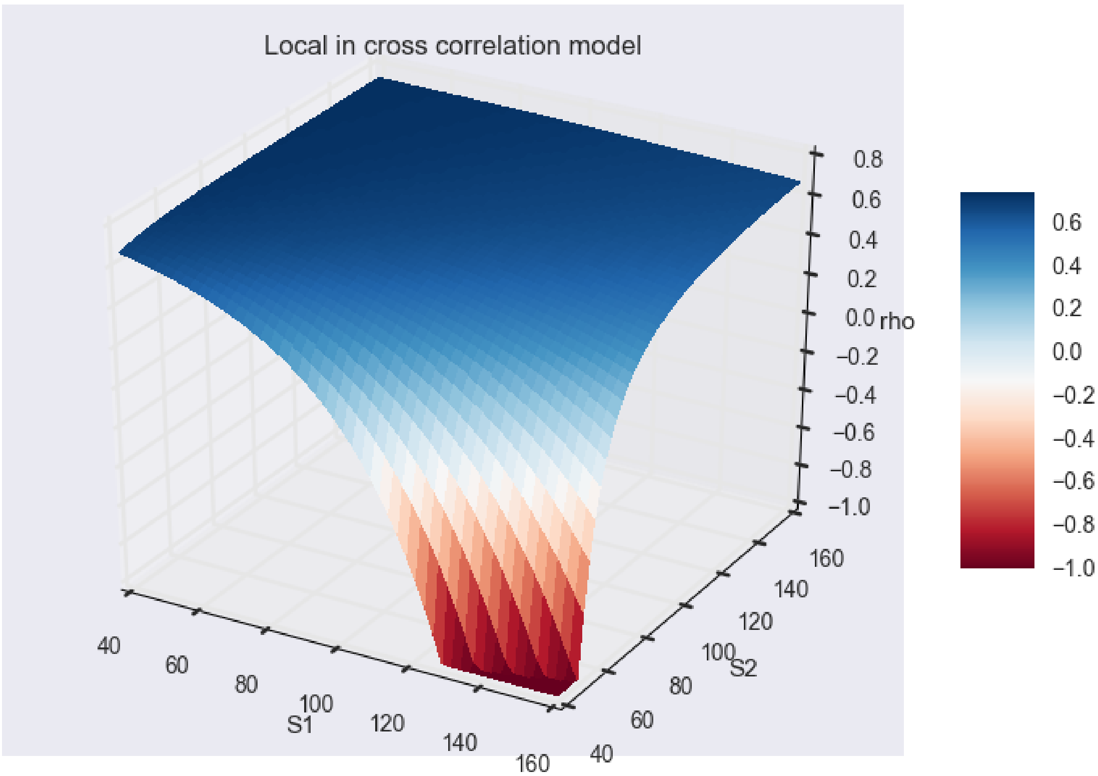
```
In [24]: # Auxiliary variables
         t = 20
         ttm = (M - t) * T / M

         x = np.linspace(40, 160, 50)
         y = np.linspace(40, 160, 50)
         s1, s2 = np.meshgrid(x, y)
```

3.8 Model local in cross correlation

We take $a = 0$ and $b = 1$

```
In [25]: def a(t,x1,x2):
             return 0
         def b(t,x1,x2):
             return 1

         flist = particle_method(S_1, S_2, S_12, sigma1, sigma2, sigma12, a, b, gri

         z = (1 / b(ttm, s1, s2)) * (flist[t](s1 / s2) - a(ttm, s1, s2))
         z = np.minimum(np.maximum(z, -1), 1)

         fig = plt.figure(figsize=(9, 6))
         ax = fig.gca(projection='3d')
```

```
surf = ax.plot_surface(s1, s2, z, rstride=1, cstride=1,
                       cmap=plt.cm.RdBu, linewidth=0,
                       antialiased=False)
ax.set_title(r'Local in cross correlation model')
ax.set_xlabel('S1')
ax.set_ylabel('S2')
ax.set_zlabel('rho')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/exp_local_corr.pdf", bbox_inches='tight')
```

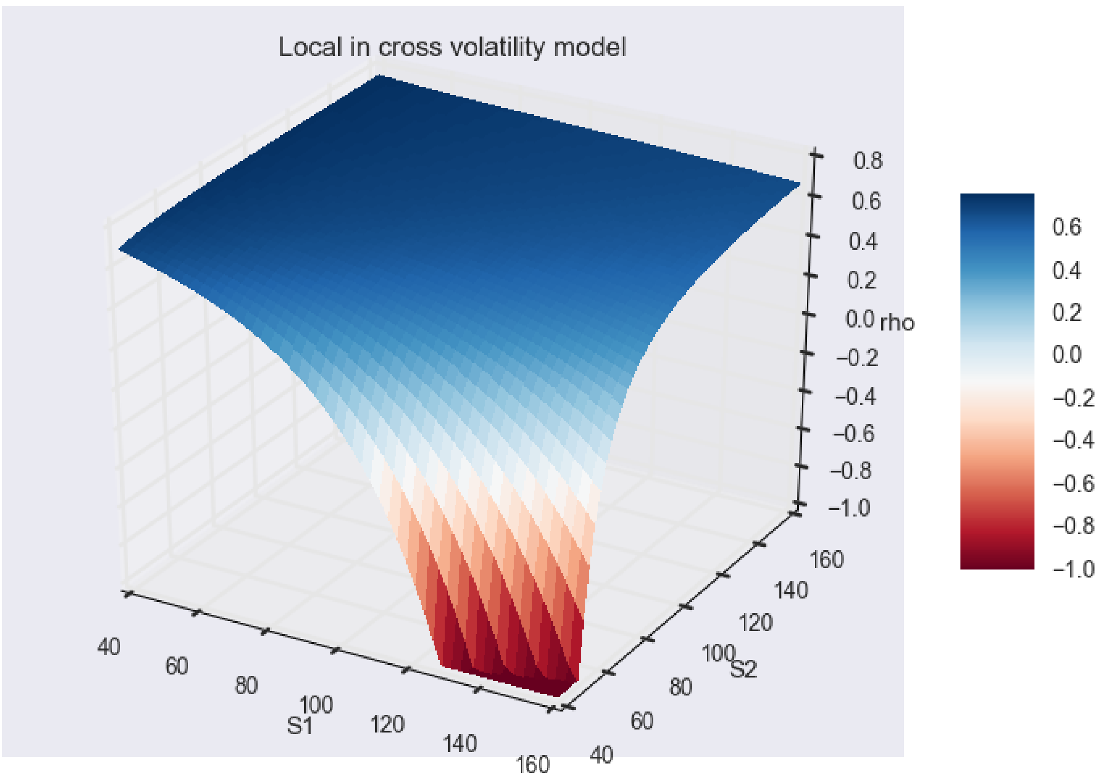


3.9 Model local in cross volatility

We take $a = \sigma_1^2 + \sigma_2^2$ and $b = -2\sigma_1\sigma_2$

```
In [26]: def a(t,x1,x2):
             return sigma1(t,x1) ** 2 + sigma2(t,x2)**2
         def b(t,x1,x2):
             return - 2 * sigma1(t,x1) * sigma2(t,x2)

         flist = particle_method(S_1, S_2, S_12, sigma1, sigma2, sigma12, a, b, gri

         z = (1/b(ttm, s1, s2)) * (flist[t](s1/s2) - a(ttm, s1, s2))
```

```
z = np.minimum(np.maximum(z, -1), 1)

fig = plt.figure(figsize=(9, 6))
ax = fig.gca(projection='3d')

surf = ax.plot_surface(s1, s2, z, rstride=1, cstride=1,
                       cmap=plt.cm.RdBu, linewidth=0,
                       antialiased=False)
ax.set_title(r'Local in cross volatility model')
ax.set_xlabel('S1')
ax.set_ylabel('S2')
ax.set_zlabel('rho')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/exp_local_vol.pdf", bbox_inches='tight')
```

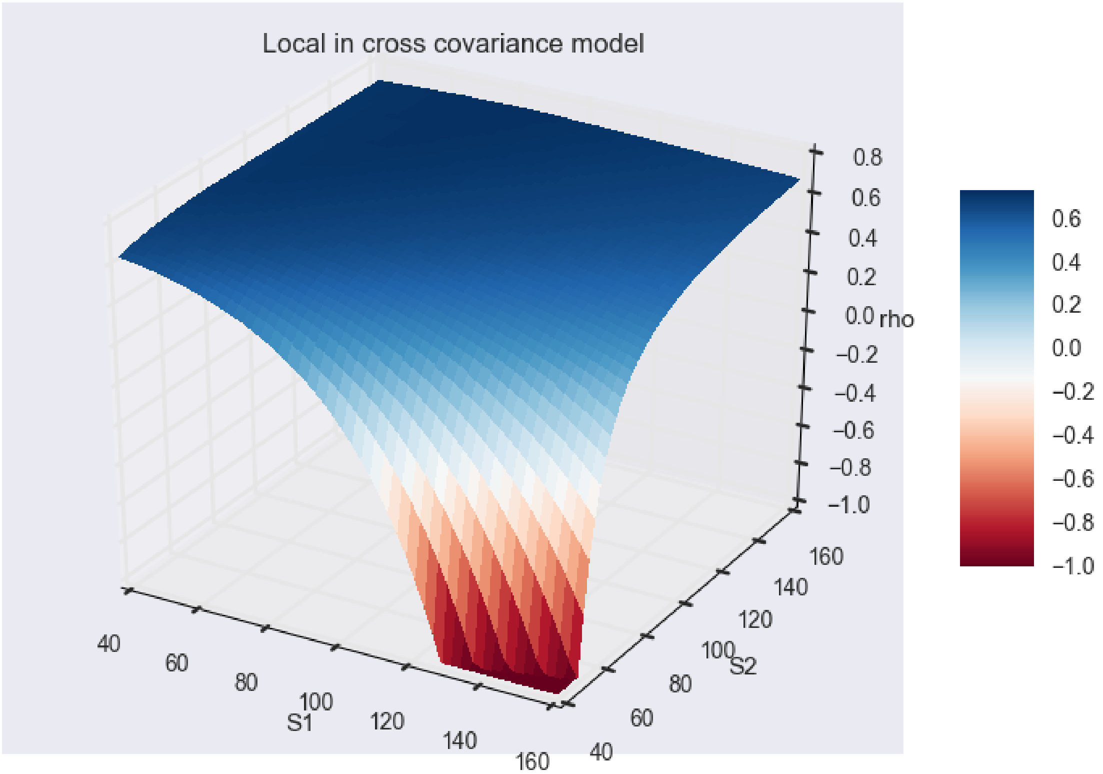


3.10 Model local in cross covariance

We take $a = 0$ and $b = \sigma_1 \sigma_2$

```
In [27]: def a(t,x1,x2):
             return 0
         def b(t,x1,x2):
```

```
    return sigma1(t,x1) * sigma2(t,x2)

flist = particle_method(S_1, S_2, S_12, sigma1, sigma2, sigma12, a, b, gri

z = (1/b(ttm, s1, s2)) * (flist[t](s1/s2) - a(ttm, s1, s2))
z = np.minimum(np.maximum(z,-1),1)

fig = plt.figure(figsize=(9, 6))
ax = fig.gca(projection='3d')

surf = ax.plot_surface(s1, s2, z, rstride=1, cstride=1,
                       cmap=plt.cm.RdBu, linewidth=0,
                       antialiased=False)
ax.set_title(r'Local in cross covariance model')
ax.set_xlabel('S1')
ax.set_ylabel('S2')
ax.set_zlabel('rho')

fig.colorbar(surf, shrink=0.5, aspect=5)
fig.savefig("images/exp_local_cov.pdf", bbox_inches='tight')
```

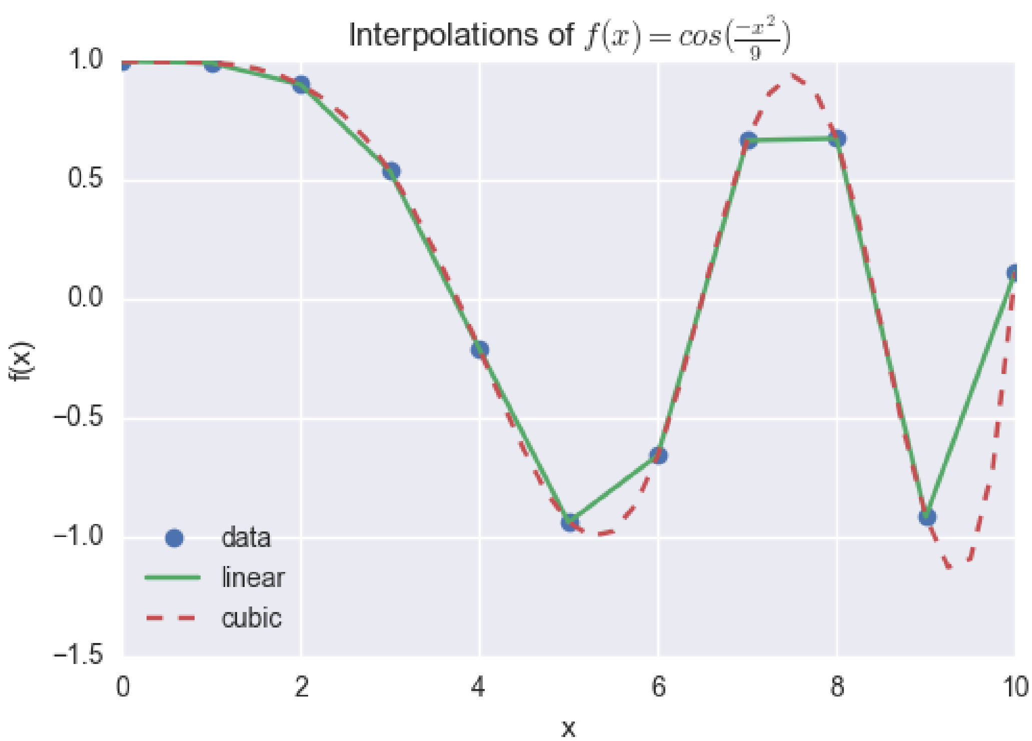


4 APPENDIX

4.1 Spline interpolation

Here we demostrate the interpolation methods

```
In [28]: from scipy.interpolate import interp1d

         x = np.linspace(0, 10, num=11, endpoint=True)
         y = np.cos(-x**2/9.0)
         f = interp1d(x, y)
         f2 = interp1d(x, y, kind='cubic')

         xnew = np.linspace(0, 10, num=41, endpoint=True)

         plt.plot(x, y, 'o', xnew, f(xnew), '-', xnew, f2(xnew), '--')
         plt.xlabel('x')
         plt.ylabel('f(x)')
         plt.legend(['data', 'linear', 'cubic'], loc='best')
         plt.title(r'Interpolations of $f(x) = cos(\frac{-x^2}{9})$')
         plt.savefig("images/spline.pdf", bbox_inches='tight')
         plt.show()
```



4.2 Delta Dirac approximation

Here we plot the delta dirac approximation function considered in the project

```
In [29]: x = np.linspace(-1, 1, num=50)
         plt.plot(x,delta(x),'-')
         plt.xlabel('$x$')
         plt.ylabel(r'$\delta (x)$')
         plt.title("Delta dirac approximation - n = 100")
         plt.savefig("images/dirac.pdf", bbox_inches='tight')
         plt.show()
```