

### Objective 1

The expression above is the standard normal pdf function.

### Objective 2

It basically means that the normal distribution pdf does not have a close form

Handwritten mathematical expressions:

- $$> \frac{1}{\sqrt{2 \cdot \pi}} \cdot e^{-\frac{x^2}{2}}$$
- $$> \text{int}(\exp(-(1/2) \cdot x^2) / \text{sqrt}(2 \cdot \pi), x)$$
$$\frac{\text{erf}\left(\frac{\sqrt{2} x}{2}\right)}{2}$$
- $$> \text{int}(\exp((1/2) \cdot x^2) / \text{sqrt}(2 \cdot \pi), x)$$
$$-\frac{1}{2} \text{erf}\left(\frac{1}{2} \sqrt{2} x\right)$$
- $$>$$

### Objective 3

I got around 30%

Desmos Graphing Calculator interface showing a complex numerical approximation of the standard normal PDF integral. The expression entered is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

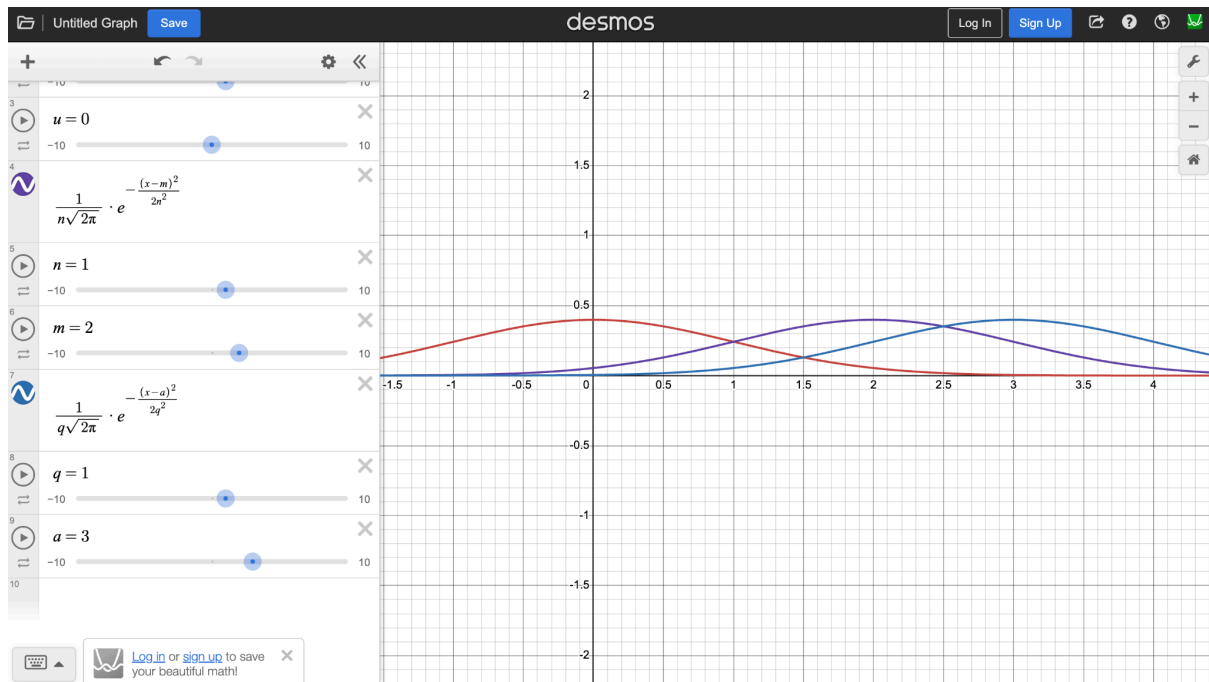
The calculator displays a long sum of terms, which is a numerical approximation of the integral of the standard normal PDF. The final result shown is:

$$0.3066659412$$

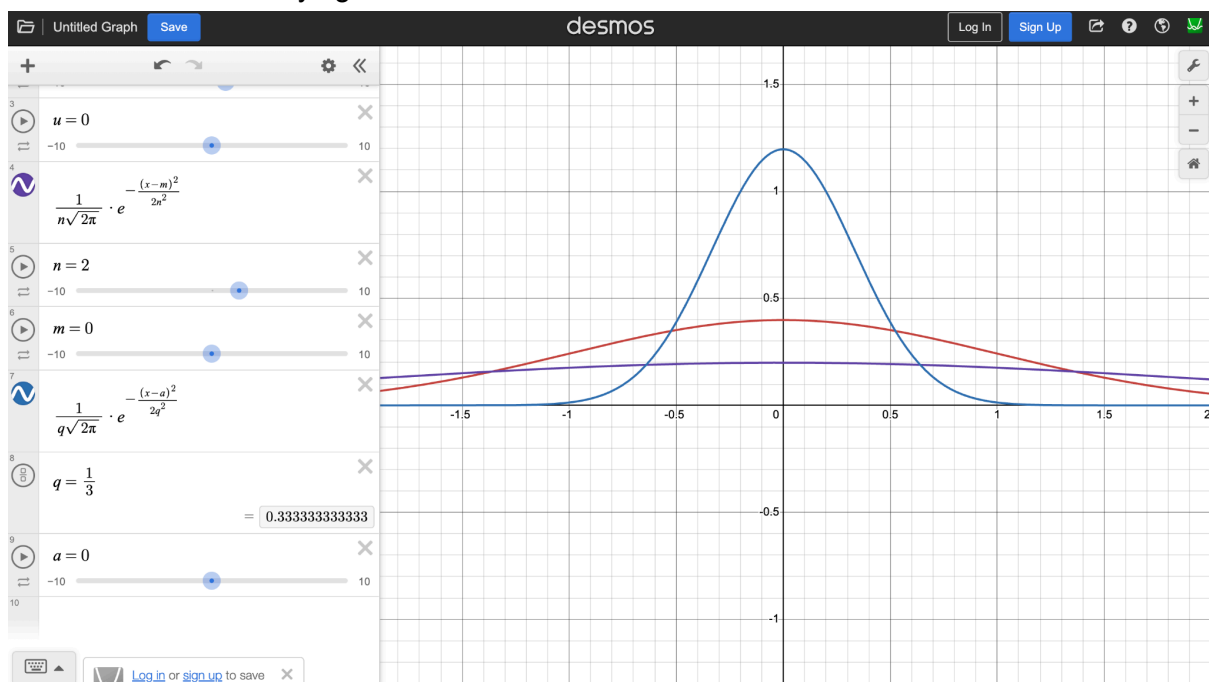
The place where I saw a similar result was on the z table. I saw a close result who is the value of 1.00 (.84134) and 0 (.50000) when you subtract these two you will get a similar result of 0.3413 and like explained in class the result of the z score table are off because of accuracy on how they were calculated.

## Objective 4

This is the curve modifying the mean



This is the curve modifying the other one



since we need to verify the mean for the normal distribution, we need to integrate using MAPLE and the output follows

$$\begin{aligned}
& \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\
& \left\{ \begin{array}{ll} \mu \operatorname{csign}\left(\frac{1}{\sigma}\right) & 0 \leq \frac{\Re\left(\frac{1}{\sigma^2}\right)}{2} \\ \infty & \text{otherwise} \end{array} \right. \quad (6) \\
& \text{simplify( (6), 'assume = real' )} \\
& \mu \operatorname{signum}(\sigma) \quad (7) \\
& \text{simplify( (7), 'assume = positive' )} \\
& \mu \quad (8)
\end{aligned}$$

This proves that mean for the normal distribution is indeed  $\mu$

### Objective 7

In this question, we need to verify the standard deviation for the normal distribution where the standard deviation for a random variable with probability density function  $f$  and mean  $\mu$  is

$$\text{defined by } \sigma = \left[ \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx \right]^{\frac{1}{2}}$$

$$\begin{aligned}
& \left[ \int_{-\infty}^{\infty} (x - 100)^2 \cdot \frac{1}{15\sqrt{2\pi}} \cdot e^{-\frac{(x-100)^2}{450}} dx \right]^{\frac{1}{2}} \\
& \sqrt{[225]} \quad (5) \\
& \sqrt{225} \\
& 15 \quad (6)
\end{aligned}$$

since  $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$  and I let  $\sigma$  as 15, and the resultant is 15, the condition where

$$\sigma = \left[ \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx \right]^{\frac{1}{2}} \text{ is met.}$$

### Objective 8

Normal distribution in some problems concerning probabilities where the value of IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15

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=
>  $\int_{85}^{115} \frac{1}{15 \cdot \sqrt{2 \cdot \pi}} \cdot e^{\frac{-(x-100)^2}{450}} dx$ 
=
> evalf[5]( (1) )
=
>  $\int_{140}^{\infty} \frac{1}{15 \sqrt{2 \pi}} e^{\frac{-(x-100)^2}{450}} dx$ 
=
> evalf[5]( (3) )

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$$\operatorname{erf}\left(\frac{\sqrt{2}}{2}\right)$$

0.68269

$$\frac{1}{2} - \frac{\operatorname{erf}\left(\frac{4\sqrt{2}}{3}\right)}{2}$$

0.00383