LANGARA COLLEGE
MATH 1273/83 LAB #10
SEQUENCES & SERIES

OVERVIEW: In this lab we will use MAPLE to study basic ideas about numerical sequences and series.

TEXT REFERENCES: sections 8.1 and 8.2.

OBJECTIVE #1: To learn the MAPLE commands for generating sequences of numbers and to use these for a variety of examples.

INSTRUCTIONS: We start by determining the first 12 terms of the sequence $a_n = \frac{(-3)^n}{n!}$ (problem #43 on page 688). The symbol $n!$ is read "n factorial" and is used to indicate the product of the numbers 1 through n with $0!$ defined to be 1. We enter this in MAPLE as we write it by pencil by using the exclamation mark. Enter **(-3)^n/n!** .

From the Context menu select *Sequence/n* (at the bottom of the menu); enter 1 for the *Lower index,* 12 for the *Upper index*, and then choose *OK.* This will give a list of exact values of the first 12 terms of the sequence.

To get decimal approximations of the terms of the sequence, you can enter -3 as $-3.$ (with a decimal point) or -3.0, and repeat the above steps. Approximate the first 12 terms of the above sequence and record your result as part of your lab report.

Use the above instructions to generate and approximate the first 12 terms of the sequence of $\left\{\frac{\ln(n^2)}{n}\right\}$ and record your result as part of your lab report.

To approximate the terms of the sequence in this latter case: make sure you are using the Worksheet mode (as opposed to Document mode). Then you can click on the command that MAPLE uses to generate the sequence, select *Apply a Command* from the top of the Context menu, and enter the **evalf** command, which numerically evaluates expressions.

Another, less convenient way to do this is to use *Coversions/List* to convert the sequence into a list for MAPLE, and then by clicking on the result, you can approximate the terms using *Approximate* from the Context menu.

OBJECTIVE #2: To get a graphical sense of a sequence by plotting it.

INSTRUCTIONS: We can use parametric equations to generate an image of a sequence as the graph of a function over the Natural Numbers. This linked video tells you how to do it

using the right-click menu:
http://www.maplesoft.com/content/TeachingConcepts/Lecture/34/Solution%204-33(b)%20SeqLim-Interactive.mp4 . Or, if you don't mind typing in a few commands, here you go:

We first load the **plots** package (execute **with(plots):** or use the *Tools/LoadPackage/Plots* from the menu bar on the top of the MAPLE window). Enter the pair $[n, a_n]$ (where a_n is the general term of your sequence). For example, to graph the first sequence in Objective #1, we start by entering **[n, (-3)^n/n!]** . Repeat the instructions from Objective 1 to obtain **seq([n, (-3)^n/factorial(n)], n = 1 .. 12)** . To plot this sequence, use the command **pointplot({seq([n, (-3)^n/factorial(n)], n = 1..12)})** . Note the curly brackets (these transform the sequence into a list of points for Maple to plot). You can change the size/color/shape of the points plotted by clicking on the graph and choosing *Symbol*, then changing the desired options.

Reproduce your plot of the first 12 terms of both sequences from Objective #1 as part of your lab report. (Note: if MAPLE "misbehaves" (i.e. doesn't want to plot), open a new worksheet from the file menu and copy your work over; that should solve the problem.)

OBJECTIVE #3: To use what we learned above to consider the behaviour of a sequence (that is, whether the sequence converges or diverges).

INSTRUCTIONS: The fundamental image we should have of a sequence is an ordered set of numbers and the fundamental issue we should address when considering a sequence is how the numbers in this set eventually behave. As such, our focus needs to be beyond the first few terms, rather it should be on large numbers of terms with large indices. The computer can help us with such a focus (to the extent of the computer's limitations).

For example, to consider the next 30 terms for the sequence we studied above, repeat the instructions from Objective #1 for approximating the values of the sequence **(-3)^n/n!** , entering 13 for the starting value and a 42 for the ending value.

Scroll through the values of the sequence to get an idea of the behaviour of these values as n increases. You do not need to include the values in the lab report, just your observations. *Plot* the result and include your plot in the lab report; the *pointplot* command discussed above automatically chooses a good viewing rectangle. In your lab report, describe what you observe about the behaviour of these numbers. Based on this observation, guess the value of the limit of this sequence.

In a similar manner use the graph of a sequence to determine the limit (if it exists) of each of the following sequences: $\left\{\arctan\left(\frac{2n}{2n+1}\right)\right\}$, $\left\{(-1)^{n+1}\frac{n+1}{n}\right\}$, $\{\sin(n)\}$, and $\left\{\sqrt[n]{3^n+5^n}\right\}$. Include your conclusion and the plots in your report.

<u>OBJECTIVE #4:</u> To explore the definition of convergence of a sequence.

<u>INSTRUCTIONS</u>: The definition on page 680 says that $\lim_{n\to\infty} a_n = L$ if we can make the terms "*as close to L as we like by taking n sufficiently large.*" For example, how many terms must we consider (i.e. how large must n be) so that $a_n = \dfrac{(-3)^n}{n!}$ differs from its limit (in absolute value) by no more than 10^{-4} ? To answer this, scan through the list of numbers that you created above. Record your conclusion as part of your lab report.

<u>OBJECTIVE #5</u>: To use MAPLE to study series.

<u>INSTRUCTIONS</u>: Any series, $\sum_{n=1}^{\infty} a_n$, has two sequences associated with it. The first is the sequence of *terms*, a_n , and the second is the sequence of *partial sums*, $s_n = \sum_{i=1}^{n} a_i$. It will be important to distinguish the roles these two sequences play in studying the behaviour of a series.

We can compute the partial sums of the terms of a given series using the same procedure as in Lab #1. For example, you can use the Expressions Palette to enter the partial sums. Study the behaviour of $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ by first calculating the partial sums $s_n = \sum_{i=1}^{n} \left(\frac{2}{3}\right)^i$,

then use the instructions from the earlier objectives to determine how the sequence $\{s_n\}$ behaves. What do you conclude about $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$?

To distinguish the roles of the terms and of the partial sums, it is helpful to plot both of these sequences together (as per figure 8.15 on page 697). Do so, and comment on what you observe about the two sequences associated with this series.

Repeat the above instructions (i.e. plot both terms and partial sums together –include your plots!) to study the behaviour of series whose terms are given by the following sequences:

$$\left\{\frac{(-3)^n}{n!}\right\} \quad , \quad \left\{\frac{1}{n^{\frac{5}{2}}}\right\} \quad , \quad \text{and} \quad \left\{\frac{1}{n}\right\}$$

That is, study the behaviour of the three series:

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n!} \quad , \quad \sum_{n=1}^{\infty} \frac{1}{n^{\frac{5}{2}}} \quad , \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n}$$

OBJECTIVE #6: To calculate the sum of series (if it exists).

INSTRUCTIONS: Repeat the above instructions to study the behaviour of series whose terms are given by the following sequences:

$$\left\{\arctan\left(\frac{2n}{2n+1}\right)\right\} \quad , \quad \left\{\ln\left(\frac{n}{n+1}\right)\right\} \quad , \quad \left\{\frac{3}{n(n+3)}\right\} \quad , \quad \text{and} \quad \left\{\left(-\frac{2}{7}\right)^{n-1}\right\}$$

That is, study the behaviour of the four series:

$$\sum_{n=1}^{\infty} \arctan\left(\frac{2n}{2n+1}\right) \quad , \quad \sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) \quad , \quad \sum_{n=1}^{\infty} \frac{3}{n(n+3)} \quad , \quad \text{and} \quad \sum_{n=1}^{\infty} \left(-\frac{2}{7}\right)^{n-1}$$

In addition: If you conclude that the series diverges, then include an analytic argument that justifies your conclusion. If you conclude that the series converges, then include an analytic argument to calculate the sum. (Note: you don't need to use the integral test. Section 8.2 will be sufficient.)