*Exercício - Relação de recorrência

a) (B) $T(1) = O(1)$
(R) $T(n) = 4T(n/2) + O(n)$

$$T(n) = 4T(n/2) + O(n)$$
$$= 4(4T(n/2) + O(n)) + O(n)$$
$$= 4^2(T(n/2^2) + 4O(\tfrac{n}{2}) + O(n)$$
$$= 4^2(4T(n/2^3) + O(\tfrac{n}{2^2}) + 2O(n) + O(n)$$
$$= 4^3 T(n/2^3) + 2^2 O(n) + 2^2 O(n) + 2O(n) + O(n)$$
$$\vdots$$
$$= 4^k T(\tfrac{n}{2^k}) + 2^{k-1} O(n) + 2^{k-2} O(n) + \dots + 2O(n) + O(n)$$
$$= 4^k T(n/2^k) + O(n) * \sum_{i=0}^{k-1} 2^i$$

$$\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow K = \ln n$$

$$= 4^{\lg n} T(1) + O(n) * \sum_{i=0}^{\lg n - 1} 2^i$$
$$= n^{\lg 4} O(1) + O(n)(\cancel{2^{\lg n}}^{\,n} - 1)$$
$$= O(n^2) + O(n^2 - n) = O(n^2) + O(n^2) = O(n^2)$$

6) (B) $T(1) = O(1)$
(R) $T(n) = 3T(n/2) + O(n)$

$T(n) = 3T(n/2) + O(n)$

$= 3(3T(n/2^2) + O(n/2)) + O(n)$

$= 3^2 T(n/2^2) + 3O(n/2) + O(n)$

$= 3^3(3T(n/2^3) + O(\frac{n}{2^2})) + 3O(n/2) + O(n)$

$= 3^4 T(n/2^3) + 3O(n/2^2) + 3O(n/2) + O(n)$

$\vdots$

$= 3^k T(n/2^k) + \underbrace{3/2^{k-1} O(n) + 3/2^{k-2} O(n) + \cdots + 3/2\, O(n) + O(n)}$

$= 3^k T(\frac{n}{2^k}) + O(n) * \sum_{i=0}^{k-1} 3/2^i$

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow k = \lg n$

$= 3^{\lg n} T(1) + O(n) * \sum_{i=0}^{\lg n - 1} 3/2^i$

$T(n) = 3^{\lg n} T(1) + O(n) \left[\frac{1[(3/2)^{\lg n} - 1]}{3/2 - 1}\right]$

$= n^{\lg 3} O(1) + O(n) \left[\left(\frac{n^{\lg 3}}{n} - 1\right) \cdot \frac{1}{2}\right]$

$= O(n^{\lg 3}) + O(n) \left[\frac{n^{\lg 3} - n}{n} \cdot \frac{1}{2}\right]$

$= O(n^{\lg 3}) + O(n) \cdot \left(\frac{2(n^{\lg 3} - n)}{n}\right)$

$= O(n^{\lg 3}) + O(n^{\lg 3} - n)$

$= O(n^{\lg 3} + n^{\lg 3} - n)$

$O(n^{\lg 3})$