TABLE A9.1 Autocovariances for Some Seasonal Models

Model	(Autocovariance of w_t)/σ_a^2	Special Characteristics
(1) $w_t = (1 - \theta B)(1 - \Theta B^s)a_t$ $w_t = a_t - \theta a_{t-1} - \Theta a_{t-s} + \theta\Theta a_{t-s-1}$ $s \geqslant 3$	$\gamma_0 = (1 + \theta^2)(1 + \Theta^2)$ $\gamma_1 = -\theta(1 + \Theta^2)$ $\gamma_{s-1} = \theta\Theta$ $\gamma_s = -\Theta(1 + \theta^2)$ $\gamma_{s+1} = \gamma_{s-1}$ All other autocovariances are zero.	(a) $\gamma_{s-1} = \gamma_{s+1}$ (b) $\rho_{s-1} = \rho_{s+1} = \rho_1\rho_s$
(2) $(1 - \Phi B^s)w_t = (1 - \theta B)(1 - \Theta B^s)a_t$ $w_t - \Phi w_{t-s} = a_t - \theta a_{t-1} - \Theta a_{t-s} + \theta\Theta a_{t-s-1}$ $s \geqslant 3$	$\gamma_0 = (1 + \theta^2)\left[1 + \frac{(\Theta - \Phi)^2}{1 - \Phi^2}\right]$ $\gamma_1 = -\theta\left[1 + \frac{(\Theta - \Phi)^2}{1 - \Phi^2}\right]$ $\gamma_{s-1} = \theta\left[\Theta - \Phi - \frac{\Phi(\Theta - \Phi)^2}{1 - \Phi^2}\right]$ $\gamma_s = -(1 + \theta^2)\left[\Theta - \Phi - \frac{\Phi(\Theta - \Phi)^2}{1 - \Phi^2}\right]$ $\gamma_{s+1} = \gamma_{s-1}$ $\gamma_j = \Phi\gamma_{j-s} \quad j \geqslant s + 2$ For $s \geqslant 4$, $\gamma_2, \gamma_3, \ldots, \gamma_{s-2}$ are all zero.	(a) $\gamma_{s-1} = \gamma_{s+1}$ (b) $\gamma_j = \Phi\gamma_{j-s} \quad j \geqslant s + 2$
(3) $w_t = (1 - \theta_1 B - \theta_2 B^2)$ $\times (1 - \Theta_1 B^s - \Theta_2 B^{2s})a_t$ $w_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \Theta_1 a_{t-s}$ $+ \theta_1\Theta_1 a_{t-s-1} + \theta_2\Theta_1 a_{t-s-2}$ $-\Theta_2 a_{t-2s} + \theta_1\Theta_2 a_{t-2s-1}$ $+ \theta_2\Theta_2 a_{t-2s-2}$ $s \geqslant 5$	$\gamma_0 = (1 + \theta_1^2 + \theta_2^2)(1 + \Theta_1^2 + \Theta_2^2)$ $\gamma_1 = -\theta_1(1 - \theta_2)(1 + \Theta_1^2 + \Theta_2^2)$ $\gamma_2 = -\theta_2(1 + \Theta_1^2 + \Theta_2^2)$ $\gamma_{s-2} = \theta_2\Theta_1(1 - \Theta_2)$ $\gamma_{s-1} = \theta_1\Theta_1(1 - \theta_2)(1 - \Theta_2)$ $\gamma_s = -\Theta_1(1 + \theta_1^2 + \theta_2^2)(1 - \Theta_2)$ $\gamma_{s+1} = \gamma_{s-1}$ $\gamma_{s+2} = \gamma_{s-2}$ $\gamma_{2s-2} = \theta_2\Theta_2$ $\gamma_{2s-1} = \theta_1\Theta_2(1 - \theta_2)$ $\gamma_{2s} = -\Theta_2(1 + \theta_1^2 + \theta_2^2)$ $\gamma_{2s+1} = \gamma_{2s-1}$ $\gamma_{2s+2} = \gamma_{2s-2}$ All other autocovariances are zero.	(a) $\gamma_{s-2} = \gamma_{s+2}$ (b) $\gamma_{s-1} = \gamma_{s+1}$ (c) $\gamma_{2s-2} = \gamma_{2s+2}$ (d) $\gamma_{2s-1} = \gamma_{2s+1}$

TABLE A9.1 (*cont.*)

Model	(Autocovariance of w_t)$/\sigma_a^2$	Special Characteristics
(3a) *Special case of model 3* $w_t = (1 - \theta_1 B - \theta_2 B^2)(1 - \Theta B^s)a_t$ $w_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \Theta a_{t-s}$ $+ \theta_1 \Theta a_{t-s-1} + \theta_2 \Theta a_{t-s-2}$ $s \geqslant 5$	$\gamma_0 = (1 + \theta_1^2 + \theta_2^2)(1 + \Theta^2)$ $\gamma_1 = -\theta_1(1 - \theta_2)(1 + \Theta^2)$ $\gamma_2 = -\theta_2(1 + \Theta^2)$ $\gamma_{s-2} = \theta_2 \Theta$ $\gamma_{s-1} = \theta_1 \Theta (1 - \theta_2)$ $\gamma_s = -\Theta(1 + \theta_1^2 + \theta_2^2)$ $\gamma_{s+1} = \gamma_{s-1}$ $\gamma_{s+2} = \gamma_{s-2}$ All other autocovariances are zero.	(a) $\gamma_{s-2} = \gamma_{s+2}$ (b) $\gamma_{s-1} = \gamma_{s-1}$
(3b) *Special case of model 3* $w_t = (1 - \theta B)(1 - \Theta_1 B^s - \Theta_2 B^{2s})a_t$ $w_t = a_t - \theta a_{t-1} - \Theta_1 a_{t-s} + \theta \Theta_1 a_{t-s-1}$ $- \Theta_2 a_{t-2s} + \theta \Theta_2 a_{t-2s-1}$ $s \geqslant 3$	$\gamma_0 = (1 + \theta^2)(1 + \Theta_1^2 + \Theta_2^2)$ $\gamma_1 = -\theta(1 + \Theta_1^2 + \Theta_2^2)$ $\gamma_{s-1} = \theta \Theta_1 (1 - \Theta_2)$ $\gamma_s = -\Theta_1(1 + \theta^2)(1 - \Theta_2)$ $\gamma_{s+1} = \gamma_{s-1}$ $\gamma_{2s-1} = \theta \Theta_2$ $\gamma_{2s} = -\Theta_2(1 + \theta^2)$ $\gamma_{2s+1} = \gamma_{2s-1}$ All other autocovariances are zero.	(a) $\gamma_{s-1} = \gamma_{s-1}$ (b) $\gamma_{2s-1} = \gamma_{2s-1}$
(4) $w_t = (1 - \theta_1 B - \theta_s B^s - \theta_{s+1} B^{s+1})a_t$ $w_t = a_t - \theta_1 a_{t-1} - \theta_s a_{t-s}$ $- \theta_{s+1} a_{t-s-1}$ $s \geqslant 3$	$\gamma_0 = 1 + \theta_1^2 + \theta_s^2 + \theta_{s+1}^2$ $\gamma_1 = -\theta_1 + \theta_s \theta_{s-1}$ $\gamma_{s-1} = \theta_1 \theta_s$ $\gamma_s = \theta_1 \theta_{s+1} - \theta_s$ $\gamma_{s+1} = -\theta_{s+1}$ All other autocovariances are zero.	(a) In general, $\gamma_{s-1} \neq \gamma_{s-1}$ $\gamma_1 \gamma_s \neq \gamma_{s-1}$
(4a) *Special case of model 4* $w_t = (1 - \theta_1 B - \theta_s B^s)a_t$ $w_t = a_t - \theta_1 a_{t-1} - \theta_s a_{t-s}$ $s \geqslant 3$	$\gamma_0 = 1 + \theta_1^2 + \theta_s^2$ $\gamma_1 = -\theta_1$ $\gamma_{s-1} = \theta_1 \theta_s$ $\gamma_s = -\theta_s$ All other autocovariances are zero.	(a) Unlike model 4, $\gamma_{s-1} = 0$

(5) $(1 - \Phi B^s)w_t = (1 - \theta_1 B - \theta_s B^s - \theta_{s+1}B^{s+1})a_t$ $w_t - \Phi w_{t-s} = a_t - \theta_1 a_{t-1} - \theta_s a_{t-s} - \theta_{s+1}a_{t-s-1}$ $s \geqslant 3$	$\gamma_0 = 1 + \theta_1^2 + \frac{(\theta_s - \Phi)^2}{1 - \Phi^2} + \frac{(\theta_{s+1} + \theta_1\Phi)^2}{1 - \Phi^2}$ $\gamma_1 = -\theta_1 + \frac{(\theta_s - \Phi)(\theta_{s+1} + \theta_1\Phi)}{1 - \Phi^2}$ $\gamma_{s-1} = (\theta_s - \Phi)\left[\theta_1 + \Phi\frac{\theta_{s+1} + \Phi\theta_1}{1 - \Phi^2}\right]$ $\gamma_s = -(\theta_s - \Phi)\left[1 - \Phi\frac{\theta_s - \Phi}{1 - \Phi^2}\right] + (\theta_{s+1} + \theta_1\Phi)\left[\theta_1 + \Phi\frac{\theta_{s+1} + \theta_1\Phi}{1 - \Phi^2}\right]$ $\gamma_{s+1} = -(\theta_{s+1} + \theta_1\Phi)\left[1 - \Phi\frac{\theta_s - \Phi}{1 - \Phi^2}\right]$ $\gamma_j = \Phi\gamma_{j-s} \quad j \geqslant s + 2$ For $s \geqslant 4$, $\gamma_2, \ldots, \gamma_{s-2}$ are all zero.	(a) $\gamma_{s-1} \neq \gamma_{s+1}$ (b) $\gamma_j = \Phi\gamma_{j-s} \quad j \geqslant s + 2$
(5a) *Special case of model 5* $(1 - \Phi B^s)w_t = (1 - \theta_1 B - \theta_s B^s)a_t$ $w_t - \Phi w_{t-s} = a_t - \theta_1 a_{t-1} - \theta_s a_{t-s}$ $s \geqslant 3$	$\gamma_0 = 1 + \frac{\theta_1^2 + (\theta_s - \Phi)^2}{1 - \Phi^2}$ $\gamma_1 = -\theta_1\left[1 - \Phi\frac{\theta_s - \Phi}{1 - \Phi^2}\right]$ $\gamma_{s-1} = \frac{\theta_1(\theta_s - \Phi)}{1 - \Phi^2}$ $\gamma_s = \frac{\Phi\theta_1^2 - (\theta_s - \Phi)(1 - \Phi\theta_s)}{1 - \Phi^2}$ $\gamma_j = \Phi\gamma_{j-s} \quad j \geqslant s + 1$ For $s \geqslant 4$, $\gamma_2, \ldots, \gamma_{s-2}$ are all zero.	(a) Unlike model 5, $\gamma_{s+1} = \Phi\gamma_1$