2) Demuestra que:

a) $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)$.

Se reescribe $\cos(3\alpha)$ como: $\cos(2\alpha+\alpha)$.

$\cos(3\alpha) = \cos(2\alpha)\cos(\alpha) - \operatorname{sen}(2\alpha)\operatorname{sen}(\alpha)$

$\cos(3\alpha) = [\cos^2(\alpha) - \operatorname{sen}^2(\alpha)]\cos\alpha - 2\operatorname{sen}(\alpha)\cos(\alpha)\operatorname{sen}(\alpha)$

$\cos(3\alpha) = \cos^3(\alpha) - \operatorname{sen}^2(\alpha)\cos(\alpha) - 2\operatorname{sen}^2(\alpha)\cos(\alpha)$.

$\cos(3\alpha) = \cos^3(\alpha) - 3\operatorname{sen}^2(\alpha)\cos(\alpha)$.

b) $\operatorname{sen}(3\alpha) = 3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)$.

$(\cos(\alpha) + i\operatorname{sen}(\alpha))^n = \cos n\alpha + i\operatorname{sen} n\alpha$.

Si $n = 3$.

$\cos 3\alpha + i\operatorname{sen} 3\alpha = (\cos\alpha + i\operatorname{sen}\alpha)^3$.

$\cos 3\alpha + i\operatorname{sen} 3\alpha = \cos^3\alpha + 3\cos^2\alpha \cdot i\operatorname{sen}\alpha + 3\cos\alpha \cdot i^2\operatorname{sen}^2\alpha \ldots$
$\ldots i^3\operatorname{sen}^3\alpha$.

$\cos 3\alpha + i\operatorname{sen} 3\alpha = \cos^3\alpha + 3i\cos^2\alpha\operatorname{sen}\alpha - 3\cos\alpha\operatorname{sen}^2\alpha \ldots$
$\ldots - i\operatorname{sen}^3\alpha$.

→ parte real.

$\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\operatorname{sen}^2\alpha$.

→ parte compleja

$\operatorname{sen} 3\alpha = -\operatorname{sen}^3\alpha + 3\cos^2\alpha\operatorname{sen}\alpha$.

Demuestre que: 6.

a) $\log(-ie) = 1 - \frac{\pi}{2} i$

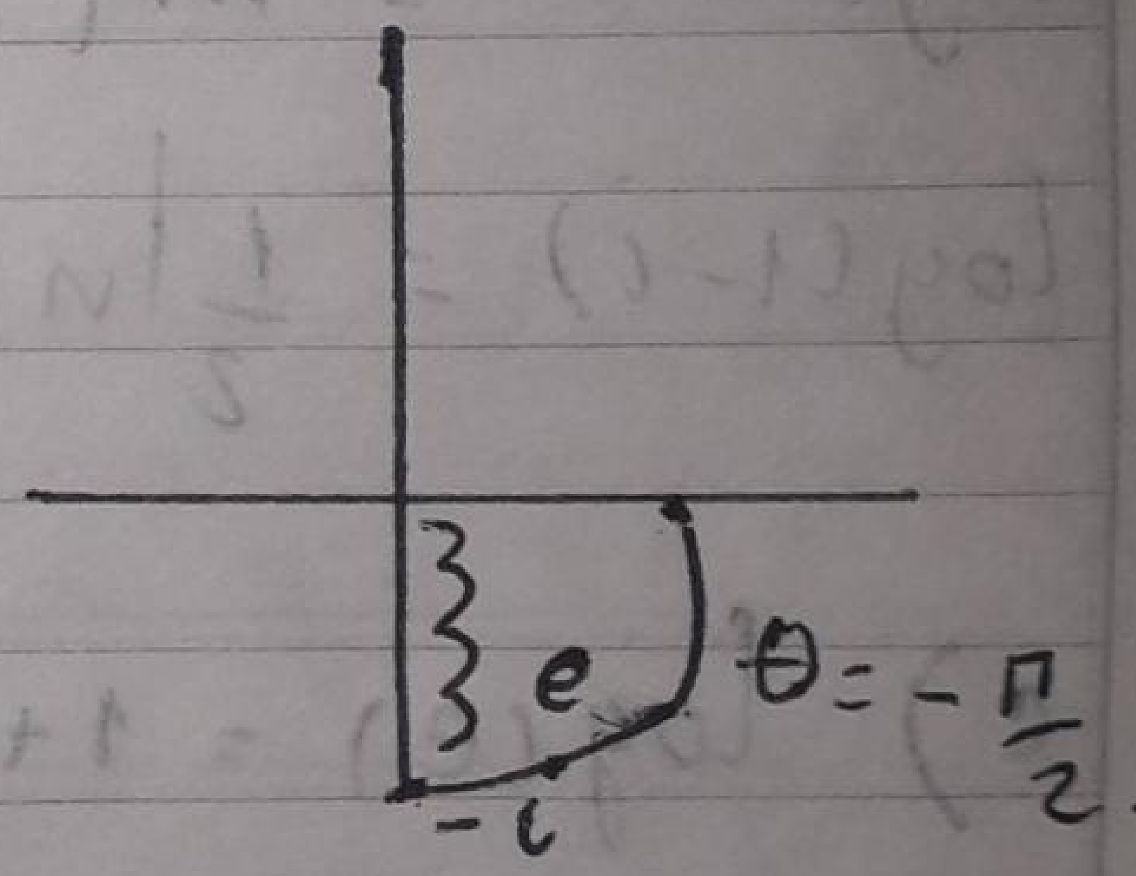
teniendo en cuenta:

$$\log(z) = \ln|z| + i\arg(z).$$

Entonces

$$|z| = \sqrt{a^2+b^2} = \sqrt{0^2+(-e)^2} = \sqrt{e^2} = e.$$

$$\theta = \operatorname{Arg} z \in (-\pi, \pi].$$



Reemplazando:

$$\log(-ie) = \ln(e) + i\left(-\frac{\pi}{2}\right).$$

$$\log(-ie) = 1 - \frac{\pi}{2} i \;//$$

b) $\log(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4}i$

→ hallando el modulo

$|z| = \sqrt{1^2+(-1)^2} = \sqrt{2}$.

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1) = -\pi/4$

reemplazando:

$\log(1-i) = \ln(\sqrt{2}) + i(-\pi/4)$.

$\log(1-i) = \ln(2^{1/2}) - \pi/4\, i$

$\log(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4}i$ //

c) $\log(e) = 1 + 2n\pi i$

$|z| = \sqrt{e^2} = e$.

$\arg(z) = \theta + 2n\pi$; $\theta = 0$.

$\arg(z) = 2n\pi$.

reemplazando:

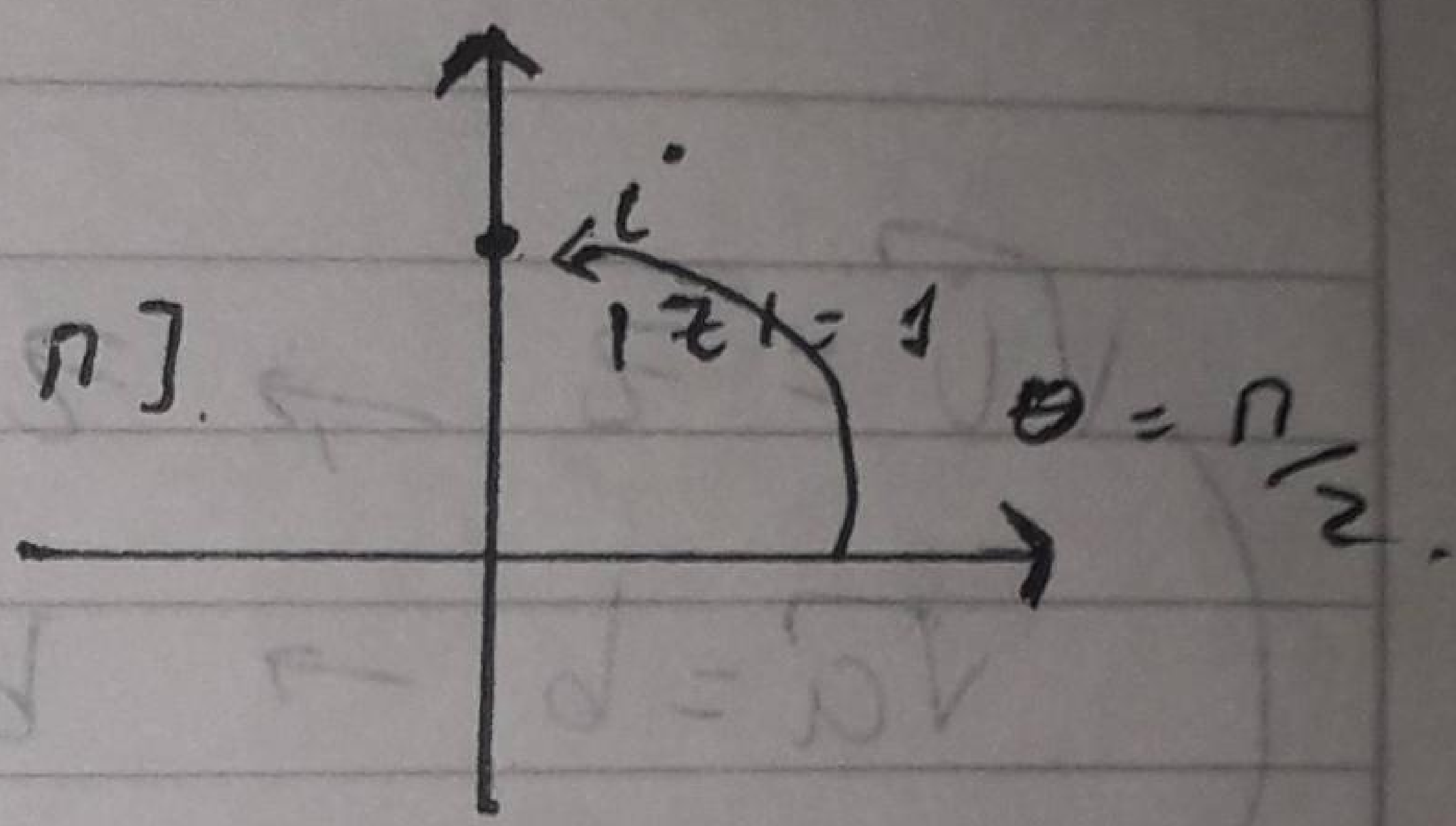
$\log(e) = \ln(e) + i(2n\pi)$.

$\log(e) = 1 + 2n\pi i$

//

d) $\log(i) = (2n + \frac{1}{2})\pi i$

$|z| = \sqrt{1^2} = 1.$



$\arg(z) = \theta + 2n\pi; \quad \theta \in (-\pi, \pi].$

$\arg(z) = \pi/2 + 2n\pi.$

remplazando:

$\log(i) = \ln(1) + i(\frac{\pi}{2} + 2n\pi).$

$\log(1) = 0 + i(\frac{\pi}{2} + 2n\pi)$

$\log(1) = i\pi(\frac{1}{2} + 2n)$ //