$0 \leq t \leq \pi$

Scribe

$\int_C \vec{f} \cdot d\vec{v}$ $\quad \vec{r}(t) = \cos(t)\hat{\imath} - \operatorname{sen}(t)\hat{\jmath}$ $\quad f = \frac{-\operatorname{sen}(t)}{1}\hat{\imath} + \frac{\cos(t)}{1}\hat{\jmath}$

$$\frac{d\vec{v}}{dt} = -\operatorname{sen}(t)\hat{\imath} - \cos(t)\hat{\jmath}$$

$$d\vec{v} = -\operatorname{sen}(t)\,dt\,\hat{\imath} - \cos(t)\,dt\,\hat{\jmath}$$

$$f = \frac{-y}{x^2+y^2}\hat{\imath} + \frac{x}{x^2+y^2}\hat{\jmath} \rightsquigarrow f(t) = \frac{-\operatorname{sen}(t)\hat{\imath}}{1} + \frac{\cos(t)\hat{\jmath}}{1}$$

a) $$\int_0^{\pi} \vec{f} \cdot d\vec{r} = \int_0^{\pi} \operatorname{sen}^2(t)\hat{\imath}\,dt - \cos^2(t)\hat{\jmath}\,dt$$

$$\Rightarrow \int_0^{\pi} \operatorname{sen}^2(t)\,dt\,\hat{\imath} = \frac{t}{2} - \frac{\operatorname{sen}(2t)}{4}\Big|_0^{\pi} - \int_0^{\pi} \cos^2(t)\,dt\,\hat{\jmath} = \frac{\cos(t)\operatorname{sen}(t) + t}{2}\Big|_0^{\pi}$$

$$= \frac{\pi}{2} - \frac{\operatorname{sen}(2\pi)}{4} - \frac{\cos(\pi)\operatorname{sen}(\pi) + \pi}{2} + \frac{\operatorname{sen}(0)}{4} + \frac{\operatorname{sen}(0)\cos(0)}{2} =$$

$\approx 1,5160{7}5023$ [J] Trabajo en sentido antihorario

Redondeando $\Rightarrow$ Cambiaremos los límites de integración por $(-\pi, 0)$

$$\frac{t}{2} - \frac{\operatorname{sen}(2t)}{4} - \frac{\cos(t)\operatorname{sen}(t) + t}{2}\Big|_{-\pi}^{0}$$

$$\frac{\cancel{\operatorname{sen}(0)}}{\cancel{4}} - \frac{\cos(0)\operatorname{sen}(0)}{2} - \left[\frac{-\pi}{2} - \frac{\operatorname{sen}(-2\pi)}{4} - \frac{\cos(-\pi)\operatorname{sen}(-\pi) - \pi}{2}\right) =$$

$\approx -0,0547213034 5$ [J] Trabajo en sentido horario.

(2a) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$$\left[\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right](\phi\psi) = \frac{\partial(\phi\psi)}{\partial x}\hat{\imath} + \frac{\partial(\phi\psi)}{\partial y}\hat{\jmath} + \frac{\partial(\phi\psi)}{\partial z}\hat{k}$$

$$= \phi\frac{\partial\psi}{\partial x}\hat{\imath} + \psi\frac{\partial\phi}{\partial x}\hat{\imath} + \phi\frac{\partial\psi}{\partial y}\hat{\jmath} + \psi\frac{\partial\phi}{\partial y}\hat{\jmath} + \phi\frac{\partial\psi}{\partial z}\hat{k} + \psi\frac{\partial\phi}{\partial z}\hat{k}$$

$$= \phi\left[\frac{\partial\psi}{\partial x}\hat{\imath} + \frac{\partial\psi}{\partial y}\hat{\jmath} + \frac{\partial\psi}{\partial z}\hat{k}\right] + \psi\left[\frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k}\right]$$

$$= \phi\left[\frac{\partial\hat{\imath}}{\partial x} + \frac{\partial\hat{\jmath}}{\partial y} + \frac{\partial\hat{k}}{\partial z}\right]\psi + \psi\left[\frac{\partial\hat{\imath}}{\partial x} + \frac{\partial\hat{\jmath}}{\partial y} + \frac{\partial\hat{k}}{\partial z}\right]\phi$$

$$= \phi\nabla\psi + \psi\nabla\phi$$

$$\therefore \nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\psi$$

(2b) $\nabla\cdot(\phi A) = (\nabla\phi)\cdot A + \phi(\nabla\cdot A)$

LI = Lado Izquierdo
LD = Lado Derecho

$$\nabla\cdot(\phi A) = \left[\frac{\partial\hat{\imath}}{\partial x} + \frac{\partial\hat{\jmath}}{\partial y} + \frac{\partial\hat{k}}{\partial z}\right]\cdot(\phi A_1\hat{\imath} + \phi A_2\hat{\jmath} + \phi A_3\hat{k}) = \frac{\partial(\phi A_1)}{\partial x} + \frac{\partial(\phi A_2)}{\partial y} + \frac{\partial(\phi A_3)}{\partial z} \; * \text{LI}$$

$$(\nabla\phi)\cdot A + \phi(\nabla A) = \left[\frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k}\right]\cdot(A_1\hat{\imath} + A_2\hat{\jmath} + A_3\hat{k}) + \phi\left[\left[\frac{\partial\hat{\imath}}{\partial x} + \frac{\partial\hat{\jmath}}{\partial y} + \frac{\partial\hat{k}}{\partial z}\right]\cdot(A_1\hat{\imath} + A_2\hat{\jmath} + A_3\hat{k})\right]$$

$$= \left[A_1\frac{\partial\phi}{\partial x} + A_2\frac{\partial\phi}{\partial y} + A_3\frac{\partial\phi}{\partial z}\right] + \phi\left[\frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}\right]$$

$$= \left[A_1\frac{\partial\phi}{\partial x} + \phi\frac{\partial A_1}{\partial x}\right] + \left[A_2\frac{\partial\phi}{\partial y} + \phi\frac{\partial A_2}{\partial y}\right] + \left[A_3\frac{\partial\phi}{\partial z} + \phi\frac{\partial A_3}{\partial z}\right]$$

$$= \frac{\partial(\phi A_1)}{\partial x} + \frac{\partial(\phi A_2)}{\partial y} + \frac{\partial(\phi A_3)}{\partial z} \; * \text{LD}$$

Tenemos que LI = LD

$$\therefore \nabla\cdot(\phi A) = (\nabla\phi)\cdot A + \phi(\nabla A)$$

Scribe

2f $\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$

$$\nabla\times(\nabla\times A) = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \times \overbrace{\left[\left[\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right]\hat{i} - \left[\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right]\hat{j} + \left[\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right]\hat{k}\right]}^{(\nabla\times A)}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z} & \frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x} & \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} \end{vmatrix} = \left[\frac{\partial^2 A_2}{\partial x\partial y} - \frac{\partial^2 A_1}{\partial y^2} - \frac{\partial^2 A_1}{\partial z^2} + \frac{\partial^2 A_3}{\partial z\partial x}\right]\hat{i} \ldots$$

$$\ldots - \left[\frac{\partial^2 A_2}{\partial x^2} - \frac{\partial^2 A_1}{\partial y\partial x} - \frac{\partial^2 A_3}{\partial y\partial z} + \frac{\partial^2 A_2}{\partial z^2}\right]\hat{j} + \left[\frac{\partial^2 A_1}{\partial z\partial x} - \frac{\partial^2 A_3}{\partial x^2} - \frac{\partial^2 A_3}{\partial y^2} + \frac{\partial^2 A_2}{\partial z\partial y}\right]\hat{k} = LI$$

$\nabla(\nabla\cdot A) - \nabla^2 A$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\left[\frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}\right] - \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\left[A_1\hat{i} + A_2\hat{j} + A_3\hat{k}\right]$$

$$= \left[\frac{\partial^2 A^2}{\partial x\partial y} - \frac{\partial^2 A_1}{\partial y^2} - \frac{\partial^2 A_1}{\partial z^2} + \frac{\partial^2 A_3}{\partial z\partial x}\right]\hat{i} - \left[\frac{\partial^2 A^2}{\partial x^2} - \frac{\partial^2 A_1}{\partial y\partial x} - \frac{\partial^2 A_3}{\partial y\partial z} + \frac{\partial^2 A_2}{\partial z^2}\right]\hat{j} \ldots$$

$$\ldots + \left[\frac{\partial^2 A_1}{\partial z\partial x} - \frac{\partial^2 A_3}{\partial x^2} - \frac{\partial^2 A_3}{\partial y^2} + \frac{\partial^2 A_2}{\partial z\partial y}\right]\hat{k} = LD$$

Tenemos que LD = LI por ende

$\therefore \nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$