5 Encuentre las raíces.

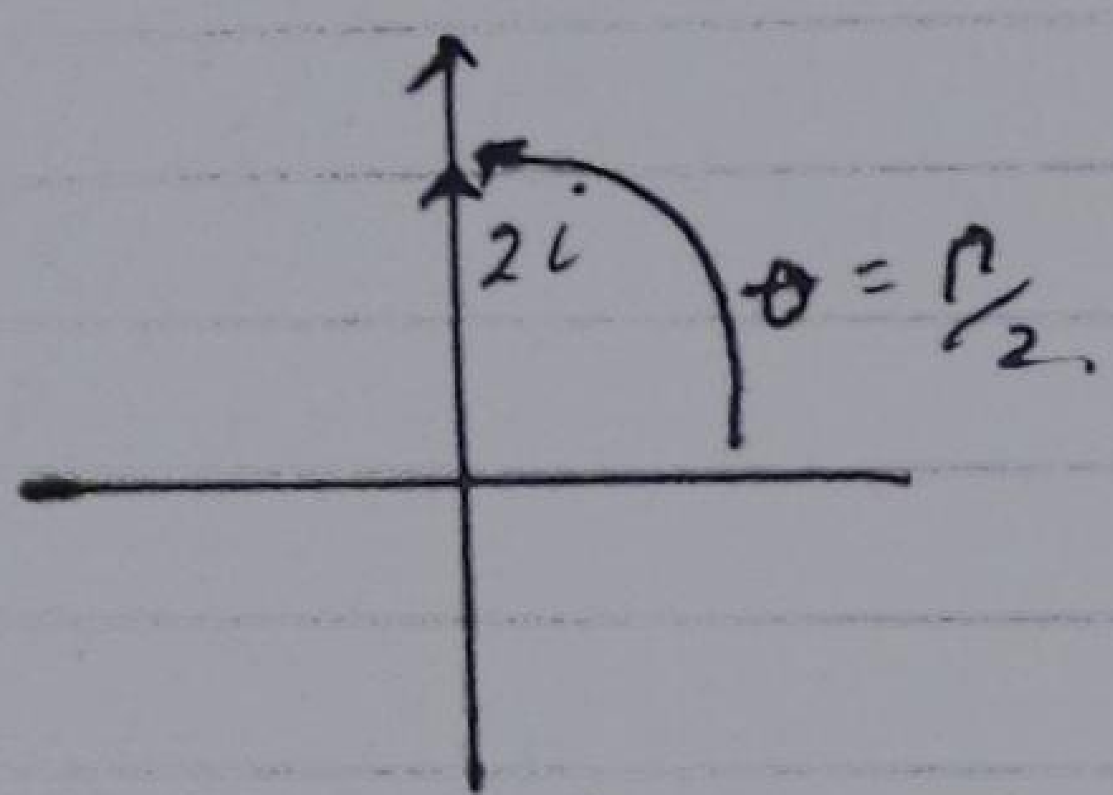
a) $(2i)^{\frac{1}{2}}$

$$\cdot\ z_k = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\,\text{Sen}\left(\frac{\theta + 2\pi k}{n}\right)\right]$$

$$r = \sqrt{a^2+b^2} = \sqrt{(2)^2} = 2.$$

$K = 0, 1..\quad n = 2.$

$$z_k = (2)^{\frac{1}{2}}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\,\text{Sen}\left(\frac{\theta + 2\pi k}{n}\right)\right].$$



reemplazando. $K=0$

$$z_k = (2)^{\frac{1}{2}}\left[\cos\left(\frac{\pi/2 + 2\pi(0)}{2}\right) + i\,\text{Sen}\left(\frac{\pi/2 + 2\pi(0)}{2}\right)\right]$$

$$z_k = (2)^{\frac{1}{2}}\left[\cos(\pi/4) + i\,\text{Sen}(\pi/4)\right].$$

$$z_0 = (2)^{\frac{1}{2}}\left[\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right].$$

$z_0 = 1 + i \rightarrow$ primera raíz.

$k = 1.$

$$z_{(1)} = (2)^{\frac{1}{2}} \left[\cos\left(\frac{\pi/2 + 2\pi}{2} \right) + i \operatorname{Sen}\left(\frac{\pi/2 + 2\pi}{2} \right) \right]$$

$$z_{(1)} = (2)^{\frac{1}{2}} \left[\cos\left(\frac{5\pi}{4} \right) + i \operatorname{Sen}\left(\frac{5\pi}{4} \right) \right].$$

$$z_{(1)} = (2)^{\frac{1}{2}} \left[-\frac{\sqrt{2}}{2} + i \left(-\frac{\sqrt{2}}{2} \right) \right]$$

$z_{(1)} = -1 - i$ a Segunda raiz.

b) $(1 - \sqrt{3}\, i)^{\frac{1}{2}}$

$$z = r_\alpha$$

$$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2.$$

$$\alpha = \tan^{-1}(-\sqrt{3}) = 120.$$

$$r_\alpha = \sqrt{2}_{120} = \begin{cases} \sqrt{2}_{\frac{120 + 360(0)}{2}} = \sqrt{2}_{60^\circ} \\ \sqrt{2}_{\frac{120 + 360(1)}{2}} = \sqrt{2}_{240^\circ} \end{cases}$$

e) $(-1)^{\frac{1}{3}}$

$|z| = r$

$|z| = \sqrt{0^2 + (-1)^2} = 1$.

$$z_k = r^{\frac{1}{n}} \left[\cos\left(\frac{\theta + 2\pi k}{n} \right) + i \operatorname{sen}\left(\frac{\theta + 2\pi k}{n} \right) \right].$$

$n = 3 \wedge k = 0, 1, 2 \quad ; \quad \theta = 0.$

para $z = 0$.

$$z_0 = \cos\left(\frac{2\pi(0)}{n} \right) + i \operatorname{sen}\left(\frac{2\pi(0)}{n} \right).$$

$$z_0 = \cos\left(\frac{2\pi(0)}{3} \right) + i \operatorname{sen}\left(\frac{2\pi(0)}{3} \right)$$

$z_0 = 1 \rightarrow$ primera raíz.

para $z = 1$.

$$z_{(1)} = \cos\left(\frac{2\pi(1)}{3} \right) + i \operatorname{sen}\left(\frac{2\pi(1)}{3} \right).$$

$z_{(1)} = -\frac{1}{2} + i\left(\frac{\sqrt{3}}{2} \right) \rightarrow$ Segunda raíz

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para $z=2$.

$$z_{(2)}= \cos\left(\frac{2\pi(2)}{3}\right)+i\,\text{Sen}\left(\frac{2\pi(2)}{3}\right).$$

$$z_{(2)}= \cos\left(\frac{4\pi}{3}\right)+i\,\text{Sen}\left(\frac{4\pi}{3}\right).$$

$z_{(2)}= -\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)$ → tercera raiz.

d) $8^{1/6}$

$|z| = r$

$|z| = \sqrt{0^2+(8)^2} = 8$

$z_k = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2\pi k}{n}\right) + i\,\text{sen}\left(\frac{\theta+2\pi k}{n}\right)\right]$

$n=6$; $k = 0, 1, 2, 3, 4, 5$; $\theta = 0$

para $z=0$

$z_k = (8)^{\frac{1}{6}}\left[\cos\left(\frac{2\pi(0)}{6}\right) + i\,\text{sen}\left(\frac{2\pi(0)}{6}\right)\right]$

$z_k = (8)^{\frac{1}{6}}[1] = (8)^{\frac{1}{6}} \rightarrow$ primera raíz

para $z=(1)$

$z_{(1)} = (8)^{\frac{1}{6}}\left[\cos\left(\frac{2\pi}{6}\right) + i\,\text{sen}\left(\frac{2\pi}{6}\right)\right]$

$z_{(1)} = (8)^{\frac{1}{6}}\left[\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right)\right] = \frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2} \Rightarrow$ segunda raíz

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para $z=2$.

$$z_{(2)}=(8)^{\frac{1}{6}}\left[\cos\left(\frac{2\pi(2)}{6}\right)+i\,\mathrm{Sen}\left(\frac{2\pi(2)}{6}\right)\right]$$

$$z_{(2)}=(8)^{\frac{1}{6}}\left[\cos\left(\frac{4\pi}{6}\right)+i\,\mathrm{Sen}\left(\frac{4\pi}{6}\right)\right].$$

$$z_{(2)}=(8)^{\frac{1}{6}}\left[-\frac{1}{2}+i\left(\frac{\sqrt{3}}{2}\right)\right]=-\frac{\sqrt{2}}{2}+i\frac{\sqrt{6}}{2}.$$

$\Downarrow$

tercera raíz.

para $z=3$.

$$z_{(3)}=(8)^{\frac{1}{6}}\left[\cos\left(\frac{2\pi(3)}{6}\right)+i\,\mathrm{Sen}\left(\frac{2\pi(3)}{6}\right)\right].$$

$$z_{(3)}=(8)^{\frac{1}{6}}\left[\cos(\pi)+i\,\mathrm{Sen}(\pi)\right]$$

$$z_{(3)}=8^{\frac{1}{6}}\left[-1\right]=-(8)^{\frac{1}{6}}$$ $\rightarrow$ cuarta raíz

para $z=4$.

$$z_{(4)}=(8)^{\frac{1}{6}}\left[\cos\left(\frac{2\pi(4)}{6}\right)+i\,\mathrm{Sen}\left(\frac{2\pi(4)}{6}\right)\right].$$

$$z_{(4)}=(8)^{\frac{1}{6}}\left[-\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right]=-\frac{\sqrt{2}}{2}-i\frac{\sqrt{6}}{2}$$

$\hookrightarrow$ Quinta raíz

para $z = 5$.

$$z_{(5)} = (8)^{\frac{1}{6}}\left[\cos\left(\frac{2\pi(5)}{6}\right) + i\,\mathrm{Sen}\left(\frac{2\pi(5)}{6}\right)\right].$$

$$z_{(5)} = (8)^{\frac{1}{6}}\left[\cos\left(\frac{10\pi}{6}\right) + i\,\mathrm{Sen}\left(\frac{10\pi}{6}\right)\right].$$

$$z_{(5)} = (8)^{\frac{1}{6}}\left[\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right] = \frac{\sqrt{2}}{2} - i\frac{\sqrt{6}}{2}.$$

$\Downarrow$

Sexta raiz

e) $(-8-8\sqrt{3}i)^{1/4}$

$z = r_\alpha$

$|z| = \sqrt{(-8)^2 + (-8\sqrt{3})^2} = \sqrt{256} = 16.$

$\alpha = \tan^{-1}\left(\frac{-8\sqrt{3}}{-8}\right) = \tan^{-1}(\sqrt{3}) = 60°$

$$\sqrt[4]{16_{60°}} = \begin{cases} 2_{\frac{60+360(0)}{4}} = 2_{15°} \\ 2_{\frac{60+360(1)}{4}} = 2_{105°} \\ 2_{\frac{60+360(2)}{4}} = 2_{195°} \\ 2_{\frac{60+360(3)}{4}} = 2_{285°} \end{cases}$$