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A Mathematical Model for Vehicle-Occupant Frontal Crash using Genetic Algorithm

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***Abstract*—In this paper, a mathematical model for vehicle - occupant frontal crash is developed. The developed model is represented as a double-spring-mass-damper system, whereby the front mass and the rear mass represent the vehicle chassis and the occupant, respectively. The springs and dampers in the model are nonlinear piecewise functions of displacements and velocities respectively. More specifically, a genetic algorithm (GA) approach is proposed for estimating the parameters of vehicle front structure and restraint system. Finally, it is shown that the obtained model can accurately reproduce the real crash test data taken from the National Highway Traffic Safety Administration (NHTSA). The maximum dynamic crash of the vehicle model is 0.05% less than that in the real crash test. The displacement of the occupant is 0.09% larger than that from the crash test. Improvement of the model accuracy is also observed from the time at maximum displacement and the rebound velocities for both the vehicle and occupant.**

Keywords–Modeling; vehicle-occupant; frontal crash; parameters estimation; genetic algorithm;

I. Introduction

Car accidents are one of the major causes of mortality in modern society. While it is desirable to maintain the crashworthiness, car manufacturers perform crash tests on a sample of vehicles for monitoring the effect of the occupant in different crash scenarios. Car crash tests are usually performed to ensure safe design standards in crashworthiness (the ability of a vehicle to be plastically deformed and yet maintains a sufficient survival space for its occupants during the crash scenario). However, this process requires a lot of time, sophisticated infrastructure and trained personnel to conduct such a test and data analysis. Therefore, to reduce the cost associated with the real crash test, it is worthy to adopt the simulation of a vehicle crash and validate the model results with the actual crash test. Nowadays, due to advanced research in simulation tools, simulated crash tests can be performed beforehand the full-scale crash test. Therefore, the cost associated with the real crash test can be reduced. Finite element method (FEM) models and lumped parameter models (LPM) are typically used to model the vehicle crash phenomena. Vehicle crashworthiness can be evaluated in four distinct modes: frontal, side, rear and rollover crashes. Several types of research have been carried out in this field, which resulted in several novel computational models of vehicle collisions in literature, and a brief review is given in this paper.

II. Literature survey and limitations of current techniques

An application of physical models composed of springs, dampers and masses joined in various arrangements for simulating a real car collision with a rigid pole was presented in [1]. In [2], a 5-DOFs lumped parameter modeling for the frontal crash was investigated to analyze the response of occupant during the impact. Ofochebe et al. in [3] studied the performance of vehicle front structure using a 4-DOFs lumped mass-spring model composed of body, engine, the cross-member and suspension and the bumper masses.

In [4] and [5], an optimization procedure to assist multi-body vehicle model development and validation was proposed. In the work of [6], the authors proposed an approach to control the seat belt restraint system force during a frontal crash to reduce thoracic injury. Klausen et al. [7] used firefly optimization method to estimate parameters of vehicle crash test based on single-mass. To reconstruct the crash event, Tørdal et al. [8] extracted the motion of a bus in an oblique crash and the kinematics of a Ford Fiesta in a pole crash from a high frame rate video. Tso-Liang et al. in [9] examined the dynamic response of the human body in a crash event and assessed the injuries sustained to the occupants head, chest and pelvic regions.To reduce the occupant injury risks in vehicle frontal crashes, mathematical models that optimize the vehicle deceleration have been developed in [10], [11]. Apart from the commonly used approaches, recently intelligent approaches have been used in the area of vehicle crash modeling. The most commonly used are Fuzzy logic in [12], Neuro-fuzzy in [13], firefly algorithm in [7] and genetic algorithm. A genetic algorithm has been used in [14] for calculating the optimized parameters of a 12-DOFs model for two vehicle types in two different frontal crashes. In [15], [16], the author used Genetic Algorithms to optimize the performance of PID, Fuzzy and Neuro-fuzzy controllers on various systems. The main challenge in accident reconstruction is the system identification described as the process of constructing mathematical models of dynamical systems using measured input-output data. In the case of a vehicle

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crash, system identification algorithm consists of retrieving the unknown parameters such as the spring stiffness and damping coefficient. A possible approach is to identify these parameters directly from experimental data. From literature, System Identification Algorithms (SIA) have been developed for different applications. Among others, we can state-space identification, eigensystem realization algorithm and data-based regressive model approaches. Typical examples where these SIA have been used can be found in [17]–[19].

In this paper, based on the previous research work [7], we develop a mathematical model for a double-spring-mass-damper system which reconstructs a vehicle-occupant frontal crash scenario and estimates structural parameters of the vehicle's front structure and the restraint system. The structural parameters estimated are spring and damping coefficients. To estimate the physical parameters of the model, a genetic algorithm is proposed. It is observed that the predicted results fit the experimental data very well.

III. The newly proposed method

The main objective of this section is to represent a dynamic model to capture the vehicle frontal crash phenomena. During the frontal crash, the vehicle is subjected to an impulsive force caused by the obstacle. The model for vehicle crash simulates a rigid barrier impact of the car, where m_1 and m_2, as shown in Figure 1 represent the frame rail (chassis) and occupant masses, respectively. In this model, the parameters to be estimated are spring stiffness constants k_l, k_{nl} and k_2, damping constants c_l, c_{nl} and c_2. When the vehicle crashes into a rigid barrier, the two masses will experience an impulsive force during the collision. The real crash phenomenon is shown in Figure 2 and it is observed that the value for the maximum dynamic crash of the vehicle is 72.69 cm, the time of the crash is 0.0894 s and the rebound velocity is -3.75 m/s. At the time of crash, the occupant experiences a forward movement of 30.3 cm making a total displacement of 103 cm. The rebound velocity of the occupant is -13.1 m/s.

In line of the model development to capture the values as mentioned earlier during the crash scenario, the 2-DOF dynamical model proposed in [20] for the free vibration analysis is adopted for solving the impact responses of the two masses. Then, the genetic algorithm is used to estimate the 2-DOF model parameters.

A. Model 1: Combination of linear and nonlinear springs and dampers

In model 1 the deforming spring and damping forces, developed at time of crash, are nonlinear cubic functions in x and $\dot{x}$ respectively. The spring stiffness and damper constants are defined as follows:

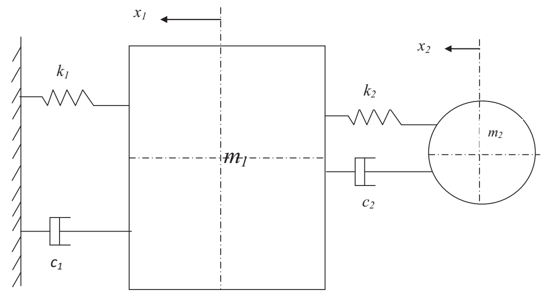

Figure 1. A double spring-mass-damper model

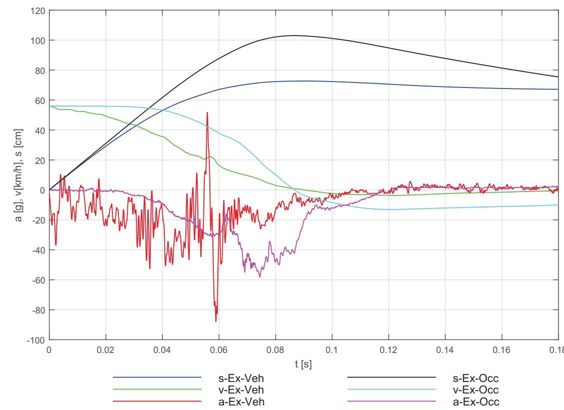

Figure 2. Crash test data

$$k_1 = k_l + k_{nl} \tag{1}$$

$$c_1 = c_l + c_{nl} \tag{2}$$

The dynamic equations of the double-mass-spring-damper model are shown in the following:

$$F_{str} = k_l x_1 + k_{nl} x_1^3 + c_l \dot{x}_1 + c_{nl} \dot{x}_1^3 \tag{3}$$

$$F_{rest} = k_2(x_2 - x_1) + c_2(\dot{x_2} - \dot{x_1}) \tag{4}$$

$$\ddot{x}_1 = (F_{rest} - F_{str})/m_1 \tag{5}$$

$$\ddot{x}_2 = (F_{rest})/m_2 \tag{6}$$

where F_{str} and F_{rest} are the deformation force of the vehicle frontal structure and the restraint system respectively. k_l and k_{nl}, are linear and nonlinear springs c_l and c_{nl}, are linear and nonlinear dampers of the front vehicle structure respectively. k_2 and c_2 are spring stiffness and damper coefficients for the restraint system respectively.

B. Model 2: Piecewise functions of springs and dampers

Figure 3 represents the vehicle- occupant model with non-linear spring and dampers, which crashes into a fixed barrier. Based on the nonlinear characteristics of velocity

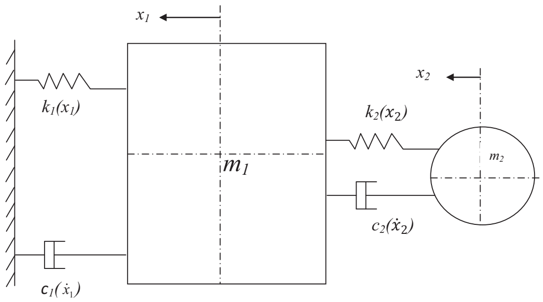

Figure 3. The Model 2

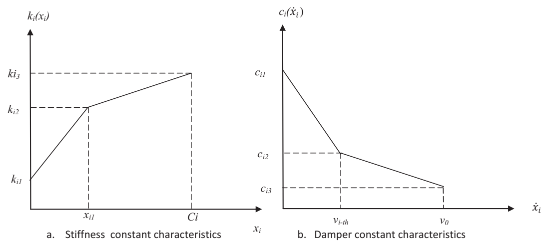

Figure 4. Stiffness and damper characteristics of vehicle frontal structure

and displacement of the vehicle and forward movement of the occupant shown in Figure 2, the springs and dampers that simulate such characteristics must also be nonlinear as predefined in Figure 4. The dynamic equation of the model is defined by:

$$F_{str} = k_1 x_1 + c_1 \dot{x}_1 \tag{7}$$

F_{rest}, $\ddot{x}_1$ and $\ddot{x}_2$ are identical to Eqs.(4) - (6). The piecewise functions for stiffness and dampers in the front structure of the vehicle and restrain system are defined as follows:

$$k_i(x_i) = \begin{cases} k_{i1} + \frac{k_{i2}-k_{i1}}{x_{i1}} x_i & x_i \leq x_{i1} \\ \\ k_{i2} + \frac{k_{i3}-k_{i2}}{C_i - x_{i1}} (x_i - x_{i1}) & x_{i1} \leq x_i \leq C_i \end{cases} \tag{8}$$

$$c_i(\dot{x}_i) = \begin{cases} c_{i1} - \frac{c_{i1}-c_{i2}}{\dot{x}_{i1}} \dot{x}_i & \dot{x}_i \leq v_{i-th} \\ \\ c_{i2} - \frac{c_{i2}-c_{v3}}{v_0 - \dot{x}_{i1}} (\dot{x}_i - v_{i-th}) & v_{i-th} \leq \dot{x}_i \leq v_0 \end{cases} \tag{9}$$

where the index $i = 1,2$ stand for 1^{st} and 2^{nd} mass respectively. C_i is the dynamic crash of the vehicle or occupant. v_0 is the initial impact velocity. v_{i-th} is the threshold velocity of i mass.

At the maximum crash, the spring stiffness is assumed to be high, but the damper coefficient is small for maintaining the shape of displacements and velocities of vehicle and occupant respectively. To get better results, the model in Figure 4 can be modified by introducing two break point on the predefined shapes of springs and dampers as shown in Figure 5 and defined in Eqs.(10) - (11).

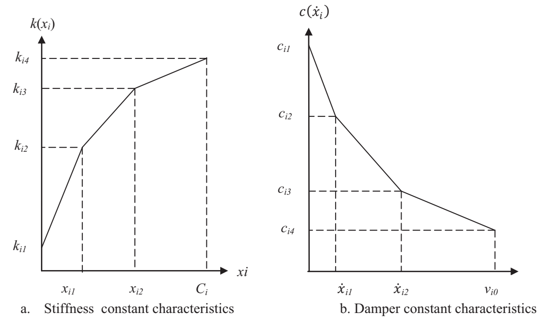

Figure 5. Stiffness and damper characteristics of the restraint system

$$k_i(x_i) = \begin{cases} k_{i1} + \frac{k_{i2}-k_{i1}}{x_{i1}} x_i & x_i \leq x_{i1} \\ \\ k_{i2} + \frac{k_{i3}-k_{i2}}{x_{i2}-x_{i1}} (x_i - x_{i1}) & x_{i1} \leq x_i \leq x_{i2} \\ \\ k_{i3} + \frac{k_{i4}-k_{i3}}{C_i - x_{i2}} (x_i - x_{i2}) & x_{i2} \leq x_i \leq C_i \end{cases} \tag{10}$$

$$c_i(\dot{x}_i) = \begin{cases} c_{i1} - \frac{c_{i1}-c_{i2}}{\dot{x}_{i1}} \dot{x}_i & \dot{x}_i \leq \dot{x}_{i1} \\ \\ c_{i2} - \frac{c_{i2}-c_{i3}}{\dot{x}_{i2}-\dot{x}_{i1}} (\dot{x}_i - \dot{x}_{i1}) & \dot{x}_{i1} \leq \dot{x}_i \leq \dot{x}_{i2} \\ \\ c_{i3} - \frac{c_{i3}-c_{i4}}{v_0 - \dot{x}_{i2}} (\dot{x}_i - \dot{x}_{i2}) & \dot{x}_{i2} \leq \dot{x}_i \leq v_0 \end{cases} \tag{11}$$

C. Optimization algorithm

The proposed algorithm seeks to find the minimum function between several variables as can be stated in a general form $minf(p)$, where p denotes the unknown variables, which are the damping and stiffness constants in the model. The cost function $f(p)$ is the objective function which should be optimized. The cost function to be minimized is the norm of the absolute error between the displacement of the simulated cash and the experimental crash data and is defined as

$$[Error] = sum(|E_{st} - E_{xp}|^T \times |E_{st} - E_{xp}) \tag{12}$$

where E_{st} and E_{xp} are the model and experimental variables (displacements, velocity and acceleration) respectively. All parameters defined in Eqs.(8) -(11) are embedded in E_{st} .

The GA method is used here for optimization of the cost function. The GA-type of search schemes is function-value comparison-based, with no derivative computation. It attempts to move points through a series of generations, each being composed of a population which has a set number of individuals, where individuals represent parameters to be estimated. The population size depends on the number of

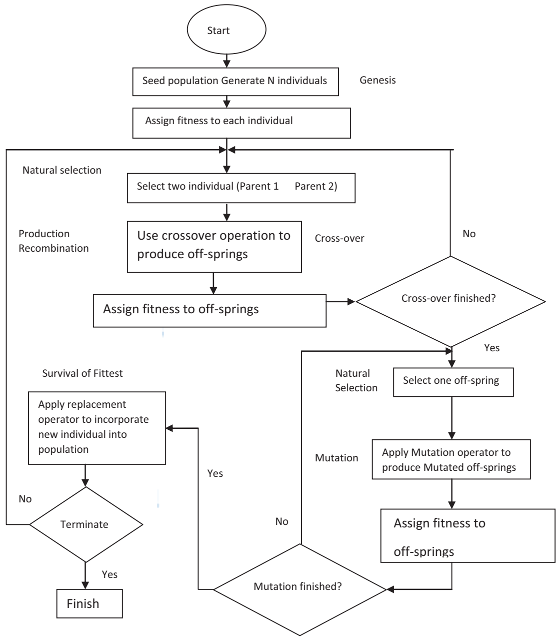

Figure 6. GA flowchart

parameters to be estimated for a given model. For example, the Model1 has six individuals, Model 2 for a one break point piecewise function, in Figure (4), has eighteen individuals and twenty-four individuals for a two break points piecewise function, in Figure (5). Each individual is a point in the parameter space (in our case, the displacement of experimental data). The schemes that are applied to the evolution of generations have some analogy to the natural genetic evolution of species, hence the term genetic.

G.A. is an adaptive heuristic search algorithm based on the evolutionary ideas of nature selection and genetics. It represents an intelligent exploitation of a random search used to solve optimization problems and consists of five operators: Initialization, Selection, crossover, mutation and replacement. Initialization is used to seed initial population randomly while selection is used to select the fittest from the population. Crossover is used to explore the search space. Mutation is used to remove the problem like genetic drift (some individuals may leave behind a few more off-springs than other individuals), and replacement is used to progress generation wise population [21]. Figure 6 shows a general flowchart of a genetic algorithm.

IV. Results and discussion

Comparisons between the model results and the experimental data are shown in Figures 7 - 9, where the stapled lines and continuous lines represent the simulation results and the experimental data respectively. The symbols s-Ex-Veh, v-Ex-Veh, a-Ex-Veh, s-Ex-Occ, v-Ex-Occ, a-Ex-Occ,

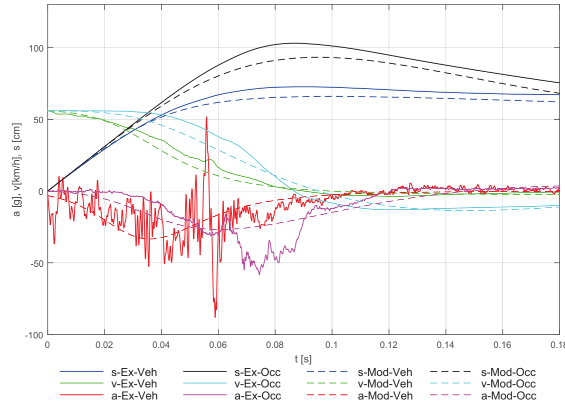

Figure 7. Comparison between vehicle model and vehicle experimental data - Nonlinear Model1

TABLE I
Parameters estimation Model1

Parameter	*Value*	*Unit*
k_l	6.4270e+04	N/m
k_{nl}	30.3830	N/m^3
c_l	6.2029e+04	Ns/m
c_{nl}	246.2119	Ns^3/m^3
k_2	4.3159e+04	N/m
c_2	2.0797e+03	Ns/m

s-Mod-Veh, v-Mod-Veh, a-Mod-Veh, s-Mod-Occ, v-Mod-Occ, a-Mod-Occ, on the legends stand for displacement (s), velocity(v) and acceleration(v) of the vehicle(Veh) and occupant(Occ) respectively. Ex and Mod stand for Experimental and Model respectively. It is noted from Figure 7 that the maximum displacements of the vehicle and occupant models are 9.2% and 9.5% less those from the experimental data respectively. The time of the dynamic crash is far from the experimental data. The maximum crash of the vehicle is 0.089 s while that from the model is 0.098 s. This is also observed on the occupant time at maximum displacement; that is 0.146 s instead of 0.12 s from data. The rebound velocity of $-2.3m/s$ for the vehicle model is slightly less than that in the real crash (i.e. $-3.75m/s$), but the occupant rebound velocity of $-13.5m/s$ is almost closer to the real crash data (i.e.$-13.1m/s$). This shows that the model presented in Figure 1, with combined linear and nonlinear force elements cannot accurately reconstruct the vehicle occupant crash scenario. The estimated parameters, linear and nonlinear springs and dampers: k_l, k_{nl}, c_l, c_{nl}, k_2 and c_2 , are shown in Table I.

An improvement is noted in Figure 8 where the stiffness and dampers in the model are piecewise functions with one break point shown in Figure 4. The maximum dynamic crash

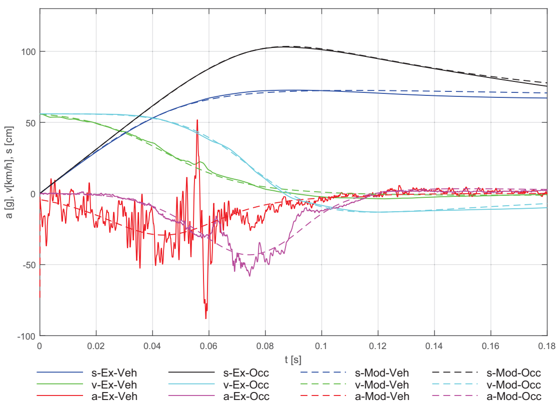

Figure 8. Comparison between vehicle model and vehicle experimental data - piecewise function with 1 break point

TABLE II
PARAMETERS ESTIMATION MODEL2- ONE BREAK POINT PIECEWISE FUNCTION

Parameter	*Value*	*Unit*	*Parameter*	*Value*	*Unit*
k_{11}	6.3993e+04	N/m	k_{21}	6.2502e+03	N/m
k_{12}	3.5743e+04	N/m	k_{22}	3.8618e+04	N/m
k_{13}	6.3669e+04	N/m	k_{23}	7.5718e+04	N/m
x_{11}	0.3034	m	x_{21}	0.0529	m
c_{11}	8.3497e+04	Ns/m	c_{21}	3.1340e+03	Ns/m
c_{12}	3.2983e+03	Ns/m	c_{22}	3.0876e+03	Ns/m
c_{13}	1.9725e+05	Ns/m	c_{23}	3.1159	Ns/m
$v_{1\text{-}th}$	15.5035	m/s	$v_{2\text{-}th}$	0.1013	m/s

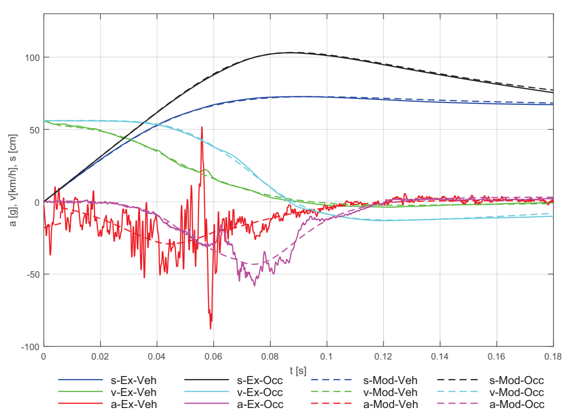

Figure 9. Comparison between vehicle model and vehicle experimental data - piecewise function with 2 break points

of the vehicle model is 0.3% less than that in the real crash test. The displacement of the occupant is 0.4% larger than that from the crash test. Improvement of the model accuracy is also observed from the time at maximum displacement and the rebound velocities for both the vehicle and occupant. The estimated parameters are shown in Table II. From Figure 9, the model accuracy is obtained by using force elements with two break point piecewise functions as shown in Figure 5. The maximum dynamic crash of the vehicle model is 0.05% less than that in the real crash test. The displacement

TABLE III
A SUMMARY OF KINEMATICS RESULTS FROM THE MODELS

Model type		C_m [cm]	T_m [s]	v_{reb} [m/s]
Model1: Mixed of linear and nonlinear springs / dampers	Vehicle	65.93	0.0978	-2.26
	Occupant	93.22	0.09473	-13.58
Model2:1 break point piecewise function	Vehicle	72.48	0.1085	-1.15
	Occupant	103.4	0.08617	-13.1
Model2: 2 break points piecewise functions	Vehicle	72.65	0.093	-2.43
	Occupant	103.2	0.0863	-12.68

TABLE IV
PARAMETERS ESTIMATION MODEL2- TWO BREAK POINTS PIECEWISE FUNCTION

Parameter	*Value*	*Unit*	*Parameter*	*Value*	*Unit*
k_{11}	7.6665e+04	N/m	k_{21}	6.8536e+03	N/m
k_{12}	7.9498e+04	N/m	k_{22}	2.5529e+04	N/m
k_{13}	9.6887e+03	N/m	k_{23}	9.9998e+04	N/m
k_{14}	9.9998e+04	N/m	k_{24}	7.2212e+04	N/m
x_{11}	0.5533	m	x_{21}	3.3735e-05	m
x_{12}	0.6711	m	x_{22}	0.7498	m
c_{11}	8.4895e+04	Ns/m	c_{21}	4.8212e+03	Ns/m
c_{12}	2.8460e+03	Ns/m	c_{22}	1.3677e+03	Ns/m
c_{13}	3.3299e+03	Ns/m	c_{23}	3.2491e+03	Ns/m
c_{14}	1.4046e+04	Ns/m	c_{24}	2.2323e+03	Ns/m
$\dot{x}_{11}$	10.4951	m/s	$\dot{x}_{21}$	7.8176	m/s
$\dot{x}_{12}$	14.6044	m/s	$\dot{x}_{22}$	15.6884	m/s

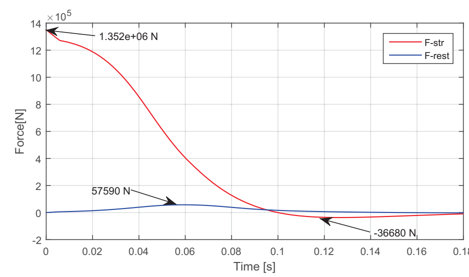

Figure 10. Deformation of the vehicle frontal structure

of the occupant is 0.09% larger than that from the crash test. Improvement of the model accuracy is also observed from the time at maximum displacement and the rebound velocities for both the vehicle and occupant. A summary of kinematics results from the models is tabulated in Table III and the optimized estimated parameters are shown in Table IV. The deformation force and loading characteristics of the vehicle front structure and restraint system are shown in Figure 10. A maximum force of $1,352,000N$ is observed at the time of collision and decreases up to $-36,680N$. The restraint system reaches the maximum force of $57,590N$ at $0.054s$. Spring and damper characteristics are shown in Figure 11 and Figure 12 respectively.

V. CONCLUSION AND FUTURE WORK

In this paper, a mathematical-based method is presented to estimate the parameters of a double-spring-mass-damper model of a vehicle-occupant frontal crash. It is observed that the model results in responses are closer to the experimental crash test. Therefore, the overall behavior of the model matches the real vehicle's crush well. Two of the main

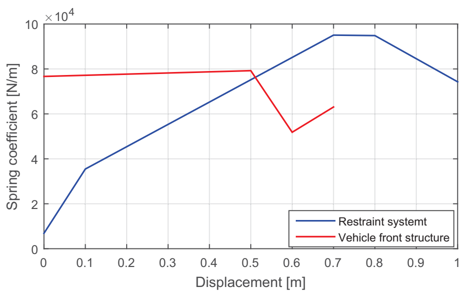

Figure 11. Spring coefficient characteristics

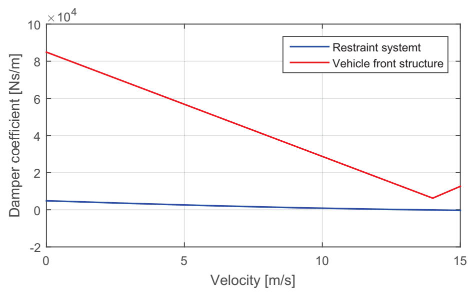

Figure 12. Damper coefficient characteristics

parameters characterizing the collision are the maximum dynamic crush - which describes the highest car's deformation and the time at which it occurs- t_m. They are pertinent to the occupant crashworthiness since they help to assess the maximum intrusion into the passenger's compartment.

The model with combined linear and nonlinear force elements showed results with a significant error. It is noted that the stepwise nonlinear springs and dampers model 2, gives better results than the model 1. Introducing more break points on the piecewise functions increases the accuracy of the model. The force due to structure deformation decreases, and the loading due to the restraint system increases and become maximum at the time of crash. These forces are almost zero after rebound phase.
The authors will extend the work by including other parts of the vehicle such as an engine in the model. Further investigations of the proposed approach to vehicle-to- vehicle crash scenario is also under study.

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