

Taller

1 Modus ponendo ponens

1.

- (1) $P \vee Q \rightarrow R$
- (2) $P \vee Q$
- (3) R pp (1,2)

$$\begin{array}{l} P \vee Q \rightarrow R \\ P \vee Q \\ \hline \therefore R \text{ pp (1,2)} \end{array}$$

2.

- (1) $\sim P \rightarrow \sim R$
- (2) $\sim P$
- (3) $\sim R$ pp (1,2)

$$\begin{array}{l} \sim P \rightarrow \sim R \\ \sim P \\ \hline \therefore \sim R \text{ pp (1,2)} \end{array}$$

3. (1) $\sim P$
- (2) $\sim P \rightarrow Q$
- (3) Q pp (2,1)

$$\begin{array}{l} \sim P \rightarrow Q \\ \sim P \\ \hline \therefore Q \text{ pp (2,1)} \end{array}$$

4. (1) $P \rightarrow Q \wedge R$
- (2) P
- (3) $Q \wedge R$ pp (1,2)

$$\begin{array}{l} P \rightarrow Q \wedge R \\ P \\ \hline \therefore Q \wedge R \text{ pp (1,2)} \end{array}$$

5. (1) $P \rightarrow Q \vee R$
- (2) P
- (3) $Q \vee R$ pp (1,2)

$$\begin{array}{l} P \rightarrow Q \vee R \\ P \\ \hline \therefore Q \vee R \text{ pp (1,2)} \end{array}$$

6. (1) $\sim R$
- (2) $\sim R \rightarrow Q \wedge P$
- (3) $Q \wedge P$ pp (2,1)

$$\begin{array}{l} \sim R \rightarrow Q \wedge P \\ \sim R \\ \hline \therefore Q \wedge P \text{ pp (2,1)} \end{array}$$

2 Resuelva los siguientes ejercicios

- a) (1) $X \wedge Y \rightarrow X + Z$
- (2) $X \wedge Y$
 - (3) $X + Z$ pp (1,2)

Si "x" es un número y "z" es un número, entonces $x+z$ es un número. "x" es un número y "z" es un número

- b) (1) $X > Y \wedge Y > Z \rightarrow X > Z$
- (2) $X > Y \wedge Y > Z$
 - (3) $X > Z$ pp (1,2)

Si $x > y$ y $y > z$, entonces $x > z$.
A la vez $x > y$ y $y > z$.

- c) (1) $X = Y \wedge Y = Z$
- (2) $X = Y \wedge Y = Z \rightarrow X = Z$
 - (3) $X = Z$ pp (2,1)

A la vez $x = y$ y $y = z$, si $x = y$ y $y = z$, entonces $x = z$

3 Demostrar la conclusión que se pide

Demostrar B

- (1) $\sim B \rightarrow E$
- (2) $E \rightarrow K$
- (3) $\sim B$
- (4) $K \rightarrow \sim L$
- (5) $\sim L \rightarrow M$
- (6) $M \rightarrow B$
- (7) E pp (1,3)
- (8) K pp (2,7)
- (9) $\sim L$ pp (4,8)
- (10) M pp (5,9)
- (11) B pp (6,10)

$$\begin{array}{l} \sim B \rightarrow E \\ \sim B \\ \hline \therefore E \text{ pp (1,3)} \end{array}$$

$$\begin{array}{l} E \rightarrow K \\ E \\ \hline \therefore K \text{ pp (2,7)} \end{array}$$

$$\begin{array}{l} K \rightarrow \sim L \\ K \\ \hline \therefore \sim L \text{ pp (4,8)} \end{array}$$

$$\begin{array}{l} \sim L \rightarrow M \\ \sim L \\ \hline \therefore M \text{ pp (5,9)} \end{array}$$

$$\begin{array}{l} M \rightarrow B \\ M \\ \hline \therefore B \text{ pp (6,10)} \end{array}$$

Demonstrar $R \vee S$

- (1) $C \vee D$
- (2) $C \vee D \rightarrow \sim F$
- (3) $\sim F \rightarrow A \wedge \sim B$
- (4) $A \wedge \sim B \rightarrow R \vee S$
- (5) $\sim F$ pp (2,1)
- (6) $A \wedge \sim B$ pp (3,5)
- (7) $R \vee S$ pp (4,6)

$$C \vee D \rightarrow \sim F$$

$$C \vee D$$

$$\therefore \sim F \text{ pp (2,1)}$$

$$\sim F \rightarrow A \wedge \sim B$$

$$\sim F$$

$$\therefore A \wedge \sim B \text{ pp (3,5)}$$

$$A \wedge \sim B \rightarrow R \vee S$$

$$A \wedge \sim B$$

$$\therefore R \vee S \text{ (4,6)}$$

① Modus tollendo tollens

1.

$$(1) Q \rightarrow R$$

$$Q \rightarrow R$$

$$(2) \sim R$$

$$\sim R$$

$$(3) \sim Q \text{ TT (1,2)}$$

$$\therefore \sim Q \text{ TT (1,2)}$$

2.

$$(1) \sim P \rightarrow Q$$

$$\sim P \rightarrow Q$$

$$(2) \sim Q$$

$$\sim Q$$

$$(3) P \text{ TT (1,2)}$$

$$\therefore P \text{ TT (1,2)}$$

3.

$$(1) R \rightarrow S$$

$$R \rightarrow S$$

$$(2) \sim S$$

$$\sim S$$

$$(3) \sim R \text{ TT (1,2)}$$

$$\therefore \sim R \text{ TT (1,2)}$$

4.

$$(1) Q \rightarrow \sim R$$

$$Q \rightarrow \sim R$$

$$(2) \sim \sim R$$

$$\sim \sim R$$

$$(3) \sim Q \text{ TT (1,2)}$$

$$\therefore \sim Q \text{ TT (1,2)}$$

5.

$$(1) P \rightarrow Q \wedge R \text{ P}$$

$$P \rightarrow Q \wedge R \text{ P}$$

$$(2) \sim (Q \wedge R) \text{ P}$$

$$\sim (Q \wedge R) \text{ P}$$

$$(3) \sim P \text{ TT (1,2)}$$

$$\therefore \sim P \text{ TT (1,2)}$$

6.

$$(1) P \vee Q \rightarrow R \text{ P}$$

$$P \vee Q \rightarrow R \text{ P}$$

$$(2) \sim R \text{ P}$$

$$\sim R \text{ P}$$

$$(3) \sim (P \vee Q) \text{ TT (1,2)}$$

$$\therefore \sim (P \vee Q) \text{ TT (1,2)}$$

Del inciso A

1 Demostrar $\sim P$

$$(1) P \rightarrow \sim Q$$

$$(2) Q$$

$$(3) \sim P \text{ TT } (1,2)$$

$$P \rightarrow \sim Q$$

$$Q$$

$$\therefore \sim P \text{ TT } (1,2)$$

2 Demostrar $\sim A$

$$(1) A \rightarrow \sim C$$

$$(2) B \rightarrow C$$

$$(3) B$$

$$(4) C \text{ pp } (2,3)$$

$$(5) \sim A \text{ TT } (1,4)$$

$$B \rightarrow C$$

$$B$$

$$\therefore C \text{ pp } (2,3)$$

$$A \rightarrow \sim C$$

$$C$$

$$\therefore \sim A \text{ TT } (1,4)$$

3 Demostrar P

$$(1) \sim P \rightarrow \sim Q$$

$$(2) Q$$

$$(3) P \text{ TT } (1,2)$$

$$\sim P \rightarrow \sim Q$$

$$Q$$

$$\therefore P \text{ TT } (1,2)$$

4 Demostrar A

$$(1) \sim A \rightarrow \sim B$$

$$(2) \sim B \rightarrow \sim C$$

$$(3) C$$

$$(4) B \text{ TT } (2,3)$$

$$(5) A \text{ TT } (1,4)$$

$$\sim B \rightarrow \sim C$$

$$C$$

$$\therefore B \text{ TT } (2,3)$$

$$\sim A \rightarrow \sim B$$

$$B$$

$$\therefore A \text{ TT } (1,4)$$

5. Demostrar $\sim S$

$$(1) P \rightarrow Q$$

$$(2) Q \rightarrow R$$

$$(3) S \rightarrow \sim R$$

$$(4) P$$

$$(5) Q \text{ pp } (1,4)$$

$$(6) R \text{ pp } (2,5)$$

$$(7) \sim S \text{ TT } (3,6)$$

$$P \rightarrow Q$$

$$P$$

$$\therefore Q \text{ pp } (1,4)$$

$$Q \rightarrow R$$

$$Q$$

$$\therefore R \text{ pp } (2,5)$$

$$S \rightarrow \sim R$$

$$R$$

$$\therefore \sim S \text{ TT } (3,6)$$

6. Demostrar $\sim A$

(1) $A \rightarrow B$	$C \rightarrow D$	$B \rightarrow C$
(2) $B \rightarrow C$	$\sim D$	$\sim C$
(3) $C \rightarrow D$	$\therefore \sim C \text{ TT } (3,4)$	$\therefore \sim B \text{ TT } (2,5)$
(4) $\sim D$		
(5) $\sim C \text{ TT } (3,4)$	$A \rightarrow B$	
(6) $\sim B \text{ TT } (2,5)$	$\sim B$	
(7) $\sim A \text{ TT } (1,6)$	$\therefore \sim A \text{ TT } (1,6)$	

Del inciso B

1 Demostrar $X=0$

(1) $X \neq 0 \rightarrow X + y \neq y$	$X \neq 0 \rightarrow X + y \neq y$
(2) $X + y = y$	$X + y = y$
(3) $X = 0 \text{ TT } (1,2)$	$\therefore X = 0 \text{ TT } (1,2)$

2 Demostrar $X \neq 0$

(1) $X = 0 \rightarrow X \neq y$	$X = z \rightarrow X = y$	$X = 0 \rightarrow X \neq y$
(2) $X = z \rightarrow X = y$	$X = z$	$X = y$
(3) $X = z$	$\therefore X = y \text{ pp } (2,3)$	$\therefore X \neq 0 \text{ TT } (1,4)$
(4) $X = y \text{ pp } (2,3)$		
(5) $X \neq 0 \text{ TT } (1,4)$		

3. Demostrar $X = y$

(1) $X \neq y \rightarrow X \neq z$	$X \neq z \rightarrow X \neq 0$	$X \neq y \rightarrow X \neq z$
(2) $X \neq z \rightarrow X \neq 0$	$X = 0$	$X = z$
(3) $X = 0$	$\therefore X = z \text{ TT } (2,3)$	$\therefore X = y \text{ TT } (1,4)$
(4) $X = z \text{ TT } (2,3)$		
(5) $X = y \text{ TT } (1,4)$		

4. Demostrar $X \neq 0$

(1) $X = y \rightarrow X = z$	$X = y \rightarrow X = z$	$X = z \rightarrow X = 1$
(2) $X = z \rightarrow X = 1$	$X = y$	$X = z$
(3) $X = 0 \rightarrow X \neq 1$	$\therefore X = z \text{ pp } (1,4)$	$\therefore X = 1 \text{ pp } (2,5)$
(4) $X = y$	$X = 0 \rightarrow X \neq 1$	
(5) $X = z \text{ pp } (1,4)$	$X = 1$	
(6) $X = 1 \text{ pp } (2,5)$	$\therefore X \neq 0 \text{ TT } (3,6)$	
(7) $X \neq 0 \text{ TT } (3,6)$		

5 Demostrar $x \neq y$

- (1) $x = y \rightarrow y = z$
- (2) $y = z \rightarrow x = w$
- (3) $y = w \rightarrow y = 1$
- (4) $y \neq 1$
- (5) $y \neq w$ TT (3,4)
- (6) $y \neq z$ TT (2,5)
- (7) $x \neq y$ TT (1,6)

$$\begin{array}{l} y = w \rightarrow y = 1 \\ \hline y \neq 1 \\ \hline \therefore y \neq w \text{ TT (3,4)} \end{array}$$

$$\begin{array}{l} y = z \rightarrow y = w \\ \hline y \neq w \\ \hline \therefore y \neq z \text{ TT (2,5)} \end{array}$$

$$\begin{array}{l} x = y \rightarrow y = z \\ \hline y \neq z \\ \hline \therefore x \neq y \text{ TT (1,6)} \end{array}$$

6 Demostrar $x = 0$

- (1) $x \neq 0 \rightarrow y = 1$
- (2) $x = y \rightarrow y = w$
- (3) $y = w \rightarrow y \neq 1$
- (4) $x = y$
- (5) $y = w$ pp (2,4)
- (6) $y \neq 1$ pp (3,5)
- (7) $x = 0$ TT (1,6)

$$\begin{array}{l} x = y \rightarrow y = w \\ \hline x = y \\ \hline \therefore y = w \text{ pp (2,4)} \end{array}$$

$$\begin{array}{l} y = w \rightarrow y \neq 1 \\ \hline y = w \\ \hline \therefore y \neq 1 \text{ pp (3,5)} \end{array}$$

$$\begin{array}{l} x \neq 0 \rightarrow y = 1 \\ \hline y \neq 1 \\ \hline \therefore x = 0 \text{ TT (1,6)} \end{array}$$

3 Modus tollendo ponens

1

$$\begin{array}{l} (1) \sim Q \vee R \\ (2) \sim R \\ \hline (3) \sim Q \text{ TP (1,2)} \end{array}$$

$$\begin{array}{l} \sim Q \vee R \\ \hline \sim R \\ \hline \therefore \sim Q \text{ TP (1,2)} \end{array}$$

2

$$\begin{array}{l} (1) T \vee (P \rightarrow Q) \\ (2) \sim T \\ \hline (3) (P \rightarrow Q) \text{ TP (1,2)} \end{array}$$

$$\begin{array}{l} T \vee (P \rightarrow Q) \\ \hline \sim T \\ \hline \therefore (P \rightarrow Q) \text{ TP (1,2)} \end{array}$$

3

$$\begin{array}{l} (1) \sim T \vee \sim R \\ (2) \sim \sim R \\ \hline (3) \sim T \text{ TP (1,2)} \end{array}$$

$$\begin{array}{l} \sim T \vee \sim R \\ \hline \sim \sim R \\ \hline \therefore \sim T \text{ TP (1,2)} \end{array}$$

4

$$\begin{array}{l} (1) P \vee Q \\ (2) \sim Q \\ \hline (3) P \text{ TP (1,2)} \end{array}$$

$$\begin{array}{l} P \vee Q \\ \hline \sim Q \\ \hline \therefore P \text{ TP (1,2)} \end{array}$$

5

$$\begin{array}{l} (1) (S \wedge T) \vee R \\ (2) \sim (S \wedge T) \\ \hline (3) R \text{ TP (1,2)} \end{array}$$

$$\begin{array}{l} (S \wedge T) \vee R \\ \hline \sim (S \wedge T) \\ \hline \therefore R \text{ TP (1,2)} \end{array}$$

6

$$\begin{array}{l} (1) (P \wedge Q) \vee S \\ (2) \sim S \\ \hline (3) (P \wedge Q) \text{ TP (1,2)} \end{array}$$

$$\begin{array}{l} (P \wedge Q) \vee S \\ \hline \sim S \\ \hline \therefore (P \wedge Q) \text{ TP (1,2)} \end{array}$$