**Control strategies**

**Frequency response**

Frequency-response analysis and design of a linear control system is based on Nyquist stability criterion, from which absolute, as well as relativity stability of linear closed-loop systems can be investigated from a knowledge of their open-loop frequency-response characteristic.

In general, when designing a closed-loop system, one adjusts the frequency-response characteristic of the open-loop transfer function by using several design criteria in order to obtain the desired transient-response characteristics for the system.

The Bode diagram, which consists of two graphs, can usefully represent the frequency-response of a system: a curve of the logarithm of the magnitude of a sinusoidal transfer function and a curve of the phase angle; both curves are plotted against the frequency on a logarithmic scale. The logarithm magnitude graph’s representation is standardized as , where the base of the logarithm is 10. Decibel is its unit and it is abbreviated dB.



Some interesting characteristics that can be extract from the Bode diagram are the cutoff frequency and the bandwidth.

The cutoff frequency can be defined as the frequency at which the magnitude of the closed-loop frequency response is 3 dB below its zero-frequency value. The closed-loop system filters out the signal components whose frequencies are greater than the cutoff frequency and transmits those signal components with frequencies lower than the cutoff frequency.

The frequency range in which the magnitude of the closed loop does not drop -3 dB is called the bandwidth of the system. The bandwidth indicates the frequency where the gain starts to fall off from its low-frequency value. Thus, the bandwidth indicates how well the system will track an input sinusoid.

The rise time and the bandwidth are inversely proportional to each other. There is, the rise time increases with the decreasing damping ratio . On the other hand, the bandwidth decreases with increasing . The specification of the bandwidth may be determined by the following factors:

1. The ability to reproduce the input signal. A large bandwidth correspond to a small rise time, or a fast response.
2. The necessary filtering characteristics for high-frequency noise.

For the system to follow arbitrary inputs accurately, it is necessary that it have a large bandwidth. From the viewpoint of noise, the bandwidth should be as narrow as possible. Thus, there are conflicting requirements for the design of a good control system and a compromise is necessary. In addition, systems with large bandwidth require high-performance components and the cost of the components usually increases.

Nyquist plots and Nyquist stability criterion

The Nyquist plot of a transfer function is the plot of the magnitude versus the phase angle of that transfer function in polar coordinates as the frequency varies from zero to infinity.

One advantage of Nyquist plot is that it shows, in only one graph, magnitude and phase of a transfer function through the whole frequency range. A disadvantage is that it does not clearly provide details of the contribution given by any of the system’s singularities.

Considering a system given by the closed-loop transfer function represented by EQUATION XXX:

For stability, all roots of the characteristic equation () must lie on the left-half s-plane. The Nyquist stability criterion relates the open-loop frequency-response to the number of zeros and poles of that lie in the right-half s-plane. This criterion can be state as follows:

If a system has p poles in the right-half s-plane, then, for stability, the system locus as a representative point *s* traces out the Nyquist path in the clockwise direction must encircle the point p times in the counterclockwise direction.

Considerations on the Nyquist stability criterion are numbered:

1. Nyquist stability criterion can be expressed as:

Where

= number of zeros of in the right-half s-plane;

= number of clockwise encirclements of the point;

= number of poles of in the right-half s-plane.

If is not zero, then, for a stable control system, z must be equal zero, or , which means that must encircle the point in the counterclockwise direction times.

If does not have any poles in the right-half s-plane, then must be equal for stability

1. Multiple-loop systems demand special attention when testing their stability by Nyquist criterion since they may include poles in the right-half s-plane. Simple inspection of the encirclements of the point may not be sufficient in determining whether these kind of systems are stable or not. In such cases, the application of Routh-Hurwitz stability criterion to the denominator of is recommended.
2. If the locus of passes through the point, then the zeros of the characteristic equation, or the closed-loop poles, are located on the imaginary axis. This is not desirable for a control system since that means the system is marginally stable and will present considerable oscillation around its operational point.

Fig. 1 shows Nyquist plots of a third-order system given by EQUATION XXX for three different values of the open-loop gain K. For a small value of the gain K, the system does not encircle the point and is, therefore, stable. For a large value of the gain , the system is unstable, since its plot encircle the point once. For a certain value of , the system’s plot passes exactly through the point . That means that, for this value of , the system is on the verge of instability and will exhibit sustained oscillations.



Fig. 1. Nyquist plot of an arbitrary third-order system

Phase margin is the amount of additional phase lag at the gain crossover frequency () required to bring the system to the verge of instability. The gain crossover frequency is the frequency at which de magnitude of the open-loop transfer function is unity. The phase margin is given by EQUATION XXX:

Gain margin is the reciprocal of the system’s magnitude at the frequency at which the phase angle crosses (). The margin gain is given by EQUATION XXX.

The gain margin of first- and second-order systems are infinite, since the Nyquist plots of such systems do not cross the negative real axis and, therefore, never crosses the point .

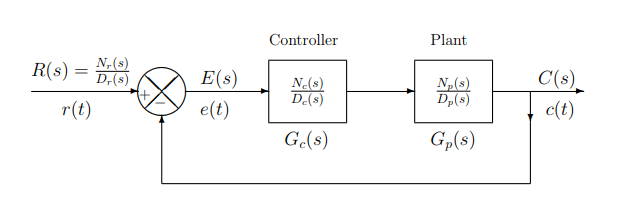
The phase and gain margin of a control system are a measure of the closeness of the Nyquist plot to the point. Therefore, these margins may be used as design criteria. For a minimum-phase system, both phase and gain margin must be positive in order for the system to be stable.

Proper gain and phase margins ensure satisfactory performance of the system against parametric variations in the system’s components. An adequate value for the gain margin is at least +6 dB. For the phase margin, it is recommended somewhere between to .

The requirement of the phase margin to be between to means that, in the Bode diagram, the slope of the log-magnitude curve will be more gradual than -40 dB/decade in the crossover frequency. It is desirable that the slope of the magnitude curve be -20 dB/decade since that assures that the system is stable. If the slope is -40 dB/decade, the system could be either stable or unstable. If the slope is -60 dB/decade or steeper, the system will be unstable.

Internal model principle

The concept of internal models plays a crucial role in regulator problems. The internal model principle can intuitively be expressed as: ’Any good regulator must create a model of the dynamic structure of the environment in the closed loop system’



In the configuration depicted in Figure 1, where the poles of R(s) are in the right half plane,

if and only if

1. the closed-loop poles are in the open left-half plane;
2. is a factor of the open-loop characteristic polynomial , that is, there is a polynomial, say , such that .

The first condition assures that the closed-loop system is asymptotically stable. The second condition means that the tracking controller must be chosen in such way that the open-loop transfer function, , contains a model of the reference signal to be tracked.

That is, if the system’s requirement is that it tracks a reference with a DC value, the open-loop transfer function of that system must contain the model of a step (); if the system’s requirement is that it tracks a sinusoidal reference, the open-loop transfer function of that system must contain the model of a sine () or cosine ();

The internal model principle can be summed up as follows: “Any good tracking controller must stabilize the closed-loop system and must contain a model of the reference signal”.

**Design of control systems in the frequency domain**

The design of control systems based on the Bode diagram approach is useful for several reasons, including:

1. The error constants Kp, Kv, or Ka can be easily identified by the low frequency asymptote of the magnitude curve;
2. Specifications of transient response can be related to characteristics of the Bode diagram like phase margin, gain margin, bandwidth, and so forth.
3. The design of a compensator to satisfy given specifications can be carried out in the Bode diagram in a simple and straightforward manner.

**Procedure for determination of a Proportional-Integral (PI) compensator**

EQUATION XXX gives the transfer function that describes a PI-compensator.

In order to design the PI-compensator, one must determine the value of the parameters and with the objective of attend a set of specifications. Two of the specifications that may be taken into consideration to determine the PI parameters are the desired crossover frequency () and the phase margin ().

As the control system is being designed following a frequency approach, the real portion of Laplace variable () can be considered to be zero and only the imaginary portion is considered (). Therefore, EQUATION XXX becomes EQUATION XXX.

The magnitude of the PI transfer function is given by EQ. X:

The phase of the PI transfer function is given by EQ. XXX.

Applying EQ.XXX and EQ.XXX into EQ XXX gives EQ.XXX and EQ.XXX.

**Procedure for determination of a Proportional-Resonant (PR) compensator**

The procedure for determination of the parameters of a PR compensator is very similar to that of a PI compensator. One must only substitute the transfer function that describes the PI compensator for the transfer function of a PR compensator.

The resonant portion of a PR compensator may be a frequency representation of a cosine function, as represented in EQUATION XXX.

A PR compensator is given by EQUATION XXX.

Where:

is the proportional gain;

is the resonant gain;

is the resonant frequency, usually

EQUATION XXX represents an alternative form of the transfer function that describes a PR-compensator.

Where:

In order to design the PR-compensator, one must determine the value of the parameters and with the objective of attend the same set of specifications cited in the case of the PI compensator.

As the control system is being designed following a frequency approach, the Laplace variable () of EQUATION XXX is substituted by the frequency variable (). Therefore, EQUATION XXX becomes EQUATION XXX.

EQ. X gives the magnitude of the PR transfer function:

EQ. XXX gives the phase of the PR transfer function.

Applying EQ.XXX and EQ.XXX into EQ XXX gives EQ.XXX and EQ.XXX.

**Procedure for determination of a Proportional-Multi-Resonant (MR) compensator**

In general, when a micro-inverter is connected to the grid, the current delivered by it to the grid is a sum of a sinusoidal waveform in the grid frequency along with several others frequencies that are multiple of the grid frequency.

The frequencies that are multiple to the grid fundamental frequency, also referred to as harmonic frequencies, cause a distortion in the current waveform that is delivered to the grid by the micro-inverter.

The idea behind the implementation of a Proportional + Multi-resonant controller, once again, based on the Internal Model Principle, is that the control should be able to compensate errors and track reference signals that have more than one frequency of interest, in this case, the harmonic frequencies of the grid voltage that cause the distortion in the current waveform aforementioned.

A proposed form for a MR compensator is given by EQUATION XXX, where the resonant gain is pondered and decreases proportionally as the degree of the harmonic implemented increases.

Where

is the frequency of interest to be tracked by the control system;

is the highest frequency to be tracked by the control system;

is the resonant frequency given by

This proposition is interesting by the point of view of design procedure of control because it has only two parameters to be determined.

Therefore, In order to design the MR-compensator, one must determine the value of the parameters and with the objective of attend the same set of specifications cited in the cases aforementioned of the PI and PR compensators.

In order to find these parameters as a function of the crossover frequency and the phase margin , EQUATION XXX can be rewrote as follows.

Rewriting EQUATION XXX in terms of products and sums depending on the harmonic orders that compose the compensator becomes EQUATION XXX.

Where

;

.

Considering the Laplace variable in the frequency domain, the term and becomes simply . EQUATION XXX can be written.

Manipulating EQUATION XX and XX yields EQUATION XX and XX which are functions for and , as function of the cutoff frequency .

**Procedure for determination of a Repetitive compensator**

Traditionally, in control system theory, the design of the controllers are made based on the internal model principle.

In order to deal with periodic references and disturbances, the internal model principle is the basic principle considered by resonant and repetitive controllers. In fact, to perfectly track or reject a sinusoidal signal with determined frequency , the controller must contain a pair of poles at in the imaginary axis.

An alternative to multi-resonant controllers to cope with periodic signals is the repetitive controller. A repetitive feedback control system is based on the concept of iterative learning control, and it has been widely used for many practical industrial systems, such as manufacturing [REFERENCE], robotics [REFERENCE], as well as in UPS (uninterruptable power source). In these controllers, error between reference signal and the measured output signal over one fundamental cycle is used to generate a new reference to the next fundamental cycle.

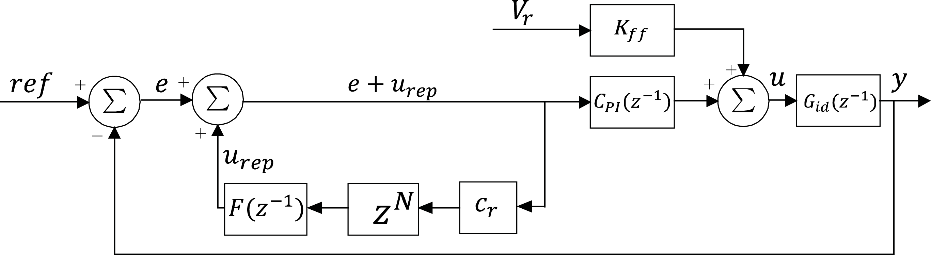
A repetitive controller is mathematically equivalent to a parallel combination of an integral controller, an infinite number of resonant controllers, and a proportional controller. [REFERENCE]

It has the advantage of being simpler to implement than it is to implement several proportional-resonant controllers in parallel. However, the microcontroller in which this control technique is implemented must storage a large number of output and error samples that are used to calculate the control variable to be applied to the plant. Another disadvantage of repetitive controllers is that it creates resonance gain peaks in harmonics of high frequencies, which can lead to instability.

A low-pass filter is generally used to attenuate these high frequency gain peaks but if the bandwidth of the plant is low, the implementation of this filter will reduce the low frequency resonance gains and deteriorate the performance of the repetitive controller. [REFERENCE]

**Direct repetitive controller**

A proposed topology for a repetitive control system is shown in FIGURE AAA, according to [REFERENCE]



The value of the repetitive controller gain needs to be carefully selected as it is a key parameter for error convergence and system stability [POWER ELECTRONIC CONVERTERS FOR MICROGRIDS, SULEIMAN M. SHARKH, MOHAMMAD A. ABUSARA, GEORGIOS I. ORFANOUDAKIS AND BABAR HUSSAIN]

A high results in fast error convergence but reduces the system’s stability. A comparison between some different values of is shown in FIGURE XXX. It can be seen that, for values of near unity, the peaks present in the magnitude plot and the abrupt changes of phase present on the phase plot of the bode diagram are very steep, while for values near zero, these characteristics are so smooth that the repetitive controller has almost no effect on the performance of the control system.

An adequate value of is selected.



The controller of instantaneous action may have any known topology which guarantees satisfactory behavior to the system dynamics. A phase-advance compensators is used; it has the following structure:

Where

is the frequency of the controller’s zero;

is the frequency of the controller’s pole, which is chosen to be higher than ;

is the controller’s gain.

The Bode diagram of the open loop transfer function of the system, with is shown in FIGURE XXX.



It can be noticed that the system is unstable due to the resonant peaks near the crossover frequency. It means that must be modified to attenuate high frequency peaks. A simple low-pass filter is used. It has the following structure:

Where:

is the cutoff frequency of the low-pass filter.

The cutoff frequency of the low-pass filter is chosen to be at 2 kHz in order to correctly attenuate the resonant peaks at the system crossover frequency. The resultant system’s bode diagram is shown in FIGURE XXX.



It can now be noticed that, at the crossover frequency, the magnitude of the bode diagram is below 0 dB, which means that the system is stable.

**Auxiliary repetitive controller**

An alternative for the repetitive control is the auxiliary repetitive control, which is used only to improve the steady-state performance of the system, while the dynamic response and stability of the system is ensured by a conventional controller (PI, PD, PR, etc.).

It is important to note that any control strategy can be used for the stability and dynamic response of the system since the auxiliary repetitive controller is not dependent of the instantaneous control action.

[INSERT AN IMAGE HERE]

The transfer function of the auxiliary repetitive control action, used to generate periodic signals of multiple harmonics of a fundamental frequency is given by [EQUATION XXX]