

# DATA 61

Beamer Template: Felipe Maldonado



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*Australian National University, March 31, 2016*



# Beamer template Data61/ANU:



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- Contains two .tex files, one where you can include your content, and the other to compile (and define whatever package you need )
- MUST compile using XeLaTeX
- Already defined 3 colours: *anugold*: **anugold**; *dataplum*: **data61 green**; *datagreen*: **data61 darker green**
- By default the footer is datagreen, the topbar (and the title) is anugold, and every slide has a background image containing both logos (+ a network).
- Support .png and .pdf images.



# Blocks

Different kinds of blocks



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## Block

Normal block. Colorless, neutral.

## Example Block

Example block. (Potential uses: list of pros, relaxing facts)

## Alert Block

Alert block. (Potential uses: list of cons, impending doom)

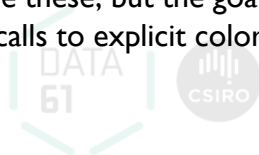
This is a transition slide, it does not increase the number of pages (see the content.tex file to learn how to use it).



More colours already defined:

- This is **crimsonred**.
- This is **paleale** / **lager**.
- This is **turtlegreen** / **green**.
- This is **paleblue**.
- This is **gray**.
- This is **charcoal**.
- This is **jeans**.
- This is **regal**.

You can use the **textcolor** command to use these, but the goal is to do things in a way where there are no calls to explicit colors, just user-adjustable values.



# Sample Feynman Diagrams

Using `tikzfeynman.sty`, you can draw Feynman diagrams with ease. The default color follows the normal text, so it automatically changes color when you swap from a light to a dark background.

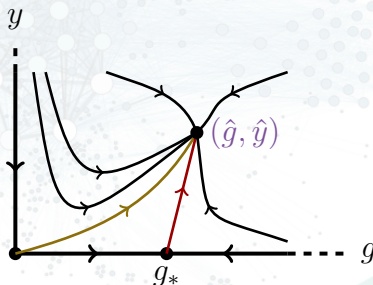


This makes it easy to copy and paste TikZ code from your paper! You can also import diagrams as images. Be sure to use an empty background and pdf/png format to ensure transparency.

# Other diagrams



Here's a nice picture illustrating Seiberg duality:



You will need to adjust the text/images to some suitable position.

# THEORETICAL BACKGROUND





## Definition: Robbins-Monro Algorithm

A *Robbins-Monro Algorithm* (RMA) is a discrete time stochastic processes  $\{x_k\}_{k \geq 0}$  whose general structure is specified by

$$x_{k+1} - x_k = \gamma_{k+1}[F(x_k) + U_{k+1}], \quad (1)$$

where

- $x_k$  takes its values in some Euclidean space (e.g.,  $\mathbb{R}^n$ );
- $\gamma_k$  is deterministic and satisfies  $\gamma_k > 0$ ,  $\sum_{k \geq 1} \gamma_k = \infty$ , and  $\lim_{k \rightarrow \infty} \gamma_k = 0$ ;
- $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a deterministic continuous vector field;
- $\mathbb{E}[U_{k+1} | \mathcal{F}^k] = 0$ , where  $\mathcal{F}^k$  is the natural filtration of the entire process.

## Theorem 2.

Let  $f(x) = x^r, 0 < r < 1$ . Then, there is a unique equilibrium in  $\text{int}(\Delta^{n-1})$ , the interior of the simplex, specified by

$$\phi^* = \frac{1}{\sum_j \bar{q}_j^{\frac{1}{1-r}}} [\bar{q}_1^{\frac{1}{1-r}}, \dots, \bar{q}_n^{\frac{1}{1-r}}]$$

The remaining equilibria are on the boundary of the simplex.

**Remark:** If assume that there exists a set  $Q \subset \{1, \dots, n\}$  ( $|Q| < n$ ) of indexes such that  $\phi_i = 0$  if  $i \in Q$ . The remaining coordinates are given as follows: If  $j \notin Q$ , then we have

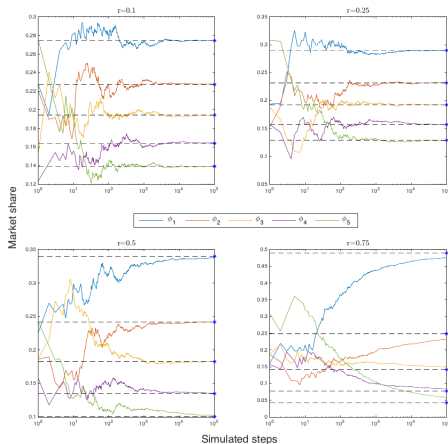
$$\phi_j = \frac{\bar{q}_j^{1/(1-r)}}{\sum_{i \notin Q} \bar{q}_i^{1/(1-r)}}.$$

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# Equilibria and stability

## Main Theorem

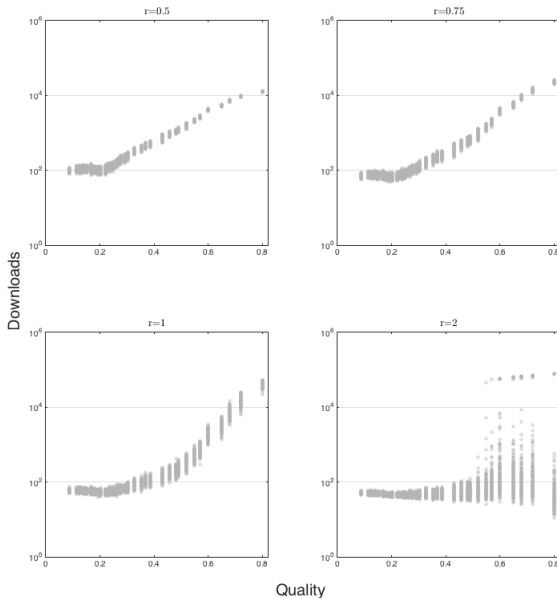
Under the social signal  $f(x) = x^r$ ,  $0 < r < 1$  with  $\phi^0 \in \text{int}(\Delta^n)$ , the RMA  $\{\phi^t\}_{t>0}$  converges to  $\phi^*$  almost surely.



# Plots



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THANKS!

# Backup slides

## Proof of Theorem 2

We know the equilibria are given by

$$p(x) = x \Leftrightarrow p_i(x) = x_i, \forall i \in \{1, \dots, K\}, \text{ then}$$

$$p_i(\Phi) = \phi_i \Leftrightarrow \frac{\bar{q}_i(\phi_i)^r}{\sum_{j=0}^K \bar{q}_j(\phi_j)^r} = \phi_i$$

if  $\phi_i > 0$  for all  $i$ , and then we have the following:

$$p_i(\Phi) = \phi_i \Leftrightarrow \bar{q}_i(\phi_i)^{r-1} = \sum_{j=0}^K \bar{q}_j(\phi_j)^r,$$

then

$$\bar{q}_i(\phi_i)^{r-1} = \bar{q}_j(\phi_j)^{r-1} \Leftrightarrow \phi_i = \left( \frac{\bar{q}_i}{\bar{q}_j} \right)^{\frac{1}{1-r}} \phi_j$$

Adding from  $i = 1$  to  $i = K$  we have

$$1 = \sum_{i=1}^K \phi_i = \frac{\phi_j}{\bar{q}_j^{1/(1-r)}} \sum_{i=1}^K \bar{q}_i^{1/(1-r)}, \text{ and in consequence}$$

$$\phi_j = \frac{\bar{q}_j^{1/(1-r)}}{\sum_{i=1}^K \bar{q}_i^{1/(1-r)}}, \quad j \in \{1, \dots, K\} \text{ are the coordinates of the equilibrium.}$$



## Proof of Theorem 3

The proof studies the asymptotic behaviour of the solutions of the following ODE:

$$\frac{d\phi^t}{dt} = p(\phi^t) - \phi^t. \quad (2)$$

Hence, we have

$$\frac{\bar{q}_i(\phi_i^t)^r}{\sum_j \bar{q}_j(\phi_j^t)^r} = \frac{d\phi_i^t}{dt} + \phi_i^t,$$

$$\frac{1}{\sum_j \bar{q}_j(\phi_j^t)^r} = \frac{1}{\bar{q}_i(\phi_i^t)^r} \left[ \frac{d\phi_i^t}{dt} + \phi_i^t \right] \quad \text{if } \phi_i^t \neq 0,$$

$$\bar{q}_i^{-1}[(\phi_i^t)^{-r} \frac{d\phi_i^t}{dt} + (\phi_i^t)^{1-r}] = \bar{q}_j^{-1}[(\phi_j^t)^{-r} \frac{d\phi_j^t}{dt} + (\phi_j^t)^{1-r}],$$

$$\frac{d}{dt} \left[ e^{(1-r)t} \bar{q}_i^{-1} (\phi_i^t)^{1-r} \right] = \frac{d}{dt} \left[ e^{(1-r)t} \bar{q}_j^{-1} (\phi_j^t)^{1-r} \right]$$

Integrating and re-ordering terms we have:

$$\frac{(\phi_i^t)^{1-r}}{\bar{q}_i} - \frac{(\phi_j^t)^{1-r}}{\bar{q}_j} = e^{(r-1)t} \left[ \frac{(\phi_i^0)^{1-r}}{\bar{q}_i} - \frac{(\phi_j^0)^{1-r}}{\bar{q}_j} \right]. \quad (3)$$

if  $\frac{(\phi_i^0)^{1-r}}{\bar{q}_i} \neq \frac{(\phi_j^0)^{1-r}}{\bar{q}_j}$ , the right-hand side of Equation (3) goes to zero as  $t \rightarrow \infty$  (because  $r < 1$ ) and hence

$$\lim_{t \rightarrow \infty} \frac{(\phi_i^t)^{1-r}}{\bar{q}_i} - \frac{(\phi_j^t)^{1-r}}{\bar{q}_j} = 0. \quad (4)$$

Now denote by  $\phi_j$  the limit of  $\phi_j^t$  for all  $j \in \{1, \dots, n\}$ . Using Equation (4), the following equation holds for all  $i, j \in \{1, \dots, n\}$ :

$$\frac{\phi_i^{1-r}}{\bar{q}_i} = \frac{\phi_j^{1-r}}{\bar{q}_j} \Leftrightarrow \phi_i = \frac{\phi_j}{\bar{q}_j^{1/(1-r)}} \bar{q}_i^{1/(1-r)} \quad (5)$$

which is the equation that defines  $\phi^*$  in Theorem 2.