DATA 61

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www.csiro.au

Beamer template Data61/ANU:

- Contains two .tex files, one where you can include your content, and the other to compile (and define whatever package you need)
- MUST compile using XeLaTeX
- Already defined 3 colours: *anugold*: anugold; *dataplum*: data61 green; *datagreen*: data61 darker green
- By default the footbar is datagreen, the topbar (and the title) is anugold, and every slide has a background image containing both logos (+ a network).
- Support .png and .pdf images.

Blocks

Different kinds of blocks

Block

Normal block. Colorless, neutral.

Example Block

Example block. (Potential uses: list of pros, relaxing facts)

Alert Block

Alert block. (Potential uses: list of cons, impending doom)

This is a transition slide, it does not increase the number of pages (see the content.tex file to learn how to use it).

Title
Subtitle

More colours already defined:

- This is crimsonred.
- This is paleale / lager.
- This is turtlegreen / green.
- This is paleblue.

- This is gray.
- This is charcoal.
- This is jeans.
- This is regal.

You can use the `textcolor` command to use these, but the goal is to do things in a way where there are no calls to explicit colors, just user-adjustable values.
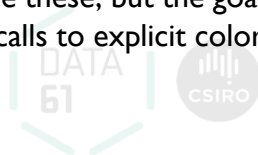
Sample Feynman Diagrams

Using `tikzfeynman.sty`, you can draw Feynman diagrams with ease. The default color follows the normal text, so it automatically changes color when you swap from a light to a dark background.
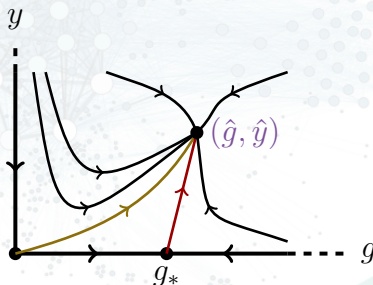

This makes it easy to copy and paste TikZ code from your paper! You can also import diagrams as images. Be sure to use an empty background and pdf/png format to ensure transparency.

Other diagrams

Here's a nice picture illustrating Seiberg duality:

You will need to adjust the text/images to some suitable position.

THEORETICAL BACKGROUND

Equations

...and theorems

Definition: Robbins-Monro Algorithm

A *Robbins-Monro Algorithm* (RMA) is a discrete time stochastic processes $\{x_k\}_{k \geq 0}$ whose general structure is specified by

$$x_{k+1} - x_k = \gamma_{k+1}[F(x_k) + U_{k+1}], \tag{1}$$

where

- x_k takes its values in some Euclidean space (e.g., $\mathbb{R}^n$);
- γ_k is deterministic and satisfies $\gamma_k > 0$, $\sum_{k \geq 1} \gamma_k = \infty$, and $\lim_{k \to \infty} \gamma_k = 0$;
- $F : \mathbb{R}^n \to \mathbb{R}^n$ is a deterministic continuous vector field;
- $\mathbb{E}[U_{k+1}|\mathcal{F}^k] = 0$, where $\mathcal{F}^k$ is the natural filtration of the entire process.

Equilibria

Theorem 2.

Let $f(x) = x^r, 0 < r < 1$. Then, there is a unique equilibrium to Equation (**??**) in $int(\Delta^{n-1})$, the interior of the simplex, specified by

$$\phi^* = \frac{1}{\sum_j \overline{q}_j^{\frac{1}{1-r}}} [\overline{q}_1^{\frac{1}{1-r}}, ..., \overline{q}_n^{\frac{1}{1-r}}]$$

The remaining equilibria are on the boundary of the simplex.

Remark: If assume that there exists a set $Q \subset \{1, .., n\}$ ($|Q| < n$) of indexes such that $\phi_i = 0$ if $i \in Q$. The remaining coordinates are given as follows: If $j \notin Q$, then we have

$$\phi_j = \frac{\overline{q}_j^{1/(1-r)}}{\sum_{i \notin Q} \overline{q}_i^{1/(1-r)}}.$$
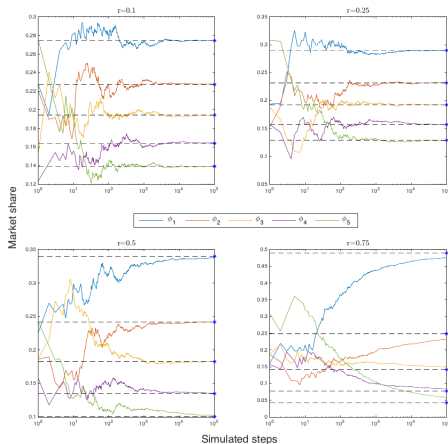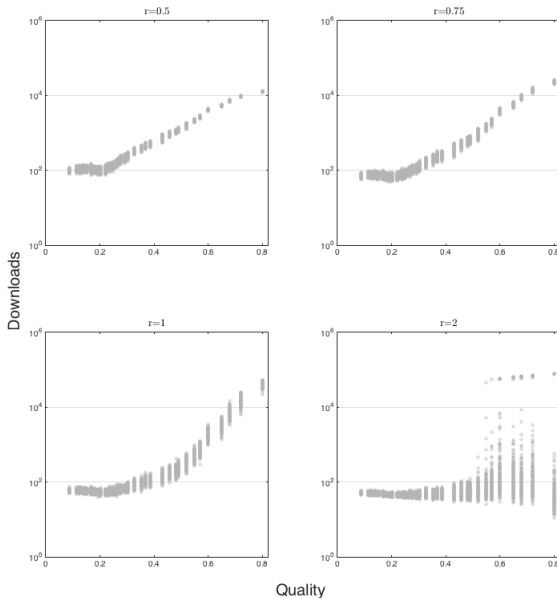
DATA
61

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Equilibria and stability

Main Theorem

Under the social signal $f(x) = x^r, 0 < r < 1$ with $\phi^0 \in int(\Delta^n)$, the RMA $\{\phi^t\}_{t>0}$ converges to ϕ^* almost surely.

Plots

DATA
61

THANKS!

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Backup slides

Proof of Theorem 2

We know the equilibria are given by
$p(x) = x \Leftrightarrow p_i(x) = x_i, \forall i \in \{1, ..., K\}$, then
$p_i(\Phi) = \phi_i \Leftrightarrow \frac{\overline{q}_i(\phi_i)^r}{\sum_{j=0}^{K} \overline{q}_j(\phi_j)^r} = \phi_i$
if $\phi_i > 0$ for all i, and then we have the following:
$p_i(\Phi) = \phi_i \Leftrightarrow \overline{q}_i(\phi_i)^{r-1} = \sum_{j=0}^{K} \overline{q}_j(\phi_j)^r$,
then

$$\overline{q}_i(\phi_i)^{r-1} = \overline{q}_j(\phi_j)^{r-1} \Leftrightarrow \phi_i = \left(\frac{\overline{q}_i}{\overline{q}_j}\right)^{\frac{1}{1-r}} \phi_j$$

Adding from $i = 1$ to $i = K$ we have
$1 = \sum_{i=1}^{K} \phi_i = \frac{\phi_j}{\overline{q}_j^{1/(1-r)}} \sum_{i=1}^{K} \overline{q}_i^{1/(1-r)}$, and in consecuence
$\phi_j = \frac{\overline{q}_j^{1/(1-r)}}{\sum_{i=1}^{K} \overline{q}_i^{1/(1-r)}}, \quad j \in \{1, ..., K\}$ are the coordinates of the
equilibrium.

Proof of Theorem 3

The proof studies the asymptotic behaviour of the solutions of the following ODE:

$$\frac{d\phi^t}{dt} = p(\phi^t) - \phi^t. \tag{2}$$

Hence, we have

$$\frac{\overline{q}_i(\phi_i^t)^r}{\sum_j \overline{q}_j(\phi_j^t)^r} = \frac{d\phi_i^t}{dt} + \phi_i^t,$$

$$\frac{1}{\sum_j \overline{q}_j(\phi_j^t)^r} = \frac{1}{\overline{q}_i(\phi_i^t)^r}\left[\frac{d\phi_i^t}{dt} + \phi_i^t\right] \quad \text{if } \phi_i^t \neq 0,$$

$$\overline{q}_i^{-1}\left[(\phi_i^t)^{-r}\frac{d\phi_i^t}{dt} + (\phi_i^t)^{1-r}\right] = \overline{q}_j^{-1}\left[(\phi_j^t)^{-r}\frac{d\phi_j^t}{dt} + (\phi_j^t)^{1-r}\right],$$

$$\frac{d}{dt}\left[e^{(1-r)t}\overline{q}_i^{-1}(\phi_i^t)^{1-r}\right] = \frac{d}{dt}\left[e^{(1-r)t}\overline{q}_j^{-1}(\phi_j^t)^{1-r}\right]$$

Integrating and re-ordering terms we have:

$$\frac{(\phi_i^t)^{1-r}}{\overline{q}_i} - \frac{(\phi_j^t)^{1-r}}{\overline{q}_j} = e^{(r-1)t} \left[\frac{(\phi_i^0)^{1-r}}{\overline{q}_i} - \frac{(\phi_j^0)^{1-r}}{\overline{q}_j} \right]. \quad (3)$$

if $\frac{(\phi_i^0)^{1-r}}{\overline{q}_i} \neq \frac{(\phi_j^0)^{1-r}}{\overline{q}_j}$, the right-hand side of Equation (??) goes to zero as $t \to \infty$ (because $r < 1$) and hence

$$\lim_{t \to \infty} \frac{(\phi_i^t)^{1-r}}{\overline{q}_i} - \frac{(\phi_j^t)^{1-r}}{\overline{q}_j} = 0. \quad (4)$$

Now denote by ϕ_j the limit of ϕ_j^t for all $j \in \{1, ..., n\}$. Using Equation (??), the following equation holds for all $i, j \in \{1, ..., n\}$:

$$\frac{\phi_i^{1-r}}{\overline{q}_i} = \frac{\phi_j^{1-r}}{\overline{q}_j} \Leftrightarrow \phi_i = \frac{\phi_j}{\overline{q}_j^{1/(1-r)}} \overline{q}_i^{1/(1-r)} \quad (5)$$

which is the equation that defines ϕ^* in Theorem 2.