Equations for Needle Probe Action	ME 415			
Due: February 22, 2011	Thermal Systems Lab	Joshua Holbrook	$\mid 1_{/}$	[/] 1

The Heat Equation, generalized:

$$-k\nabla^2 T = \rho C \frac{\partial T}{\partial t} \tag{1}$$

The exact solution to the heat equation with an infinite line source with constant heat flux:

$$T(r,t) = -\frac{q}{4\pi k} \operatorname{Ei}\left(-\frac{r^2}{4kt}\right) \tag{2}$$

Note that ${\sf q}$ is the heat flux per linear distance.

The "long-time" solution to the same problem:

$$T(r,t) = \frac{q}{4\pi k} \ln \left(\frac{4kt}{r^2} - \frac{\gamma q}{4\pi k} \right) \tag{3}$$

Note that γ is the "Euler-Masceroni constant."

Using this solution to solve for k:

$$k = \frac{q}{4\pi} \left(\frac{dT}{d \left(\ln(t) \right)} \right)^{-1} \tag{4}$$

 $\frac{dT}{d(\ln(t))}$ may be found using a linear curve fit.

An Analogous Approach applied to the Cooling Curve:

$$k = -\frac{q}{4\pi} \left(\frac{dT}{d \left(\ln(t_{cool}) \right)} \right)^{-1} \tag{5}$$

 $t_{\rm cool} = t - t_{\rm heated}$, where $t_{\rm heated}$ is the length of time the sample was heated. In other words, for the cooling curve, reset t_0 .

Calculating q, heat flux per linear distance (W/m) from the given voltage:

$$\left(\frac{V^2}{R}\right) / l$$
 (6)

The "voltage" column is given in millivolts. $R=0.7902\Omega,$ and l=0.120m. V should be averaged over the heating curve portion of the experiment.