Cálculo Diferencial e Integral - Prof. Luciano O. Condori

Tabela de Trigonometria

1) $\operatorname{cossec}(x) = \frac{1}{\operatorname{sen}(x)}$

2) $\sec(x) = \frac{1}{\cos(x)}$

3) $\operatorname{tg}(x) = \frac{\operatorname{sen}(x)}{\cos(x)}$

4) $\operatorname{cotg}(x) = \frac{\cos(x)}{\operatorname{sen}(x)}$

5) $\operatorname{sen}^2(x) + \cos^2(x) = 1, \forall x \in \mathbb{R}$

6) $\sec^2(x) = 1 + \operatorname{tg}^2(x), \forall x \in \mathbb{R}$

7) $\operatorname{cosec}^2(x) = 1 + \operatorname{cotg}^2(x), \forall x \in \mathbb{R}$

8) $\operatorname{sen}(x \pm y) = \operatorname{sen}(x)\cos(y) \pm \cos(x)\operatorname{sen}(y), \forall x, y \in \mathbb{R}$

9) $\cos(x \pm y) = \cos(x)\cos(y) \mp \operatorname{sen}(x)\operatorname{sen}(y), \forall x, y \in \mathbb{R}$

10) $\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x), \forall x \in \mathbb{R}$

11) $\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x), \forall x \in \mathbb{R}$

12) $\operatorname{sen}(-x) = -\operatorname{sen}(x),\ \cos(-x) = \cos(x)$

Tabela de Derivadas

1) $\Big(c\Big)' = 0, c \in \mathbb{R}$

2) $\Big(x^n\Big)' = nx^{n-1}, x \in \mathbb{R}, n \in \mathbb{Q}$

3) $\Big(e^x\Big)' = e^x, x \in \mathbb{R}$

4) $\Big(\ln(x)\Big)' = \frac{1}{x}, x \in \mathbb{R}, x > 0$

5) $\Big(\operatorname{sen}(x)\Big)' = \cos(x), x \in \mathbb{R}$

6) $\Big(\cos(x)\Big)' = -\operatorname{sen}(x), x \in \mathbb{R}$

7) $\Big(\operatorname{tg}(x)\Big)' = \sec^2(x)$

8) $\Big(\operatorname{cotg}(x)\Big)' = -\operatorname{cossec}^2(x)$

9) $\Big(\sec(x)\Big)' = \sec(x)\operatorname{tg}(x)$

10) $\Big(\operatorname{cossec}(x)\Big)' = -\operatorname{cossec}(x)\operatorname{cotg}(x)$

11) $\Big(a^x\Big)' = a^x \ln(a), x \in \mathbb{R}$

12) $\Big(\log_a(x)\Big)' = \frac{1}{x}\frac{1}{\ln(a)}, x \in \mathbb{R}, x > 0$

Equação da Reta Tangente em $(x_0, f(x_0))$

$T_{x_0} : y - f(x_0) = f'(x_0)(x - x_0)$

Derivada da Função Composta ($u = f(x)$ e $v = g(x)$)

1) $\Big(u \pm v\Big)' = u' \pm v'$

2) $\Big(uv\Big)' = uv' + vu'$

3) $\Big(\frac{u}{v}\Big)' = \frac{vu' - uv'}{v^2}$

4) $\Big(cu\Big)' = cu'$

5) $\Big(u^n\Big)' = nu^{n-1}u', n \in \mathbb{Q}$

6) $\Big(\frac{1}{u}\Big)' = -\frac{u'}{u^2}$

7) $\Big(\ln(u)\Big)' = \frac{u'}{u}$

8) $\Big(e^u\Big)' = e^u u'$

9) $\Big(\sqrt{u}\Big)' = \frac{u'}{2\sqrt{u}}$

10) $\Big(\frac{1}{\sqrt{u}}\Big)' = -\frac{u'}{2u\sqrt{u}}$

11) $\Big(\sqrt[n]{u}\Big)' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$

12) $\Big((\frac{1}{u})^n\Big)' = -\frac{nu'}{u^{n+1}}$

13) $\Big(a^u\Big)' = a^u \ln(a) u'$

14) $\Big(\log_a(u)\Big)' = \frac{u'}{u}\frac{1}{\ln(a)}$

15) $\Big(\operatorname{sen}(u)\Big)' = \cos(u)u'$

16) $\Big(\cos(u)\Big)' = -\operatorname{sen}(u)u'$

17) $\Big(\operatorname{tg}(u)\Big)' = \sec^2(u)u'$

18) $\Big(\operatorname{cotg}(u)\Big)' = -\operatorname{cossec}^2(u)u'$

19) $\Big(\sec(u)\Big)' = \sec(u)\operatorname{tg}(u)u'$

20) $\Big(\operatorname{cossec}(u)\Big)' = -\operatorname{cossec}(u)\operatorname{cotg}(u)u'$

21) $\Big(\operatorname{arcsen}(u)\Big)' = \frac{u'}{\sqrt{1-u^2}}$

22) $\Big(\arccos(u)\Big)' = -\frac{u'}{\sqrt{1-u^2}}$

23) $\Big(\operatorname{arctg}(u)\Big)' = \frac{u'}{1+u^2}$

24) $\Big(\operatorname{arccotg}(u)\Big)' = -\frac{u'}{1+u^2}$

25) $\Big(\operatorname{arcsec}(u)\Big)' = \frac{u'}{u\sqrt{u^2-1}}$

26) $\Big(\operatorname{arccossec}(u)\Big)' = -\frac{u'}{u\sqrt{u^2-1}}$

27) $\Big(u^v\Big)' = vu^{v-1}u' + u^v v' \ln(u)$

Tabela 1: **Tabela de Derivação e Integração - Prof. Luciano O. Condori**

Cálculo Diferencial e Integral - Prof. Luciano O. Condori

Tabela de Potênciação e Logaritmo

1) $\log_a(N) = x \Leftrightarrow N = a^x$ 2) $\ln(N) = x \Leftrightarrow N = e^x$ 3) $\ln(MN) = \ln(M) + \ln(N)$

4) $\ln(\frac{M}{N}) = \ln(M) - \ln(N)$ 5) $\ln(M^x) = x\ln(M)$ 6) $\ln(1) = 0$ e $\ln(e) = 1$

Tabela de Integrais

1) $\int u^n du = \frac{u^{n+1}}{n+1}, n \neq -1, n \in \mathbb{Q}$	2) $\int \frac{1}{u} du = \ln\|u\|$
3) $\int \frac{1}{au+b} du = \frac{1}{a}\ln\|au+b\|$	4) $\int \frac{1}{u^2} du = -\frac{1}{u}$
5) $\int \frac{1}{\sqrt{u}} du = 2\sqrt{u}$	6) $\int \sqrt{u} du = \frac{2}{3}\sqrt{u^3}$
7) $\int e^u du = e^u$	8) $\int \ln(u) du = u(\ln\|u\| - 1)$
9) $\int a^u du = a^u \frac{1}{\ln(a)}$	10) $\int \log_a(u) du = \frac{1}{\ln(a)} \int \ln(u) du$
11) $\int \frac{1}{u^2+a^2} du = \frac{1}{a}\text{arctg}(\frac{u}{a})$	12) $\int \frac{1}{\sqrt{a^2-u^2}} du = \text{arcsen}(\frac{u}{a})$
13) $\int \frac{1}{u^2-a^2} du = \frac{1}{2a}\ln\|\frac{u-a}{u+a}\|$	14) $\int \sqrt{a^2-u^2} du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\text{arcsen}(\frac{u}{a})$
15) $\int \frac{1}{\sqrt{u^2 \pm a^2}} du = \ln\|u \pm \sqrt{u^2 \pm a^2}\|$	16) $\int \sqrt{u^2 \pm a^2} du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\|u + \sqrt{u^2 \pm a^2}\|$
17) $\int \text{sen}(u) du = -\cos(u)$	18) $\int \cos(u) du = \text{sen}(u)$
19) $\int \text{tg}(u) du = \ln\|\sec(u)\|$	20) $\int \text{cotg}(u) du = \ln\|\text{sen}(u)\|$
21) $\int \sec(u) du = \ln\|\sec(u) + \text{tg}(u)\|$	22) $\int \text{cossec}(u) du = \ln\|\text{cossec}(u) - \text{cotg}(u)\|$
23) $\int \text{sen}^2(u) du = \frac{u}{2} - \frac{\text{sen}(u)\cos(u)}{2}$	24) $\int \cos^2(u) du = \frac{u}{2} + \frac{\text{sen}(u)\cos(u)}{2}$
25) $\int \text{tg}^2(u) du = \text{tg}(u) - u$	26) $\int \text{cotg}^2(u) du = -\text{cotg}(u) - u$
27) $\int \sec^2(u) du = \text{tg}(u)$	28) $\int \text{cossec}^2(u) du = -\text{cotg}(u)$
29) $\int \sec(u)\text{tg}(u) du = \sec(u)$	30) $\int \text{cossec}(u)\text{cotg}(u) du = -\text{cossec}(u)$
31) $\int \text{arcsen}(u) du = u\ \text{arcsen}(u) + \sqrt{1-u^2}$	32) $\int \arccos(u) du = u\ \arccos(u) - \sqrt{1-u^2}$
33) $\int \text{arctg}(u) du = u\ \text{arctg}(u) - \frac{\ln\|1+u^2\|}{2}$	34) $\int \text{arccotg}(u) du = u\ \text{arccotg}(u) + \frac{\ln\|u^2+1\|}{2}$

Integração por partes ($u = f(x)$ e $v = g(x)$)

35) $\int u dv = uv - \int v du$

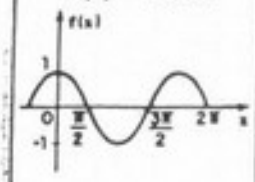
Integral definida

36) $\int_a^b f(u) du = F(u)\Big]_a^b = F(b) - F(a),\ F(u) = \int f(u) du$

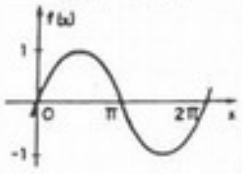
Tabela 2: **Tabela de Derivação e Integração - Prof. Luciano O. Condori**

1. GRÁFICOS, ÁREAS, PERÍMETROS E VOLUMES

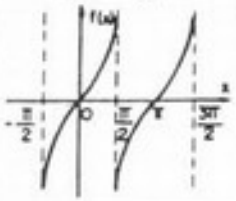
$f(x) = \cos x$



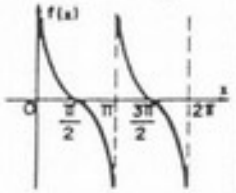
$f(x) = \operatorname{sen} x$



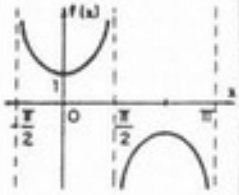
$f(x) = \operatorname{tg} x$



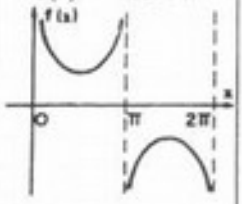
$f(x) = \operatorname{cotg} x$



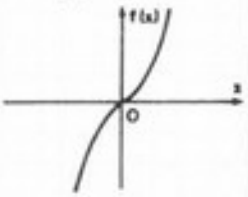
$f(x) = \sec x$



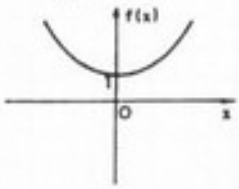
$f(x) = \operatorname{cosec} x$



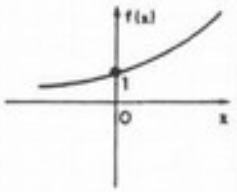
$f(x) = \operatorname{senh} x$



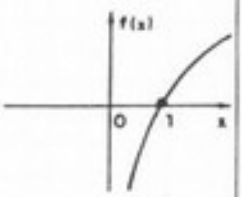
$f(x) = \cosh x$



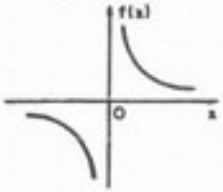
$f(x) = e^x$



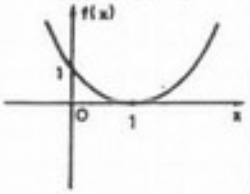
$f(x) = \ell nx$



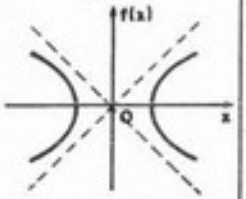
$f(x) = 1/x$



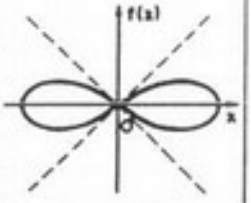
$f(x) = (x-1)^2$



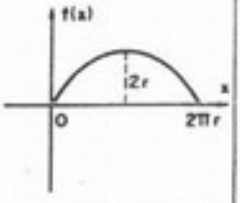
Hipérbole



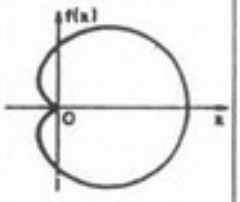
Leminiscata



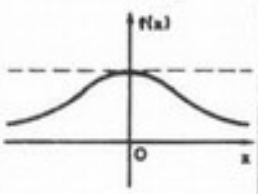
Ciclóide



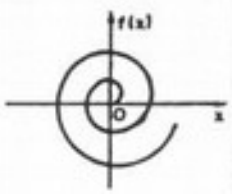
Cardióide



Curva de Agnesi



Espiral de Arquimedes



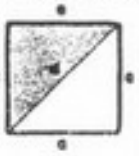
Círculo

Área = πr^2
Perímetro = $2\pi r$

Triângulo equilátero

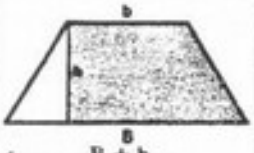
Área = $a^2\sqrt{3}/4$
Altura = $a\sqrt{3}/2$

Quadrado



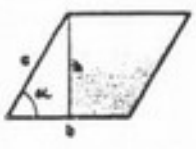
Área = a^2
Diagonal = $a\sqrt{2}$

Trapézio



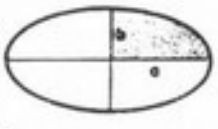
Área = $(\frac{B+b}{2})h$
h = altura

Losango



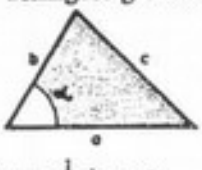
Área = bh = ab sen α
Perímetro = 2a + 2b

Elipse



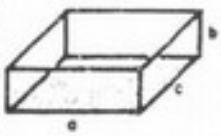
Área = πab
Perímetro $\cong \pi\sqrt{2(a^2+b^2)}$

Triângulo genérico



Área = $\frac{1}{2}$ ab sen α
Perímetro = a + b + c

Paralelepípedo



Volume = abc
Área superficial =
= 2(ab + ac + bc)

Cilindro

Volume = $\pi r^2 h$
Área lateral = $2\pi rh$

Cone

Volume = $\frac{1}{3}\pi r^2 h$
Área
lateral = $\pi r\sqrt{r^2+h^2}$

Esfera

Volume = $\frac{4}{3}\pi r^3$
Área superficial = $4\pi r^2$

Tronco de cone

Volume =
$= \frac{1}{3}\pi h(a^2+ab+b^2)$
Área lateral =
$= \pi(a+b)\sqrt{h^2+(b-a)^2}$

2. Símbolos Matemáticos

≠	diferente
=	igual
∪	contém
⊂	contido
!	fatorial
<	menor que
>	maior que
≤	menor ou igual
≥	maior ou igual
+	adição
−	subtração
÷	divisão
×	multiplicação
∝	proporcional
≅	aproximado
%	porcentagem
⇔	se e somente se
→	então
⇒	implicação
∃	existe
∄	não existe
∈	pertence
∉	não pertece
∀	qualquer
||	módulo
∴	portanto
⊥	ortogonal
∧	e
∨	ou
i	imaginário
Σ	somatória
Π	produtoria
∞	infinito
∪	união
∩	interseção
∇	operador nabla
Δ	diferença
∇^2	laplaciano
$\int$	integral
$\vec{A}$	vetor
$\vec{A} \cdot \vec{B}$	prod. escalar
$\vec{A} \times \vec{B}$	prod. vetorial
$\frac{d}{dx}$	derivada
lim	limite
Z	complexo
$\bar{Z}$	conjugado
C	complementar
/	tal que
R*	operador destrutivo
Γ	função gama
β	função beta

3. Constantes matemáticas

π ≅ 3,141593	$\log_{10} 3 \cong 0{,}477121$
π/2 ≅ 1,570796	$\log_{10} e \cong 0{,}434294$
π/3 ≅ 1,047198	$\log_{10} \pi \cong 0{,}497149$
π/4 ≅ 0,785398	$\ell n\ 2 \cong 0{,}693147$
1 rad ≅ 57,29578°	$\ell n\ 3 \cong 1{,}098612$
1° ≅ 0,017453 rad	$\gamma \cong 0{,}577215$
$e \cong 2{,}718282$	$e^{\gamma} \cong 1{,}781072$
$e^2 \cong 7{,}389056$	$\sqrt{e} \cong 1{,}648721$
$1/e \cong 0{,}367879$	$\sqrt{\pi} \cong 1{,}772453$
$\pi^e \cong 22{,}459157$	$\sqrt{2} \cong 1{,}414213$
$e^{\pi} \cong 15{,}154262$	$\sqrt{3} \cong 1{,}732050$
$e^{\pi} \cong 23{,}140692$	$\Gamma(1/3) \cong 2{,}678938$
$\log_{10} 2 \cong 0{,}301029$	$\Gamma(1/2) \cong \sqrt{\pi}$

4. Valores fundamentais da Trigonometria

α	0°	30°	45°	60°	90°
Seno	0	1/2	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Coseno	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	1/2	0
Tangente	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

5. Letras gregas

A α	Alfa	N ν	Ni
B β	Beta	Ξ ξ	Xi
Γ γ	Gama	O o	Omicron
Δ δ	Delta	Π π	Pi
E ε	Epsilon	P ρ	Rô
Z ζ	Zeta	Σ σ	Sigma
H η	Eta	T τ	Tau
Θ θ	Teta	Y υ	Ipsilon
I ι	Iota	Φ φ	Fi
K κ	Capa	X χ	Qui
Λ λ	Lambda	Ψ ψ	Psi
M μ	Mi	Ω ω	Ômega

6. Potenciação

$a^{x+y} = a^x . a^y \qquad (a^x)^y = a^{xy} \qquad (ab)^x = a^x b^x$

$\sqrt[n]{a.b} = \sqrt[n]{a}.\sqrt[n]{b} \qquad \sqrt[n]{a} = a^{1/n} \qquad \sqrt{-a} = i\sqrt{a}$

$a^{-x} = \frac{1}{a^x} \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}} \qquad \sqrt{-1} = i$

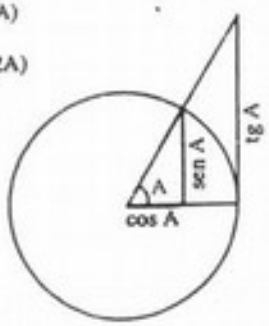
$x^2-y^2 = (x-y)(x+y)$
$x^3-y^3 = (x-y)(x^2+xy+y^2)$
$x^3+y^3 = (x+y)(x^2-xy+y^2)$
$(x+y)^2 = x^2+2xy+y^2$
$(x-y)^2 = x^2-2xy+y^2$
$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$
$(x-y)^3 = x^3-3x^2y+3xy^2-y^3$
$(x+y+z)^2 = x^2+y^2+z^2+2xy+2xz+2yz$
$(x+y-z)^2 = x^2+y^2+z^2+2xy-2xz-2yz$
$(x-y+z)^2 = x^2+y^2+z^2-2xy+2xz-2yz$

9. Logaritmos

$\log_c A.B = \log_c A + \log_c B$
$\log_c A/B = \log_c A - \log_c B$
$\log_c A^B = B\log_c A \qquad \log_c A = \log_b A/\log_b C$
$\log_e A = \ln A \qquad \log_{10} A = \log A$

10. Trigonometria

$\text{sen}^2B+\cos^2B = 1$
$\sec^2B = 1+\text{tg}^2B$
$\text{cosec}^2B = 1+\text{cotg}^2B$
$\cos(A+B) = \cos A\cos B - \text{sen } A \text{ sen } B$
$\cos(A-B) = \cos A\cos B + \text{sen } A \text{ sen } B$
$\text{sen}(A+B) = \text{sen } A\cos B + \cos A \text{ sen } B$
$\text{sen}(A-B) = \text{sen } A\cos B - \cos A \text{ sen } B$
$\text{sen } 2A = 2 \text{ sen } A \cos A$
$\cos 2A = \cos^2 A - \text{sen}^2 A$
$\cos A + \cos B = 2\,\text{sen}\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$
$\cos A - \cos B = -2\,\text{sen}\frac{1}{2}(A+B)\,\text{sen}\frac{1}{2}(A-B)$
$\text{sen } A + \text{sen } B = 2\,\text{sen}\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$
$\text{sen } A - \text{sen } B = 2\,\text{sen}\frac{1}{2}(A-B)\cos\frac{1}{2}(A+B)$
$\cos A\cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$
$\text{sen } A\cos B = \frac{1}{2}\{\text{sen}(A-B) + \text{sen}(A+B)\}$
$\text{sen } A \text{ sen } B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$
$\text{sen}(-A) = -\text{sen } A \text{ (impar)}$
$\cos(-A) = \cos A \text{ (par)}$
$\text{sen}^2A = \frac{1}{2}(1-\cos 2A)$
$\cos^2A = \frac{1}{2}(1+\cos 2A)$
$\text{tg } A = \frac{\text{sen } A}{\cos A}$
$\text{cotg } A = \frac{\cos A}{\text{sen } A}$
$\sec A = \frac{1}{\cos A}$
$\text{cosec } A = \frac{1}{\text{sen } A}$



$\text{tg}(A \pm B) = \frac{\text{tg } A \pm \text{tg } B}{1 \mp \text{tg } A \text{ tg } B}$
$\text{tg } 2A = \frac{2\text{tg } A}{1-\text{tg}^2 A}$
$\cos hA = \frac{1}{2}(e^A + e^{-A})$
$\text{sen } hA = \frac{1}{2}(e^A - e^{-A})$
$\cos^2 hA - \text{sen}^2 hA = 1$

7. Séries de Maclaurin

$f(x) = f(o) + \frac{f'(o)x}{1!} + \frac{f''(o)x^2}{2!} + \frac{f'''(o)x^3}{3!} + \ldots$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$a^x = 1 + x\ln a + \frac{(x\ln a)^2}{2!} + \frac{(x\ln a)^3}{3!} + \ldots$

$\text{sen}x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \; -1 < x \leq 1$

$\ln x = 2\{[\frac{x-1}{x+1}] + \frac{1}{3}[\frac{x-1}{x+1}]^3 + \frac{1}{5}[\frac{x-1}{x+1}]^5 + \ldots\} \; x > 0$

$\text{tg}x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots \qquad |x| < \frac{\pi}{2}$

$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \ldots \qquad |x| < \frac{\pi}{2}$

$\text{senh}x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$

$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$

8. Séries numéricas

$1 + 2 + 3 + 4 + \ldots + n = \frac{n^2+n}{2}$

$1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots = \frac{\pi}{4}$

$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots = \ln 2$

$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \frac{1}{9.11} + \ldots = \frac{1}{2}$

$\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \frac{1}{4.6} + \frac{1}{5.7} + \ldots = \frac{3}{4}$

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \ldots = \frac{\pi^2}{8}$

$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \frac{1}{9^4} + \ldots = \frac{\pi^4}{96}$

11. Limites fundamentais do cálculo

$\lim_{x\to 0}\frac{\text{sen}x}{x} = 1$	$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$	$\lim_{x\to 0}\frac{\text{tg}x}{x} = 1$
$\lim_{x\to\infty}(1+\frac{1}{x})^x = e$	$\lim_{x\to\infty}(1+\frac{y}{x})^x = e^y$	$\lim_{x\to\infty}\frac{a^x-1}{x} = \ln a$
$\lim_{x\to\infty}\sqrt[x]{x} = 1$	$\lim_{x\to 0}\frac{e^x-1}{x} = 1$	$\lim_{x\to 1}\frac{x-1}{\ln x} = 1$
$\lim_{x\to 3}\frac{x^2-9}{x-3} = 6$	$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} = \frac{1}{2}$	$\lim_{x\to\infty} x^x = \infty$

12. Transformação de coordenadas

Polar

$\begin{cases} x = \rho\cos\theta \\ y = \rho\text{sen}\,\theta \end{cases}$
$dx\,dy = \rho\,d\rho\,d\theta$

Esférica

$\begin{cases} x = \rho\,\text{sen}\,\phi\cos\theta \\ y = \rho\,\text{sen}\,\phi\,\text{sen}\,\theta \\ z = \rho\cos\phi \end{cases}$
$dx\,dy\,dz = \rho^2\,\text{sen}\,\phi\;d\theta\;d\phi\;d\rho$

Cilíndrica

$\begin{cases} x = \rho\cos\theta \\ y = \rho\text{sen}\,\theta \\ z = z \end{cases}$
$dx\,dy\,dz = \rho\,d\rho\,d\theta\,dz$

13. Derivadas de funções

$\frac{df}{dx} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

$\frac{df}{dx} = \frac{df}{dt}.\frac{dt}{dx}$

$df = (\frac{\partial f}{\partial x})dx + (\frac{\partial f}{\partial y})dy$

$\frac{d}{dx}[c] = 0$
$\frac{d}{dx}[x^n] = nx^{n-1}$
$\frac{d}{dx}[\frac{1}{x}] = -\frac{1}{x^2}$
$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$
$\frac{d}{dx}[e^{nx}] = ne^{nx}$
$\frac{d}{dx}[e^x] = e^x$
$\frac{d}{dx}[b^x] = b^x\ln b$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$
$\frac{d}{dx}[\ln ax] = \frac{1}{x}$
$\frac{d}{dx}[\log_b^x] = \frac{1}{x}\log_b^e$
$\frac{d}{dx}[\text{sen}x] = \cos x$
$\frac{d}{dx}[\text{sen } ax] = a\cos ax$
$\frac{d}{dx}[\cos x] = -\text{sen}x$
$\frac{d}{dx}[\cos ax] = -a\,\text{sen } ax$

$\frac{d}{dx}[\text{tg}x] = 1+\text{tg}^2x$
$\frac{d}{dx}[\text{cotg}x] = -1-\text{cotg}^2x$
$\frac{d}{dx}[\text{sen}^2x] = 2\text{sen}x\cos x$
$\frac{d}{dx}[\cos^2x] = -2\cos x\,\text{sen}x$
$\frac{d}{dx}[\cosh x] = \text{senh}x$
$\frac{d}{dx}[\text{senh}x] = \cosh x$
$\frac{d}{dx}[\text{arc cos}x] = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}[\text{arc sen}x] = \frac{1}{\sqrt{1-x^2}}$

14. Integrais

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad (n \neq -1)$$

$$\int ax\,dx = \frac{ax^2}{2}$$

$$\int \frac{1}{x} dx = \ln x$$

$$\int \frac{dx}{ax} = \frac{1}{a} \ln x$$

$$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b|$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b|$$

$$\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln\left|\frac{x}{ax+b}\right|$$

$$\int \frac{x^2 dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3} \ln|ax+b|$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

$$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln|ax+b|$$

$$\int \frac{x^2\,dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3} \ln|ax+b|$$

$$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2} \ln\left|\frac{x}{ax+b}\right|$$

$$\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3} \ln\left|\frac{x}{(ax+b)}\right|$$

$x^2 \pm a^2$

$$\int \frac{x^3 dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2} \ln|x^2+a^2|$$

$$\int \frac{x^3 dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2} \ln|x^2-a^2|$$

$$\int \frac{x^2 dx}{x^2+a^2} = x - a \arctg \frac{x}{a}$$

$$\int \frac{x^2 dx}{x^2-a^2} = x + \frac{a}{2} \ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2} \ln|x^2+a^2|$$

$$\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2} \ln|x^2-a^2|$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctg \frac{x}{a}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2} \ln\left|\frac{x^2-a^2}{x^2}\right|$$

$$\int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2} \ln\left|\frac{x^2}{x^2+a^2}\right|$$

$$\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2 x} + \frac{1}{2a^3} \ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{dx}{x^2(x^2+a^2)} = -\frac{1}{a^2 x} - \frac{1}{a^3} \arctg \frac{x}{a}$$

$$\int \frac{x^2 dx}{(x^2+a^2)^2} = \frac{-x}{2(x^2+a^2)} + \frac{1}{2a} \arctg \frac{x}{a}$$

$$\int \frac{x\,dx}{(x^2+a^2)^2} = \frac{-1}{2(x^2+a^2)}$$

$$\int \frac{x\,dx}{(x^2-a^2)^2} = \frac{-1}{2(x^2-a^2)}$$

$\sqrt{a+bx}$

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b}(a+bx)^{3/2}$$

$$\int x\sqrt{a+bx}\,dx = \frac{2}{15b^2}(3bx-2a)(a+bx)^{3/2}$$

$$\int \frac{x^2 dx}{\sqrt{a+bx}} = \frac{2}{15b^3}(3b^2x^2 - 4abx + 8a^2)\sqrt{a+bx}$$

$$\int \frac{x\,dx}{\sqrt{a+bx}} = \frac{2}{3b^2}(bx-2a)\sqrt{a+bx}$$

$$\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$$

$\sqrt{a^2-x^2}$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}(a^2-x^2)^{3/2}$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1} \frac{x}{a}$$

$$\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

$$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2 x}$$

$$\int \frac{x^2}{\sqrt{a^2-x^2}} dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{x}{a}$$

$$\int \frac{\sqrt{a^2-x^2}}{x^2} dx = -\frac{\sqrt{a^2-x^2}}{x} - \operatorname{sen}^{-1} \frac{x}{a}$$

$$\int \frac{\sqrt{a^2-x^2}}{x^2} dx = \sqrt{a^2-x^2} - a \ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$$

$$\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

$\sqrt{x^2 \pm a^2}$

$$\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{x^2\,dx}{\sqrt{x^2 \pm a^2}} = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{\sqrt{x^2-a^2}}{x} dx = \sqrt{x^2-a^2} - a \sec^{-1}\left|\frac{x}{a}\right|$$

$$\int \frac{\sqrt{x^2+a^2}}{x} dx = \sqrt{x^2+a^2} - a \ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$$

$$\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right|$$

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a} \ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$$

tg x, cotg x, sec x, cosec x

$$\int \operatorname{sen} ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\operatorname{sen} ax$$

$$\int \operatorname{sen}^2 ax\,dx = \frac{x}{2} - \frac{\operatorname{sen} 2ax}{4a}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a}$$

$$\int \operatorname{sen} ax \operatorname{sen} bx\,dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} - \frac{\operatorname{sen}(a+b)x}{2(a+b)} \quad (a \neq \pm b)$$

$$\int \operatorname{sen} ax \cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} \quad (a \neq \pm b)$$

$$\int \cos ax \cos bx\,dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} + \frac{\operatorname{sen}(a+b)x}{2(a+b)} \quad (a \neq \pm b)$$

$$\int x \operatorname{sen} ax\,dx = \frac{\operatorname{sen} ax}{a^2} - \frac{x\cos ax}{a}$$

$$\int x\cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x \operatorname{sen} ax}{a}$$

$$\int \operatorname{sen} ax \cos ax\,dx = -\frac{\cos 2ax}{4a}$$

$$\int \frac{\cos ax}{\operatorname{sen} ax} dx = \frac{1}{a} \ln(\operatorname{sen} ax)$$

$$\int \frac{\operatorname{sen} ax}{\cos ax} dx = -\frac{1}{a} \ln(\cos ax)$$

$$\int \frac{dx}{1-\operatorname{sen} ax} = \frac{1}{a} \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$$

$$\int \frac{dx}{1+\operatorname{sen} ax} = -\frac{1}{a} \operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right)$$

$$\int \frac{dx}{1+\cos ax} = \frac{1}{a} \operatorname{tg} \frac{ax}{2}$$

$$\int \frac{dx}{1-\cos ax} = -\frac{1}{a} \operatorname{cotg} \frac{ax}{2}$$

$$\int \operatorname{sen}^{-1} ax\,dx = x \operatorname{sen}^{-1} ax + \frac{1}{a}\sqrt{1-a^2x^2}$$

$$\int \cos^{-1} ax\,dx = x\cos^{-1} ax - \frac{1}{a}\sqrt{1-a^2x^2}$$

$$\int \operatorname{tg} ax\,dx = -\frac{1}{a} \ln|\cos ax|$$

$$\int \operatorname{tg}^n ax \sec^2 ax\,dx = \frac{\operatorname{tg}^{n+1} ax}{(n+1)a}$$

$$\int \operatorname{tg}^3 ax\,dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a} \ln\cos ax$$

$$\int \operatorname{tg}^2 ax\,dx = \frac{\operatorname{tg} ax}{a} - x$$

$$\int x \operatorname{tg}^2 ax\,dx = \frac{x \operatorname{tg} ax}{a} + \frac{1}{a^2} \ln\cos ax - \frac{x^2}{2}$$

$$\int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a} \ln \operatorname{sen} ax$$

$$\int \frac{\sec^2 ax}{\operatorname{tg} ax}\,dx = \frac{1}{a}\ln \operatorname{tg} ax$$

$$\int \operatorname{cotg}^3 ax\,dx = -\frac{\operatorname{cotg}^2 ax}{2a} - \frac{1}{a}\ln \operatorname{sen} ax$$

$$\int \operatorname{cotg}^2 ax\,dx = -\frac{\operatorname{cotg}^2 ax}{a} - x$$

$$\int \operatorname{cotg} ax\,dx = \frac{1}{a}\ln \operatorname{sen} ax$$

$$\int x \operatorname{cotg}^2 ax\,dx = -\frac{x \operatorname{cotg} ax}{a} + \frac{1}{a^2}\ln \operatorname{sen} ax - \frac{x^2}{2}$$

$$\int \frac{dx}{\operatorname{cotg} ax} = -\frac{1}{a}\ln \cos ax$$

$$\int \sec^3 ax\,dx = \frac{\sec ax \operatorname{tg} ax}{2a} + \frac{1}{2a}\ln(\sec ax + \operatorname{tg} ax)$$

$$\int \sec^2 ax\,dx = \frac{\operatorname{tg} ax}{a}$$

$$\int \sec ax\,dx = \frac{1}{a}\ln(\sec ax + \operatorname{tg} ax)$$

$$\int x \sec^2 ax\,dx = \frac{x}{a}\operatorname{tg} ax + \frac{1}{a^2}\ln \cos ax$$

$$\int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a}$$

$$\int \operatorname{cosec}^2 ax\,dx = -\frac{\operatorname{cotg} ax}{a}$$

$$\int \operatorname{cosec} ax\,dx = \frac{1}{a}\ln \operatorname{tg} \frac{ax}{2}$$

$$\int \frac{dx}{\operatorname{cosec} ax} = -\frac{\operatorname{cosec} ax}{a}$$

$$\int x \operatorname{cosec}^2 ax\,dx = -\frac{x \operatorname{cotg} ax}{a} + \frac{1}{a^2}\ln \operatorname{sen} ax$$

$$\int \operatorname{tg}^{-1} ax\,dx = x \operatorname{tg}^{-1} ax - \frac{1}{2a}\ln(1+a^2x^2)$$

Exp. e log.

$$\int e^{bx}\,dx = \frac{e^{bx}}{b}$$

$$\int x^2 e^{bx}\,dx = \frac{e^{bx}}{b}\left(x^2 - \frac{2x}{b} + \frac{2}{b^2}\right)$$

$$\int x e^{bx}\,dx = \frac{e^{bx}}{b}\left(x - \frac{1}{b}\right)$$

$$\int \frac{e^{bx}}{x}\,dx = \ln x + \frac{bx}{1.1!} + \frac{(bx)^2}{2.2!} + \frac{(bx)^3}{3.3!} + \frac{(bx)^4}{4.4!} + \cdots$$

$$\int \frac{dx}{a+be^{cx}} = \frac{x}{a} - \frac{1}{ca}\ln(a+be^{cx})$$

$$\int e^{ax}\operatorname{sen} bx\,dx = \frac{e^{ax}(a \operatorname{sen} bx - b\cos bx)}{a^2+b^2}$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b \operatorname{sen} bx)}{a^2+b^2}$$

$$\int x \ln x\,dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right)$$

$$\int \ln x\,dx = x\ln x - x$$

$$\int \ln ax\,dx = x\ln ax - x$$

$$\int \ln^2 x\,dx = x\ln^2 x - 2x\ln x + 2x$$

$$\int x^n \ln ax\,dx = \frac{x^{n+1}\ln ax}{n+1} - \frac{x^{n+1}}{(n+1)^2} \quad (n \neq -1)$$

$$\int \frac{dx}{x \ln nx} = \ln(\ln nx)$$

$$\int \frac{\ln nx}{x^2}\,dx = -\frac{\ln nx}{x} - \frac{1}{x}$$

$$\int \frac{\ln nx\,dx}{x} = \frac{1}{2}\ln^2 x$$

$$\int \frac{\ln^a x\,dx}{x} = \frac{\ln^{a+1} x}{a+1} \quad (a \neq -1)$$

$$\int \frac{(\ln bx)^a}{x}\,dx = \frac{(\ln bx)^{a+1}}{a+1} \quad (a \neq -1)$$

$$\int \ln(a^2+x^2)dx = x\ln(a^2+x^2) - 2x + 2a \operatorname{arc tg} \frac{x}{a}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b \operatorname{sen} bx)$$

$$\int e^{ax}\operatorname{sen} bx\,dx = \frac{e^{ax}}{a^2+b^2}(a \operatorname{sen} bx - b\cos bx)$$

Hiperbólicas

$$\int \cosh ax\,dx = \frac{1}{a}\operatorname{senh} ax$$

$$\int \operatorname{senh} ax\,dx = \frac{1}{a}\cosh ax$$

$$\int \cosh^2 ax\,dx = \frac{\operatorname{senh} 2ax}{4a} + \frac{x}{2}$$

$$\int \operatorname{senh}^2 ax\,dx = \frac{\operatorname{senh} 2ax}{4a} + \frac{x}{2}$$

$$\int x\cosh ax\,dx = \frac{x}{a}\operatorname{senh} ax - \frac{1}{a^2}\cosh ax$$

$$\int x \operatorname{senh} ax\,dx = \frac{x}{a}\cosh ax - \frac{1}{a^2}\operatorname{senh} ax$$

$$\int \operatorname{tgh} ax\,dx = \frac{1}{a}\ln(\cosh ax)$$

$$\int \operatorname{tgh}^2 ax\,dx = -\frac{1}{a}\operatorname{tg} ax + x$$

$$\int \operatorname{cotgh} ax\,dx = \frac{1}{a}\ln|\operatorname{senh} ax|$$

$$\int \operatorname{cotgh}^2 ax\,dx = x - \frac{1}{a}\operatorname{cotgh} ax$$

$$\int \operatorname{sech} ax\,dx = \frac{1}{a}\operatorname{sen}^{-1}(\operatorname{tgh} ax)$$

$$\int \operatorname{sech}^2 ax\,dx = \frac{1}{a}\operatorname{tgh} ax$$

$$\int x^2 \operatorname{senh} ax\,dx = \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\cosh ax - \frac{2x}{a^2}\operatorname{senh} ax$$

$$\int x \operatorname{senh}^2 ax\,dx = \frac{x \operatorname{senh} 2ax}{4a} - \frac{\cosh 2ax}{8a^2} - \frac{x^2}{4}$$

$$\int x\cosh^2 ax\,dx = \frac{x \operatorname{senh} 2ax}{4a} - \frac{\cosh ax}{8a^2}$$

$$\int x^2 \cosh ax\,dx = -\frac{2x\cosh 2ax}{a^2} + \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\operatorname{senh} ax$$

$$\int \frac{dx}{\operatorname{senh} ax} = \frac{1}{a}\ln \operatorname{tgh} \frac{ax}{2}$$

$$\int \frac{dx}{\operatorname{senh}^2 ax} = -\frac{\operatorname{cotgh} ax}{a}$$

$$\int \frac{dx}{\cosh ax} = \frac{2}{a}\operatorname{arc tg} e^{ax}$$

$$\int \operatorname{senh} ax \cosh bx\,dx = \frac{\cosh(a+b)x}{2(a+b)} + \frac{\cosh(a-b)x}{2(a-b)}$$

$$\int \operatorname{senh} ax \cosh ax\,dx = \frac{\operatorname{senh}^2 ax}{2a}$$

$$\int e^{ax}\cosh bx\,dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b}\right] \qquad a^2 \neq b^2$$

$$\int e^{ax}\operatorname{senh} bx\,dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b}\right] \qquad a^2 \neq b^2$$

15. Propriedades ∫

$$\int_a^b u\,v' = uv\Big|_a^b - \int_a^b uv'$$

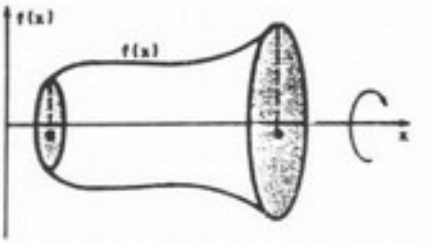
$$\int_a^b f(x)dx = -\int_b^a f(x)\,dx$$

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\cdot f'(x)dx = \frac{1}{2}[f(x)]^2\Big|_a^b$$

$$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

16. Sólido de revolução

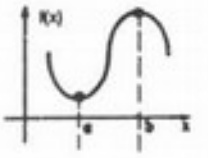


$$\text{Volume} = \int_a^b \pi[f(x)]^2\,dx$$

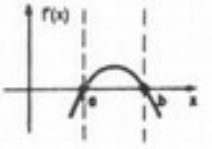
$$\text{Comprimento} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$\text{Área lateral} = \int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

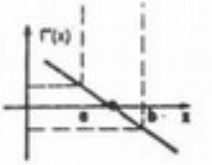
17. Máximo e mínimo



a: mínimo
b: máximo



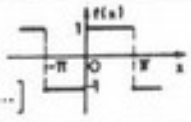
f'(a) = 0 : mínimo
f'(b) = 0 : máximo



f''(a) > 0 : mínimo
f''(b) < 0 : máximo

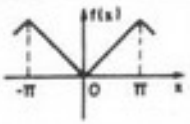
18. Séries de Fourier

A) $f(x) = \begin{cases} 1 \text{ se } 0<x<\pi \\ -1 \text{ se } -\pi<x<0 \end{cases}$



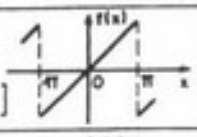
$$f(x) = \frac{4}{\pi}\cdot\left[\frac{\operatorname{sen}1x}{1} + \frac{\operatorname{sen}3x}{3} + \frac{\operatorname{sen}5x}{5} + \ldots\right]$$

B) $f(x) = \begin{cases} x \text{ se } 0<x<\pi \\ -x \text{ se } -\pi<x<0 \end{cases}$



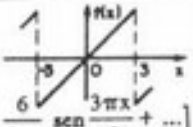
$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\cdot\left[\frac{\cos 1x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right]$$

C) $f(x) = x$ para $-\pi<x<\pi$



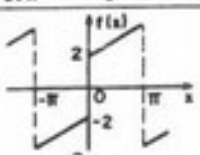
$$f(x) = 2\left[\frac{\operatorname{sen}1x}{1} - \frac{\operatorname{sen}2x}{2} + \frac{\operatorname{sen}3x}{3} - \ldots\right]$$

D) $f(x) = x$ para $-3<x<3$



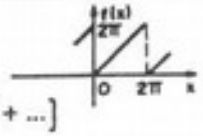
$$f(x) = \left[\frac{6}{1.\pi}\operatorname{sen}\frac{\pi x}{3} - \frac{6}{2.\pi}\operatorname{sen}\frac{2\pi x}{3} + \frac{6}{3.\pi}\operatorname{sen}\frac{3\pi x}{3} + \ldots\right]$$

E) $f(x) = \begin{cases} x+2 \text{ se } 0<x<\pi \\ x-2 \text{ se } -\pi<x<0 \end{cases}$



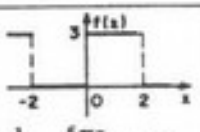
$$f(x) = \left[\frac{8+2\pi}{\pi}\right]\operatorname{sen}x - \frac{2}{2}\operatorname{sen}2x + \left[\frac{8+2\pi}{3\pi}\right]\operatorname{sen}3x - \frac{2}{4}\operatorname{sen}4x + \ldots$$

F) $f(x) = x$ para $0<x<2\pi$



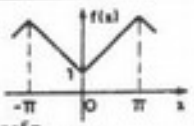
$$f(x) = \pi - 2\cdot\left[\frac{\operatorname{sen}x}{1} + \frac{\operatorname{sen}2x}{2} + \frac{\operatorname{sen}3x}{3} + \ldots\right]$$

G) $f(x) = \begin{cases} 0 \text{ se } -2<x<0 \\ 3 \text{ se } 0<x<2 \end{cases}$



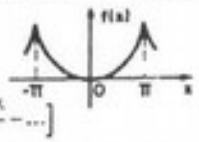
$$f(x) = \frac{3}{2} + \frac{6}{\pi}\cdot\left[\operatorname{sen}\frac{\pi x}{2} + \frac{1}{3}\operatorname{sen}\frac{3\pi x}{5} + \frac{1}{5}\operatorname{sen}\frac{5\pi x}{2} + \ldots\right]$$

H) $f(x) = \begin{cases} x+1 \text{ se } 0<x<\pi \\ -x+1 \text{ se } -\pi<x<0 \end{cases}$



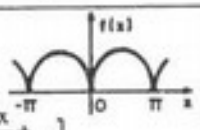
$$f(x) = \frac{\pi}{2} + 1 - \frac{4}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right]$$

I) $f(x) = x^2$ para $-\pi<x<\pi$



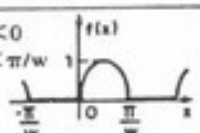
$$f(x) = \frac{\pi^2}{3} - 4\cdot\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{2^2} - \ldots\right]$$

J) $f(x) = |\operatorname{sen}x|$ para $-\pi<x<\pi$



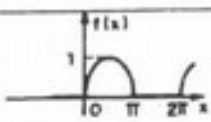
$$f(x) = \frac{2}{\pi} - \frac{4}{\pi}\left[\frac{\cos 2x}{1.3} + \frac{\cos 4x}{3.5} + \frac{\cos 6x}{5.7} + \ldots\right]$$

L) $f(x) = \begin{cases} 0 & \text{se } -\pi/w<x<0 \\ \operatorname{sen}wx & \text{se } 0<x<\pi/w \end{cases}$



$$f(x) = \frac{1}{\pi} + \frac{1}{2}\operatorname{sen}wx - \frac{2}{\pi}\cdot\left[\frac{1}{1.3}\cos 2wx + \frac{1}{3.5}\cos 4wx + \ldots\right]$$

M) $f(x) = \begin{cases} \operatorname{sen}x & \text{se } 0<x<\pi \\ 0 & \text{se } \pi<x<2\pi \end{cases}$



$$f(x) = \frac{1}{\pi} + \frac{1}{2}\operatorname{sen}x - \frac{2}{\pi}\cdot\left[\frac{\cos 2x}{1.3} + \frac{\cos 4x}{3.5} + \ldots\right]$$

19. Transformada de Laplace

$f_1(x) = 1$	$\to \mathcal{L}[f_1(x)] = \frac{1}{s}$	$f_{12}(x) = \cos ax$	$\to \mathcal{L}[f_{12}(x)] = \frac{s}{s^2+a^2}$
$f_2(x) = x$	$\to \mathcal{L}[f_2(x)] = \frac{1}{s^2}$	$f_{13}(x) = e^{bx}.\cos ax$	$\to \mathcal{L}[f_{13}(x)] = \frac{s-b}{(s-b)^2+a^2}$
$f_3(x) = x^2$	$\to \mathcal{L}[f_3(x)] = \frac{2}{s^3}$	$f_{14}(x) = x\cos ax$	$\to \mathcal{L}[f_{14}(x)] = \frac{s^2-a^2}{(s^2+a^2)^2}$
$f_4(x) = x^n$	$\to \mathcal{L}[f_4(x)] = \frac{n!}{s^{n+1}}$	$f_{15}(x) = 1-\cos ax$	$\to \mathcal{L}[f_{15}(x)] = \frac{a^2}{s(s^2+a^2)}$
$f_5(x) = e^{ax}$	$\to \mathcal{L}[f_5(x)] = \frac{1}{s-a}$	$f_{16}(x) = e^{-ax}\cos bx$	$\to \mathcal{L}[f_{16}(x)] = \frac{s+a}{(s+a)^2+b^2}$
$f_6(x) = xe^{ax}$	$\to \mathcal{L}[f_6(x)] = \frac{1}{(s-a)^2}$	$f_{17}(x) = \cosh ax$	$\to \mathcal{L}[f_{17}(x)] = \frac{s}{s^2-a^2}$
$f_7(x) = x^n e^{-ax}$	$\to \mathcal{L}[f_7(x)] = \frac{n!}{(s+a)^{n+1}}$	$f_{18}(x) = x\cosh ax$	$\to \mathcal{L}[f_{18}(x)] = \frac{s^2+a^2}{(s^2-a^2)^2}$
$f_8(x) = \operatorname{sen}ax$	$\to \mathcal{L}[f_8(x)] = \frac{a}{s^2+a^2}$	$f_{19}(x) = \frac{x^2}{2}\cosh ax$	$\to \mathcal{L}[f_{19}(x)] = \frac{s^3+3a^2s}{(s^2-a^2)^3}$
$f_9(x) = x\operatorname{sen}ax$	$\to \mathcal{L}[f_9(x)] = \frac{2as}{(s^2+a^2)^2}$	$f_{20}(x) = \operatorname{senh}ax$	$\to \mathcal{L}[f_{20}(x)] = \frac{a}{s^2-a^2}$
$f_{10}(x) = \frac{e^{ax}\operatorname{sen}bx}{b}$	$\to \mathcal{L}[f_{10}(x)] = \frac{1}{(s-a)^2+b^2}$	$f_{21}(x) = \frac{x\operatorname{senh}ax}{2a}$	$\to \mathcal{L}[f_{21}(x)] = \frac{s}{(s^2-a^2)}$
$f_{11}(x) = \frac{e^{ax}-e^{bx}}{a-b}$	$\to \mathcal{L}[f_{11}(x)] = \frac{1}{(s-a)(s-b)}$	$f_{22}(x) = \ln x$	$\to \mathcal{L}[f_{22}(x)] = \frac{-\gamma-\ln s}{s}$

20. Geometria Analítica

- **Triângulo retângulo**

$a^2 = b^2 + c^2$ $\quad c^2 = a.m$

$h^2 = m.n$ $\quad b.c = a.h$

$b^2 = a.n$ $\quad b.m = c.h$

$c.n = b.h$

- **Triângulo qualquer**

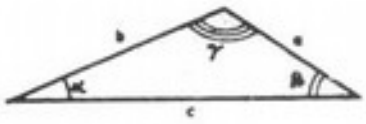
$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$$\frac{a}{\operatorname{sen}\alpha} = \frac{b}{\operatorname{sen}\beta} = \frac{c}{\operatorname{sen}\gamma}$$

$$\frac{a+b}{a-b} = \frac{\operatorname{tg}1/2(\alpha+\beta)}{\operatorname{tg}1/2(\alpha-\beta)}$$



- **Reta**

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Distância entre dois pontos

$m = \operatorname{tg}\alpha = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$ Coeficiente angular

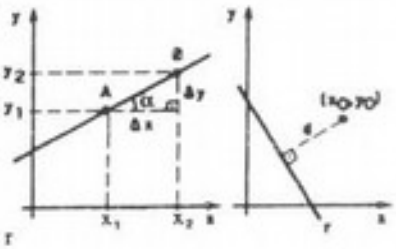
$y-y_1 = m(x-x_1)$ Equação da reta para 2 pontos

$m_r = m_s$ As retas r e s são paralelas

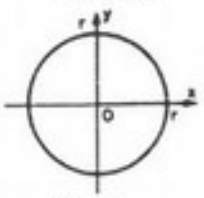
$m_r \cdot m_s = -1$ As retas r e s são perpendiculares

$ax+by+c=0$ Equação geral da reta

$d = \frac{|axo+byo+c|}{\sqrt{a^2+b^2}}$ Distância de um ponto (xo, yo) à reta r

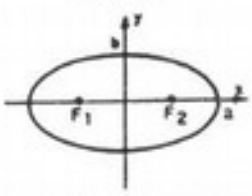


- **Círculo**



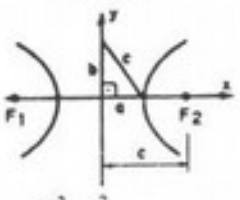
$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

- **Elipse**



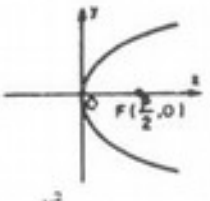
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- **Hipérbole**



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- **Parábola**



$$\frac{y^2}{2p} = x$$

21. Progressões

- **Aritmética** – Seqüência de números $a_1, a_2, a_3, \ldots a_n, a_n+1, \ldots$ cujo termo a_n+1 é igual a a_n somado de uma constante r chamada razão

- $a_n = a_1 + (n-1)r$ Termo geral • $S_n = (\frac{a_1+a_n}{2})n$ Soma de n termos

- **Geométrica** – Seqüência de números $a_1, a_2, a_3, \ldots, a_n, a_n+1, \ldots$ cujo termo a_n+1 é igual a a_n multiplicado por uma constante q chamada razão.

- $a_n = a_1 \cdot q^{n-1}$ Termo geral • $S_n = \frac{a_1 q^n - a_1}{q-1}$ Soma de n termos

22. Constantes fundamentais

Velocidade do som no ar (°C)	s	$331{,}36\ ms^{-1}$
Velocidade da luz no vácuo	c	$2{,}997 \times 10^{8}$ m/s
Carga elementar	e	$1{,}602 \times 10^{-19}$ c
Massa de repouso – elétron	m_e	$9{,}109 \times 10^{-31}$ kg
Massa de repouso – próton	m_p	$1{,}672 \times 10^{-27}$ kg
Massa de repouso – nêutron	m_n	$1{,}674 \times 10^{-27}$ kg
Permissividade do vácuo	ϵ_o	$8{,}854 \times 10^{-12} c^2\ N^{-1}/m^2$
Permeabilidade do vácuo	μ_o	$4\pi \times 10^{-7}$ Tm/A
Constante de Planck	h	$6{,}626 \times 10^{-34}$ j.s
Constante de Avogrado	N	$6{,}022 \times 10^{23}\ mol^{-1}$
Constante estrutura fina	α	$7{,}297 \times 10^{-3}$
Constante de gases	R	$8{,}314\ jk^{-1}\ mol^{-1}$
Constante de Boltzmann	K	$1{,}380 \times 10^{-23}\ jk^{-1}$
Constante Stefan-Boltzmann	σ	$5{,}669 \times 10^{-8}\ wm^{-2}/x^4$
Constante gravitacional	G	$6{,}673 \times 10^{-11}\ Nm^2/kg^2$
Massa da Terra	M_t	$5{,}977 \times 10^{24}$ kg
Massa da Lua	M_t	$73{,}8 \times 10^{21}$ kg
Massa do Sol	M_s	$1{,}989 \times 10^{30}$ kg
Raio da Terra	R_t	6.380 km
Raio da Lua	R_t	1.738 km
Raio do Sol	R_s	$6{,}959 \times 10^{5}$ km
Distância média Sol-Terra	ST	$149{,}6 \times 10^{9}$ m
Distância média Lua-Terra	LT	$3{,}844 \times 10^{8}$ m
Aceleração da gravidade - Lua	$\mathcal{L}$	$1{,}621\ ms^{-2}$
Aceleração da gravidade - Sol	S	$273{,}2\ ms^{-2}$
Aceleração da gravidade - Terra	g	$9{,}806\ ms^{-2}$
Massa específica média da Terra	d	$5{,}522 \times 10^{3}$ kg/m³
Velocidade angular da Terra	w	$7{,}29 \times 10^{-5}$ rad/s

23. Dimensões

Massa	kg	Potência (w)	kgm^2/s^3	Diferença de potencial (V)	j/c
Comprimento	m	Pressão (Pa)	N/m^2	Resistência elétrica	Ω
Área	m^2	Fluxo de massa	kg/s	Capacitância (F)	c/v
Volume	m^3	Vazão	m^3/s	Campo magnético	T
Tempo	s	Massa específica	kg/m^3	Campo elétrico	N/c
Velocidade	m/s	Momento de inércia	kgm^2	Condutância (s)	A/V
Aceleração	m/s^2	Freqüência (Hz)	1/s	Indutância (H)	vs^2/c
Força (N)	kgm/s^2	Carga elétrica	c	Capacidade térmica	cal/°C
Energia (j)	kgm^2/s^2	Corrente elétrica (A)	c/s	Calor específico	cal/g°C

24. Relação das escalas de temperatura

$\frac{T_C}{5} = \frac{T_F - 32}{9}$	$\frac{T_C}{5} = \frac{T_R}{4}$	$T_K = T_C + 273$	$\frac{T_F - 32}{9} = \frac{T_R}{4}$
Celsius-Fahrenheit	Celsius-Reaumur	Kelvin-Celsius	Fahrenheit-Reaumur

Material	Temperatura de fusão (T)	Massa específica (ρ)	Condutividade térmica (K)	Calor específico (c)
	°C	kg/m³	j/m°C	kj/kg°C
Alumínio	660	2.706	203	0,897
Aço	1.545	7.852	55	0,465
Ferro	1.532	7.898	72	0,451
Cobre	1.080	8.953	385	0,384
Bismuto	272	9.781	7,85	0,123
Magnésio	648	1.750	171	1,014
Níquel	1.454	8.906	91	0,445
Prata	962	10.523	420	0,234
Tungstênio	3.388	19.348	64	0,134
Zinco	420	7.145	113	0,383
Urânio	1.130	19.050	28	0,117
Molibdênio	2.620	10.230	123	0,250

25. Conversão de unidades

Pascal	Pa	$1\ N/m^2$
Atmosfera	atm	$1{,}013 \times 10^{5}$ Pa
Bar	bar	10^{5} Pa
Caloria	cal	4,184 j
Cavalo-vapor	cv	735,5 w
Calor inglês	Btu	1.055 j
HP	HP	745,702 j/s
Roentgen	R	$2{,}58 \times 10^{-4}$ c/kg
Angstrom	Å	10^{-10} m
Quilograma-força	kgf	9,807 N
Metro	m	10^{2} cm
Quilômetro	km	10^{3} m
Hectare	ha	$10^{4}\ m^2$
Acre	Ac	0,404 ha
Polegada	in	2,54 cm
Pé	ft	30,481 cm
Jarda	yd	91,440 cm
Milha	mi	1.609 m
Milha marítima	mi	1.853 m
Légua	lc	4,82 km
Légua marítima	lc	5,59 km
Quilo	kg	10^{3} g
Libra	lb	0,4536 kg
Onça	oz	28,35 g

26. Óptica

c: vel. da luz no vácuo

n: índice de refração

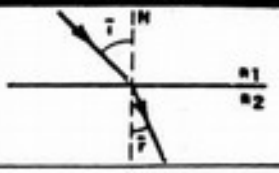
v: vel. da luz no meio

$$n = \frac{c}{v}$$

Reflexão

$$i = r$$

Refração



$$\frac{\text{sen } i}{\text{sen } r} = \frac{n_2}{n_1}$$

27. Mudanças de estado

28. Gás perfeito

Estado inicial	Estado final	
P_1, V_1, T_1	P_2, V_2, T_2	$PV = NRT$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$n = \frac{m}{M}$$

m: massa
M: mol

29. Tabela periódica

1 H 1,00																	2 He 4,00
3 Li 6,93	4 Be 9,01											5 B 10,81	6 C 12,01	7 N 14,00	8 O 15,99	9 F 18,99	10 Ne 20,18
11 Na 22,99	12 Mg 24,31											13 Al 26,98	14 Si 28,08	15 P 30,97	16 S 32,06	17 Cl 35,45	18 Ar 39,94
19 K 39,10	20 Ca 40,08	21 Sc 44,95	22 Ti 47,90	23 V 50,94	24 Cr 51,99	25 Mn 54,93	26 Fe 55,84	27 Co 58,93	28 Ni 58,71	29 Cu 63,54	30 Zn 65,37	31 Ga 69,72	32 Ge 72,59	33 As 74,92	34 Se 78,96	35 Br 79,90	36 Kr 83,80
37 Rb 85,47	38 Sr 87,62	39 Y 88,90	40 Zr 91,22	41 Nb 92,90	42 Mo 95,94	43 Tc 99	44 Ru 101,07	45 Rh 102,91	46 Pd 106,4	47 Ag 107,87	48 Cd 112,40	49 In 114,82	50 Sn 118,69	51 Sb 121,75	52 Te 127,60	53 I 126,90	54 Xe 131,30
55 Cs 132,91	56 Ba 137,34	57 La 138,91	72 Hf 178,49	73 Ta 180,95	74 W 183,85	75 Re 186,2	76 Os 190,2	77 Ir 192,2	78 Pt 195,09	79 Au 196,97	80 Hg 200,59	81 Tl 204,37	82 Pb 207,19	83 Bi 208,98	84 Po 210	85 At 210	86 Rn 222
87 Fr 223	88 Ra 226	89 Ac 227															

58 Ce 140,12	59 Pr 140,91	60 Nd 144,24	61 Pm 145	62 Sm 150,35	63 Eu 151,96	64 Gd 157,25	65 Tb 158,92	66 Dy 162,50	67 Ho 164,93	68 Er 167,26	69 Tm 168,93	70 Yb 173,04	71 Lu 174,97	
90 Th 232,04	91 Pa 231	92 U 138,03	93 Np 237	94 Pu 242	95 Am 243	96 Cm 247	97 Bk 249	98 Cf 251	99 Es 254	100 Fm 253	101 Md 256	102 No 253	103 Lr 257	

30. Massa específica (kg/m³)

Ar (-40°C)	1,429
Ar (-20°C)	1,351
Ar (0°C)	1,250
Ar (20°C)	1,171
Ar (40°C)	1,082
Ar (60°C)	1,031
Ar (100°C)	0,912
Ar (200°C)	0,720
Ar (800°C)	0,320
Hidrogênio (-50°C)	0,1062
Hidrogênio (0°C)	0,0871
Hidrogênio (50°C)	0,0731
Nitrogênio (-50°C)	1,491
Nitrogênio (0°C)	1,209
Nitrogênio (50°C)	1,022
Oxigênio (-55°C)	1,701
Oxigênio (0°C)	1,379
Oxigênio (55°C)	1,172
Hélio (0°C)	0,178
Hélio (50°C)	0,171
Dióxido de Carbono (40°C)	1,731
Monóxido de Carbono (40°C)	1,189