

LISTA EXTRA

Atenção: Essa lista consiste em alguns exercícios resolvidos referentes ao conteúdo da 1ª prova (1º bimestre). Além desse conteúdo, na prova final terá matéria da 2ª prova (2º bimestre). Oriente resolver a lista avaliativa e exercícios dos slides. **Bons estudos e qualquer dúvida estou à disposição!**

1. Calcule a integral abaixo:

$$\int_0^2 \int_0^1 (1 + 2x + 2y) dy dx$$

$$\int_0^1 (1 + 2x + 2y) dy = 2 + 2x$$

$$\int_0^2 (2 + 2x) dx = 4 + 4 = 8$$

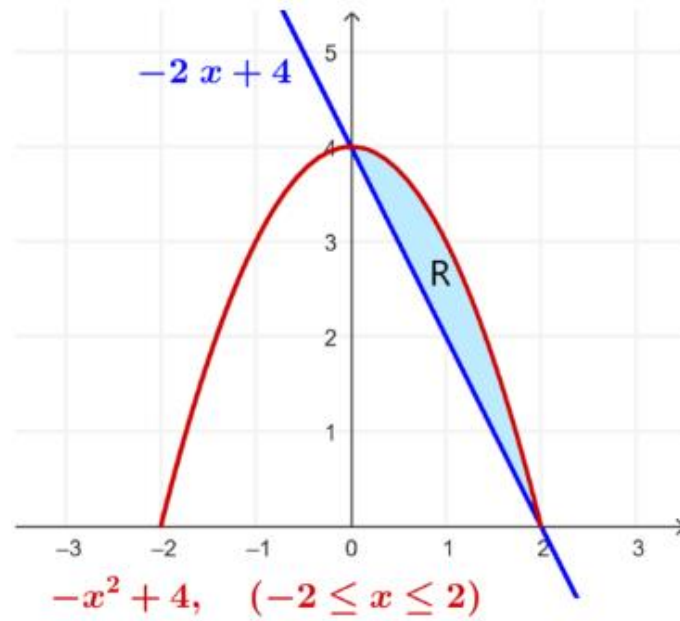
2. Calcular a integral dupla:

$$\iint_R x y \, dx dy$$

Tendo o seguinte:

$$R = \{(x, y) \in \mathbb{R}^2 / 4 - 2x \leq y \leq 4 - x^2\}$$

LISTA EXTRA



$$\int_0^2 \left[\int_{4-2x}^{4-x^2} x y \, dy \right] dx$$

$$\left[x \int_{4-2x}^{4-x^2} y \, dy \right] = \frac{1}{2} x^2 (x-2)^2 (x+4)$$

$$\begin{aligned} \int_0^2 \left[\frac{1}{2} x^2 (x-2)^2 (x+4) \right] dx &= \int_0^2 \left(\frac{1}{2} x^5 - 6x^3 + 8x^2 \right) dx \\ &= \left[\frac{x^6}{12} - \frac{6x^4}{4} + \frac{8x^3}{3} \right]_0^2 = \frac{8}{3} \end{aligned}$$

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3. Calcular a integral abaixo:

$$\int_{y=0}^2 \int_{y=0}^{x^2} y dy dx$$

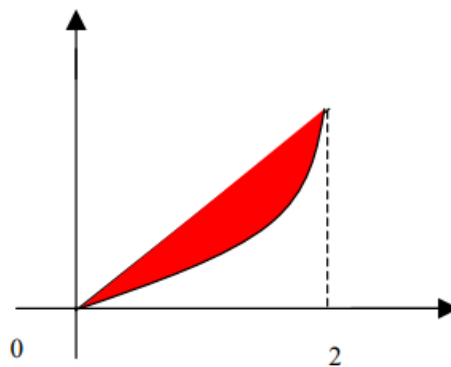
$$\frac{1}{2} \int_0^2 y^2 \Big|_0^{x^2} dx$$

$$\frac{1}{2} \int_0^2 x^4 dx$$

$$\frac{1}{2} \cdot \frac{1}{5} x^5 \Big|_0^2$$

$$\frac{1}{10} (2)^5 = \frac{32}{10} = \frac{16}{5}$$

4. Determinar a área da região limitada:



$$\begin{cases} y = x^3 \\ y = 4x \end{cases}$$

$$x^3 - 4x = 0 \begin{cases} 0 \\ +2 \\ -2 \end{cases}$$

$$R = \begin{cases} 0 \leq x \leq 2 \\ x^3 \leq y \leq 4x \end{cases}$$

$$A = \int_{x=0}^2 \int_{y=x^3}^{4x} dy dx$$

$$A = \int_0^2 y \Big|_{x^3}^{4x} dx$$

$$A = \int_0^2 (4x - x^3) dx = 4 \frac{x^2}{2} - \frac{x^4}{4} \Big|_0^2 = 4$$

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5. Calcular a integral tripla:

$$\begin{aligned}
 & \int_0^1 \int_0^{1-x} \int_0^{1-x-z} x^2 \, dz \, dy \, dx \\
 &= \int_0^1 \int_0^{1-x} x^2(1-x-y) \, dy \, dx = \int_0^1 \int_0^{1-x} (x^2 - x^3 - x^2 y) \, dy \, dx \\
 &= \int_0^1 \left(x^2 y - x^3 y - x^2 \frac{y^2}{2} \right) \Big|_0^{1-x} dx = \int_0^1 \left(x^2(1-x) - x^3(1-x) - \frac{x^2}{2}(1-x)^2 \right) dx \\
 &= \int_0^1 \left(\frac{x^2}{2} - x^3 + \frac{x^4}{2} \right) dx = \left[\frac{1}{2} \cdot \frac{x^3}{3} - \frac{x^4}{4} + \frac{1}{2} \cdot \frac{x^5}{5} \right] \Big|_0^1 = \frac{1}{60}.
 \end{aligned}$$

6. Calcular a integral tripla:

① $\iiint \kappa y z \, dx \, dy \, dz$ $R = [0,1] \times [1,2] \times [0,3]$

$\int_0^3 \int_1^2 \int_0^1 \kappa y z \, dx \, dy \, dz$

① $\int_0^1 \kappa y z \, dx = \frac{x^2}{2} y z \Big|_0^1 = \boxed{\frac{1}{2} y z}$

② $\int_1^2 \frac{yz}{2} \, dy = \frac{1}{2} \cdot \frac{y^2}{2} \cdot z \Big|_1^2 = \frac{y^2}{4} \cdot z = \frac{1}{4} z = \boxed{\frac{3}{4} z}$

③ $\int_0^3 \frac{3}{4} z \, dz = \frac{3z^2}{8} \Big|_0^3 = \boxed{\frac{27}{8} \text{ u.v.}}$