BPSA PRODUCTION SCHEDULING

Model

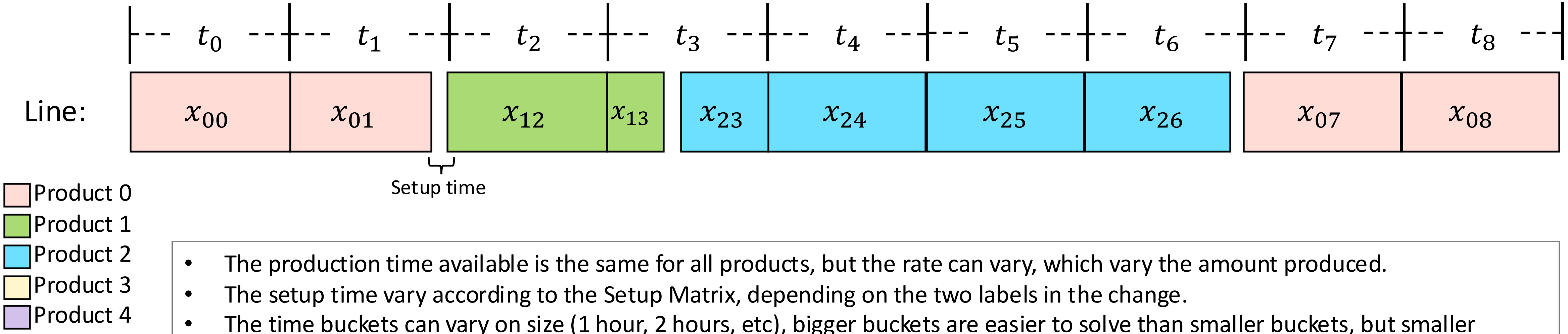
February 2024

VARIABLES

Values to be determined by the solver

VARIABLES – 1

X_{it}: The number of the product **i** (in thousands) that shall be produced in period **t**.



- The production time available is the same for all products, but the rate can vary, which vary the amount produced.
- The setup time vary according to the Setup Matrix, depending on the two labels in the change.
- The time buckets can vary on size (1 hour, 2 hours, etc), bigger buckets are easier to solve than smaller buckets, but smaller buckets facilitate the "sequencing approach", since we have the maximum of 1 setup in a small bucket and we don't need to track exactly the sequence (which is a non-linear model).

Example:

- X_{00}: Thousands of Product 0 shall be produced in period 0.
- X_{45}: Thousands of Product 4 shall be produced in period 5.

VARIABLES – 2

- I_{it}: **Inventory of product i (in thousands) stored at end of period t**
  - To be able to fulfill all the demand, the solver can produce the respective products before the demand period, keeping the products in inventory until the demand periods reached.
  - This variable represents how much of each product (in thousands) is stored in inventory in this kind of situation. It tracks the accumulated inventory along the periods.
- B_{it}: **Backlog of product i (in thousands) at period t**
  - When the solver is not able to produce the products to meet a demand, it is considered as backlog, which can be partial backlog or full backlog. The backlog is cumulative through the periods and can be met in the following periods (postponement) or never be met in the planning horizon ("real" backlog).
  - This variable tracks the accumulated backlog of each product in each period.

VARIABLES – 3

- **Z_{ijt}: Setup of the product i from product j in period t**
  - This binary variable (1|0) enables the control of the products setups in line, while connects the two products involved in a new setup: **i** (starting production) **j** (was on production) at period **t**.
  - To start the production of a product **i** one setup shall happen in the period **t** or in periods before, without the setup of other products in between.
- **ZA_{it}: Setup carry-over of product i in period t**
  - This binary variable (1|0) represents the situation where a product **i** keeps being produced through periods after a single setup. It does not apply any kind of cost or production time consumption, and it is used uniquely to control what product can be produced in the period.

PARAMETERS

Static scenario input data or configuration

PARAMETERS – 1

- **D_{it}: Demand of product i in period t**
  - The demand (in thousands) of the product **i** the need to be "served" in the period **t**. To meet a demand volume, a production can happen before the demand period (stock), in the demand period (no stock) or after the demand period (backlog/postponement – with extra "cost" penalty).
- **$C_{\mathbf{t}}$: Time available to produce (capacity) at period t**
  - How much production time is available at each period. This time capacity is consumed by production itself or by setups for new products in line.
- **$T_{\mathbf{t}}$: Time to produce 1000 products i in the period t**
  - This represents the production time consumed to produce each thousand products at period **t**, it is calculated by $\mathrm{T}_{\mathbf{t}} = \frac{1}{r_{\mathbf{t}}}$, where $\mathbf{r}_{\mathbf{t}}$ is the line rate (1000/min) at period *t*.
  - Currently we consider that the production rate varies only by the period, being the same for all products.

PARAMETERS – 2

- ***SC_{ij}: Setup "cost" changing from product*** i ***to product*** j
  - The "cost" represents the complexity to change from product **i** to product **j** in the line.
  - It do not necessary represents the money expend, or the time taken, since we can have a setup with a small "setup time" but with a high amount of preparation work to start the label change, which can generate issues on the execution.
- ***ST_{ij}: Setup time for changing from product*** i ***to product*** j
  - The time (in minutes) for changing from the product **I** to the product **j** in line.

OBJECTIVE FUNCTION

Mathematical function to drive the minimization/maximization

OBJECTIVE FUNCTION

- The goal is to minimize Backlog, Setup Cost and Number of Setups.
- Backlog component drives to meet demands being in the right periods. Backlogs that happens in periods different from the last period (postponements) costs 50% of the backlog cost ($0.5 * \boldsymbol{w_B}$)
- Setup component (Setup Cost and Number of Setups) drives to minimize the number setups and select the best setups when one need to happen. Using only the setup cost (without the number of setups) can drive to unnecessary setups, when transitioning from A to B and then B to C (with no production of B) is cheaper than from A to C.

$$\textbf{MIN} \sum_{i=0}^{n} \sum_{t=0}^{m} (\boldsymbol{w_B} * \boldsymbol{B_{it}}) + \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{t=0}^{m} [(\boldsymbol{SC_{ji}} * \boldsymbol{Z_{ijt}}) + \boldsymbol{Z_{ijt}}]$$

<u>Glossary:</u>
$\boldsymbol{i}, \boldsymbol{j}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{B_{it}}$: Backlog of product **i** in **t**
$\boldsymbol{SC_{ij}}$: Setup cost from product **i** to product **j**
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.
$\boldsymbol{w_B}$: Backlog weight/cost per thousands of products. Postponements (backlog in periods different from the last period cost 50%)
n: Number of products
m: Number of periods

RESTRICTIONS

Mathematical equations which represent the problem being solved

RESTRICTIONS 1 - DEMAND FULFILLMENT

- The demand of each period shall be fulfilled by production or inventory
- If not fulfilled, the demand is a backlog on the period.
- The Inventory and Backlog quantities can be accumulated through the periods

$$\boxed{\boldsymbol{X_{it} + I_{it-1} - I_{it} + B_{it} - B_{it-1} = D_{it} \qquad\qquad \forall_i, \forall_t}}$$

Glossary:
$\boldsymbol{i}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{X_{it}}$: Production of product **i** in **t**
$\boldsymbol{B_{it}}$: Backlog of product **i** in **t**
$\boldsymbol{I_{it}}$: Inventory of product **i** in **t**
$\boldsymbol{D_{it}}$: Demand of product **i** in **t**

RESTRICTIONS 2 - ENSURE SETUP TO PRODUCE

- The production of product **i** can only occurs in the period **t** if there is a setup or setup carry-over for the product in the period.
- The setup of the product **i** could happen from any other product **j**

$$\boldsymbol{X_{it} \le (\sum_{t=0}^{m} D_{it}) * ((\sum_{j=0}^{n} Z_{ijt}) + ZA_{it})} \qquad \forall_{i}, \forall_{t}$$

<u>**Glossary:**</u>
$\boldsymbol{i, j}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{X_{it}}$: Production of product **i** in **t**
$\boldsymbol{D_{it}}$: Demand of product **i** in **t**
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.
$\boldsymbol{ZA_{it}}$: If there is a setup being carry over for the product **i** in the period **t**.
n: Number of products
m: Number of periods

RESTRICTIONS 3 - ENSURE SETUP ONLY WHEN PRODUCE

- Ensure that a setup will be made only when a production will happen in the same period or in the next one. This avoids useless setups (not used) that can happen in a local optimum.

$$\sum_{j=0}^{n} Z_{ijt} \leq X_{it} + X_{i(t+1)} \qquad \forall_i, \forall_t$$

Glossary:
i, j : Product
t: Period
X_{it}: Production of product **i** in **t**
Z_{ijt}: If a setup from product **j** to product **i** had happen in the period **t**.
n: Number of products

RESTRICTIONS 4 - CAPACITY RESTRICTION

The time consumed by producing **X** products + the time to setup a new product, must be less or equal to the production time available in the period **t**.

$$\left(\sum_{i=0}^{n} \boldsymbol{X_{it}} * \boldsymbol{T_t}\right) + \left(\sum_{i=0}^{n} \sum_{j=0}^{n} \boldsymbol{Z_{ijt}} * \boldsymbol{ST_{ji}}\right) \leq \boldsymbol{C_t} \qquad \forall_t$$

Glossary:

$\boldsymbol{i}, \boldsymbol{j}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{X_{it}}$: Production of product **i** in **t**
$\boldsymbol{T_t}$: Time to produce thousands of products in **t**
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.
$\boldsymbol{ST_{ij}}$: Setup time to change from product **i** to product **j**.
$\boldsymbol{C_t}$: Production time available (capacity) at period **t**.
n: Number of products

RESTRICTIONS 5 - SINGLE SETUP PER PERIOD

Ensure at maximum one setup carry for each period. **t**

$$\boxed{\sum_{i=0}^{n} \sum_{j=0}^{n} Z_{ijt} \leq 1 \qquad\qquad \forall_t}$$

<u>Glossary:</u>

i, j : Product

t: Period

Z_{ijt}: If a setup from product **j** to product **i** had happen in the period **t**.

n: Number of products

RESTRICTIONS 6 - SINGLE SETUP CARRY-OVER PER PERIOD

Ensure at maximum one setup carry-over for each period **t**.

$$\boxed{\sum_{i=0}^{n} ZA_{it} \leq 1 \qquad\qquad \forall_t}$$

Glossary:

i : Product

t: Period

ZA_{it}: If there is a setup being carry over for the product **i** in the period **t**.

n: Number of products

RESTRICTIONS 7 - ENSURE SETUP CARRY-OVER CONSISTENCY

Only setup carry-over if the product had a setup or a setup carry-over at the previous period.

$$\boxed{\boldsymbol{ZA_{it} \leq \left(\sum_{j=0}^{n} Z_{ijt-1}\right) + ZA_{it-1} \qquad \forall_i, \forall_t}}$$

Glossary:
$\boldsymbol{i, j}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.
$\boldsymbol{ZA_{it}}$: If there is a setup being carry over for the product **i** in the period **t**.
n: Number of products

RESTRICTIONS 8 - ENSURE SETUP CARRY-OVER CONSISTENCY

Only have a setup carry-over if had setup (or setup carry-over) in the previous period and **do not** had setup of a different product in the previous period.

$$\boldsymbol{ZA_{it} \leq 1 + \left(\sum_{j=0}^{n} Z_{ijt-1}\right) - \left(\sum_{j=0}^{n} Z_{kjt-1}\right) \qquad \forall_{i}, \forall_{t}, \forall_{k \neq i}}$$

<u>**Glossary:**</u>
$\boldsymbol{i, j, k}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.
$\boldsymbol{ZA_{it}}$: If there is a setup being carry over for the product **i** in the period **t**.
n: Number of products

RESTRICTIONS 9 - SETUP PRODUCTS CONNECTION

To setup the product **i** from the product **j** at period **t**, must have a setup carry-over of the product **j** at t. Since we can have a single setup at each period and have a setup carry-over every time a setup occurred in the last period, we can use the setup carry over to identify the "product from".

$$\boxed{\boldsymbol{Z_{ijt} \leq ZA_{jt} \qquad\qquad \forall_i\ ,\forall_j,\ \forall_t}}$$

Glossary:
$\boldsymbol{i}, \boldsymbol{j}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.
$\boldsymbol{ZA_{it}}$: If there is a setup being carry over for the product **i** in the period **t**.

RESTRICTIONS 10 – NO SETUP BETWEEN SAME PRODUCT

Ensure that we don't have setup from product i to I, which don't affect the cost but makes the output less clear.

$$\boxed{\boldsymbol{Z_{iit} = 0 \qquad\qquad \forall_{i}, \forall_{t}}}$$

Glossary:
$\boldsymbol{i}, \boldsymbol{k}$: Product
$\boldsymbol{t}$: Period
$\boldsymbol{Z_{ijt}}$: If a setup from product **j** to product **i** had happen in the period **t**.

RESTRICTIONS 11 – PERIOD “0”

- The period 0 represents the “initial period” and is out of the planning horizon.
- It is necessary for mathematical consistency.
- Production, Inventory, Backlog and Setup are set to 0 at this period

$\boldsymbol{X_{i0} = 0}$ $\forall_{\boldsymbol{i}}$: No production of any product at period 0
$\boldsymbol{I_{i0} = 0}$ $\forall_{\boldsymbol{i}}$: No inventory of any product at period 0
$\boldsymbol{B_{i0} = 0}$ $\forall_{\boldsymbol{i}}$: No backlog of any product at period 0
$\boldsymbol{Z_{ij0} = 0}$ $\forall_{\boldsymbol{i}}, \forall_{\boldsymbol{j}}$: No setup of any product from any product at period 0

RESTRICTIONS 12 – INITIAL SETUP

For the first setup (transition) to happen we need to have an initial setup running at the line in period 0. This is done setting the setup carry-over of the running product at period 0 to 1 and all others to 0.

$\boldsymbol{ZA_{i0} = 0} \quad \forall_{\boldsymbol{i \neq k}}$: Setup carry-over for all products set to 0, except for the product **k** (product currently running at line)

Glossary:
$\boldsymbol{i, k}$: Product
$\boldsymbol{ZA_{it}}$: If there is a setup being carry over for the product **i** in the period **t**.

RESTRICTIONS 13 – AT LEAST ONE PRODUCTION

To ensure the production of small demands, we can ensure that there is a minimum of production for all production in the scenario.

$$\sum_{t=1}^{m} X_{it} \geq M, \qquad \forall_i$$

Glossary:
i: Product
t: Period
X_{it}: Production of product **i** at period **t**.
n: Number of products
m: Number of periods
M: Min production in thousands.

RESTRICTIONS 14 – NON-NEGATIVITY

Production, Inventory and Backlog are positive variables

$$\boxed{\begin{array}{ll} \boldsymbol{X_{it} \geq 0} & \boldsymbol{\forall_i, \forall_t} \\ \boldsymbol{I_{it} \geq 0} & \boldsymbol{\forall_i, \forall_t} \\ \boldsymbol{B_{it} \geq 0} & \boldsymbol{\forall_i, \forall_t} \end{array}}$$

RESTRICTIONS 15 – BINARY VARIABLES

Setup and Setup Carry-Over are binary variables

$$\boxed{\begin{array}{lr} \boldsymbol{Z_{ijt} \in \{0,1\}} & \boldsymbol{\forall_i, \forall_j, \forall_t} \\ \boldsymbol{ZA_{it} \in \{0,1\}} & \boldsymbol{\forall_i, \forall_t} \end{array}}$$

FULL MODEL

$$\text{MIN} \sum_{i=0}^{n}\sum_{t=0}^{m}(w_B * B_{it}) + (w_I * I_{it}) + \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{t=0}^{m}(SC_{ji} * Z_{ijt})$$

$$X_{it} + I_{it-1} - I_{it} + B_{it} - B_{it-1} = D_{it} \qquad \forall_i, \forall_t$$

$$X_{it} \le (\textstyle\sum_{t=0}^{m} D_{it}) * ((\sum_{j=0}^{n} Z_{ijt}) + ZA_{it}) \qquad \forall_i, \forall_t$$

$$\sum_{j=0}^{n} Z_{ijt} \le X_{it} + X_{i(t+1)} \qquad \forall_i, \forall_t$$

$$(\textstyle\sum_{i=0}^{n} X_{it} * T_t) + (\sum_{i=0}^{n}\sum_{j=0}^{n} Z_{ijt} * ST_{ji}) \le C_t \qquad \forall_t$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} Z_{ijt} \le 1 \qquad \forall_t$$

$$\sum_{i=0}^{n} ZA_{it} \le 1 \qquad \forall_t$$

$$ZA_{it} \le (\textstyle\sum_{j=0}^{n} Z_{ijt-1}) + ZA_{it-1} \qquad \forall_i, \forall_t$$

$$ZA_{it} \le 1 + (\textstyle\sum_{j=0}^{n} Z_{ijt-1}) - (\sum_{j=0}^{n} Z_{kjt-1}) \qquad \forall_i, \forall_t, \forall_{k \ne i}$$

$$Z_{ijt} \le ZA_{jt} \qquad \forall_i, \forall_j, \forall_t$$

$$Z_{iit} = 0 \qquad \forall_i, \forall_t$$

$$\sum_{j=0}^{n}\sum_{t=1}^{m} Z_{ijt} + \sum_{t=1}^{m} ZA_{it} > 0, \qquad \forall_i$$

$$X_{i0} = 0 \quad \forall_i$$
$$I_{i0} = 0 \quad \forall_i$$
$$B_{i0} = 0 \quad \forall_i$$
$$Z_{ij0} = 0 \quad \forall_i, \forall_j$$

$$ZA_{i0} = 0 \quad \forall_{i \ne k}$$

$$X_{it} \ge 0 \quad \forall_i, \forall_t$$
$$I_{it} \ge 0 \quad \forall_i, \forall_t$$
$$B_{it} \ge 0 \quad \forall_i, \forall_t$$

$$Z_{ijt} \in \{0, 1\} \quad \forall_i, \forall_j, \forall_t$$
$$ZA_{it} \in \{0, 1\} \quad \forall_i, \forall_t$$

THANK YOU