Parameter Translation:

Cronos Discrete-time model:

$$Y_t = \sigma_t Z_t + \mu$$

$$\sigma_t^2 = \exp(X_t + h)$$

$$X_t = \phi X_{t-1} + \varepsilon_t$$

$$\varepsilon_t \overset{iid}{\sim} N(0, \nu^2)$$

Assumption:

$$Y_t = \log\left(\frac{S_{t\delta}}{S_{(t-1)\delta}}\right) = \int_{(t-1)\delta}^{t\delta} d(\log S_u),$$

$$dS_t = \mu' S_t\, dt + \sigma_t' S_t\, dW_{1,t}$$

$$\Downarrow$$

$$d\log S_t = (\mu' - \sigma_t'^2/2)\, dt + \sigma_t'\, dW_{1,t}$$

$$\sigma_t' = \exp(X_t')$$

$$dX_t' = \alpha(X_t' - \beta) + \gamma\, dW_{2,t}$$

Translation

1. $e^{\alpha\delta} = \phi$

2. $\sigma_t' \cdot \sqrt{\delta} = \sigma_t$

$$\Downarrow$$

$$\frac{X_t + k}{2} = X_t' + \log\left[\sqrt{\delta}\right]$$

take expectations:

$$\beta = \frac{k}{2} - \log\sqrt{\delta} \qquad \left(= E \log \sigma_t'\right)$$

3. (matching variance of steps in AR eqn)

$$\frac{\nu^2}{4} = \gamma^2\left(\frac{e^{2\alpha\delta} - 1}{2\alpha}\right)$$

4. $\left(\mu' - \frac{\sigma_t'^2}{2}\right)\delta = \mu$, replacing $\sigma_t'^2$ by $\underbrace{E\left(\sigma_t'^2\right)}$,

$$= E\left(e^{2X_t'}\right)$$

$$= e^{2\beta + \frac{\nu^2}{2(1-\phi^2)}}$$

$$\downarrow$$

$$\mu'\delta - \frac{\delta}{2} e^{\left[2\beta + \frac{\nu^2}{2(1-\phi^2)}\right]} = \mu.$$