Nome: Felipe Barroso de Castro
RA: 2311292
Curso: Engenharia de Software

## Questão 1

Calcule o valor das integrais iteradas abaixo:

a)

$$\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy$$

1.A) $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy$

$$\int_0^{\sqrt{y}} xy^2 \, dx = y^2 \int_0^{\sqrt{y}} x \, dx = y^2 \left[\frac{x^2}{2}\right]_0^{\sqrt{y}}$$

$$y^2 \left(\frac{(\sqrt{y})^2}{2}\right) = y^2 \left(\frac{y}{2}\right) = \frac{y^3}{2}$$

$$\int_0^4 \frac{y^3}{2} \, dy = \frac{1}{2} \int_0^4 y^3 \, dy = \frac{1}{2} \left[\frac{y^4}{4}\right]_0^4$$

$$\frac{1}{2} \left(\frac{4^4}{4} - 0\right) = \frac{1}{2} \left(\frac{256}{4}\right) = \frac{1}{2} \cdot 64$$

$R = \underline{32}$

b)

$$\int_0^1 \int_{2x}^2 (x-y)\ dy\ dx$$

1) B) $\displaystyle\int_0^1 \int_{2x}^2 (x-y)\ dy\ dx$

$$\int_{2x}^{2} (x-y)\,dy = \left[xy - \frac{y^2}{2}\right]_{2x}^{2} = \left(x\cdot 2 - \frac{2^2}{2}\right) - \left(x\cdot 2x - \frac{(2x)^2}{2}\right)$$

$$(2x-2) - \cancel{(2x^2 - 2x^2)}$$

$$\int_0^1 (2x-2)\,dx = \left[x^2 - 2x\right]_0^1 = (1^2 - 2\cdot 1) = -1$$

$R = -1$

c)

$$\int_0^1 \int_{x^2}^{x} (1+2y)dydx$$

c) $\int_0^1 \int_{x^2}^{x} (1+2y)\, dydx$

$$\int_{x^2}^{x} (1+2y)\, dy = \left[ y + y^2 \right]_{x^2}^{x} = (x + x^2) - (x^2 + (x^2)^2)$$

$$(x + x^2) - (x^2 + x^4) = x - x^4$$

$$\int_0^1 (x - x^4)\, dx = \left[ \frac{x^2}{2} - \frac{x^5}{5} \right]_0^1 = \left( \frac{1}{2} - \frac{1}{5} \right) = \frac{3}{10}$$

R = 3/10

d)

$$\int_0^2 \int_y^{2y} xy\,dx\,dy$$

D) $\int_0^2 \int_0^{2y} xy\;dxdy$

$\int_y^{2y} xy\,dx = y\int_y^{2y} x\,dx = y\left[\frac{x^2}{2}\right]_y^{2y}$

$y\left(\frac{(2y)^2}{2} - \frac{y^2}{2}\right) = \frac{3y^3}{2}$

$\int_0^2 \frac{3y^3}{2}\,dy = \frac{3}{2}\int_0^2 y^3\,dy = \frac{3}{2}\left[\frac{y^4}{4}\right]_0^2 = \frac{3}{2}\left(\frac{2^4}{4} - 0\right)$

$\frac{3}{2} \cdot 4 = 6$

$R = \underline{6}$

e)

$$\int_0^1 \int_0^{s^2} \cos\left(s^3\right)\, dt\, ds$$

e) $\displaystyle\int_0^1\int_0^{s^2} \cos(s^3)\, dt\, ds$

$$\int_0^{s^2} \cos(s^3)\, dt = \cos(s^3)\int_0^{s^2} 1\, dt = s^2\cos(s^3)$$

$$\int_0^1 s^2\cos(s^3)\, ds \rightarrow \int_0^1 \cos(u)\cdot\frac{du}{3} = \frac{1}{3}\int_0^1 \cos(u)\, du$$

$$\frac{1}{3}\Big[\sin(u)\Big]_0^1 = \frac{1}{3}(\sin(1)-\sin(0)) = \frac{1}{3}(\sin(1)-0)$$

$$R = \frac{\sin(1)}{3} \;//$$

## Questão 2

Calcule a integral dupla:

a)

$$\iint_D y^2 \, dA, \quad D = \{(x,\, y) \mid -1 \leqslant y \leqslant 1,\ -y-2 \leqslant x \leqslant y\}$$

2. A)

$$\iint_D y^2 \, dA$$

$$D = \{(x, y) \mid -1 \leq y \leq 1,\ -y-2 \leq x \leq y\}$$

$$\int_{-1}^{1} \int_{-y-2}^{y} y^2 \, dx \, dy$$

$$\int_{-1}^{1} \left[y^2 x\right]_{-y-2}^{y} dy \rightsquigarrow \int_{-1}^{1} y^2(y-(-y-2))\, dy$$

$$\int_{-1}^{1} y^2(y+y+2)\, dy \rightsquigarrow \int_{-1}^{1} y^2(2y+2)\, dy$$

$$\int_{-1}^{1} (2y^3+2y^2)\, dy = 2\int_{-1}^{1} y^3 dy + 2 \int_{-1}^{1} y^2 \, dy$$

$$2\int_{-1}^{1} y^3 dy = 2\left[\frac{y^4}{4}\right]_{-1}^{1} = 2\left(\frac{1}{4} - \frac{(-1)^4}{4}\right) = 0$$

$$2\int_{-1}^{1} y^2 dy = 2\left[\frac{y^3}{3}\right]_{-1}^{1} = 2\left(\frac{1^3}{3} - \frac{(-1)^3}{3}\right) = 2 \cdot \frac{2}{3}$$

$$R = 4/3$$

b)

$$\iint_D \frac{y}{x^5+1}\,dA, \quad D = \{(x,\ y) \mid 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant x^2\}$$

2. B) $\displaystyle\iint_D \frac{y}{x^5+1}\,dA$, $D = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le x^2\}$

$$\int_0^1\int_0^{x^2} \frac{y}{x^5+1}\,dy\,dx$$

$$\int_0^1 \left[\frac{y^2}{2(x^5+1)}\right]_0^{x^2} dx = \int_0^1 \frac{(x^2)^2}{2(x^5+1)}\,dx$$

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$u = x^5+1$, $du = 5x^4$

$$\int_1^2 \frac{1}{10u}\,du = \frac{1}{10}\int_1^2 \frac{1}{u}\,du = \frac{1}{10}\Big[\ln u\Big]_1^2$$

$$\frac{1}{10}(\ln 2 - \ln 1) = \frac{1}{10}\ln 2$$

$$R = \frac{\ln 2}{10}$$

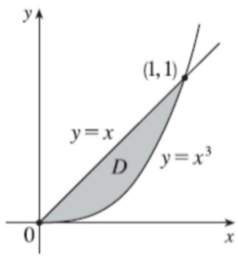
**Questão 3**

Calcule a integral dupla

a)

$$\int_D (x^2+2y)\,dA,$$

D é limitada por y = x, $y = x^3$, $x \geqslant 0$. Dica: considere a figura abaixo para ajudar:




3. A) $\int_D (x^2 + 2y)\,dA$ $\left|\begin{array}{l} y = x \\ y = x^3 \\ x \geq 0 \end{array}\right.$

$$\int_0^1 \int_{x^3}^{x} (x^2 + 2y)\,dy\,dx$$

$$\int_{x^3}^{x} (x^2 + 2y)\,dy = \left[x^2 y + y^2\right]_{x^3}^{x}$$

$$(x^2 x + x^2) - (x^2 x^3 + (x^3)^2) = (x^3 + x^2) - (x^5 + x^6)$$

$$x^3 + x^2 - x^5 - x^6$$

$$\int_0^1 (x^3 + x^2 - x^5 - x^6)\,dx$$

$\downarrow$

$$\underbrace{\int_0^1 x^3\,dx + \int_0^1 x^2\,dx - \int_0^1 x^5\,dx \int_0^1 x^6\,dx}_{R = \frac{23}{84}}$$