Questão 1

Determine a área da superfície:

a) A parte do plano $z = 2 + 3x + 4y$ que está acima do retângulo $[0,\ 5] \times [1,\ 4]$

1.A)

$z = 2 + 3x + 4y$

$[0,5] \times [1,4]$

Montando a integral:

$$= \int_1^4 \int_0^5 2 + 3x + 4y \, dx\,dy$$

$$= \left[2x + \frac{3x^2}{2} + 4yx\right]_0^5 \rightsquigarrow \underbrace{\left(2\cdot 5 + \frac{3\cdot 5^2}{2} + 4y\cdot 5\right) - 0}_{10 + \frac{75}{2} + 20y}$$

$$= \int_1^4 10 + \frac{75}{2} + 20y \, dy \rightsquigarrow \left[10y + \frac{75y}{2} + \frac{20y}{2}\right]_1^4$$

$$\underbrace{\left[10\cdot 4 + \frac{75\cdot 4}{2} + \frac{20\cdot 4^2}{2}\right]}_{} - \underbrace{\left[10 + \frac{75}{2} + \frac{20}{2}\right]}_{}$$

$$\underbrace{40 + 150 + 160 - 10}_{} - \frac{95}{2}$$

$$340 - \frac{95}{2} = 292{,}5$$

$R = \underline{292{,}5}$

b) A parte do plano $2x + 5y + z = 10$ que está dentro do cilindro $x^2 + y^2 = 9$

1. B)

$2X + 5Y + Z = 10$

$Z = 10 - 2X - 5Y$

$$\int_0^3\int_0^3 10 - 2x - 5y \, dxdy$$

$$\left[10x - \frac{\cancel{2}x^2}{\cancel{2}} - 5xy\right]_0^3 \sim> \left(10 \cdot 3 - 3^2 - 5 \cdot 3y\right) - 0$$

$$\int_0^3 30 - 9 - 15y \, dy \sim> \int_0^3 21 - 15y \, dy$$

$$\underbrace{\left[21 \cdot 3 - \frac{15 \cdot 3^2}{2}\right] - 0}$$

$$63 - \frac{15 \cdot 3^2}{2} \sim> 63 - 67{,}5$$

$R = -4{,}5$

c) A parte da superfície $z = 1 + 3x + 2y^2$ que está acima do triângulo com vértices (0, 0), (0, 1) e (2, 1)

1) c)

$z = 1 + 3x + 2y^2$

$[0, 2] \times [0, 1]$

$= \int_0^1 \int_0^2 1 + 3x + 2y^2 \, dx \, dy$

$=$

$\left[x + \frac{3x^2}{2} + 2xy^2 \right]_0^2 \rightsquigarrow \left(2 + \frac{3 \cdot 2^2}{2} + 2 \cdot 2y^2 \right) - 0$

$2 + \frac{3 \cdot \cancel{4}^{2}}{\cancel{2}} + 4y^2 \rightsquigarrow 8 + 4y^2$

$\int_0^1 8 + 4y \, dy \rightsquigarrow \left[8y + \frac{4y^3}{3} \right]_0^1 \rightsquigarrow 8 + \frac{4}{3}$

$R = 9,\bar{3}$

d) A parte do paraboloide hiperbólico $z = y^2 - x^2$ que está entre os cilindros $x^2 + y^2 = 1$ e $x^2 + y^2 = 4$

1. D)

$z = y^2 - x^2$

$[1, 2] \times [1, 2]$

$$= \int_1^2 \int_1^2 y^2 - x^2 \, dx \, dy$$

$$= \left[xy^2 - \frac{x^3}{3} \right]_1^2 \rightarrow \underbrace{\left(2y^2 - \frac{2^3}{3}\right) - \left(y^2 - \frac{1}{3}\right)}_{y^2 - \frac{7}{3}}$$

$$= \int_1^2 y^2 - \frac{7}{3} \, dy \rightarrow \left[\frac{y^3}{3} - \frac{7}{3} y \right]_1^2$$

$$\underbrace{\left(\frac{2^3}{3} - \frac{7}{3} \cdot 2 \right) - \left(\frac{1}{3} - \frac{7}{3} \right)}$$

$$\frac{-6}{3} + \frac{6}{3} = 0$$

ou

$$\frac{6}{3} + \frac{6}{3} = 4$$

$R = 4$

Questão 2

Determine a área exata da superfície $z = 1 + 2x + 3y + 4y^2$, $1 \leqslant x \leqslant 4$, $0 \leqslant y \leqslant 1$.

2.

$z = 1 + 2x + 3y + 4y^2$

$[1, 4] \times [0, 1]$

$$= \int_0^1 \int_1^4 1 + 2x + 3y + 4y^2 \, dx \, dy$$

$$= \left[x + \cancel{\frac{2x^2}{2}} + 3xy + 4xy^2 \right]_1^4 \rightsquigarrow \left[x + x^2 + 3xy + 4xy^2 \right]_1^4$$

$$\underbrace{\left[4 + 4^2 + 3\cdot 4y + 4\cdot 4y^2\right] - \left[1 + 1^2 + 3y + 4y^2\right]}_{18 + 9y + 12y^2}$$

$$\int_0^1 18 + 9y + 12y^2 \, dy \rightsquigarrow \left[18y + \frac{9y^2}{2} + \frac{\cancel{12}^{\,4}\, y^3}{\cancel{3}} \right]_0^1$$

$$\left(18 \cdot 1 + \frac{9 \cdot 1^2}{2} + 4 \cdot 1^3 \right) - 0 \to 18 + \frac{9}{2} + 4$$

$22 + 4,5 = 26,5$

R = 26,5