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Questão 1

Calcule a integral de linha, onde C é a curva dada:

a)

$$\int_C y\,ds, \quad C:\ x = t^2, \quad y = 2t, \quad 0 \leqslant t \leqslant 3$$

$$\int_c y\,ds,\ C: X = t^2,\ y = 2t,\ 0 \leq t \leq 3$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$\frac{dx}{dt} = \frac{d}{dt}(t^2) = 2t, \frac{dy}{dt} = \frac{d}{dt}(2t) = 2$$

$$ds = \sqrt{(2t^2) + 2^2}dt = \sqrt{4(t^2+1)}\,dt = 2\sqrt{t^2+1}\,dt$$

$$\int_c y\,ds = \int_0^3 2t \,.\, 2\sqrt{t^2+1}\,dt = 4\int_0^3 t\sqrt{t^2+1}\,dt$$

$$u = t^2 + 1$$
$$du = 2t\,dt$$

$$t\sqrt{t^2+1}\,dt = t\sqrt{u}\,.\,\frac{du}{2t} = \frac{\sqrt{u}}{2}du$$

$$4\int_0^3 t\sqrt{t^2+1}\,dt = 4\,.\,\frac{1}{2}\int_1^{10} u^{\frac{1}{2}}du = 2\int_1^{10} u^{\frac{1}{2}}du$$

$$2\int_1^{10} u^{\frac{1}{2}}\,du = 2\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{10} = \frac{4}{3}\left[10^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{4}{3}\left[10\sqrt{10} - 1\right]$$

$R =$

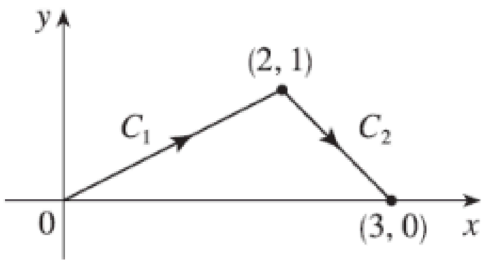
$$\int_c y\,ds = \frac{4}{3}\left(10\sqrt{10} - 1\right)$$

b)

$$\int_C (x+2y)dx + x^2dy,$$

onde C consiste nos segmentos de reta de (0, 0) a (2, 1) e de (2, 1) a (3, 0).
Dicas:

- *Considere o gráfico abaixo:*



- *Considere* $C = C_1 + C_2$
- *Considere para* $C_1:\ x = x,\ y = \frac{1}{2}x$ *e para* $C_2:\ x = x,\ y = 3 - x$

$$\int_C (x + 2y)\ dx + x^2 dy \qquad x = 2t \qquad y = t \qquad 0 \le t \le 1$$

$$\int_C (x + 2y)\ dx + x^2\, dy = \int_0^1 \left[(2t + 2t)\,.2\,(2t)^2\right] dt$$

$$= \int_0^1 \left(8t + 4t^2\right)\ dt$$

$$\int_0^1 8t\ dt + \int_0^1 4t^2\ dt = 8\left[\frac{t^2}{2}\right]_0^1 + 4\left[\frac{t^3}{3}\right]_0^1 = \frac{16}{3}$$

$$x = 2 + t \qquad y = 1 - t \qquad 0 \le t \le 1$$

$$\int_0^1 (x + 2y)\ dx + x^2\, dy = \int_0^1 \left[(2 + t + 2(1 - t))\ .\ 1\ (2 + t)^2\ .\ (-1)\right] dt$$

$$\int_0^1 t^2\, dt$$

$$\int_0^1 \left(4 - t - 4 - 4t - t^2\right) dt = \int_0^1 \left(-5t - t^2\right) dt =$$

$$= -5\int_0^1 t\ dt - \int_0^1 t^2\ dt$$

$$-5\left[\frac{t^2}{2}\right]_0^1 - \left[\frac{t^3}{3}\right]_0^1 = -\frac{17}{6}$$

$$\frac{16}{3} - \frac{17}{6} = \frac{5}{2}$$

c)

$$\int_C xe^{yz}\,ds,$$

onde C é o segmento de reta de $(0, 0, 0)$ a $(1, 2, 3)$.
Dica: Considere a parametrização de C como $x = t,\ y = 2t,\ z = 3t,\ 0 \le t \le 1$

$$\int_C xe^{yz}ds$$

$x = t \qquad y = 2y \qquad z = 3t \qquad 0 \le t \le 1$

$dx = dt \quad dy = 2dt \quad dz = 3dt$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}dt = \sqrt{1 + 4 + 9} = \sqrt{14}\,dt$$

$u = 4t^2 \qquad du = 8t\ dt$

$$te^{4t^2}dt = \frac{1}{8}\ e^u du$$

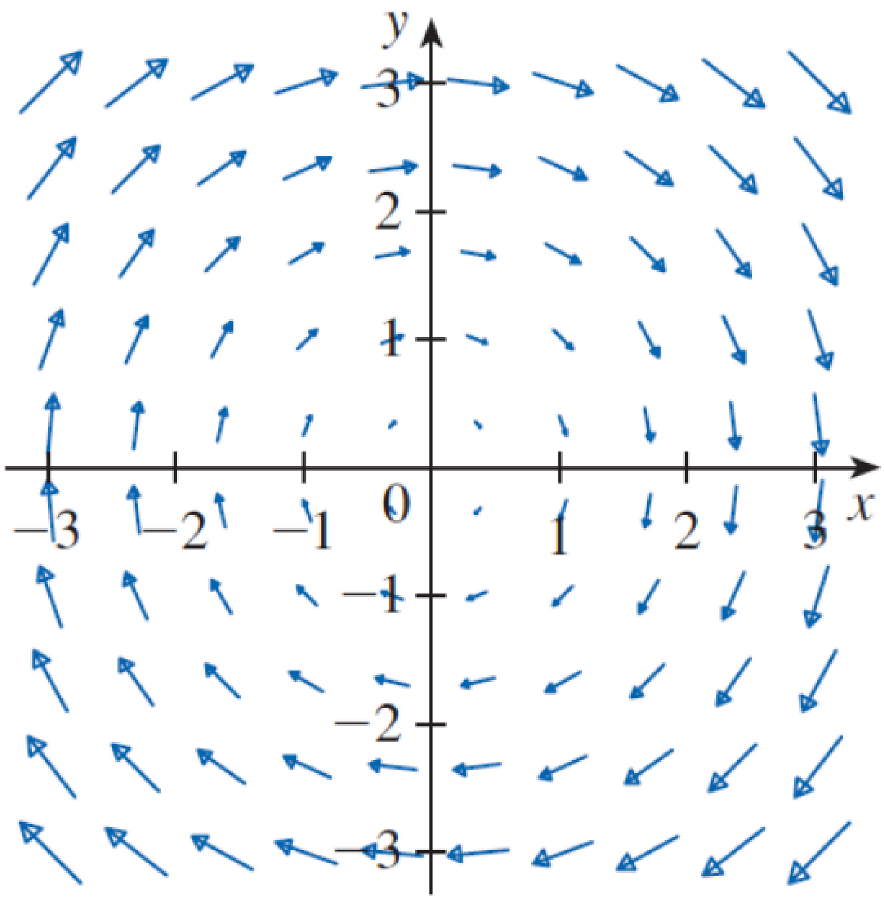
$$\sqrt{14}\int_0^1 te^{4t^2}dt = \frac{\sqrt{14}}{8}\int_0^4 e^u du$$

$$[e^u]_0^4 = \frac{\sqrt{14}}{8}\left(e^4 - 1\right)$$

$$\int_C x\,e^{y^2}\,ds = \frac{\sqrt{14}}{8}\left(e^4 - 1\right)$$

Questão 2

Seja $\vec{\mathbf{F}}$ o campo vetorial mostrado na figura:



a) Se C_1 é o segmento de reta vertical de (-3, -3) a (-3, 3), determine se $\int_{C_1} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ é positivo, negativo ou zero. **Explique a sua resposta.**

$$\int_{C1} \vec{F} \cdot \vec{dr}$$

$X = -3 \qquad y = 3a - 3 \qquad \vec{dr} = (0,\ dy)$

$\vec{F} \cdot \vec{dr} = (-y,\ -3) \cdot (0,\ dy) = -3dy$

$$\int_{C1} \vec{F} \cdot \vec{dr} = \int_{-3}^{3} -3dy = -3[y]_{-3}^{3} = -3(3 - (-3)) = -18$$

$Logo, \int_{C1} \vec{F} \cdot \vec{dr}$ *é negativo*

b) Se C_2 é o círculo de raio 3 e centro na origem percorrido no sentido anti-horário, determine se $\int_{C_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ é positivo, negativo ou zero. **Explique a sua resposta.**

$$\int_{C2} \vec{F} \,.\, \vec{dr}$$

$X = 3\cos\theta \qquad y = 3sen\theta$

θ *varia de* 0 *a* 2π

$\vec{dr} = (-3\,\text{sen}\,\theta d\theta,\ 3\cos\theta d\theta)$

$\vec{F} = (-y,\ x) = (-3\,\text{sen}\,\theta,\ 3\cos\theta)$

$\vec{F}\,.\,\vec{dr} = (-3\,\text{sen}\,\theta,\ 3\cos\theta)\ .\ (-3\,\text{sen}\,\theta d\theta,\ 3\cos\theta d\theta) =$

$= \left(9\text{sen}^2\theta + 9\cos^2\theta\right)d\theta = 9\left(\text{sen}^2\theta + \cos^2\theta\right)d\theta = 9d\theta$

$$\int_{c2} \vec{F}\,.\,\vec{dr} = \int_0^{2\pi} 9d\theta = 9\left[\theta\right]_0^{2\pi} = 9(2\pi - 0) = 18\pi$$

Logo, $\int_{C2} \vec{F}\,.\,\vec{dr}$ é positivo

Questão 3

Calcule a integral de linha $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, onde C é dada pela função vetorial $\vec{\mathbf{r}}(t)$:

a)

$$\vec{\mathbf{F}}(x,\ y) = xy\hat{\boldsymbol{\imath}} + 3y^2\hat{\boldsymbol{\jmath}},\ \vec{\mathbf{r}}(t) = 11t^4\hat{\boldsymbol{\imath}} + t^3\hat{\boldsymbol{\jmath}},\ 0 \leqslant t \leqslant 1$$

$r(t) = 11t^4\hat{i} + t^3\hat{j}$

$r'(t) = \frac{d}{dt}\left(11t^4\right)\hat{i} + \frac{d}{dt}\left(t^3\right)\hat{j} = 44t^3\hat{i} + 3t^2\hat{j} = F(x,\ y) = xy\hat{i} + 3y^2\hat{j}$

$F(r(t)) = \left(11t^4\right)\left(t^3\right)\hat{i} + 3\left(t^3\right)^2\hat{j} = 11t^7\hat{i} + 3t^6\hat{j}$

$F(r(t))\ .\ r'(t) = \left(11t^7\hat{i} + 3t^6\hat{j}\right)\ .\ \left(44t^3\hat{i} + 3t^2\hat{j}\right) = 11t^7.\ 44t^3 + 3t^6\ .\ 3t^2$

$= 484t^{10} + 9t$

$$\int_0^1 F(r(t))\ .\ r'(t)\,dt = \int_0^1 \left(484t^{10} + 9t\right)dt$$

$$\int_0^1 484t^{10}\,dt = 484\int_0^1 t^{10}\,dt = 484\left[\frac{t^{11}}{11}\right]_0^1 =$$

$484\ .\ \frac{1}{11} = 44$

$$\int_0^1 9t^8\,dt = 9\int_0^1 t^8\,dt = 9\left[\frac{t^9}{9}\right]_0^1 = 9\ .\ \frac{1}{9} = 1$$

Logo, $44 + 1 = 45$

b)

$$\vec{\mathbf{F}}(x,\ y,\ z) = (x+y)\hat{\boldsymbol{i}} + (y-z)\hat{\boldsymbol{j}} + z^2\hat{\boldsymbol{k}},\ \vec{\mathbf{r}}(t) = t^2\hat{\boldsymbol{i}} + t^3\hat{\boldsymbol{j}} + t^2\hat{\boldsymbol{k}},\ 0 \leqslant t \leqslant 1$$

$\vec{F}(x,\ y,\ z) = (x,\ y)\,\hat{i} + (y - z)\,\hat{j} + z^2\,\widehat{k}$

$\vec{r}(t) = t^2\hat{i} + t^3\hat{j} + t^2\,\widehat{k}$

$0 \leqslant t \leqslant 1$

$F(r(t)) = F\left(t^2,\ t^3,\ t^2\right)$

$F\left(t^2,\ t^3,\ t^2\right) = \left(t^2 + t^3\right)i + \left(t^3 - t^2\right)j + \left(t^2\right)^2 k$

$= \left(t^2 + t^3\right)i + \left(t^3 - t^2\right)j + t^4 k$

$\dfrac{dr}{dt} = \dfrac{d}{dt}\left(t^2 i + t^3 j + t^2 k\right) = 2ti + 3t^{3j} + 2tk$

$dr = \left(2ti + 3t^{3j} + 2tk\right)\,dt$

$F(r(t))\ \dfrac{dr}{dt} = \left(t^2 + t^3\right)(2t) + \left(t^3 - t^2\right)\left(3t^2\right) + t^4(2t) =$

$= \left(t^2 + t^3\right)\,.\,2t + \left(t^3 - t^2\right)\,.\,3t^2 + t^4\,.\,2t$

$= 2t^3 + 2t^4 + 3t^5 - 3t^4 + 2t^5 =$

$= 2t^3 + 2t^5 - t^4 + 5t^5$

$\displaystyle\int_0^1 \left(2t^3 + 2t^5 - t^4 + 5t^5\right)\,dt = \int_0^1 \left(2t^3 + 7t^5 - t^4\right)$

$\displaystyle\int_0^1 2t^3\,dt - \int_0^1 7t^5\,dt + \int_0^1 t^4\,dt$

$\displaystyle\int_0^1 2t^3\,dt = 2\left[\frac{t^4}{4}\right]_0^1 = 2\,.\,\frac{1}{4} = \frac{1}{2}$

$\displaystyle\int_0^1 t^4\,dt = \left[\frac{t^5}{5}\right]_0^1 = \frac{1}{5}$

$\displaystyle\int_0^1 7t^5\,dt = 7\left[\frac{t^6}{6}\right]_0^1 = \frac{7}{6}$

$\dfrac{1}{2} - \dfrac{1}{5} + \dfrac{7}{6} = \dfrac{22}{15}$

$Logo,\ A\ integral\ da\ linha\ é : \dfrac{22}{15}$

c)

$$\vec{\mathbf{F}}(x,\ y,\ z) = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} - xy\hat{\boldsymbol{k}},\ \vec{\mathbf{r}}(t) = \cos t\hat{\boldsymbol{\imath}} + \operatorname{sen} t\hat{\boldsymbol{\jmath}} + t\hat{\boldsymbol{k}},\ 0 \leqslant t \leqslant \pi$$

$r(t) = \cos(t)i + sen(t)j + tk, \quad 0 \le t \le \pi$

$\mathrm{F(x, y, z) = xi + yj - xyk}$

$\mathrm{F(r(t)) = F(\cos(t), sen(t), t)}$

$\mathrm{F(\cos(t), sen(t), t) = \cos(t)\,i + sen(t)\,j - \cos(t)\,sen(t)\,k}$

$$\frac{\mathrm{dr}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}(\cos(\mathrm{t})\,\mathrm{i} + \mathrm{sen(t)\,j + tk}) = -\mathrm{sen(t)\,i} + \cos(\mathrm{t})\,\mathrm{j + k}$$

$\mathrm{dr} = (-\mathrm{sen(t)\,i} + \cos(\mathrm{t})\,\mathrm{j + K})\,\mathrm{dt}$

$$\mathrm{F(r(t))} \cdot \frac{\mathrm{dr}}{\mathrm{dt}} = (\cos(\mathrm{t})\,\mathrm{i + sen(t)\,j} - \cos(\mathrm{t})\,\mathrm{sen(t)\,k}).(-\mathrm{sen(t)\,j} + \cos(\mathrm{t})\,\mathrm{j + k})$$

$= \cos(\mathrm{t})(-\mathrm{sen(t)}) + \mathrm{sen(t)}\cos(t) + (-\cos(t)\,sen(t))(1)$

$= -2\cos(t)\,sen(t) + sen(t)\cos(t)$

$= -\cos(t)\,sen(t)$

$$\int_0^{\pi} -\cos(t)\,sen(t)\,dt$$

$$\int_0^{\pi} -\cos(t)\,sen(t)\,dt = \frac{-1}{2}\int_0^{\pi} sen(2t)\,dt$$

$$-\frac{1}{2}\int_0^{\pi} sen(2t)\,dt = -\frac{1}{2}\int_0^{2\pi} sen(u) \cdot \frac{1}{2}\,du = -\frac{1}{4}\int_0^{2\pi} sen(u)\,du$$

$sen(u) = -\cos(u)$

$$-\frac{1}{4}\left[-\cos(u)\right]_0^{2\pi} = -\frac{1}{4}(-\cos(2\pi) + \cos(0)) =$$

$$-\frac{1}{4}(-1 + 1) = -\frac{1}{4} \cdot 0 = 0$$

Portanto, o valor da integral da linha é 0.

Questão 4

Determine o trabalho realizado pelo campo de força $F(x,\ y,\ z) = \langle x - y^2,\ y - z^2,\ z - x^2\rangle$ sobre uma partícula que se move ao longo do segmento de reta de (0, 0, 1) a (2, 1, 0).

$r(t) = (1 - t)(0,0,1) + t(2,1,0) = (2t, t, 1 - t)$

$r(t) = <2t,\ t,\ 1 - t>$

$$r'(t) = \frac{d}{dt}<2t,\ t,\ 1-t> = <2,\ 1,\ -1>$$

$$F(r(t)) = F(2t,\ t,\ 1-t) = <2t - t^2,\ t - (1-t)^2,\ 1 - t - 4t^2>$$

$$F(r(t)) = <2t - t^2,\ 3t - 1 - t^2,\ 1 - t - 4t^2>$$

$$F(r(t))\ .\ r'(t) = \left(2t - t^2\right).2 + \left(3t - 1 - t^2\right).1 + \left(1 - t - 4t^2\right).(-1)$$

$$= 2\left(2t - t^2\right) + \left(3t - 1 - t^2\right) + \left(-1 + t + 4t^2\right)$$

$$= 4t - 2t^2 + 3t - 1 - t^2 - 1 + t + 4t^2$$

$$= 4t + 3t + t - 2t^2 - t^2 + 4t^2 - 2$$

$$= 8t + t^2 - 2$$

$$\int_0^1 \left(8t + t^2 - 2\right) dt$$

$$\int_0^1 8t\ dt = 8\int_0^1 t\ dt = 8\left[\frac{t^2}{2}\right]_0^1 = 8\ .\ \frac{1}{2} = 4$$

$$\int_0^1 t^2\ dt = \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\int_0^1 -2dt = -2[t]_0^1 = -2\ .\ 1 = -2$$

Somando os resultados:

$$4 + \frac{1}{3} - 2 = 2 + \frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$$

Questão 5

A força exercida pela carga elétrica colocada na origem sobre uma partícula carregada em um ponto (x, y, z) com vetor posição $\vec{\mathbf{r}} = \langle x,\ y,\ z\rangle$ é

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = K\frac{\vec{\mathbf{r}}}{|\vec{\mathbf{r}}|}|\vec{\mathbf{r}}|^3,$$

onde K é uma constante (veja o Exemplo 5 da Seção 16.1). Encontre o trabalho feito quando a partícula se move ao longo de uma linha reta de (2, 0, 0) a (2, 1, 5).

$$r(t) = (1-t)(2,\ 0,\ 0) + t(2,1,5) = (2,\ t,\ 5t)$$

$$r(t) = <2,\ t,\ 5t>$$

$$r'(t) = \frac{d}{dt}<2,\ t,\ 5t> = <0,\ 1,\ 5>$$

$$F(r(t)) = K\frac{r(t)}{|r(t)|^3}$$

$$|r(t)| = \sqrt{2^2 + t^2 + (5t)^2} = \sqrt{4 + t^2 + 25t^2} = \sqrt{4 + 26t^2}$$

Logo,

$$F(r(t)) = K\frac{<2,\ t,\ 5t>}{\left|4 + 26t^2\right|^{\frac{3}{2}}}$$

$$u = 4 + 26t^2 \qquad du = 52t\,dt \qquad t\,dt = \frac{du}{52}$$

$Quando\ t = 0,\ u = 4$

$Quando\ t = 1,\ u = 30$

$$\int_4^{30} K\frac{26}{52}u^{\frac{-3}{2}}du = \frac{K}{2}\int_4^{30} u^{-\frac{3}{2}}du$$

$$\int u^{\frac{-3}{2}}\,du = -2u^{-\frac{1}{2}}$$

$$\frac{K}{2}\left[-2u^{-\frac{1}{2}}\right]_4^{30} = K\left[-u^{-\frac{1}{2}}\right]_4^{30} = K\left[-\frac{1}{\sqrt{30}} + \frac{1}{2}\right]$$

$$K\left[\frac{1}{2} - \frac{1}{\sqrt{30}}\right]$$