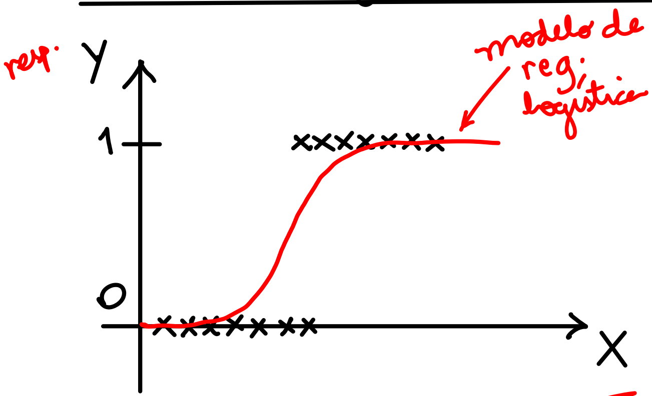
Modelo de regressão logística

MLG com resposta Bernoulli (p) fc. lig. logito



$$\rightarrow Y_i = \begin{cases} 1, \text{ se a característica está presente} \\ 0, \text{ c.c.} \end{cases}$$

$$\rightarrow P(Y_i = 1 \mid x_i) = \pi_i$$

$$\rightarrow P(Y_i = 0 \mid x_i) = 1 - \pi_i$$

preditora

$$\eta_i = \log\left(\frac{\pi_i}{1-\pi_i}\right) = \alpha + \beta x_i, \; i = 1, \dots, n$$

De forma geral, com p preditoras, podemos escrever

β: parâmetro

$\hat{\beta}$: estimador de β

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = x_i^T \beta, \; i = 1, \dots, n$$

$$= \beta_0 + \beta_1 x_{1i} + \dots + \beta_p x_{pi}$$

MLG

modelo ajustado:

$$\log\left(\frac{\hat{\pi}_i}{1-\hat{\pi}_i}\right) = x_i^T \hat{\beta} \Rightarrow \frac{\hat{\pi}_i}{1-\hat{\pi}_i} = \exp(x_i^T \hat{\beta})$$

$\widehat{\text{Chance}}$:

$$\frac{\hat{\pi}_i}{1-\hat{\pi}_i} = \exp(x_i^T \hat{\beta})$$

$$\hat{\pi}_i = (1-\hat{\pi}_i)\exp(x_i^T\hat{\beta}) = \exp(x_i^T\hat{\beta}) - \hat{\pi}_i \exp(x_i^T\hat{\beta})$$

$$\hat{\pi}_i\left(1 + \exp(x_i^T\hat{\beta})\right) = \exp(x_i^T\hat{\beta})$$

$$\boxed{\hat{\pi}_i = \frac{\exp(x_i^T\hat{\beta})}{1 + \exp(x_i^T\hat{\beta})}}$$

$\hat{\beta}$ EMV de β.

Razão de chances

Chance: $\dfrac{\pi_i}{1-\pi_i}$

$$\eta_i = \log\left(\frac{\hat{\pi}_i}{1-\hat{\pi}_i}\right) = \hat{\alpha} + \hat{\beta}_1 x_{1i} + \dots + \hat{\beta}_p x_{pi}$$

Qual o efeito de aumentar uma unidade em X_1? (mantenho todas as demais preditoras fixadas)

$$\eta_i^* = \hat{\alpha} + \hat{\beta}_1 (x_{1i}+1) + \dots + \hat{\beta}_p x_{pi}$$

$$\log\left(\frac{\hat{\pi}_i^*}{1-\hat{\pi}_i^*}\right) = \hat{\alpha} + \hat{\beta}_1 x_{1i} + \hat{\beta}_1 + \hat{\beta}_2 x_{2i} + \dots + \hat{\beta}_p x_{pi}$$

Razão de chances (aumentando uma unidade em x_{1i})

$$\frac{\dfrac{\hat{\pi}_i^*}{1-\hat{\pi}_i^*}}{\dfrac{\hat{\pi}_i}{1-\hat{\pi}_i}} = \frac{\exp\{\cancel{\hat{\alpha}} + \cancel{\hat{\beta}_1 x_{1i}} + \hat{\beta}_1 + \cancel{\hat{\beta}_2 x_{2i}} + \dots + \cancel{\hat{\beta}_p x_{pi}}\}}{\exp\{\cancel{\hat{\alpha}} + \cancel{\hat{\beta}_1 x_{1i}} + \cancel{\hat{\beta}_2 x_{2i}} + \dots + \cancel{\hat{\beta}_p x_{pi}}\}}$$

$$= \exp(\hat{\beta}_1)$$

Se X_1 é binária, basta repetir as contas da ausência da característica $(X_1 = 0)$ p/ a presença $(X_1 = 1)$