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Calculation of distance to default

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Abstract

Evaluation of the probability of default of the company is one of the fundamental issues of credit risk analysis. The probability of default is an important inputs into many types of credit risk management processes at the single name and portfolio level, as well as in the pricing and hedging of credit risk. Credit risk is an unseparated part of financial risk. The credit risk of the company is often discussed also as the risk of the default of the company. Default of the company is usually associated with the bankruptcy of the company. We are interested in the credit event or default event which is defined as a failure to accomplish a predetermined liabilities or to meet requirements detailed in the agreement. Modelling of credit risk for the prediction of the default should be in attention of many individuals and companies. Various credit rating agencies such as Standard and Poor, Fitch and Moody's were made for this case. The article is dedicated to the calculation of distance to default as a variable introduced in the KMV model.

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1. Introduction

Prediction of the default of the company can be made only with a certain degree of probability, it is never sure. Probability of the default of the company can be very small, but is never equal to zero. Conversely, if default event happens, it will cause the lender financial losses so the identification of the likelihood of default is an important issue (Kollár, 2014). People and companies have been forecasting the probability of the default for decades (Saunders & Allen, 2002). The probability of default can be modelled in various ways and also by the use of different models. These models evaluate the probability by the use of the market data and basically can be divided

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into two groups based on different assumptions, so we distinguish between structural and reduced – form models (Lehutová, 2011). There have been developed also hybrid models which try to integrate assumptions from both previous mentioned models, the structural and the reduced – form approach (Cisko & Klieštik, 2013).

Pricing models see the company´s equity as a call option on the underlying asset. Because at the maturity of debt bondholders receive their debts and equity holders take the rest. The assumptions are based on the use of (Delianedis & Geske, 2003): observable value *VE* and volatility of equity *E*, unobservable value *VA* and volatility of company´s asset *A,* Black and Scholes option pricing theory, Equity is a call option on the value of assets of the company – *VA is considered as C, VE as C,* Debt *D* is taken as a strike price, so *D considered as K* (Michaliková, Spuchľáková & Cúg, 2014).

Roots of structural models as is outlined come from the work of Black and Scholes (1973) and Merton (1974). Geske (1977, 1979) later drawn – out Merton´s assumptions by viewing that multiple default options for coupons, sinking funds, junior debt, safety covenants, or other payment obligations could be treated as compound options (Majerčák & Majerčáková, 2013). Another way of extension proved Black and Cox (1976) who allows reduction arrangements and limits on refinancing. On the other side Turnbull (1979) included corporate taxes and bankruptcy costs. Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Leland (1995) and Leland and Toft (1996), Eom, Helwege, and Huang (2003) also introduced another assumptions of structural models (Lando, 2004).

The article deals with the calculation of distance to default which is part of the KMV model introduced by Kealhofer, McQuown and Vasicek in 1974 also as an extension of Merton´s model and represents structural approach (Valášková, Gavláková & Dengov, 2014).

2. KMV model

Model KMV was established as is mentioned above by Keaholfer, McQuown and Vasicek in 1974 and is founded on assumptions of Merton´s bond pricing model. Later in 2002 was bought by the company Moody´s. KMV is the name set to a successful practical implementation of structural credit modelling (Boďa & Kanderová, 2010). They made some expectations in order to produce commercially acceptable credit methods. The main difficulty, as well as in other structural models, is how to assign dynamics to the company value, which is an unobserved process (Bielicky, Jeanblanc & Rutkowski, 2009).

The KMV model estimates a likelihood of default for each company in the example at any specified time. To estimate the likelihood of default the model deducts the face value of the debt of the company from an evaluation of the market value of the company and then splits this difference by an approximation of the volatility of the company (Bartošová, 2005). The result of this which is called *z-score* and is referred to as the distance to default, is then replaced into a cumulative density function to compute the possibility that the value of the company will be less than the face value of debt while forecasting horizon (Klieštik & Birtus, 2013). The market value of the company is than just the total amount of the market values of the debt of the company and the value of its equity. Calculation of the probability of default is simpler while both of these quantities are readily observable. While values of the equity are usually readily available, the reliable data on the market value of the company´s debt is commonly unobtainable (Kim, Ramaswamy & Sunderesan, 1993).

This model is based on the estimation of the quantities of company´s asset which means the current value and the volatility from the market value of the company´s equity and the equity's instantaneous volatility, along with knowing the outstanding and maturity of debt. The maturity of debt is selected and the book value of the debt is set equally to the face value of the debt. The default of the company then occurs when the value of the asset of the company falls under the *default point* (DD), which is the face value of the debt (Kollár & Cisko, 2014)

For the quantification of the probability of default by the use of KMV model is introduced the new variable *distance to default*, which represents the distance between the expected value of assets of the company and the default point and then splits this difference by an estimation of the volatility of the company in a time horizon. At that time, the distance to default is replaced into a cumulative density function to compute the likelihood that the value of the company will be fewer than the face value of debt at the maturity of the debt (Ammann, 2001).

Based on the assumption of Merton the equity is considered as a call option on the value of assets of the company and the time. So it monitors the subsequent stochastic differential equation:

$$dE = \mu_E E dt + \sigma_E E dW \tag{1}$$

Where: μE and σE are the instaneous anticipated rate of return on this equity and its volatility.
The dynamics of the equity can be written by the use of Ito´s Lemma as:

$$\begin{aligned} dE &= \frac{\partial E}{\partial F} dF + \frac{\partial E}{\partial t} dt + \frac{1}{2}\sigma_E^2 F^2 \frac{\partial^2 E}{\partial F^2}(dF)^2 \dots \\ &= \left(\frac{1}{2}\sigma_F^2 F^2 \frac{\partial^2 E}{\partial F^2} + \mu_F F \frac{\partial E}{\partial F} + \frac{\partial E}{\partial t}\right) dt + \sigma_F F \frac{\partial E}{\partial F} dW \end{aligned} \tag{2}$$

When we compare diffusion expressions shown in equations (1) and (2), then we can regain the relationship that:

$$E\sigma_E = F\sigma_F \frac{\partial E}{\partial F} \tag{3}$$

There is a need to add equation from Merton´s option pricing theory to continue with derivation:

$$\begin{aligned} E(F,\mathrm{t}) &= \mathrm{F(t)\,N(d_1)} - e^{-r(T-t)} D N(d_2) \\ d_{1,2} &= \frac{\log\left(D/F(t)\right) \pm \left(r - \frac{1}{2}\sigma_F^2\right)(T-t)}{\sigma_F\sqrt{T-t}} \end{aligned} \tag{4}$$

Additionally after this we can derive $Equity\ Delta \Delta^E = \frac{\partial E}{\partial F} = N(d_1) > 0$ from the equation (4).

Later we can find the new relation between the volatility of the company and that of the equity (Spuchľáková & Cúg, 2014):

$$E\sigma_E = F\sigma_F N(d_1) \tag{5}$$

Equally associating drift terms in equations (1) and (2) there is:

$$\mu_E E = \frac{1}{2}\sigma_F^2 F^2 \frac{\partial^2 E}{\partial F^2} + \mu_F F \frac{\partial E}{\partial F} + \frac{\partial E}{\partial t} \tag{6}$$

Additionally can be derived from the equation (4):

$$\begin{aligned} &Equity\ Gamma\ \Gamma^E = \frac{\partial^2 E}{\partial F^2} = \frac{n(d_1)}{F\sigma_F\sqrt{T-t}} > 0 \\ &Equity\ Theta\quad \theta^E = \frac{\partial E}{\partial t} = -\frac{Fn(d_1)\sigma_F}{2\sqrt{T-t}} - rDe^{-r(T-t)} N(d_2) \end{aligned} \tag{7}$$

So we can have:

$$\mu_F = \frac{\mu_E E - \theta^E - \frac{1}{2}\sigma_F^2 F^2 \Gamma^E}{F\Delta^E} \tag{8}$$

Value of equity for the public companies can be directly observed from stock exchange market, because it directly suggests that the value of the option written on the underlying value of the asset of the company can be observed (Bielicki & Rutkowski, 2002). Additionally the volatility of the equity can be gained by approximating the implied volatility from an observed option value or by using a historical stock returns data. Once we have risk – free interest rate and the time horizon of the debt, the only unknown quantities are the value of assets of the company F(t) and the volatility of the company σF. So then can be solved two nonlinear simultaneous equation (3) and (4) to determine F(t) and σF by the equity value, volatility value and capital structure (Šukalová & Ceniga, 2013).

3. Calculation of distance to default

KMV model violates the assumption of Merton that assets of the company are tradable so it is aware of this point. KMV model uses the Black – Scholes and Merton structures as inspiration to compute an intermediate phase called *Distance to Default* **(DD)** and after this calculate the probability of default. Firstly the Distance to Default has to be calculated and then can be developed and estimated the probability of default of a specific company by results of the values of assets of the company and company's volatility (Black & Scholes, 1973).

The default event occurs when the value of asset of the company is under the *default point*.

$$d^* = short - term\ debt + \frac{1}{2} \times long - term\ debt \tag{9}$$

In Merton´s model is the face value of the debt observed as the default point but by the use of the volatility of the asset of the company to measure can be calculated the Distance to Default. The chance that the company will default is less when the number of calculated Distance to Default is higher (Klieštik, 2009).

So the calculation of Distance to Default under the risk – neutral probability measure is:

$$Dis\tan ce\ to\ Default = \frac{E(F(t)) - d^*}{\sigma_F} = \frac{\ln\left(F(t)/D\right) + \left(r - \frac{1}{2}\sigma_F^2\right)(T-t)}{\sigma_F\sqrt{T-t}} \tag{10}$$

Where:

r	-	the risk – free rate of the return of the asset of the company,
F(t)	-	the current value of the asset of the company,
D	-	the face value of the debt,
σ_F	-	the annualized company value volatility

After this can be developed the equation for the probability of default. It is based on the classification that the value of the asset of the company is below the value of the debt (11).

This is default probability based on the underlying asset with some risk so the r (risk – free interest rate) is replaced by the expected return on the asset of the company μF.

$$PD(t) = P\left[F(t) \le D\right] = N(-DD) = N\left(-\frac{\ln\left(\frac{F(t)}{D}\right) + \left(\mu_F - \frac{\sigma_F^2}{2}\right)T}{\sigma_F \sqrt{T}} \right) \tag{11}$$

3.1. Example of calculation of Distance to Default

For the calculation of distance to default and then expected default probability is necessary to gain these characteristics (Brigo & Tarenghi, 2004): risk – free interest rate based on the data of EURIBOR, price and number of stocks of analysed companies, balance sheets of companies. Then the process of calculation continue the derivation of parameters: returns and volatility of equity based on the historical data, market value of equity (stock price x number of stock), risk – free interest rate from Euribor, time liabilities with maturity in one year, short – term liabilities and half of long – term liabilities, solve two nonlinear equations to get value and volatility of the asset, calculate Distance to Default and Probability to Default (Búc, Križanová & Klieštik, 2013).

The example was calculated on four companies: a) bank that needed help during the existence, b) company that produces cars and had no problems during existence, c) stock company that bankrupted, d) financial company without problems.

Based on the calculations we can assume that some financial problems can be anticipated one or sometimes one and half year before.

Analysed model is more valuable for rating then predicting the default probability. It can be assumed also that financial companies have higher probability to get into the problems or be defaulted during the crisis.

4. Conclusion

The articles deals with the calculation of distance to default, which is the variable introduced in model KMV and based on this variable can be computed probability of default. KMV model uses expectations and assumptions of original paper of Merton for quantification of credit risk, so the equity value of the company is seen as a call option with the underlying asset corresponding with the company´s value and with the strike price is equal to the level of foreign sources of analysed company. KMV model defines that the failure of the analysed company occurs at a time when the market value of the business assets derived from the market price of the equity falls below the payable debt. At the end of the article is example of calculation of distance to default.

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