Homework 3

Due March 15,2023

Problem 1

Let

$$X_1, ..., X_n \stackrel{iid}{\sim} N(0,1) = F_X, \quad Y_1, ..., Y_n \stackrel{iid}{\sim} t_{10} = F_Y,$$

where t_{10} is the t-distribution with 10 degrees of freedom. Use simulation to answer the following:

1.1

Suppose $n = 1000$, does the Kolmogorov-Smirnov Two-Sample Test rejects the hypothesis at $\alpha = 0.05$?

1.2

On average, how large does n has to be for the null hypothesis to be rejected?

1.3

Suppose we change the degree of freedom from 10 to d. How does n in 1.2 change with d? Plot a graph to illustrate their relationship.

Problem 2

2.1

Generate samples $(n = 30)$ according to the following model

$$X_1, ..., X_n \stackrel{iid}{\sim} N(0,1) = F_X, \quad Y_1, ..., Y_n \stackrel{iid}{\sim} N(0,2^2) = F_Y,$$

Does the Mann-Whitney U test reject the null hypothesis $H_0 : F_X = F_Y$ at $\alpha = 0.1$?

2.2

Generate samples $(n = 30)$ according to the following model

$$X_1, ..., X_n \overset{iid}{\sim} N(0, 1) = F_X, \quad Y_1, ..., Y_n \overset{iid}{\sim} N(0.5, 1) = F_Y,$$

Does the Mann-Whitney U test reject the null hypothesis $H_0 : F_X = F_Y$ at $\alpha = 0.1$?

Problem 3

Suppose you have the following sample

$$1, \ 2, \ 2.5, \ 3.3, \ 10, \ 15, \ 15.5, \ 17, \ 20.$$

Let the population median be M. Does the Wilcoxon signed rank test reject $H_0 : M = 14$ at $\alpha = 0.1$?