非参 2023.

Problem ①:

(1) $X_1 \cdots X_n \overset{iid}{\sim} F$, derive the delta ~~est~~ method estimation of Variance

(2) Write the definition of influence function of F.

(3) Derive the asymptotic distribution of $\sqrt{n}(T(\hat{F}_n) - T(F))$, $T(F)$ is linear functional.

Problem ②:

(1) Write the definition of kernel density estimator.

(2) $\hat{f}_n^h(x)$ is the kernel density estimator, write the bias and variance of $\hat{f}_n^h(x)$

(3) Give a practical way to pick out an h in $\hat{f}_n^h(x)$, to minimize $E[\int (\hat{f}_n^h(x) - f(x))^2 dx]$

Problem ③:

(1) Write the definition of linear smoother.

(2) Write the effective degree of freedom of linear smoother, explain why it is a reasonable measure of degree of freedom.

(3) write the definition of Nadaraya-Watson kernel estimator, explain why it's called as local constant estimator.

(4). Write the definition of natural spline. ~~Exp~~ When does natural spline come into nonparametric regression, why we need natural spline?

(5) $Y_i = r(X_i) + \sigma(X_i)\varepsilon_i$, explain how to derive $\hat{\sigma}^2$ here.

Problem ④:

$Z_i = \theta_i + \sigma \varepsilon_i$, $\varepsilon_i \sim N(0,1)$; $\theta \in \{\theta : \sum_{i=1}^{n} \theta_i^2 \le c^2\}$.

(1) How does normal-means problem related to nonparametric regression?

(2) Write the definition of J-S estimator.

(3) Derive the J-S estimator. (hint: use SURE or Bayes).

(4) Explain why J-S estimator is good (hint: use pinsker's theorem).

(5) ~~Exp~~ Explain why the assumption $\theta \in \{\theta : \sum_{i=1}^{n} \theta_i^2 \le c^2\}$ is reasonable.

Problem ⑤:

(1) Write the definition of fourier transform.

(2) Explain one limitation of fourier transform in detail.

(3) Explain why wavelet can solve the limitation you explained in (2), explain in detail.

(4) let $f_{jk}(x) = 2^{j/2} f(2^j x - k)$ $\phi(x) = I(0 \le x \le 1)$ $\psi(x) = -I(0 \le x \le \frac{1}{2}) + I(\frac{1}{2} < x \le 1)$

and $W_j = \{\psi_{jk}(x), k = 0, 1, \cdots 2^j - 1\}$.

Prove: $\{\phi, W_0, W_1, \cdots W_j\}$ is an orthonormal basis.

(5) when using wavelet in nonparametric regression, how will you estimate the coefficient? explain in detail.