Homework 4

Due March 22 2023

Problem 1

Let

$$X_1, ..., X_n \overset{iid}{\sim} N(0, \sigma_X^2), \quad Y_1, ..., Y_n \overset{iid}{\sim} N(0, \sigma_Y^2),$$

Use simulation to answer the following:

1.1

Suppose $n = 100$, $\sigma_X^2 = 1$ and $\sigma_Y^2 = 1.2$, use the Mood test and the Sukhatme test to test $H_0 : \sigma_X^2 = \sigma_Y^2$ at level $\alpha = 0.05$.

1.2

Can you think of a scenario where the Mood test is more powerful than the Sukhatme test?

1.3

Can you think of a scenario where the Sukhatme test is more powerful than the Mood test?

Problem 2

Suppose X and Y follows the same continuous distribution and are independent. If the median of the distribution is 0, show that $\mathbb{P}(\{Y < X < 0\} \cup \{0 < X < Y\}) = \frac{1}{4}$.

Problem 3

Let

$$X_1, ..., X_n \overset{iid}{\sim} N(0, 1) = F_X, \quad Y_1, ..., Y_n \overset{iid}{\sim} N(\mu, 1) = F_Y,$$

Use simulation to answer the following:

3.1

Suppose $n = 100$ and $\mu = 0.1$, use the Wilcoxon Rank-Sum test and the Terry-Hoeffding test to test $H_0 : F_X = F_Y$ at level $\alpha = 0.05$.

3.2

Can you think of a scenario where the Wilcoxcon Rank-Sum test is more powerful than the Terry-Hoeffding test?

3.3

Can you think of a scenario where the Terry-Hoeffding test is more powerful than the the Wilcoxcon Rank-Sum test?