Homework 1

Due March 1 2023

Problem 1

A manufacturer wants to market a new brand of heat-resistant tiles which may be used on the space shuttle. A random sample of m of these tiles is put on a test and the heat resistance capacities of the tiles are measured. Let $X_{(1)}$ denote the smallest of these measurements. The manufacturer is interested in finding the probability that in a future test (performed by, say, an independent agency) of a random sample of n of these tiles, at least k,$k = 1, 2, .., n$ will have a heat resistance capacity exceeding $X_{(1)}$ units. Assume that the heat resistance capacities of these tiles follows a continuous distribution with cdf F. Show that the probability of interest is given by $\sum_{r=k}^{n} P(r)$, where $P(r) = \dfrac{mn!(r+m-1)!}{r!(n+m)!}$.

Problem 2

Let $X_{(1)} < X_{(2)} < ... < X_{(n)}$ be order statistics for a random sample from the exponential distribution $F_X(x) = \exp(-x)$ for $x \geq 0$

1. Show that $X_{(r)}$ and $X_s - X_{(r)}$ are independent for any $s > r$.
2. Find the distribution of $X_{(r+1)} - X_{(r)}$.
3. Show that $E(X_{(i)}) = \sum_{j=1}^{i} \frac{1}{n+1-j}$.
4. Interpret the significance of these results if the sample arose from a life test on n light bulbs with exponential lifetimes.

Problem 3

If X is a continuous random variable with pdf $f_X(x) = 2(1-x)$, $0 < x < 1$, find the transformation $Y = g(X)$ such that $Y \sim U(0, 2)$.