

MATH130165h: Homework 2

Due Apr 11, 2024

Problem 1. [20 pt] Let $A \in \mathbb{R}^{m \times k}$, $B \in \mathbb{R}^{k \times n}$ and $C = AB$. Show that there exists δC such that,

$$\tilde{C} = AB + \delta C,$$

and an upper bound for δC (element-wise or matrix norm upper bound, you could also show both).

Lemma A. Let P be an exact Householder transformation, and \hat{P} be the computed one. Then

$$\mathfrak{fl}(\hat{P}A) = P(A + E)$$

with

$$\|E\|_2 = \|A\|_2 O(\epsilon_{\text{machine}}).$$

Problem 2. [20 pt] Prove Lemma A.

Problem 3. [30 pt] Implement a memory-efficient LU factorization with partial pivoting, i.e., store L and U together in a $m \times m$ matrix and permutation matrix P as a vector of size m .

Problem 4. [15 pt] Implement the solve function associated with the memory-efficient LU factorization with partial pivoting.

Problem 5. [5 pt] Apply your LU factorization code to the following matrices,

$$A = \begin{pmatrix} 1 & & & & & & 1 \\ -1 & 1 & & & & & 1 \\ -1 & -1 & 1 & & & & 1 \\ \vdots & \vdots & \vdots & \ddots & & & \vdots \\ -1 & -1 & -1 & \cdots & -1 & 1 & 1 \\ -1 & -1 & -1 & \cdots & -1 & -1 & 1 \end{pmatrix}$$

of size 8×8 and 100×100 .