

Problem 2 $\nearrow i=1, \dots, n$ # sign changes in $p_i(a) = \#$ zeros of $p_n(a)$ Evaluate polynomials at $x=1$:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$P_0(1) = 0 \quad P_1(1) = 0 \quad P_2(1) = -1$$

$$P_3(1) = -1 \quad P_4(1) = -1$$

 \angle There's 1 eigenvalue smaller than 1.Evaluate polynomials at $x=2$:

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$P_0(2) = 0 \quad P_1(2) = -1 \quad P_2(2) = 0$$

$$P_3(2) = 1 \quad P_4(2) = 1$$

 \angle There're 2 eigenvalues smaller than 2. \Rightarrow There's 1 eigenvalue in the interval $[1, 2]$.