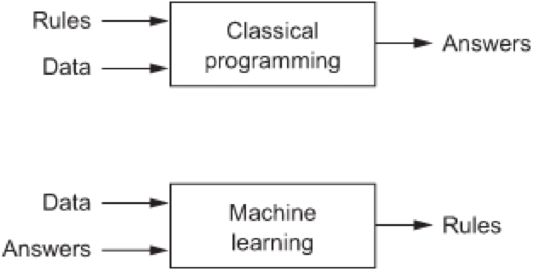
Natural Language Processing for Law and Social Science

4. Supervised Learning with Text

Weekly Q&A Page

What is machine learning?



- In classical computer programming, humans input the rules and the data, and the computer provides answers.
- In machine learning, humans input the data and the answers, and the computer learns the rules.

What do ML Algorithms do? Fit a function to data points

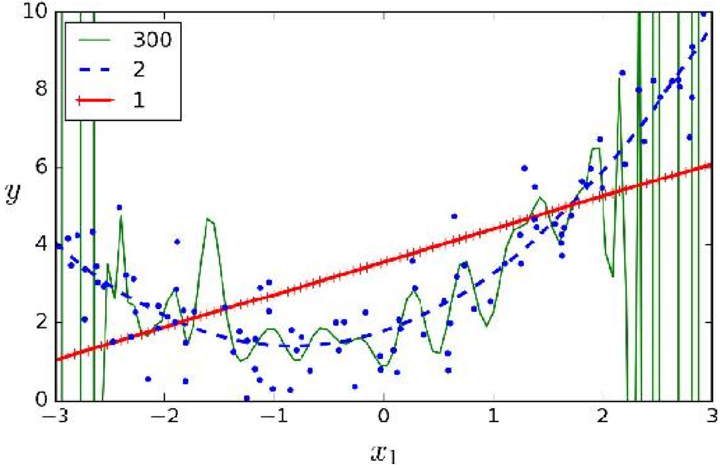


Figure 4-14. High-degree Polynomial Regression

What do ML Algorithms do? Minimize a cost function

- A typical cost function (or loss function) for regression problems is Mean Squared Error (MSE):

$$\text{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(x_i;\theta) - y_i)^2$$

  - n_D, the number of rows/observations
  - x, the matrix of predictors, with row x_i
  - y, the vector of outcomes, with item y_i
  - $h(x_i;\theta) = \hat{y}$ the model prediction (hypothesis)

The **data** (x, y) are taken as given, and the ML algorithm searches for **parameters** θ to minimize the cost function.

Linear Regression is Machine Learning

- Ordinary Least Squares Regression (OLS) assumes the functional form $f(x;\theta) = x_i'\theta$ and minimizes the mean squared error (MSE)

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- This minimand has a closed form solution

$$\hat{\theta} = (\boldsymbol{x}'\boldsymbol{x})^{-1}\boldsymbol{x}'\boldsymbol{y}$$

  - most machine learning models do **not** have a closed form solution $\rightarrow$ use numerical optimization instead (gradient descent).

$$\mathrm{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(\theta; \boldsymbol{x}_i) - y_i)^2$$

- The partial derivative for feature j is

$$\frac{\partial \mathrm{MSE}}{\partial \theta_j} = \frac{2}{n_D} \sum_{i=1}^{n_D} (\underbrace{h(\theta; \boldsymbol{x}_i) - y_i}_{\text{error for this obs}}) \underbrace{\frac{\partial h(\theta; \boldsymbol{x}_i)}{\partial \theta_j}}_{\text{how } \theta_j \text{ shifts } h(\cdot)}$$

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- The ***gradient*** ∇ gives the vector of these partial derivatives for all features:

$$\nabla_\theta \text{MSE} = \begin{bmatrix} \frac{\partial \text{MSE}}{\partial \theta_1} \\ \frac{\partial \text{MSE}}{\partial \theta_2} \\ \vdots \\ \frac{\partial \text{MSE}}{\partial \theta_{n_x}} \end{bmatrix}$$

- ***Gradient descent*** nudges θ against the gradient (the direction that reduces MSE):

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta \text{MSE}$$

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- Each document i has an associated outcome or label $\boldsymbol{y}_i$ with dimensions $n_y \geq 1$
- Some documents are labeled and some are unlabeled $\rightarrow$
  - we would like to learn a function $\hat{\boldsymbol{y}}(d_i)$ based on the labeled data ...
  - ... to machine-classify the unlabeled data.

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- The solution: all the methods from Weeks 1, 2, 3 for extracting informative numerical information from documents:
  - style features
  - counts over dictionary patterns
  - tokens
  - n-grams
  - principal components
  - topic shares
  - etc.
- documents can thus be **featurized** – represented as a matrix of vectors $\boldsymbol{x}$ with $n_x \geq 1$ features.

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Determined by the data type of the outcome variable (or label):

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- **Regression**: a one-dimensional, continuous, real-valued outcome.
  - e.g., number of days of prison assigned
- **Multinomial Classification**: Three or more discrete, un-ordered outcomes.
  - e.g., predict what judge is assigned to a case: Alito, Breyer, or Cardozo

Loss functions, more generally

- The loss function $L(\hat{\boldsymbol{y}}, \boldsymbol{y})$ assigns a score based on prediction and truth:
  - Should be bounded from below, with the minimum attained only for cases where the prediction is correct.
- The average loss for the test set is

$$\mathcal{L}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i)$$

- The estimated parameter matrix θ solves

$$\hat{\theta} = \arg\min_{\theta} \mathcal{L}(\theta)$$

$\hookrightarrow$ optimizes over parameter space; treats the data as constants.

Gradient Descent

- even when cost function is not convex (eg neural nets), gradient descent often gets decent results.
- ***Stochastic* gradient descent (SGD)** computes the gradient for a single randomly sampled instance (at each iteration).
  - Much faster, still works well.

Data Prep for Machine Learning

- Data Pre-Processing: See Geron Chapter 2 for pandas and sklearn syntax:
  - imputing missing values.
  - feature scaling (often helpful/necessary for ML models to work well)
    - if predictors are sparse (e.g. bag-of-words), use `StandardScaler(with_mean=False)`.
  - encoding categorical variables.
  - **Best practice: reproducible data pipeline**.

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- Train/Test Split:
  - ML models can achieve arbitrarily high accuracy in-sample, so performance should be evaluated out-of-sample.
  - standard approach: randomly sample 80% training dataset to learn parameters, form predictions in 20% testing dataset for evaluating performance.

Use Cross-Validation During Model Training

- Within the training set:
  - Use cross-validation with grid search to get model performance metrics across subsets of data using different hyperparameter specs.
  - Find the best hyperparameters for out-of-fold prediction in the training set.
- Then evaluate model performance in the test set using these hyperparameters.

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- Then evaluate model performance in the test set using these hyperparameters.
- Cross-validation is less common in deep learning, where training multiple models is too computationally expensive.
  - instead, use dropout and early stopping (next week).

Model Evaluation in Test Set

Evaluating a "good" model is context-dependent. Here are some basics.

Regression:

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Classification:

- more complicated, but accuracy is a good baseline:
  accuracy $=$ ($\#$ correct test-set predictions) / ($\#$ of test-set observations)
- What if one of the outcomes is over-represented – e.g., 19 out of 20? Then I can guess the modal class and get 95% accuracy.
  - Some alternative classifier metrics designed to address class imbalance (more below).

ML Essentials

Overview

Regression / Regularization

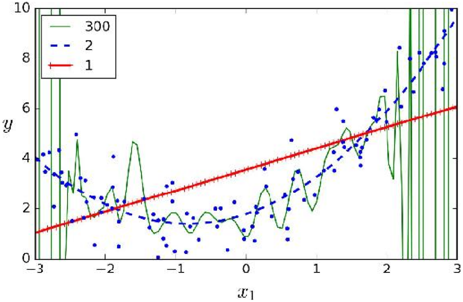
Binary Classification

Multi-Class Models

Osnabruegge, Ash, and Morelli 2021

Regression models $\leftrightarrow$ Continuous outcome

- If the outcome is continuous (e.g., Y = tax revenues collected, or criminal sentence imposed in months of prison):
  - Need a regression model.
- Problems with OLS:
  - tends to over-fit training data.
  - cannot handle multicollinearity.



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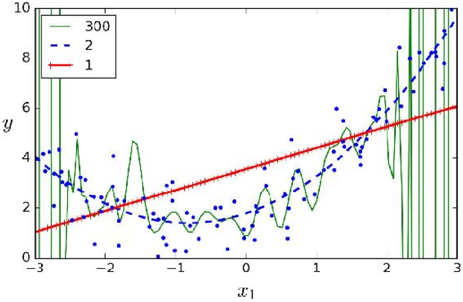

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- ***Regularization***: model training methods designed to reduce/prevent over-fitting.

Regularized Loss Function

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i) + \lambda R(\theta)$$

- $R(\theta)$ is a "regularization function" or "regularizer", designed to reduce over-fitting.
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- Elastic Net: $R_{enet} = \lambda_1 R_1 + \lambda_2 R_2$

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  - The standard approach is to apply a transformation (e.g. sigmoid/logit) to normalize $\hat{y} \in [0, 1]$.
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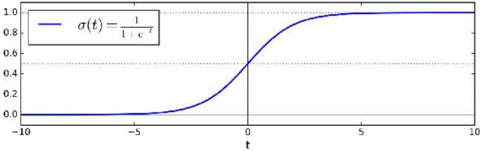
- The binary cross-entropy (or log loss) is:

$$L(\theta) = \underbrace{-\frac{1}{n_D}}_{\text{negative}} \sum_{i=1}^{n_D} [\underbrace{y_i}_{y_i=1} \underbrace{\log(\hat{y}_i)}_{\text{log prob} y_i=1} + \underbrace{(1-y_i)}_{y_i=0} \underbrace{\log(1-\hat{y}_i)}_{\text{log prob} y_i=0}]$$

- In **logistic regression** we use a sigmoid transformation:

$$\hat{y} = \text{sigmoid}(\boldsymbol{x} \cdot \theta) = \frac{1}{1 + \exp(-\boldsymbol{x} \cdot \theta)}$$



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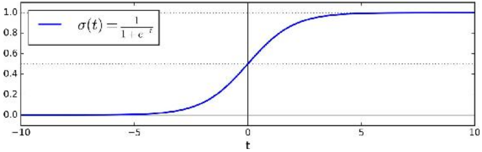

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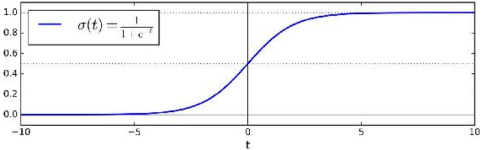

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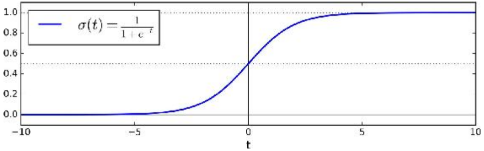

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- Like linear regression, logistic regression can be regularized with L1 or L2 penalties.

A **Confusion Matrix** is a nice way to visualize classifier performance:

		Predicted Class	
		Negative	Positive
True Class	Negative	**# True Negatives**	# False Positives
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$$\text{Recall (for positive class)} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$

- Recall decreases with false negatives. "When this outcome occurs, I don't miss it."

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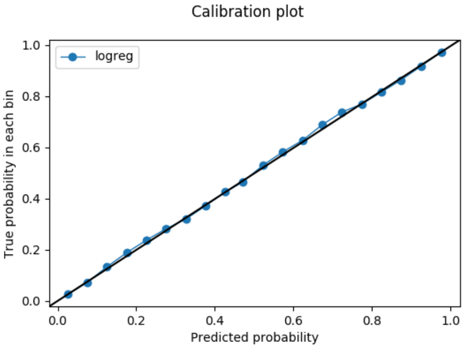
AUC-ROC = Area Under the Receiver Operating Characteristic Curve

- provides an aggregate measure of performance across all possible classification thresholds.
- Interpretation: randomly sample one positive and one negative example. AUC = probability that the model correctly guesses which is which.

Evaluating Classification Models: Calibration Curves



- Plotting the binned fraction in a category (Y axis) against the predicted probability in a category (X axis):
- Provides evidence of whether the classifer is replicating the conditional distribution of the outcome.

```
from seaborn import regplot
regplot(y_test, y_pred, x_bins=20)
```

Application: Predicting Political Party from Text

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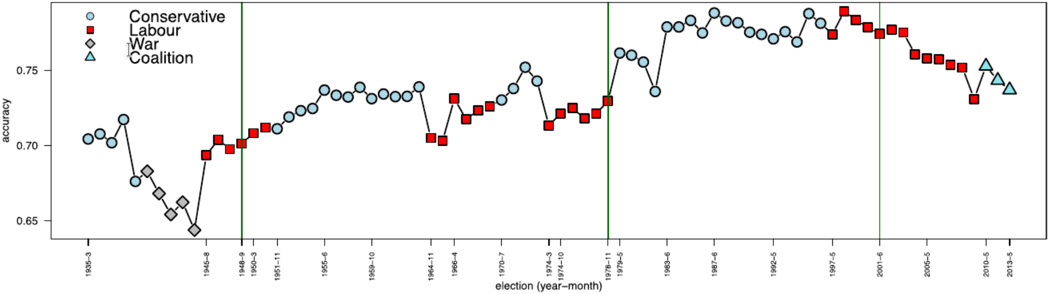
In years that classifier is more accurate, speech is more polarized:



Multiple Classes: Setup

- The outcome is $y_i \in \{1, ..., k, ..., n_y\}$ output classes, which can also be represented as a one-hot vector
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- We want to learn a vector function
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \theta)$$
taking text features $\boldsymbol{x}$ as inputs and outputting a vector of probabilities across outcome classes:
$$\hat{\boldsymbol{y}} = \{\hat{y}^1, ..., \hat{y}^{n_y}\}, \sum_{k=1}^{n_y} \hat{y}^k = 1, \hat{y}^k \geq 0 \ \forall k$$
- for prediction step, can select the highest-probability class:
$$\tilde{y} = \arg\max_k \hat{y}_{[k]}$$

Categorical Cross Entropy

- The standard loss function in multinomial classification is **categorical cross entropy**:

$$L(\theta) = -\sum_{k=1}^{n_y} \boldsymbol{y}^k \log(\hat{y}^k(\boldsymbol{x}, \theta))$$

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- Since there is just one true class ($y = 1$ for one class k^*, and zero for others), simplifies to

$$L(\theta) = -\log(\hat{y}^{k^*}(\boldsymbol{x}, \theta))$$

  - Rewards putting higher probability on the true class, ignores distribution of probabilities on other classes.
  - function is convex $\rightarrow$ gradient descent will find the optimum.

Multinomial Logistic Regression

Multinomial logistic regression computes probabilities for each class k using the softmax transformation

$$\hat{y}_k(\boldsymbol{x}_i) = \Pr(y_i = k) = \frac{\exp(\theta_k' \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta_l' \boldsymbol{x}_i)}$$

- softmax is the multiclass generalization of sigmoid $\rightarrow$ can then interpret $\hat{y}$ as probabilities.
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The **L2-penalized logistic regression** has loss function

$$\mathcal{L}(\theta) = -\frac{1}{n_D} \sum_{i=1}^{n_D} \log \frac{\exp(\theta'_{k^*} \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta'_l \boldsymbol{x}_i)} + \lambda \sum_{j=1}^{n_x} \sum_{k=1}^{n_y} (\theta_{[j,k]})^2$$

- λ = strength of L2 penalty (could also add lasso penalty)
  - as before, predictors should be scaled to the same variance.

		Predicted Class		
		Class A	Class B	Class C
True Class	Class A	Correct A	A, classed as B	A, classed as C
	Class B	B, classed as A	Correct B	B, classed as C
	Class C	C, classed as A	C, classed as B	Correct C

More generally, with **multi-class confusion matrix** M with items M_{ij} (row i, column j):

$$\text{Precision for } k = \frac{\text{True Positives for } k}{\text{True Positives for k} + \text{False Positives for } k} = \frac{M_{kk}}{\sum_l M_{lk}}$$

$$\text{Recall for } k = \frac{\text{True Positives for } k}{\text{True Positives for } k + \text{False Negatives for } k} = \frac{M_{kk}}{\sum_l M_{kl}}$$

$$F_1(k) = 2 \times \frac{\text{precision}(k) \times \text{recall}(k)}{\text{precision}(k) + \text{recall}(k)}$$

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Can average these metrics across classes to get aggregate metrics.

- e.g., balanced accuracy = unweighted average of recalls across classes.
- can weight classes by their frequency in dataset

Cross-Domain (Transfer) Learning

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  - train a model in a big labeled dataset
  - apply in a smaller (mostly) unlabeled dataset

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- In NLP:
  - transfer learning is intuitive because NLP tasks share common knowledge about language.
  - labeled data is scarce/expensive, so learn tasks on tons of unlabeled data.
  - reflected in success of pre-trained models, e.g. BERT and GPT.

Osnabruegge, Ash, and Morelli 2021

This paper takes the idea of transfer learning to the political science context.

- Learn to predict political topics from text in a labeled corpus (party manifestos from Comparative Manifesto Project)

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- Use for empirical analysis of electoral institutions and speech content.

Overview of Text Analysis Methods (Osnabruegge et al 2021

	Dictionaries (Custom)	Dictionaries (Generic)	Topic Modeling	Within-Domain Supervised Learning	Cross-Domain Supervised Learning
Design/Annotation Costs	High	Low	Low	High	Moderate
Specificity	High	Moderate	Low	High	Moderate
Interpretability	High	High	Moderate	High	High
Validatability	Low	Low	Low	High	High

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Group Activity