ELEC2205 Electronic Design:
D3 – Analogue Circuit Design Exercise

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Abstract

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1 Theoretical Design

The multi-stage amplifier circuit can be separated into two circuits; a common emitter stage followed by a common collector stage. Each stage can be analysed separately to calculate appropriate resister and capacitor values before adding the circuits together.

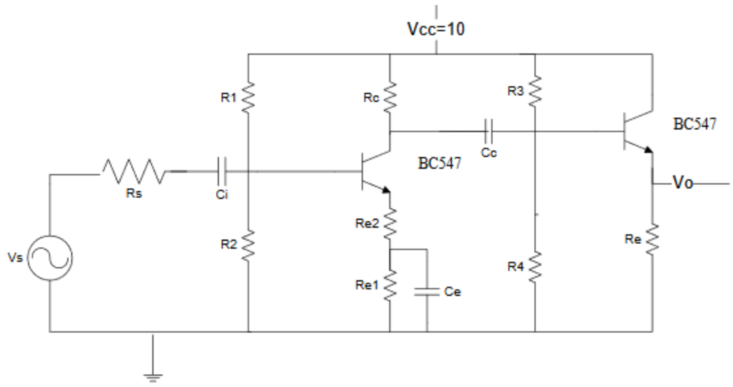

Figure 1: Two stage amplifier circuit

1.1 First stage: Common Emitter Circuit

The first circuit can be identified as a common emitter stage with partially by-passed emitter resistance.

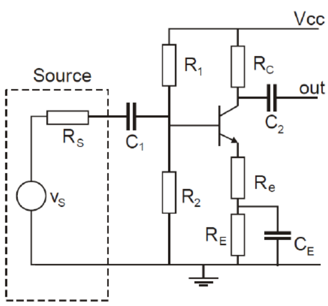

Figure 2: Common emitter stage

1.1.1 Mid-band Gain

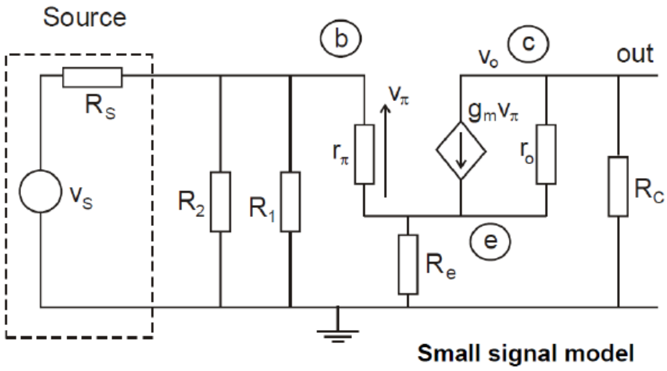

Figure 3: Small signal model of common emitter

By performing nodal analysis on Figure 3, the following equations can be obtained.

$$
\begin{aligned}
\text{Base: } & \frac{v_b - v_s}{R_s} + \frac{v_b}{R_1} + \frac{v_b}{R_2} + \frac{v_b - v_e}{r_\pi} && = 0 \\
& \frac{v_b - v_s}{R_s} + v_b\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{v_b - v_e}{r_\pi} && = 0 \qquad (1a) \\
\text{Emitter: } & \frac{v_e - v_b}{r_\pi} + g_m v_\pi + \frac{v_e}{R_e} && = 0 \\
& \frac{v_e - v_b}{r_\pi} - g_m(v_b - v_e) + \frac{v_e}{R_e} && = 0 \qquad (1b) \\
\text{Collector: } & \frac{v_c}{R_c} + g_m(v_b - v_e) && = 0 \qquad (1c)
\end{aligned}
$$

Rearranging equation 1b

$$
\begin{aligned}
v_e[\frac{1}{r_\pi} + \frac{1}{R_e} + g_m] &= \frac{v_b}{r_\pi} + g_m v_b \\
v_e[R_e(1 + g_m r_\pi) + r_\pi] &= v_b R_e(1 + g_m r_\pi)
\end{aligned}
$$

Using the fact that $g_m r_\pi = \beta$

$$v_e = v_b R_e(\frac{1 + \beta}{R_e(1 + \beta) + r_\pi}) \qquad (2)$$

Modifying equation 1c as such

$$
\begin{aligned}
\frac{v_c}{R_c} + g_m(v_b - v_e) &= 0 \\
\frac{v_c}{R_c} + g_m v_b &= g_m v_e \\
v_e &= \frac{v_c}{g_m R_c} + v_b
\end{aligned}
$$

gives a value of v_e that can be substituted back into equation 2

$$\begin{aligned}
v_b R_e\left(\frac{1+\beta}{R_e(1+\beta)+r_\pi}\right) &= \frac{v_c}{g_m R_c} + v_b \\
v_b R_e(1+\beta) &= \frac{v_c(R_e(1+\beta)+r_\pi)}{g_m R_c} + v_b(R_e(1+\beta)+r_\pi) \\
-v_b r_\pi &= \frac{v_c(R_e(1+\beta)+r_\pi)}{g_m R_c} \\
\frac{v_c}{v_b} &= -\frac{r_\pi g_m R_c}{R_e(1+\beta)+r_\pi} \\
\frac{v_c}{v_b} &= -\frac{\beta R_c}{R_e(1+\beta)+r_\pi}
\end{aligned}$$

This may be approximated as

$$A = -\frac{\beta R_c}{R_e(1+\beta)+r_\pi} \approx -\frac{R_c}{R_e} \tag{3}$$

1.1.2 Input Impedance

The impedance into the base terminal of the common emitter circuit is given by

$$\begin{aligned}
R_b &= \frac{v_b}{i_b} \\
\text{Where } i_b &= \frac{v_b - v_e}{r_\pi}
\end{aligned}$$

Therefore

$$\begin{aligned}
R_b &= \frac{v_b r_\pi}{v_b - v_e} \\
&= \frac{r_\pi}{1 - \frac{v_e}{v_b}}
\end{aligned} \tag{4}$$

From equation 2

$$\frac{v_e}{v_b} = R_e\left(\frac{1+\beta}{R_e(1+\beta)+r_\pi}\right) \tag{5}$$

Substituting equation 5 into 4 and rearranging gives gives

$$R_b = r_\pi + R_e(\beta+1)$$

Therefore input impedance is the parallel combination of R_1, R_2 and $R_b = r_\pi + R_e(\beta+1)$

$$R_i = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_\pi + R_e(\beta+1)}\right)^{-1} \tag{6}$$

1.1.3 Output Impedance

By examining 3 it can be seen that the output impedance will be

$$R_o = R_c \tag{7}$$

1.2 Common Collector Circuit

The second stage of the circuit consists of a straight forward common collector circuit as shown in figure 4.

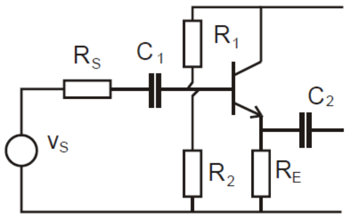

Figure 4: Common emitter stage

1.2.1 Mid-band Gain

To determine the mid-band gain, the circuit should be redraw with respect to the small signal model, as in 5

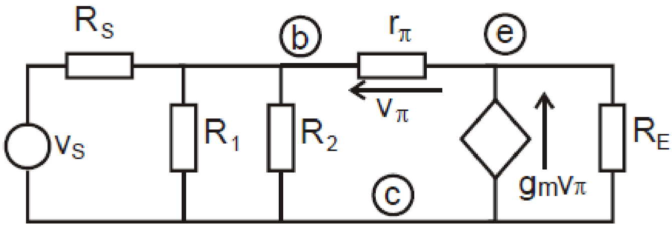

Figure 5: Common emitter stage

By using KVL on the emitter terminal we can establish that

$$\frac{v_e - v_s}{r_\pi} + g_m v_\pi + \frac{v_e}{R_E} = 0 \tag{8}$$

$$\frac{v_e - v_s}{r_\pi} + g_m(v_e - v_s) + \frac{v_e}{R_E} = 0$$

$$\frac{v_e - v_s}{r_\pi} + \frac{\beta(v_e - v_s)}{r_\pi} + \frac{v_e}{R_E} = 0$$

$$v_e(\beta R_E + R_E + r_\pi) = v_s R_E(\beta + 1)$$

$$\frac{v_e}{v_s} = \frac{R_E(\beta + 1)}{R_E(\beta + 1) + r_\pi} \tag{9}$$

Therefore gain ≈ 1

1.3 Input Impedance

Input impedance

$$R_i = \frac{v_s}{i_b} \tag{10}$$

Neglecting R_1, R_2 and R_s gives

$$i_b = \frac{v_s - v_e}{r_\pi}$$

Substituting equation ?? into this gives

$$\begin{aligned} i_b &= \frac{v_s}{r_\pi} - \frac{v_s}{r_\pi}\left(\frac{R_E(\beta+1)}{R_E(\beta+1)+r_\pi}\right) \\ &= \frac{v_s}{r_\pi}\left(1 - \frac{R_E(\beta+1)}{R_E(\beta+1)+r_\pi}\right) \\ &= \frac{v_s}{R_E(\beta+1)+r_\pi} \qquad (11) \end{aligned}$$

Therefore combining 10 and 11 gives

$$R_i = \frac{v_s}{i_b} = R_E(\beta+1) + r_\pi \qquad (12)$$

1.3.1 Output Impedance

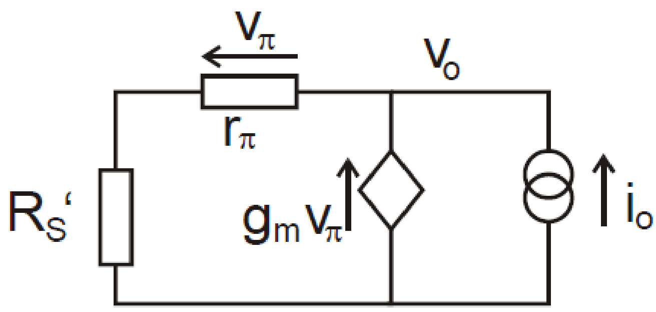

Figure 6: Common emitter stage

In figure 6 the Thevenin equivalent of the source resistances has been taken (R_s')

$$R_s' = R_s||R_1||R_2$$

By KVL

$$i_o + g_m v_\pi - \frac{v_o}{r_\pi R_s'} = 0 \qquad (13)$$

$$v_\pi = -\frac{r_\pi v_o}{r_\pi + R_s'} \qquad (14)$$

Substituting equation 13 into 14 gives

$$\begin{aligned} i_o - g_m \frac{r_\pi v_o}{r_\pi + R_s'} - \frac{v_o}{r_\pi R_s'} &= 0 \\ \frac{v_o(1+g_m r_\pi)}{r_\pi + R_s'} &= i_b \\ R_o = \frac{v_o}{i_b} &= \frac{r_\pi + R_s'}{1+\beta} \qquad (15) \end{aligned}$$

This shows that the output impedance is dependant on the source and bias impedance.