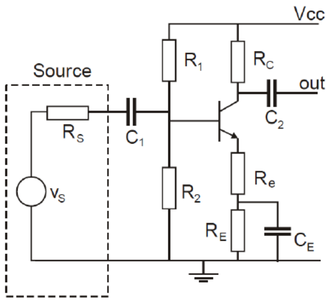
ELEC2205 Electronic Design:
D3 – Analogue Circuit Design Exercise

Jemma Watson
jw14g15
Tutor: Dr Tracy Melvin

1 Derivations

1.1 First stage: Common Emitter Circuit

The first circuit can be identified as a common emitter stage with partially by-passed emitter resistance.



1.1.1 Mid-band Gain

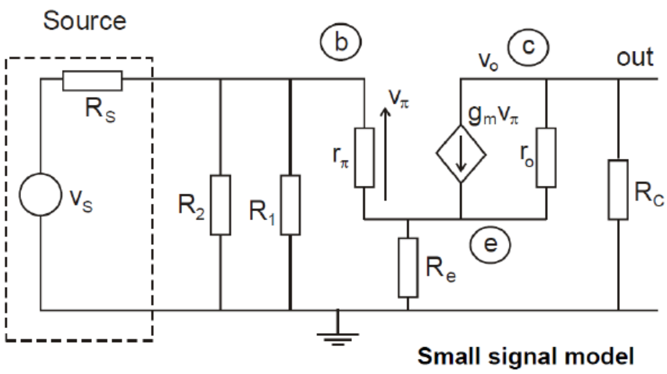


Figure 1: Small signal model of common emitter

By performing nodal analysis on Figure 1, the following equations can be obtained.

$$
\begin{aligned}
\text{Base: } & \frac{v_b - v_s}{R_s} + \frac{v_b}{R_1} + \frac{v_b}{R_2} + \frac{v_b - v_e}{r_\pi} && = 0 \\
& \frac{v_b - v_s}{R_s} + v_b\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{v_b - v_e}{r_\pi} && = 0 \qquad \text{(1a)} \\
\text{Emitter: } & \frac{v_e - v_b}{r_\pi} + g_m v_\pi + \frac{v_e}{R_e} && = 0 \\
& \frac{v_e - v_b}{r_\pi} - g_m(v_b - v_e) + \frac{v_e}{R_e} && = 0 \qquad \text{(1b)} \\
\text{Collector: } & \frac{v_c}{R_c} + g_m(v_b - v_e) && = 0 \qquad \text{(1c)}
\end{aligned}
$$

Rearranging equation 1b

$$
\begin{aligned}
v_e[\frac{1}{r_\pi} + \frac{1}{R_e} + g_m] &= \frac{v_b}{r_\pi} + g_m v_b \\
v_e[R_e(1 + g_m r_\pi) + r_\pi] &= v_b R_e(1 + g_m r_\pi)
\end{aligned}
$$

Using the fact that $g_m r_\pi = \beta$

$$
v_e = v_b R_e(\frac{1+\beta}{R_e(1+\beta) + r_\pi}) \tag{2}
$$

Modifying equation 1c as such

$$
\begin{aligned}
\frac{v_c}{R_c} + g_m(v_b - v_e) = 0 \\
\frac{v_c}{R_c} + g_m v_b = g_m v_e \\
v_e = \frac{v_c}{g_m R_c} + v_b
\end{aligned}
$$

gives a value of v_e that can be substituted back into equation 2

$$
\begin{aligned}
v_b R_e(\frac{1+\beta}{R_e(1+\beta) + r_\pi}) &= \frac{v_c}{g_m R_c} + v_b \\
v_b R_e(1+\beta) &= \frac{v_c(R_e(1+\beta) + r_\pi)}{g_m R_c} + v_b(R_e(1+\beta) + r_\pi) \\
-v_b r_\pi &= \frac{v_c(R_e(1+\beta) + r_\pi)}{g_m R_c} \\
\frac{v_c}{v_b} &= -\frac{r_\pi g_m R_c}{R_e(1+\beta) + r_\pi} \\
\frac{v_c}{v_b} &= -\frac{\beta R_c}{R_e(1+\beta) + r_\pi}
\end{aligned}
$$

This may be approximated as

$$
A = -\frac{\beta R_c}{R_e(1+\beta) + r_\pi} \approx -\frac{R_c}{R_e} \tag{3}
$$

1.1.2 Input Impedance

The impedance into the base terminal of the common emitter circuit is given by

$$R_b = \frac{v_b}{i_b}$$
$$\text{Where } i_b = \frac{v_b - v_e}{r_\pi}$$

Therefore

$$R_b = \frac{v_b r_\pi}{v_b - v_e}$$
$$= \frac{r_\pi}{1 - \frac{v_e}{v_b}} \tag{4}$$

From equation 2

$$\frac{v_e}{v_b} = R_e\left(\frac{1+\beta}{R_e(1+\beta)+r_\pi}\right) \tag{5}$$

Substituting equation 5 into 4 and rearranging gives gives

$$R_b = r_\pi + R_e(\beta+1)$$

Therefore input impedance is the parallel combination of R_1, R_2 and $R_b = r_\pi + R_e(\beta+1)$

$$R_i = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_\pi + R_e(\beta+1)}\right)^{-1} \tag{6}$$

1.1.3 Output Impedance

By examining 1 it can be seen that the output impedance will be

$$R_o = R_c \tag{7}$$

1.2 Common Collector Circuit

The second stage of the circuit consists of a straight forward common collector circuit as shown in figure 2.

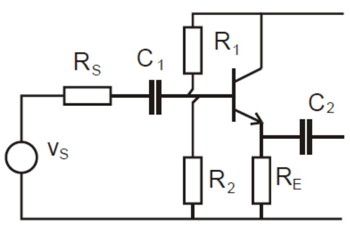


Figure 2: Common emitter stage

1.2.1 Mid-band Gain

To determine the mid-band gain, the circuit should be redraw with respect to the small signal model, as in 3

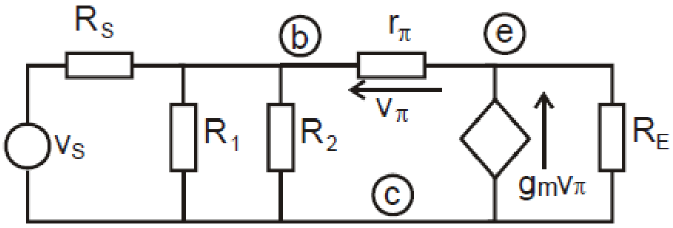


Figure 3: Common emitter stage

By using KVL on the emitter terminal we can establish that

$$\frac{v_e - v_s}{r_\pi} + g_m v_\pi + \frac{v_e}{R_E} = 0 \tag{8}$$

$$\frac{v_e - v_s}{r_\pi} + g_m(v_e - v_s) + \frac{v_e}{R_E} = 0$$

$$\frac{v_e - v_s}{r_\pi} + \frac{\beta(v_e - v_s)}{r_\pi} + \frac{v_e}{R_E} = 0$$

$$v_e(\beta R_E + R_E + r_\pi) = v_s R_E(\beta + 1)$$

$$\frac{v_e}{v_s} = \frac{R_E(\beta + 1)}{R_E(\beta + 1) + r_\pi} \tag{9}$$

Therefore gain ≈ 1

1.3 Input Impedance

Input impedance

$$R_i = \frac{v_s}{i_b} \tag{10}$$

Neglecting R_1, R_2 and R_s gives

$$i_b = \frac{v_s - v_e}{r_\pi}$$

Substituting equation 9 into this gives

$$i_b = \frac{v_s}{r_\pi} - \frac{v_s}{r_\pi}\left(\frac{R_E(\beta + 1)}{R_E(\beta + 1) + r_\pi}\right)$$

$$= \frac{v_s}{r_\pi}\left(1 - \frac{R_E(\beta + 1)}{R_E(\beta + 1) + r_\pi}\right)$$

$$= \frac{v_s}{R_E(\beta + 1) + r_\pi} \tag{11}$$

Therefore combining 10 and 11 gives

$$R_i = \frac{v_s}{i_b} = R_E(\beta + 1) + r_\pi \tag{12}$$

1.3.1 Output Impedance

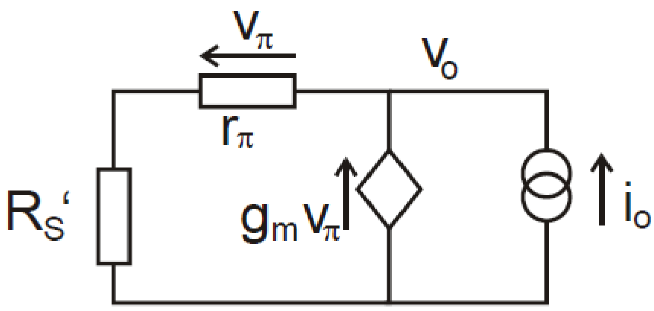


Figure 4: Common emitter stage

In figure 4 the Thevenin equivalent of the source resistances has been taken (R_s')

$$R_s' = R_s||R_1||R_2$$

By KVL

$$i_o + g_m v_\pi - \frac{v_o}{r_\pi R_s'} = 0 \tag{13}$$

$$v_\pi = -\frac{r_\pi v_o}{r_\pi + R_s'} \tag{14}$$

Substituting equation 13 into 14 gives

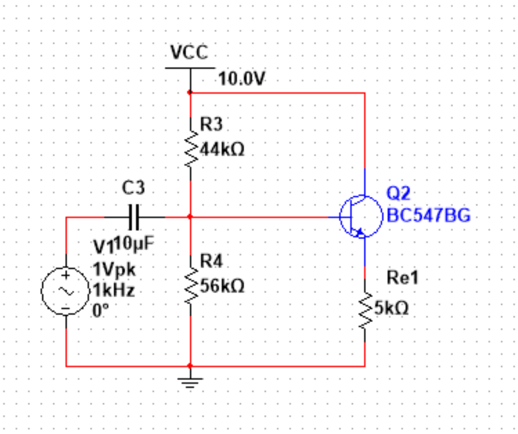
$$i_o - g_m \frac{r_\pi v_o}{r_\pi + R_s'} - \frac{v_o}{r_\pi R_s'} = 0$$

$$\frac{v_o(1 + g_m r_\pi)}{r_\pi + R_s'} = i_b$$

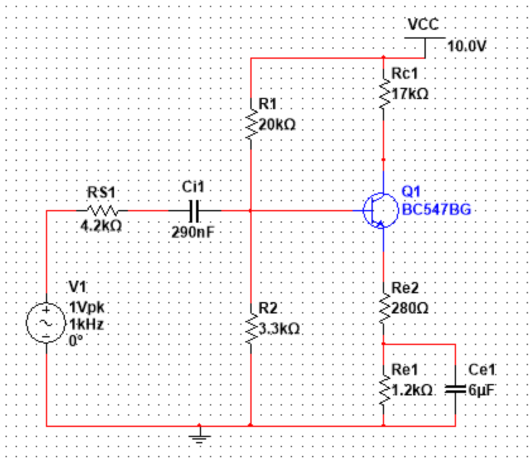
$$R_o = \frac{v_o}{i_b} = \frac{r_\pi + R_s'}{1 + \beta} \tag{15}$$

2 Simulation

Used MATLAB program to calculate values for each of the resistors and capacitors.



(a) Schematic for common collector amplifier



(b) Schematic for common emitter amplifier

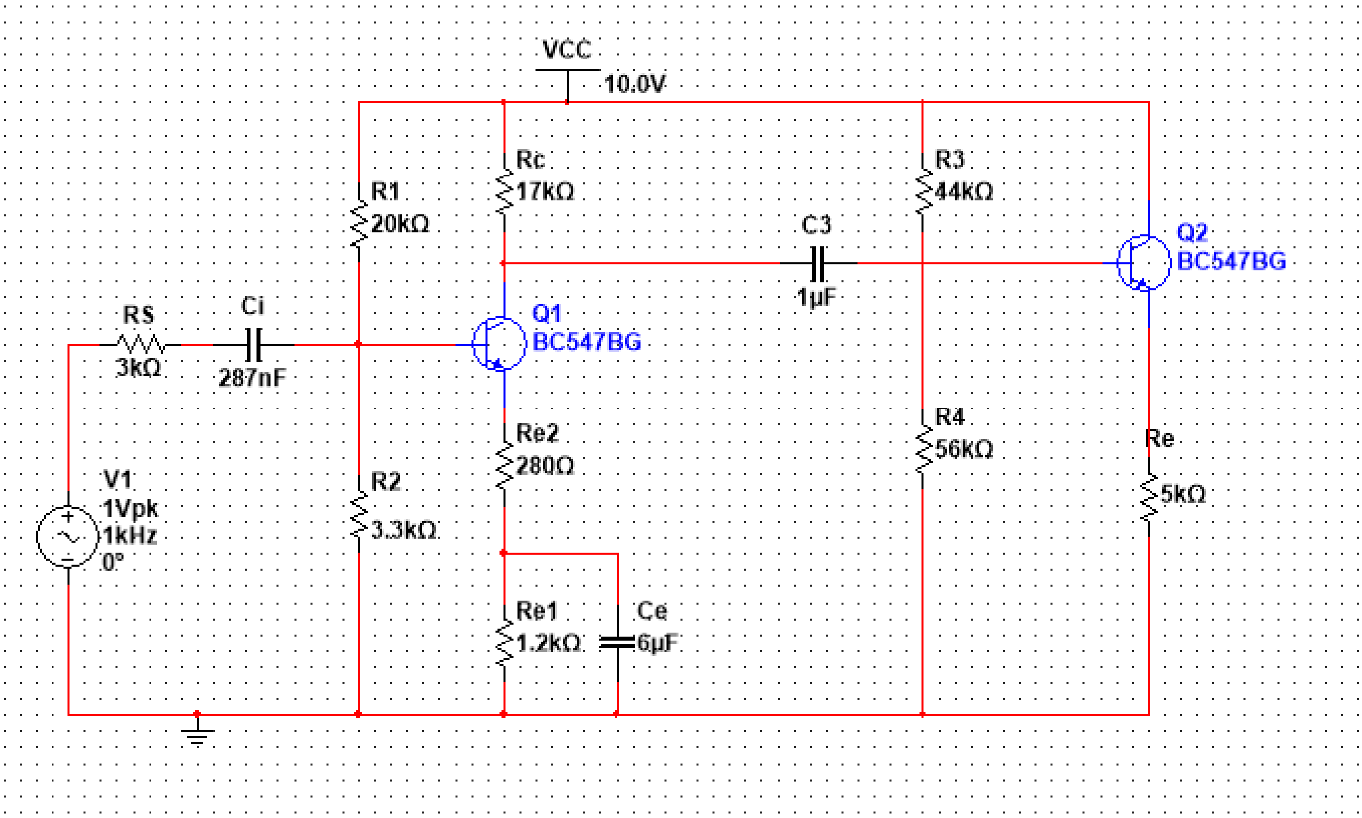


Figure 5: Schematic for full amplifier featuring both stages connected

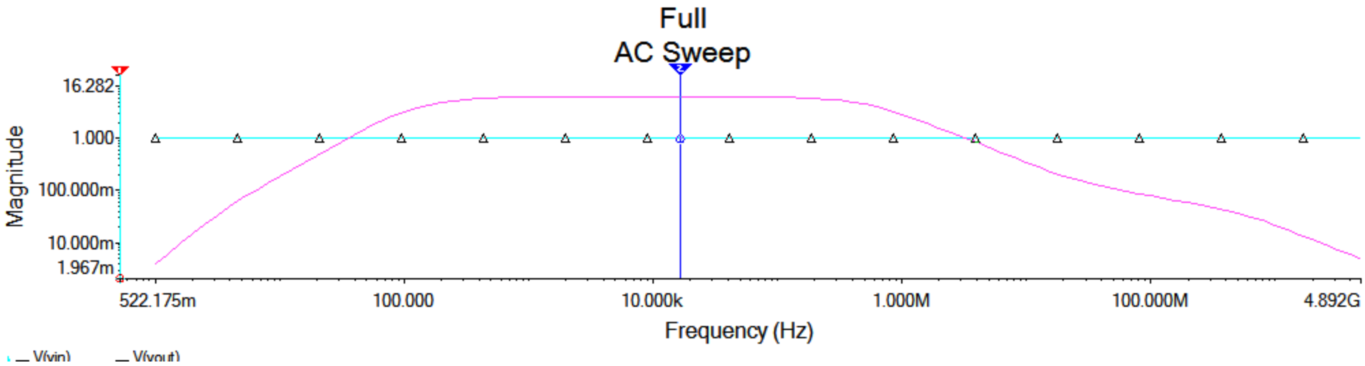


Figure 6: Simulation for the combined amplifier

	V(vin)	V(vout)
x1		
y1		
x2	16.6131k	16.6131k
y2	1.0000	6.2133
dx		
dy		

Gain of 6.2133 from the simulation.
Input impedance 3.1150×10^5
Output impedance 143.2382

Output impedance stage 1 17×10^3
Input impedance stage 2 1×10^6

Therefore $10R_{o1} \ll R_{i2}$