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TO DO

CAPITA PHI EINFÜGEN und y(x)

1 Homework Lecture 3

1.1 Excercise 1

To show: $(D + q\Phi^T\Phi)w_{Bayes} = q\Phi^T t + D\mu$

Starting from the likelihood Function and simplify it:

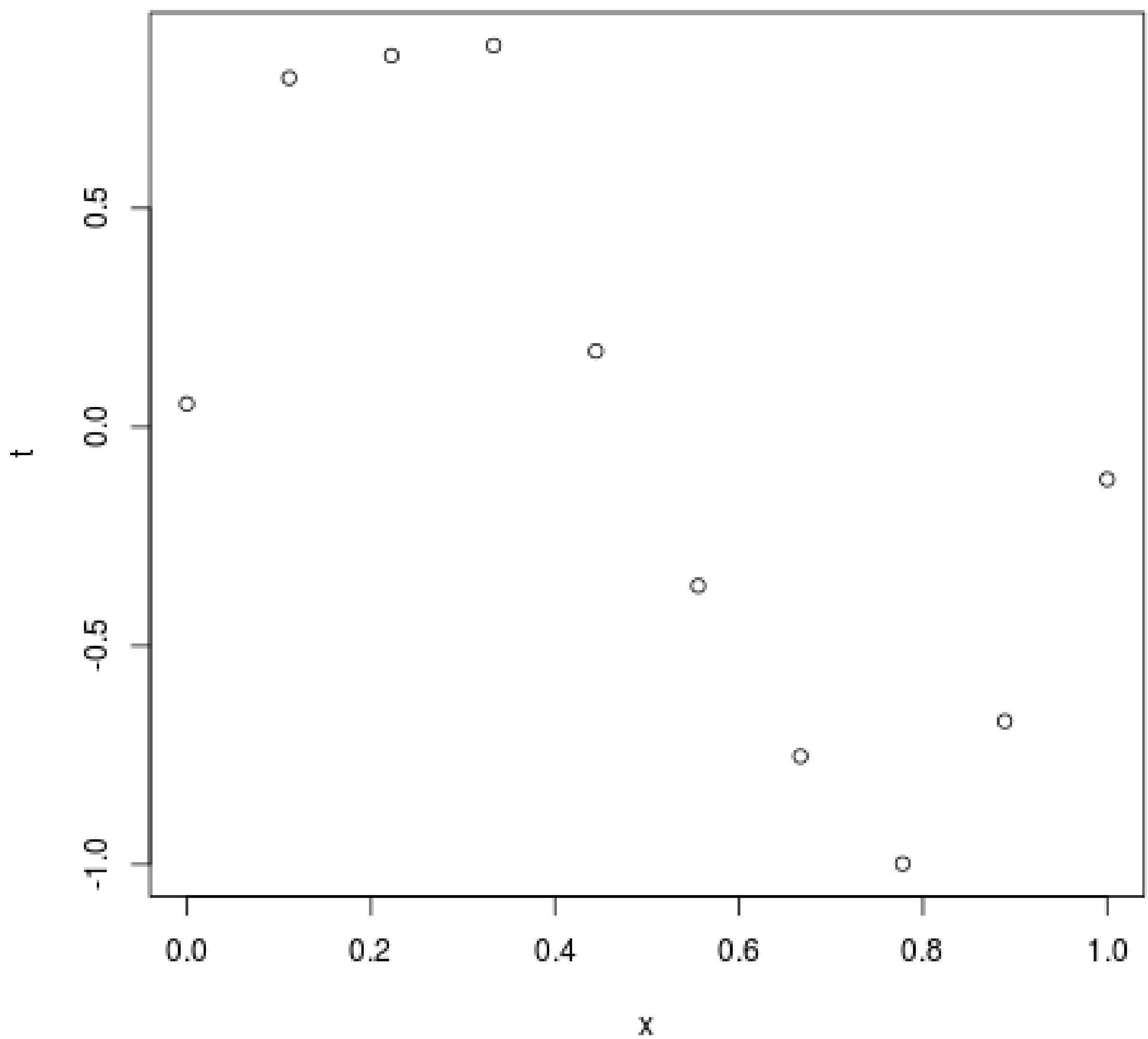
$$\begin{aligned}
l &= (t - \Phi w)^T qI(t - \Phi w) + (w - \mu)^T D(w - \mu) + Const \\
&= t^T qt - t^T q\Phi w - w^T \Phi^T qt + w^T \Phi^T q\Phi w + w^T Dw - w^T D\mu - \mu^T Dw + \mu^T D\mu + Const \\
&= t^T qt - 2t^T q\Phi w + qw^T \Phi^T \Phi w + w^T Dw - 2\mu^T Dw + \mu^T D\mu + Const
\end{aligned}$$

Take the derivative and set it to zero

$$\begin{aligned}
\frac{\partial l}{\partial w} &\overset{!}{=} 0 \\
&= -2t^T q\Phi + 2qw^T \Phi^T \Phi + 2w^T D - 2\mu^T D = 0 \\
2qw^T \Phi^T \Phi + 2w^T D &= 2t^T q\Phi + -2\mu^T D \\
qw^T \Phi^T \Phi + w^T D &= t^T q\Phi + -\mu^T D \\
\Phi^T \Phi qw + Dw &= \Phi^T qt + D\mu \\
(\Phi^T \Phi q + D)w &= \Phi^T qt + D\mu
\end{aligned}$$

1.2 Excercise in R

1.2.1 Plot the data from the data set



1.2.2 Phix function

Listing 1: ”R-Code for the phix function”

```
    phix <- function(x,M,categorial){
       if (categorial=="poly"){
         num.basis <- 0:M
         poly      <- sapply(num.basis,function(b){x^b})
         return(poly)
         } else if (categorial=="Gauss"){
           gaus <- sapply(0:(M-1), function(i){exp(-(x-i/(M-1))^2/.1)})
         return(gaus)
         }
       }
```

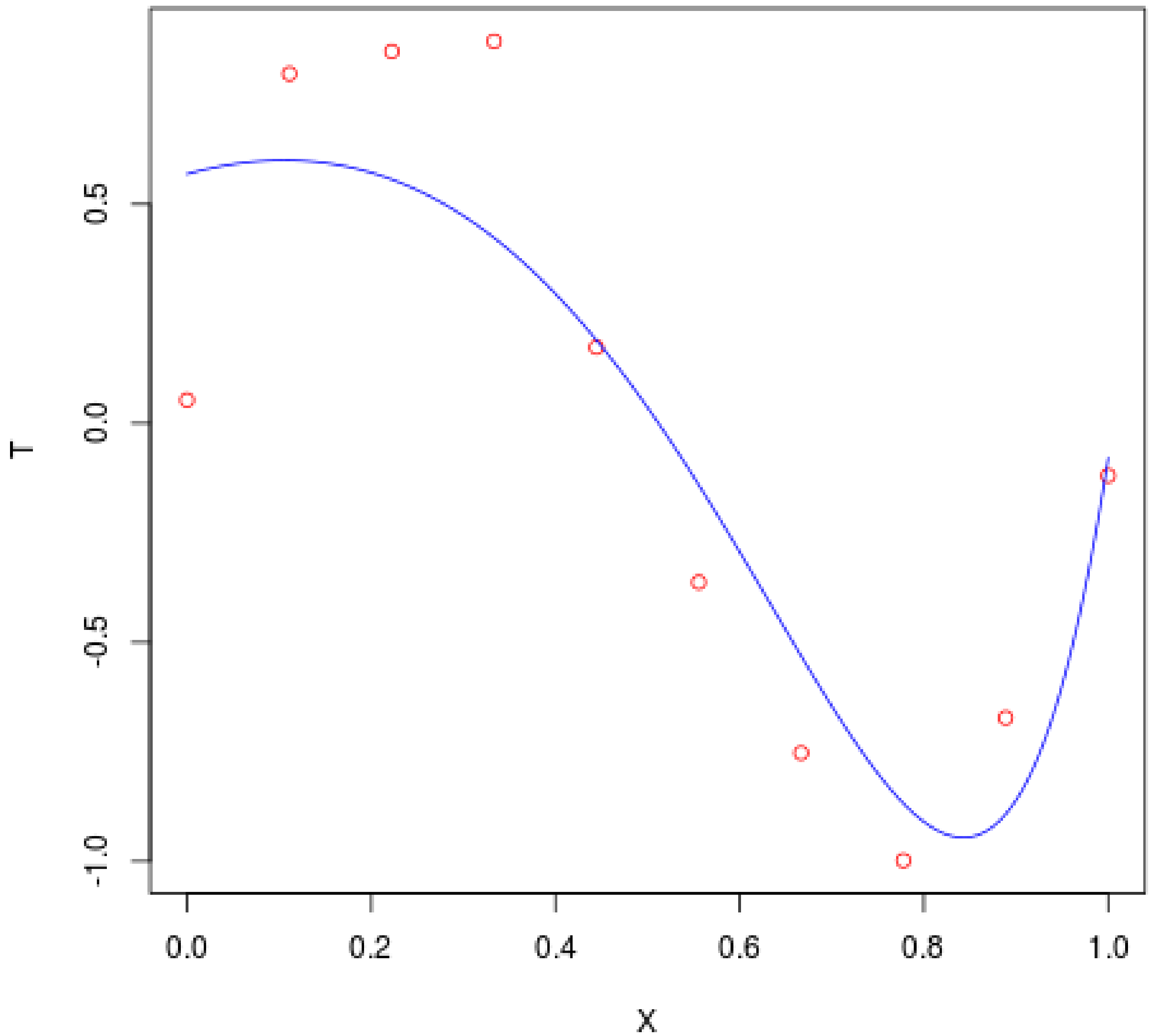
1.2.3 post.param function

Listing 2: ”R-Code for the post.param function”

```
    post.params <- function(dependent,independent,M,cat,delta,q){
       phi        <<- phix(independent,M,cat)
       lambda      <- delta/q
       lambda.mat  <- diag( ncol( t(phi) %*% phi ) ) * lambda
       t           <- dependent
       wbayes      <- solve((lambda.mat + t(phi) %*% phi)) %*% t(phi) %*% t
       Q           <- q * (lambda.mat + t(phi) %*% phi)
       l           <- list(wbayes,Q)
       return(l)
      }
 }
```

1.2.4 Plot the simulation vs. observed values

Excercise 3


0.5
0.0
-0.5
-1.0
T
0.0
0.2
0.4
0.6
0.8
1.0
X

2 Homework Lecture 4

2.1 Excercise 1

To show: $\phi^T w_{Bayes} = \sum_{n=1}^{N} q\phi^T Q_n^{-1}$

Prerquisite information:

$$\begin{aligned} w_{Bayes} &= (\lambda I + \Phi^T \Phi)^{-1} \Phi^T t \\ Q &= q(\lambda I + \Phi^T \Phi) \\ \phi^T &= \Phi \end{aligned}$$

Following that:

$$\begin{aligned} \phi^T w_{Bayes} &= \phi^T (\lambda I + \Phi^T \Phi)^{-1} \Phi^T t \\ &= \Phi(\lambda I + \Phi^T \Phi)^{-1} \Phi^T t \\ &= \Phi q Q^{-1} \Phi^T t \\ &= \Phi q Q^{-1} \sum_{n=1}^{N} \Phi_n^T * t_n \\ &= \sum_{n=1}^{N} \Phi q Q^{-1} \Phi_n^T * t_n \\ &= \sum_{n=1}^{N} q\phi^T Q_n^{-1} \end{aligned}$$

2.2 Excercise 2

To show: The weight of t_n ist $k(x, x_n)$

Prerquisite information:

$$k := q\phi^T Q^{-1} \Phi(y)$$

Starting with the insights from the last excersice:

$$\phi^T w_{Bayes} = \sum_{n=1}^{N} q\phi^T Q_n^{-1}$$

Notice that $q\phi^T Q^{-1} \phi$ fits the former definition of k. Substituting it into the last equation leads to:

$$\phi^T w_{Bayes} = \sum_{n=1}^{N} k(x, x_n) t_n$$

2.3 Excercise 3

To show: $K = q\Phi Q^{-1}\Phi^T$

Starting from the insights of the last excersice:

$$k(x, x_n) = \sum_{n=1}^{N} q\phi^T Q^{-1}\phi$$

Extend this to the first variable x_k to get an entry of K

$$k(x_k, x_n) = \sum_{n=1}^{N} \sum_{k=1}^{M+1} q\Phi(x_k)^T Q^{-1}\phi$$

Rewrite this sumation form of a matrix product to obtain this fact in matrix terms

$$\sum_{k=1}^{M+1} \sum_{n=1}^{N} k(x_k, x_n) = K = q\Phi Q^{-1}\Phi^T$$

Then an element

2.4 Excercise 4

To show: K is equal to the hat matrix of the linear regression if $\lambda = 0$:

Prerquisite information:

$$\lambda = \frac{\delta}{q}$$

Proof:

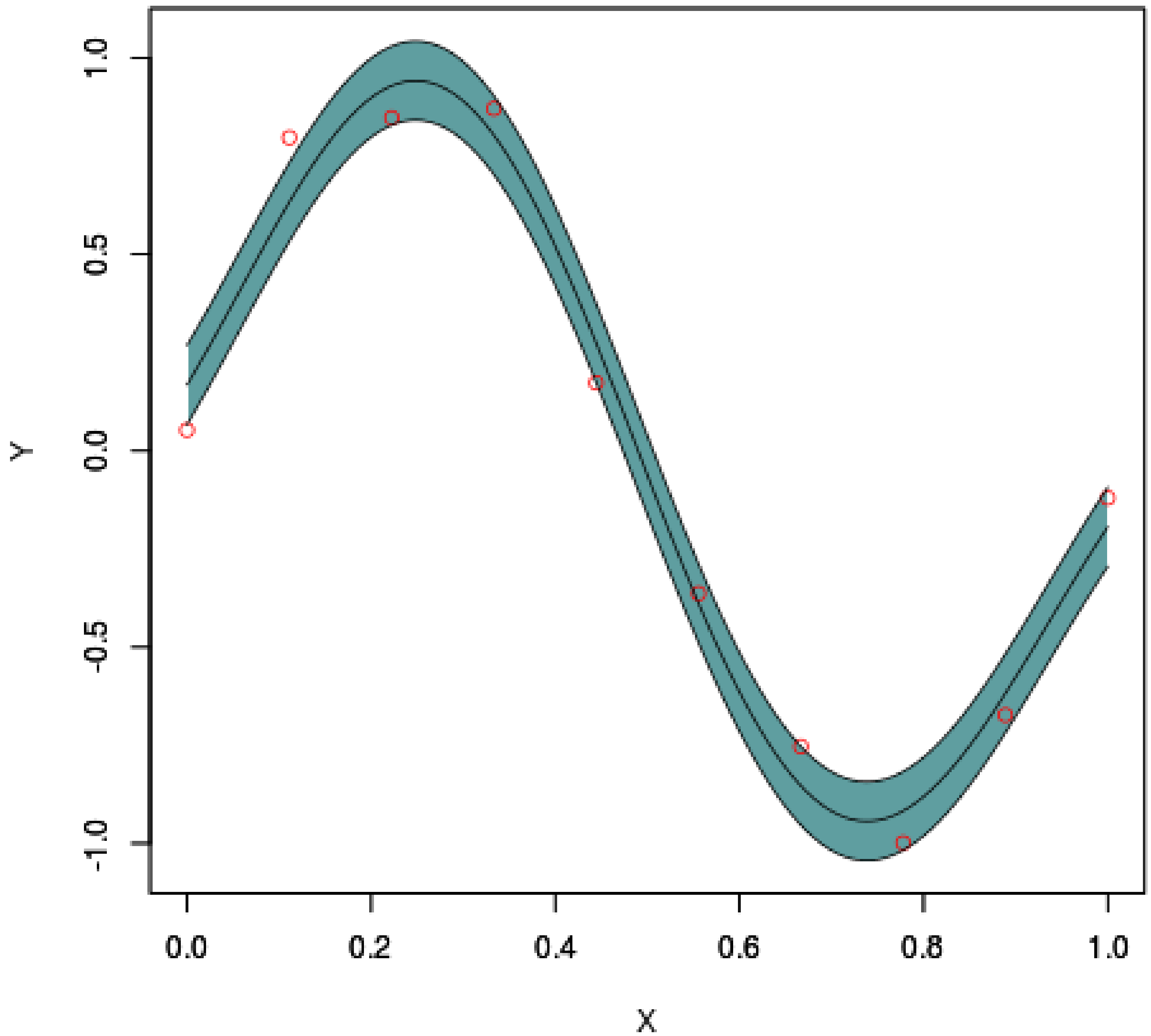
$$\begin{aligned}
K &= q\Phi(\delta I + q\Phi^T\Phi)^{-1}\Phi^T \\
&= q * \frac{1}{q}\Phi(\delta\frac{1}{q}I + \Phi^T\Phi)^{-1}\Phi^T \\
&= \Phi(\lambda I + \Phi^T\Phi)^{-1}\Phi^T \\
&= \Phi(0 + \Phi^T\Phi)^{-1}\Phi^T \\
&= \Phi(\Phi^T\Phi)^{-1}\Phi^T = H
\end{aligned}$$

2.5 Excercise in R

2.5.1 Phix function

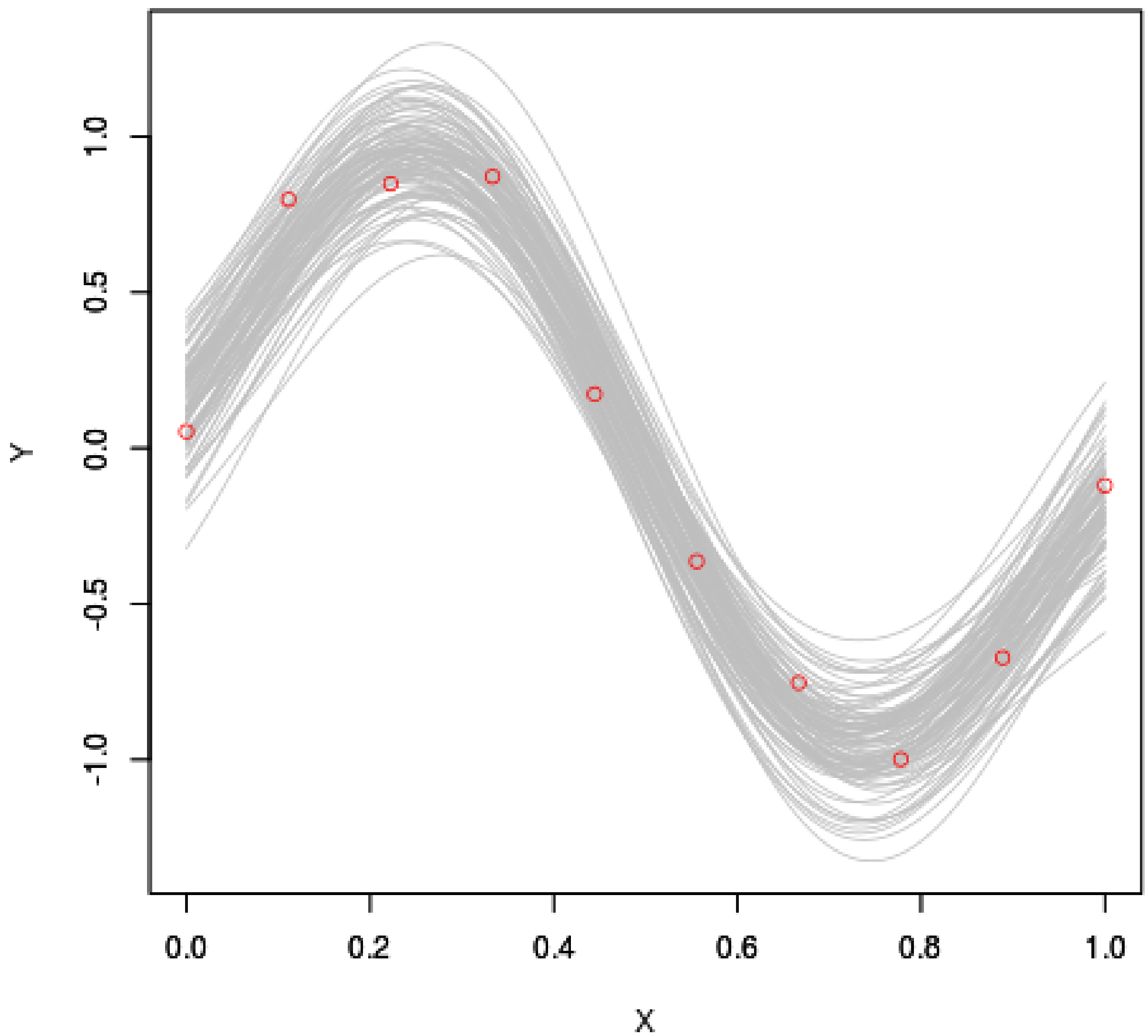
Listing 3: "R-Code for a function for predictive mean an precision"

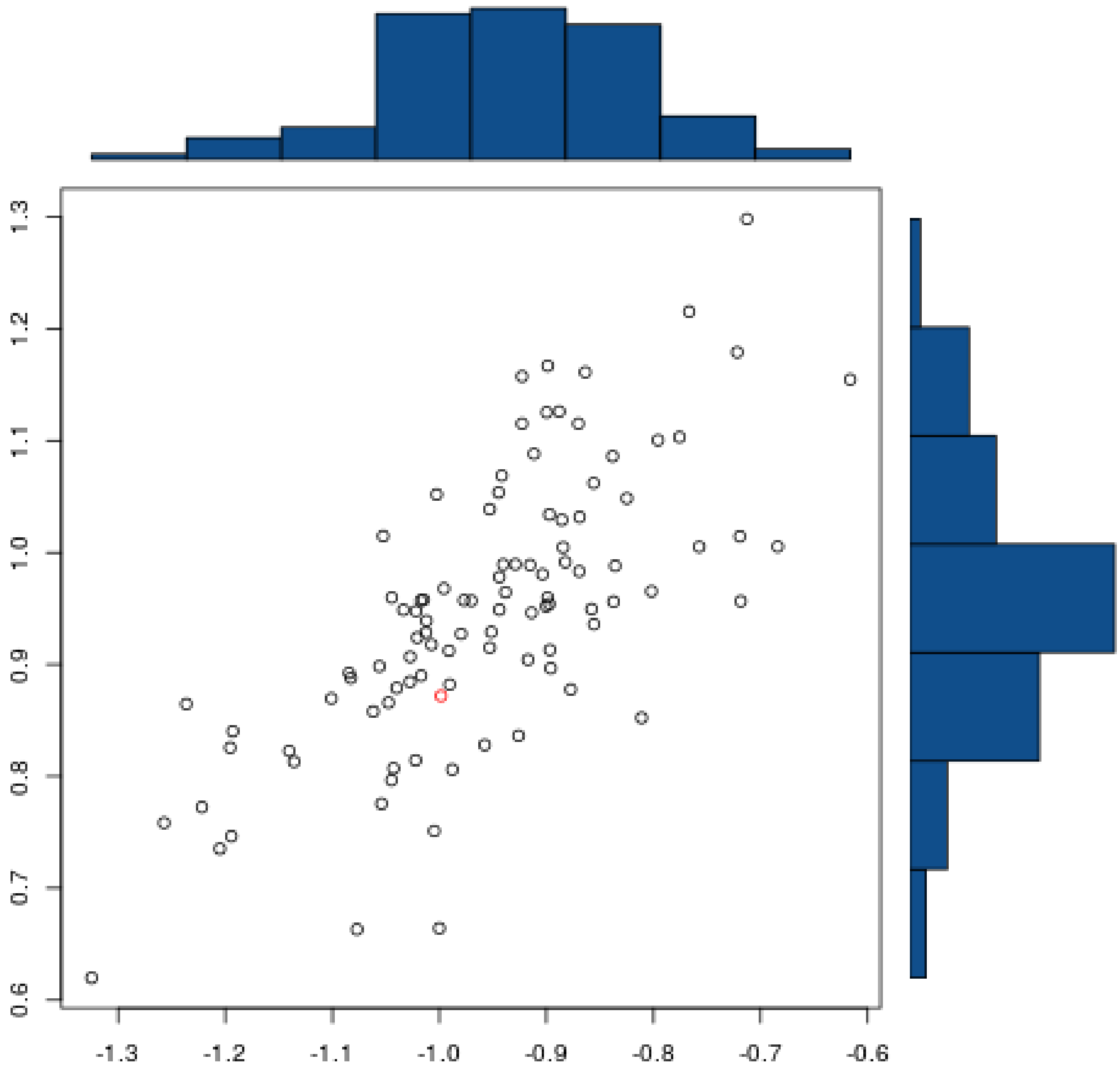
```
simulate.function <- function(input){
  training.sim <- post.params(out.data,in.data,9,"Gauss",1,(1/0.1)^2 )
  estimators <- training.sim[[1]]
  dot.numb <- seq(0,1,0.001)
  phi <- phix(dot.numb.new, 9, "Gauss")
  fitted <- phi %*% estimators
  simulation <- post.params(fitted,dot.numb,9,"Gauss",1,(1/0.1)^2 )
  Q <- simulation[[2]]
  Qinv <- solve(Q)
  variance <- 1/(1/0.1)^2 + diag( phi %*% Qinv %*% t(phi) )
  precision <- solve(variance)
  output <- list(fitted,precision)
  return(output)
}
```



1.0
0.5
0.0
-0.5
-1.0
Y
0.0
0.2
0.4
0.6
0.8
1.0
X

2.5.2 Simulations





1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
-1.3
-1.2
-1.1
-1.0
-0.9
-0.8
-0.7
-0.6