Problem Set 2

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Problem 1

a)

(1)

Define the utilities as:

$$U_i(A) := \alpha + d_i^A a$$
$$U_i(B) := \beta + d_i^B b,$$

where α, $\beta \geq 0$ and $d_i^* a$ is the number ...

Best reply:

$$U_i(A) > U_i(B)$$

Substitute the definition of the utilities in the last equation:

$$\alpha + d_i^A a > \beta + d_i^B b$$

Define d_i and rephrase it:

$$d_i := d_i^A + d_i^B$$
$$\underbrace{\frac{d_i^A}{d_i}}_{f_i} + \underbrace{\frac{d_i^B}{d_i}}_{(1-f_i)} = 1$$

Using the last identity we can write the the inequality in the following way:

$$\alpha + f_i a > \beta + (1 - f_i)b$$
$$f_i(a + b) \geq \beta + b - \alpha$$
$$f_i \geq \frac{(\beta - \alpha) + b}{b + a} \in [0, 1]$$

Let $p_A = \frac{(\beta-\alpha)+b}{b+a}$ and $p_B = (1 - p_A) = \frac{(\alpha-\beta)+a}{b+a}$. So best response is to choose A **iff** at least p_A of its friends chhose A.

(b)

The Nash-Equilibrium (NE) appears when:

- any node i that chooses A has at least p_A friends who also choose A
- any node i that chooses B has at least p_B friend who also choose B

(c)

Let C_p a p-cluster and $C_p \subset G$. In the p-clustering sense we can say that NE occurs if i chooses A then $i \in C_A$, where C_A is a p_A-cluster and if i chooses B then $i \in C_B$, where C_B is a p_B-cluster.

b)

Set $\alpha = \beta = 0$. Each nodes starts with B. Let $t = q = \frac{1}{2}$ for switching to A.

(b1)

Let $S = \{e - f\}$, where S is the starting set meaning that the two nodes initially adopt behaviour A.

!!!!! FOR FELIX !!!!!! put in a graph or leave some space for drawing one #Denisisthebest

(b2)

The Blocking set is $BS := \{g, j\}$, with density $D = \frac{2}{3}$.

c)

!!!! DENI WILL PAINT SOME MORE STUFF!!!

d)

No we can't have the same Nash-Equilibria, because when we have a tie we are more likely to choose one of them namely to choose the one which has bigger initial values (α or β).

Problem 2

(2.1.a)

Step 1: $h_{A_i}^{(1)} = 2h_{C_i}^{(1)} = \frac{1}{8}$ $a_{B_i}^{(1)} = \frac{3}{8}$ $a_D^{(1)} = \frac{5}{8}$ Step 2: $h_{A_i}^{(2)} = \frac{9}{8}$ $a_{B_i}^{(2)} = \frac{27}{8}$ $h_{C_i}^{(2)} = \frac{5}{8}$ $a_D^{(2)} = \frac{25}{8}$ Conclusion: $a_{B_i}^{(2)} > a_D^{(2)}$ even from the second iteration there is a difference between the authority meassures of the two different nodes. ***(2.1.b)***

We define the formulas for the two sub-graphs separatly. The first sub-graph consists of A's and B's. Let $b :=$ number of B's and $a :=$ number of A's and $c =$ constant $= \frac{1}{8}$. For Step $k = 1$: $h^{(1)} = a^0 b^0 c = c$ $a^{(1)} = ab^0 c$ Step $k = 2$: $h^{(2)} = a^{(1)} b = bab^0 c = b^{(k-1)} a^{(k-1)} c$ $a^{(2)} = abab^0 c = a^{(k)} b^{(k-1)} c$ Step $k = 3$: $h^{(3)} = a^{(2)} b = a^{(k-1)} b^{(k-2)} bc = a^{(k-1)} b^{(k-1)} bc$ $a^{(3)} = aa^{(k-1)} b^{(k-1)} c = a^{(k)} b^{(k-1)} c$ In general we define: $h_{A_i}^{(k)} := a^{(k-1)} b^{(k-1)} bc$ $a_{B_i}^{(k)} := a^{(k)} b^{(k-1)} c$

for the secon sub-graph Step $k = 1$: $h^{(1)} = c$ $a^{(1)} = 5c$ Step $k = 2$: $h^{(2)} = 5c = 5^{(k-1)} c$ $a^{(2)} = 5 * 5c = 5^k c$ In general we define: $h_{C_i}^{(k)} = 5^{(k-1)} c$ $a_D^{(k)} = 5^k c$

(2.1.c)

Let the normalizing constant be

$$\begin{aligned} const_h &= 3[a^{k-1} b^{k-1} c] + 55^{k-1} c \\ &= 33^{2k-2} c + 5^k c \\ &= (3^{2k-1} + 5^k) c \end{aligned}$$

The normalized version of the hub meassure is:

$$\begin{aligned} \widehat{h_{A_i}} &= \frac{h_{A_i}}{const_h} \\ &\propto \frac{9^k}{(3^{2k-1} + 5^k)} \end{aligned}$$