Back-calculation of ICU admissions from time-series of occupied beds

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Methods

We are interested in estimating the daily number of new ICU admissions I_t in a specific region based on the observed number of occupied ICU beds per day $B_t = b_t$, where $t = 1, \dots, T$.

Let N be the number of individuals $i = 1, \dots, N$ that occupied a bed at some time-point $t < T$, and LOS_i be the length of the ICU stay (number of days an individual occupied a bed in the ICU) of individual I.

We assume, that the LOS follows a discrete distribution F_θ, where $P(LOS_i = k) = \theta_k,\ k = 0, ..., K$ and $\sum_{k=0}^{K} \theta_k = 1$. Additionally we define the probability $P(LOS_i > k) = 1 - \sum_{l=1}^{k} \theta_l = \eta_k$.

Let A_i be the admission day of individual i, and S_i be an individual-specific binary vector of length T with entry $S_{i,t} = 1$ if individual i is occupying a bed at the ICU on day t and $S_{i,t} = 0$, if not:

$$S_{i,t} = \begin{cases} 1, & \text{if } A_i \leq t \leq A_i + LOS_i \\ 0, & \text{otherwise} \end{cases}$$

Based on these definitions, the observed number of occupied ICU beds on day t corresponds to

$$b_t = \sum_{i=1}^{N} S_{i,t} = \sum_{i=1}^{N} \mathbb{1}(A_i \leq t \leq A_i + LOS_i) = \sum_{i=1}^{N} \mathbb{1}(A_i \leq t)\mathbb{1}(LOS_i \geq t - A_i)$$

and the unobserved number of new ICU admissions I_t are

$$I_t = \sum_{i=1}^{N} \mathbb{1}(A_i = t).$$

The estimation of I_t based on the observed $B_t = b_t$ is closely related to the estimation of an inverse convolution or *backprojection* methods. We assume, that the latent number of new ICU admissions on day t, I_t, are Poisson distributed with expectation λ_t. The observed number of occupied beds on day t are then Poisson distributed as well:

$$B_t \sim \text{Pois}(\sum_{k=0}^{K} \lambda_{t-k}\eta_k),$$

where η_k is known based on the assumed LOS-distribution F_θ.

Based on this assumed distribution of the observed data $B_t = b_t$ we set up a Bayesian hierarchical model to estimate the expected number of new ICU admissions based on the data $B = b = (b_1, \ldots, b_T)$. Since the number of to be estimated parameters $\lambda = (\lambda_1, \ldots, \lambda_T)$ is the same as the number of observed data points b, this is close to not identifiable. We therefore impose regularization on the estimated λ_t assuming a smooth change in the daily number of new ICU admissions by setting up a Bayesian hierarchical model

$$\begin{aligned}
\lambda_1 &\sim \text{Prior}(\phi) \\
\sigma_{rw} &\sim \text{Half-N}(0, 10) \\
\lambda_t|\lambda_{t-1} &\sim N(\lambda_{t-1}, \sigma_{rw}),\ t = 2, \ldots, T \\
B_t|\lambda &\sim \text{Pois}(\textstyle\sum_{k=0}^{K} \lambda_{t-k}\eta_k)
\end{aligned}$$

and estimate λ by MCMC sampling.